"Chapter 13"

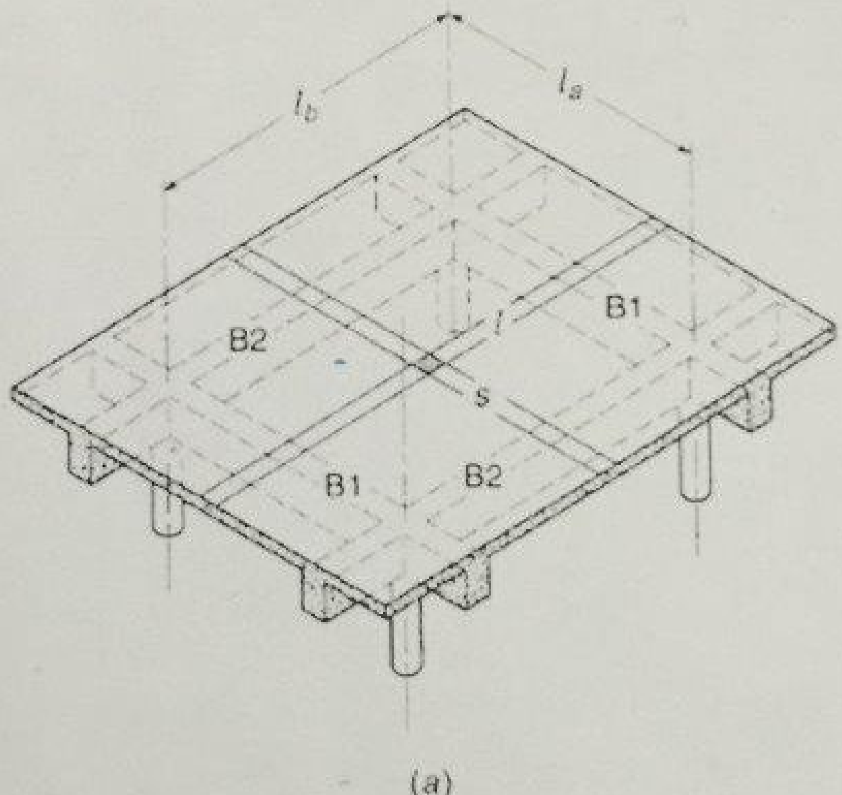
'Analysis and Design of Slabs'

Two-Way Column Supported Slabs:

When two-way slabs are supported by relatively shallow, flexible beams or if column-line beams are omitted altogether, as for flat plates, flat slabs, or two-way joist systems, then a number of new considerations are introduced.

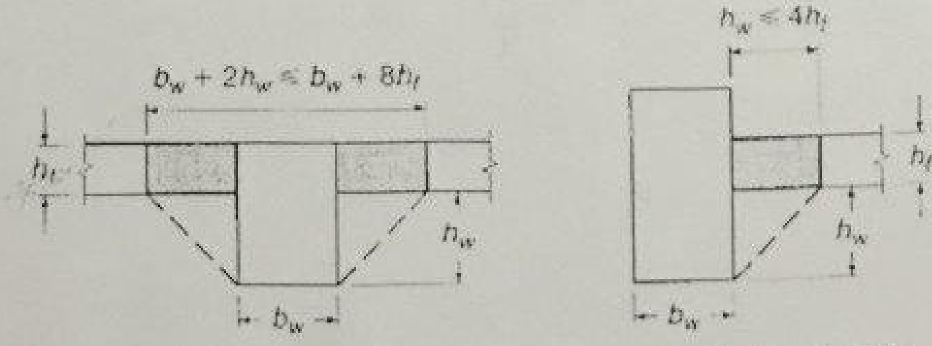
Figure 13.8a shows a portion of a floor system in which a rectangular slab panel is supported by relatively shallow beams on four sides. The beams, in turn, are carried by columns at the intersection of their centerlines. If a surface load q is applied, that load is shared between imaginary slab strips l_a in the short direction and l_b in the long direction. The portion of the load that is carried by the long strips l_b is delivered to the beams B1 spanning in the short direction of the panel. The portion carried by the beams B1 plus that carried directly in the short direction by the slab strips l_a sums up to 100 percent of the load applied to the panel. Similarly, the short-direction slab strips l_a deliver a part of the load to long direction beams B2. That load, plus the load carried directly in the long direction by the slab, includes 100% of the applied load.

It is clearly a requirement of statics that, for column-supported construction, 100 percent of the applied load must be carried in each direction, jointly by the slab & its supporting beams.



(a)

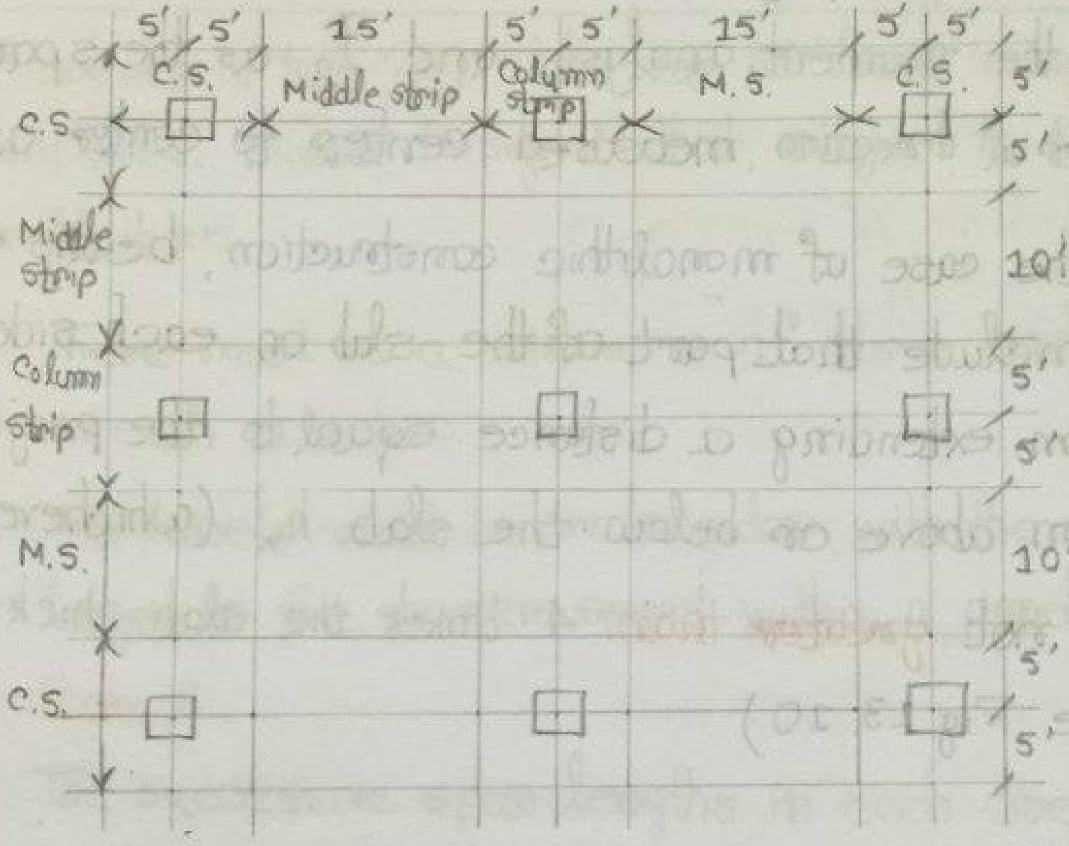
FIGURE 13.8
Column-supported two-way slabs: (a) two-way slab with beams; (b) two-way slab without beams.



(a) Symmetric slab

(b) Single side slab

FIGURE 13.10
Portion of slab to be included with beam.



A typical panel is divided, for purposes of design into column strips & middle strips.

A Column strip is defined as a strip of slab having a width on each side of the column centerline equal to one-fourth the smaller of the panel dimensions l_1 & l_2. Such a strip includes colum-line beams, if present.

A Middle strip is bounded by two column strips.

In all cases, l_1 is defined as the span in the direction of the moment analysis and l_2 as the span in the lateral direction measured center to center of support.

In the case of monolithic construction, beams are defined to include that part of the slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab h_w (whichever is greater) but not greater than 4 times the slab thickness.

(See Fig 13.10)

Limitations/Restrictions of Direct Design Method :

Moments in two-way slabs can be found using the semi-empirical Direct Design Method, subject to the following restrictions :

1. There must be a minimum of three continuous spans in each direction.
2. The panels must be rectangular, with the ratio of the longer to the shorter spans within a panel not greater than 2.
3. The successive span lengths in each direction must not differ by more than one-third of the longer span.
4. Columns may be offset a maximum of 10% of the span in the direction of the offset from either axis between centerlines of successive columns.
5. Loads must be due to gravity only, and the unfactored live load must not exceed 2 times the unfactored dead load.
6. If beams are used on the column lines, the relative stiffness of the beams in the two perpendicular directions, given by the ratio $\frac{\alpha_{f_1} l_2^2}{\alpha_{f_2} l_1^2}$, must be between 0.2 & 5.0

Total Static Moment at Factored Loads:

The total factored moment in a span, for a strip bounded laterally by the centerline of the panel on each side of the centerline of supports, is

$$M_o = \frac{q_u l_2 l_n^2}{8}$$

where, q_u = surface load

l_n = clear span in the direction of moments

l_2 = centerline span in the perpendicular direction of moments

Assignment of Moments to Critical Sections:

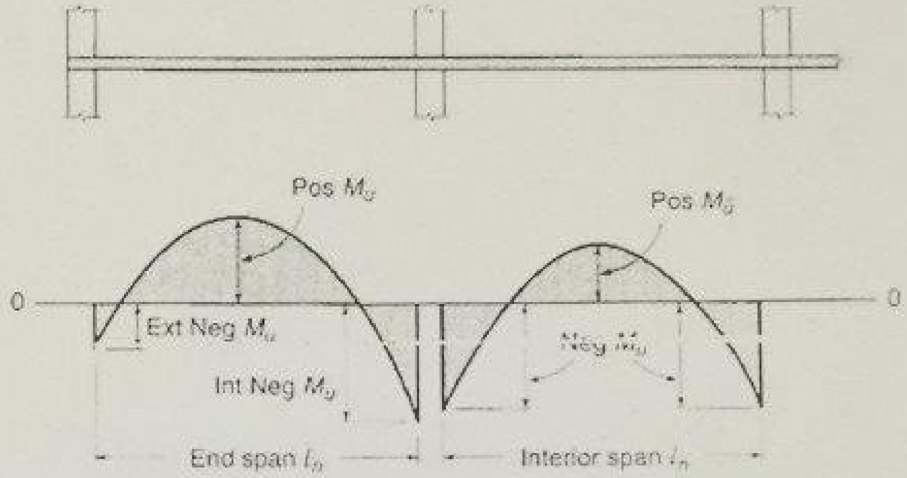
For interior spans, the total static moment is apportioned between the critical positive and negative bending sections according to the following ratios:

Negative Factored Moment: Neg $M_u = 0.65\, M_o$

Positive Factored Moment: Pos $M_u = 0.35\, M_o$

The critical section for negative bending is taken at the face of rectangular supports, or at the face of an equivalent square support having the same cross-sectional area as round support.

FIGURE 13.11
Distribution of total static moment M_u to critical sections for positive and negative bending.



In the case of end spans, the apportionment of the total static moment among the three critical sections ('interior negative', positive and exterior 'negative') depends upon,

1. The flexural restraint provided for the slab by the exterior column or the exterior wall, as the case may be.
2. The presence or absence of beams on the column lines.

ACI Code 13.6.3 specifies five alternative sets of moment distribution coefficients for end spans as shown in Table 13.3 and illustrated in Fig 13.2.

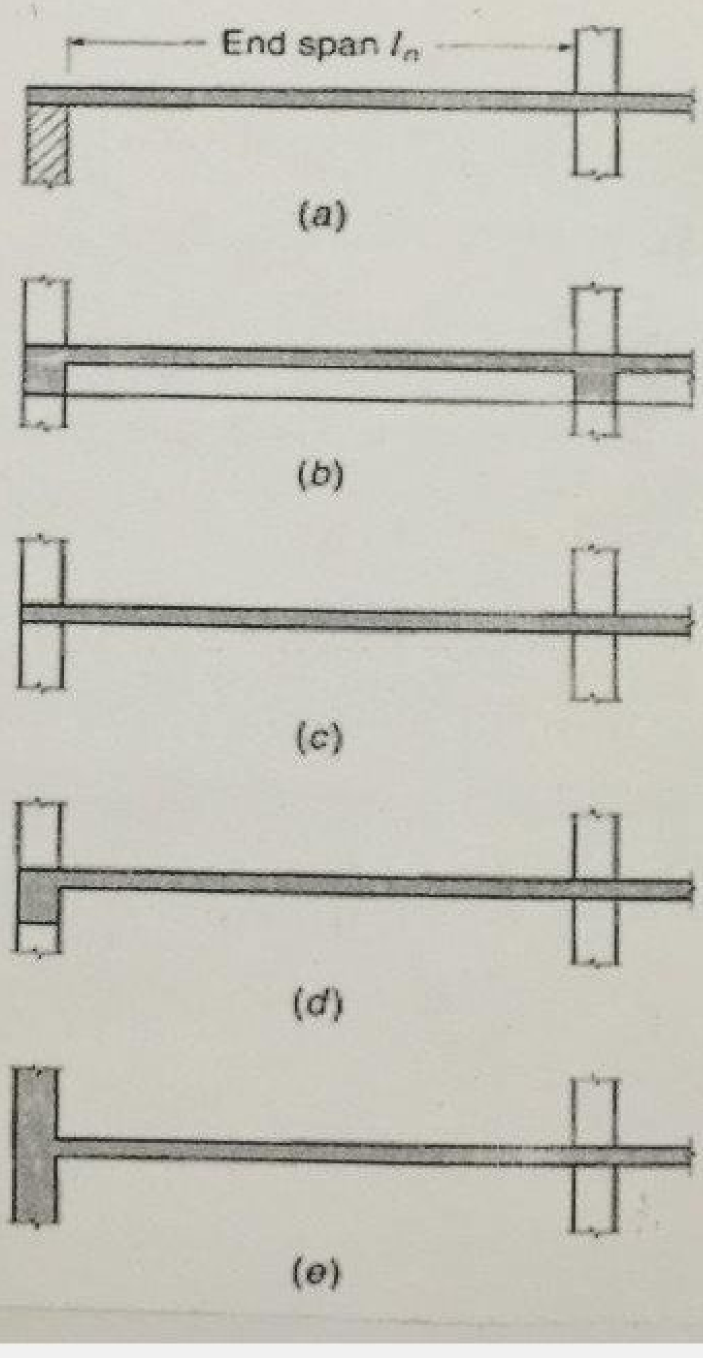
TABLE 13.3

Distribution factors applied to static moment M_o for positive and negative moments in end span

	(a)	(b)	(c)	(d)	(e)
	Exterior Edge Unrestrained	Slab with Beams between All Supports	Slab without Beams between Interior Supports		Exterior Edge Fully Restrained
			Without Edge Beam	With Edge Beam	
Interior negative moment	0.75	0.70	0.70	0.70	0.65
Positive moment	0.63	0.57	0.52	0.50	0.35
Exterior negative moment	0	0.16	0.26	0.30	0.65

FIGURE 13.12

Conditions of edge restraint considered in distributing total static moment M_o to critical sections in an end span (*a*) exterior edge unrestrained, e.g., supported by a masonry wall, (*b*) slab with beams between all supports, (*c*) slab without beams, i.e., flat plate, (*d*) slab without beams between interior supports but with edge beam, (*e*) exterior edge fully restrained, e.g., by monolithic concrete wall.



In Case a, the exterior edge has no moment restraint, such as would be the condition with a masonry wall, which provides vertical support but no rotational restraint.

Case b represents a two-way slab with beams on all sides of the panels.

Case c is a flat plate, with no beams at all.

Case d is a flat plate in which a beam is provided along the exterior edge.

Case e represents a fully restrained edge, such as that obtained if the slab is monolithic with a very stiff reinforced concrete wall.

Lateral Distribution of Moments:

For design purposes, it is convenient to consider the moments constant within the bounds of a middle strip or column strip unless there is a beam present on the column line. In the latter case, because of its greater stiffness, the beam will tend to take a larger share of the column strip moment than the adjacent slab.

The distribution of total "negative" or "positive" moment between slab middle strips, slab column strips, and beams depends upon

1. The ratio $\frac{l_2}{l_1}$
2. The relative stiffness of the beam and the slab
3. The degree of torsional restraint provided by the edge beam

A convenient parameter defining the relative stiffness of the beam and slab spanning in either direction is,

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

in which E_{cb} and E_{cs} are the moduli of elasticity of the beam and slab concrete (usually the same) and I_b & I_s are the moments of inertia of the effective beam and slab. Subscripted parameters α_{f_1} & α_{f_2} are used to identify α computed for the directions of l_1 & l_2 respectively.

The flexural stiffness of the beam and slab may be based on the gross concrete section, neglecting reinforcement and possible cracking, and variations due to column capitals and drop panels may be neglected.

For the beam, if present, I_b is based on the effective cross-section defined as in Fig 13.10, such as

For the edge beams : $I_b = \frac{bh^3}{12} \times 1.5$

For the interior beams : $I_b = \frac{bh^3}{12} \times 2.0$

For the slab, I_s is taken equal to $\frac{bh^3}{12}$, where b in this case is the width between panel centerlines on each side of the beam.

The relative restraint provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter β_t, defined as

$$\beta_t = \frac{E_{cb} \cdot C}{2 E_{cs} I_s}$$

where, I_s as before, is calculated for the slab spanning in direction l_1 and having width bounded by panel centerlines in the l_2 direction.

The constant C pertains to the torsional rigidity of the effective transverse beam, which is defined according to ACI Code 13.7.5 as the largest of the following:

1. A portion of the slab having a width equal to that of the column, bracket or capital in the direction in which moments are taken.
2. The portion of the slab specified in 1. plus that part of any transverse beam above and below the slab.
3. The transverse beam defined as in Fig 13.10.

The constant C is calculated by dividing the section into its component rectangles, each having smaller dimension x and larger dimension y, and summing the contributions off all the parts by means of the equation,

$$C = \sum \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$$

The subdivision can be done in such a way as to maximize C.

With these parameters defined, ACI Code 13.6.4 distributes the negative and positive moments between column strips and middle strips, assigning to the column strips the percentages of positive and negative moments shown in Table 13.4. Linear interpolation can be made between the values shown.

TABLE 13.4
Column-strip moment, percent of total moment at critical section

		l_2/l_1 0.5	l_2/l_1 1.0	l_2/l_1 2.0
Interior negative moment				
$\alpha_{f1} l_2/l_1 = 0$		75	75	75
$\alpha_{f1} l_2/l_1 \geq 1.0$		90	75	45
Exterior negative moment				
$\alpha_{f1} l_2/l_1 = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$\alpha_{f1} l_2/l_1 \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45
Positive moment				
$\alpha_{f1} l_2/l_1 = 0$		60	60	60
$\alpha_{f1} l_2/l_1 \geq 1.0$		90	75	45

- The column-line beam spanning in the direction l_1 is to be proportioned to resist 85 percent of the column strip moment if $\frac{\alpha_{f1} l_2}{l_1}$ is equal to or greater than 1.0. For values between 1 & 0, the proportion to be resisted by the beam may be obtained by linear interpolation.

