CE 317: Design of Concrete Structures II

3.00 credit, 3hrs/week

Dr. Tahsin Reza Hossain

Professor, Room No-536

Email: tahsin@ce.buet.ac.bd

Syllabus

New

- Design of column supported slabs
- Introduction to floor systems
- Design of columns under uniaxial and biaxial loading, Introduction to slender column
- Structural design of footings, pile caps
- Seismic detailing
- Shear wall
- Structural forms
- Introduction of prestressed concrete, Analysis and preliminary design of prestressed beam section

Class routine

- A section- Saturday (3)
- B section- Sunday (3)
- C section- Monday (3)

Books

- Design of Concrete Structures
  - Nilson, Darwin, Dolan 14th Ed
- Structural Concrete- Theory and Design
  - Hassoun, Al-Manaseer 4th Ed
- Reinforced Concrete- Mechanics & Design
  - Wight & McGregor 5th Ed

Many more……..

COLUMNS

Short columns
Chapter 8

Introduction: Axial Compression

- Columns are members that carry loads chiefly in compression
- Usually also carry bending moment. Tensile stresses may be produced over a part of the cross section
- Columns are generally referred to as compression member. Compression dominates behavior

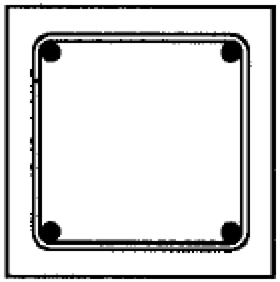
- Generally vertical members
- Also arches, truss members,
- Column= compression member

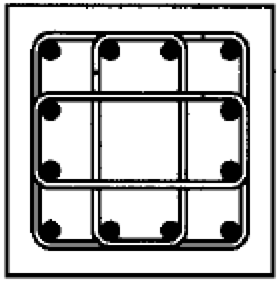
Three types of column

1. Members reinforced with longitudinal bars and lateral ties
2. Members reinforced with longitudinal bars and continuous spirals
3. Composite compression members

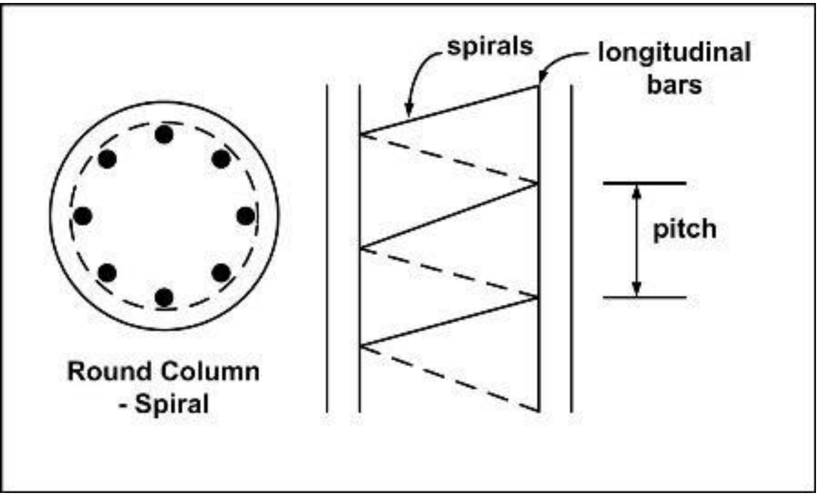
- 1 and 2 are common

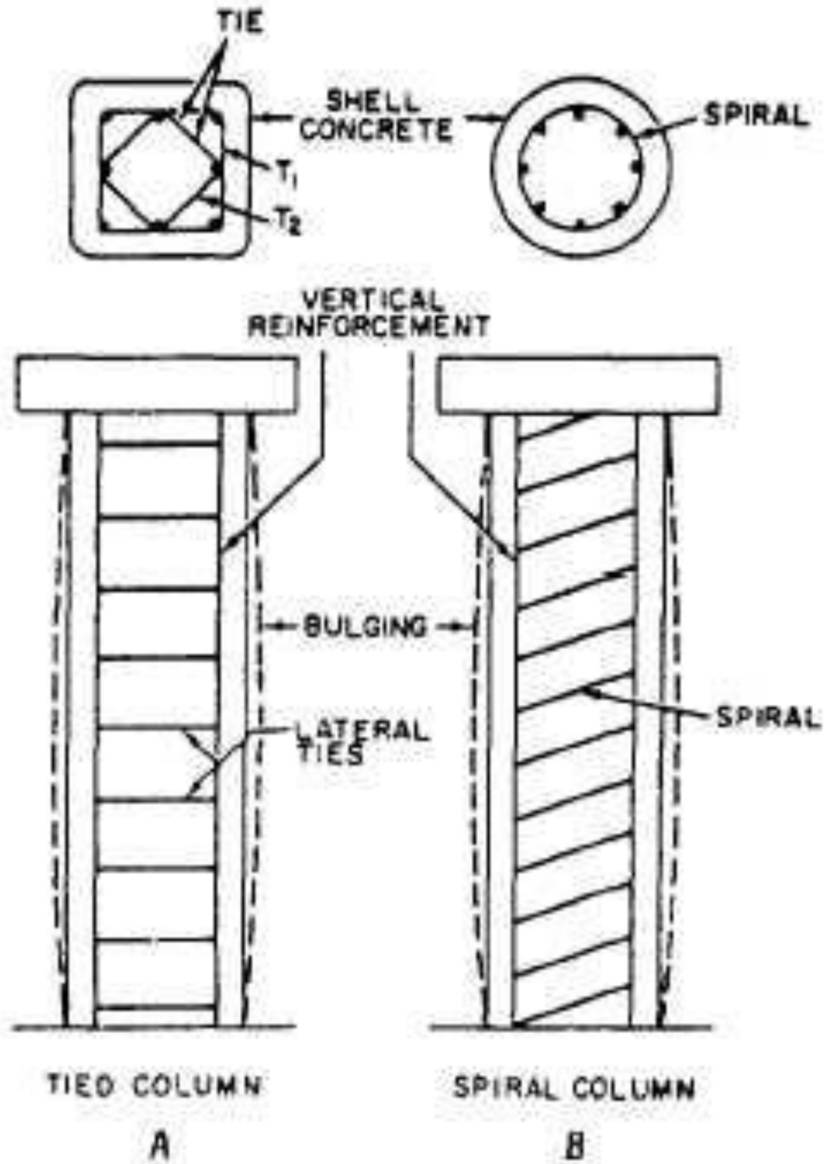
Types- 1. Tied Column





2. Spirally reinforced column




TIE
SHELL CONCRETE
SPIRAL
T_1
T_2
VERTICAL REINFORCEMENT
BULGING
LATERAL TIES
SPIRAL
TIED COLUMN
A
SPIRAL COLUMN
B

3. Composite columns

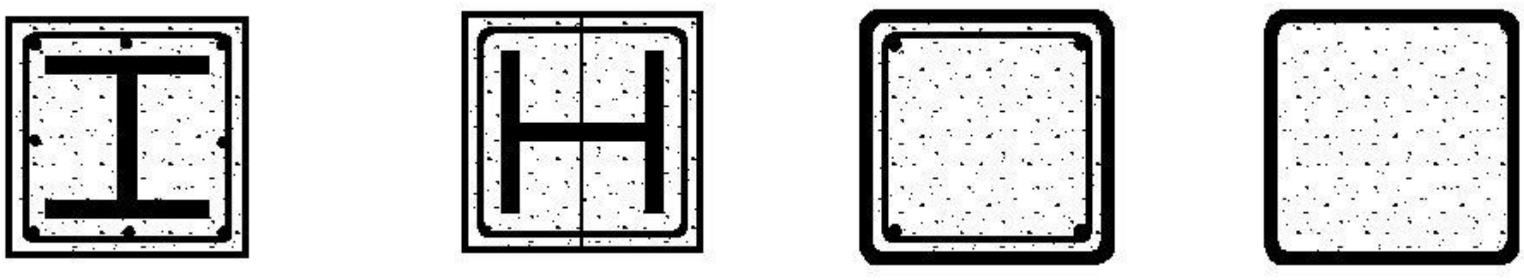
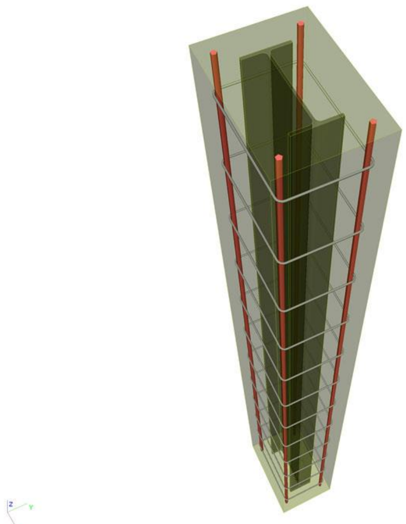


Figure 11 Typical cross-sections of composite columns



Main reinforcement

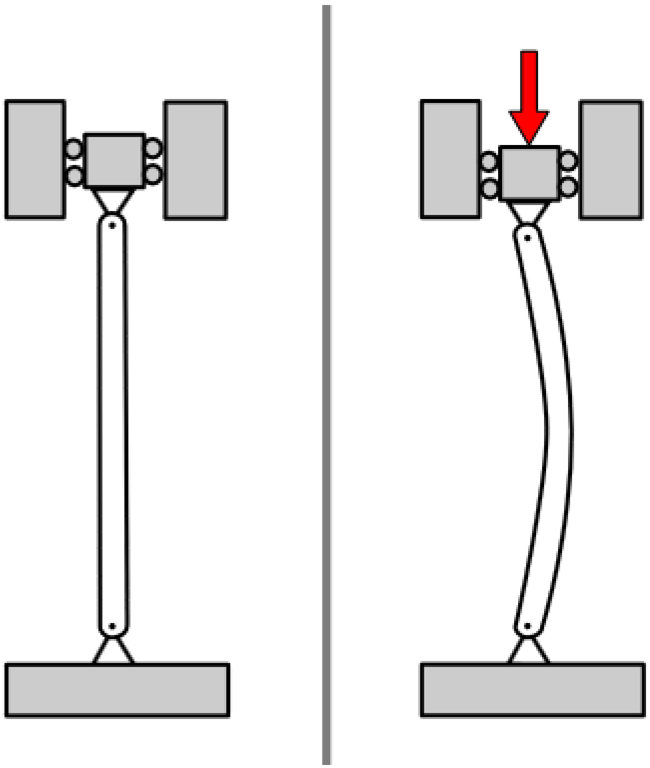
- Main reinforcement is longitudinal, parallel to load, square, rectangular or circular arrangement
- Reinf ratio- 0.01 to 0.08 (1 to 8 %)
- Lower limit to ensure resistance to bending not accounted for and reduce effects of creep and shrinkage
- Higher limit – not economical and difficult due to congestion
- Normally 2-3 %, preferably not exceed 4%.

Main reinforcement

- Higher size (No 5 and more) used
- Even No 14 and No 18, even bundled
- Minimum 4 longitudinal bars when rectangular or circular ties
- Minimum 6 bars when continuous spiral is used

Short column and Slender column

- Secondary effects - buckling



Reinforcements –usual sizes

- Slab- No 3, 4, 5 (10mm, 12mm, 16mm)
- Beam- No 5,6, 7, 8 (16 20 ~~22~~ 25mm)
- Stirrup/tie- No, 3 4 (10 12mm)
- Column –No 5, 6 7 8 9 10 11 14 18 (16 20 ~~22~~ 25 28 32 ….)
- Mat- No 4,5,6,8 (12 16 20 25 mm)
- Smaller sizes preferred as long as there is no congestion

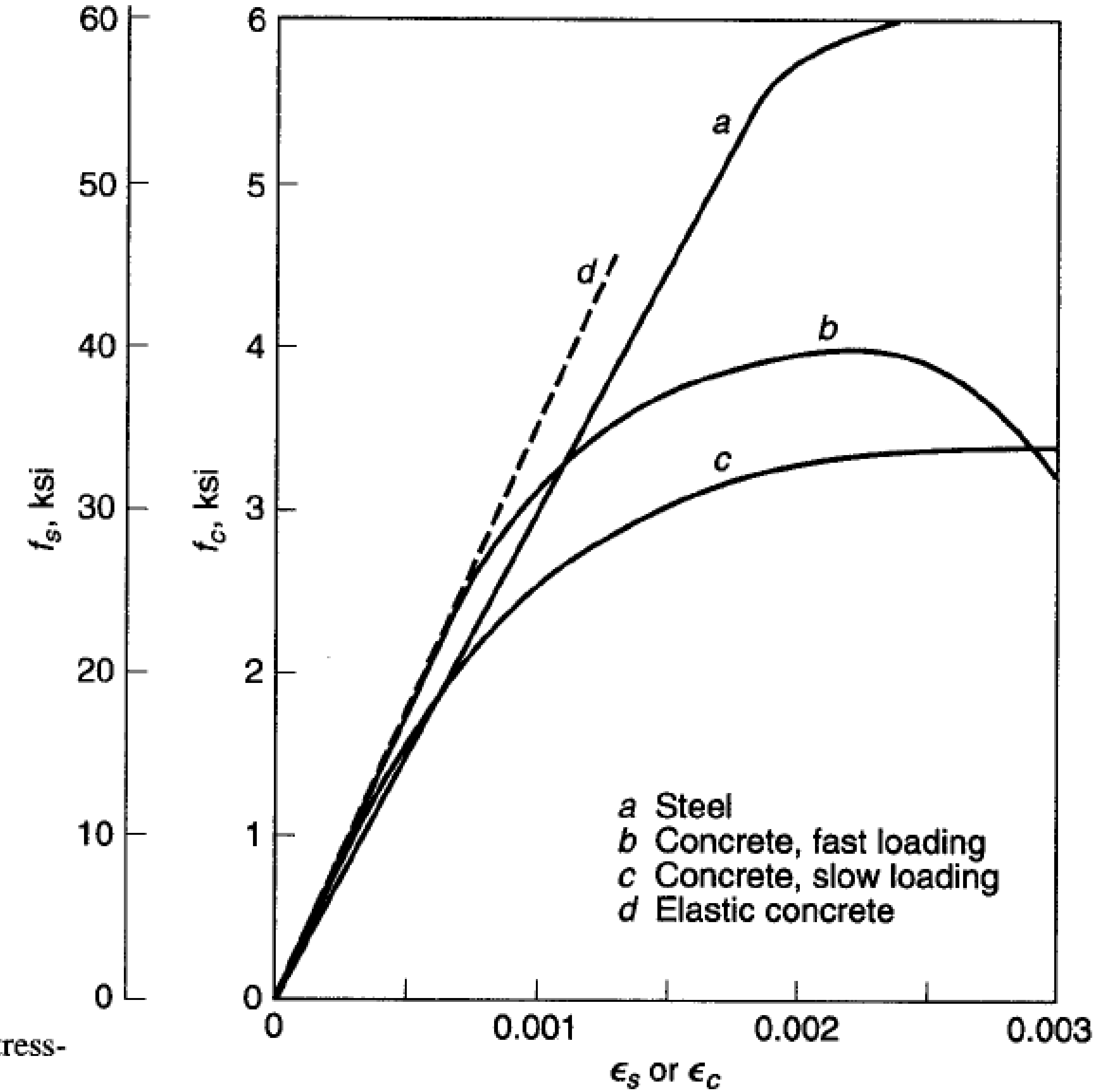


FIGURE 1.16
Concrete and steel stress-strain curves.

