# SLENDER COLUMN

## Introduction

- Chapter 8 deals with short columns
- Strength is governed entirely by strength of material and geometry of cross section
- With increase in high strength material, column sections are getting smaller
- A column is slender if cross section dimensions are small compared to its length
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- The degree of slenderness is expressed in terms of slenderness ratio *l/r*
- *l* is unsupported length and *r* is smallest radius of gyration of the section, *√I/A*
- With increase in *l/r* ratio, there is possibility of buckling of column.
- If a axially loaded column fails in buckling i.e. by sudden lateral displacement of the member between ends, consequent overstressing of steel and concrete occur by bending stresses that are superimposed on the axial compressive stresses.

- Most columns are subjected to bending as well as axial force.
- These moments produce lateral deflections between ends. Associated with this lateral displacements are *secondary moments* that add to primary moments and that may become very large for slender column.

- A practical definition of a slender column is one for which there is a significant reduction in axial load capacity because of these secondary moments
- According to ACI code column provisions, any reduction greater than 5 percent is considered significant requiring consideration of slenderness effects.
- Most columns are short columns

**FIGURE 9.2**
Effect of slenderness on strength of axially loaded columns.

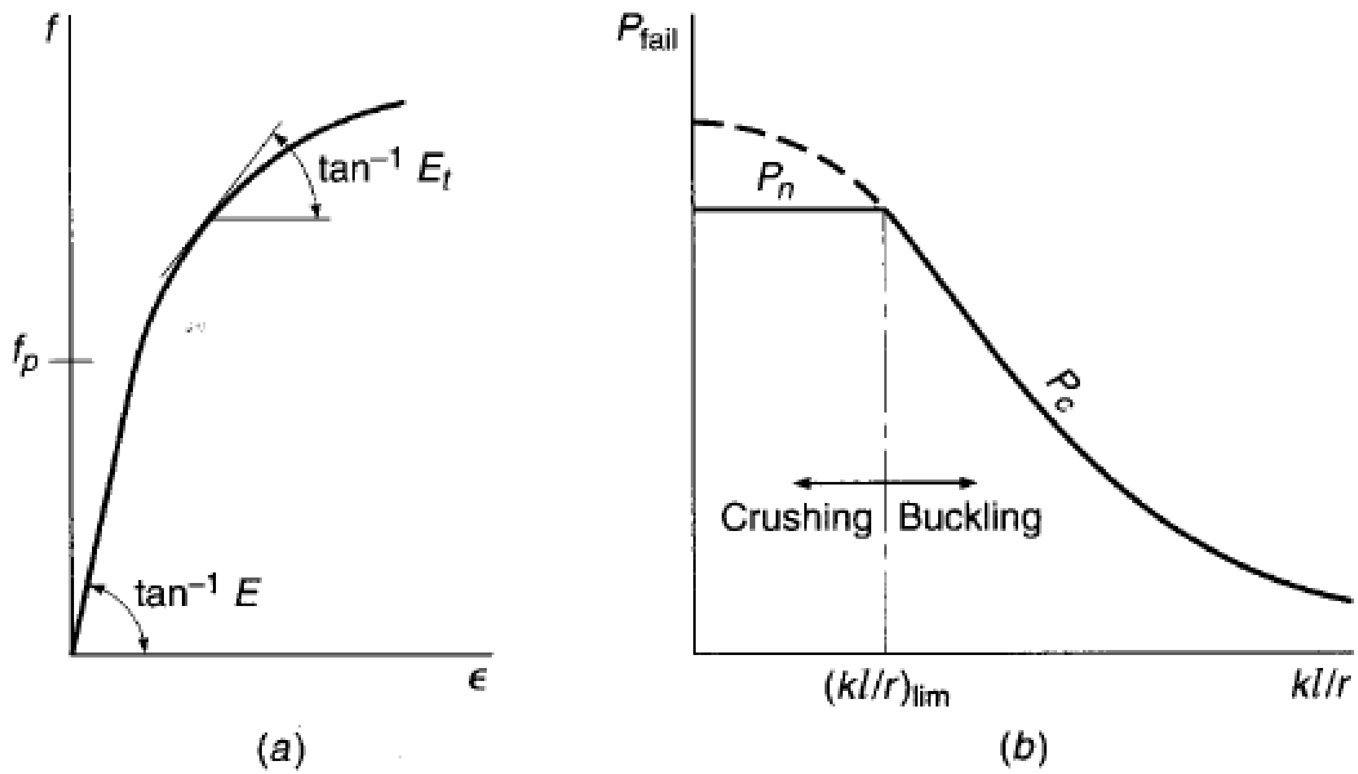
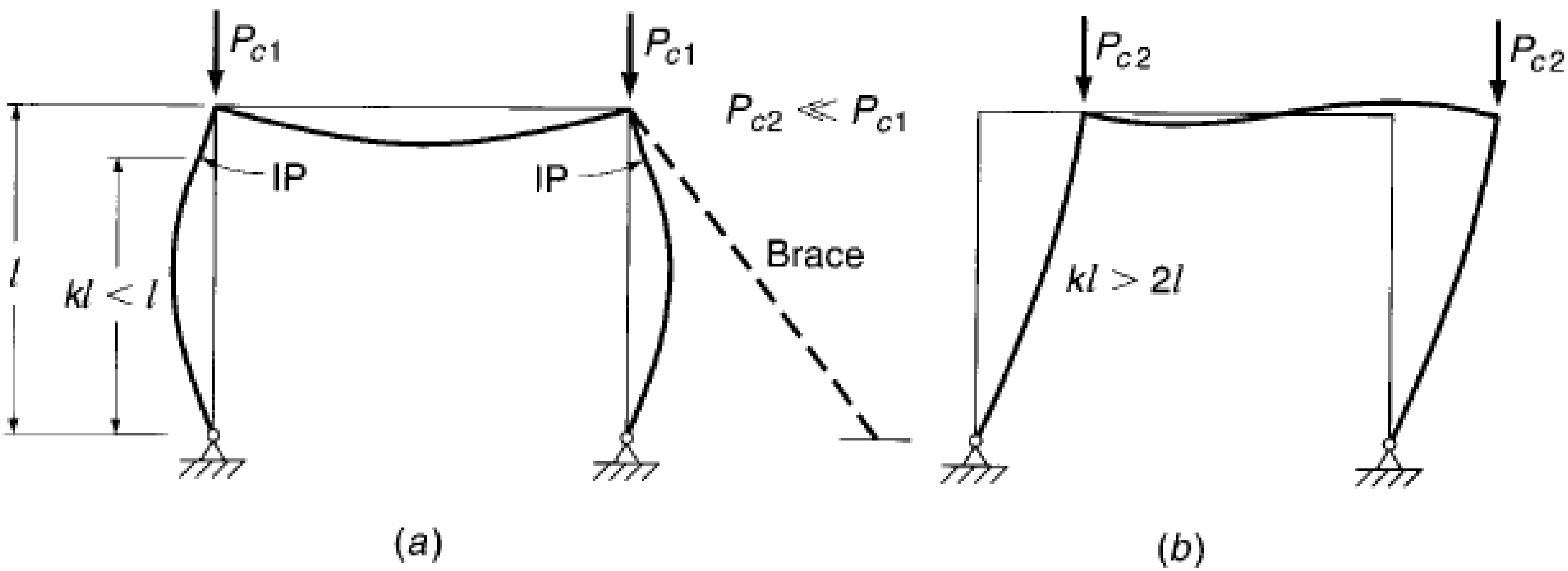


**FIGURE 9.3**
Rigid-frame buckling:
(*a*) laterally braced;
(*b*) unbraced.



In summary, the following can be noted:

1. The strength of concentrically loaded columns decreases with increasing slenderness ratio $kl/r$.
2. In columns that are *braced against sidesway* or that are parts of frames braced against sidesway, the effective length $kl$, i.e., the distance between inflection points, falls between $l/2$ and $l$, depending on the degree of end restraint.
3. The effective lengths of columns that are *not braced against sidesway* or that are parts of frames not so braced are always larger than $l$, the more so the smaller the end restraint. In consequence, the buckling load of a frame not braced against sidesway is always substantially smaller than that of the same frame when braced.