- The portion of the moment not resisted by the column strip is proportionately assigned to the adjacent half middle strips. Each middle strip is designed to resist the sum of the moments assigned to its two half-middle strips. A middle strip adjacent and parallel to a wall is designed for twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

Flexural Reinforcement For Column-Supported Slabs:

- To provide for local concentrated loads, as well as to ensure that tensile cracks are narrow and well-distributed, a maximum bar spacing at critical sections of 2 times the total slab thickness (2h) is specified by ACI Code 13.3.2 for two-way slabs.

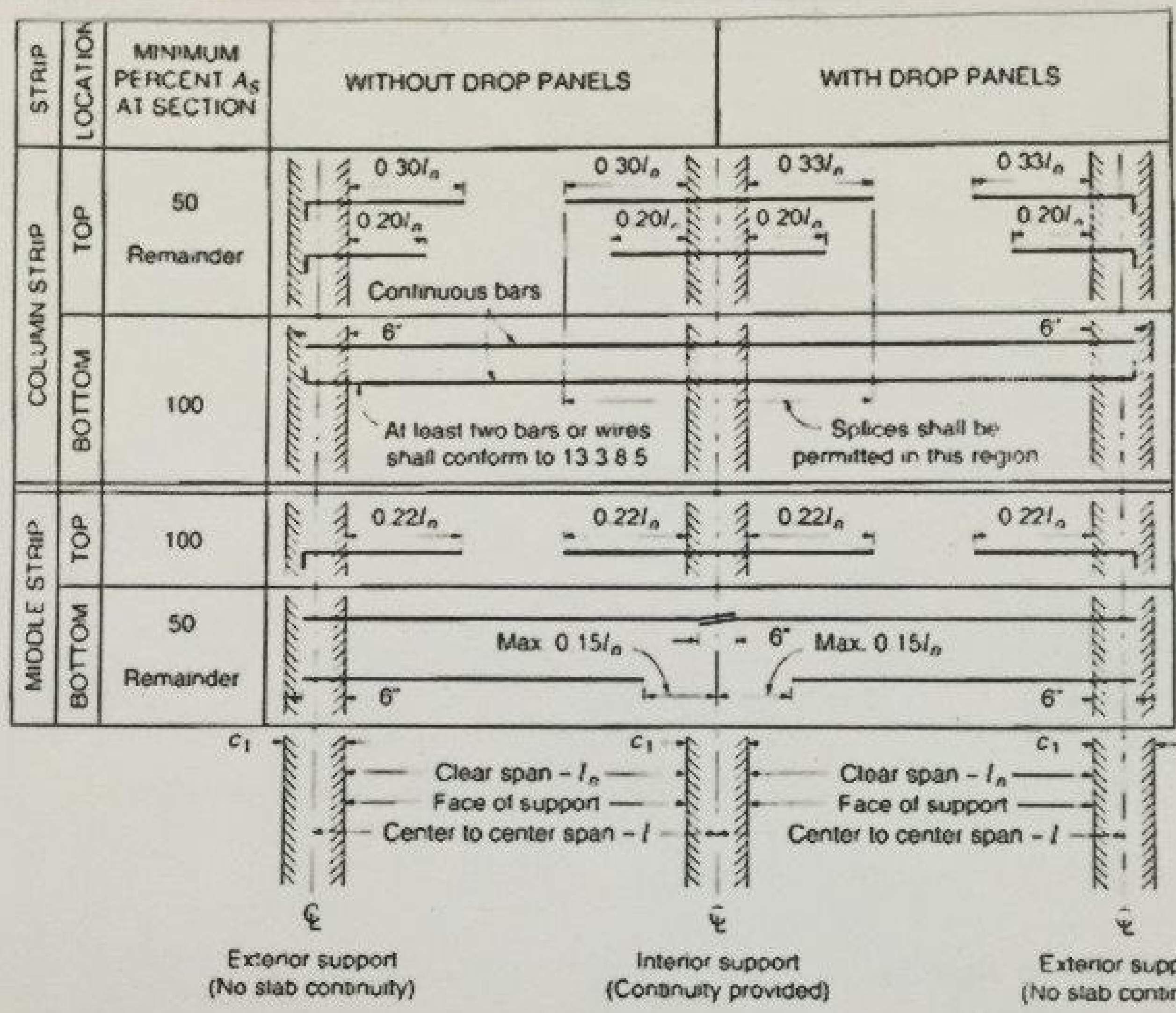
- At least the minimum steel required for temperature & shrinkage crack control must be provided

  Grade 40 or 50 deformed bar : $0.0020\,bh$

  Grade 60 deformed bar : $0.0018\,bh$

  > Grade 60 deformed bar: $\dfrac{0.0018 \times 60{,}000}{f_y} \times bh$

- For protection of steel against damage from fire or corrosion, at least $\frac{3}{4}$ in. concrete cover must be maintained.
- Short-direction bars are normally placed closer to the top or bottom surface of the slab, with the larger effective depth, $d = h$ - clear cover.
- Long-direction bars are placed inside these, with smaller $d' = d$ - bar diameter



For Seismic Detailing: To ensure ductile behavior throughout two-way slabs without beams, at least one-quarter of the top reinforcement at the support in column strips must be continuous throughout the span, as must bottom reinforcement equal to at least one-third of the top reinforcement at the support in column strips.

A minimum of one half of all bottom reinforcement at midspan in both column and middle strips must be continuous and develop its yield strength at the face of the support.

Depth Limitations of the ACI Code:

The minimum thickness of two-way slabs without interior beams must not be less than provided by Table 13.5

TABLE 13.5
Minimum thickness of slabs without interior beams

Yield Stress f_y, psi	Without Drop Panels			With Drop Panels		
	Exterior Panels		Interior Panels	Exterior Panels		Interior Panels
	Without Edge Beams	With Edge Beams[a]		Without Edge Beams	With Edge Beams[a]	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
75,000	$l_n/28$	$l_n/31$	$l_n/31$	$l_n/31$	$l_n/34$	$l_n/34$

[a] Slabs with beams along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

a. Slabs without Interior Beams:

Minimum thickness

For slabs without drop panels : 5 in

For slabs with drop panels : 4 in

b. Slabs with Beams on All Sides :

According to ACI Code 9.5.3.3 for $\alpha_{fm} \le 0.2$, the minimum thickness of Table 13.5 shall apply.

For $\alpha_{fm} > 0.2$ but $\alpha_{fm} < 2.0$, the slab thickness must not be less than

$$h = \frac{l_n\left(0.8 + \frac{f_y}{2,00,000}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \quad \text{\& not less than } 5.0 \text{ in.}$$

For $\alpha_{fm} > 2.0$, the thickness must not be less than,

$$h = \frac{l_n\left(0.8 + \frac{f_y}{2,00,000}\right)}{36 + 9\beta} \quad \text{\& not less than } 3.5 \text{ in.}$$

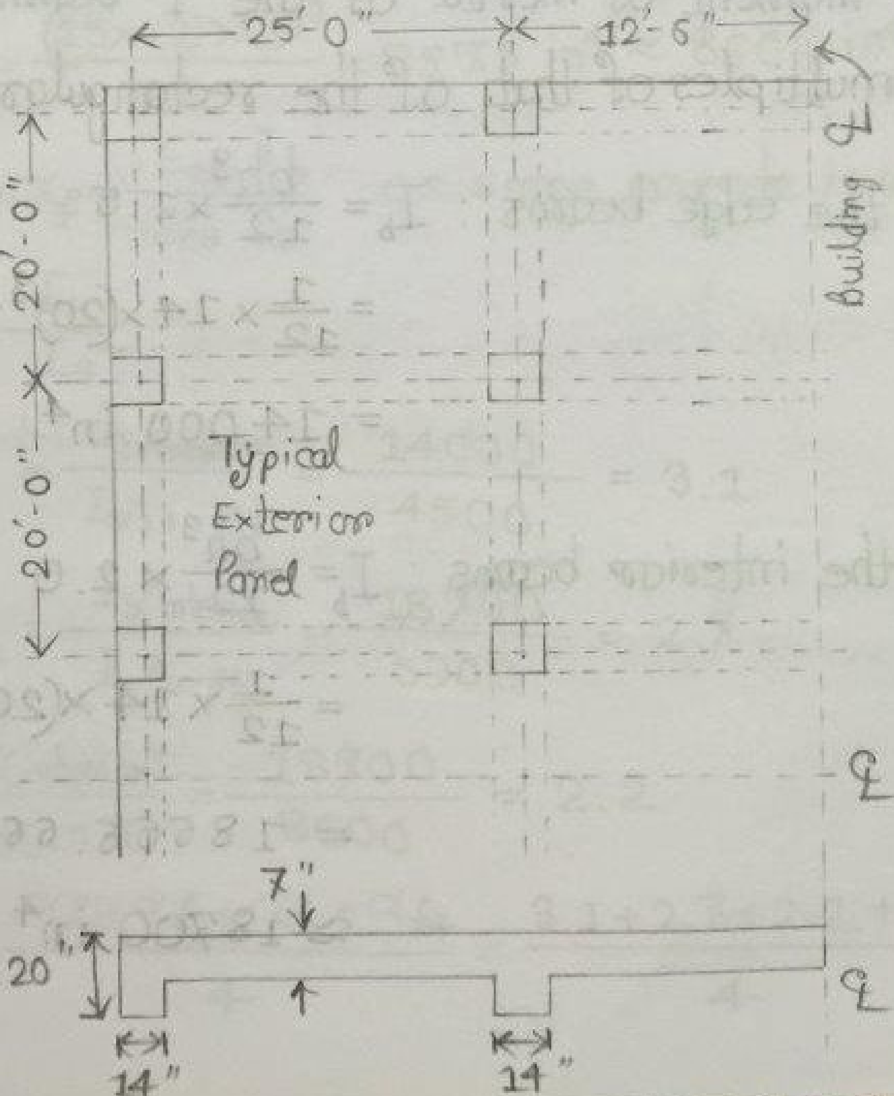
Where, l_n = clear span in long direction, inch

α_{fm} = average value of α_f for all beams on edges of a panel

β = ratio of clear span in long direction to clear span in short direction

Example 13.2: Design of two-way slab with edge beams:

A two-way reinforced concrete building floor system is composed of slab panels measuring 20×25 ft in plan, supported by shallow column-line beams cast monolithically with the slab, as shown in figure. Using concrete with $f_c' = 4000$ psi and steel with $f_y = 60{,}000$ psi, design a typical exterior panel to carry a service live load of 144 psf in addition to the self-weight of the floor.



Sol^n: Trial value for slab thickness for exterior panel,

$$h = \frac{\text{Perimeter}}{180} = \frac{(20+25)\times 2}{180} = 0.5' \approx 6''$$

So, take thickness $= h+1 = (6+1) = 7''$

Now, the effective flange projection beyond the face of the beam webs is the lesser of

$4h_f = 4\times 7 = 28''$

or $h_w = (20-7) = 13''$ (governs)

The moment of inertia of the T-beams will be estimated as multiples of that of the rectangular portion as follows:

For the edge beams : $I_b = \frac{bh^3}{12}\times 1.5$

$$= \frac{1}{12}\times 14\times(20)^3\times 1.5$$

$$= 14{,}000 \text{ in}^4$$

For the interior beams : $I_b = \frac{bh^3}{12}\times 2.0$

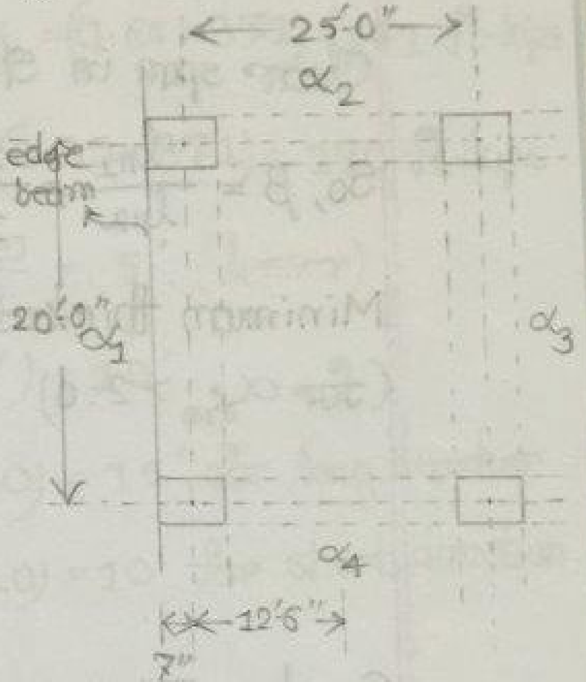
$$= \frac{1}{12}\times 14\times(20)^3\times 2.0$$

$$= 18666.6667 \text{ in}^4$$

$$\approx 18700 \text{ in}^4$$

For the edge beam, $b = 12'\text{-}6'' + \frac{14}{2}'' = 13'\text{-}1'' \approx 13.1'$

For the slab strips,



For the 13.1 ft edge width,

$$I_s = \frac{bh^3}{12} = \frac{(13.1\times12)\times(7)^3}{12} = 4493.3 \text{ in}^4$$
$$\approx 4500 \text{ in}^4$$

For the 20 ft width,

$$I_s = \frac{bh^3}{12} = \frac{(20\times12)\times(7)^3}{12} = 6860 \text{ in}^4$$
$$\approx 6900 \text{ in}^4$$

For the 25 ft width,

$$I_s = \frac{bh^3}{12} = \frac{(25\times12)\times(7)^3}{12} = 8575 \text{ in}^4 \approx 8600 \text{ in}^4$$

We know, $\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$ as same concrete is used, $E_{cb} = E_{cs}$

So, $\alpha_f = \frac{I_b}{I_s}$

Now, $\alpha_{f_1} = \frac{I_{b(edge)}}{I_{s(13.1 ft)}} = \frac{14000}{4500} = 3.1$

$$\alpha_{f_2} = \alpha_{f_4} = \frac{I_{b(interior)}}{I_{s(20 ft)}} = \frac{18700}{6900} = 2.7$$

$$\alpha_{f_3} = \frac{I_{b(interior)}}{I_{s(25 ft)}} = \frac{18700}{8600} = 2.2$$

So, $\alpha_{f_m} = \frac{\alpha_{f_1} + \alpha_{f_2} + \alpha_{f_3} + \alpha_{f_4}}{4} = \frac{3.1 + 2.7 + 2.2 + 2.7}{4} = 2.7$

Clear span in Long direction, $l_{n1} = \left(25' - \frac{14''}{12}\right) = 23.8'$ or $286''$

Clear span in Short direction, $l_{n2} = \left(20' - \frac{14''}{12}\right) = 18.8'$

So, $\beta = \frac{l_{n1}}{l_{n2}} = \frac{23.8'}{18.8'} = 1.27$

Minimum thickness, $h_{min} = \frac{l_{n1}\left(0.8 + \frac{f_y}{2,00,000}\right)}{36 + 9\beta}$ [l_n in long direction, inch]

(for $\alpha_{fm} > 2.0$)

$$= \frac{286 \times \left(0.8 + \frac{60000}{200000}\right)}{36 + 9 \times 1.27}$$

$$= 6.63''$$

So, $h = 7''$ is ok.

Now, For a $7''$ slab, Dead Load $= h \times 150$ pcf

$$= \left(\frac{7}{12} \times 150\right) \text{ or } 87.5 \text{ psf}$$

$$\approx 88 \text{ psf}$$

So, Design load, $\omega_u = 1.2\,DL + 1.6\,LL$

$$= 1.2 \times 88 + 1.6 \times 144$$

$$= 336 \text{ psf}$$

For short-span direction, $M_o = \frac{\omega_u l_1 (l_{n2})^2}{8}$

$$= \frac{0.336 \times 25 \times (18.8)^2}{8}$$

$$= 371 \text{ ft-kips}$$

This is distributed as follows:

Negative Design Moment $= 0.65\, M_o = (0.65 \times 371) = 241$ ft-kips

Positive Design Moment $= 0.35\, M_o = (0.35 \times 371) = 130$ ft-kips

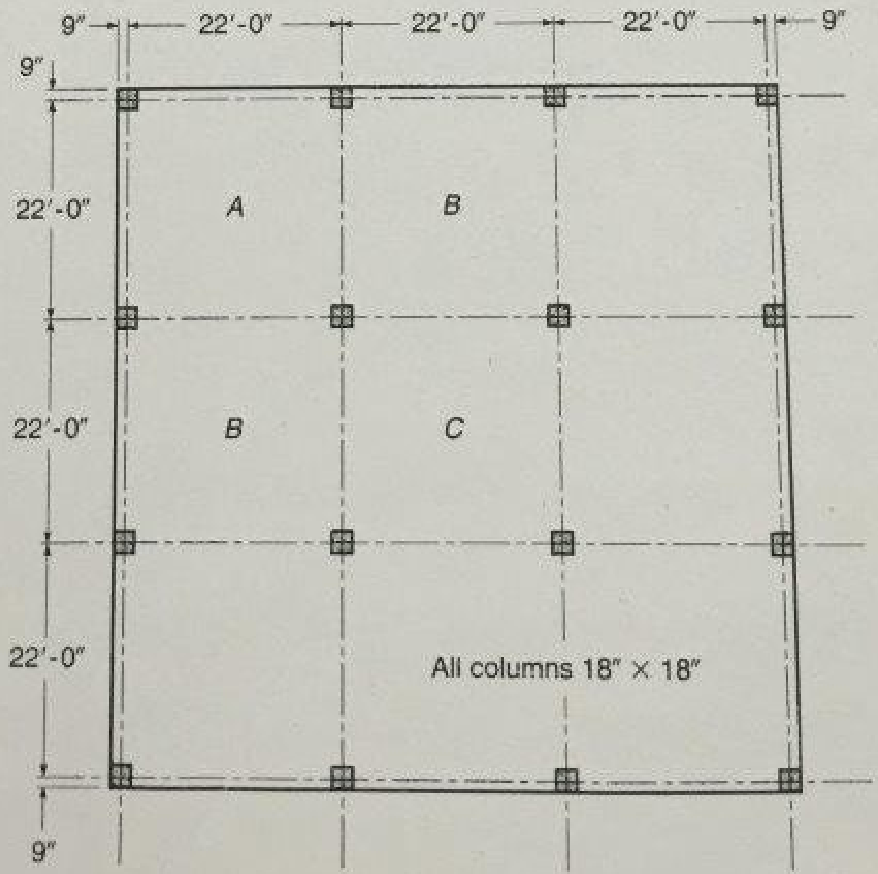
Here, $\frac{l_1}{4} = \frac{25}{4} = 6.25' > \frac{l_2}{4} = \frac{20}{4} = 5'$ (lesser)

So, Width of column-strip $= (2 \times 5') = 10'$

& Width of middle-strip $= (25 - 10) = 15'$ for long direction

& $(20 - 10) = 10'$ for short direction

Example 13.3 : Design of flat plate floor by Direct Design Method : An office building is planned using a flat plate floor system with the column layout as shown in figure. No beams, drop panels, or column capitals are permitted. Specified live load is 100 psf, and dead load will include the weight of the slab plus an allowance of 20 psf for finish floor plus suspended loads. The columns will be 18 in. square and the floor-to-floor height of the structure will be 12 ft. Design the interior panel C, using material strengths $f_y = 60000$ psi and $f_c' = 4000$ psi. Straight bar reinforcement will be used.



