CE 315: Design of Concrete Structures I

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Syllabus

New

- Fundamental behavior of reinforced concrete
- Introduction to strength design and alternate design methods
- Flexural design of beams (singly reinforced, doubly reinforced, T-beams) using strength design method
- Shear, diagonal tension and torsion of beams
- Bond and anchorage
- Design of one-way slab
- Design of two-way edge supported slabs: using strip and alternate methods

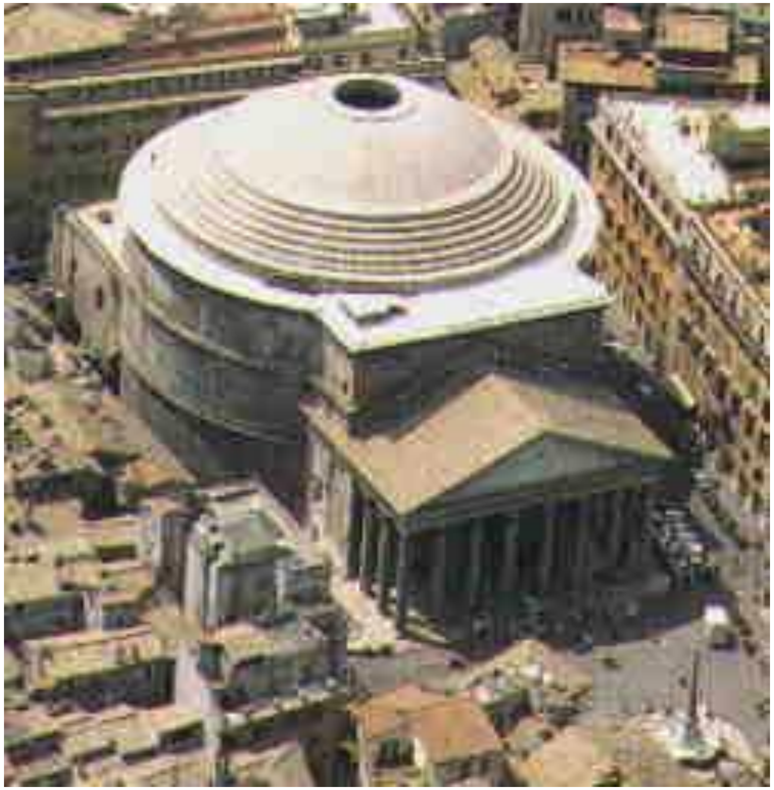
Books

- Design of Concrete Structures
  - Nilson, Darwin, Dolan 14th Ed
- Structural Concrete- Theory and Design
  - Hassoun, Al-Manaseer 4th Ed
- Reinforced Concrete- Mechanics & Design
  - Wight & McGregor 5th Ed

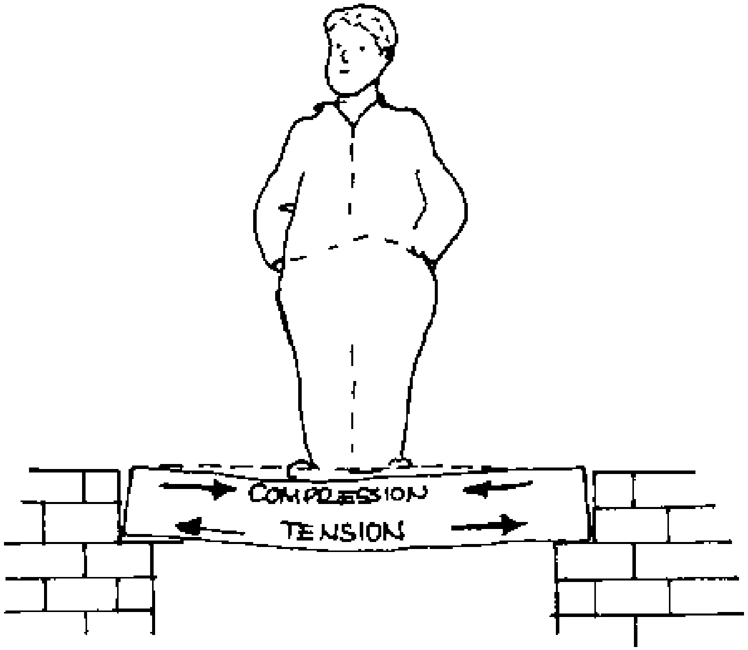
Many more……..

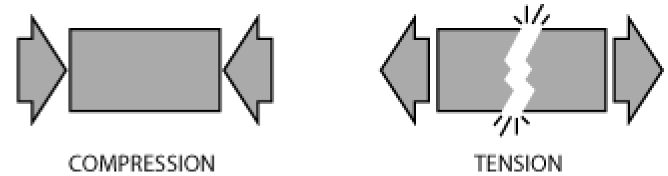
Concrete, Reinforced Concrete (RC), Prestressed Concrete (PC)

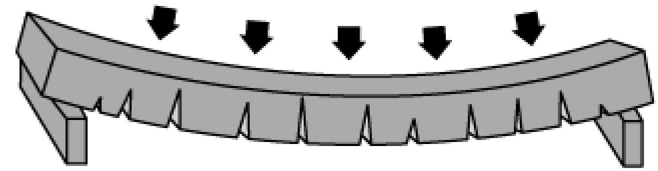
- What is concrete? Constituents?
  - Stone like material, cement, coarse and fine aggregate, water, admixture
- A bit of history
- Advantages, disadvantages
  - Easy to make, relatively low-cost, formabilty, weather and fire resistant, good comp strength
  - Weak in tension
- Reinforced concrete-mild steel
- Where to place the reinforcement-examples
- Prestressed concrete



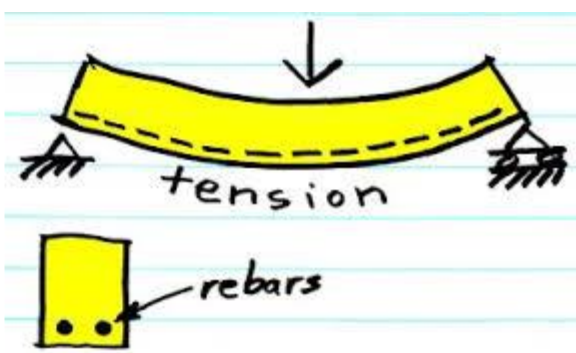
Roman Pantheon, unreinforced concrete dome, diameter 43.3m, 25BC, 125AD







Tensile forces pull apart the bottom of this concrete slab when it bends



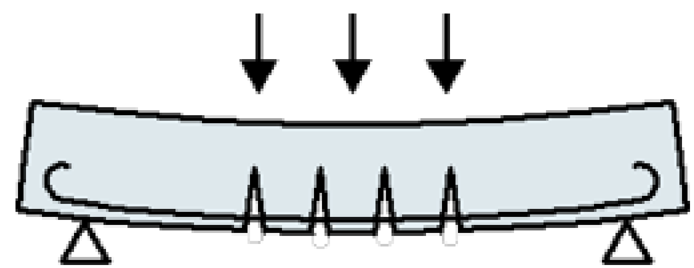
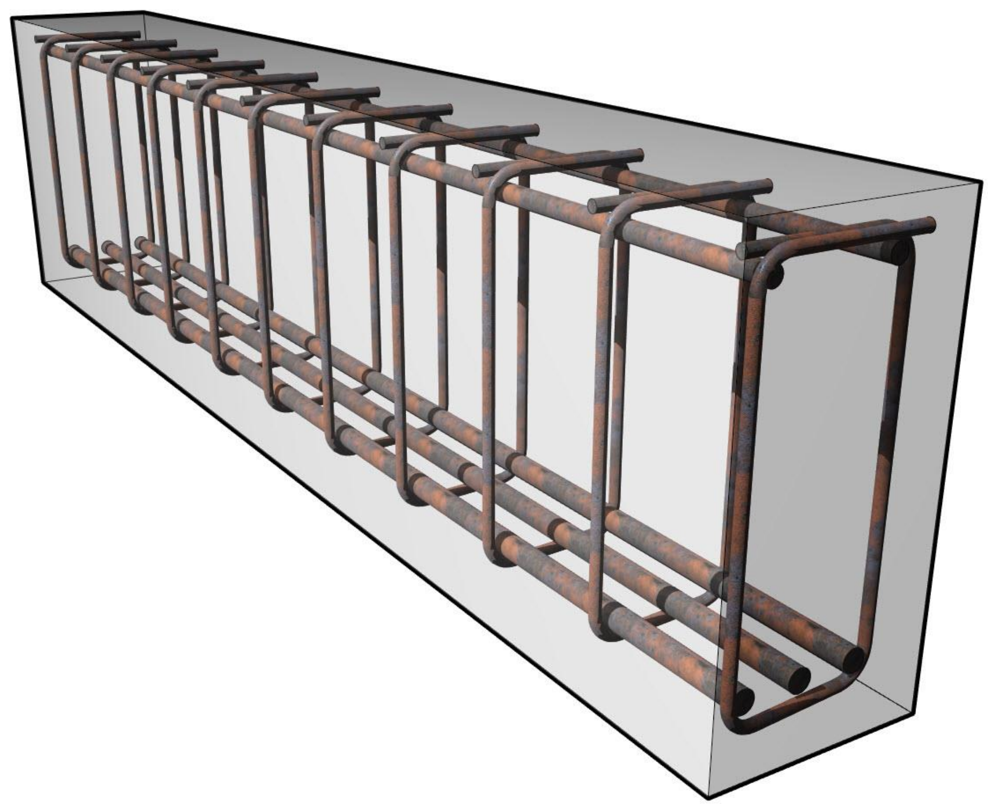