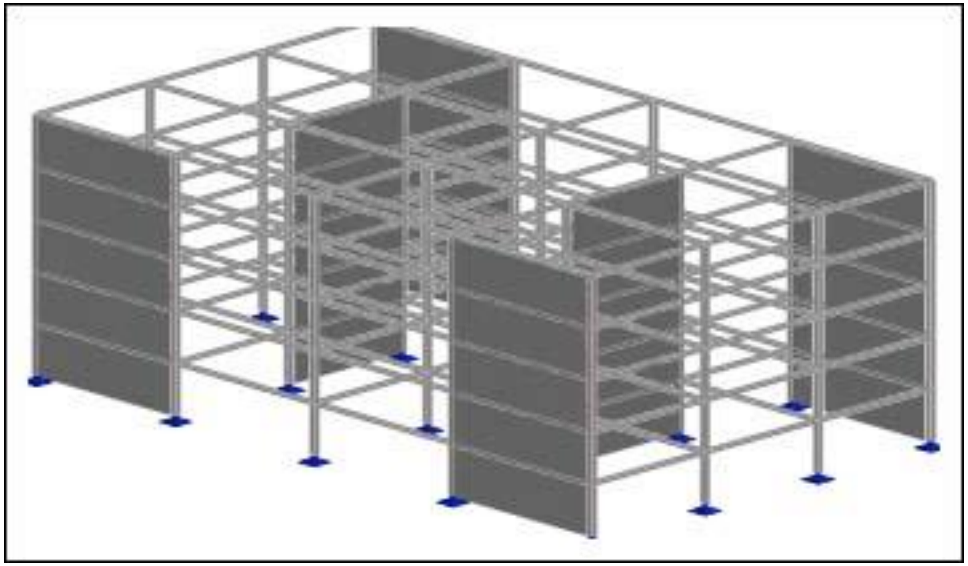
# ACI criteria for neglecting slenderness effects

The procedure of designing slender columns is inevitably lengthy, particularly because it involves a trial-and-error process. At the same time, studies have shown that most columns in existing buildings are sufficiently stocky that slenderness effects reduce their capacity only a few percent. As stated in Chapter 8, an ACI-ASCE survey indicated that 90 percent of columns braced against sway, and 40 percent of unbraced columns, could be designed as short columns; i.e., they could develop essentially the full cross-sectional strength with little or no reduction from slenderness (Ref. 9.3). Furthermore, lateral bracing is usually provided by shear walls, elevator shafts, stairwells, or other elements for which resistance to lateral deflection is much greater than for the columns of the building frame. It can be concluded that in most cases in reinforced concrete buildings, slenderness effects may be neglected.

To permit the designer to dispense with the complicated analysis required for slender column design for these ordinary cases, ACI Code 10.10.1 provides limits below which the effects of slenderness are insignificant and may be neglected. These limits are adjusted to result in a maximum unaccounted reduction in column capacity of no more than 5 percent. Separate limits are applied to braced and unbraced structures, alternately described in the ACI Code as *nonsway* and *sway* frames, respectively. For the purpose of determining if slenderness effects may be neglected, ACI Code 10.10.1 permits compression members to be considered as braced against sidesway if the total stiffness of the bracing elements resisting lateral movement of a story is at least 12 times the stiffness of all columns in that story. The Code provisions are as follows:

1. For compression members braced against sidesway (i.e., in nonsway structures), the effects of slenderness may be neglected when $kl_u/r \leq 34 - 12M_1/M_2$, where $34 - 12M_1/M_2$ is not taken greater than 40.
2. For compression members not braced against sidesway (i.e., in sway structures), the effects of slenderness may be neglected when $kl_u/r$ is less than 22.

# Shear Walls



Horizontal forces acting on buildings, e.g., those due to wind or seismic action, can be resisted by different means. Rigid-frame resistance of the structure, augmented by the contribution of ordinary masonry walls and partitions, can provide for wind loads in many cases. However, when heavy horizontal loading is likely, such as would result from an earthquake, reinforced concrete shear walls are used. These may be added solely to resist horizontal forces, or concrete walls enclosing stairways or elevator shafts may also serve as shear walls.

Figure 18.13 shows a building with wind or seismic forces represented by arrows acting on the edge of each floor or roof. The horizontal surfaces act as deep beams to transmit loads to vertical resisting elements *A* and *B*. These shear walls, in turn, act as cantilever beams fixed at their base to carry loads down to the foundation. They are subjected to (1) a variable shear, which reaches a maximum at the base; (2) a bending moment, which tends to cause vertical tension near the loaded edge and compression at the far edge; and (3) a vertical compression due to ordinary gravity loading from the structure. For the building shown, additional shear walls *C* and *D* are provided to resist loads acting in the long direction of the structure.

Shear is apt to be critical for walls with a relatively low ratio of height to length. High shear walls are controlled mainly by flexural requirements.

**FIGURE 18.13**
Building with shear walls subject to horizontal loads: (*a*) typical floor; (*b*) front elevation; (*c*) end elevation.

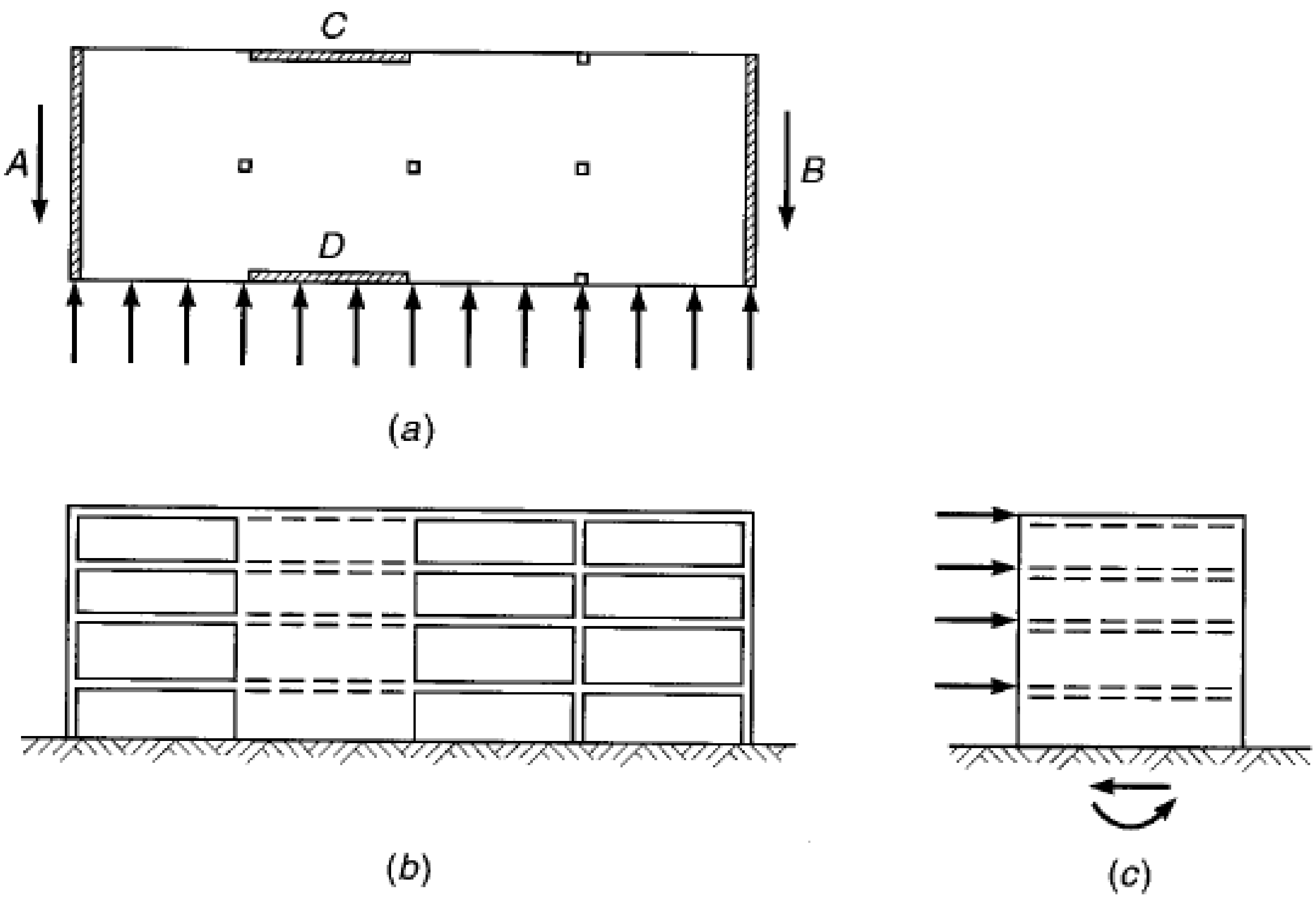
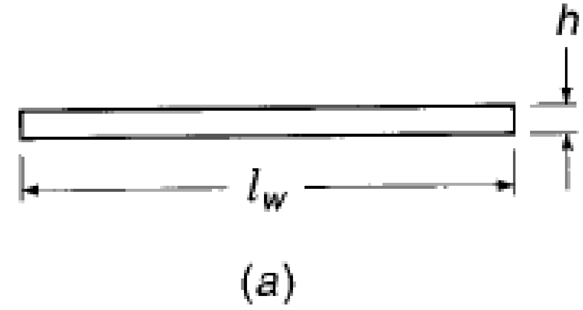
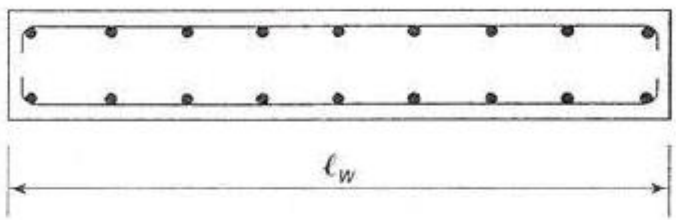