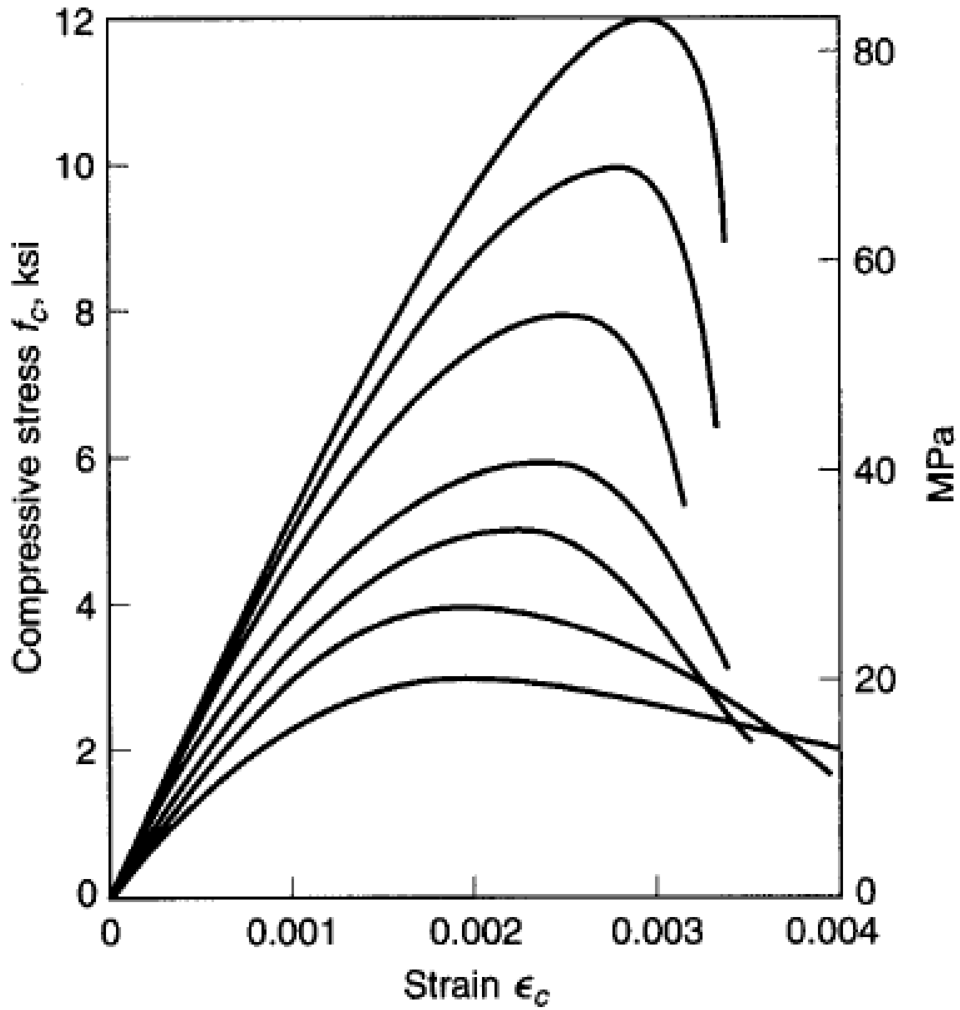


FIGURE 2.3
Typical compressive stress-strain curves for normal-density concrete with w_c = 145 pcf. *(Adapted from Refs. 2.23 and 2.24.)*

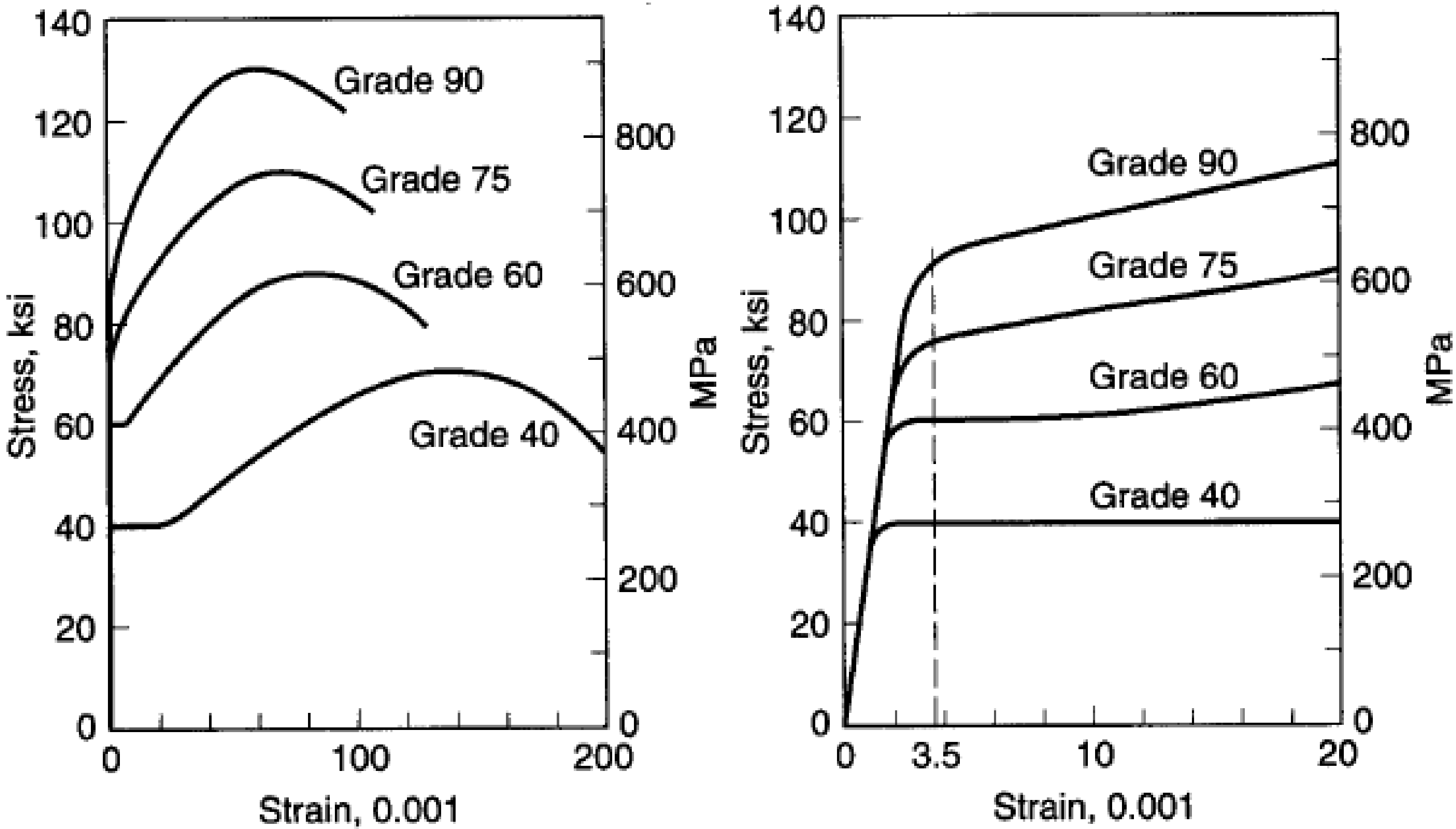
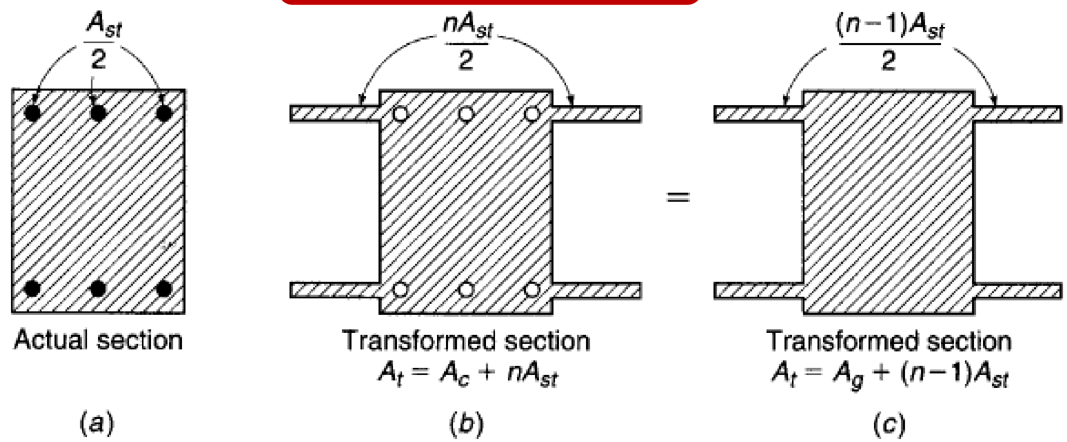


FIGURE 2.15
Typical stress-strain curves for reinforcing bars.

The behavior of short, axially loaded compression members was discussed in Section 1.9 in introducing the basic aspects of reinforced concrete. It is suggested that the earlier material be reviewed at this point. In Section 1.9, it was demonstrated that, for lower loads at which both materials remain elastic, the steel carries a relatively small portion of the total load. The steel stress f_s is equal to n times the concrete stress:

$$f_s = nf_c \tag{8.1}$$

Elastic range



where $n = E_s/E_c$ is the modular ratio. In this range the axial load P is given by

$$P = f_c[A_g + (n-1)A_{st}] \tag{8.2}$$

where A_g is the gross area of the cross section, A_{st} is the total area of the reinforcing steel, and the term in brackets is the area of the transformed section (see Fig. 8.2). Equations (8.2) and (8.1) can be used to find concrete and steel stresses, respectively, for given loads, provided both materials remain elastic. Example 1.1 demonstrated the use of these equations.

In Section 1.9, it was further shown that the nominal strength of an axially loaded column can be found, recognizing the nonlinear response of both materials, by

Nominal Strength

$$P_n = 0.85f'_c A_c + A_{st}f_y \tag{8.3a}$$

where A_c = net area of concrete, or

$$P_n = 0.85f'_c (A_g - A_{st}) + A_{st}f_y \tag{8.3b}$$

Design strength

According to ACI Code 10.3.6, the *design strength* of an axially loaded column is to be found based on Eq. (8.3*b*) with the introduction of certain strength reduction factors. The ACI factors are lower for columns than for beams, reflecting their greater importance in a structure. A beam failure would normally affect only a local region, whereas a column failure could result in the collapse of the entire structure. In addition, these factors reflect differences in the behavior of tied columns and spirally reinforced columns that will be discussed in Section 8.2. A basic ϕ factor of 0.75 is used for spirally reinforced columns and 0.65 for tied columns, vs. $\phi = 0.90$ for most beams.

A further limitation on column strength is imposed by ACI Code 10.3.6 to allow for accidental eccentricities of loading not considered in the analysis. This is done by imposing an upper limit on the axial load that is less than the calculated design strength. This upper limit is taken as 0.85 times the design strength for spirally

reinforced columns and 0.80 times the calculated strength for tied columns. Thus, according to ACI Code 10.3.6, for spirally reinforced columns

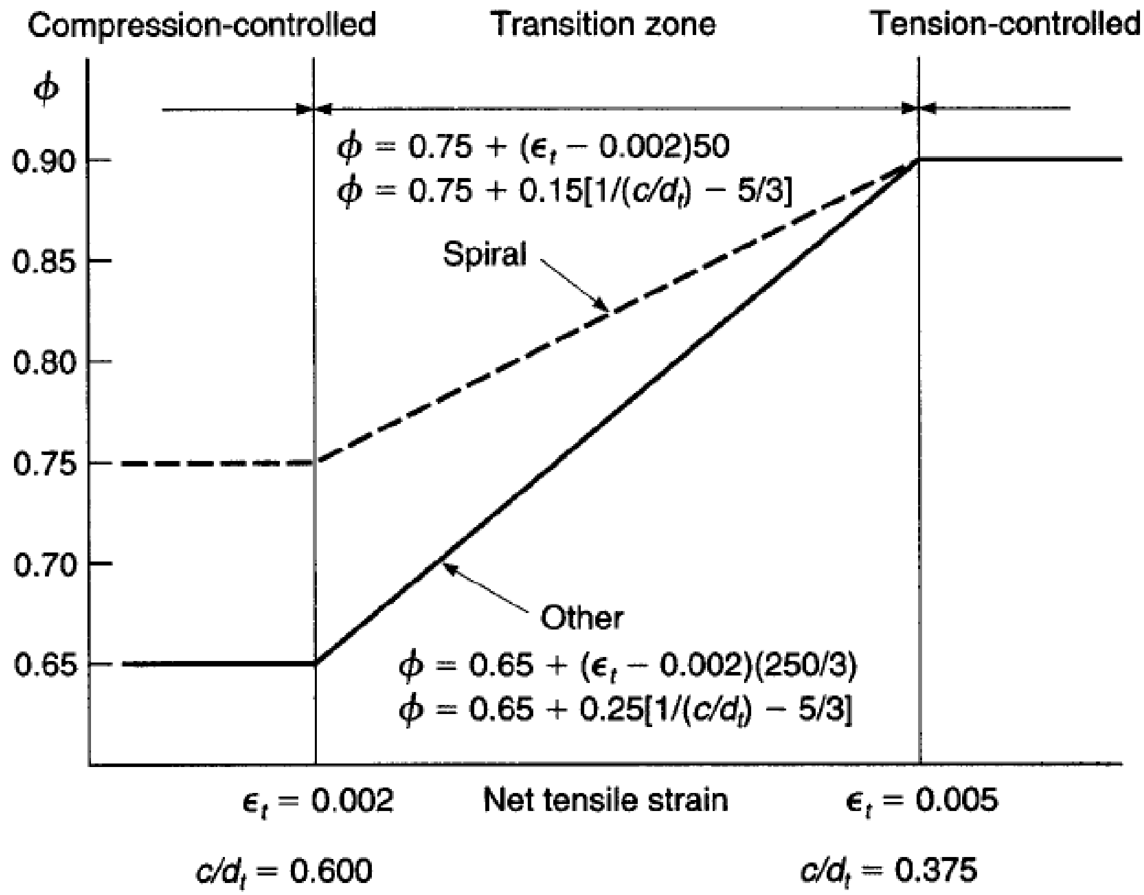
$$\phi P_{n(\max)} = 0.85\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}] \tag{8.4a}$$

with $\phi = 0.75$. For tied columns

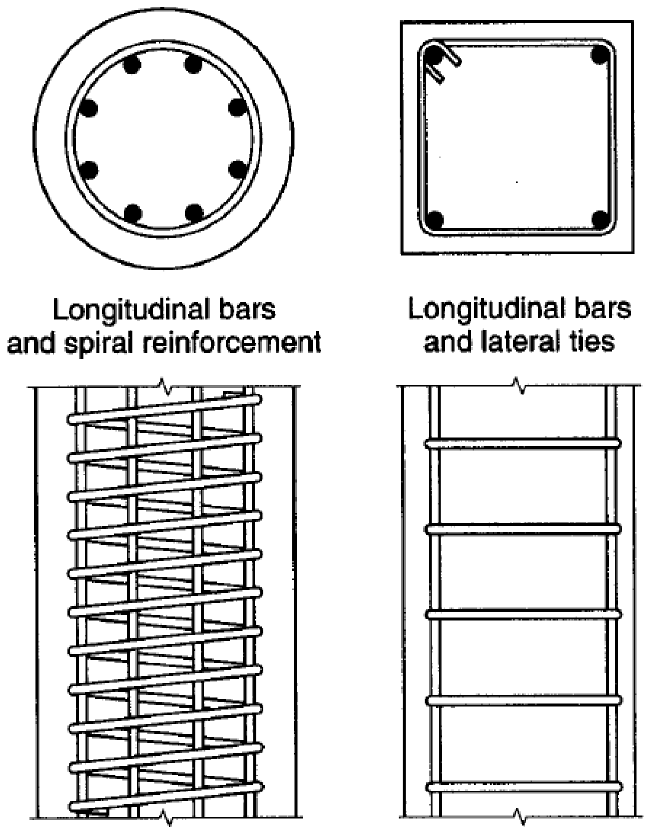
$$\phi P_{n(\max)} = 0.80\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}] \tag{8.4b}$$

with $\phi = 0.65$.

FIGURE 3.9
Variation of strength reduction factor with net tensile strain in the steel.



8.2 Lateral ties and spiral



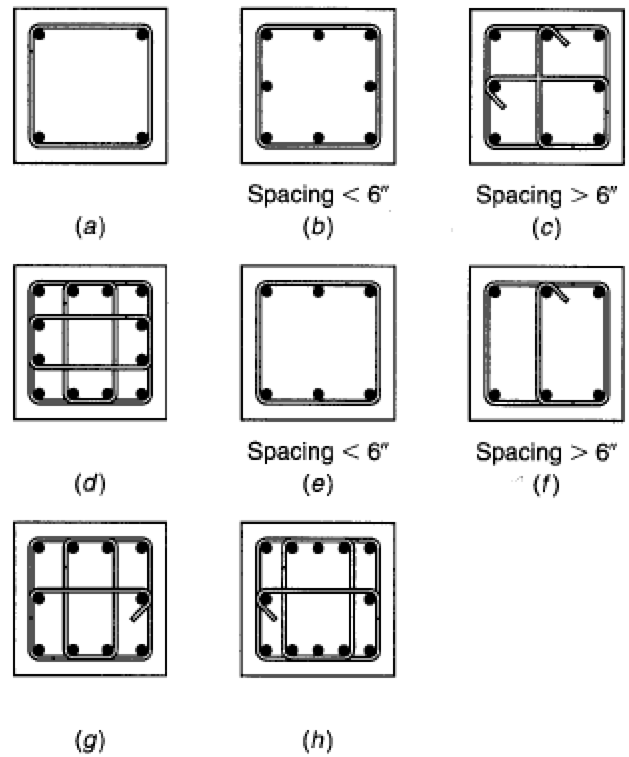


FIGURE 8.3
Tie arrangements for square and rectangular columns.

- Large P, small M – longitudinal bars uniform (a to d)
- Large M – bars at maximum distance from axis of bending
- Bundled bar- 2,3,4
- Bundled bars act as a unit
-

Purpose of Lateral ties and spiral

- Hold longitudinal bar in position while concrete is placed
- Prevent longitudinal bars from buckling

ACI provisions for ties

All bars of tied columns shall be enclosed by *lateral ties*, at least No. 3 (No. 10) in size for longitudinal bars up to No. 10 (No. 32), and at least No. 4 (No. 13) in size for Nos. 11, 14, and 18 (Nos. 36, 43, and 57) and bundled longitudinal bars. The spacing of the ties shall not exceed 16 diameters of longitudinal bars, 48 diameters of tie bars, nor the least dimension of the column. The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135°, and no bar shall be farther than 6 in. clear on either side from such a laterally supported bar. Deformed wire or welded wire fabric of equivalent area may be used instead of ties. Where the bars are located around the periphery of a circle, complete circular ties may be used.

ACI provisions for spirals

Spirals shall consist of a continuous bar or wire not less than $\frac{3}{8}$ in. in diameter, and the clear spacing between turns of the spiral must not exceed 3 in. nor be less than 1 in.

Problem 1

- Determine nominal and design axial compression capacity of a column 12″X12″reinforced with 4 No 9 bars. Also check the ties No. 3 @ 12in c/c. Given: f_c'= 4ksi and f_y=60 ksi.

Problem 2

- Design a tied column for
  - P_{DL}= 300 kip and P_{LL}= 200 kip
  - f_c'= 3 ksi and f_y=60 ksi

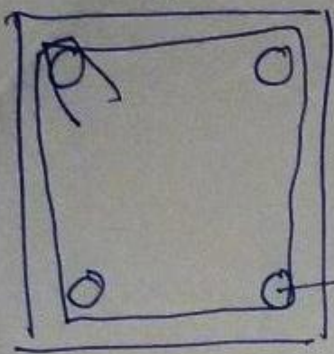
Problem 3

- Design a tied column with section 10″x10″ for
  - P_{DL}= 60 kip and P_{LL}= 30 kip
  - f_c'= 3 ksi and f_y=60 ksi

Review Problem

1.