Figure 1.1 – Reinforced concrete beam under load

RC Beam



Structural forms: buildings

- Beam
- Column
- Slab

FIGURE 1.1
One-way reinforced concrete floor slab with monolithic supporting beams. (*Portland Cement Association.*)

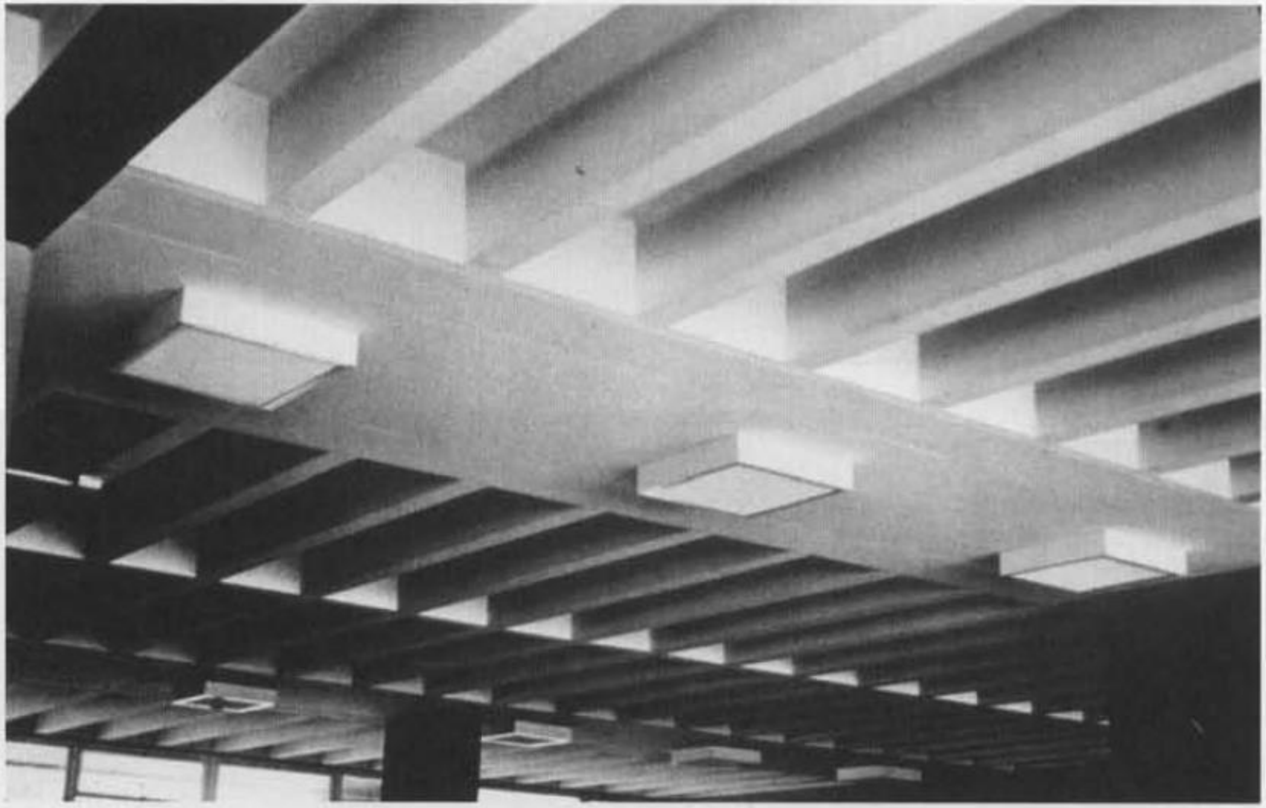


FIGURE 1.2
One-way joist floor system, with closely spaced ribs supported by monolithic concrete beams; transverse ribs provide for lateral distribution of localized loads. (*Portland Cement Association.*)

FIGURE 1.3
Flat plate floor slab, carried directly by columns without beams or girders. (*Portland Cement Association.*)

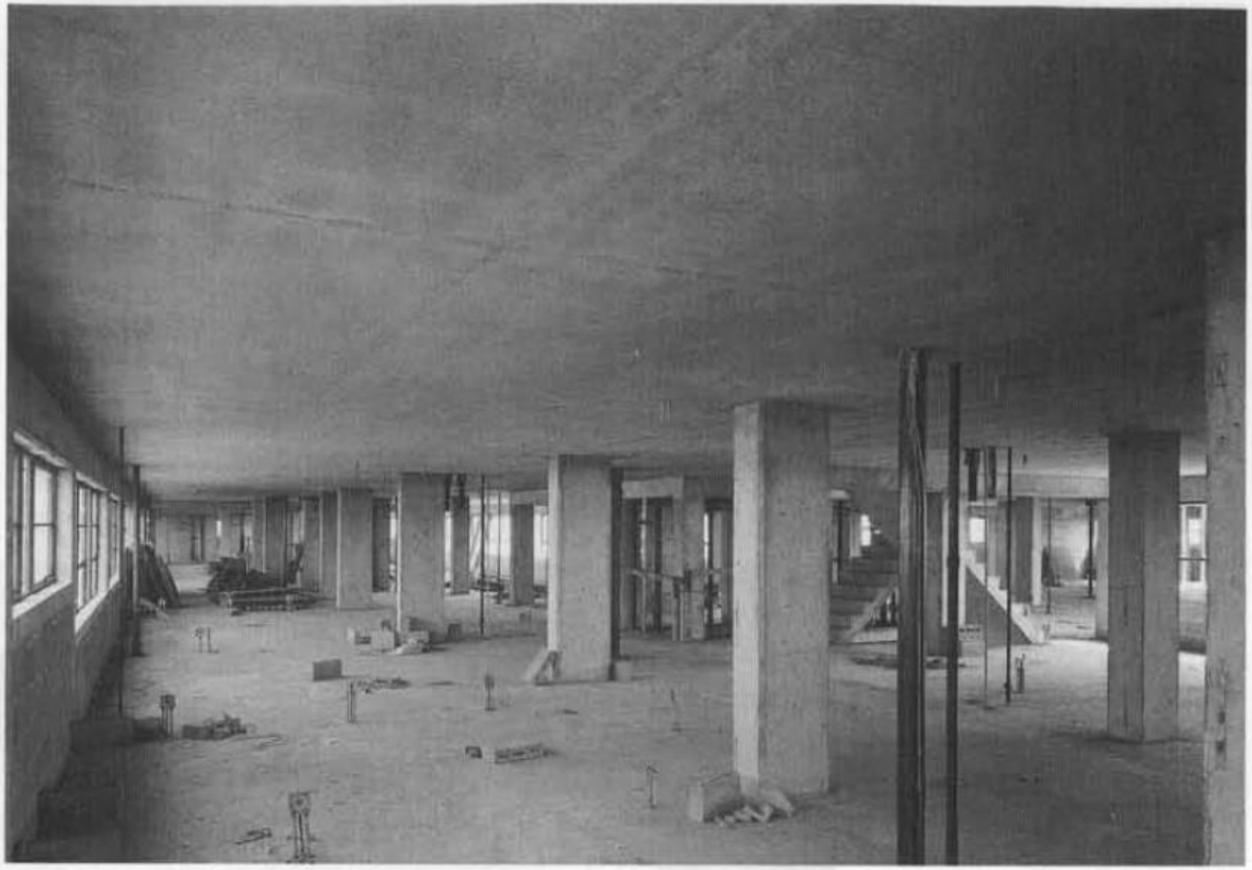


FIGURE 1.4
Flat slab floor, without beams but with slab thickness increased at the columns and with flared column tops to provide for local concentration of forces. (*University of Southern Maine.*)