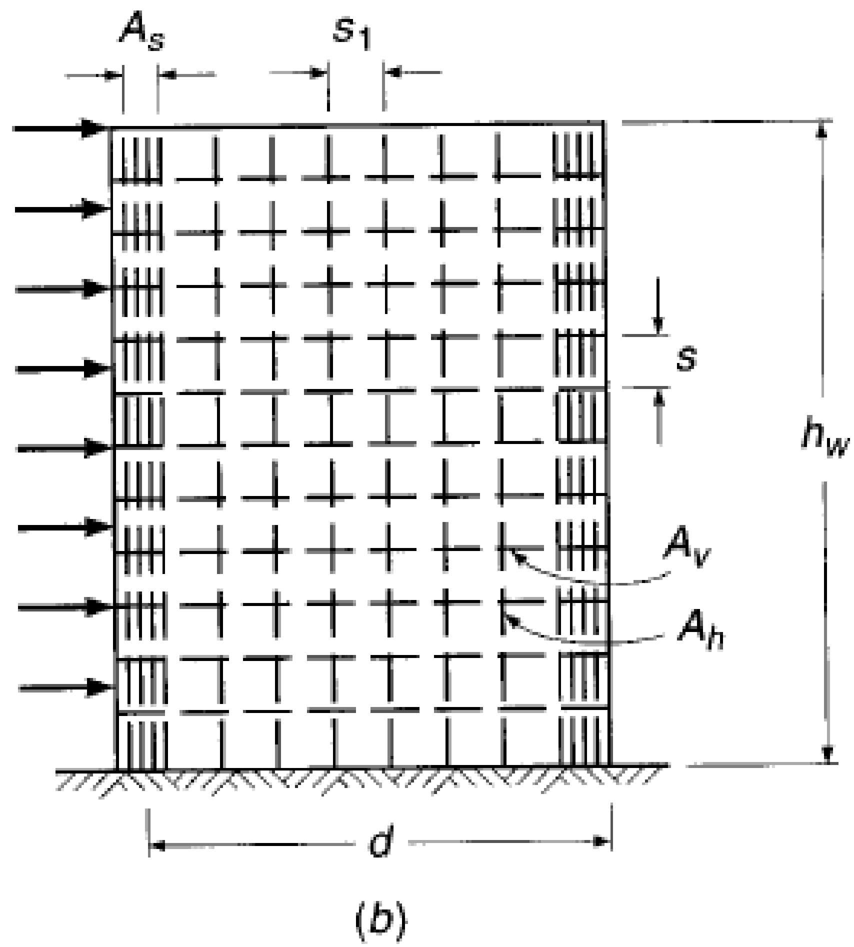
Figure 18.14 shows a typical shear wall with height $h_w$, length $l_w$, and thickness $h$. It is assumed to be fixed at its base and loaded horizontally along its left edge. Vertical flexural reinforcement of area $A_s$ is provided at the left edge, with its centroid a distance $d$ from the extreme compression face. To allow for reversal of load, identical reinforcement is provided along the right edge. Horizontal shear reinforcement with area $A_v$ at spacing $s$ is provided, as well as vertical shear reinforcement with area $A_h$ at spacing $s_1$. Such distributed steel is normally placed in two layers, parallel to the faces of the wall.

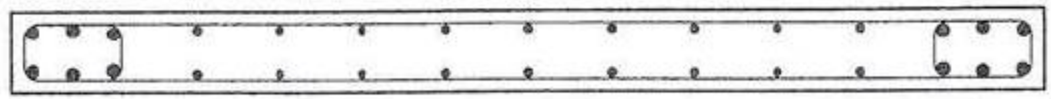
**FIGURE 18.14**
Geometry and reinforcement of a typical shear wall:
(*a*) cross section;
(*b*) elevation.



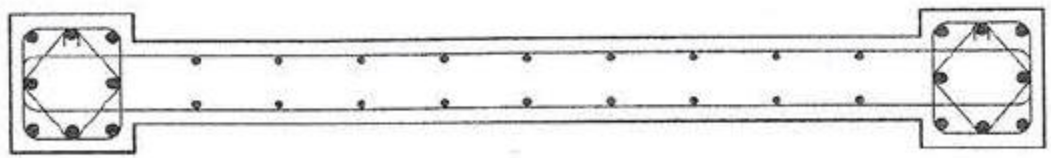



(a) Typical wall section.

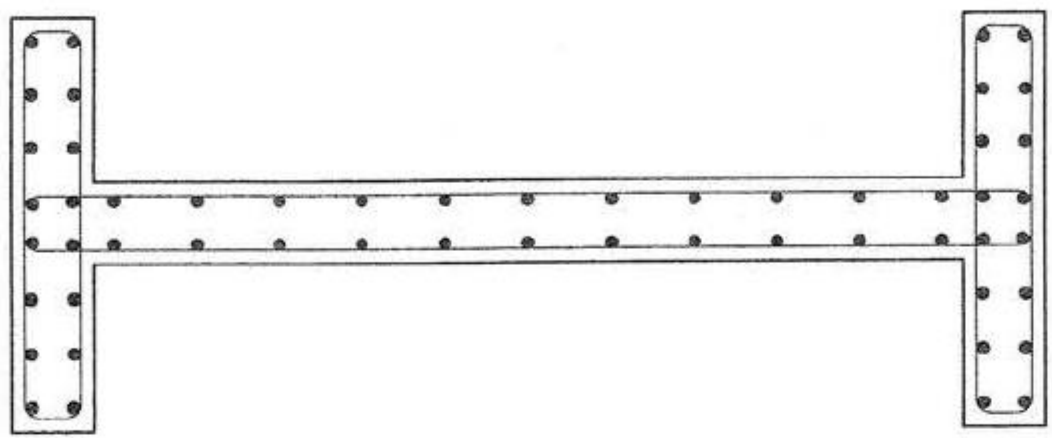




(a) Boundary element within dimensions of wall.