12"



12"

4 #9

3 tie @ 12" c/c

$$P_n = 0.85 f_c' (A_g - A_{st}) + A_{st} f_y$$

$$= 0.85 * 4 (144 - 4) + 4 * 60$$

$$= 476 + 240 = \underline{716^k}$$

nominal strength ↑

design strength $= \alpha \phi P_n = 0.80 * 0.65 P_n$

$$= \underline{\underline{372.3^k}}$$

<u>Tie spacing</u>

16 bar dia $= 16 * \frac{9}{8} = 18"$

48 tie dia $= 48 * \frac{3}{8} = 18"$

Least dim $= 12$ "

<u>Tie size</u>

#3 upto #10 ba

3 @ 12" c/c tie is ok

$P_u = 200^k$

3 @ 12 ...

2. Design problem

$P_{DL} = 300^k \quad P_{LL} = 200^k$

$P_u = 1.2 * 300 + 1.6 * 200 = 680^k$

$\alpha \phi P_n \geq P_u$

$680 = \alpha \phi * [0.85 f_c' (A_g - A_{st}) + A_{st} f_y]$

$= \alpha \phi A_g [0.85 f_c' (1 - \rho_g) + \rho_g f_y]$

Let $\rho_g = 2\% = 0.02$

$\Rightarrow 680 = 0.8 * 0.65 * A_g [0.85 * 3 (1 - .02) + .02 * 60]$

$A_g = 353.5 \, in^2 \Rightarrow 18.8 \times 18.8$

18×18, 19×19, more or less

Let $19'' \times 19''$

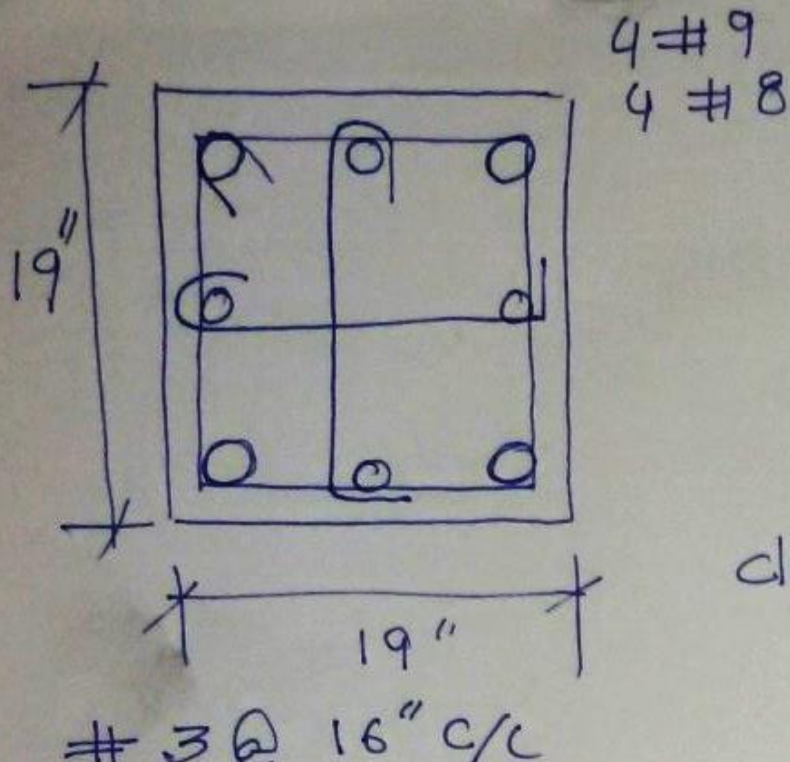
$680 = \alpha \phi [.85 f_c' (A_g - A_{st}) + A_{st} f_y]$

$= 0.8 * 0.65 [.85 * 3 (19 * 19 - A_{st}) + A_{st} * 60]$

$57.45 A_{st} = 387.14 \quad A_{st} = 6.73 \, in^2$

$4 * .79$

$4 * 1.0$



Tie size ⇒ #3

Tie spacing $16 * \frac{8}{8}" = 16"$

$48 * \frac{3}{8} = 18"$

$19" = 19$

clear spacing

$= 19 - 2*1.5 - 2*\frac{9}{8} - \frac{8}{8} - 2*\frac{3}{8}$

$= 12" \div 2 = 6"$

3. $P_{DL} = 60^k$ $P_{LL} = 30^k$ $P_U = 1.2 * 60 + 1.6 * 30$

$= 120^k.$

$$120 = 0.8 * .65 \left[.85 * 3 * (10 * 10 - A_{st}) + A_{st} * 60 \right]$$

$$230.76 = 255 + 57.45 A_{st}$$

$A_{st} = -ve.$

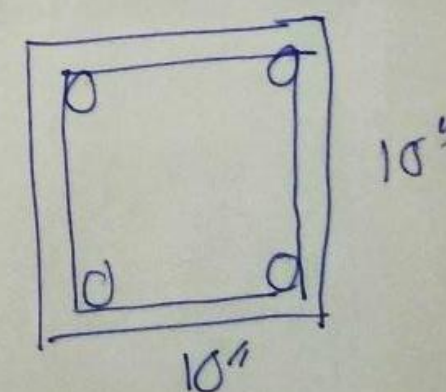
if 1% ⇒ 1.0 in^2.

min ⇒ 4 Nos # 5

Tie # 3 ⇒ $16 * \frac{5}{8} = 10"$

$48 * \frac{3}{8} = 18"$

Least $= 10"$



3 @ 10" c/c

Spirally reinforced column

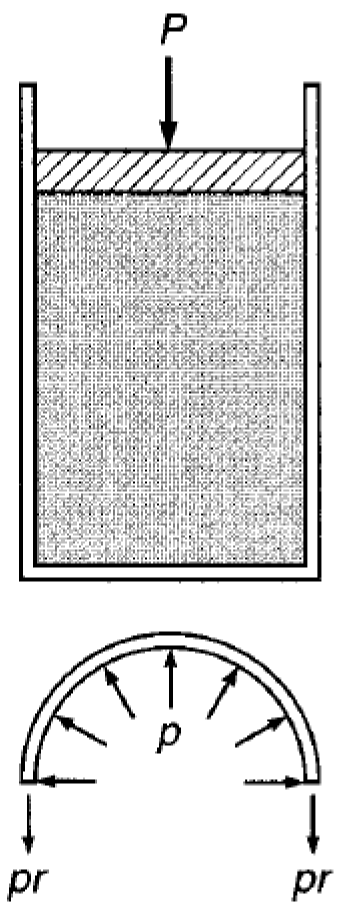


FIGURE 8.4
Model for action of a spiral.

- If filled with sand, load carrying capacity is due to hoop tension only
- If filled with concrete, it can carry without confinement
- Closely spaced spiral behaves like this drum, ie. it counteracts expansion of concrete
- Capacity of the core greatly increased
- Failure occurs when spiral yields and confinement greatly reduces

Tied and spiral behavior

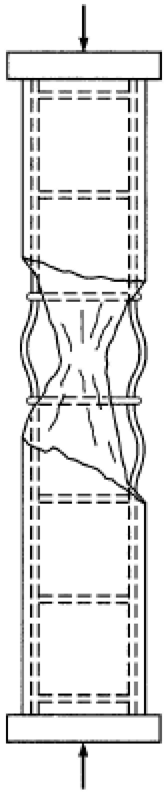
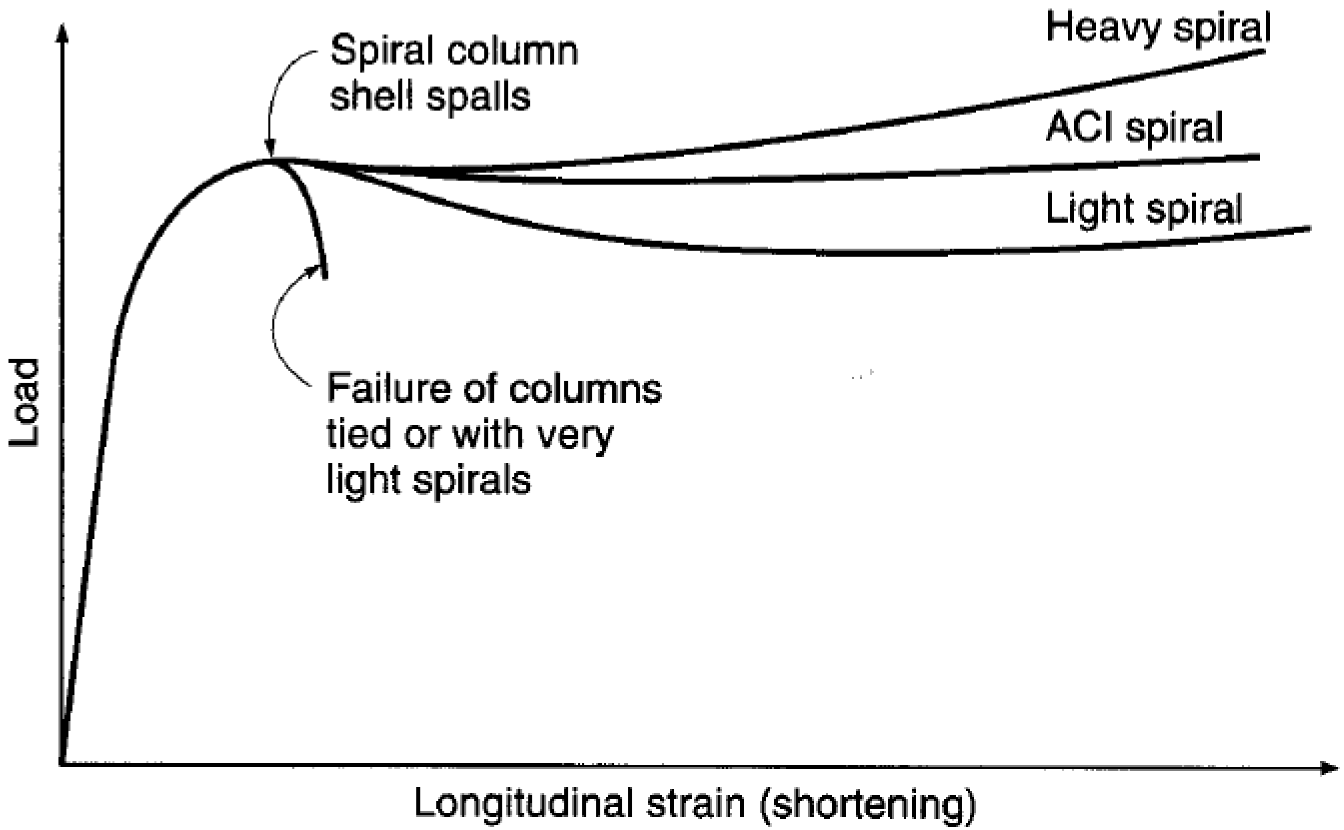


FIGURE 8.5
Failure of a tied column.

- A tied column fails when load reaches P_n
- Concrete fails in crushing and shearing in inclined plane
- Longitudinal steel buckles between ties
- A spirally rein column, the outer shell spalls off at the same load P_n
- Depending on the amount of spiral, the failure load can be much higher than P_n
- Axial strain will be much higher- higher toughness



Heavy spiral
Spiral column
shell spalls
ACI spiral
Light spiral
Load
Failure of columns
tied or with very
light spirals
Longitudinal strain (shortening)

ACI spiral

- Excess capacity is wasted
- ACI provides minimum amount of spiral that contributes to capacity slightly higher than that concrete shell
- This is ACI spiral-
- It hardly increases P_n
- It prevents instantaneous crushing, buckling of long steel, produces a gradual and ductile failure- a tougher column

ACI spiral derivation

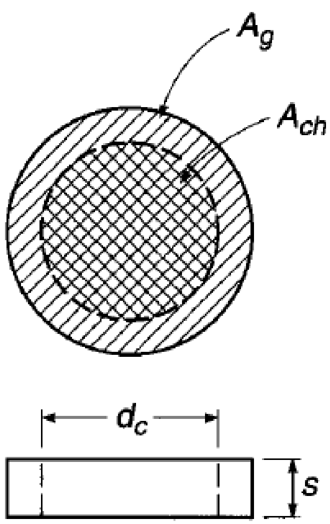


FIGURE 8.7
Confinement of core concrete due to hoop tension.

It has been found experimentally (Refs. 8.3 to 8.5) that the increase in compressive strength of the core concrete in a column provided through the confining effect of spiral steel is closely represented by the equation

$$f_c^* - 0.85f_c' = 4.0f_2' \tag{a}$$

where f_c^* = compressive strength of spirally confined core concrete
$0.85f_c'$ = compressive strength of concrete if unconfined
f_2' = lateral confinement stress in core concrete produced by spiral

The confinement stress f_2' is calculated assuming that the spiral steel reaches its yield stress f_y when the column eventually fails. With reference to Fig. 8.7, a hoop tension analysis of an idealized model of a short segment of column confined by one turn of lateral steel shows that

$$f_2' = \frac{2A_{sp}f_{yt}}{d_c s} \tag{b}$$

where A_{sp} = cross-sectional area of spiral wire
f_{yt} = yield strength of spiral steel
d_c = outside diameter of spiral
s = spacing or pitch of spiral wire

A *volumetric ratio* is defined as the ratio of the volume of spiral steel to the volume of core concrete:

$$\rho_s = \frac{2\pi d_c A_{sp}}{2} \frac{4}{\pi d_c^2 s}$$

from which

$$A_{sp} = \frac{\rho_s d_c s}{4} \qquad (c)$$

Substituting the value of A_{sp} from Eq. (*c*) into Eq. (*b*) results in

$$f_2' = \frac{\rho_s f_{yt}}{2} \qquad (d)$$

To find the right amount of spiral steel, one calculates

$$\text{Strength contribution of the shell} = 0.85 f_c'(A_g - A_{ch}) \qquad (e)$$

where A_g and A_c are, respectively, the gross and core concrete areas. Then substituting the confinement stress from Eq. (*d*) into Eq. (*a*) and multiplying by the core concrete area, one finds

$$\text{Strength provided by spiral} = 2\rho_s f_{yt} A_{ch} \tag{f}$$

The basis for the design of the spiral is that the strength gain provided by the spiral should be at least equal to that lost when the shell spalls, so combining Eqs. (*e*) and (*f*) yields

$$0.85 f_c'(A_g - A_{ch}) = 2\rho_s f_{yt} A_{ch}$$

from which

$$\rho_s = 0.425\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}} \tag{g}$$

According to the ACI Code, this result is rounded upward slightly, and ACI Code 10.9.3 states that the ratio of spiral reinforcement shall not be less than

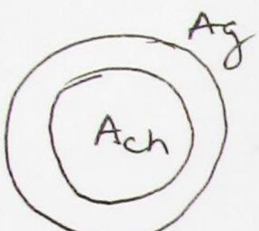
$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f_c'}{f_{yt}} \tag{8.5}$$

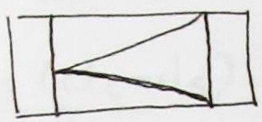
It is further stipulated in the ACI Code that f_{yt} not be taken greater than 100,000 psi and that spiral reinforcement not be spliced if f_{yt} is greater than 60,000 psi.

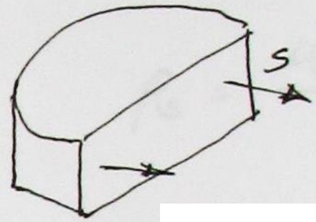
ACI Spiral

$$f_c^* = 0.85 f_c' + 4.0 f_2'$$

↑ core — ↑ unconfined — ↑ confinement







where A_{sp} = cross-sectional area of spiral wire
f_{yt} = yield strength of spiral steel
d_c = outside diameter of spiral
s = spacing or pitch of spiral wire

$$2 A_{sp} * f_y = f_2' \cdot d_c \cdot s$$

$$f_2' = \frac{2 A_{sp} \cdot f_y}{d_c \cdot s}$$

$$\rho_s = \frac{\text{vol. of spiral in one loop}}{\text{vol. of core of concrete}} = \text{volumetric ratio}$$

$$= \frac{\not{2} \pi d_c \cdot A_{sp}}{\not{2}} \cdot \frac{4}{\pi d_c^2 \cdot s}$$

$$A_{sp} = \frac{\rho_s d_c s}{4}$$

$\rho_s f_y$

$$A_{sp} = \frac{\rho_s d_c s}{4}$$

$$f_2' = 2 \cdot \frac{\rho_s d_c s}{4} \cdot \frac{f_y}{d_c \cdot s} \Rightarrow \boxed{f_2' = \frac{\rho_s f_y}{2}}$$

strength of conc. shell (outer) = strength prov by spiral

$$\Rightarrow 0.85 f_c' (A_g - A_{ch}) = 4 * \frac{\rho_s f_y}{2} * A_{ch}$$

$$\rho_s = 0.425 \left(\frac{A_g}{A_{ch}} - 1 \right) \frac{f_c'}{f_y}$$

ACI $\Rightarrow$ $$\boxed{\rho_s = 0.45 \left(\frac{A_g}{A_{ch}} - 1 \right) \frac{f_c'}{f_y}}$$

Problem 4

- Design a circular spiral column to support
  - P_{DL}= 475 kip and P_{LL}= 250 kip
  - f_c'= 4 ksi and f_y=60 ksi
  - Steel ratio of about 3%.
  - Also design necessary spiral

$P_u = 1.2 * 475 + 1.6 * 250 = 970^k$.