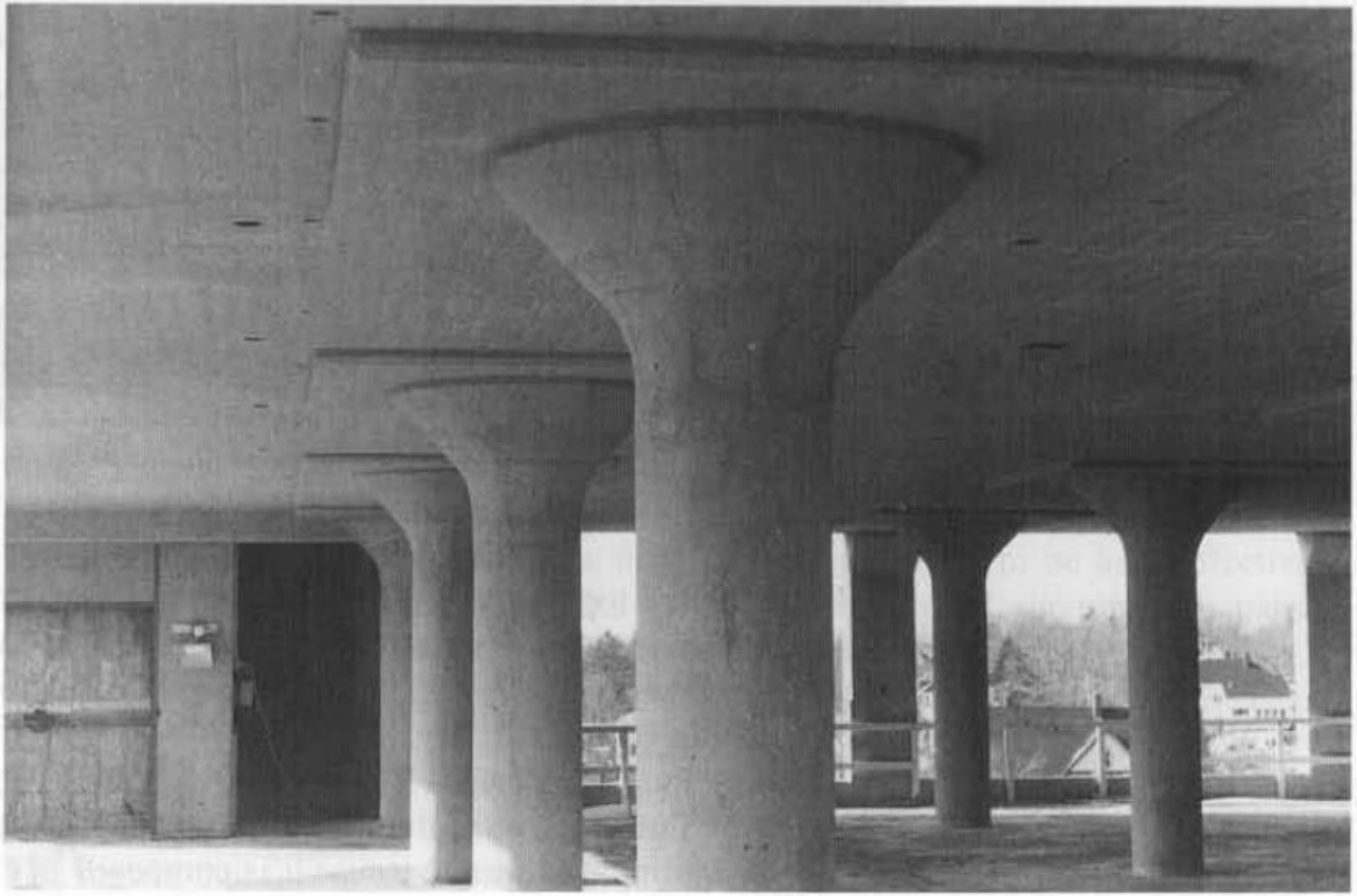
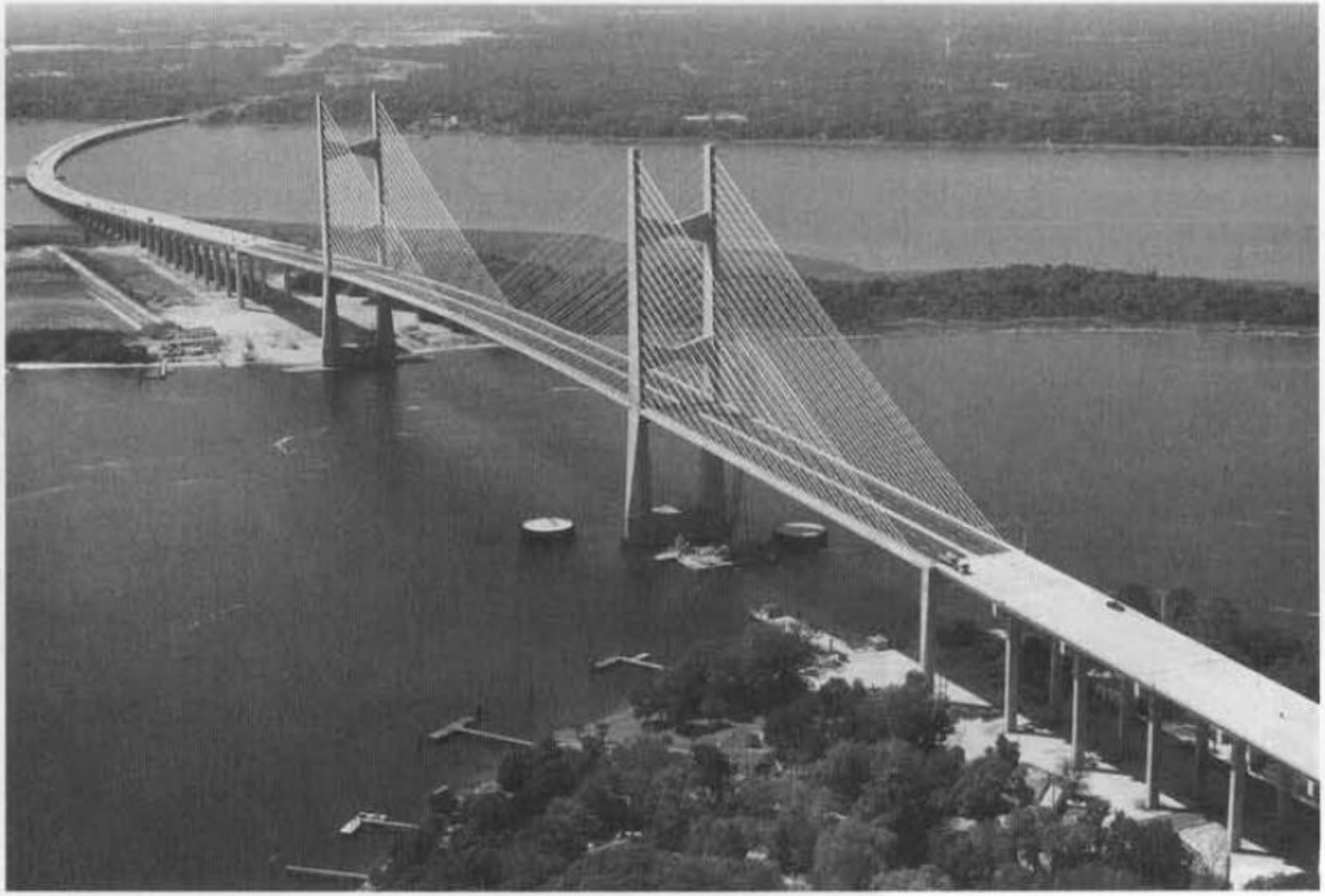