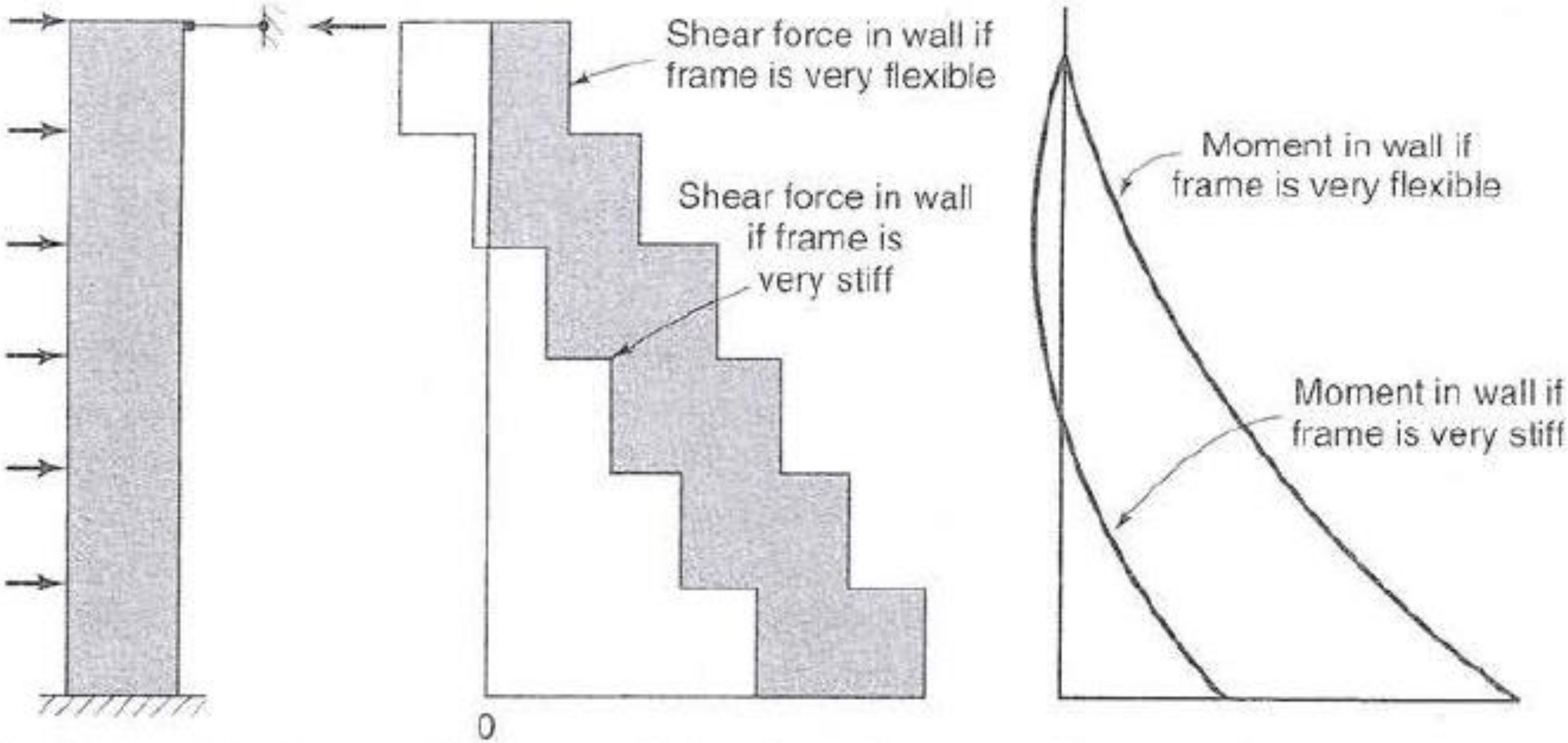


(b) Wall with enlarged boundary element.



(c) Wall with reinforcement concentrated in flanges.

Fig. 18-16
Structural walls with concentrated reinforcement at their edges.




(a) Idealization of the frame from a wall–frame building as a propped cantilever.

(b) Range of shear-force diagrams for wall.

(c) Range of moment diagrams for wall.

Fig. 18-7
Effect of frame stiffness on shear and moment in the shear wall.

# Failure mode: high-rise

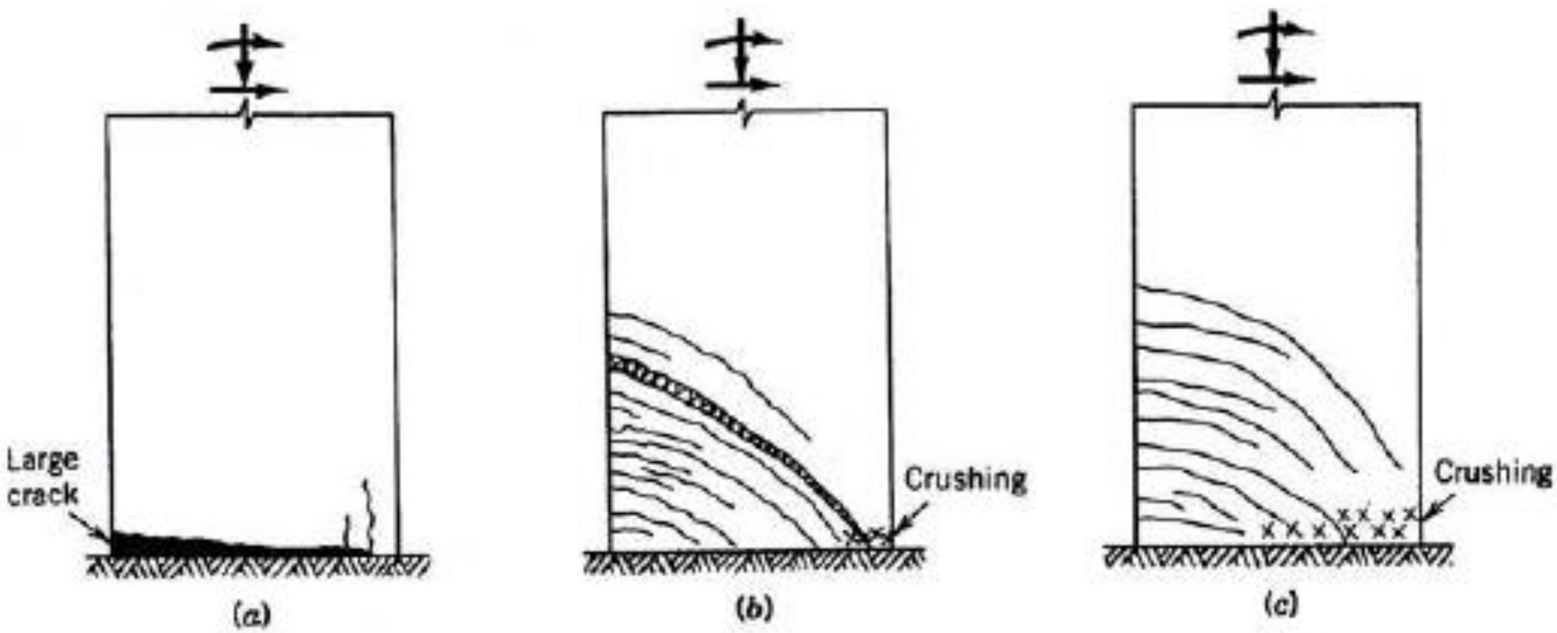



**Figure 22.6** Failure modes: high-rise walls. (*a*) Fracture of steel. (*b*) Flexure-shear failure. (*c*) Failure by concrete crushing.

# Failure mode: low-rise

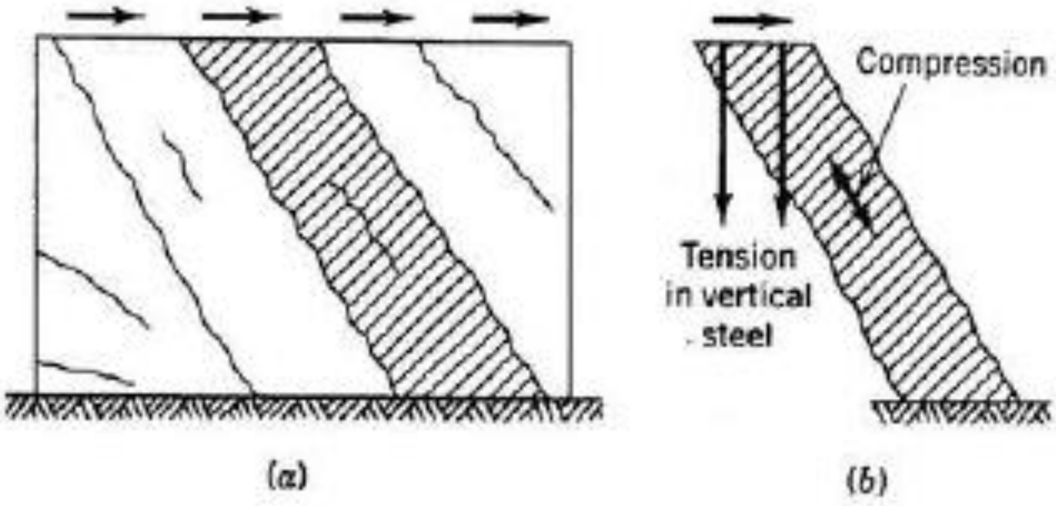



**Figure 22.8** Failure mode: low-rise wall. (*a*) Shear crack pattern. (*b*) Compression strut between cracks.

## 22.3 Design of Walls: Shear

The design of shear walls follows the same approach as that for beams. The provisions for shear design of walls are contained in Code 11.10—Special provisions for walls. Design of the horizontal section for shear in the plane of wall is based on

$$V_u \leq \phi V_n \qquad \text{(Code Eq. 11.1)}$$

$$V_n = V_c + V_s \qquad \text{(Code Eq. 11.2)}$$

An upper limit on $V_n$ is based on tests of walls[1,7]

$$V_n \leq 10\sqrt{f'_c}hd \tag{22.1}$$

where $h$ is the thickness of the wall and $d$ is the effective depth that is taken as 0.8 of the horizontal length of the wall $\ell_w$. A larger value of $d$ may be used if determined by a strain compatibility analyses of the wall.

The shear strength provided by the concrete for walls subjected to a net axial compression is

$$V_c = 2\sqrt{f'_c}hd \tag{22.2}$$

For walls subjected to a net axial tension

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f'_c}hd \tag{22.3}$$

when $N_u$ is the factored axial tensile force (in pounds) taken as negative for tension and $A_g$ is the gross cross-sectional area of the wall.

Code 11.10.6 provides some alternative (more detailed) equations for calculating $V_c$. These alternative equations may yield a value of $V_c$ that is greater than is computed using Eqs. 22.2 or 22.3, but is not used here.