Sol^{n}: Minimum thickness, $h = \frac{l_n}{30}$ [For exterior panel criterion]

$$= \frac{\left(22 - \frac{18}{12}\right) \times 12}{30}$$

$$= 8.2'' \approx 8.5''$$ (0.5" upper rounding)

Self-weight of slab = $h \times 150$ pcf

$$= \left(\frac{8.5}{12} \times 150\right) = 106.25 \text{ psf} \approx 106 \text{ psf}$$

Factored Design Load, $\omega_u = 1.2\,DL + 1.6\,LL$

$$= 1.2 \times (106 + 20) + 1.6 \times 100$$

$$= 311.2 \text{ psf}$$

$$\approx 0.312 \text{ ksf}$$

As the dimension of panel is same,

Factored Moment, $M_o = \frac{\omega_u l_2 l_n^2}{8} = \frac{0.312 \times 22 \times (20.5)^2}{8}$

$$= 360.5745 \text{ ft-kips}$$

$$\approx 361 \text{ ft-kips}$$

Now, $\frac{l_2}{l_1} = \frac{22}{22} = 1$ & $\alpha_{f_1} \frac{l_2}{l_1} = 0 \times 1 = 0$ [No Beam Present]

From, Graph A.4, for interior panel,

Column-strip interior negative moment = 75 percent

positive moment = 60 percent

Middle-strip negative moment = 25 percent
positive moment = 40 percent

Again,

Negative Design Moment = $0.65\ M_o$

Positive Design Moment = $0.35\ M_o$

Now, For Column-strip,

$+M\ (c.s.) = 0.35\ M_o \times 0.6$

$= 0.35 \times 361 \times 0.6$

$= 75.81$ k-ft

$= \dfrac{75.81}{11}$ k-ft/ft

$= 6.89$ k-ft/ft

Here, $\dfrac{l_1}{4} = \dfrac{l_2}{4} = \dfrac{22'}{4} = 5.5'$

Width of Column-strip $= 2 \times 5.5$
$= 11'$

Width of Middle-strip $= 22' - 11'$
$= 11'$

$-M\ (c.s.) = 0.65\ M_o \times 0.75$

$= 0.65 \times 361 \times 0.75$

$= 175.9875$ k-ft

$= \dfrac{175.9875}{11}$ k-ft/ft

$= 15.99887 \approx 16$ k-ft/ft

Here, $t = 8.5''$ so, $d = (t - 1.5) = (8.5'' - 1.5'') = 7''$

Let, $a = 1''$

So, $+A_s(C.S.) = \frac{M_u}{\phi f_y(d-\frac{a}{2})} = \frac{6.89\times12}{0.9\times60\times(7-\frac{1}{2})}$

$= 0.2356 \text{ in}^2$

Corresponding $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.2356\times60}{0.85\times4\times12} = 0.3465''$

Revised $+A_s(C.S.) = \frac{M_u}{\phi f_y(d-\frac{a}{2})} = \frac{6.89\times12}{0.9\times60\times(7-\frac{0.3465}{2})}$

$= 0.2243 \text{ in}^2$

Provide #3 bar with spacing $= \frac{0.11\times12}{0.2243} = 5.88''$ c/c

So, use #3 bar @ 5.5" c/c

Now, $-A_s(C.S.) = \frac{M_u}{\phi f_y(d-\frac{a}{2})} = \frac{16\times12}{0.9\times60\times(7-\frac{0.3465}{2})}$

$= 0.5208 \text{ in}^2$

Provide #4 bar with spacing $= \frac{0.20\times12}{0.5208} = 4.61''$ c/c

So, use #4 bar @ 4.5" c/c

Now, $+A_s(M.S.) = \frac{+A_s(C.S.)\times0.4}{}$

$= \frac{0.2243\times0.4}{0.6}$

$= 0.1495 \text{ in}^2$

$$-A_s(M.S.) = \frac{-A_s(C.S.) \times 0.25}{0.75}$$

$$= \frac{0.5208 \times 0.25}{0.75}$$

$$= 0.1736 \text{ in}^2$$

Now, $A_{s(min)} = A_{s(temp.\ \&\ shrinkage)} = 0.0018\, bh$

$$= 0.0018 \times 12 \times 8.5$$

$$= 0.1836 \text{ in}^2$$

Here $+A_s(M.S.)$ & $-A_s(M.S.) < A_{s(min)}$

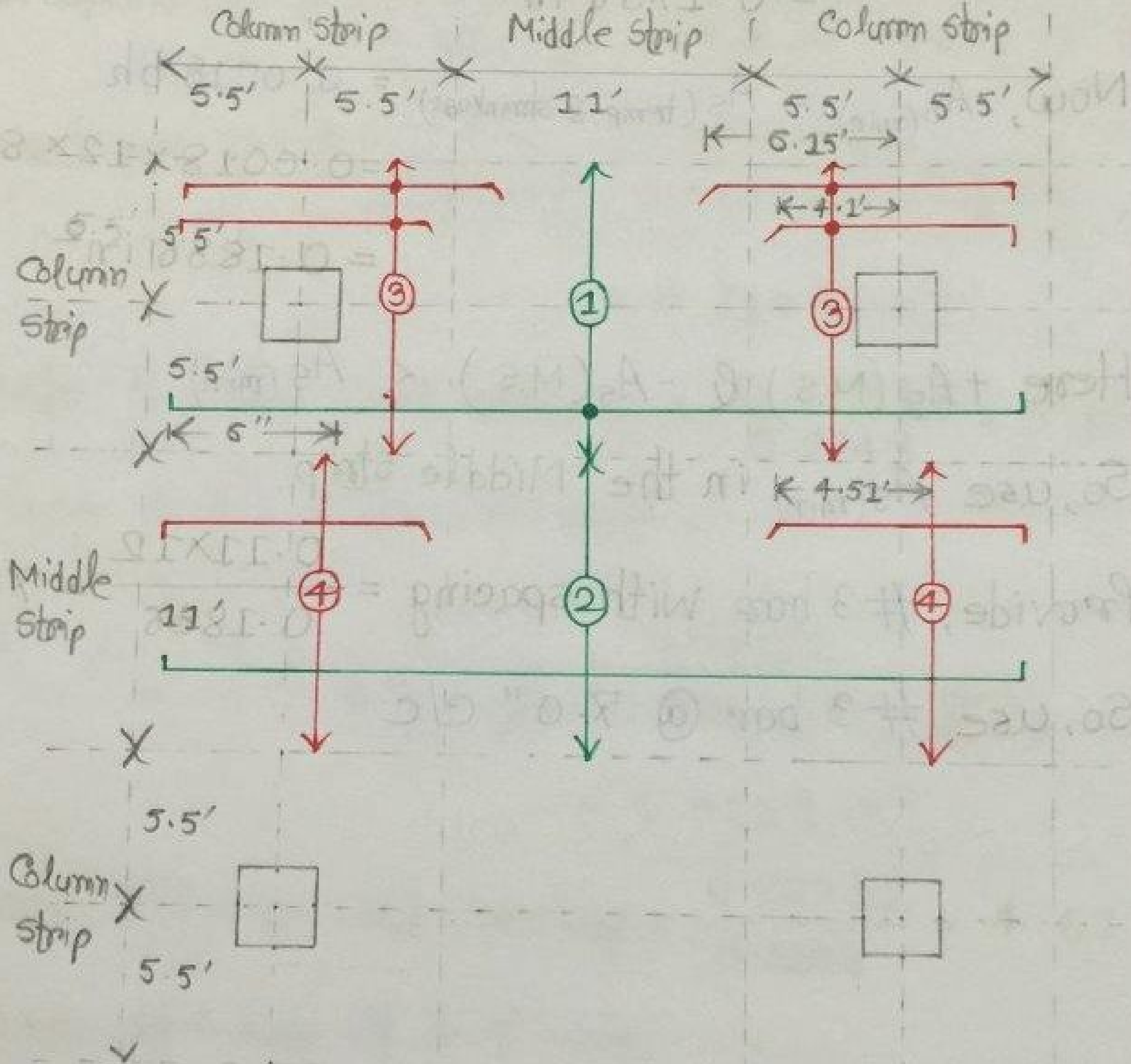
So, use $A_{s(min)}$ in the Middle strip.

Provide, #3 bar with spacing $= \frac{0.11 \times 12}{0.1836} = 7.19$ in c/c

So, use #3 bar @ 7.0" c/c

① #3 @ 5.5" c/c　　③ #4 @ 9" c/c

② #3 @ 7.0" c/c　　④ #3 @ 7" c/c



Shear Design in Flat Plates and Flat slabs :

When two-way slabs are supported directly by columns, as in flat slabs and flat plates, or when slabs carry concentrated loads, as in footings, shear near the columns is of critical importance.

a. Slabs Without Special Shear Reinforcement :

Two kinds of shear may be critical in the design of flat slabs, flat plates, or footings.

1. Beam Type Shear : The first is the familiar beam-type shear leading to diagonal tension failure.

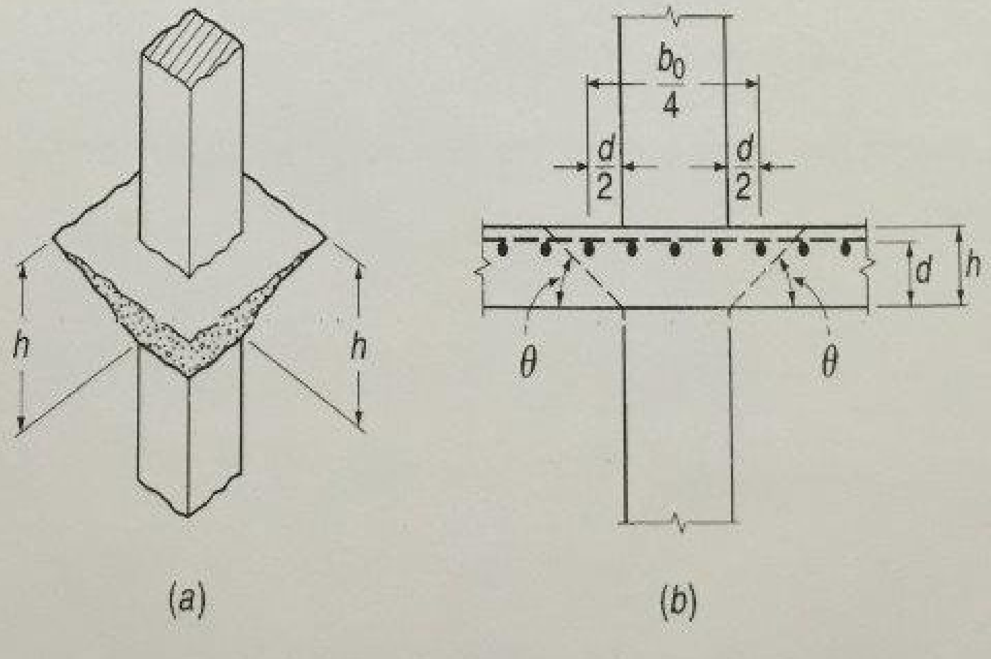
- Applicable particularly to long narrow slabs or footings, this analysis considers the slab to act as a wide beam, spanning between supports provided by the perpendicular column strips.
- A potential diagonal crack extends in a plane across the entire width l_2 of the slab.
- The critical section is taken a distance d from the face of the column or capital.
- The design shear strength ϕV_c must be at least equal to the required strength V_u at factored loads.

- The nominal shear strength, $V_c = 2\lambda\sqrt{f_c'}\, b_w d$ where, b_w is equal to the panel width l_2.

2. Punching Shear : Alternatively, failure may occur by punching shear, with the potential diagonal crack following the surface of a truncated cone or pyramid around the column, capital or drop panel as shown in Fig 13.21 a

- The failure surface extends from the bottom of the slab, at the support, diagonally upward to the top surface.
- The angle of inclination with the horizontal, θ depends upon the nature and amount of reinforcement in the slab. It may range between about 20 & 45°.

FIGURE 13.21
Failure surface defined by punching shear.



- The "critical section" for shear is taken perpendicular to the plane of the slab and a distance $\frac{d}{2}$ from the periphery of the support, as shown in Fig 13.21 b
- The shear force V_u to be resisted can be calculated as the total factored load on the area bounded by panel centerlines around the column less the load applied within the area defined by the critical shear perimeter, unless significant moments must be transferred from the slab to the column.

- For slabs supported by columns having a ratio of long to short sides not greater than 2, the nominal shear strength may be taken equal to,

$$V_c = 4\lambda\sqrt{f_c'}\, b_o d$$

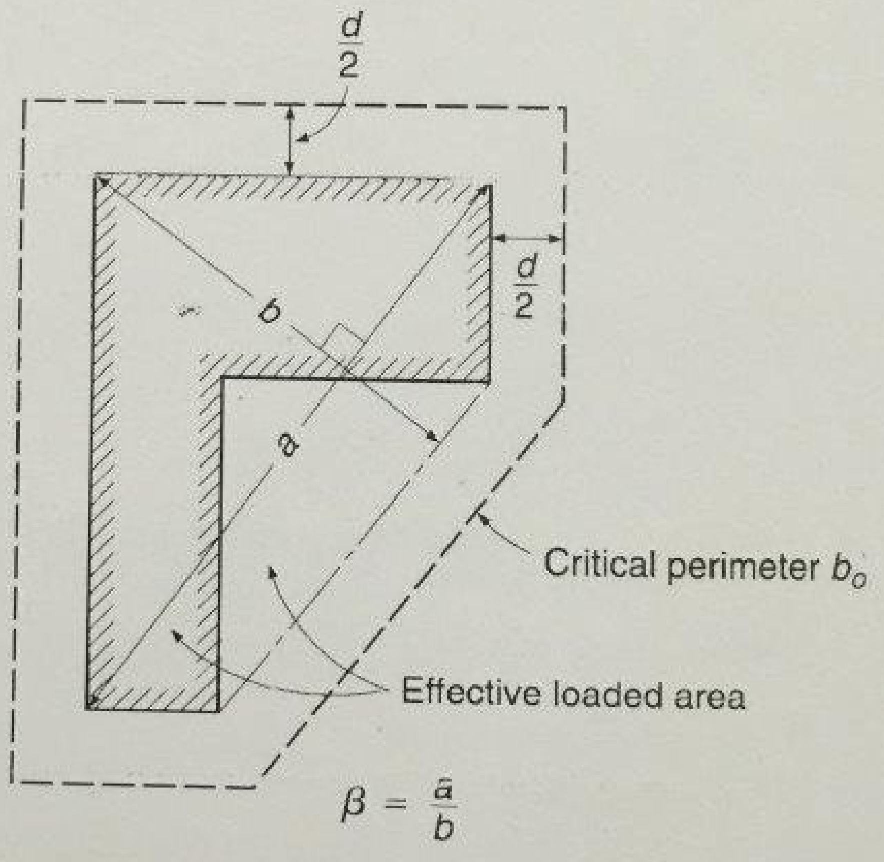
where, b_o = the perimeter along the critical section

λ = the light-weight concrete factor

- The design strength is taken as ϕV_c, where, $\phi = 0.75$ for shear. The basic requirement is, $V_u \leq \phi V_c$.

• For columns with non-rectangular cross-sections the ACI Code indicates that the perimeter b_o must be of minimum length, but need not approach closer than $\frac{d}{2}$ to the perimeter of the reaction area. The manner of defining the critical perimeter b_o and the ratio β for such irregular support configurations is illustrated in Fig 13.23.

FIGURE 13.23
Punching shear for columns of irregular shape.



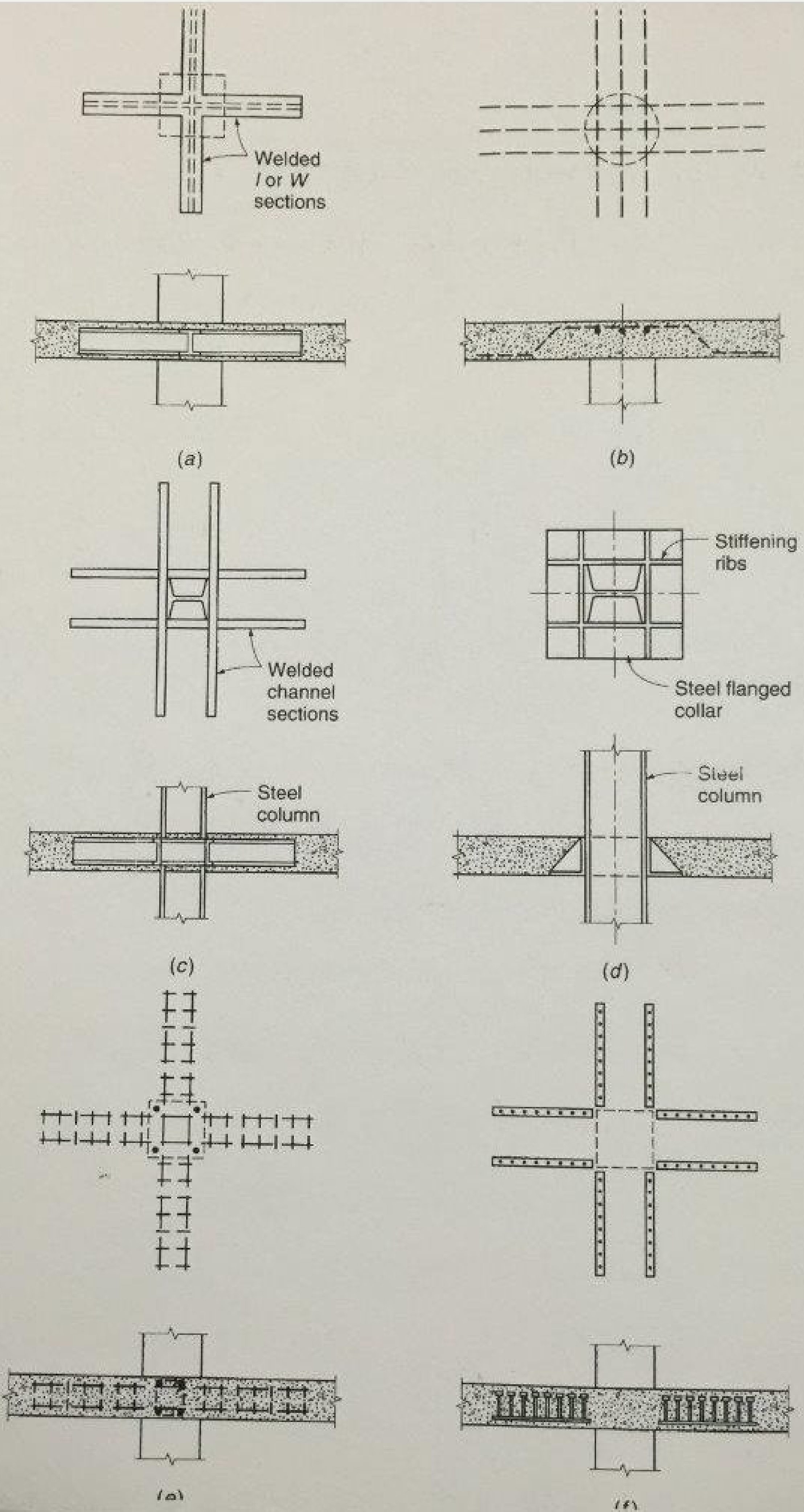
Types of Shear Reinforcement

Special shear reinforcement is often used at the supports for flat plates, and sometimes for flat slabs as well. It may take several forms. A few common types are shown in Fig. 13.24.