FIGURE 1.8
Napoleon Bonaparte Broward Bridge, with a 1300 ft center span at Dame Point, Jacksonville, Florida. (*HNTB Corporation, Kansas City, Missouri.*)



Loads

- Dead load attached
- Live load not attached
- Environmental load
  - Wind
  - Earthquake
  - Snow, soil pressure, temperature

  - Building codes- ACI, BNBC, IS, EuroCode

TABLE 1.1
Minimum uniformly distributed live loads

Occupancy or Use	Live Load, psf[a]	Occupancy or Use	Live Load, psf[a]
Apartments (see residential)		Dining rooms and restaurants	100
Access floor systems		Dwellings (see residential)	
Office use	50	Fire escapes	100
Computer use	100	On single-family dwellings only	40
Armories and drill rooms	150	Garages (passenger cars only)	40
Assembly areas and theaters		Trucks and buses[b]	
Fixed seats (fastened to floor)	60	Grandstands (see stadium and arena bleachers)	
Lobbies	100	Gymnasiums, main floors and balconies[c]	100
Movable seats	100	Hospitals	
Platforms (assembly)	100	Operating rooms, laboratories	60
Stage floors	150	Patient rooms	40
Balconies (exterior)	100	Corridors above first floor	80
On one and two-family residences only, and not exceeding 100 ft^2	60	Hotels (see residential)	
		Libraries	
Bowling alleys, poolrooms, and similar recreational areas	75	Reading rooms	60
		Stack rooms[d]	150
Catwalks for maintenance access	40	Corridors above first floor	80
Corridors		Manufacturing	
First floor	100	Light	125
Other floors, same as occupancy served except as indicated		Heavy	250
		Marquees and canopies	75
Dance halls and ballrooms	100	Office buildings	
Decks (patio and roof)		File and computer rooms shall be designed for heavier loads based on anticipated occupancy	
Same as area served, or for the type of occupancy accommodated		Lobbies and first-floor corridors	100

(continued)

TABLE 1.1
(Continued)

Occupancy or Use	Live Load, psf[a]
Offices	50
Corridors above first floor	80
Penal institutions	
Cell blocks	40
Corridors	100
Residential	
Dwellings (one and two-family)	
Uninhabitable attics without storage	10
Uninhabitable attics with storage	20
Habitable attics and sleeping areas	30
All other areas except stairs and balconies	40
Hotels and multifamily houses	
Private rooms and corridors serving them	40
Public rooms and corridors serving them	100
Reviewing stands, grandstands, and bleachers[c]	
Roofs	
Ordinary flat, pitched, and curved roofs	20
Roofs used for promenade purposes	60
Roofs used for roof gardens or assembly purpose	100
Roofs used for other special purposes[f]	
Awnings and canopies	
Fabric construction supported by a lightweight rigid skeleton structure[g]	5
All other construction	20
Schools	
Classrooms	40
Corridors above first floor	80
First-floor corridors	100
Sidewalks, vehicular driveways, and yards subject to trucking[e]	250
Stadiums and arenas	
Bleachers[c]	100
Fixed seats (fastened to floor)[c]	60
Stairs and exit ways	100
One and two-family residences only	40
Storage areas above ceilings	20
Storage warehouses (shall be designed for heavier loads if required for anticipated storage)	
Light	125
Heavy	250
Stores	
Retail	
First floor	100
Upper floors	73
Wholesale, all floors	125
Walkways and elevated platforms (other than exitways)	60
Yards and terraces, pedestrians	100