The critical section for shear is taken at a distance equal to $\frac{1}{2}$ the horizontal length of the wall $\ell_w/2$ or $\frac{1}{2}$ the height of the wall $h_w/2$, whichever is less. Sections between the base of the wall and the critical section should be designed for the shear at the critical section.

If the factored shear force $V_u$ exceeds the shear strength $\phi V_c$, horizontal shear reinforcement must be provided to satisfy Code Eqs. 11.1 and 11.2. The shear strength provided by the horizontal reinforcement is computed in the same way as in beams.

$$V_s = \frac{A_{vh} f_y d}{s_2} \qquad \text{(Code Eq. 11.34)}$$

The terms in Code Eq. 11.34 are defined in Fig. 22.10. Combining Code Eqs. 11.1, 11.2, and 11.34, the area of horizontal reinforcement in each layer spaced at $s_2$ is $A_{vh} = (V_u - \phi V_c)s_2/(\phi f_y d)$. The minimum horizontal reinforcement $A_{vh} = 0.0025 s_2 h$. Minimum vertical reinforcement must be provided to satisfy the greater of

$$A_{vv} \geq \left[0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)\left(\frac{A_{vh}}{s_2 h} - 0.0025\right)\right] s_1 h$$

$$A_{vv} \geq 0.0025 s_1 h \qquad (22.4)$$

but $A_{vv}/s_1h$ does not need to be greater than $A_{vh}/s_2h$. $A_{vv}$ and $A_{vh}$ represent the area of steel in a layer at a spacing of $s_1$ or $s_2$, respectively. As $h_w/\ell_w$ is reduced, the amount of vertical shear reinforcement needed increases, as discussed previously.

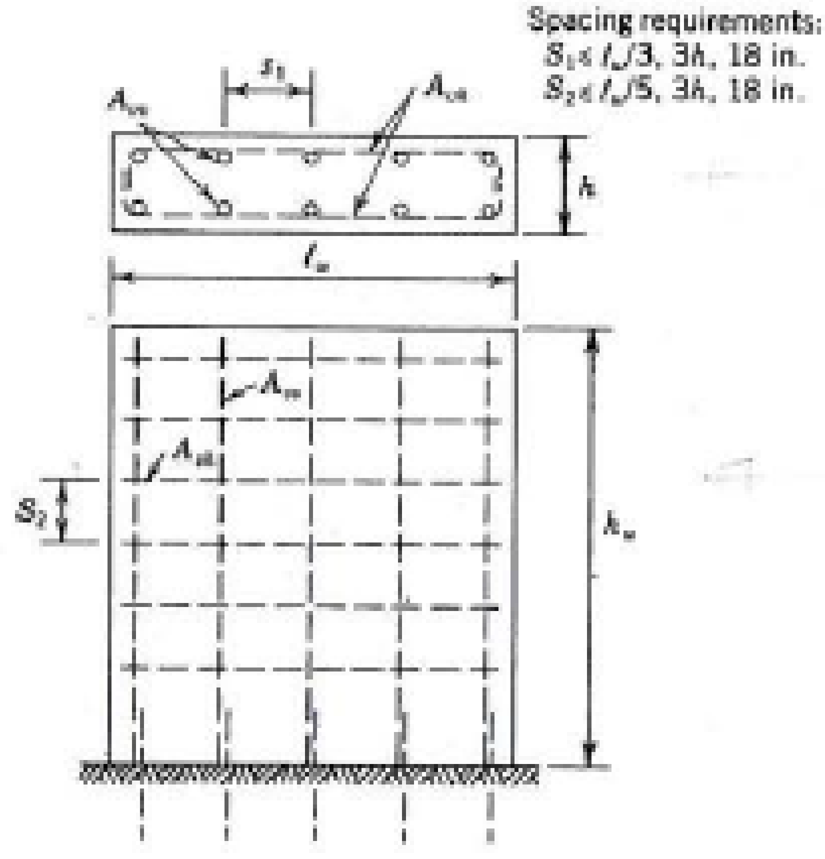



**Figure 22.10** Reinforcement in a wall.

If $V_u$ is less than $\phi V_c$, the minimum reinforcement previously specified is supplied, except when $V_u < \phi V_c/2$. The minimum reinforcement also must be checked against the requirements of Code Chapter 14. Code 14.3 requires that

$$A_{vv} \geq 0.0012 s_1 h \text{ for \#5 or smaller bars}$$

$$A_{vv} \geq 0.0015 s_1 h \text{ for \#6 or larger bars}$$

The design basis for shear walls, according to ACI Code 11.9, is of the same general form as that used for ordinary beams:

$$V_u \le \phi V_n \tag{18.1}$$

where

$$V_n = V_c + V_s \tag{18.2}$$

Based on tests (Refs. 18.9 and 18.10), an upper limit has been established on the nominal shear strength of walls:

$$V_n \le 10\lambda\sqrt{f'_c}\, hd \tag{18.3}$$

where $\lambda$ is the lightweight concrete strength modification factor (see Section 4.5a). In this and all other equations pertaining to the design of shear walls, the distance $d$ is taken equal to $0.8l_w$. A larger value of $d$, equal to the distance from the extreme compression face to the center of force of all reinforcement in tension, may be used when determined by a strain compatibility analysis.

The value of $V_c$, the nominal shear strength provided by the concrete, may be based on the usual equations for beams, according to ACI Code 11.9.5. For walls subject to vertical compression,

$$V_c = 2\lambda\sqrt{f'_c}\,hd \tag{18.4}$$

and for walls subject to vertical tension $N_u$,

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\lambda\sqrt{f'_c}\,hd \tag{18.5}$$

Here, $N_u$ is the factored axial load in pounds, taken negative for tension, and $A_g$ is the gross area of horizontal concrete section in square inches. Alternately, the value of $V_c$

When the factored shear force $V_u$ does not exceed $\phi V_c/2$, a wall may be reinforced according to minimum requirements. When $V_u$ exceeds $\phi V_c/2$, reinforcement for shear is to be provided according to the following requirements.

The nominal shear strength $V_s$ provided by the horizontal wall steel is determined on the same basis as for ordinary beams:

$$V_s = \frac{A_v f_y d}{s} \tag{18.8}$$

where $A_v$ = area of horizontal shear reinforcement within vertical distance $s$, $in^2$
$s$ = vertical distance between horizontal reinforcement, in.
$f_y$ = yield strength of reinforcement, psi

Substituting Eq. (18.8) into Eq. (18.2), then combining with Eq. (18.1), one obtains the equation for the required area of horizontal shear reinforcement within a distance $s$:

$$A_v = \frac{(V_u - \phi V_c)s}{\phi f_y d} \tag{18.9}$$

The minimum permitted ratio of horizontal shear steel to gross concrete area of vertical section is

$$\rho_t = 0.0025 \tag{18.10}$$

and the maximum spacing $s$ is not to exceed $l_w/5$, $3h$, or 18 in.

Test results indicate that for low shear walls, vertical distributed reinforcement is needed as well as horizontal reinforcement. Code provisions require vertical steel of area $A_h$ within a spacing $s_1$, such that the ratio of vertical steel to gross concrete area of horizontal section will be not less than

$$\rho_l = 0.0025 + 0.5\left(2.5 - \frac{h_w}{l_w}\right)(\rho_t - 0.0025) \tag{18.11}$$

nor less than 0.0025. However, the vertical reinforcement ratio need not be greater than the required horizontal reinforcement ratio. The spacing of the vertical bars is not to exceed $l_w/3$, $3h$, or 18 in.

TABLE 18-1 Minimum Reinforcement in Walls Compared to Other Members[a]

| Reason | ACI Code Section | Requirement | Maximum Spacing |
|---|---|---|---|
| Shrinkage and temperature | 7.12.2.1 | (b) Slabs where Grade-60 deformed bars or welded-wire fabric (plain or deformed) are used: 0.0018 | Five times the slab thickness, no farther apart than 18 in. |
| Minimum flexural steel in slabs | 10.5.4 | The minimum area of tensile reinforcement in the direction of the slab span is the same as by 7.12.2.1 | Three times the slabthickness, no farther apart than 18 in. |
| Deep beams | 11.7.4 | The area of shear reinforcement perpendicular to the span shall not be less than $0.0025b_w s$ | $s$ shall not exceed d/5 or 12 in. |
| | 11.7.5 | The area of shear reinforcement parallel to the span not be less than $0.0015b_w s_2$ | $s_2$ shall not exceed d/5 or 12 in. |
| Walls | 11.9.9.2 | Ratio $\rho_t$ of horizontal shear reinforcement area to gross concrete area shall not be less than 0.0025. | Spacing of horizontal shear reinforcement shall not exceed $\ell_w/5$, $3h$, or 18 in. |
| | 11.9.9.4 | Ratio $\rho_\ell$ of vertical shear reinforcement shall not be less than $\rho_\ell = 0.0025 + 0.5(2.5 - h_w/\ell_w) \times (\rho_t - 0.0025)$ nor 0.0025. | Spacing of vertical shear reinforcement shall not exceed $\ell_w/3$, $3h$, or 18 in. |
| Slab reinforcement | 13.3.1 | Area of reinforcement in each direction shall be determined from moments at critical sections, but shall not be less than required by 7.12.2.1 | Spacing of reinforcement at critical sections shall not exceed two times the slab thickness |
| Minimum reinforcement—Walls | 14.3.2 | Minimum ratio of vertical reinforcement area to gross concrete area shall be: (a) 0.0012 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi. | 14.3.5 Vertical and horizontal reinforcement shall not be spaced farther apart than three times the wall thickness, nor farther apart than 18 in. |
| | 14.3.3 | Minimum ratio of horizontal reinforcement area to gross concrete area shall be: (a) 0.0020 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi. | |

[a]If more than one of these sections apply, the sections requiring the largest minimum area and the smallest spacing shall govern.