The *shearheads* shown in Fig. 13.24*a* and *c* consist of standard structural steel shapes embedded in the slab and projecting beyond the column. They serve to increase the effective perimeter b_o of the critical section for shear. In addition, they may contribute to the negative bending resistance of the slab. The reinforcement shown in Fig. 13.24*a* is particularly suited for use with concrete columns. It consists of short lengths of I or wide-flange beams, cut and welded at the crossing point so that the arms are continuous through the column. Normal negative slab reinforcement passes over the top of the structural steel, while bottom bars are stopped short of the shearhead. Column bars pass vertically at the corners of the column. The effectiveness of this type of shearhead has been documented by tests by Corley and Hawkins (Ref. 13.18). The channel frame in Fig. 13.24*c* is very similar in its action, but is adapted for use with steel columns. The bent-bar arrangement in Fig. 13.24*b* is suited for use with concrete columns. The bars are usually bent at 45° across the potential diagonal tension crack, and extend along the bottom of the slab a distance sufficient to develop their strength by bond. The flanged collar in Fig. 13.24*d* is designed mainly for use with lift-slab construction (see Chapter 18). It consists of a flat bottom plate with vertical stiffening ribs. It may incorporate sockets for lifting rods, and usually is used in conjunction with shear pads welded directly to the column surfaces below the collar to transfer the vertical reaction.

Another type of shear reinforcement is illustrated in Fig. 13.24*e*, where vertical stirrups have been used in conjunction with supplementary horizontal bars radiating outward in two perpendicular directions from the support, to form what are termed *integral beams* contained entirely within the slab thickness. These beams act in the same general way as the shearheads shown in Fig. 13.24*a* and *c*. Adequate anchorage of the stirrups is difficult in slabs thinner than about 10 in. ACI Code 11.11.3 requires the slab effective depth d to be at least 6 in., but not less than 16 times the diameter of the shear reinforcement. In all cases, closed hoop stirrups should be used, with a large-diameter horizontal bar at each bend point, and the stirrups must be terminated with a standard hook (Ref. 13.19).

Headed shear stud reinforcement, shown in Fig. 13.24*f*, is governed by ACI Code 11.11.5. This consists of large-head studs welded to steel strips. The strips are supported on wire chairs during construction to maintain the required concrete cover to the bottom of the slab below the strip, and the usual cover is maintained over the top of the head. Because of the positive anchorage provided by the stud head and the steel strip, these devices are more effective, according to tests, than either the bent-bar or integral beam reinforcement (Refs. 13.20 and 13.21). In addition, they can be placed more easily, with less interference with other reinforcement, than other types of shear steel.


Welded
I or W
sections
(a)
(b)
Stiffening
ribs
Welded
channel
sections
Steel flanged
collar
Steel
column
Steel
column
(c)
(d)

Design of Bent-Bar Reinforcement:

- If shear reinforcement in the form of bars is used (Fig. 13.24 b), the limit value of V_n, calculated at the critical section $\frac{d}{2}$ from the support face, may be increased to $6\sqrt{f_c'}\, b_o d$ according to ACI Code 11.11.3.

- The shear resistance of the concrete V_c is reduced to $2\lambda\sqrt{f_c'}\, b_o d$, and reinforcement must provide for the excess shear above ϕV_c.

- The total bar area A_v crossing the critical section at slope angle α is easily obtained by equating the vertical component of the steel force to the excess shear force to be accomodated:

$$\phi A_v f_y \sin\alpha = V_u - \phi V_c$$

- Where inclined shear reinforcement is all bent at the same distance from a support, $V_s = A_v f_y \sin\alpha$ is not to exceed $3\sqrt{f_c'}\, b_o d$, according to ACI Code 11.4.7.

- The required area of reinforcement for shear is found by transposing the preceding equation:

$$A_V = \frac{V_u - \phi V_c}{\phi f_y \sin\alpha}$$

- Successive sections at increasing distances from the support must be investigated and reinforcement provided where V_u exceeds ϕV_c.
- Only the center three-quarters of the inclined portion of the bent bars can be considered effective in resisting shear, and full development length must be provided past the location of peak stress in the steel, which is assumed to be at slab mid-depth, $\frac{d}{2}$.

Example 13.4 : Design of Bar reinforcement for punching shear : A flat plate floor has thickness $h = 7\frac{1}{2}$ in. and is supported by 18 in. square columns spaced 20 ft on centers each way. The floor will carry a total factored load of 300 psf. check the adequacy of the slab in resisting punching shear at a typical interior column, and provide shear reinforcement, if needed, using bent bars. An average effective depth d = 6 in may be used. Material strengths are $f_y = 60,000$ psi and $f_c' = 4000$ psi.

Sol^n: Here, $l_n = \left(20' - \frac{18}{12}\right) = 18.5'$ — If h is not given

$\therefore h = \frac{l_n}{30} = \frac{18.5 \times 12}{30} = 7.4'' \approx 7.5''$

So, $d = h - 1.5 = (7.5 - 1.5) = 6''$

DL = Floor Finish (FF) + Partition Wall (PW) + Self-Weight (S.W.)

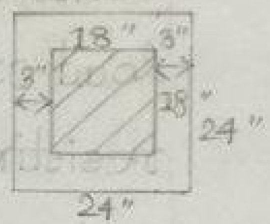
LL = Live Load

Factored Load, $\omega_u = 1.2DL + 1.6LL$

Here, $\omega_u = 300$ psf (given)

The first critical section for punching shear is a distance $\frac{d}{2} = \frac{6''}{2} = 3''$ from the column face, providing a Shear perimeter, $b_0 = (4 \times 24'') = 96''$

Based on the tributary area of loaded floor, the factored shear is,



$$V_u = 300 \times \left[(20)^2 - \frac{24 \times 24}{(12)^2}\right]$$

$$= 1{,}18{,}800 \text{ lb}$$

and if no shear reinforcement is used, the design strength of the slab,

$$\phi V_c = \phi 4 \sqrt{f_c'}\, b_o d$$

$$= 0.75 \times 4 \times \sqrt{4000} \times 96 \times 6$$

$$= 1{,}09{,}288.3159 \approx 1{,}09{,}300 \text{ lb}$$

Since $V_u > \phi V_c$, shear reinforcement is required.

Bars bent at 45° will be used in two directions.

When shear strength is provided by a combination of reinforcement and concrete, the concrete contribution is reduced to,

$$\phi V_c = \phi\, 2\sqrt{f'_c}\, b_o d$$

$$= 0.75 \times 2 \times \sqrt{4000} \times 96 \times 6$$

$$= 54,644.15797 \approx 54,650 \text{ lb}$$

and so the shear V_s to be resisted by the reinforcement is,

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{118800 - 54650}{0.75} = 85,533.3333$$

$$\approx 85,600 \text{ lb}$$

This is below the maximum permissible value of

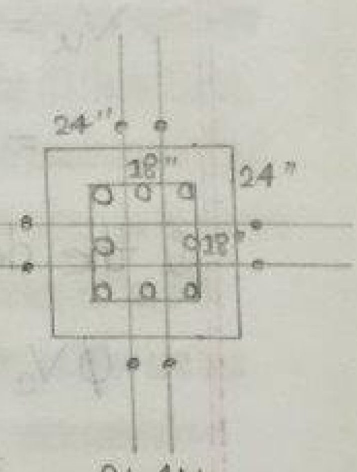
$$V_{s(max)} = 3\sqrt{f'_c}\, b_o d$$

$$= 3 \times \sqrt{4000} \times 96 \times 6$$

$$= 1,09,288.3159 \approx 1,09,300 \text{ lb}$$

Now, the required bar area,

$$A_v = \frac{V_u - \phi V_c}{\phi f_y \sin\alpha}$$



$$= \frac{118800 - 54650}{0.75 \times 60000 \times \sin(45^\circ)}$$

$$= 2.016 \text{ in}^2$$

A total of four bars will be used (two in each direction), and with "eight legs" crossing the critical section, the necessary area per bar is $= \frac{A_v}{8} = \frac{2.016}{8} = 0.252 \text{ in}^2$

Use, #5 bars with $A_v = 0.31 \, in^2$

The upper limit of $V_n = 6\sqrt{f_c'}\, b_o d$ is automatically satisfied in this case, given the more stringent limit on V_s.

With bars bent at 45° and effective through the center three-fourths of the inclined length, the next critical section is approximately $\frac{3}{4}$ times the effective depth, $\frac{3}{4}d = \left(\frac{3}{4} \times 6\right) = 4.5''$, past the first critical section,

Giving a perimeter of, $b_o = (24 + 2 \times 4.5) \times 4$
$$= (33 \times 4) = 132''$$

The factored shear at that critical section,

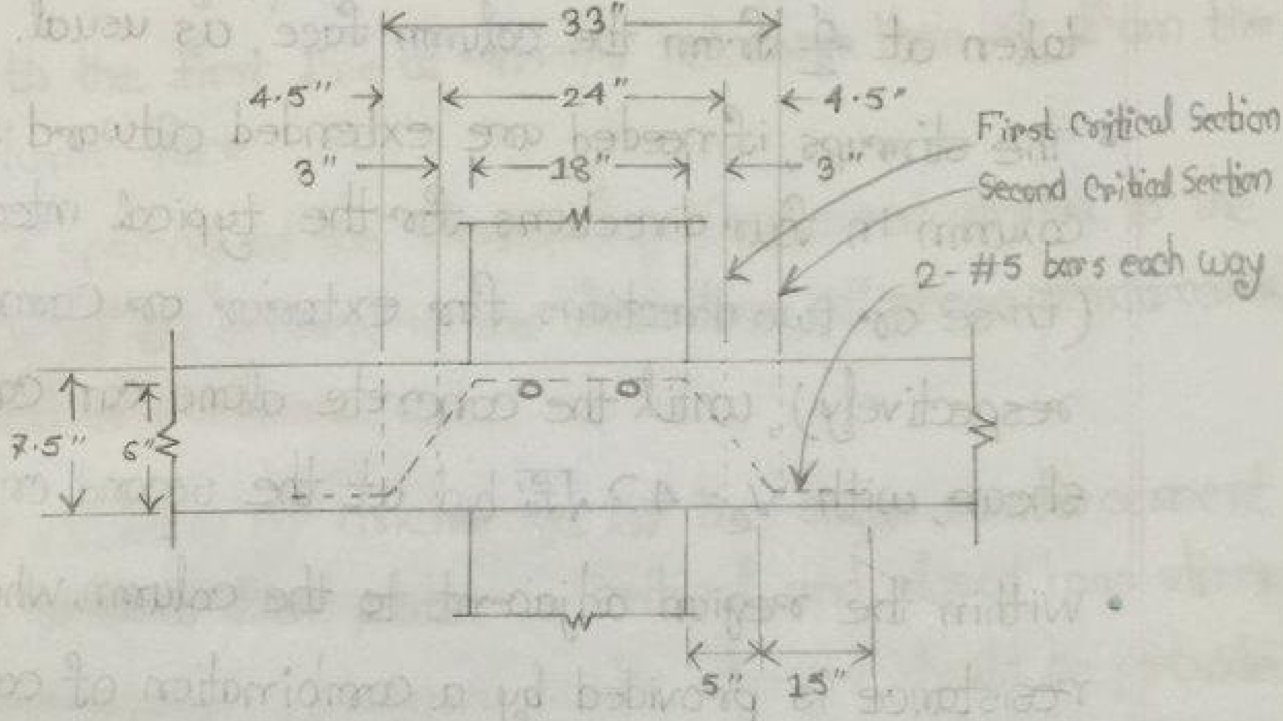
$$V_u = 300 \times \left[(20)^2 - \frac{33 \times 33}{(12)^2}\right]$$
$$= 117731.25 \approx 117750 \text{ lb}$$

& the design capacity of the concrete is,

$$\phi V_c = \phi 4\sqrt{f_c'}\, b_o d$$
$$= 0.75 \times 4 \times \sqrt{4000} \times 132 \times 6$$
$$= 150271.4344 \approx 150300 \text{ lb}$$

So, $V_u < \phi V_c$ confirming that no additional bent bars are needed.

The #5 bars will be extended along the bottom of the slab the full development length of 15".



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† ACI Code 11.11.3 and ACI Commentary 11.11.3 are ambiguous regarding the value of V_c to be used for flat plate slabs beyond the region where shear reinforcement is required. In general, for slabs where shear reinforcement is not required, V_c is calculated from Eqs. (13.11*a*) to (13.11c), with V_c in most cases equal to $4\lambda\sqrt{f'_c}\,b_o d$. When shear reinforcement is provided, the limiting shear may be increased to a maximum of $6\sqrt{f'_c}\,b_o d$; however, the shear reinforcement must be designed to carry all shear in excess of ϕV_c with $V_c = 2\lambda\sqrt{f'_c}\,b_o d$. This seems to imply that the reduction in V_c to one-half its normal value applies only where there is a sharing of the force between concrete and steel reinforcement and that, in the region where shear reinforcement is *not* required, the full concrete contribution of $4\lambda\sqrt{f'_c}\,b_o d$ can be used. The examples that follow have been prepared on that basis. The alternative interpretation is that if shear reinforcement is required at the column, then the concrete contribution is reduced to $2\lambda\sqrt{f'_c}\,b_o d$ throughout the slab. This more conservative interpretation could be adopted in many cases without significant cost increase, because of the rapid increase in V_c with increasing distance from the column resulting from the increase in concrete shear perimeter b_o, as well as the reduction in net shear force V_u.

Design of Integral Beams With Vertical Stirrups :

- The first critical section for shear design in the slab is taken at $\frac{d}{2}$ from the column face, as usual.
- The stirrups, if needed, are extended outward from the column in four directions for the typical interior case (three or two directions for exterior or corner columns, respectively), until the concrete alone can carry the shear, with $V_c = 4\lambda\sqrt{f_c'}\, b_o d$ at the second critical section.
- Within the region adjacent to the column, where shear resistance is provided by a combination of concrete and steel, the nominal shear strength V_n must not exceed $6\sqrt{f_c'}\, b_o d$ according to ACI Code 11.11.3.
- In this region, the concrete contribution is reduced to $V_c = 2\lambda\sqrt{f_c'}\, b_o d$.
- The second critical section crosses each integral beam at a distance $\frac{d}{2}$ measured outward from the last stirrup and is located so that its perimeter b_o is a minimum (i.e. for the typical case, defined by 45° lines between the integral beams).

- The required spacing of the vertical stirrups,

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c}$$ but must not exceed $\frac{d}{2}$,

with the first line of stirrups not more than $\frac{d}{2}$ from the column face.

- The spacing of the stirrup legs (measured parallel to the face of the column) in the first line of shear reinforcement must not exceed $2d$.

- The problem of anchorage of the shear reinforcement in shallow flat plates is critical, and closed hoop stirrups, terminating in standard hooks, always should be provided with interior corner bars to improve pullout resistance.

Example 13.5 : Design of an integral beam with vertical stirrups : The flat plate slab with 7.5 in total thickness and 6 in effective depth shown in figure is carried by 12 in square column 15 ft on centers in each direction. A factored load of 120 kips must be transmitted from the slab to a typical interior column. Concrete and steel strengths used are, respectively, $f_c' = 4000$ psi and $f_y = 60000$ psi. Determine if shear reinforcement is required for the slab, and if so, design integral beams with stirrups to carry the excess shear.

Sol^n: At the critical section $\frac{d}{2}$ from the face of the column, perimeter $b_o = \left(12 + 2 \times \frac{6}{2}\right) \times 4$ $\left[b_o = \left(12 + 2 \times \frac{d}{2}\right) \times 4\right]$

$$= 72''$$

The design shear strength,

$$\phi V_c = \phi 4 \lambda \sqrt{f_c'}\, b_o d$$

$$= 0.75 \times 4 \times 1 \times \sqrt{4000} \times 72 \times 6$$

$$= 81966.24 \text{ lb}$$

$$\approx 82 \text{ kips}$$

Now, $\phi V_c < V_u = 120$ kips (given)

So, shear reinforcement is required.

The effective depth, $d = 6''$ just satisfies the minimum allowed to use vertical stirrups.

The maximum design strength allowed,

$$\phi V_n = \phi\, 6\sqrt{f_c'}\, b_o d$$

$$= 0.75 \times 6 \times \sqrt{4000} \times 72 \times 6$$

$$= 122949.36\ lb$$

$$\approx 123\ kips$$

satisfactorily above the actual V_u.

When shear is resisted by combined action of concrete and bar reinforcement, the concrete contribution is reduced to,

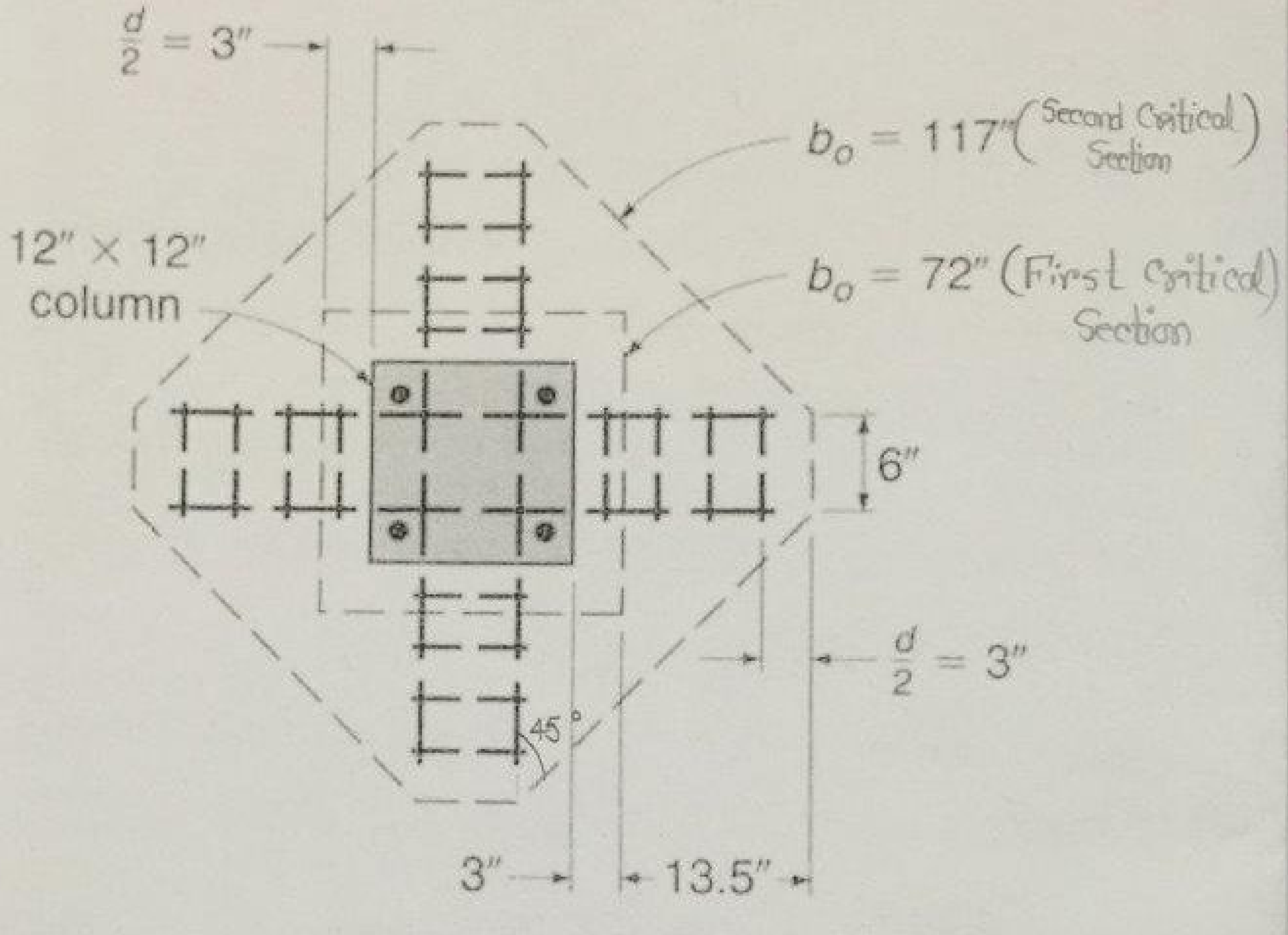
$$\phi V_c = \phi\, 2\sqrt{f_c'}\, b_o d$$