Wind Load

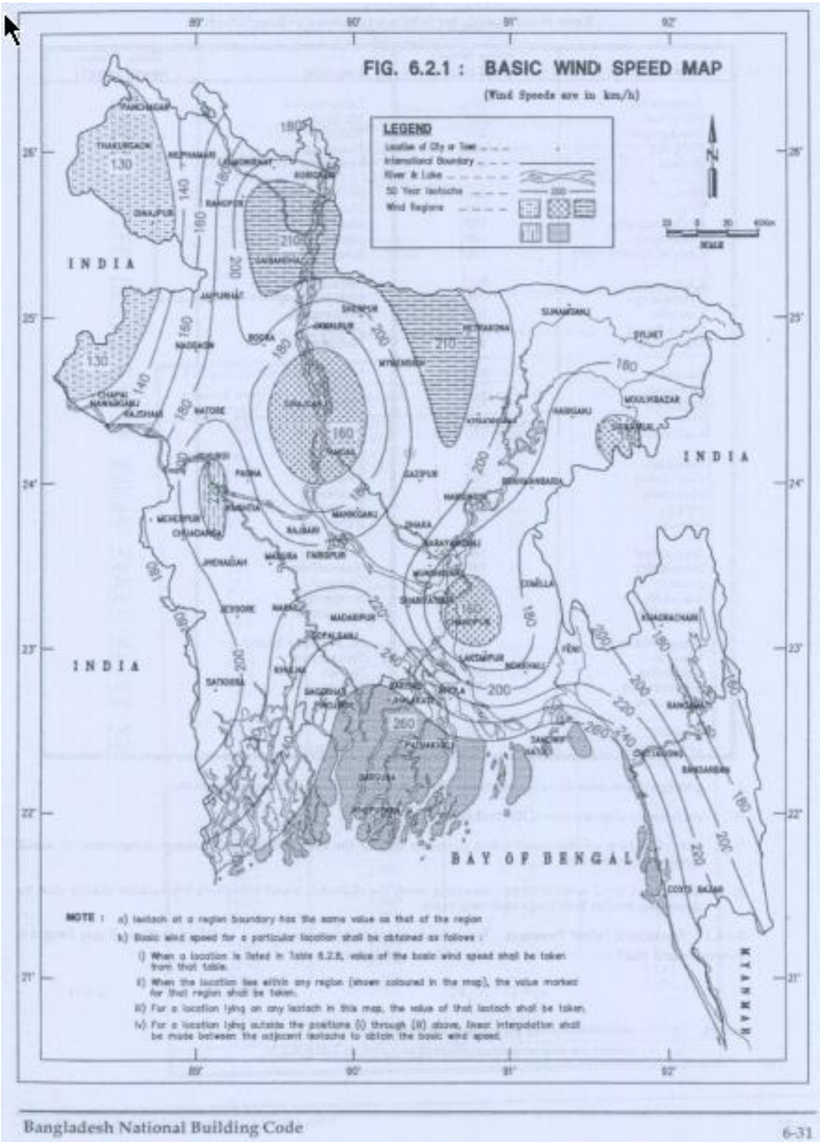
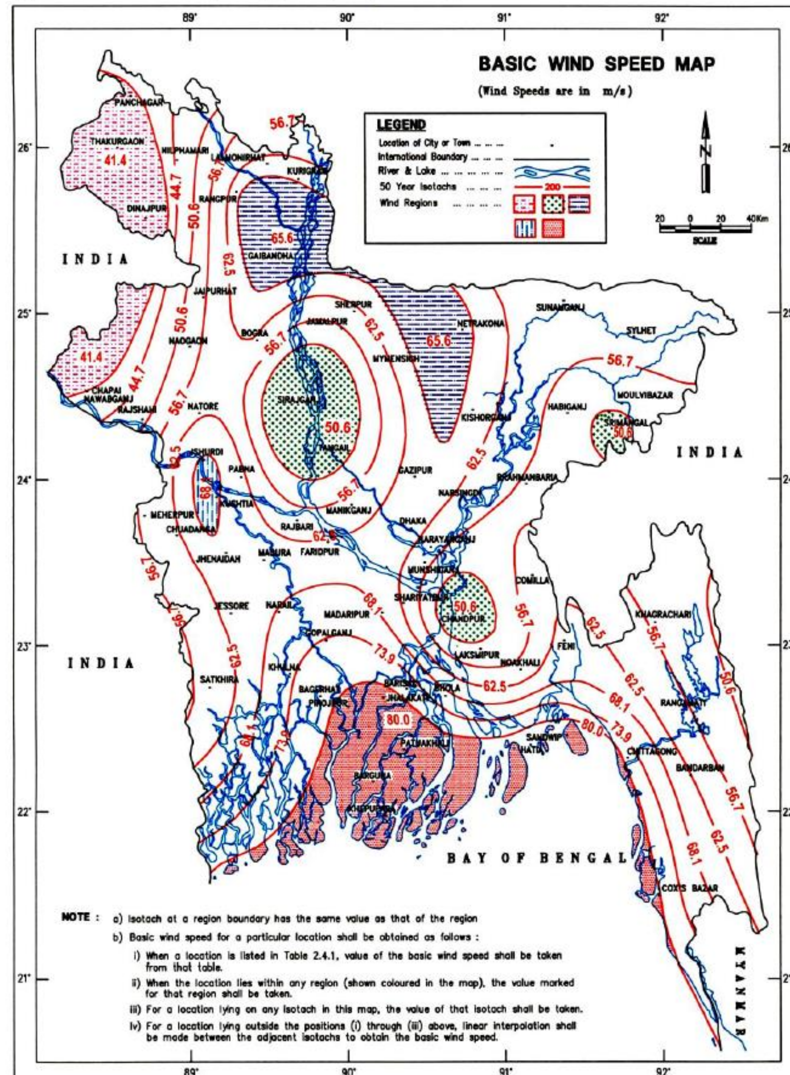
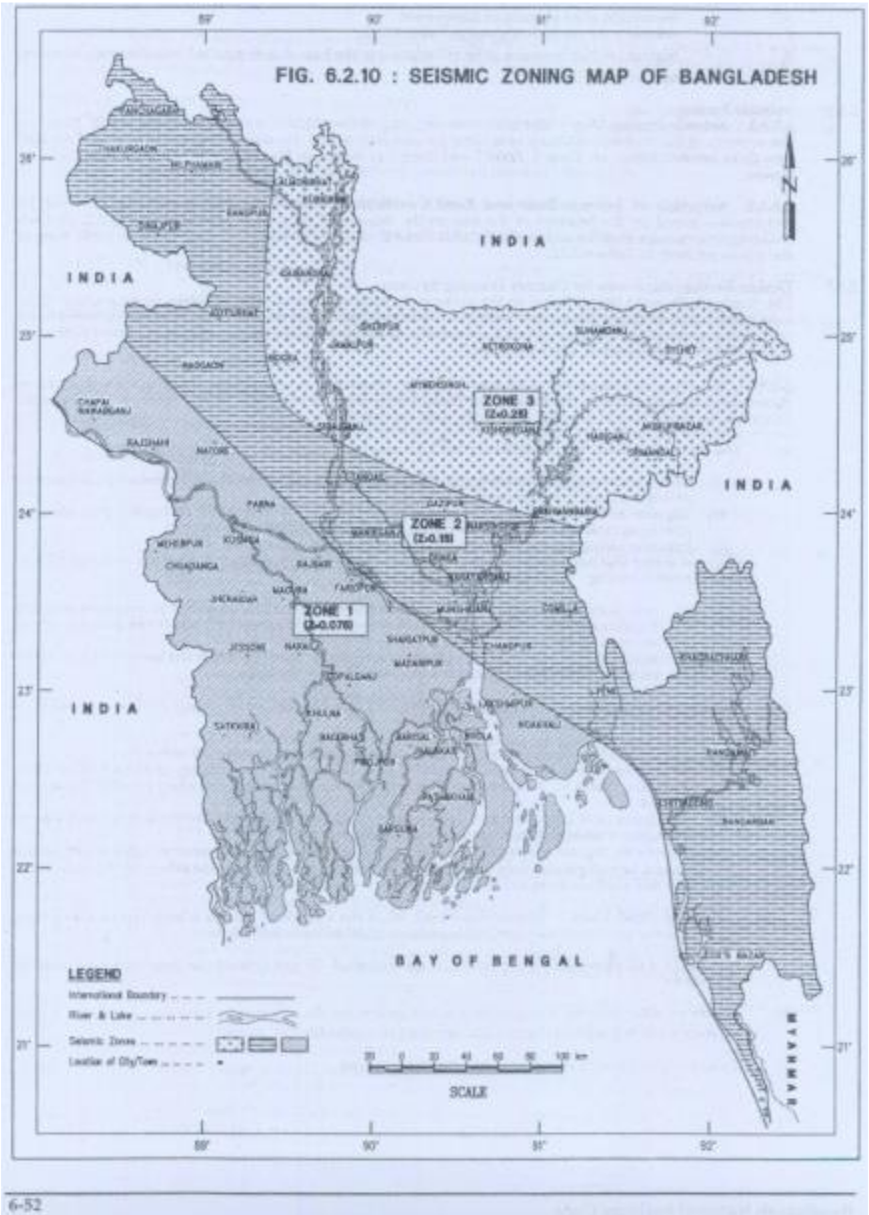
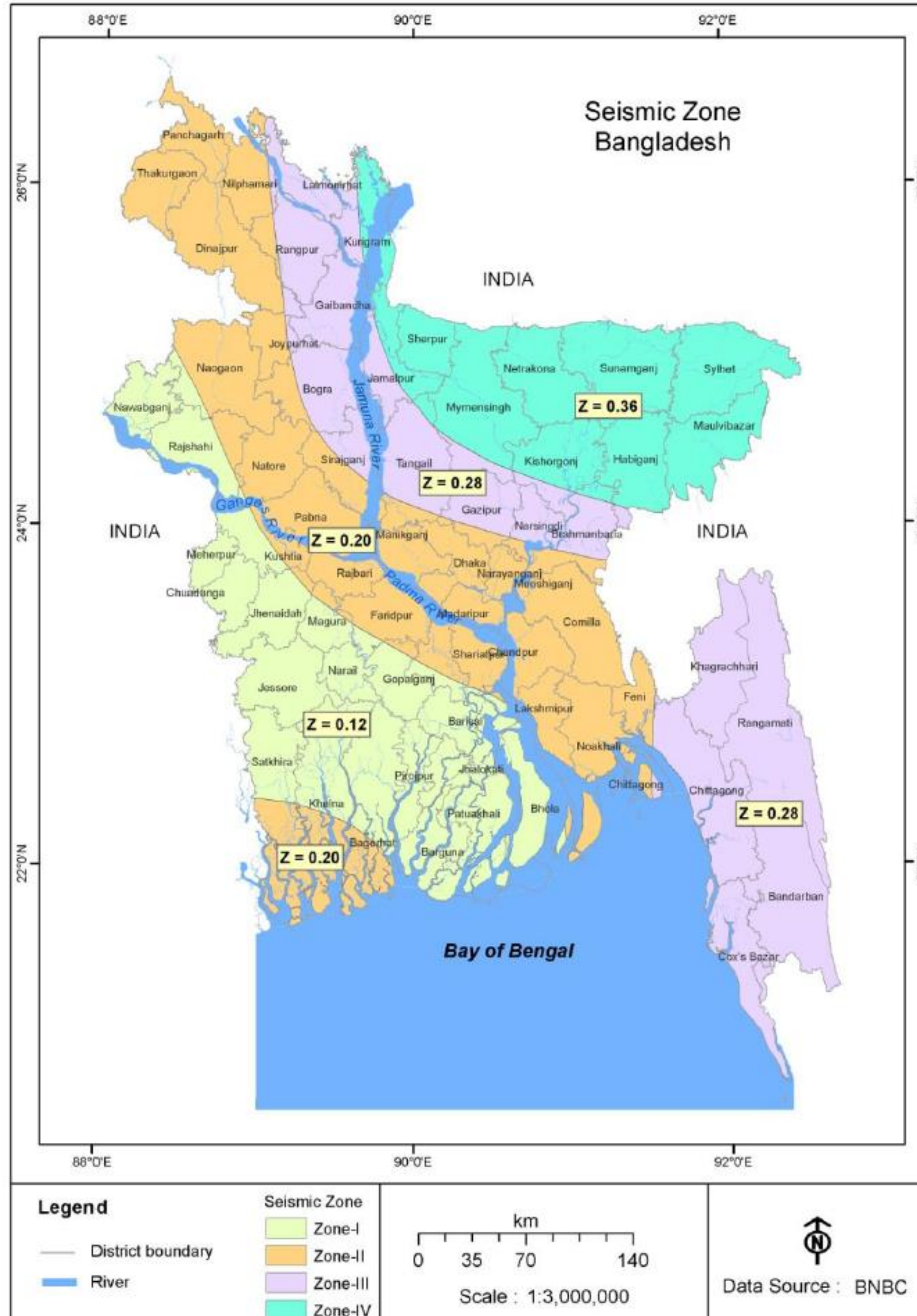