## 22.4 Design of Walls: Flexure

As shown in Fig. 22.5$c$, the wall must be designed to resist the moment at the base. The section at the base is subjected to an axial load produced by gravity loads on the wall. To consider the effect of both forces in combination, an interaction diagram (Chapter 6) could be developed. In low-to-moderate rise structures, however, the axial compressive load is generally quite low, that is, less than the balanced load on the section. Therefore, it is conservative to consider only the moment at the base. The problem can be further simplified by ignoring the distributed steel across the length of the wall and providing enough reinforcement at the ends to resist the moment. If boundary elements are used, a very close approximation of the longitudinal (vertical) steel required in the boundary element is $A_s = M_u/f_y(\ell_w - c_w)$, where $\ell_w$ and $c_w$ are as shown in Fig. 22.7. The same area of steel should be provided in both boundary elements because the lateral force can generally act in either direction. If the wall is a constant thickness, the same approach can be used by concentrating more reinforcement at the ends over a distance $c_w$.

In many cases the distributed reinforcement provides sufficient moment capacity without any additional reinforcement at the ends or in boundary elements. In this case a flexural analysis of the section is carried out to determine the moment capacity at the base.[1] The reinforcement is represented by a "plate" of length $\ell_w$ and a thickness such that the area $A_{st}$ is the same as that provided by reinforcing bars uniformly distributed across the wall section as shown in Fig. 22.11. The moment capacity of the section is given by

$$M_u = \phi\left[0.5A_{st}f_y\ell_w\left(1 - \frac{z}{\ell_w}\right)\right] \tag{22.5}$$

where $z/\ell_w = 1/(2 + 0.85\beta_1\ell_w h f'_c/A_{st}f_y)$ and $\beta_1$ is as defined for the equivalent rectangular stress block in Chapter 3 (0.85 for $f'_c = 4$ ksi). Equation 22.5 can be modified for cases in which axial load is included.[1] For low axial loads, strength is controlled by flexure and $\phi$ can be taken as 0.9.

## 22.5 Design Example

A three-story wall is subjected to factored wind forces as shown in Fig. 22.12. The wall is 15-ft long and 8-in. thick. Design reinforcement for the wall at the first level between the base and second floor. Grade 60 reinforcement and $f'_c = 3000$ psi are used.

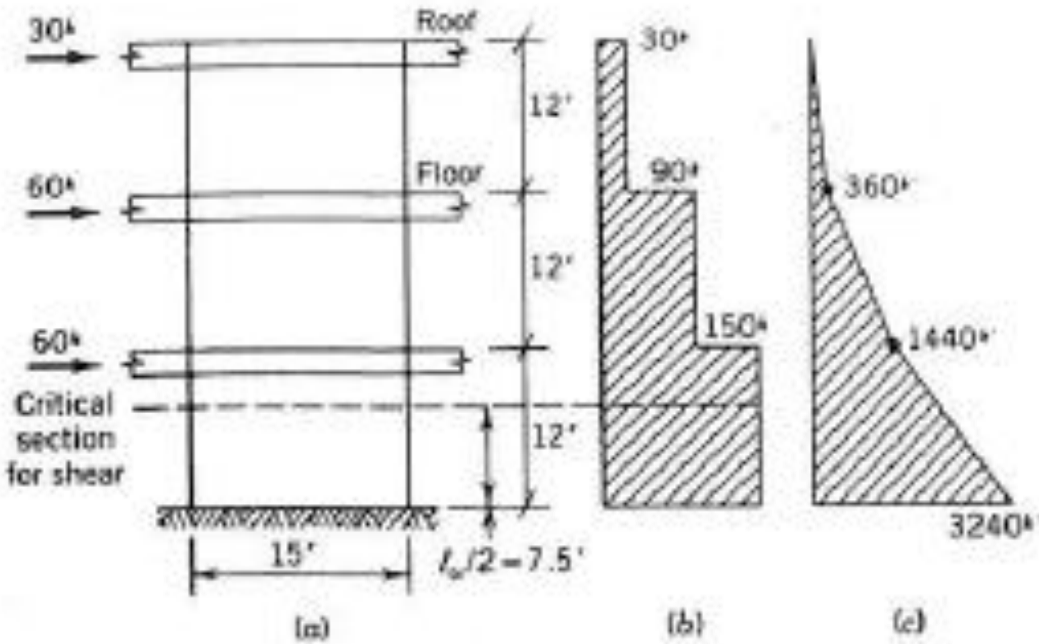



**Figure 22.12** (*a*) Factored lateral wind loads, $u = 0.75(1.3W)$, Code 9.3. (*b*) Shear diagram. (*c*) Moment diagram.

## Solution

Because shear is constant over the first level, the entire 12-ft height is designed for the same shear force. Check maximum shear strength permitted.

$$V_u = \phi V_n \leq \phi 10\sqrt{f'_c}hd \; (h = 0.8\ell_w)$$
$$V_u = 150^k \leq 0.85 \times 10\sqrt{3000} \times 0.8 \times 15 \times 12 \times 8/1000$$
$$150^k \leq 536^k \qquad \textbf{O.K.}$$

Determine shear strength provided by concrete $V_c$.

$$V_c = 2\sqrt{f'_c}hd = 2\sqrt{3000} \times 0.8 \times 15 \times 12 \times 8/1000 = 126^k$$

$V_u = 150^k$, which is greater than $\phi V_c = 107^k$, so shear reinforcement must be provided.

Determine the required horizontal shear reinforcement $A_{vh}$

$$V_s = V_u/\phi - V_c = 150/0.85 - 126^k = 50.5^k$$

From Code 11.34,

$$A_{vh}/s_2 = V_s/f_y d = 50.5/60 \times 0.8 \times 15 \times 12 = 0.0058 \text{ in.}^2\text{/in.}$$

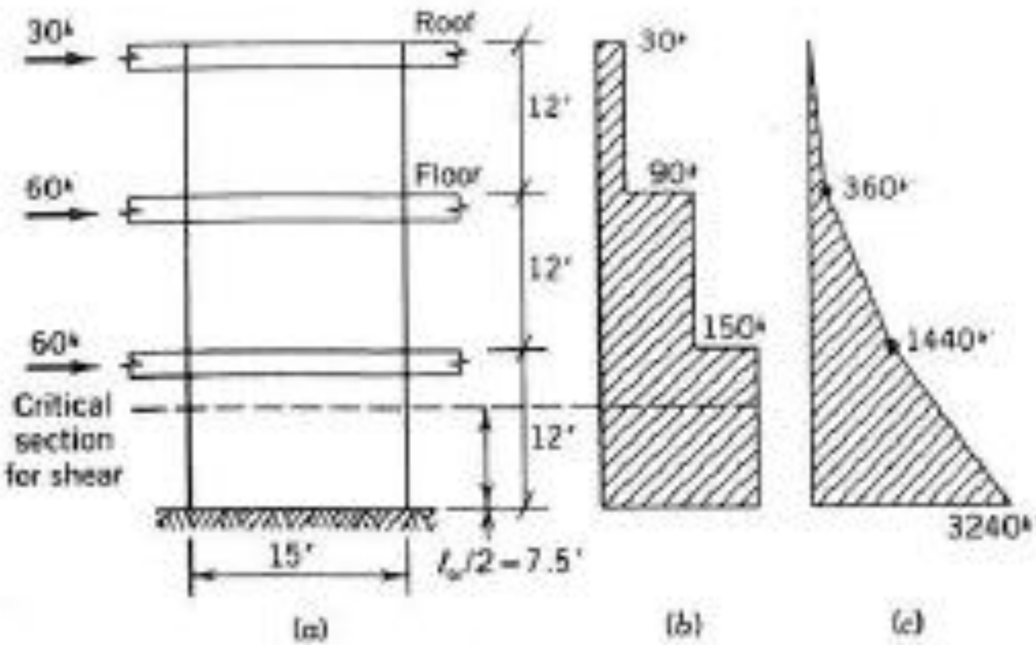



**Figure 22.12** (*a*) Factored lateral wind loads, $u = 0.75(1.3W)$, Code 9.3. (*b*) Shear diagram. (*c*) Moment diagram.