$$= 0.75 \times 2 \times \sqrt{4000} \times 72 \times 6$$

$$= 40983.12\ lb$$

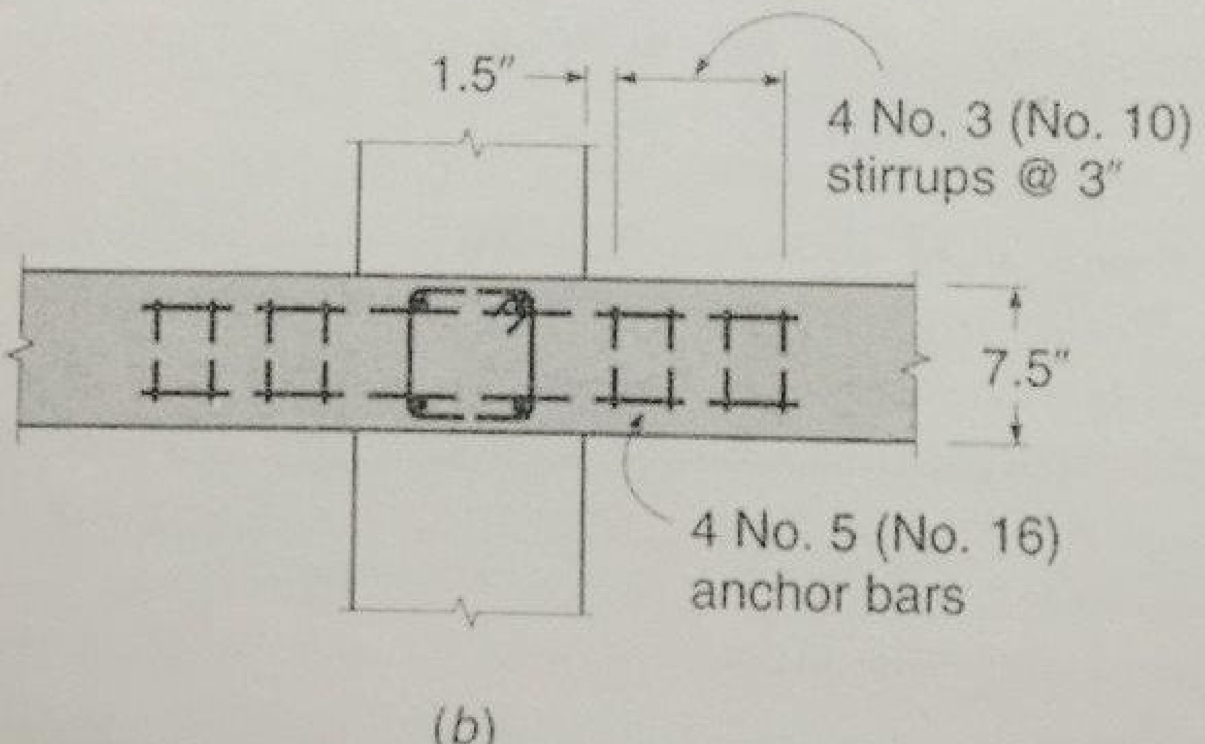
$$\approx 41\ kips$$

The #3 vertical closed hoop stirrups will be used since $d \geq 16$ times stirrup diameter $\left(\frac{d}{16} = \frac{6}{16} = \frac{3}{8}''\right)$ and arranged along four integral beams as shown in figure.



d/2 = 3″
b_o = 117″ (Second Critical Section)
12″ × 12″ column
b_o = 72″ (First Critical Section)
6″
d/2 = 3″
45°
3″
13.5″

(a)



1.5″
4 No. 3 (No. 10) stirrups @ 3″
7.5″
4 No. 5 (No. 16) anchor bars

(b)

So, total 8 legs are used, thus

$A_{s(provided)} = (8 \times 0.11) = 0.88\ in^2$ at the first critical section, a distance $\frac{d}{2}$ from the column face,

The required spacing,

$$S = \frac{\phi A_v f_y d}{V_u - \phi V_c} = \frac{0.75 \times 0.88 \times 60 \times 6}{120 - 41} = 3.0076''$$

However, the maximum spacing, $S_{max} = \frac{d}{2} = \frac{6}{2} = 3''$ (controls)

So, #3 stirrups at a constant spacing of 3" will be used.

Now, the required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from,

$$\phi V_c = \phi 4 \sqrt{f'_c}\, b_o d$$

$$\Rightarrow 120000 = 0.75 \times 4 \times \sqrt{4000} \times b_o \times 6 \quad [\phi V_c = V_u]$$

$$\therefore b_o = 105.41''$$

This requires a minimum projection of critical section past the face of the column of $\left(\frac{105.41}{4} - 15''\right) = 11.35''$

Four stirrups at a constant 3" spacing gives $(4 \times 3) = 12''$, the first being placed at $\frac{S}{2} = \frac{3}{2} = 1.5'' \leq \frac{d}{2} = 3''$ from the column face.

This provides a perimeter at the second critical section of

$$b_o = \left(\frac{(1.5+12)+3}{\sin 45°} + 6\right) \times 4 = 117.34'' \text{ exceeding}$$

the requirement.

Four longitudinal #5 bars will be provided inside the corners of each closed hoop stirrup, as shown, to provide anchorage of the shear reinforcement.

'Chapter 16'

"Footings And Foundations"

For Footing Design three parameters need to be known

① f_c'

② f_y

③ Soil Bearing Capacity.

→ q_a, allowable bearing capacity

→ q_e, effective bearing capacity

→ q_u, ultimate bearing capacity

Footing Design means determination of

① Area ⇒ for unfactored load

② Thickness

③ Reinforcement

$\left.\begin{matrix}\text{② Thickness}\\ \text{③ Reinforcement}\end{matrix}\right\}$ for factored load

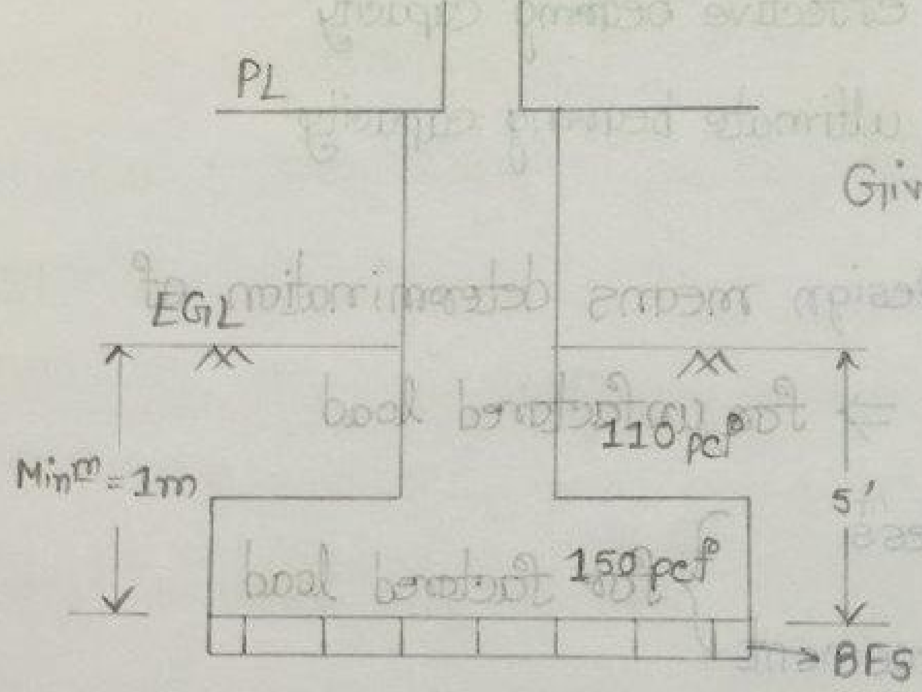
Area of footing,

$$A = \frac{DL + LL + \text{Self-weight}}{q_a} \quad \text{or} \quad \frac{DL + LL + WL/EL}{1.33\, q_a}$$ [whichever is larger]

We will use, $A = \dfrac{DL + LL}{q_e}$

- Minimum depth of footing = 1m

For determining Footing Area, the loads must be calculated at the level of the base of the footings, i.e. at the contact plane between soil and footing. This means that the weight of the footing & surcharge (i.e. fill and possible liquid pressure on top of the footing) must be included.



Given, $q_a = 4000$ psf

Effective pressure at the base of footing,

$$q_e = q_a - \gamma h$$

Assumed Avg. of 110 & 150 pcf (unit weight)

$$= 4000 - 125 \times 5$$

$$= 3375 \text{ psf}$$

- Always provide greater footing area than required.
- Rounding dimensions to 3" e.g. 10'-3"; 8'-6" etc.
- Most of the time thickness is governed by Punching Shear.
- If $\frac{\text{Length}}{\text{Width}} >> 2$, then Beam Shear may govern.
- For assuming Effective Depth, d, we will use the footing dimensions,

  $d_{(assumed)} = (\text{Length} + \text{Width})$ of footing, in Inches.

e.g. for $10' \times 10' \rightarrow d = (10+10)'' = 20''$

- Check Punching Shear first, then check Beam Shear.
- Critical section for,

  Punching Shear : At a distance $\frac{d}{2}$ from all faces

  Beam Shear : At a distance d from all faces

- In footing, main bar is placed in Long Direction

  In flat plate, main bar is placed in Long Direction

  In two-way slab, main bar is placed in Short Direction

Example 16.2 : Design of a square footing : A column 18 in square with $f_c' = 4$ ksi, reinforced with eight No. 8 (No. 25) bars of $f_y = 60$ ksi, supports a dead load of 225 kips and a live load of 175 kips. The soil (fill) has a unit weight of 100 pcf. The allowable soil pressure q_a is 5 kips/ft^2. Design a square footing with base 5 ft below grade, using $f_c' = 4$ ksi and $f_y = 60$ ksi.

Sol^n: Since the space between the bottom of the footing and the surface will be occupied partly by concrete and partly by soil (fill), an average unit weight of $\frac{100+150}{2} = 125$ pcf will be assumed.

So, effective pressure at the base of footing 5 ft below grade,

$$q_e = q_a - \gamma h = (5000 - 125 \times 5) \text{ psf or, } 4375 \text{ psf}$$

Hence, required footing area, $A_{req} = \frac{DL+LL}{q_e}$

$$\Rightarrow A_{req} = \frac{(225+175) \text{ kips}}{4.375 \text{ ksf}} = 91.4286 \text{ ft}^2$$

So, base of $\sqrt{91.4286} = 9.56' \approx 9'-9''$ (3" upper rounding) is selected, furnishing a footing area of, $A = (9.75 \times 9.75) \text{ft}^2$

$= 95.0625 \text{ ft}^2$

For strength design, the upward pressure caused by the factored column loads is,

$$q_u = \frac{1.2DL + 1.6LL}{A_{provided}}$$

$$= \frac{1.2 \times 225 + 1.6 \times 175}{9.75 \times 9.75}$$

$$= \frac{270 + 280}{95.0625}$$

$$= 5.7857 \text{ kips/ft}^2$$

Check for Punching Shear: Trial value for effective depth,

d = (length + width) of footing in inches

$$= (9.75 + 9.75)''$$

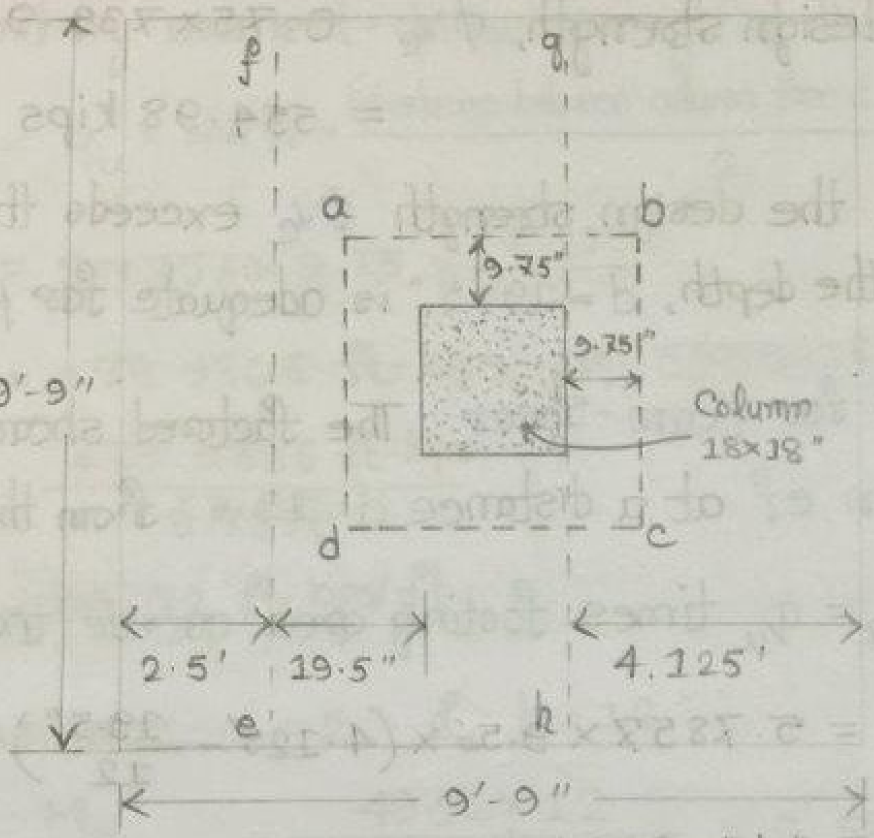
$$= 19.5''$$

Hence, the length of the critical perimeter is,

$$b_0 = 4 \times (18'' + d)$$

$$= 4 \times (18 + 19.5)$$

$$= 150''$$



The shear force acting on this perimeter being equal to the total upward pressure minus that acting within the perimeter abcd is,

$$V_{u1} = 5.7857 \times \left[(9.75)^2 - \left(\frac{18+19.5}{12}\right)^2\right]$$

$$= 493.5 \text{ kips}$$

The corresponding nominal shear strength,

$$V_c = 4\lambda\sqrt{f_c'}\, b_o d$$

$$= 4 \times 1 \times \sqrt{4000} \times 150 \times \frac{19.5}{1000}$$

$$= 739.97 \text{ kips}$$

The design strength, $\phi V_c = 0.75 \times 739.97$

$= 554.98$ kips

Since the design strength ϕV_c exceeds the factored shear, V_{u1}, the depth, $d = 19.5''$ is adequate for punching shear.

Check for Beam-Shear : The factored shear acting on section ef at a distance $d = 19.5''$ from the column face,

$V_{u2} = q_u$ times footing area outside that section

$$= 5.7857 \times 9.75 \times \left(4.125' - \frac{19.5''}{12}\right)$$

$= 141.0264$ kips

& the nominal shear strength is,

$$V_c = 2\lambda\sqrt{f_c'}\, b_w d$$

$$= 2 \times 1 \times \sqrt{4000} \times (9.75' \times 12) \times \frac{19.5''}{1000}$$

$= 288.5895$ kips

So, the design shear strength, $\phi V_c = 0.75 \times 288.5895$

$= 216.44$ kips

So, design shear strength ϕV_c is larger than the factored shear V_{u2}. so that $d = 19.5''$ is adequate for Beam-shear.

The bending moment on section gh at the column face,

$$M_u = q_u \times \text{width} \times \frac{(\text{distance between column face \& end of footing})^2}{2}$$

$$= 5.7857 \times 9.75' \times \frac{(4.125')^2}{2}$$

$$= 479.9306 \text{ ft-kips or, } 5759.1672 \text{ in-kips}$$

$$= \frac{479.9306 \text{ ft-kips}}{9.75 \text{ ft}}$$

$$= 49.22 \text{ ft-kips/ft}$$

Now, assume, $a = 2"$ & $b = 12"$

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{49.22 \times 12}{0.9 \times 60 \times (19.5 - \frac{2}{2})} = 0.5912 \text{ in}^2/\text{ft}$$

Corresponding, $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.5912 \times 60}{0.85 \times 4 \times 12} = 0.8694"$

Revised $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{49.22 \times 12}{0.9 \times 60 \times (19.5 - \frac{0.8694}{2})} = 0.5737 \text{ in}^2/\text{ft}$

& $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.5737 \times 60}{0.85 \times 4 \times 12} = 0.8437"$ close to previous assumption

Now, $A_{s(min)} = A_{s(\text{temp \& shrinkage})} = 0.0018\, bt$

For concrete in contact with ground, a minimum cover of 3" is required for corrosion protection