Fig. 2.4.1 Basic wind speed (V, m/s) map of Bangladesh



Earthquake Loads





Serviceability, Strength and Structural Safety

- To serve its purpose, a structure must be safe against collapse and serviceable in use

- Strength of the structure be adequate for all loads

- Serviceability – deflection small, hairline cracks, minimum vibration

Strength and safety

- If loads and moments, shears, axial force can be predicted **accurately**, safety can be ensured by providing a carrying capacity just barely in excess of the known demand.
- Capacity= Demand

Uncertainty

- There are a number of sources of uncertainty in Analysis, Design and Construction
- Read 7 points
  - Actual load may differ
  - Actual load distribution may be different
  - Assumption, simplification in analysis
  - Actual structural behaviour may differ
  - Actual member dimensions may differ
  - Reinforcement may not be in proper position
  - Actual material strength may differ
- Consideration given to consequence of failure
- Nature of failure is also important

Variability of Loads, Strength, safety

Load can be considered as random variable

Form of distribution curve (probability density function) can b determined from large scale load survey

Probability of occurrence

Area under curve is probability of occurrence

Q_d specified service load for design

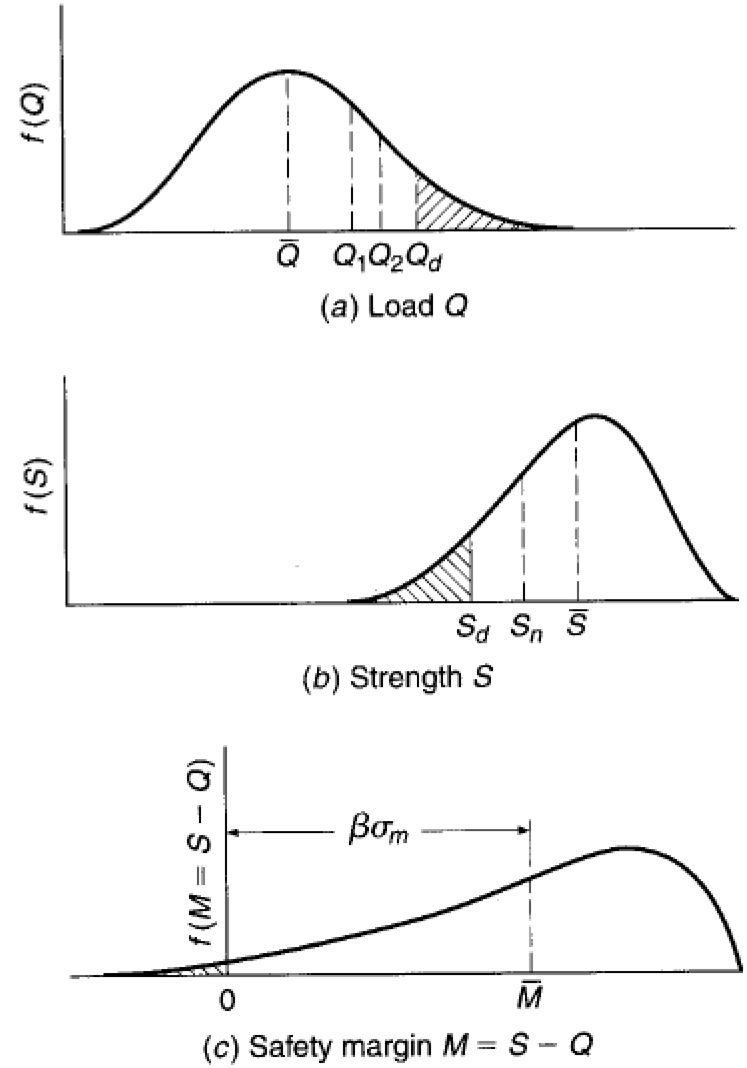
S_d Design strength

$$\psi_s \bar{S} \geq \psi_L \bar{Q}$$

M is also a random variable

Beta between 3 and 4 corresponds a probability of failure of 1:100,000

FIGURE 1.14 Frequency curves for (*a*) loads *Q*, (*b*) strengths *S*, and (*c*) safety margin *M*.



Partial safety factor

$$\phi S_n \geq \gamma Q_d$$

$$\phi S_n \geq \gamma_d D + \gamma_l L$$

$$\phi S_n \geq \gamma_{d_i} D + \gamma_{l_i} L + \gamma_{w_i} W + \cdots$$

- Strength reduction factor X Nominal Strength >

  Load Factor X

  Specified Service Load

Why partial factors are different

Concrete

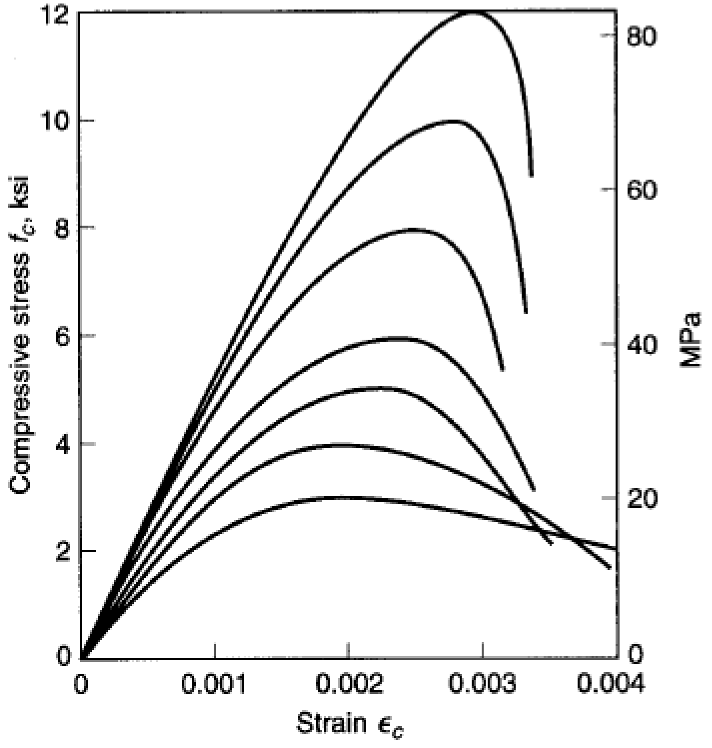
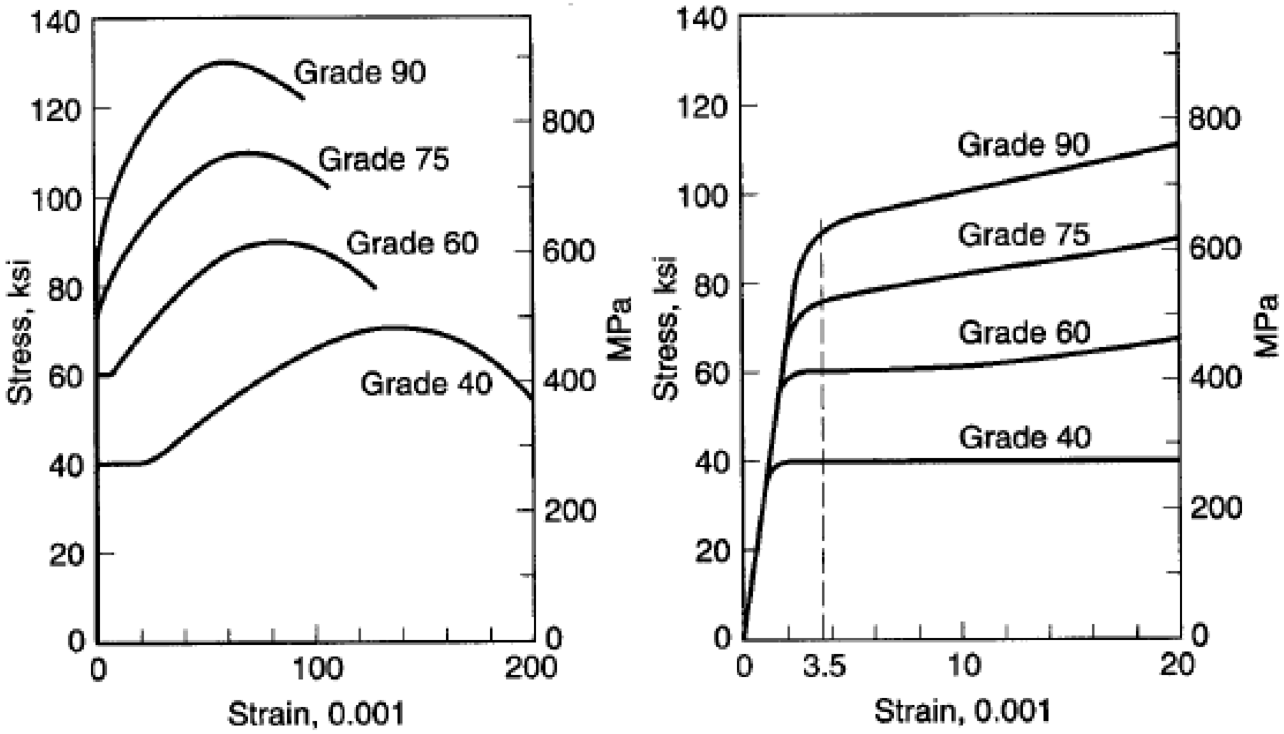