The minimum horizontal reinforcement is $A_{vh}/s_2 \geq 0.0025h = 0.0025 \times 8 = 0.02$ in.$^2$/in., which is greater than 0.0058, so minimum requirements control. Because $s_2$ must not exceed $\ell_w/5 = 15 \times 12/5 = 36$ in., $3h = 3 \times 8 = 24$ in., or 18 in., use $s_2 \leq 18$ in. Using two curtains of steel and #3 bars, $A_{vh} = 2 \times 0.11 = 0.22$ in.$^2$ and $s_2 = A_{vh}/0.02 = 0.22/0.02 = 11$-in. spacing of layers of horizontal reinforcement. If #4 bars are used, $s_2 = 0.40/0.02 = 20$ in. Use #4 bars at 18-in. spacing.

Determine the required vertical shear reinforcement $A_{vv}$ (Eq. 22.4).

$$\frac{A_{vv}}{s_1} \geq \left[0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)\left(\frac{A_{vh}}{s_2 h} - 0.0025\right)\right] h$$

$$\frac{A_{vv}}{s_1} \geq \left[0.0025 + 0.5\left(2.5 - \frac{36}{15}\right)(0.0025 - 0.0025)\right] 8$$

$$\frac{A_{vv}}{s_1} = 0.0025 \times 8 = 0.02 \text{ in.}^2/\text{in.}$$

which is the same as the horizontal reinforcement. If $A_{sh}$ is controlled by minimum requirements, $A_{sv}$ also will be controlled by the minimum values. Therefore, vertical shear reinforcement also can be provided by using #4 bars at 18 in.

Determine moment capacity with vertical steel provided for shear. Vertical shear reinforcement is effective in flexure. Using Eq. 22.4

$$M_u = \phi\left[0.5A_{st}f_y\ell_w\left(1 - \frac{z}{\ell_w}\right)\right]$$

The area of steel $A_{st} = 0.4 \times 15 \times 12/18 = 4.0$ in.$^2$ and $z/\ell_w = 1/[2 + 0.85 \times 0.85 \times 15 \times 12 \times 8 \times 3/(4 \times 60)] = 0.07$ so that

$$M_u = 0.9 \times 0.5 \times 4.0 \times 60 \times 15 \times (1 - 0.07) = 1507 \text{ k-ft.}$$

Moment from factored wind loads is 3240 k-ft so additional vertical steel is needed at end sections for flexure. If steel is added over a 1-ft section at each end to provide an additional capacity $M'_u = 3240 - 1507 = 1733$ k-ft, the area of steel is

$$A_s = M'_u/f_y(\ell_w - c_w) = 1733 \text{ k-ft}/60 \text{ ksi} \times (18 - 1 \text{ ft}) = 1.69 \text{ in.}^2$$

This additional capacity can be provided with 4-#7 bars ($4 \times 0.6 = 2.4$ in.$^2$) that meet the flexural requirements and replace 2-#4 bars at the end of the wall ($1.69 + 0.4 = 2.1$ in.$^2$). The vertical steel should be spliced at the base to meet the splice requirements of the Code discussed in Chapter 8. The wall reinforcement detail for the first level is shown in Fig. 22.13.

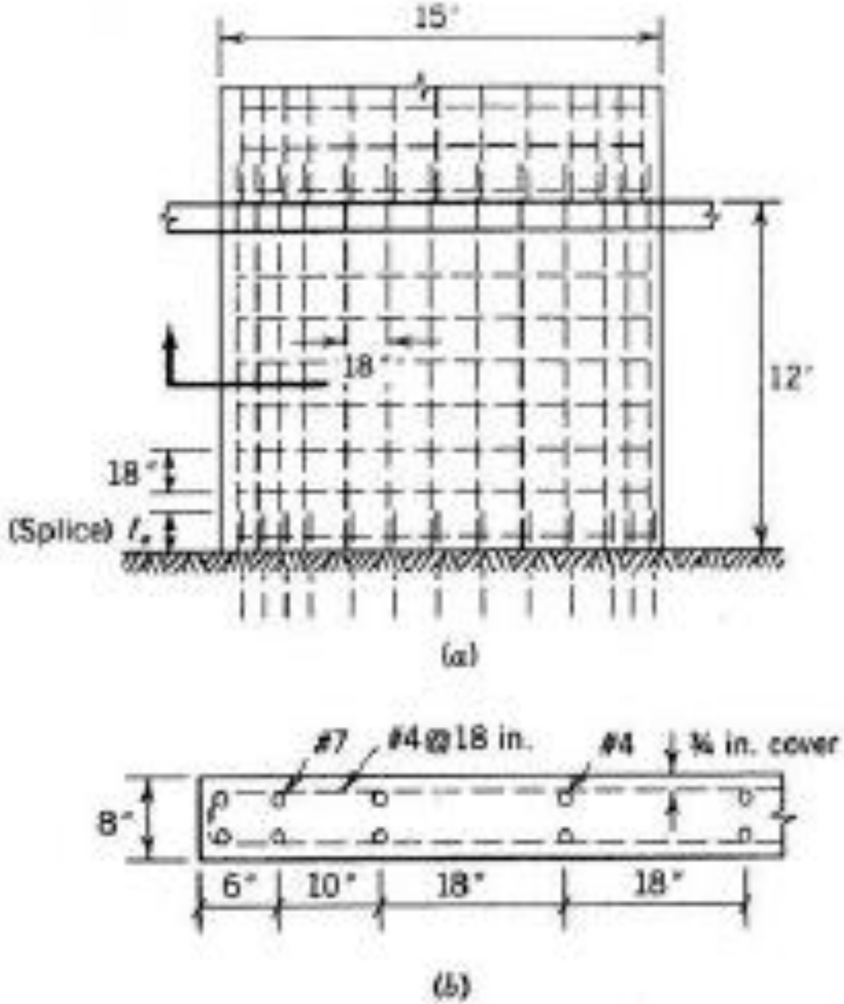
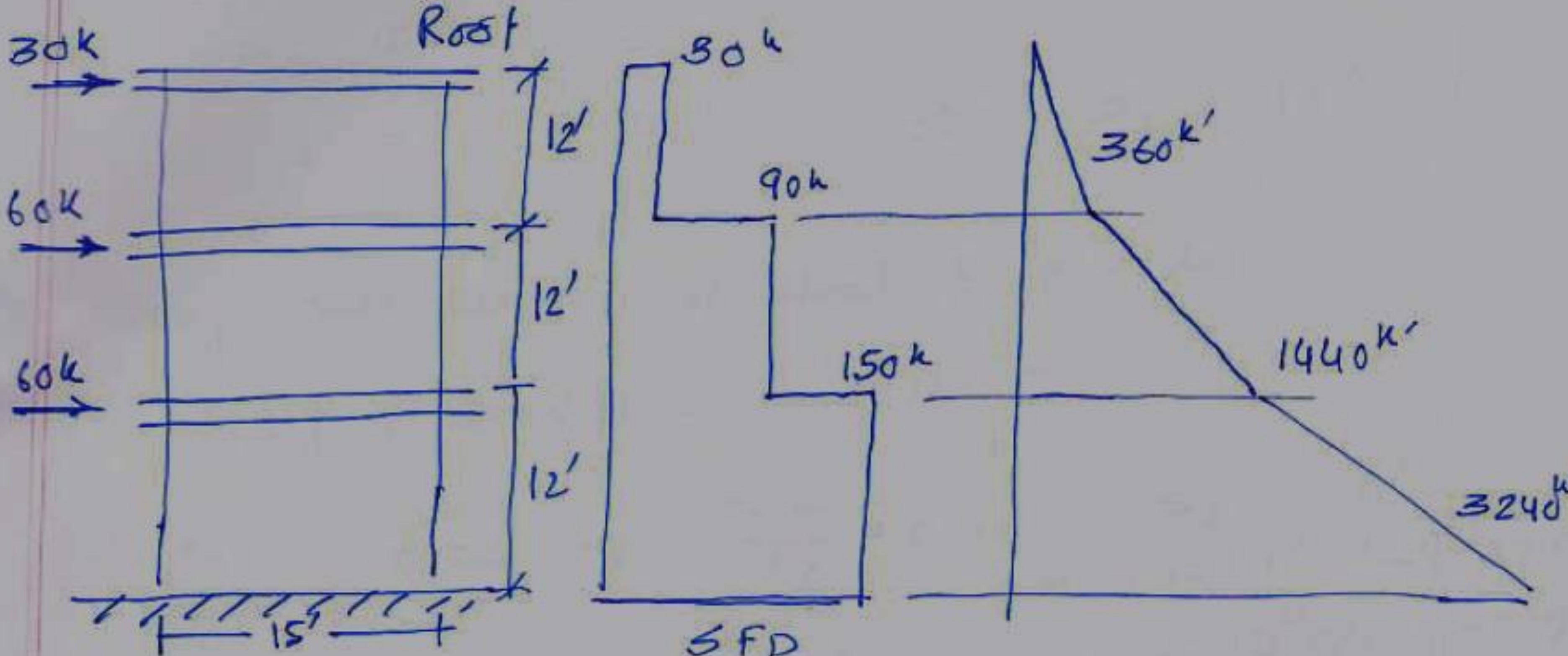