With, $d = 19.5"$ measured from the top of the footing to the center of the upper layer of bars, the total thickness of the footing that is required,

$$t = 19.5 + 1.5 \times 1 + 3 = 24" \approx 2'-0"$$

So, $A_{s(min)} = 0.0018\,bt = 0.0018 \times 12 \times 24"$

$$= 0.5184 \ in^2/ft$$

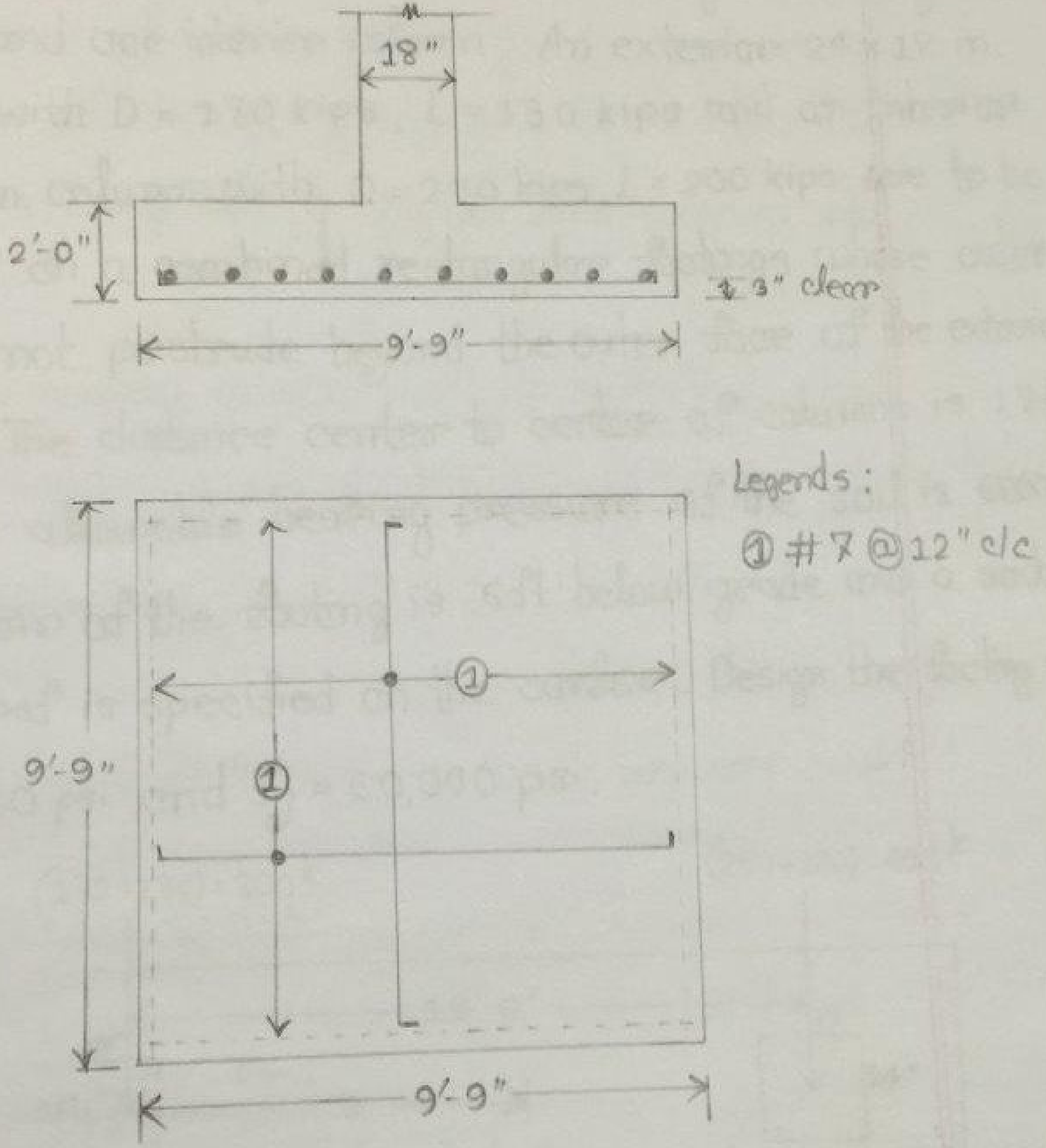
Here, $A_{s(required)} > A_{s(min)}$

Provide #7 bars with spacing $= \dfrac{0.60 \times 12}{0.5737} = 12.55"$ c/c

$$\approx 12" \text{ c/c}$$

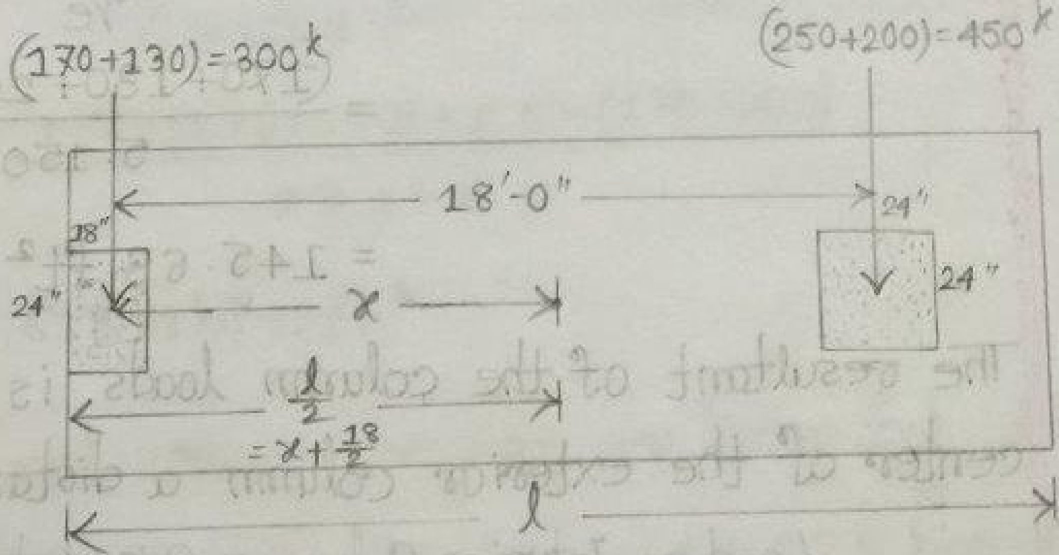
So, use #7 bars @ 12" c/c [Alternatively, 10- #7 bars]

in both direction.



Reinforcement Details

Example 16.3 : Design of a combined footing supporting one exterior and one interior column : An exterior 24 × 18 in. column with $D = 170$ kips, $L = 130$ kips and an interior 24 × 24 in. column with $D = 250$ kips, $L = 200$ kips are to be supported on a combined rectangular footings whose outer end cannot protrude beyond the outer face of the exterior column. The distance center to center of columns is 18ft. and the allowable bearing pressure of the soil is 6000psf. The bottom of the footing is 6ft below grade and a surcharge of 100 psf is specified on the surface. Design the footing for $f_c' = 3000$ psi and $f_y = 60{,}000$ psi.



Sol^n: The space between the bottom of the footing and the surface will be occupied partly by concrete (footing, concrete floor) and partly by backfill.

An average unit weight of $\left(\frac{150+100}{2}\right)$ or, 125 pcf can be assumed.

Now, Effective allowable bearing pressure at base of footing

$$q_e = q_u - \gamma h - \text{surcharge load}$$

$$= 6000 - (6 \times 125) - 100 \quad [100 \text{ psf for surcharge at surface}]$$

$$= 5150 \text{ psf}$$

Now, required area, $A_{req} = \dfrac{\text{Sum of Column loads}}{q_e}$

$$= \frac{(170+130+250+200)\text{ kips}}{5.150 \text{ ksf}}$$

$$= 145.63 \text{ ft}^2$$

The resultant of the column loads is located from the center of the exterior column a distance,

$$x = \frac{\text{Load on Interior Column} \times \text{c/c distance between columns}}{\text{Total Load on Columns}}$$

$$= \frac{(250+200) \times 18}{250+200+170+130} = 10.8'$$

in direction of Length

Hence, the length of footing, $l = 2 \times \left(x + \frac{\text{Column Dimension}}{2}\right)$

$$= 2 \times \left(10.8 + \frac{18}{2 \times 12}\right)$$

$= 23.1'$ [Can't be rounded, because of property line restrictions]

The required width, $b = \frac{\text{Area}}{l} = \frac{145.63}{23.1} = 6.30'$

$\approx 6.5'$ or $6'\,6''$

Strength design in Longitudinal Direction:

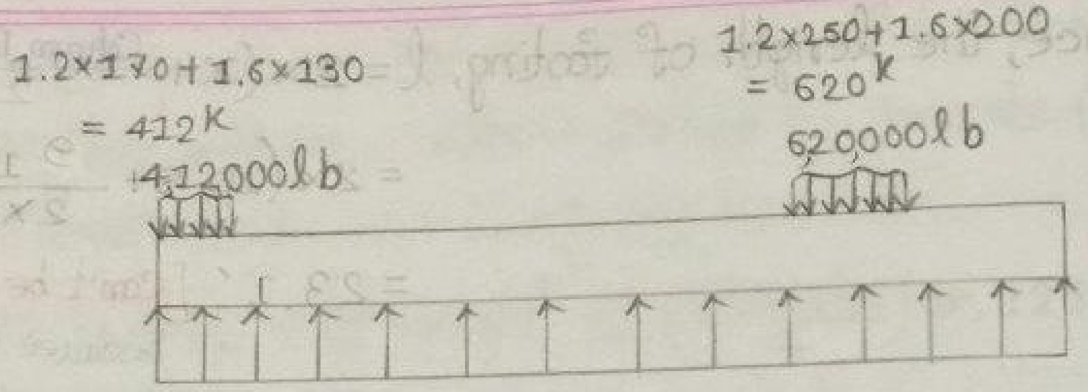
The net upward pressure caused by the factored column loads is,

$$q_u = \frac{1.2DL + 1.6LL}{A_{provided}}$$

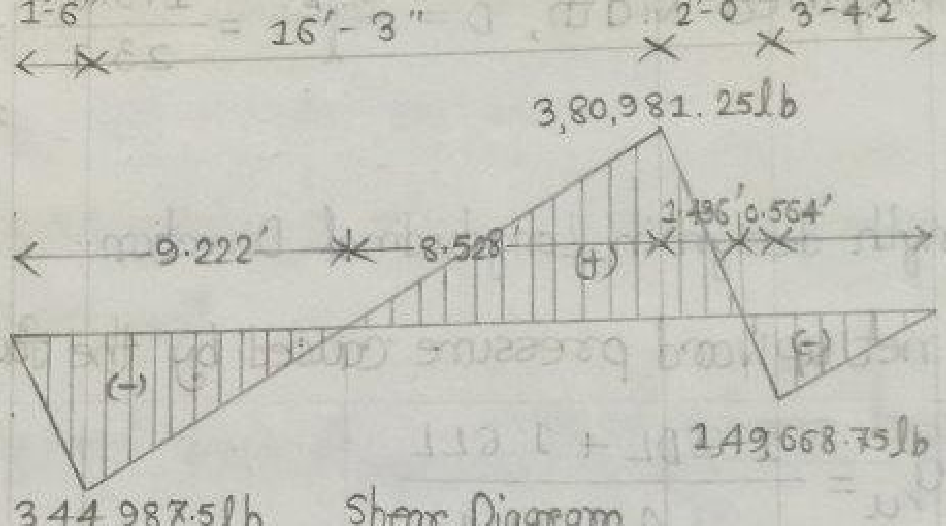
$$= \frac{1.2 \times (170+250) + 1.6 \times (130+200)}{23.1 \times 6.5}$$

$$= 6.873 \text{ kips/ft}^2$$

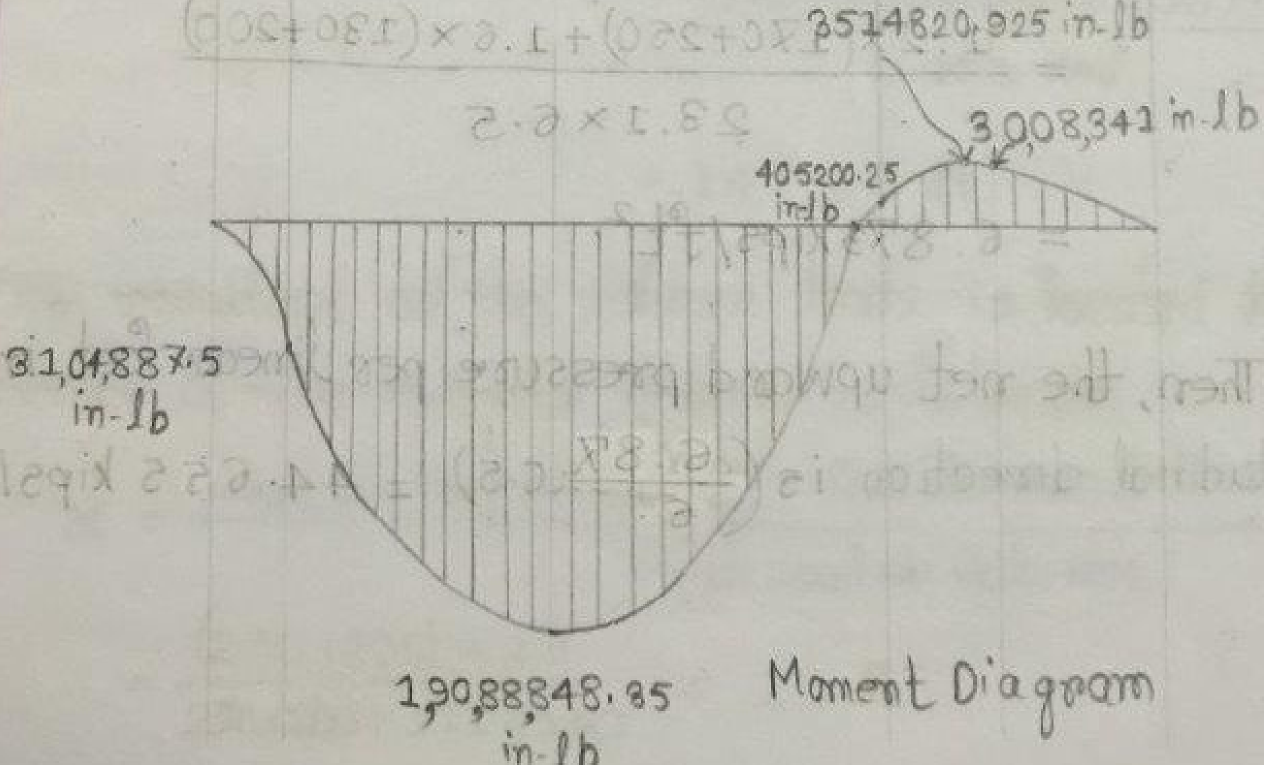
Then, the net upward pressure per linear foot in the longitudinal direction is $(6.873 \times 6.5) = 44.675$ kips/ft



$$\frac{1032,000}{23.1} = 44,675 \text{ lb/ft}$$



Shear Diagram



Moment Diagram

The maximum negative moment between the columns occurs at the section of zero shear.

Let, x be the distance from the outer edge of the exterior column to this section. Then,

$$V_{u-x} = 44,675x - 4,12,000$$

$$\Rightarrow 0 = 44675x - 412000$$

$$\therefore x = 9.2222'$$

The moment at this section is,

$$M_{u-x} = \left[44675 \times \frac{(9.2222)^2}{2} - 412000\left(9.2222 - \frac{1.5}{2}\right)\right] \times 12$$

$$= -19089179.63 \text{ in-lb}$$

The moment at the right edge of the interior column is,

$$M_u = \omega_u \frac{l^2}{2}$$

$$= \left[44675 \times \frac{(3.35)^2}{2}\right] \times 12$$

$$= 3008191.125 \text{ in-lb}$$

Trial value of $d = \left[(\text{Length} + \text{Width}) \text{ of footing in inches}\right] \times 1.25$

$$= (6.5 + 23.1)'' \times 1.25$$

$$= 29.6'' \times 1.25$$

$$\cong 30'' \times 1.25$$

$$= 37.5''$$

From the shear diagram, it is seen that the critical section for flexural shear occurs at a distance d to the left of the left face of the interior column (max^{m} shear at a distance 'd' from either columns).

At that point, the factored shear is,

$$V_u = 380981.25 - \frac{37.5''}{12} \times 44675$$

$$= 241371.875 \text{ lb}$$

& the design shear strength,

$$\phi V_c = \phi 2 \lambda \sqrt{f_c'}\, b_w d$$

$$= 0.75 \times 2 \times 1 \times \sqrt{3000} \times (6.5 \times 12) \times 37.5$$

$$= 240313.2721 \text{ lb}$$

Here, $\phi V_c > V_u$, indicating that $d = 37.5''$ is adequate.

Additionally, as in single footings, punching shear should be checked on a perimeter section a distance $\frac{d}{2}$ around the column, on which the;

Nominal shear stress, $v_c = 4 \lambda \sqrt{f_c'}$

$$= 4 \times 1 \times \sqrt{3000}$$

$$= 219.09 \text{ psi}$$

Of the two columns, the exterior one with a three-sided perimeter a distance $\frac{d}{2}$ from the column is more critical in regard to this punching shear. The perimeter is,

$$b_o = 2\times\left[18'' + \frac{37.5''}{2}\right] + \left[24'' + 37.5''\right] = 2\times 36.75 + 61.5$$

$$= 135'' \text{ or, } 11.25'$$

and, the shear force being the column load minus the soil pressure within the perimeter is,

$$V_u = 412000 - \left(\frac{36.75\times 61.5}{12\times 12}\right)\times 6873\text{ psf}$$

$$= 304126.1172 \text{ lb}$$

On the other hand, the design shear strength on the perimeter section is,

$$\phi V_c = \phi v_c b_o d$$

$$= 0.75\times 219.09\times(11.25\times 12)\times 37.5$$

$$= 831857.3438 \text{ lb}$$

considerably larger than the factored shear V_u.

With $d = 37.5''$ and with $3.5''$ cover from the center of the bars to the top surface of the footing, the total thickness is, $t = (37.5 + 3.5) = 41''$

Let, $a = 1''$ & $b = 6.5'$

Required Steel area, $A_{S\,(top)} = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{19089179.63 \text{ in-lb}}{0.9 \times 60 \times (37.5 - \frac{1}{2}) \times 1000}$

$= 9.5541 \text{ in}^2$

$= \frac{9.5541 \text{ in}^2}{6.5 \text{ ft}}$

$= 1.4699 \text{ in}^2/\text{ft}$

Provide #9 bars with spacing $= \frac{12 \times 1.0}{1.4699} = 8.16''$ c/c

$\approx 8.0''$ c/c

Use #9 bars @ 8" c/c [Or, 10 - #9 bars] as Top bars.

For the portion of the longitudinal beam that cantilevers beyond the interior column, $M_u = 3008191.125$ in-lb [At right face]

Let, $a = 1''$ & $b = 6.5'$

Required Steel Area, $A_S = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{3008191.125}{0.9 \times 60000 \times (37.5 - \frac{1}{2})}$

$= 1.5056 \text{ in}^2$

Corresponding, $a = \frac{A_S f_y}{0.85 f_c' b} = \frac{1.5056 \times 60}{0.85 \times 3 \times (6.5 \times 12)} = 0.4542''$ close to assumption

So, $A_{S(required)\,(bottom)} = \frac{1.5056 \text{ in}^2}{6.5 \text{ ft}} = 0.2316 \text{ in}^2/\text{ft}$

Now, $A_{s(min)} = A_{s(temp\ \&\ shrinkage)} = 0.0018\,bt$

$$= 0.0018 \times 12 \times 41$$

$$= 0.8856\ in^2/ft$$

$A_{s(bottom)} < A_{s(min)}$

So, provide minimum reinforcement at the bottom of column.