FIGURE 2.3
Typical compressive stress-strain curves for normal-density concrete with w_c = 145 pcf. (*Adapted from Refs. 2.23 and 2.24.*)

Steel

FIGURE 2.15
Typical stress-strain curves for reinforcing bars.



Design Basis

- Strength Design
- Load factored- hypothetical overload stage
- Material stress level
  - Nonlinear inelastic
  - Concrete $f_{c'}$
  - Steel reaches f_y
  - Both or one
- USD
  - Ultimate Strength Design

- Service load design
- Load unfactored
  - Service load
- Material stress level
  - At allowable stresses
  - Half of $f_{c'}$
  - Half of f_y
- WSD
  - Working Stress Design

Design Codes and Specifications

- International Building Code- consensus code
- American Concrete Institute ACI Code- *Building Code requirement for Structural Concrete -318-2008*
- AASHTO- American Association of State Highway and Transportation Officials- for bridges
- American Railway Engineering and Maintenance of Way Association –AREMA-*Manual of Railway Engineering*

Bangladesh National Building Code

- BNBC
- First in 1993
- Up-gradation is in progress

Safety provision of ACI/BNBC Code

Design strength ≥ required strength

$$\phi S_n \geq U$$

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

$$\phi P_n \geq P_u$$

Load factors

TABLE 1.2
Factored load combinations for determining required strength *U* in the ACI Code

Condition[a]	Factored Load or Load Effect U
Basic[b]	$U = 1.2D + 1.6L$
Dead plus fluid[b]	$U = 1.4(D + F)$
Snow, rain, temperature, and wind	$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$
	$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$
	$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
	$U = 0.9D + 1.6W + 1.6H$
Earthquake	$U = 1.2D + 1.0E + 1.0L + 0.2S$
	$U = 0.9D + 1.0E + 1.6H$

[a] Where the following represent the loads or related internal moments or forces resulting from the listed factors: D = dead load; E = earthquake; F = fluids; H = weight or pressure from soil; L = live load; L_r = roof live load; R = rain; S = snow; T = cumulative effects of temperature, creep, shrinkage, and differential settlement; W = wind.

[b] The ACI Code includes F or H loads in the load combinations. The "basic" load condition of $1.2D + 1.6L$ reflects the fact that most buildings have neither F nor H loads present and that $1.4D$ rarely governs design.

Probability of overload 1/1000

Strength reduction factor

TABLE 1.3
Strength reduction factors in the ACI Code

Strength Condition	Strength Reduction Factor ϕ
Tension-controlled sections[a]	0.90
Compression-controlled sections[b]	
Members with spiral reinforcement	0.75
Other reinforced members	0.65
Shear and torsion	0.75
Bearing on concrete	0.65
Post-tensioned anchorage zones	0.85
Strut-and-tie models[c]	0.75

[a] Chapter 19 discusses reductions in ϕ for pretensioned members where strand embedment is less than the development length.

[b] Chapter 3 contains a discussion of the linear variation of ϕ between tension and compression-controlled sections. Chapter 8 discusses the conditions that allow an increase in ϕ for spirally reinforced columns.

[c] Strut-and-tie models are described in Chapter 10.

Probability of understrength 1/100

- Probability of Structural failure

  1/100,000

Fundamental Assumption for RC Behavior

1. Equilibrium
2. Strain in steel=Strain in surrounding concrete
3. Plane cross section remain plane
4. Concrete does not resist any tension
5. The theory is based on the actual stress-strain relationship of concrete and steel or some simplified equivalent.

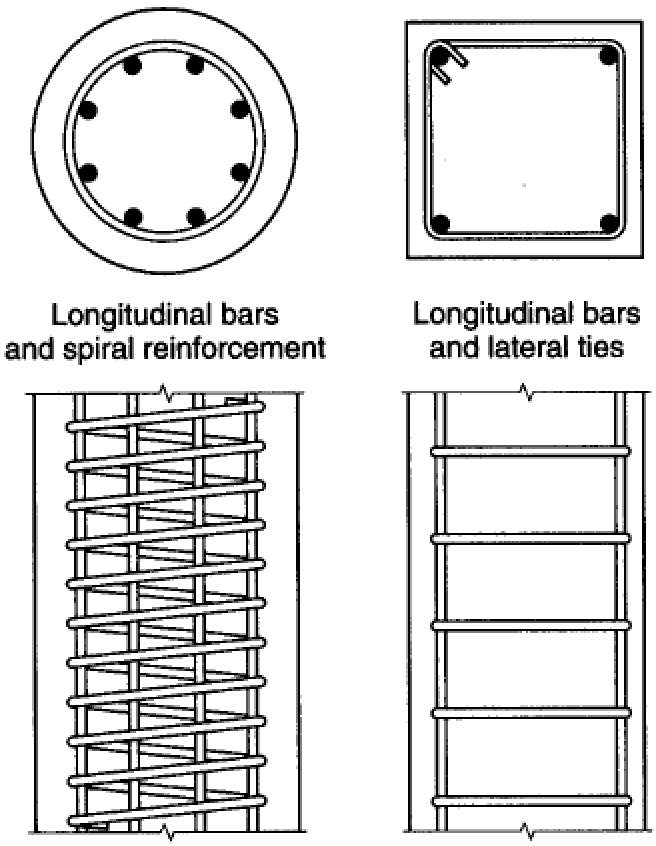
Read last para

Behaviour of members subject to Axial Loads

- Fundamental behaviour illustrated

- Axial Compression
  - Economical to make concrete carry most loads
  - Steel reinforcement is always provided
    - Bending may exist
    - Cross section reduced