$= \alpha \phi [.85 f_c' (A_g - A_{st}) + A_{st} f_y]$

$= \alpha \phi A_g [.85 f_c' (1 - \rho_g) + \rho_g f_y]$

$970 = 0.85 * .75 A_g [.85 * 4 (1 - .03) + .03 * 60]$

$A_g = 298.5 \text{ in}^2 = \frac{\pi d^2}{4}$

$d = 19.49 \text{ in}^2$.

Let $d = 20''$. $A_g = 314.15 \text{ in}^2$

$970 = 0.85 * .75 [.85 * 4 (314.15 - A_{st}) + A_{st} * 60]$

$1521.5 = 1068.11 - 3.4 A_{st} + 60 A_{st}$

$56.6 A_{st} = 453.39$ $A_{st} = 8.01 \text{ in}^2$.

8 #9 or.

10 #8

Dia of core $= 20 - 2*1.5 = 17$

$A_{ch} = \frac{\pi}{4} * 17^2 = 226.98$

$A_g = 314.15$

Min $\rho_s = 0.45 \left(\frac{A_g}{A_{ch}} - 1 \right) \frac{f_c'}{f_y}$

$= 0.45 \left(\frac{314.15}{226.98} - 1 \right) \frac{4}{60} = 0.01152$

#3 spiral $A_{sp} = 0.11 \text{ in}^2$ $d_c = 17$

$d_s = 0.375$ in

$$\rho_s = \frac{A_{sp} * \pi d_c}{\frac{\pi}{4} * d_c^2 * s} = \frac{4 A_{sp}}{d_c * s}$$

$\Rightarrow 0.01152 = \frac{4 * .11}{17 * s}$ $\Rightarrow$ $s = 2.2$

#3 @ 2" c/c

TABLE A.14
Size and pitch of spirals, ACI Code

Diameter of Column, in.	Out to Out of Spiral, in.	f'_c, psi 3000	4000	5000
f_y = 40,000 psi				
14, 15	11, 12	$\frac{3}{8}$–$1\frac{3}{4}$	$\frac{1}{2}$–$2\frac{1}{2}$	$\frac{1}{2}$–$1\frac{3}{4}$
16	13	$\frac{3}{8}$–$1\frac{3}{4}$	$\frac{1}{2}$–$2\frac{1}{2}$	$\frac{1}{2}$–2
17–19	14–16	$\frac{3}{8}$–$1\frac{3}{4}$	$\frac{1}{2}$–$2\frac{1}{2}$	$\frac{1}{2}$–2
20–23	17–20	$\frac{3}{8}$–$1\frac{3}{4}$	$\frac{1}{2}$–$2\frac{1}{2}$	$\frac{1}{2}$–2
24–30	21–27	$\frac{3}{8}$–2	$\frac{1}{2}$–$2\frac{1}{2}$	$\frac{1}{2}$–2
f_y = 60,000 psi				
14, 15	11, 12	$\frac{3}{8}$–$2\frac{3}{4}$	$\frac{3}{8}$–2	$\frac{1}{2}$–$2\frac{3}{4}$
16–23	13–20	$\frac{3}{8}$–$2\frac{3}{4}$	$\frac{3}{8}$–2	$\frac{1}{2}$–3
24–29	21–26	$\frac{3}{8}$–3	$\frac{3}{8}$–$2\frac{1}{4}$	$\frac{1}{2}$–3
30	27	$\frac{3}{8}$–3	$\frac{3}{8}$–$2\frac{1}{4}$	$\frac{1}{2}$–$3\frac{1}{4}$

Compression plus Bending

Rectangular columns

- Columns chiefly carries compression
- But bending is almost always present
  - By continuity, part of monolithic frames
  - By transverse loads, wind, earthquake
  - By eccentric on bracket
  - By inevitable construction imperfection
  - Arch axis not coincides with pressure line

Statically equivalent

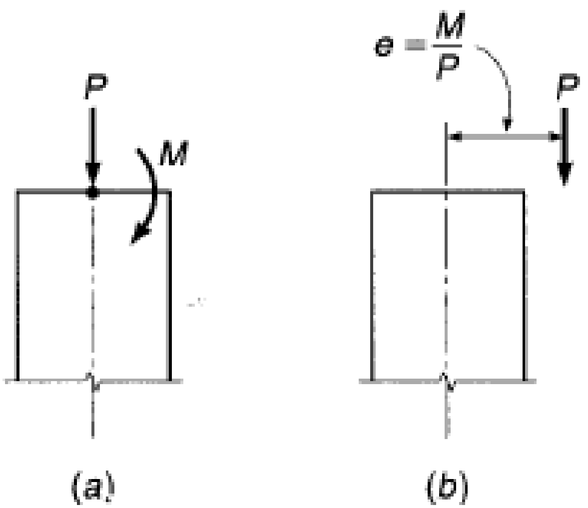


FIGURE 8.8
Equivalent eccentricity of column load.

- Two loads are statically equivalent
- Columns can be classified by *e*

Small e

- Comp over entire section
- If overloaded, fails by crushing of concrete and yielding of steel in comp in overloaded side

Large *e*

- Some part in tension
- If overloaded, may fail in yielding of steel in tension at the farthest side from load

For columns, load stages below the ultimate are generally not important. Cracking of concrete, even for columns with large eccentricity, is usually not a serious problem, and lateral deflections at service load levels are seldom, if ever, a factor. Design of

columns is therefore based on the factored load, which must not exceed the design strength, as usual, i.e.,

$$\phi M_n \geq M_u \tag{8.6a}$$

$$\phi P_n \geq P_u \tag{8.6b}$$

8.4 Strain Compatibility Analysis and Interaction diagram

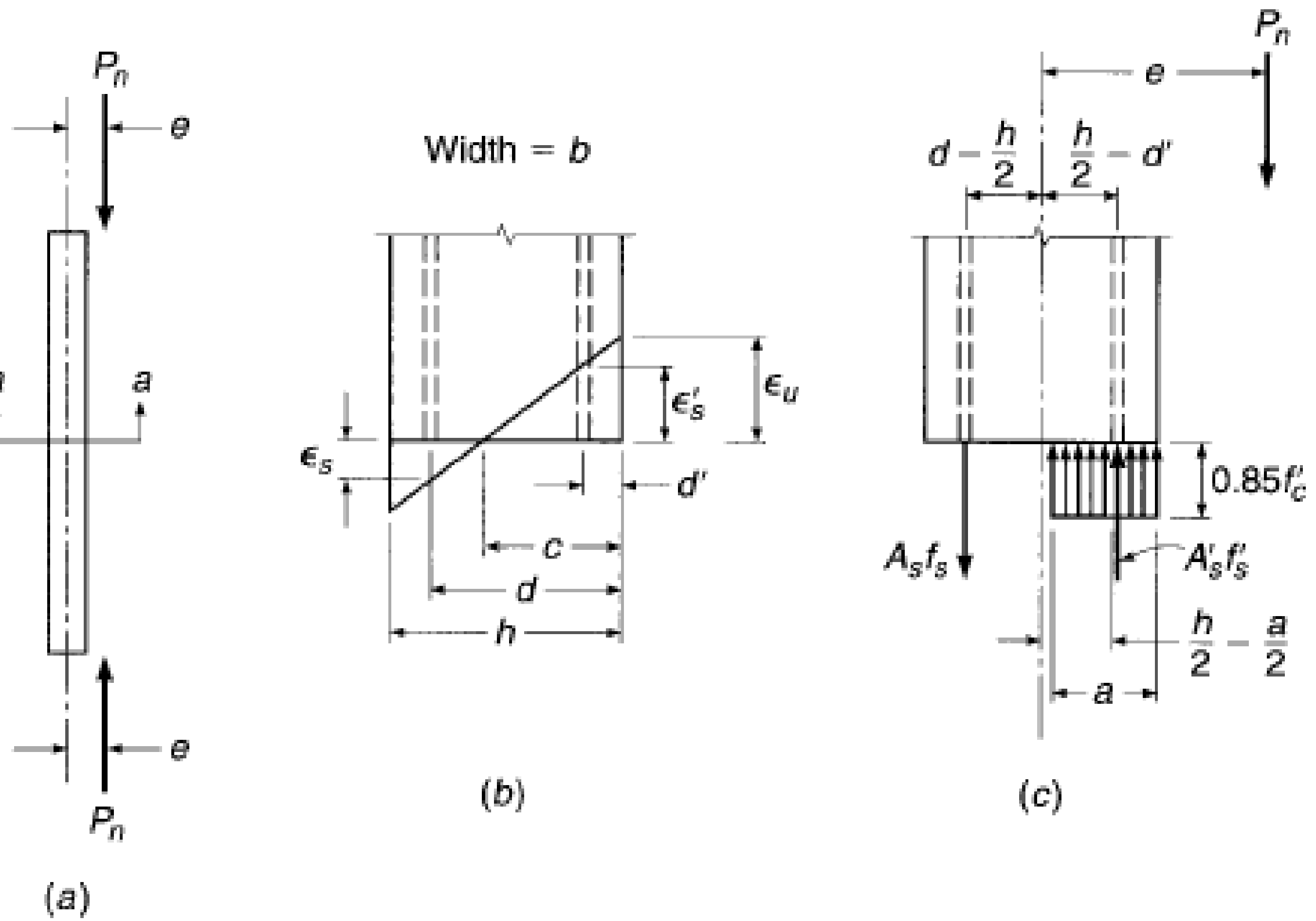


FIGURE 8.9
Column subject to eccentric compression: (*a*) loaded column; (*b*) strain distribution at section *a-a*; (*c*) stresses and forces at nominal strength.

Equilibrium between external and internal axial forces shown in Fig. 8.9*c* requires that

$$P_n = 0.85 f'_c ab + A'_s f'_s - A_s f_s \tag{8.7}$$

Also, the moment about the centerline of the section of the internal stresses and forces must be equal and opposite to the moment of the external force P_n, so that

$$M_n = P_n e = 0.85 f'_c ab\left(\frac{h}{2} - \frac{a}{2}\right) + A'_s f'_s\left(\frac{h}{2} - d'\right) + A_s f_s\left(d - \frac{h}{2}\right) \tag{8.8}$$

These are the two basic equilibrium relations for rectangular eccentrically compressed members.

- For large reinforcement ratio

$$A_s' \text{ by } f_s' - 0.85f_c' \text{ rather than by } f_s'.$$

- For large *e*
  - Failure is initiated by yielding of tension steel $f_s = f_y$
  - When ε_u is reached, compression steel may or may not have yielded, can be found from compatibility of strain
- For small *e*
  - ε_u is reached before tension steel yields
  - Stress in the other side of load may also be in compression, not in tension.

For a given eccentricity determined from the frame analysis (i.e., $e = M_u/P_u$) it is possible to solve Eqs. (8.7) and (8.8) for the load P_n and moment M_n that would result in failure as follows. In both equations, f_s', f_s, and a can be expressed in terms of a single unknown c, the distance to the neutral axis. This is easily done based on the geometry of the strain diagram, with ϵ_u taken equal to 0.003 as usual, and using the stress-strain curve of the reinforcement. The result is that the two equations contain only two unknowns, P_n and c, and can be solved for those values simultaneously. However, to do so in practice would be complicated algebraically, particularly because of the need to incorporate the limit f_y on both f_s' and f_s.

A better approach, providing the basis for practical design, is to construct a *strength interaction diagram* defining the failure load and failure moment for a given column for the full range of eccentricities from zero to infinity. For any eccentricity, there is a unique pair of values of P_n and M_n that will produce the state of incipient failure. That pair of values can be plotted as a point on a graph relating P_n and M_n,

Interaction diagram

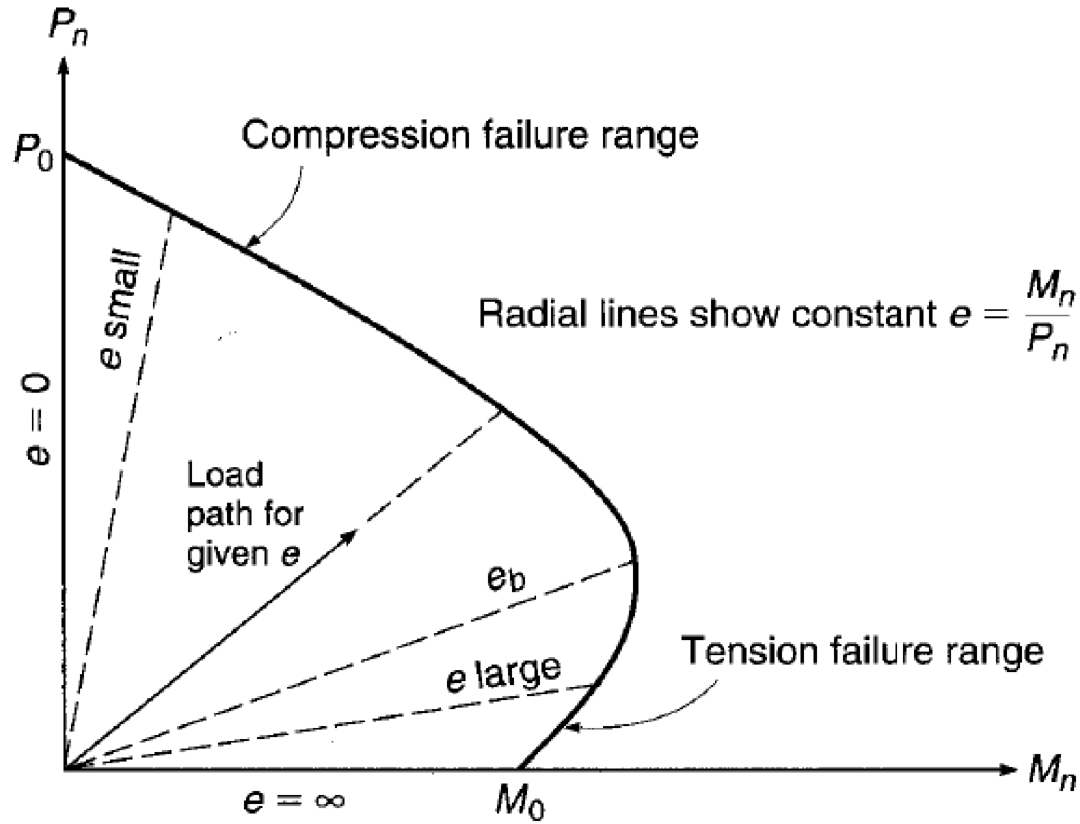


FIGURE 8.10
Interaction diagram for nominal column strength in combined bending and axial load.

On such a diagram, any radial line represents a particular eccentricity $e = M/P$. For that eccentricity, gradually increasing the load will define a load path as shown, and when that load path reaches the limit curve, failure will result. Note that the vertical axis corresponds to $e = 0$, and P_0 is the capacity of the column if concentrically loaded, as given by Eq. (8.3*b*). The horizontal axis corresponds to an infinite value of e, i.e., pure bending at moment capacity M_0. Small eccentricities will produce failure governed by concrete compression, while large eccentricities give a failure triggered by yielding of the tension steel.

For a given column, selected for trial, the interaction diagram is most easily constructed by selecting successive choices of neutral axis distance c, from infinity (axial load with eccentricity 0) to a very small value found by trial to give $P_n = 0$ (pure bending). For each selected value of c, the steel strains and stresses and the concrete force are easily calculated as follows. For the tension steel,

$$\epsilon_s = \epsilon_u \frac{d - c}{c} \tag{8.9}$$

$$f_s = \epsilon_u E_s \frac{d - c}{c} \leq f_y \tag{8.10}$$

while for the compression steel,

$$\epsilon_s' = \epsilon_u \frac{c - d'}{c} \tag{8.11}$$

$$f_s' = \epsilon_u E_s \frac{c - d'}{c} \leq f_y \tag{8.12}$$

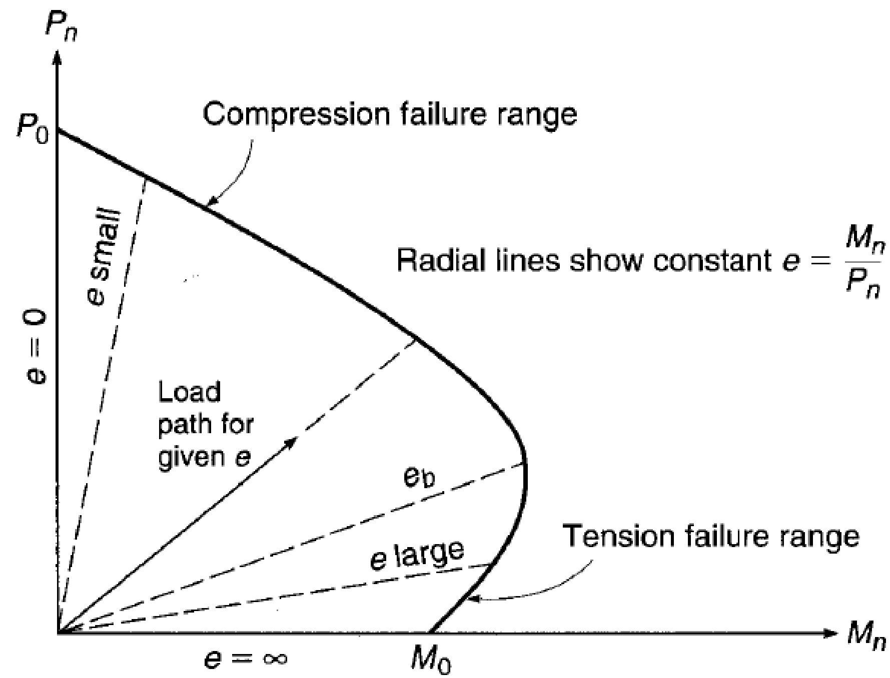
The concrete stress block has depth

$$a = \beta_1 c \leq h \tag{8.13}$$

and consequently the concrete compressive resultant is

$$C = 0.85 f_c' ab \tag{8.14}$$

The nominal axial force P_n and nominal moment M_n corresponding to the selected neutral axis location can then be calculated from Eqs. (8.7) and (8.8), respectively, and thus a single point on the strength interaction diagram is established. The calculations are then repeated for successive choices of neutral axis to establish the curve defining the strength limits, such as Fig. 8.10. The calculations, of a repetitive nature, are easily programmed for the computer or performed using a spreadsheet.