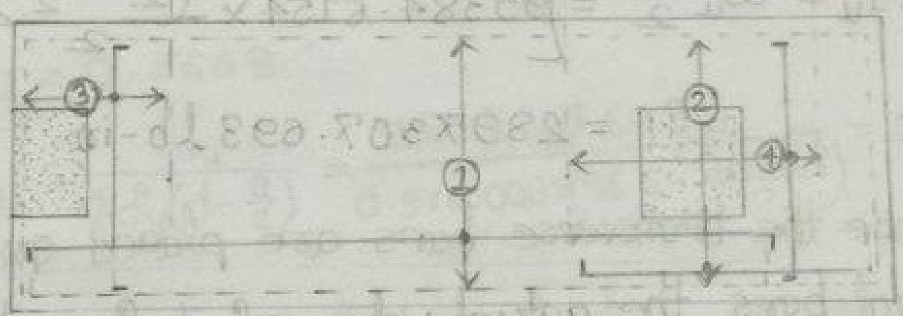
Provide, #7 bars with spacing $= \dfrac{0.6 \times 12}{0.8856} = 8.13''\ c/c$

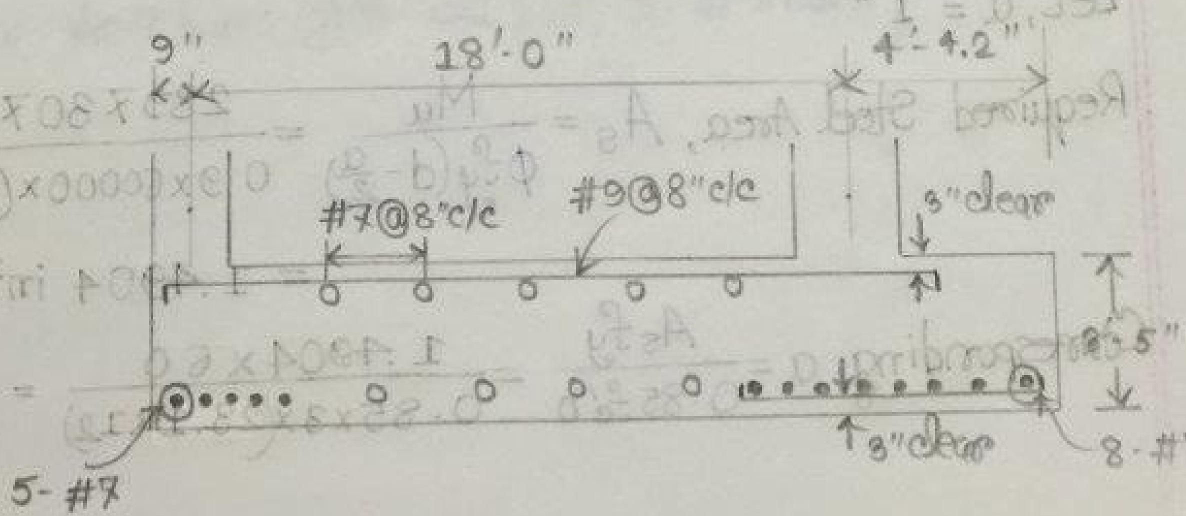
$$\approx 8.0''\ c/c$$

So, use #7 bars @ 8" c/c [Or, 10- #7 bars] as Bottom bars

① #9 @ 8" c/c ③ #7 @ 8" c/c

② #7 @ 8" c/c ④ #7 @ 8" c/c





Design of transverse beam under interior column:

The effective width of the transverse beam is assumed to be equal to the width of the column plus $\frac{d}{2}$ on either side of the column,

So, Width $= 24 + 2\left(\frac{d}{2}\right) = 24 + 2\times\left(\frac{37.5}{2}\right) = 61.5''$ or, $5.125'$

The net upward load per linear foot of the transverse beam is $\frac{620000\,lb}{6.5\,ft} = 95384.6154\ lb/ft$.

The moment at the edge of the interior column is, from Transverse direction

$$M_u = \omega_u \frac{l^2}{2} = \left[95384.6154 \times \frac{\left(\frac{6.5'}{2} - \frac{2'}{2}\right)^2}{2}\right] \times 12$$

$$= 2897307.693\ lb\text{-}in$$

Since the transverse bars are placed on top of the longitudinal bars, the actual value of d furnished is $= 37.5 - 1.0$
$= 36.5''$

Let, $a = 1''$

Required Steel Area, $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{2897307.693}{0.9\times 60000\times(36.5-\frac{1}{2})}$

$= 1.4904\ in^2$

Corresponding $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.4904\times 60}{0.85\times 3\times(5.125\times 12)} = 0.5702''$

Revised $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{2897307.693}{0.9 \times 60000 \times (36.5 - \frac{0.5702}{2})} = 1.4815 \text{ in}^2$

& $a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1.4815 \times 60}{0.85 \times 3 \times (5.125 \times 12)} = 0.5668''$ close to previous assumption

So, $A_{s(required)} = \frac{1.4815 \text{ in}^2}{5.125 \text{ ft}} = 0.2891 \text{ in}^2/\text{ft} < A_{s(min)}$

Provide, minimum reinforcement in transverse beam.

Use #7 bars @ 8" c/c [or, 8- #7 bars]

For transverse beam under exterior column,

Width $= \left(18'' + \frac{37.5}{2}\right) = 36.75''$

Using, $a = 0.5668''$

$A_{s(required)} = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{2897307.693}{0.9 \times 60000 \times (36.5 - \frac{0.5668}{2})} = 1.4815 \text{ in}^2/\text{ft}$

$< A_{s(min)}$

Provide, minimum reinforcement, $A_{s(min)} = 0.8856 \text{ in}^2/\text{ft}$

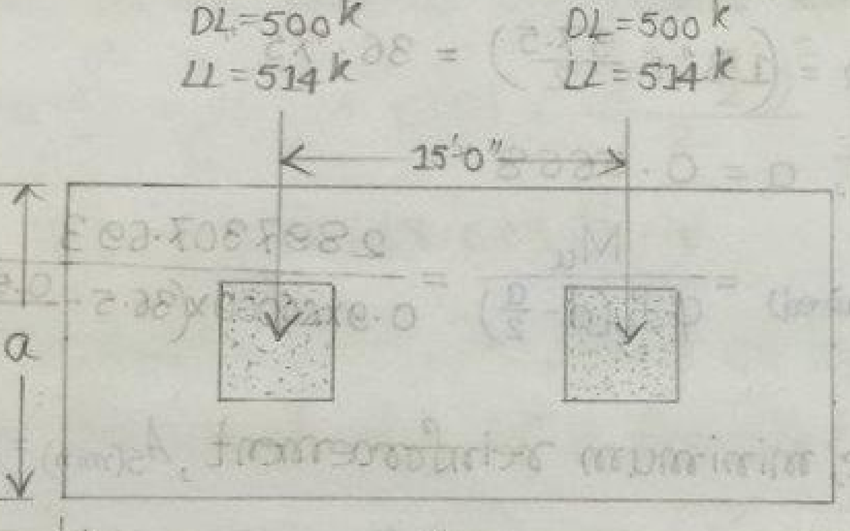
Use, #7 bars @ 8" c/c [or, 5- #7 bars]

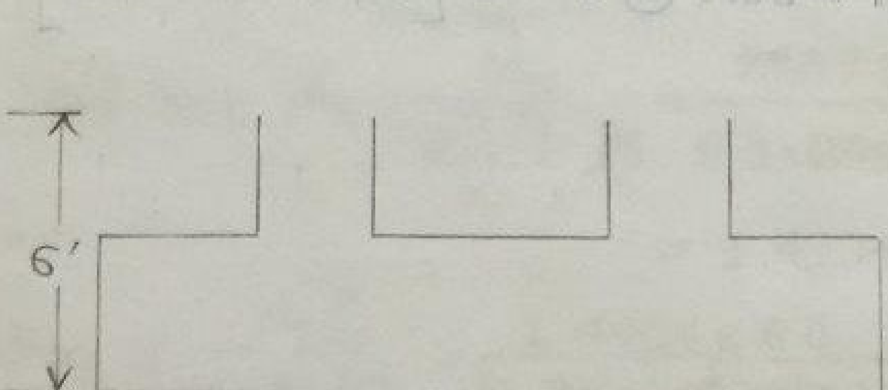
2012-2013

7(b)

Problem: Two interior columns of a high-rise building are 15 ft apart and each carries $DL = 500^k$ and $LL = 514^k$. The columns are $22'' \times 22''$ in cross-section. The columns will be supported on a rectangular combined footing with a long-side dimension twice that of the short side. The allowable soil bearing pressure is 8000 psf. The bottom of the footing will be 6 ft below grade. Design the footing and show the reinforcement with neat sketches. $f_c' = 3$ ksi and $f_y = 60$ ksi.

Sol^n:





Effective allowable bearing pressure at the base of footing,

$$q_e = q_a - \gamma h = [8000 - 125\times 6] = 7250 \text{ psf}$$

Required footing area, $A_{req} = \dfrac{DL+LL}{q_e} = \dfrac{2\times(500+514)\text{ kip}}{7.250 \text{ ksf}}$

$= 279.72 \text{ ft}^2$

$\approx 280 \text{ ft}^2$

Now, $2a \times a = A_{req}$

$\Rightarrow 2a^2 = 280$

$\therefore a = 11.83' \approx 12'$

So, Length = $12'$

& Width = $2a = 2\times 12' = 24'$

The net upward pressure caused by the factored column loads,

$$q_u = \frac{1.2DL + 1.6LL}{A_{req}} = \frac{(1.2\times 500 + 1.6\times 514)\times 2}{12\times 24}$$

$= 9.8778 \text{ ksf}$

Factored load on each column, $P_u = 1.2\,DL + 1.6\,LL$

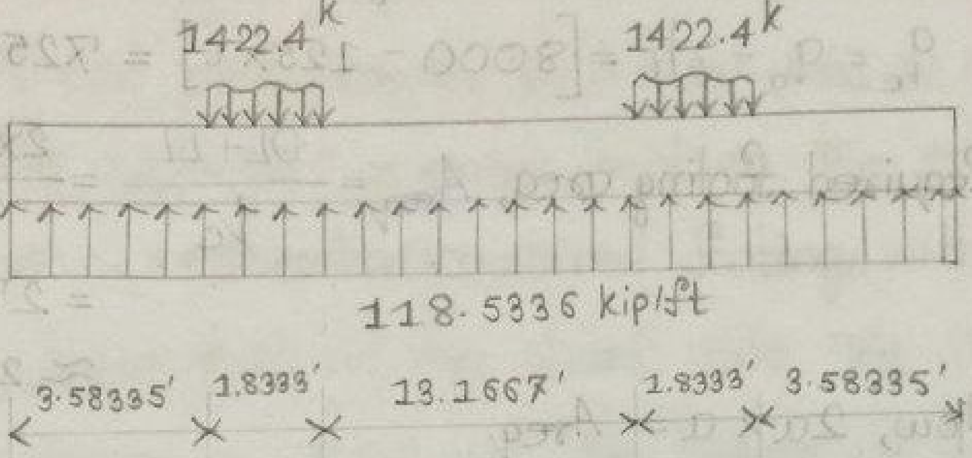
$= (1.2\times 500 + 1.6\times 514)$

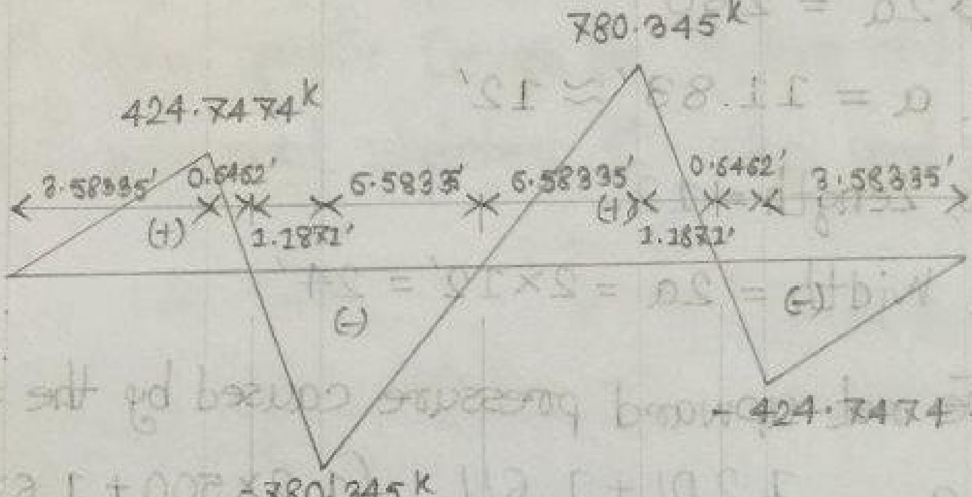
$= 1422.4 \text{ kip}$

Trial value of d = (Length + Width) of footing, inches

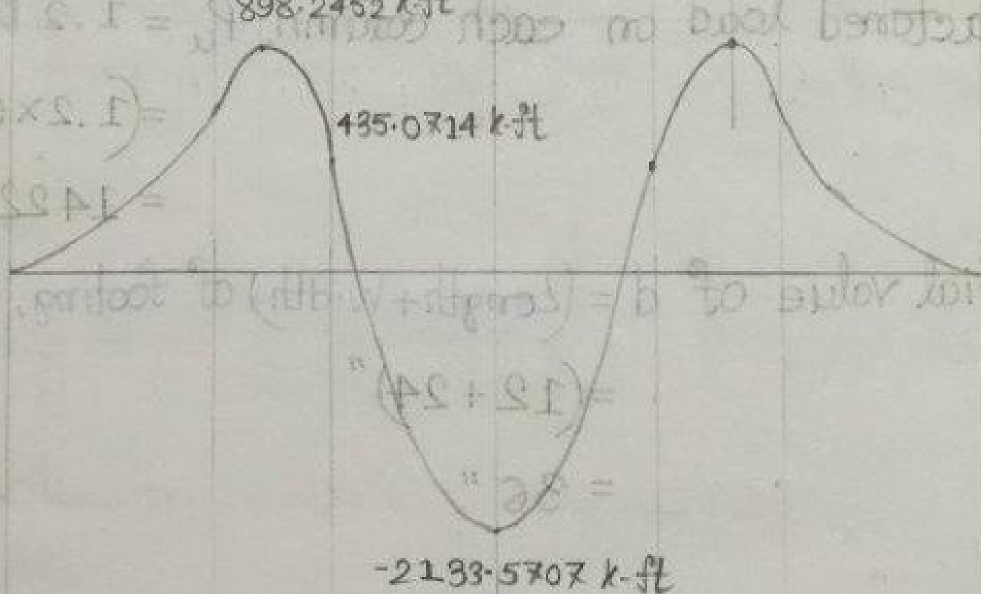
$= (12+24)''$

$= 36''$





Shear Diagram



Moment Diagram

Check for Punching Shear: At a section $\frac{d}{2}$ from column face,

Perimeter, $b_o = \left[22 + 2\times\frac{36}{2}\right]\times 4 = (58\times 4) = 232''$

Factored Shear, $V_u = 1422.4 - \frac{58\times 58}{12\times 12}\times 9.8778$

$= 1191.6436\, kip \approx 1192\, kip$

Design Shear Strength, $\phi V_c = \phi 4\sqrt{f_c'}\, b_o d$

$= 0.75\times 4\times\sqrt{3000}\times 232\times\frac{36}{1000}$

$= 1372.3736\, kip$

As, $\phi V_c > V_u$, indicating that $d = 36''$ is adequate.

Check for Beam Shear: The net upward pressure per linear foot in the longitudinal direction is $(9.8778\times 12) = 118.5336$ kip/ft

Factored Shear at a distance d from right face of column,

$V_{u-d} = \left[-780.345 + 118.5336\times\frac{36''}{12}\right] = -424.7442$ kips

Design Shear Strength, $\phi V_c = \phi 2\lambda\sqrt{f_c'}\, b_w d$

$= 0.75\times 2\times\sqrt{3000}\times(12\times 12)\times\frac{36}{1000}$

$= 425.9091\, kip$

Here, $V_{u-d} < \phi V_c$ but very close. So, d needs to be reassumed.

Let us assume, $d = 38''$

$$\therefore \phi V_c = \frac{425.9091 \times 38}{36} = 449.5707 \text{ kips}$$

Factored shear at a distance $d = 38''$,

$$V_{u-d} = \left[-780.345 + 118.5336 \times \frac{38}{12}\right] = -404.9886 \text{ kip}$$

Here, $V_{u-d} < \phi V_c$ so, $d = 38''$ is adequate.

From Bending Moment Diagram,

Maximum Negative Moment, $-M_u = -2133.5707$ k-ft

$$= -\frac{2133.5707 \text{ k-ft}}{12 \text{ ft}}$$

$$= -177.7976 \text{ k-ft/ft}$$

Let, $a = 1''$ & $b = 12''$

Steel Area, $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{177.7976 \times 12}{0.9 \times 60 \times (38 - \frac{1}{2})} = 1.0536 \text{ in}^2/\text{ft}$

Corresponding, $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.0536 \times 60}{0.85 \times 3 \times 12} = 2.0659''$

Revised, $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{177.7976 \times 12}{0.9 \times 60 \times (38 - \frac{2.0659}{2})} = 1.0688 \text{ in}^2/\text{ft}$

Corresponding, $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.0688 \times 60}{0.85 \times 3 \times 12} = 2.0957''$ close to previous assumption

So, $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{177.7976 \times 12}{0.9 \times 60 \times (38 - \frac{2.0957}{2})} = 1.0692 \text{ in}^2/\text{ft}$

$A_{s(min)} = A_{s(temp\ \&\ shrinkage)} = 0.0018\,bt$

Providing 4" clear cover, total thickness, $t = (38+4) = 42"$

Now, $A_{s(min)} = (0.0018 \times 12 \times 42) = 0.9072\ in^2/ft$

Here, $A_{s(top)} > A_{s(min)}$

Provide #9 bars with spacing $= \frac{12 \times 1.0}{1.0692} = 11.22"\ c/c$

$\approx 11"\ c/c$

Use, #9 bars @ 11" c/c [Or, 14-#9 bars]

as Top rebars

Maximum Positive Moment, $+M_u = 898.2452$ kip-ft

$= \frac{898.2452\ k\text{-}ft}{12\ ft}$

$= 74.8538\ k\text{-}ft/ft$

Using, $a = 2.0957"$

Steel Area, $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{74.8538 \times 12}{0.9 \times 60 \times (38 - \frac{2.0957}{2})} = 0.4502\ in^2/ft$

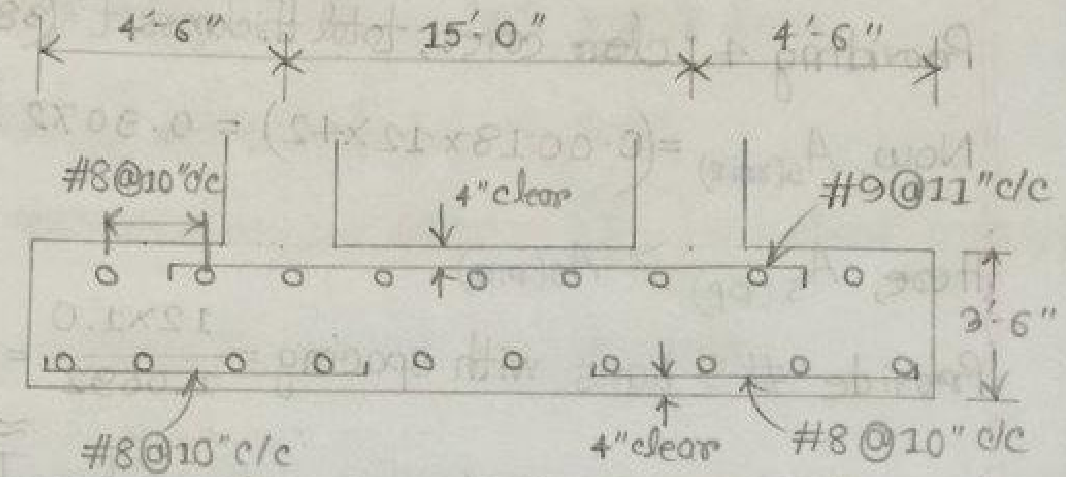
Here, $A_{s(bottom)} < A_{s(min)}$; provide minimum reinforcement.