**Figure 22.13** Design example: reinforcement. (*a*) Elevation of wall. (*b*) Section through wall.

A three-story shear wall is subjected to factored wind forces as shown in fig. The wall is 15 ft long and 8 in thick. Design reinforcement at first level. $f_c' = 3$ ksi, $f_y = 60$ ksi.



Factored wind load = $U = 0.75 * 1.3 W$

$d = 0.8 l_w$
$= 144$ in

$V_u = \phi V_n \leq \phi 10 \sqrt{f_c'}\, h d$

$= 150^k \leq 0.75 * 10 \sqrt{3000} * 144 * 8 = 473^k$ (ok.)

$V_c = 2\sqrt{f_c'}\, h d = 2\sqrt{3000} * 144 * 8 = 126^k$.

$\phi V_c = 0.75 * 126 = 94.6^k$.

$V_u > \phi V_c$ shear reinf needed.

$S = \frac{\phi A_v f_y d}{V_u - \phi V_c}$

$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_y d}$

$\frac{A_v}{s} = \frac{150 - 94.6}{0.75 * 60 * 144} = 0.00855 \text{ in}^2/\text{in} = \frac{A_{vn}}{s_2}$

Min^m horizontal reinforcement

$$\frac{A_{vh}}{s_2} \geq 0.0025h$$

$$= 0.0025 * 8$$

$$= 0.02 \text{ in}^2/\text{in}$$

$L_w/5$, $3h$, $18''$ — $18''$ controls.

$\frac{L_w}{5} = \frac{15*12}{5} = 36''$

$3h = 3*8 = 24''$ so, $s_2 \leq 18''$

$18''$

Using two curtain of steel of #3 bar

$A_{vh} = 2*0.11 = 0.22 \text{ in}^2$

$11''$

$A_{vh}$

$$\frac{A_{vh}}{s_2} = 0.02 \Rightarrow \frac{.22}{s_2} = 0.02 \qquad s_2 = 11''$$

# 3 @ 11" c/c two layer

# 4 @ 18" c/c two lay

if # 4 $\frac{0.4}{s_2} = .02 \qquad s_2 = 20''$

vertical reinf

$$\frac{A_{vv}}{s_1} \geq \left[0.0025 + 0.5\left(2.5 - \frac{h_w}{l_w}\right)\left(\frac{A_{vh}}{s_2 h} - 0.0025\right)\right] h$$

$$= \left[.0025 + 0.5\left(2.5 - \frac{36}{15}\right)\left(0.0025 - .0025\right)\right] 8$$

$$= 0.0025 \times 8 = 0.02 \text{ in}^2/\text{in}$$

# 4 @ 18" c/c

Moment capacity of vertical steel provided for shear

$$\phi M_n = \phi\left[0.5 A_{st} f_y L_w \left(1 - \frac{Z}{L_w}\right)\right]$$

$$A_{st} = 0.4 * 15 * 12/18 = 4 \text{ in}^2.$$

$$Z/L_w = \frac{1}{2 + 0.85\beta_1 \frac{L_w h f_c'}{A_{st} f_y}}$$

$$= \frac{1}{2 + .85 * .85 * \frac{15 * 12 * 8 * 3}{4 * 60}} = 0.07$$

K-

.7 1507

$4 * 60$ $k$

$$\phi M_n = 0.9\left[0.5 * 4 * 60 * 15\,(1 - 0.07)\right] = 1507^{k'} < 3240^{k'}$$

Additional vert steel reqd.

$$\phi M_{n2} = 3240 - 1507 = 1733^{k'}$$

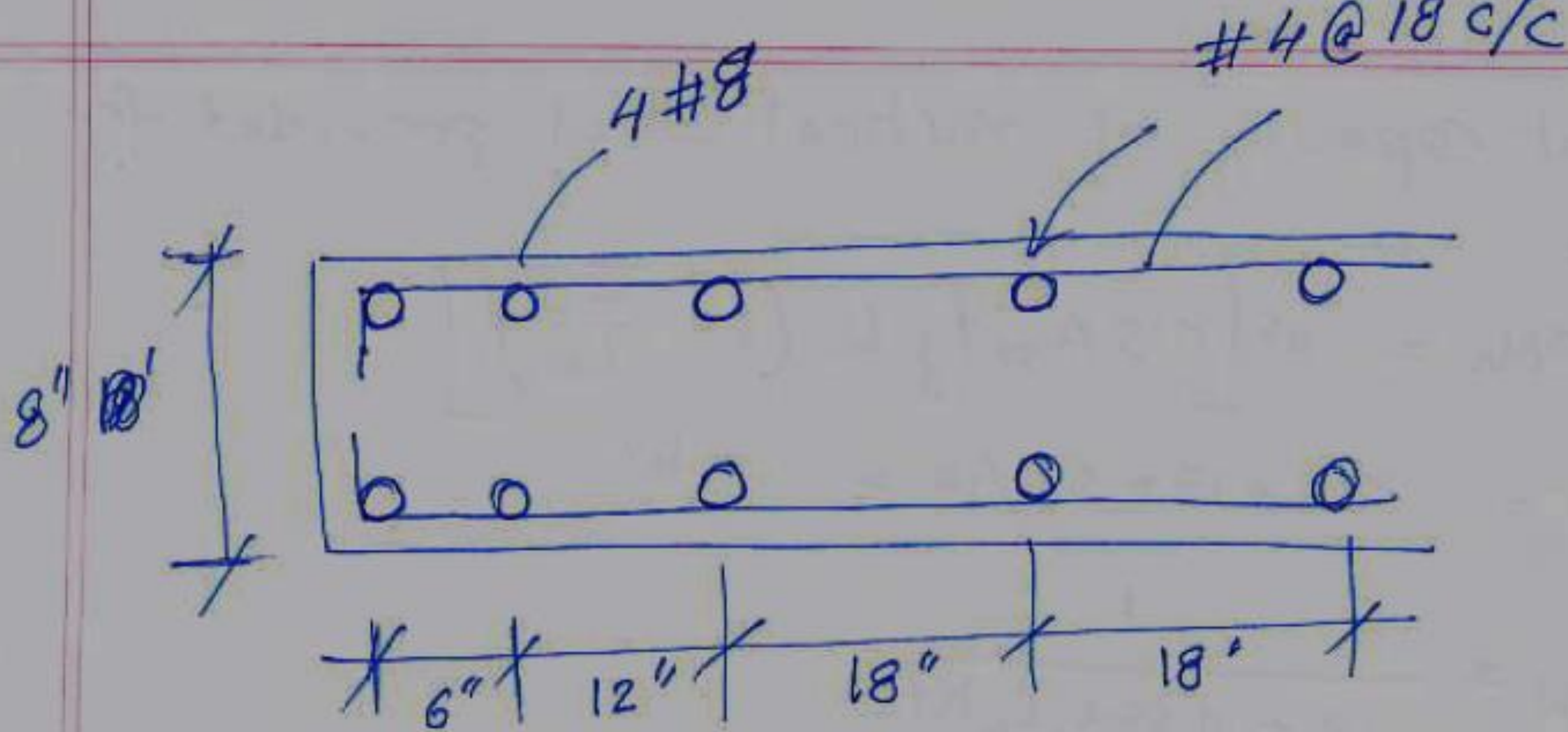
Extra steel added over 1 ft section at each end.

$$A_s = \frac{1733}{.9 * 60 * (L_w - C_w)} = \frac{1733}{.9 * 60 * (15 - 1)} = 2.29\ in^2.$$

4 #8

add area of 2#4 (replaced) $= 0.4$

$$2.29 + 0.4 = 2.69\ in^2.$$



#4 @ 18" c/c
4#8
8"
6"
12"
18"
18"