Balanced Failure

As already noted, the interaction curve is divided into a compression failure range and a tension failure range.† It is useful to define what is termed a *balanced failure mode* and corresponding eccentricity e_b with the load P_b and moment M_b acting in combination to produce failure, with the concrete reaching its limit strain ϵ_u at precisely the same instant that the tensile steel on the far side of the column reaches yield strain. This point on the interaction diagram is the dividing point between compression failure (small eccentricities) and tension failure (large eccentricities).

The values of P_b and M_b are easily computed with reference to Fig. 8.9. For balanced failure,

$$c = c_b = d \frac{\epsilon_u}{\epsilon_u + \epsilon_y} \tag{8.15}$$

and

$$a = a_b = \beta_1 c_b \tag{8.16}$$

Equations (8.9) through (8.14) are then used to obtain the steel stresses and the compressive resultant, after which P_b and M_b are found from Eqs. (8.7) and (8.8).

Note that, in contrast to beam design, one cannot restrict column designs such that yielding failure rather than crushing failure would always be the result of overloading. The type of failure for a column depends on the value of eccentricity e, which in turn is defined by the load analysis of the building or other structure.

It is important to observe, in Fig. 8.10, that in the region of compression failure the larger the axial load P_n, the smaller the moment M_n that the section is able to sustain before failing. However, in the region of tension failure, the reverse is true; the larger the axial load, the larger the simultaneous moment capacity. This is easily understood. In the compression failure region, failure occurs through overstraining of the concrete. The larger the concrete compressive strain caused by the axial load alone, the smaller the margin of additional strain available for the added compression caused by bending. On the other hand, in the tension failure region, yielding of the steel initiates failure. If the member is loaded in simple bending to the point at which yielding begins in the tension steel, and if an axial compression load is then added, the steel compressive stresses caused by this load will superimpose on the previous tensile stresses. This reduces the total steel stress to a value below its yield strength. Consequently, an additional moment can now be sustained of such magnitude that the combination of the steel stress from the axial load and the increased moment again reaches the yield strength.

The typical shape of a column interaction diagram shown in Fig. 8.10 has important design implications. In the range of tension failure, a *reduction in axial load* may produce failure for a given moment. In carrying out a frame analysis, the designer must consider all combinations of loading that may occur, including that which would produce minimum axial load paired with a given moment (the specific load combinations are specified in ACI Code 8.10 and described in Section 12.3). Only that amount of compression that is certain to be present should be used in calculating the capacity of a column subject to a given moment.

Problem

Column strength interaction diagram. A 12 × 20 in. column is reinforced with four No. 9 (No. 29) bars of area 1.0 in^2 each, one in each corner as shown in Fig. 8.11*a*. The concrete cylinder strength is f'_c = 4000 psi and the steel yield strength is 60 ksi. Determine (*a*) the load P_b, moment M_b, and corresponding eccentricity e_b for balanced failure; (*b*) the load and moment for a representative point in the tension failure region of the interaction curve; (*c*) the load and moment for a representative point in the compression failure region; (*d*) the axial load strength for zero eccentricity. Then (*e*) sketch the strength interaction diagram for this column. Finally, (*f*) design the transverse reinforcement, based on ACI Code provisions.

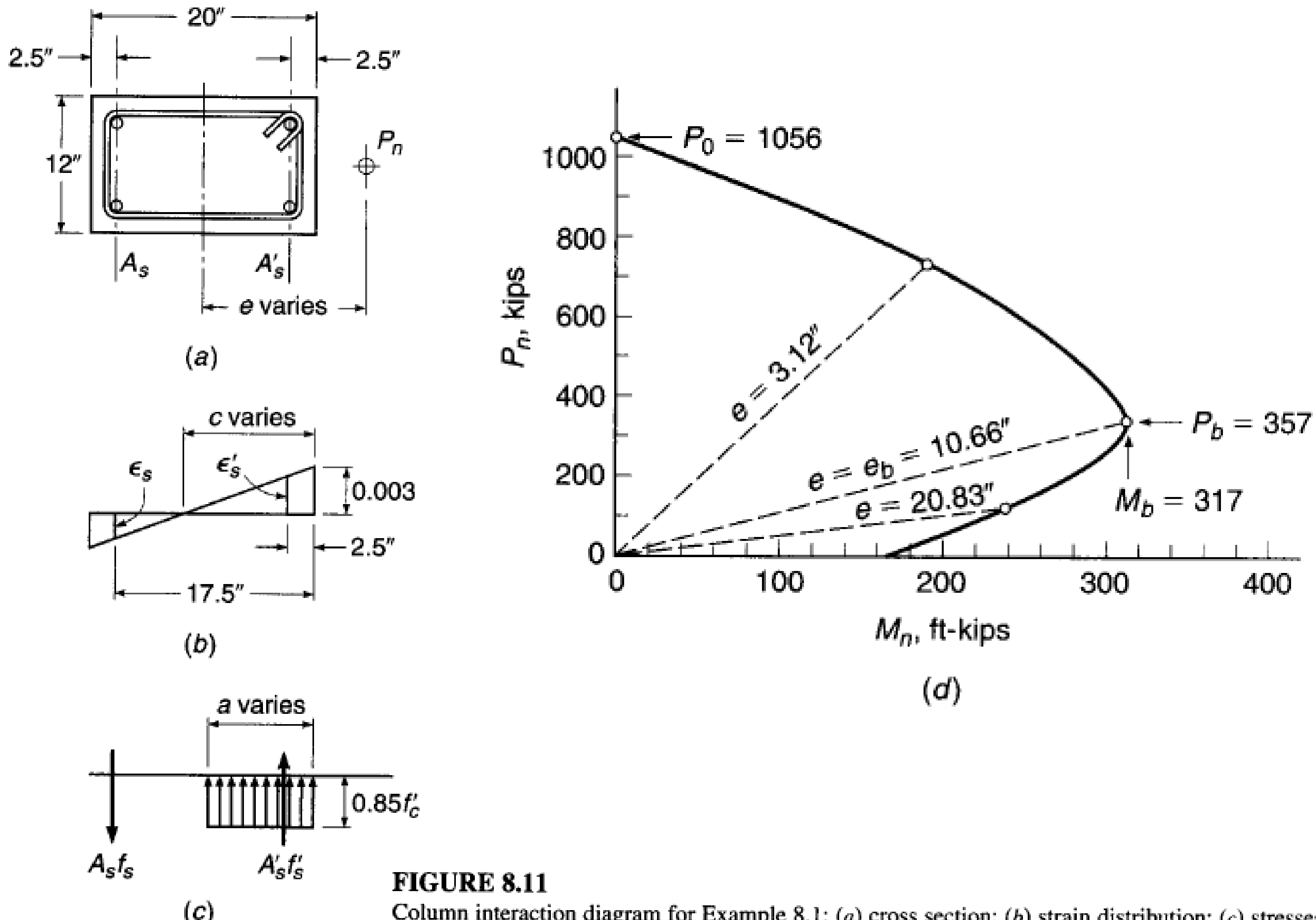


FIGURE 8.11
Column interaction diagram for Example 8.1: (*a*) cross section; (*b*) strain distribution; (*c*) stresses and forces; (*d*) strength interaction diagram.

Solution.

(*a*) The neutral axis for the balanced failure condition is easily found from Eq. (8.15) with $\epsilon_u = 0.003$ and $\epsilon_y = 60/29{,}000 = 0.0021$

$$c_b = 17.5 \times \frac{0.003}{0.0051} = 10.3 \text{ in.}$$

giving a stress-block depth $a = 0.85 \times 10.3 = 8.76$ in. For the balanced failure condition, by definition, $f_s = f_y$. The compressive steel stress is found from Eq. (8.12):

$$f_s' = 0.003 \times 29{,}000 \,\frac{10.3 - 2.5}{10.3} = 65.9 \text{ ksi} \qquad \textbf{but} \qquad \leq 60 \text{ ksi}$$

confirming that the compression steel, too, is at the yield. The concrete compressive resultant is

$$C = 0.85 \times 4 \times 8.76 \times 12 = 357 \text{ kips}$$

The balanced load P_b is then found from Eq. (8.7) to be

$$P_b = 357 + 2.0 \times 60 - 2.0 \times 60 = 357 \text{ kips}$$

and the balanced moment from Eq. (8.8) is

$$\begin{aligned} M_b &= 357(10 - 4.38) + 2.0 \times 60(10 - 2.5) + 2.0 \times 60(17.5 - 10) \\ &= 3806 \text{ in-kips} = 317 \text{ ft-kips} \end{aligned}$$

The corresponding eccentricity of load is $e_b = 10.66$ in.

(*b*) Any choice of c *smaller than* $c_b = 10.3$ in. will give a point in the tension failure region of the interaction curve, with eccentricity larger than e_b. For example, choose $c = 5.0$ in. By definition, $f_s = f_y$. The compressive steel stress is found to be

$$f_s' = 0.003 \times 29{,}000 \frac{5.0 - 2.5}{5.0} = 43.5 \text{ ksi}$$

With the stress-block depth $a = 0.85 \times 5.0 = 4.25$, the compressive resultant is $C = 0.85 \times 4 \times 4.25 \times 12 = 173$ kips. Then from Eq. (8.7), the thrust is

$$P_n = 173 + 2.0 \times 43.5 - 2.0 \times 60 = 140 \text{ kips}$$

and the moment capacity from Eq. (8.8) is

$$\begin{aligned} M_n &= 173(10 - 2.12) + 2.0 \times 43.5(10 - 2.5) + 2.0 \times 60(17.5 - 10) \\ &= 2916 \text{ in-kips} = 243 \text{ ft-kips} \end{aligned}$$

giving eccentricity $e = 2916/140 = 20.83$ in., well above the balanced value.

(*c*) Now selecting a c value *larger than* c_b to demonstrate a compression failure point on the interaction curve, choose $c = 18.0$ in., for which $a = 0.85 \times 18.0 = 15.3$ in. The compressive concrete resultant is $C = 0.85 \times 4 \times 15.3 \times 12 = 624$ kips. From Eq. (8.10) the stress in the steel at the left side of the column is

$$f_s = 0.003 \times 29{,}000 \frac{17.5 - 18.0}{18.0} = -2 \text{ ksi}$$

Note that the negative value of f_s indicates correctly that A_s is in compression if c is greater than d, as in the present case. The compressive steel stress is found from Eq. (8.12) to be

$$f'_s = 0.003 \times 29{,}000 \frac{18.0 - 2.5}{18.0} = 75 \text{ ksi} \quad \text{but} \quad \leq 60 \text{ ksi}$$

Then the column capacity is

$$P_n = 624 + 2.0 \times 60 + 2.0 \times 2 = 748 \text{ kips}$$

$$M_n = 624(10 - 7.65) + 2.0 \times 60(10 - 2.5) - 2.0 \times 2(17.5 - 10)$$

$$= 2336 \text{ in-kips} = 195 \text{ ft-kips}$$

giving eccentricity $e = 2336/748 = 3.12$ in.

(*d*) The axial strength of the column if concentrically loaded corresponds to $c = \infty$ and $e = 0$. For this case,

$$P_n = 0.85 \times 4 \times 12 \times 20 + 4.0 \times 60 = 1056 \text{ kips}$$

Note that, for this as well as the preceding calculations, subtraction of the concrete displaced by the steel has been neglected. For comparison, if the deduction were made in the last calculation,

$$P_n = 0.85 \times 4(12 \times 20 - 4) + (4.0 \times 60) = 1042 \text{ kips}$$

The error in neglecting this deduction is only 1 percent in this case; the difference generally can be neglected, except perhaps for columns with reinforcement ratios close to the maximum of 8 percent. In the case of design aids, however, such as those presented in Refs. 8.2 and 8.7 and discussed in Section 8.10, the deduction is usually included for all reinforcement ratios.

(*e*) From the calculations just completed, plus similar repetitive calculations that will not be given here, the strength interaction curve of Fig. 8.11*d* is constructed. Note the characteristic shape, described earlier, the location of the balanced failure point as well as the "small eccentricity" and "large eccentricity" points just found, and the axial load capacity.
In the process of developing a strength interaction curve, it is possible to select the values of steel strain ϵ_s, as done in step *a*, for use in steps *b* and *c*. Selecting ϵ_s uniquely establishes the neutral axis depth *c*, as shown by Eqs. (8.9) and (8.15), and is useful in determining M_n and P_n for values of steel strain that correspond to changes in the strength reduction factor ϕ, as will be discussed in Section 8.9.

(*f*) The design of the column ties will be carried out following the ACI Code restrictions. For the minimum permitted tie diameter of $\frac{3}{8}$ in., used with No. 9 (No. 29) longitudinal bars having a diameter of 1.128 in a column the least dimension of which is 12 in., the tie spacing is not to exceed

$$48 \times \frac{3}{8} = 18 \text{ in.}$$

$$16 \times 1.128 = 18.05 \text{ in.}$$

$$b = 12 \text{ in.}$$

The last restriction controls in this case, and No. 3 (No. 10) ties will be used at 12 in. spacing, detailed as shown in Fig. 8.11*a*. Note that the permitted spacing as controlled by the first and second criteria, 18 in., must be reduced because of the 12 in. column dimension.

EXAMPLE 8.2

Analysis of eccentrically loaded column with distributed reinforcement. The column in Fig. 8.12*a* is reinforced with ten No. 11 (No. 36) bars distributed around the perimeter as shown. Load P_n will be applied with eccentricity e about the strong axis. Material strengths are $f_c' = 6000$ psi and $f_y = 75$ ksi. Find the load and moment corresponding to a failure point with neutral axis $c = 18$ in. from the right face.

SOLUTION. When the concrete reaches its limit strain of 0.003, the strain distribution is that shown in Fig. 8.12*b*, the strains at the locations of the four bar groups are found from similar triangles, after which the stresses are found by multiplying strains by $E_s = 29{,}000$ ksi applying the limit value f_y:

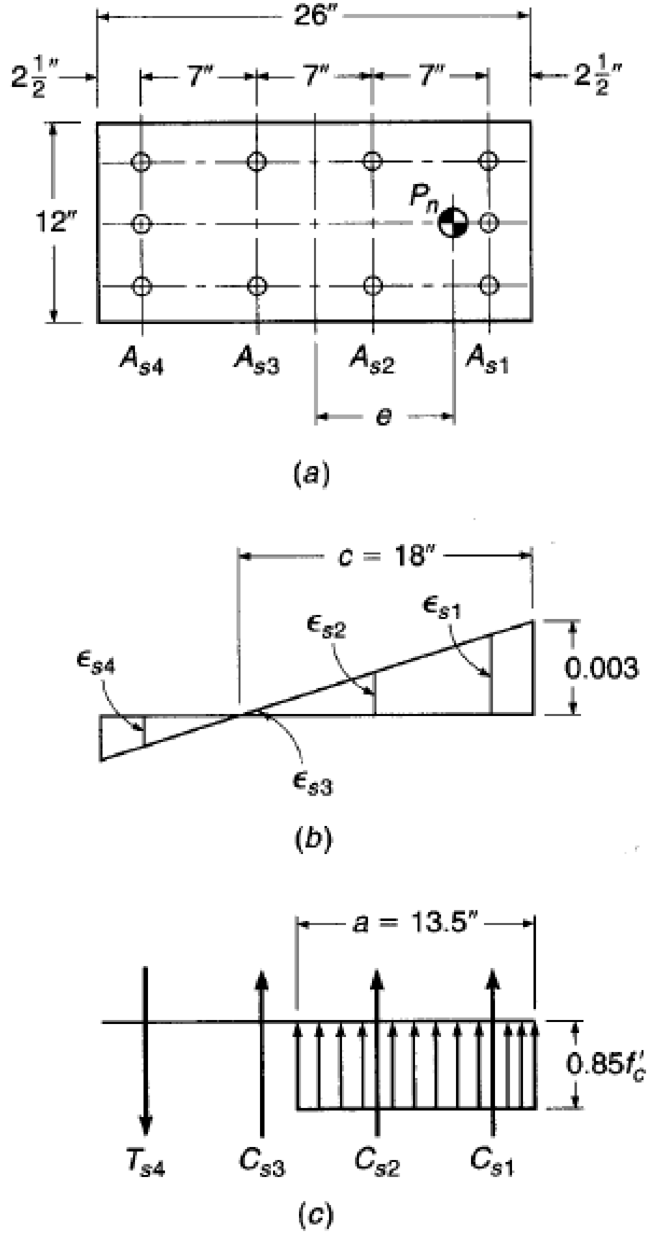
$$\epsilon_{s1} = 0.00258 \qquad f_{s1} = 75.0 \text{ ksi compression}$$

$$\epsilon_{s2} = 0.00142 \qquad f_{s2} = 41.2 \text{ ksi compression}$$

$$\epsilon_{s3} = 0.00025 \qquad f_{s3} = 7.3 \text{ ksi compression}$$

$$\epsilon_{s4} = 0.00091 \qquad f_{s4} = 26.4 \text{ ksi tension}$$

FIGURE 8.12
Column in Example 8.2: (*a*) cross section; (*b*) strain distribution; (*c*) stresses and forces.