$(0.9072\ in^2/ft)$

Provide #8 bars with spacing $= \frac{12 \times 0.79}{0.9072} = 10.45"\ c/c$

$\approx 10"\ c/c$

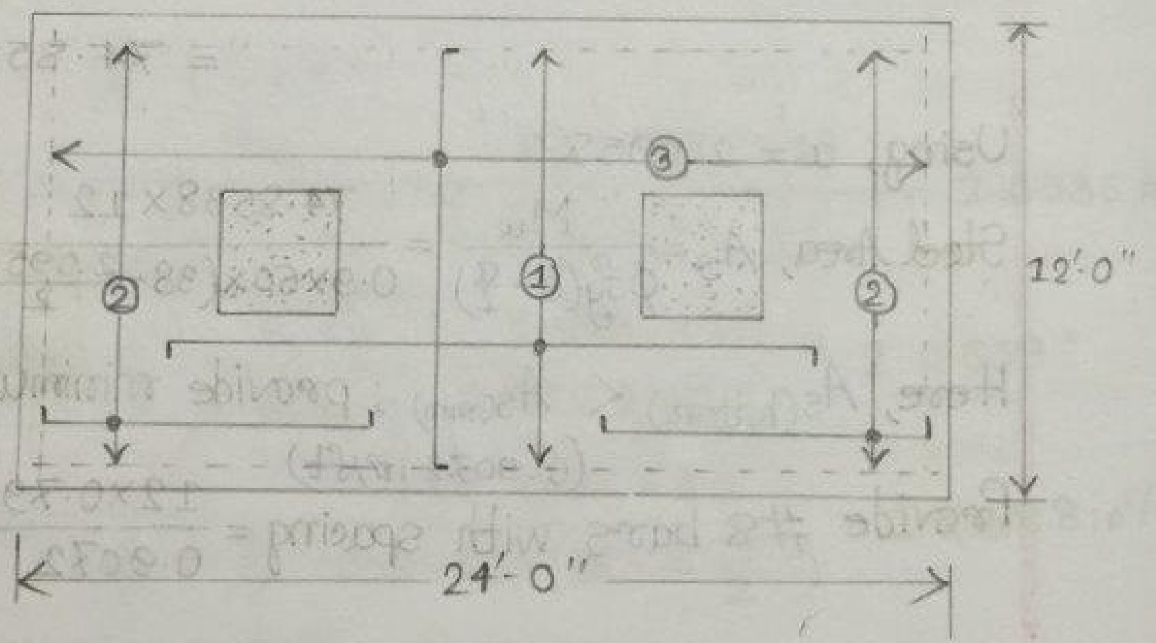
Use, #8 bars @ 10" c/c [Or, 15-#8 bars]



① #9@11"c/c

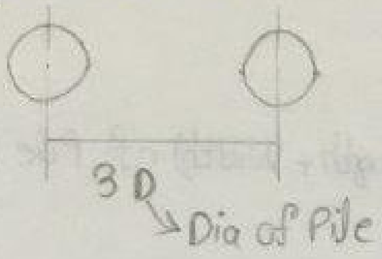
② #8@10"c/c

③ #8@10"c/c

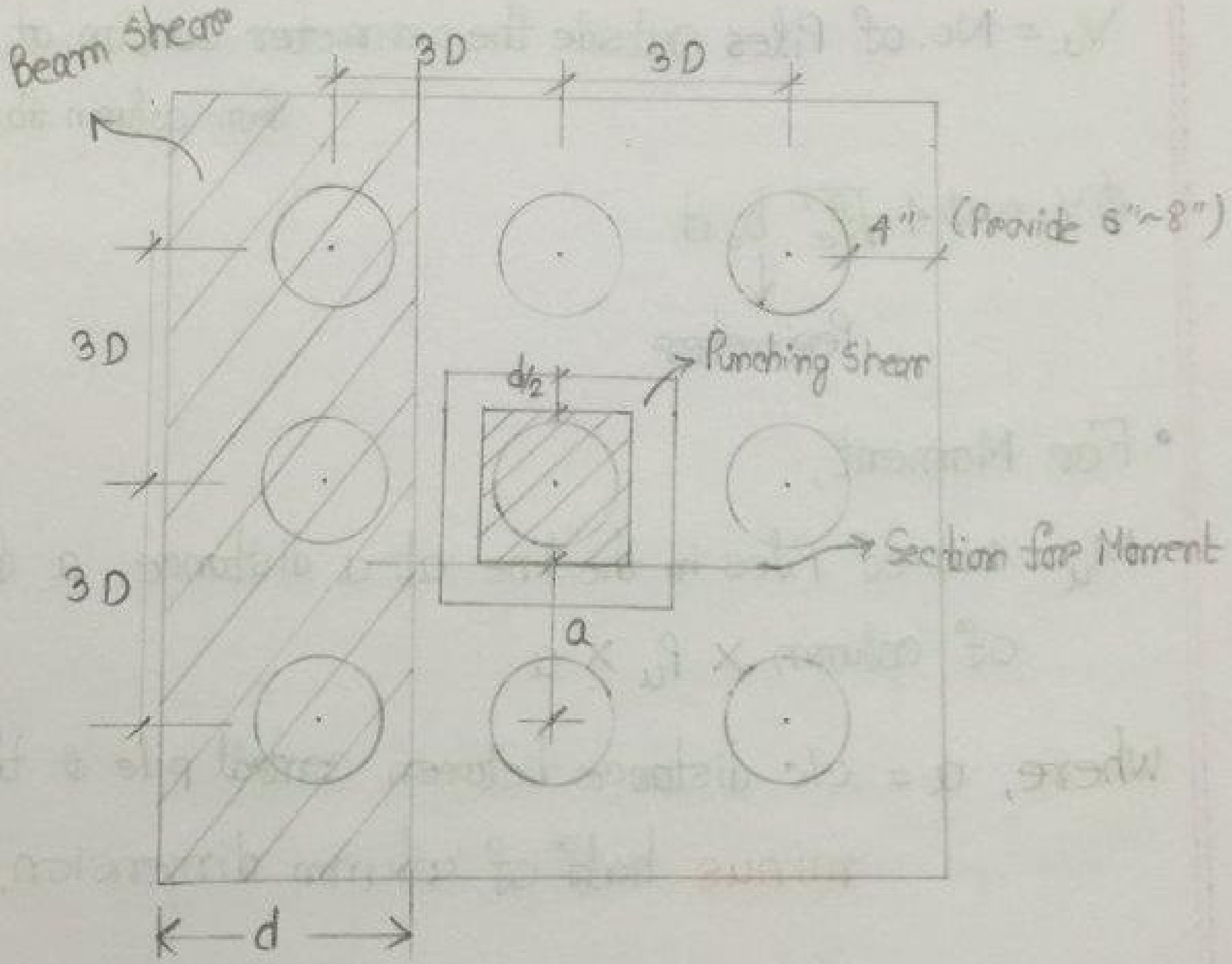


Pile Caps :

Arrangement of Piles :



min^m dia of Pile, $D = 18'', 20'', 24''$



- For Beam Shear, at a distance d from the edge of pile cap.

V_u = No. of Piles within section at distance d $\times P_u$

$\phi V_c = \phi 2 \sqrt{f_c'}\, b\, d$

b: Width of Pile Cap

d: 1.5x(Length + Width) of Pile Cap in Inches

- For Punching Shear, at a distance $\frac{d}{2}$ from column face.

V_u = No. of Piles Outside the perimeter section at $\frac{d}{2}$ from column face $\times P_u$

$\phi V_c = \phi 4 \sqrt{f_c'}\, b_o\, d$

b_o: Perimeter

- For Moment,

M_u = No. of Piles in the line at a distance a from the face of column $\times P_u \times a$

where, a = c/c distance between central pile & the closest pile minus half of column dimension.

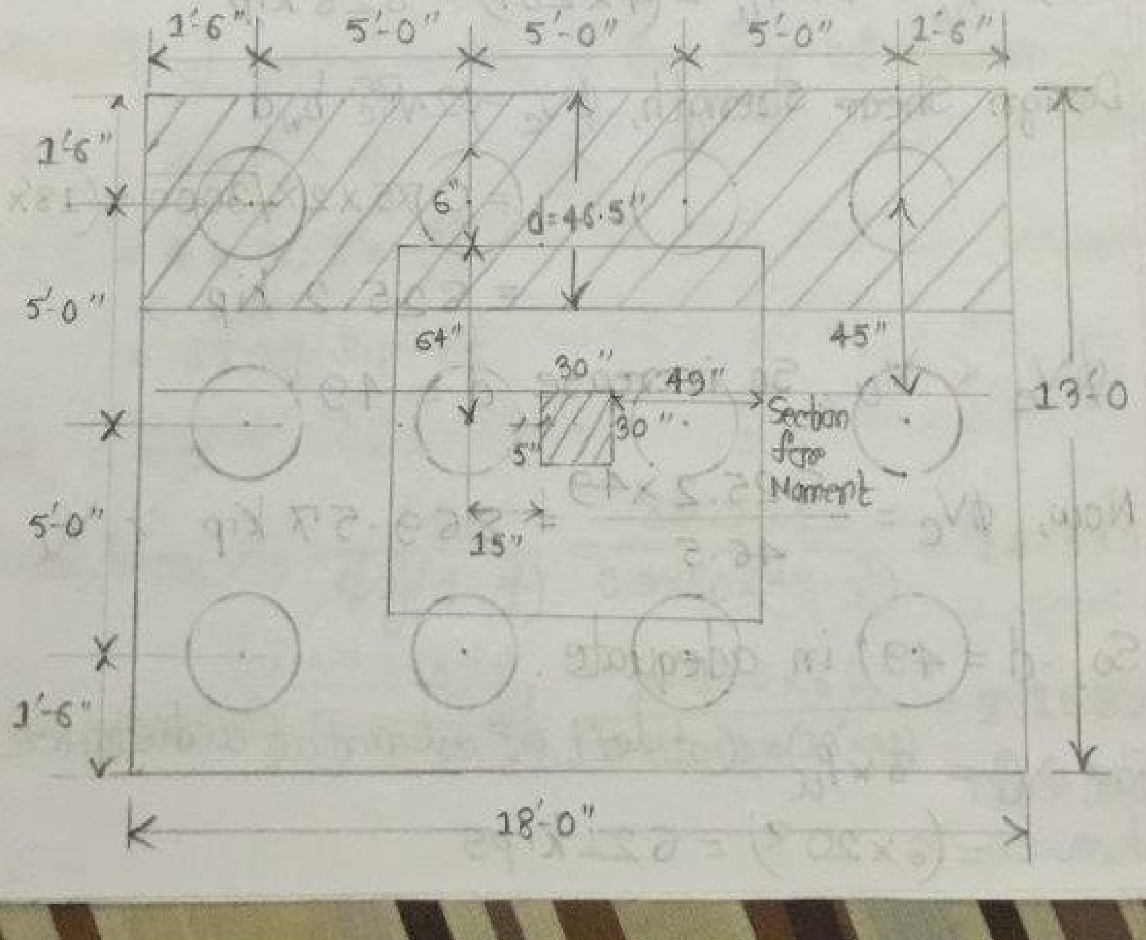
"2009-2010"

6(c): 20" dia cast-in-situ piles shall be provided for a RC column 30" × 30" in section carrying working loads DL = 1000 kip and LL = 800 kip. The allowable load carrying capacity of each pile is 150 kip. Pile spacing shall be 3 times the pile diameter i.e. = 5 ft. c/c each direction. Design the pile cap by USD. Make neat sketches (Plan and section). Given: $f_c' = 3$ ksi and $f_y = 60$ ksi.

Sol^n: Number of piles, $n = \dfrac{DL+LL}{\text{Load Carrying Capacity of Pile}}$

$= \dfrac{1000+800}{150}$

$= 12$



Effective depth for Pile Cap, $d = 1.5 \times (\text{Length} + \text{Width})$

$= 1.5 \times (18 + 13)$

$= 46.5''$

Factored Load per pile, $P_u = \dfrac{1.2\,DL + 1.6\,LL}{n}$

$= \dfrac{1.2 \times 1000 + 1.6 \times 800}{12}$

$= 206.6667$ kip

≈ 207 kip

Check for Beam Shear: At a distance d from the edge of pile cap, 4 pile caps are present. (at top of column)

So, $V_u = 4 \times P_u = (4 \times 207) = 828$ kip

Design Shear Strength, $\phi V_c = \phi 2\sqrt{f_c'}\, b_w d$

$= 0.75 \times 2 \times \sqrt{3000} \times (18' \times 12) \times \dfrac{46.5}{1000}$

$= 825.2$ kip

$\phi V_c < V_u$, So increase $d = 49''$

Now, $\phi V_c = \dfrac{825.2 \times 49}{46.5} = 869.57$ kip $> V_u$

So, $d = 49''$ in adequate.

(at left of column, at a distance d)

For, $V_u = 3 \times P_u$

$= (3 \times 207) = 621$ kips

Now, $\phi V_c = \phi 2\sqrt{f_c'} b_w d = 0.75 \times 2 \times \sqrt{3000} \times (13 \times 12) \times \frac{49}{1000}$

$= 628.02$ kip $> V_u$

Check for Punching Shear: At a distance $\frac{d}{2}$ from column face, the perimeter, $b_o = 4 \times (30'' + 2 \times \frac{49}{2})$

$= 4 \times 79$

$= 316''$

No. of piles active $= \left[6 + 4 \times \frac{6}{20} \right] = 7.2$

So, $V_u = 7.2 \times P_u = (7.2 \times 207) = 1490.4$ kip

Design shear strength, $\phi V_c = \phi 4 \sqrt{f_c'} b_o d$

$= 0.75 \times 4 \times \sqrt{3000} \times 316 \times \frac{49}{1000}$

$= 2544.28$ kip

Here, $\phi V_c > V_u$, so $d = 49''$ is adequate.

Moment at critical section at the face of column (top), for short direction

$M_u = 4 \times P_u \times a$

$= 4 \times 207 \times \frac{45''}{12}$

$= 3105$ kip-ft

Let, $a = 2''$,

Steel Area, $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{3105 \times 12}{0.9 \times 60 \times (49 - \frac{2}{2})} = 14.375 \text{ in}^2$

Corresponding $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{14.375 \times 60}{0.85 \times 3 \times (13' \times 12)} = 2.1682''$

close to previous assumption

Revised $A_s = \dfrac{M_u}{\phi f_y (d - \frac{a}{2})} = \dfrac{3105 \times 12}{0.9 \times 60 \times (49 - \frac{2.1682}{2})} = 14.4 \text{ in}^2$

or, $\dfrac{14.4 \text{ in}^2}{18 \text{ ft}}$

$= $ [illegible] in^2/ft

Now, $A_{s(min)} = 0.0018\, bt$

Providing 9" clear cover, total thickness $= (49" + 9")$

$= 58"$

So, $A_{s(min)} = (0.0018 \times 12 \times 58) = 1.2528 \text{ in}^2/\text{ft}$

$A_{s(req)} < A_{s(min)}$

So, provid minimum reinforcement, $A_{s(min)} = 1.2528 \text{ in}^2/\text{ft}$

Provide, #9 bars with spacing $= \dfrac{12 \times 1}{1.2528} = 9.58"$ c/c

$\approx 9.5"$ c/c

Use, #9 bars @ 9.5" c/c in short direction.

Moment at left face of column, for long direction,

$M_u = 3 \times P_u \times a_1 + 3 \times P_u \times a_2$

$= 3 \times 207 \times \dfrac{15"}{12} + 3 \times 207 \times \left[\dfrac{15"}{12} + 5'\right]$

$= 4657.5$ kip-ft

Using, $a = 2.1682"$,

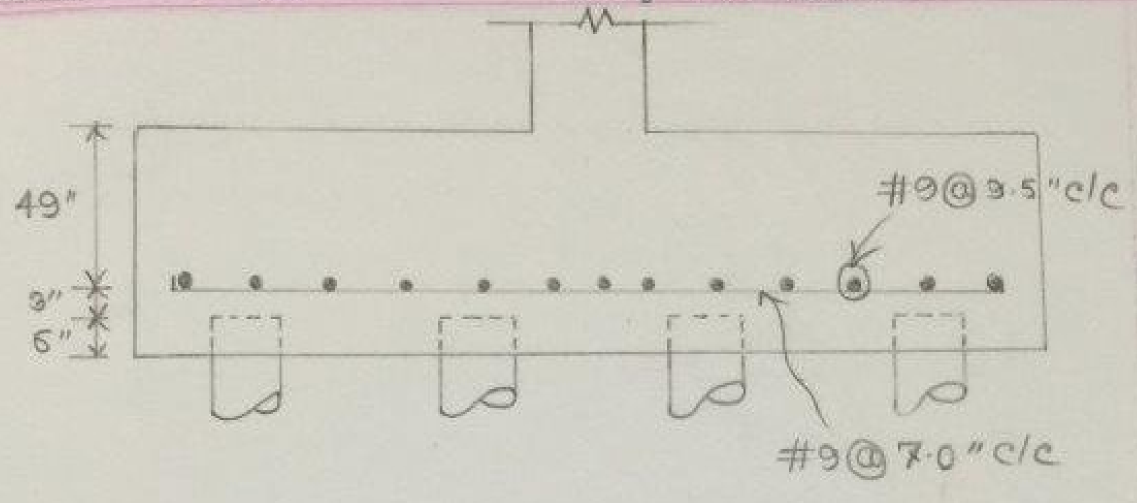
Steel Area, $A_s = \dfrac{M_u}{\phi f_y (d - \frac{a}{2})} = \dfrac{4657.5 \times 12}{0.9 \times 60 \times (49 - \frac{2.1682}{2})} = 21.6 \text{ in}^2$

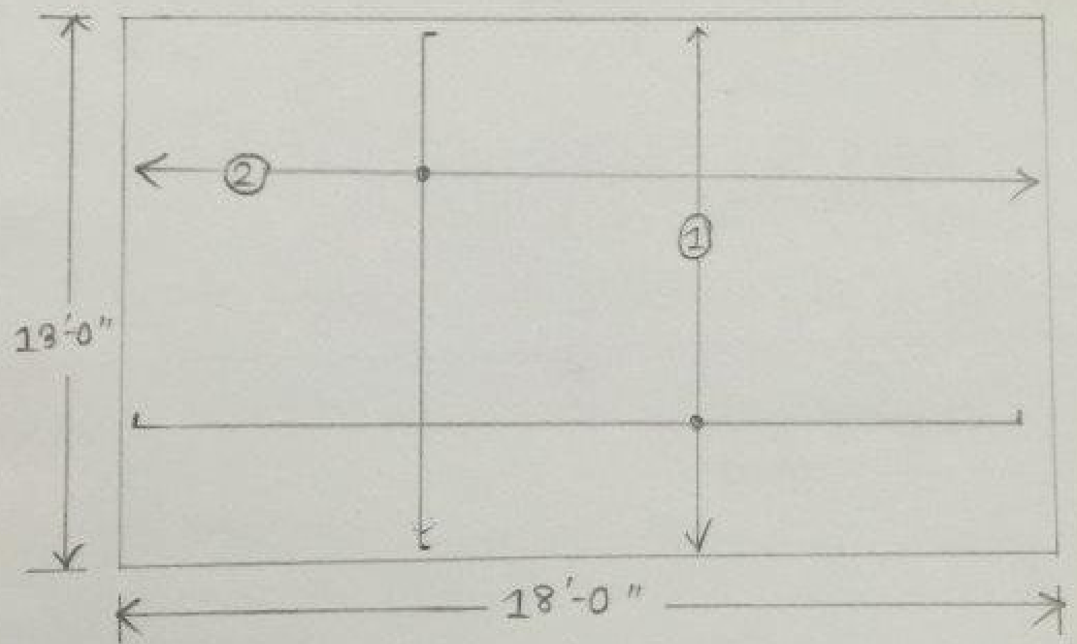
$= \dfrac{21.6 \text{ in}^2}{13 \text{ ft}}$

$= 1.6615 \text{ in}^2/\text{ft}$

Provide #9 bars with spacing $= \dfrac{12 \times 1.0}{1.6615} = 7.22"$ c/c $\approx 7"$ c/c

Use, #9 bars @ 7" c/c in long direction.





1 #9 @ 7" c/c

2 #9 @ 9.5" c/c