For $f'_c = 6000$ psi, $\beta_1 = 0.75$ and the depth of the equivalent rectangular stress block is $a = 0.75 \times 18 = 13.5$ in. The concrete compressive resultant is $C = 0.85 \times 6 \times 13.5 \times 12 =$ 826 kips, and the respective steel forces in Fig. 8.12*c* are

$$C_{s1} = 4.68 \times 75.0 = 351 \text{ kips}$$
$$C_{s2} = 3.12 \times 41.2 = 129 \text{ kips}$$
$$C_{s3} = 3.12 \times 7.3 = 23 \text{ kips}$$
$$T_{s4} = 4.68 \times 26.4 = 124 \text{ kips}$$

The axial load and moment that would produce failure for a neutral axis 18 in. from the right face are found by the obvious extensions of Eqs. (8.7) and (8.8):

$$P_n = 826 + 351 + 129 + 23 - 124 = 1205 \text{ kips}$$
$$\begin{aligned} M_n &= 826(13 - 6.75) + 351(13 - 2.5) + 129(13 - 9.5) - 23(13 - 9.5) \\ &\quad + 124(13 - 2.5) \\ &= 10{,}520 \text{ in-kips} \\ &= 877 \text{ ft-kips} \end{aligned}$$

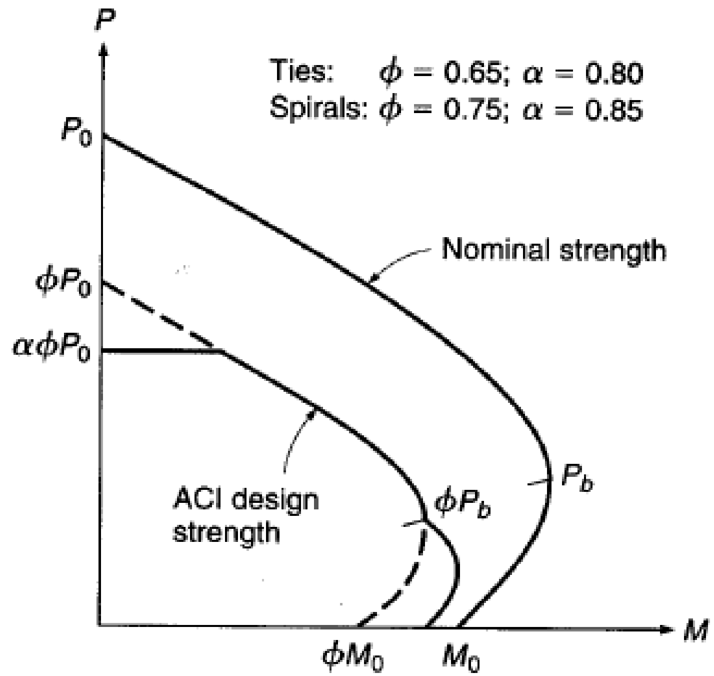
The corresponding eccentricity is $e = 10{,}520/1205 = 8.73$ in. Other points on the interaction diagram can be computed in a similar way.

Two general conclusions can be made from this example:

1. Even with the relatively small eccentricity of about one-third of the depth of the section, only the bars of group 1 just barely reached their yield strain, and consequently their yield stress. All other bar groups of the relatively high-strength steel that was used are stressed far below their yield strength, which would also have been true for group 1 for a slightly larger eccentricity. It follows that the use of the more expensive high-strength steel is economical in symmetrically reinforced columns only for very small eccentricities, e.g., in the lower stories of tall buildings.
2. The contribution of the intermediate bars of groups 2 and 3 to both P_n and M_n is quite small because of their low stresses. Again, intermediate bars, except as they are needed to hold ties in place, are economical only for columns with very small eccentricities.

8.9 ACI CODE PROVISIONS FOR COLUMN DESIGN

FIGURE 8.15
ACI safety provisions superimposed on column strength interaction diagram.



Read article- important

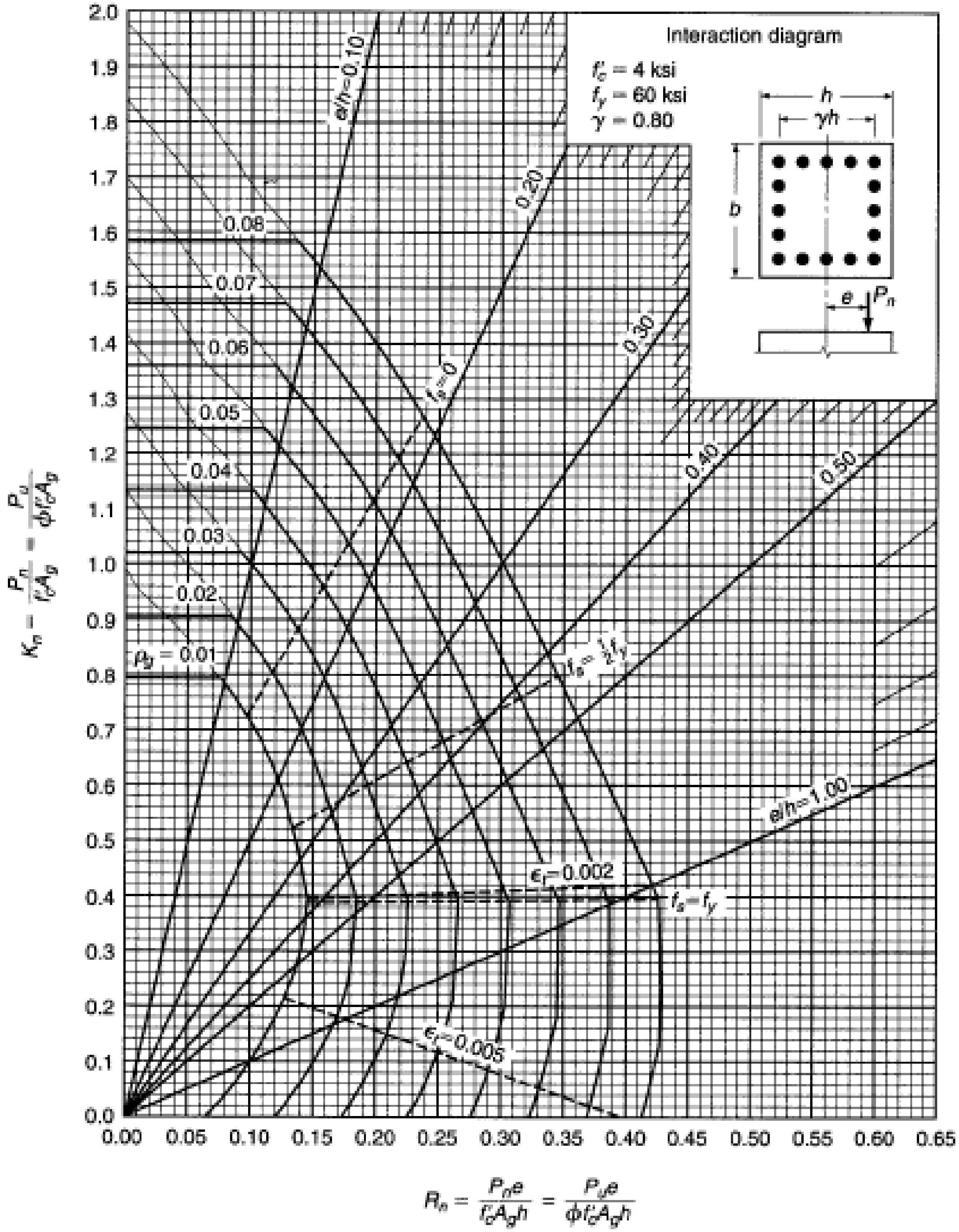
Why phi is low for column?

There are various reasons why the ϕ values for columns are lower than those for flexure or shear (0.90 and 0.75, respectively). One is that the strength of underreinforced flexural members is not much affected by variations in concrete strength, since it depends primarily on the yield strength of the steel, while the strength of axially loaded members depends strongly on the concrete compressive strength. Because the cylinder strength of concrete under site conditions is less closely controlled than the yield strength of mill-produced steel, a larger occasional strength deficiency must be allowed for. This is particularly true for columns, in which concrete, being placed from the top down in a long, narrow form, is more subject to segregation than in horizontally cast beams. Moreover, electrical and other conduits are frequently located in building columns; this reduces their effective cross sections, often to an extent unknown to the designer, even though this is poor practice and restricted by the ACI Code. Finally, the consequences of a column failure, say in a lower story, would be more catastrophic than those of a single beam failure in the same building.

Why alpha?

At the other extreme, for columns with very small or zero calculated eccentricities, the ACI Code recognizes that accidental construction misalignments and other unforeseen factors may produce actual eccentricities in excess of these small design values. Also, the concrete strength under high, sustained axial loads may be somewhat smaller than the short-term cylinder strength. Therefore, regardless of the magnitude of the calculated eccentricity, ACI Code 10.3.6 limits the maximum design strength to $0.80\phi P_0$ for tied columns (with $\phi = 0.65$) and to $0.85\phi P_0$ for spirally reinforced columns (with $\phi = 0.75$), where P_0 is the nominal strength of the axially loaded column with zero eccentricity [see Eq. (8.4)].

DESIGN CHARTS



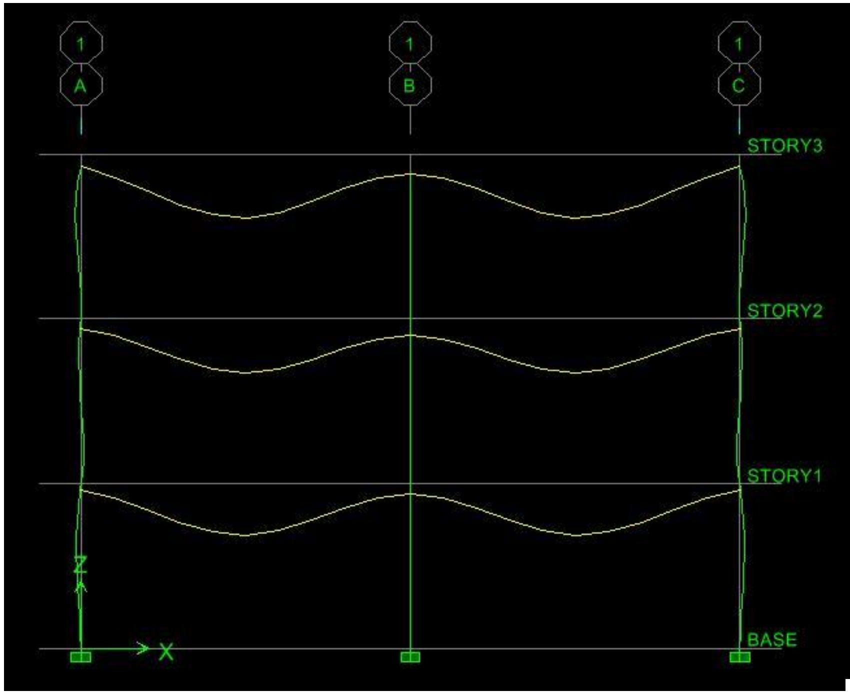
GRAPH A.7

Column strength interaction diagram for rectangular section with bars on four faces and γ = 0.80.

EXAMPLE 8.3

Selection of reinforcement for column of given size. In a three-story structure, an exterior column is to be designed for a service dead load of 222 kips, maximum live load of 297 kips, dead load moment of 136 ft-kips, and live load moment of 194 ft-kips. The minimum live load compatible with the full live load moment is 166 kips, obtained when no live load is placed on the roof but a full live load is placed on the second floor. Architectural considerations require that a rectangular column be used, with dimensions $b = 20$ in. and $h = 25$ in.

(*a*) Find the required column reinforcement for the condition that the full live load acts.
(*b*) Check to ensure that the column is adequate for the condition of no live load on the roof.
Material strengths are $f_c' = 4000$ psi and $f_y = 60{,}000$ psi.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A
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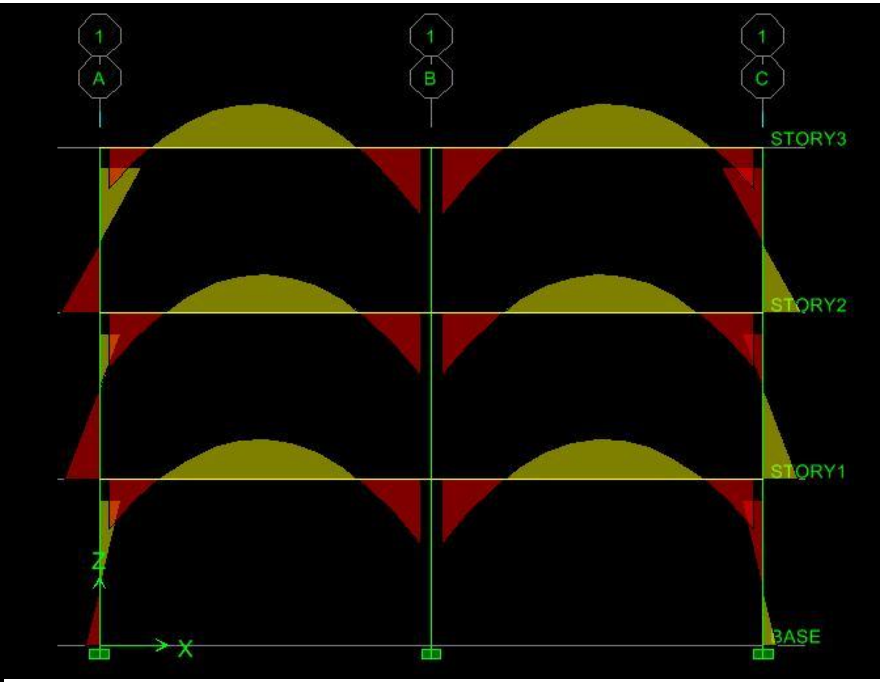
1
A
1
B
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C
STORY3
STORY2
STORY1
BASE
Z
X

Solution.

(*a*) The column will be designed initially for full load, then checked for adequacy when live load is partially removed. According to the ACI safety provisions, the column must be designed for a factored load $P_u = 1.2 \times 222 + 1.6 \times 297 = 742$ kips and a factored moment $M_u = 1.2 \times 136 + 1.6 \times 194 = 474$ ft-kips. A column 20 × 25 in. is specified, and reinforcement distributed around the column perimeter will be used. Bar cover is estimated to be 2.5 in. from the column face to the steel centerline for each bar. The column parameters (assuming bending about the strong axis) are

$$K_n = \frac{P_u}{\phi f_c' A_g} = \frac{742}{0.65 \times 4 \times 500} = 0.570$$

$$R_n = \frac{M_u}{\phi f_c' A_g h} = \frac{474 \times 12}{0.65 \times 4 \times 500 \times 25} = 0.175$$

With 2.5 in. cover, the parameter $\gamma = (25 - 5)/25 = 0.80$. For this column geometry and material strengths, Graph A.7 of Appendix A applies. From that figure, with the calculated values of K_n and R_n, $\rho_g = 0.024$. Thus, the required reinforcement is $A_{st} = 0.024 \times 500 = 12.00$ in^2. Twelve No. 9 (No. 29) bars will be used, one at each corner and two evenly spaced along each face of the column, providing $A_{st} = 12.00$ in^2.

(*b*) With the roof live load absent, the column will carry a factored load $P_u = 1.2 \times 222 + 1.6 \times 166 = 532$ kips and factored moment $M_u = 566$ ft-kips, as before. Thus, the column parameters for this condition are

$$K_n = \frac{P_u}{\phi f_c' A_g} = \frac{532}{0.65 \times 4 \times 500} = 0.409$$

$$R_n = \frac{M_u}{\phi f_c' A_g h} = \frac{474 \times 12}{0.65 \times 4 \times 500 \times 25} = 0.175$$

and $\gamma = 0.80$ as before. From Graph A.7 it is found that a reinforcement ratio of $\rho_g = 0.017$ is sufficient for this condition, less than that required in part (*a*), so no modification is required.

Selecting No. 3 (No. 10) ties for trial, the maximum tie spacing must not exceed $48 \times 0.375 = 18$ in., $16 \times 1.128 = 18.05$ in., or 20 in. Spacing is controlled by the diameter of the ties, and No. 3 (No. 10) ties will be used at 18 in. spacing, in the pattern shown in Fig. 8.3*d*.

EXAMPLE 8.4

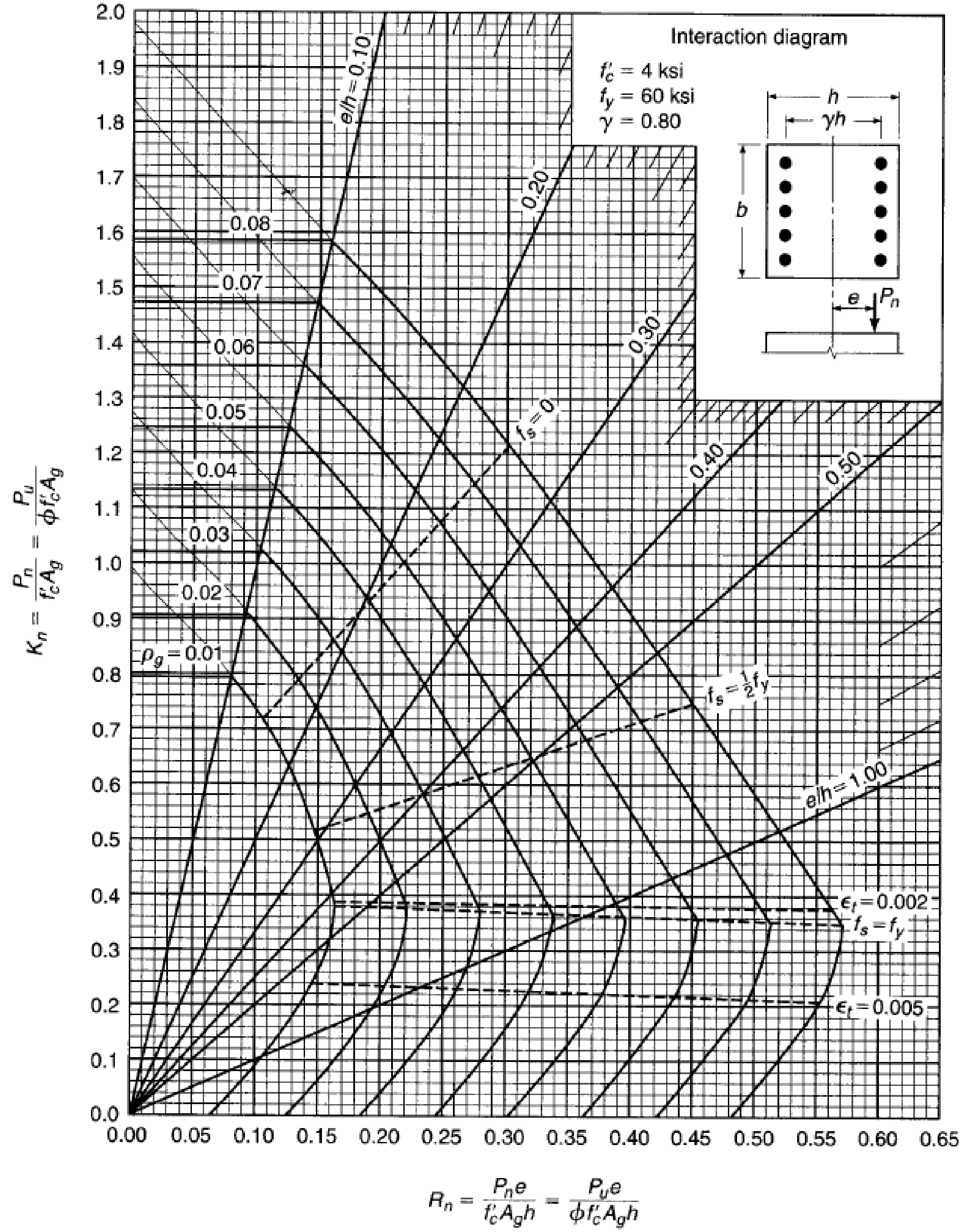
Selection of column size for a given reinforcement ratio. A column is to be designed to carry a factored load $P_u = 481$ kips and factored moment $M_u = 492$ ft-kips. Material strengths $f_y = 60{,}000$ psi and $f'_c = 4000$ psi are specified. Cost studies for the particular location indicate that a reinforcement ratio ρ_g of about 0.03 is optimum. Find the required dimensions b and h of the column. Bending will be about the strong axis, and an arrangement of steel with bars concentrated in two layers, adjacent to the outer faces of the column and parallel to the axis of bending, will be used.

Solution. It is convenient to select a trial column dimension h, perpendicular to the axis of bending; a value of $h = 25$ in. will be selected, and assuming a concrete cover of 2.5 in. to the bar centers, the parameter $\gamma = 0.80$. Graph A.11 of Appendix A applies. For the stated loads the eccentricity is $e = 492 \times 12/481 = 12.3$ in., and $e/h = 12.3/25 = 0.49$. From Graph A.11

with $e/h = 0.49$ and $\rho_g = 0.03$, $K_n = P_w/\phi f'_c A_g = 0.51$. For the trial dimension $h = 25$ in., the required column width is

$$b = \frac{P_u}{\phi f'_c K_n h} = \frac{481}{0.65 \times 4 \times 0.51 \times 25} = 14.5 \text{ in.}$$

A column 15 × 25 in. will be used, for which the required steel area is $A_{st} = 0.03 \times 15 \times 25 = 11.25 \text{ in}^2$. Eight No. 11 (No. 36) bars will be used, providing $A_{st} = 12.48 \text{ in}^2$, arranged in two layers of four bars each, similar to the sketch shown in Graph A.11.



GRAPH A.11

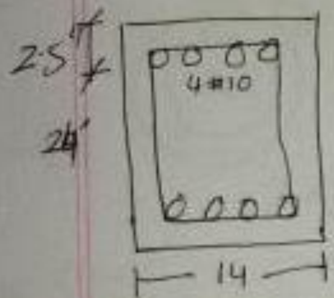
Column strength interaction diagram for rectangular section with bars on end faces and $\gamma = 0.80$.

Review problem

- Use the charts to determine the column strength ϕP_n, of the short column shown in Fig (14x24, reinforced with 8 No 10 bars), Use f_c'=4ksi and f_y=60 ksi, e=12in

Nadeem Ex 11.12 (Changed)

Use the charts to determine the column strength ϕP_n of the short col shown in fig. acting at an eccentricity $e = 12$ in. Use $f_c' = 4$ ksi, $f_y = 60$ ksi.



$h = 24''$

$\gamma h = 24 - 5 = 19''$

$\gamma = \frac{19}{24} = 0.79$

$\rho = \frac{8 * 1.27}{24 * 14} = 0.03$

A.11

$e/h = \frac{12}{24} = 0.5$

$K_n = 0.5$

$R_n = 0.25$

$\frac{P_u}{\phi f_c' A_g} = 0.5 \Rightarrow \frac{P_u}{.65 * 4 * 14 * 24} = .5$

$P_u = 436.8^k$

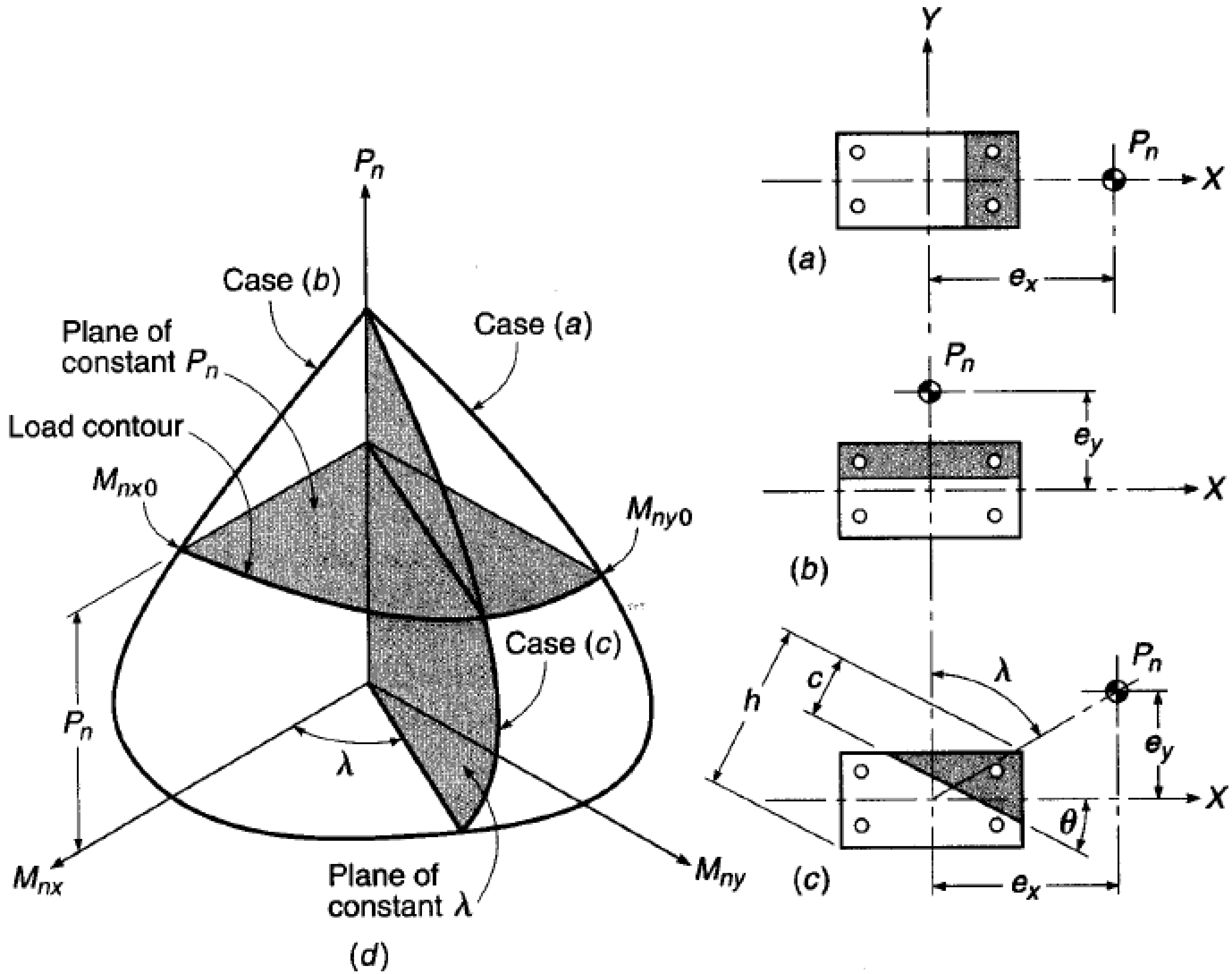
$\frac{M_u}{\phi f_c' A_g h} = 0.25 \Rightarrow \frac{M_u \times 12}{.65 * 4 * 14 * 24 * 24} = 0.25$

$M_u = 436.8^{k'}$

BIAXIAL BENDING

- There are situations when axial compression is associated with simultaneous bending is present about both principal axes of the section
- Corner column is such a case
- Interior column may also experience biaxial bending-irregular grid, lateral load

FIGURE 8.16
Interaction diagram for compression plus biaxial bending: (*a*) uniaxial bending about *Y* axis; (*b*) uniaxial bending about *X* axis; (*c*) biaxial bending about diagonal axis; (*d*) interaction surface.



Load contour method

The load contour method is based on representing the failure surface of Fig. 8.16*d* by a family of curves corresponding to constant values of P_n (Ref. 8.8). The general form of these curves can be approximated by a nondimensional interaction equation

$$\left(\frac{M_{nx}}{M_{nx0}}\right)^{\alpha 1} + \left(\frac{M_{ny}}{M_{ny0}}\right)^{\alpha 2} = 1.0 \tag{8.18}$$

where

$$M_{nx} = P_n e_y$$

$$M_{nx0} = M_{nx} \qquad \text{when } M_{ny} = 0$$

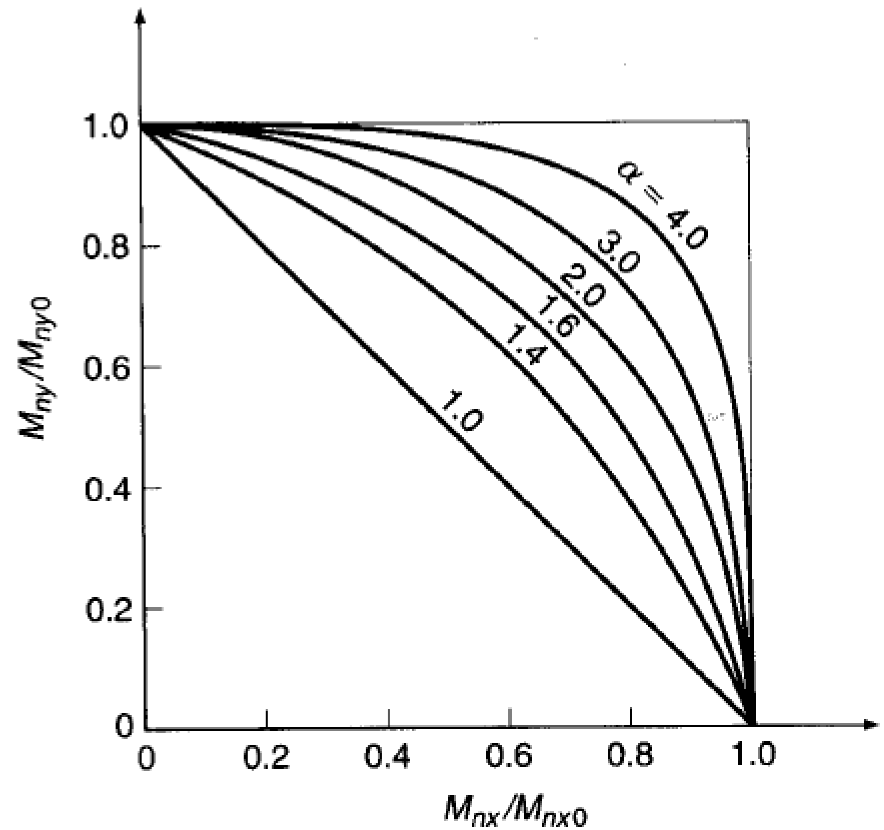
$$M_{ny} = P_n e_x$$

$$M_{ny0} = M_{ny} \qquad \text{when } M_{nx} = 0$$

and α_1 and α_2 are exponents depending on column dimensions, amount and distribution of steel reinforcement, stress-strain characteristics of steel and concrete, amount of concrete cover, and size of lateral ties or spiral. When $\alpha_1 = \alpha_2 = \alpha$, the shapes of such interaction contours are as shown in Fig. 8.17 for specific α values.

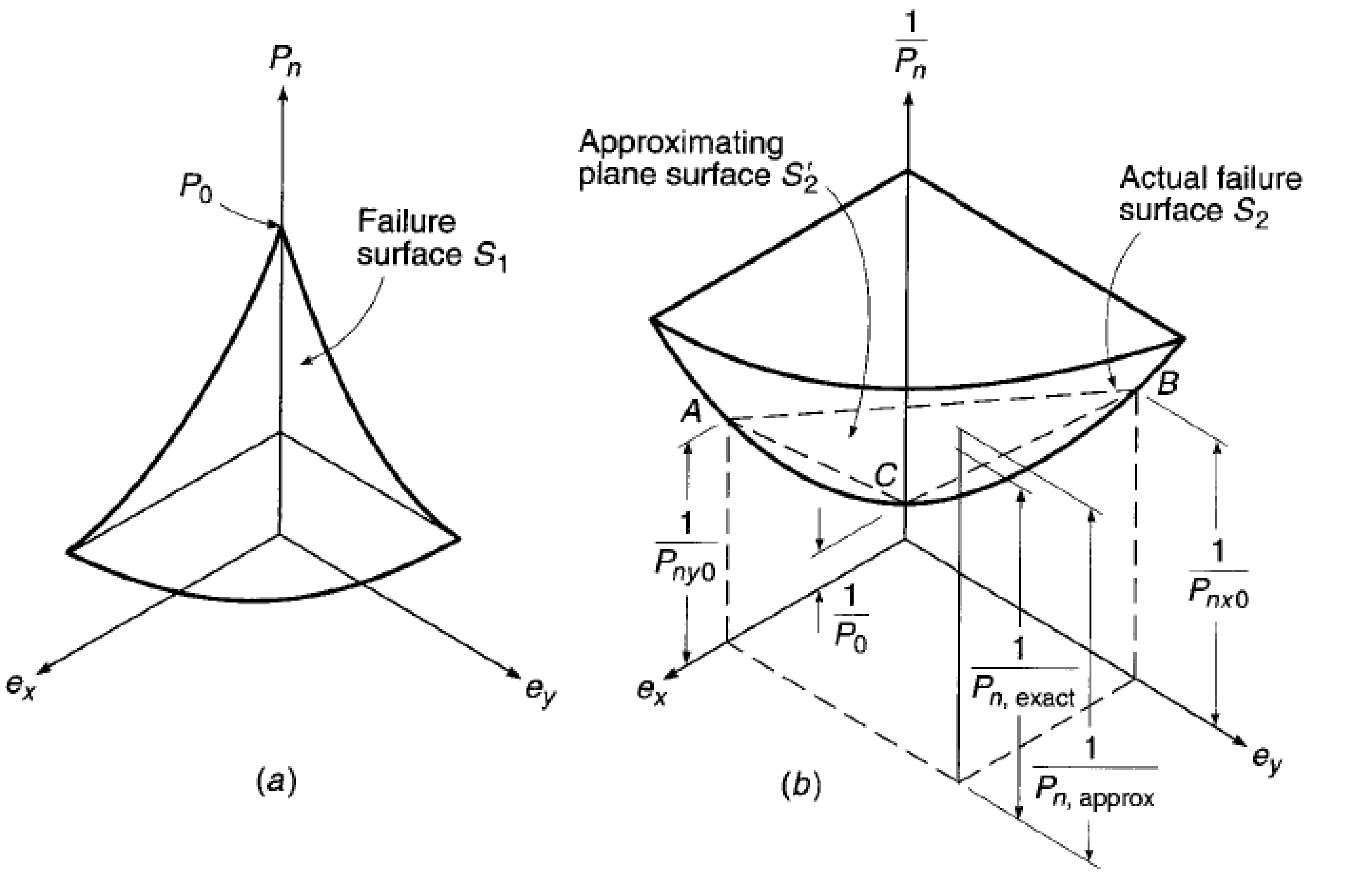
Calculations reported by Bresler in Ref. 8.9 indicate that α falls in the range from 1.15 to 1.55 for square and rectangular columns. Values near the lower end of that range are the more conservative. Methods and design aids permitting a more defined estimation of α are found in Ref. 8.7.

FIGURE 8.17
Interaction contours at constant P_n for varying α. (*Adapted from Ref. 8.8.*)



Reciprocal Load method

FIGURE 8.18
Interaction surfaces for the reciprocal load method.



Bresler's reciprocal load equation derives from the geometry of the approximating plane. It can be shown that

$$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_0} \tag{8.19}$$

where P_n = approximate value of nominal load in biaxial bending with eccentricities e_x and e_y

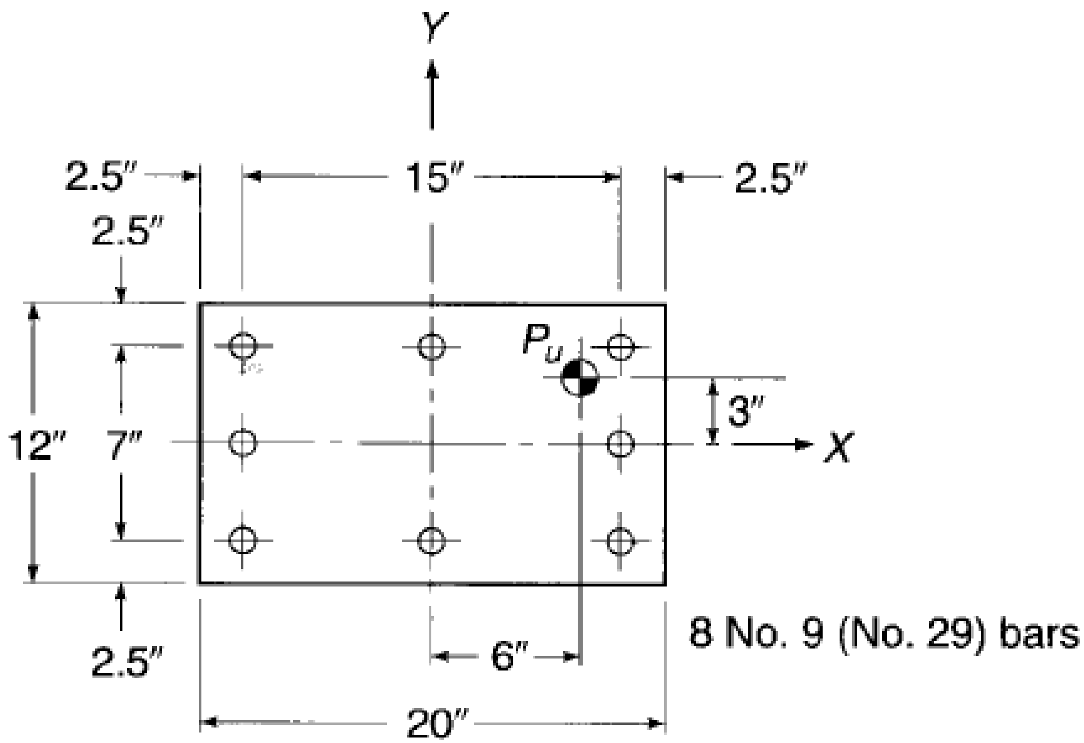
P_{ny0} = nominal load when only eccentricity e_x is present ($e_y = 0$)

P_{nx0} = nominal load when only eccentricity e_y is present ($e_x = 0$)

P_0 = nominal load for concentrically loaded column

Example 8.5

Design of column for biaxial bending. The 12 × 20 in. column shown in Fig. 8.19 is reinforced with eight No. 9 (No. 29) bars arranged around the column perimeter, providing an area $A_{st} = 8.00 \text{ in}^2$. A factored load P_u of 255 kips is to be applied with eccentricities $e_y = 3$ in. and



$e_x = 6$ in., as shown. Material strengths are $f'_c = 4$ ksi and $f_y = 60$ ksi. Check the adequacy of the trial design (*a*) using the reciprocal load method and (*b*) using the load contour method.

SOLUTION

(*a*) By the reciprocal load method, first considering bending about the Y axis, $\gamma = 15/20 = 0.75$, and $e/h = 6/20 = 0.30$. With the reinforcement ratio of $A_{st}/bh = 8.00/240 = 0.033$, using the average of Graphs A.6 ($\gamma = 0.70$) and A.7 ($\gamma = 0.80$),

$$\frac{P_{ny0}}{f_c' A_g}\,(\text{avg}) = \frac{0.62 + 0.66}{2} = 0.64 \qquad P_{ny0} = 0.64 \times 4 \times 240 = 614 \text{ kips}$$

$$\frac{P_0}{f_c' A_g} = 1.31 \qquad P_0 = 1.31 \times 4 \times 240 = 1258 \text{ kips}$$

Then for bending about the X axis, $\gamma = \frac{7}{12} = 0.58$ (say 0.60), and $e/h = \frac{3}{12} = 0.25$. Graph A.5 of Appendix A gives

$$\frac{P_{nx0}}{f_c' A_g} = 0.65 \qquad P_{nx0} = 0.65 \times 4 \times 240 = 624 \text{ kips}$$

$$\frac{P_0}{f_c' A_g} = 1.31 \qquad P_0 = 1.31 \times 4 \times 240 = 1258 \text{ kips}$$

Substituting these values in Eq. (8.19) results in

$$\frac{1}{P_n} = \frac{1}{624} + \frac{1}{614} - \frac{1}{1258} = 0.00244$$

from which $P_n = 410$ kips. Thus, according to the Bresler method, the design load of $P_u = 0.65 \times 410 = 267$ kips can be applied safely.

(*b*) By the load contour method, for Y axis bending with $P_u/(\phi f'_c A_g) = 255/(0.65 \times 4 \times 240) = 0.41$. The average from Graphs A.6 and A.7 of Appendix A is

$$\frac{M_{ny0}}{f'_c A_g h}\,(\text{avg}) = \frac{0.212 + 0.235}{2} = 0.224$$

Hence, $M_{ny0} = 0.224 \times 4 \times 240 \times 20 = 4300$ in-kips. Then for X axis bending, with $P_u/(\phi f'_c A_g) = 0.41$, as before, from Graph A.5,

$$\frac{M_{nx0}}{f'_c A_g h} = 0.186$$

So $M_{nx0} = 0.186 \times 4 \times 240 \times 12 = 2140$ in-kips. The factored load moments about the Y and X axes, respectively, are

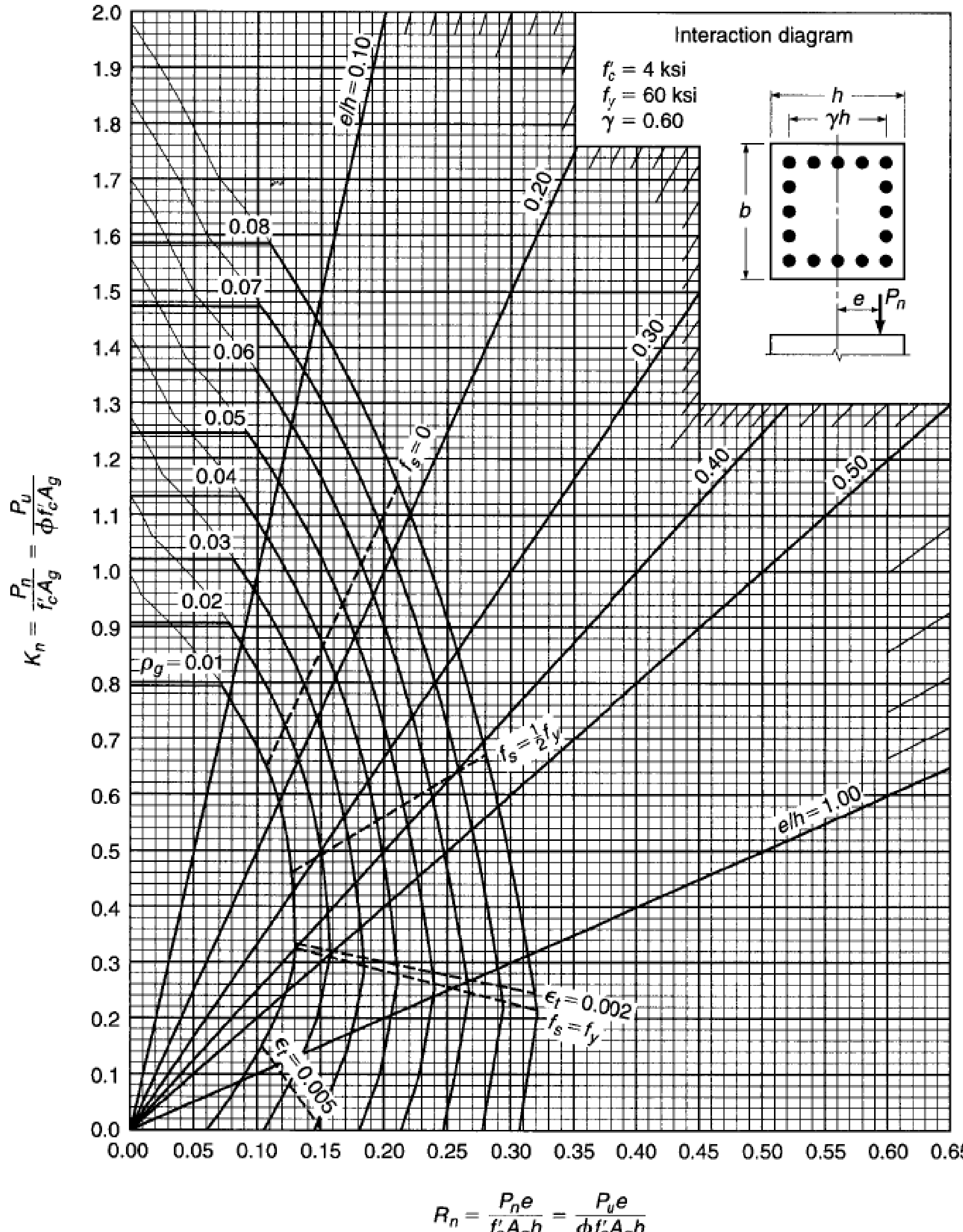
$$M_{uy} = 255 \times 6 = 1530 \text{ in-kips}$$

$$M_{ux} = 255 \times 3 = 765 \text{ in-kips}$$

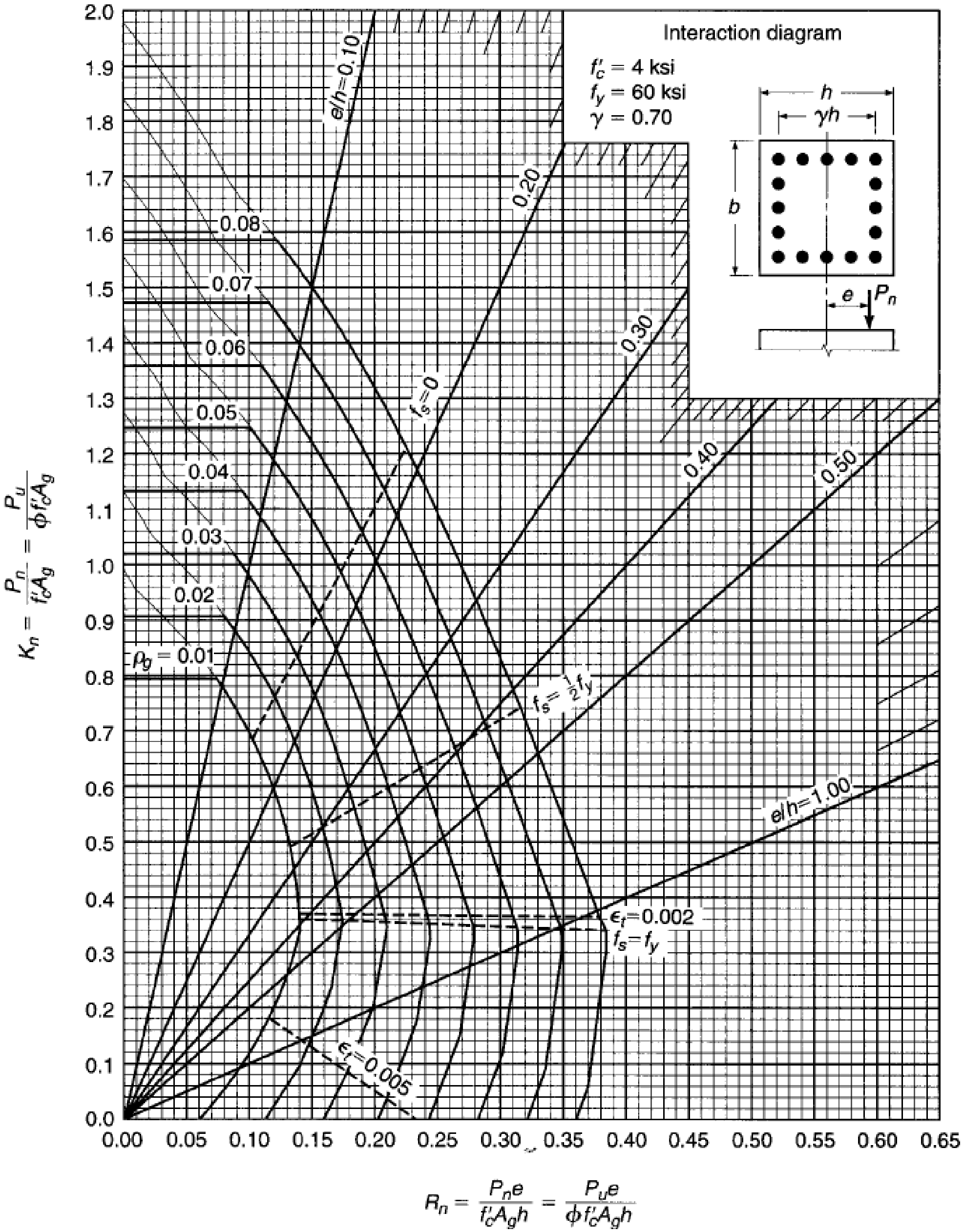
Adequacy of the trial design will now be checked using Eq. (8.18) with an exponent α conservatively taken equal to 1.15. Then with $M_{nx} = M_{ux}/\phi$ and $M_{ny} = M_{uy}/\phi$, that equation indicates

$$\left(\frac{765/0.65}{2140}\right)^{1.15} + \left(\frac{1530/0.65}{4300}\right)^{1.15} = 0.502 + 0.500 = 1.002$$

This is close enough to 1.0 that the design would be considered safe by the load contour method also.

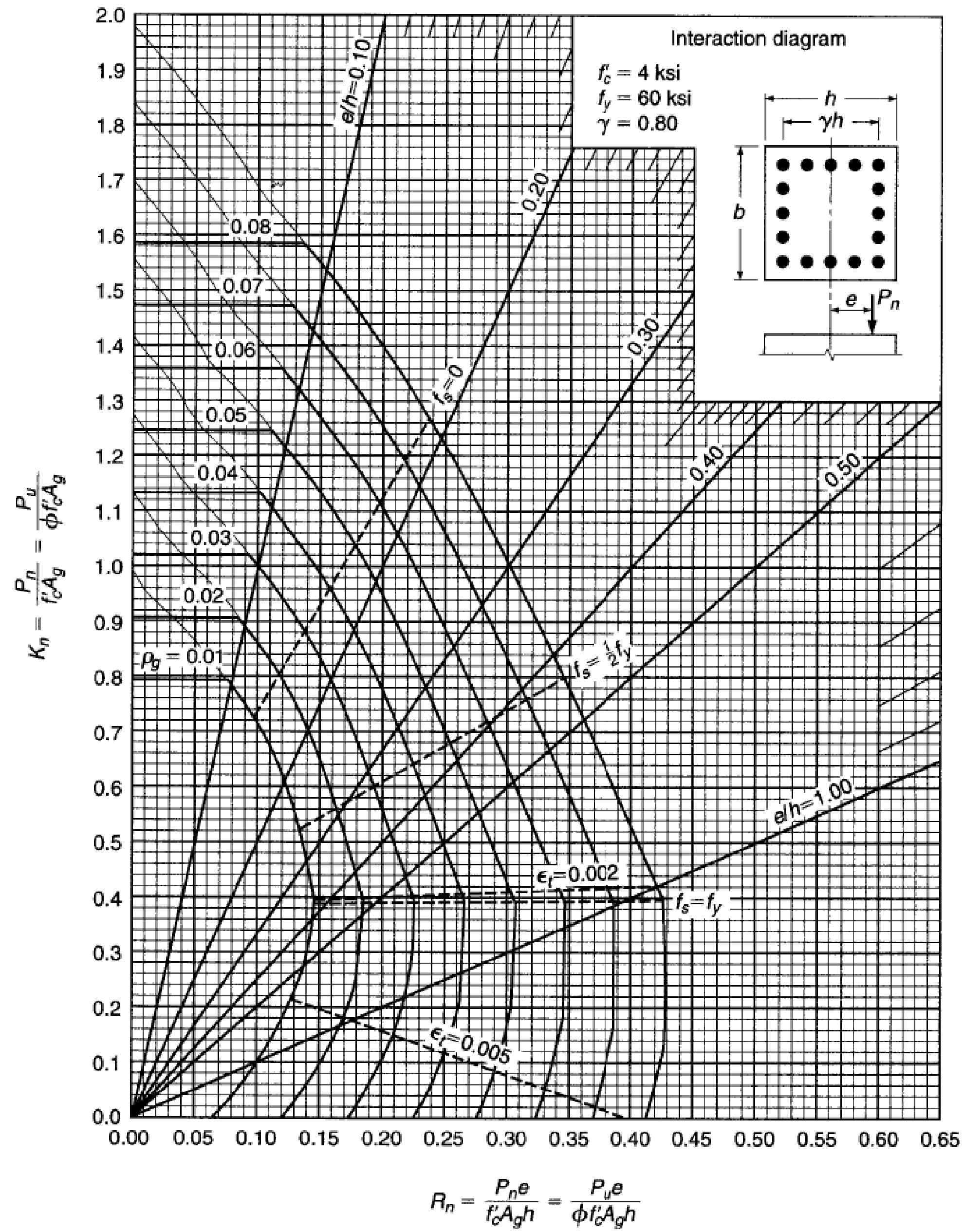


GRAPH A.5
Column strength interaction diagram for rectangular section with bars on four faces and $\gamma = 0.60$.



GRAPH A.6

Column strength interaction diagram for rectangular section with bars on four faces and $\gamma = 0.70$.



GRAPH A.7

Column strength interaction diagram for rectangular section with bars on four faces and $\gamma = 0.80$.

End of short Column