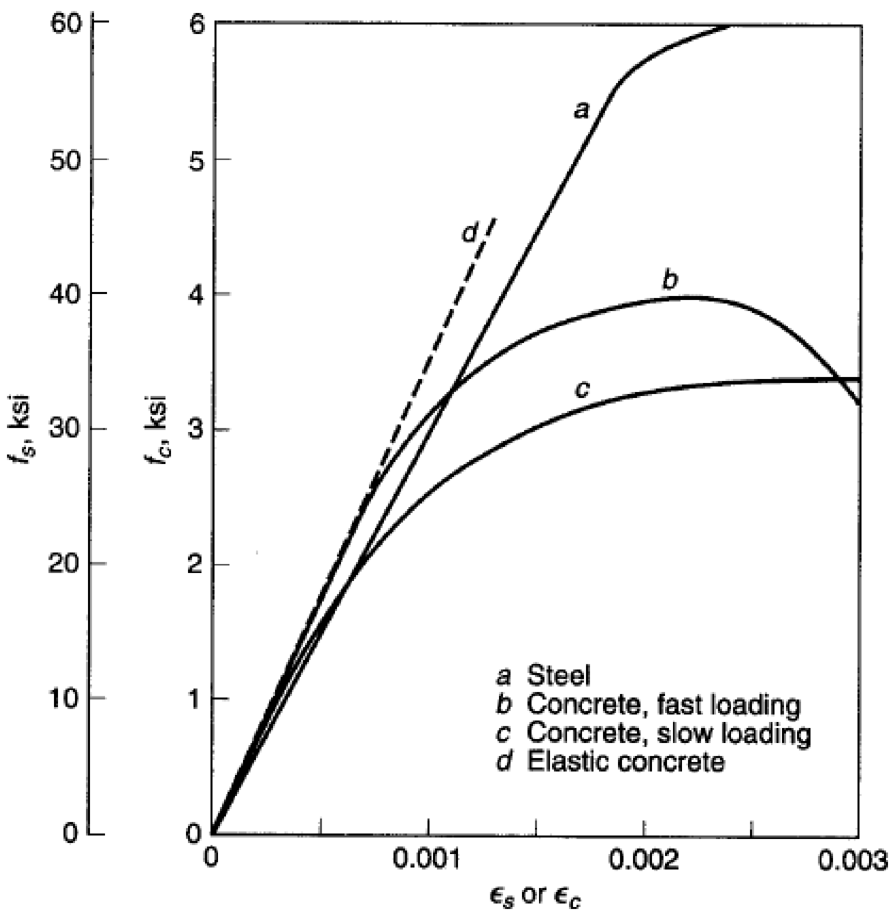
RC Column



- Square, tied column
  - Tie
    - Hold longitudinal bar during construction
    - Prevent bucking under load
  - Circular spirally reinforced column
  - Spiral
    - same
    - confinement to concrete

$f_{c'}$=4,000 psi

f_y= 60,000psi



- Slow loading
- Fast loading
- $0.85f_c'$

Elastic behaviour

- Up to fc'/2, concrete behave elastic
- Also stress and strain proportional
- Range extends to a strain of 0.0005

- Steel is elastic nearly to yield 60 ksi, strain 0.002

Because the compression strain in the concrete, at any given load, is equal to the compression strain in the steel,

$$\epsilon_c = \frac{f_c}{E_c} = \epsilon_s = \frac{f_s}{E_s}$$

Hooke's law

from which the relation between the steel stress f_s and the concrete stress f_c is obtained as

$$f_s = \frac{E_s}{E_c} f_c = nf_c \tag{1.6}$$

where $n = E_s/E_c$ is known as the *modular ratio.*

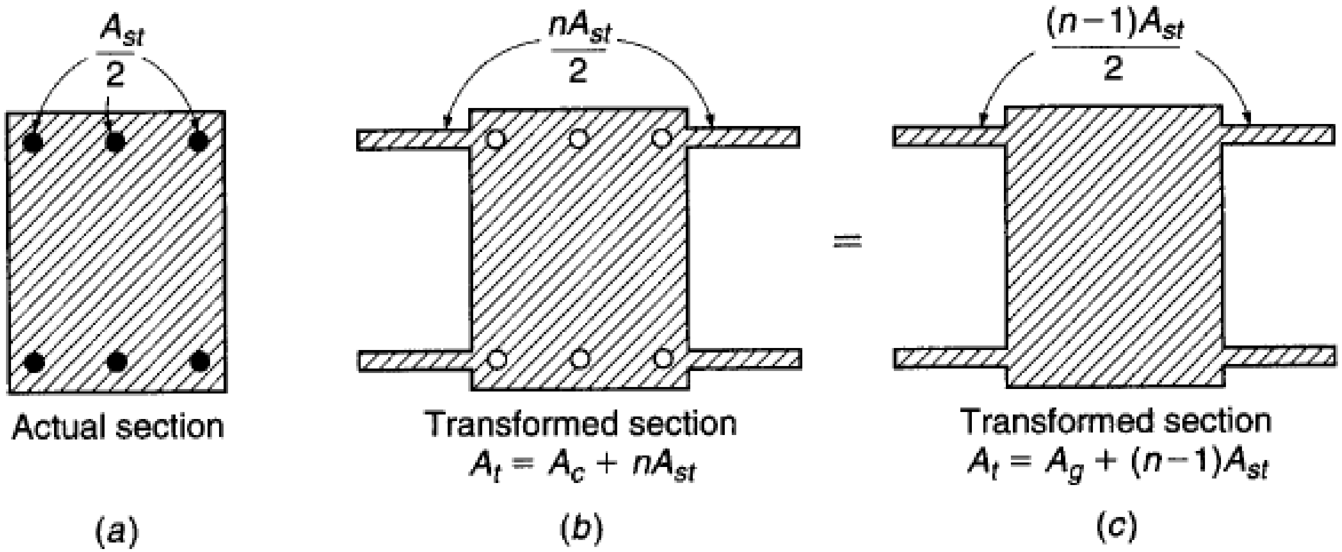
Let

A_c = net area of concrete, i.e., gross area minus area occupied by reinforcing bars
A_g = gross area
A_{st} = total area of reinforcing bars
P = axial load

Then

$$P = f_c A_c + f_s A_{st} = f_c A_c + n f_c A_{st}$$

$$P = f_c(A_c + nA_{st}) \tag{1.7}$$



$$P = f_c[A_g + (n - 1)A_{st}] \tag{1.8}$$

Valid up to 50 to 60 percent of f_c'

A column made of the materials defined in Fig. 1.16 has a cross section of 16 × 20 in. and is reinforced by six No. 9 (No. 29) bars, disposed as shown in Fig. 1.17. (See Tables A.1 and A.2 of Appendix A for bar diameters and areas and Section 2.14 for a description of bar size designations.) Determine the axial load that will stress the concrete to 1200 psi. The modular ratio n may be assumed equal to 8. (In view of the scatter inherent in E_c, it is customary and satisfactory to round off the value of n to the nearest integer.)

SOLUTION. One finds $A_g = 16 \times 20 = 320\ \text{in}^2$, and from Appendix A, Table A.2, two No. 9 (No. 29) bars provide steel area $A_{st} = 6.00\ \text{in}^2$ or 1.88 percent of the gross area. The load on the column, from Eq. (1.8), is $P = 1200[320 + (8 - 1)6.00] = 434{,}000$ lb. Of this total load, the concrete is seen to carry $P_c = f_c A_c = f_c(A_g - A_{st}) = 1200(320 - 6) = 377{,}000$ lb, and the steel $P_s = f_s A_{st} = (nf_c)A_{st} = 9600 \times 6 = 57{,}600$ lb, which is 13.3 percent of the total axial load.

TABLE A.1

Designations, diameters, areas, and weights of standard bars

Bar No.				
Inch-Pound[a]	SI[b]	Diameter, in.	Cross-Sectional Area, in^2	Nominal Weight, lb/ft
3	10	$\frac{3}{8} = 0.375$	0.11	0.376
4	13	$\frac{1}{2} = 0.500$	0.20	0.668
5	16	$\frac{5}{8} = 0.625$	0.31	1.043
6	19	$\frac{3}{4} = 0.750$	0.44	1.502
7	22	$\frac{7}{8} = 0.875$	0.60	2.044
8	25	$1 = 1.000$	0.79	2.670
9	29	$1\frac{1}{8} = 1.128$[c]	1.00	3.400
10	32	$1\frac{1}{4} = 1.270$[c]	1.27	4.303
11	36	$1\frac{3}{8} = 1.410$[c]	1.56	5.313
14	43	$1\frac{3}{4} = 1.693$[c]	2.25	7.650
18	57	$2\frac{1}{4} = 2.257$[c]	4.00	13.600

[a]Based on the number of eighths of an inch included in the nominal diameter of the bars. The nominal diameter of a deformed bar is equivalent to the diameter of a plain bar having the same weight per foot as the deformed bar.

[b]Bar number approximates the number of millimeters included in the nominal diameter of the bar. Bars are marked with this designation.

Inelastic range

One may want to calculate the magnitude of the axial load that will produce a strain or unit shortening $\epsilon_c = \epsilon_s = 0.0010$ in the column of Example 1.1. At this strain the steel is seen to be still elastic, so that the steel stress $f_s = \epsilon_s E_s = 0.001 \times 29{,}000{,}000 = 29{,}000$ psi. The concrete is in the inelastic range, so that its stress cannot be directly calculated, but it can be read from the stress-strain curve for the given value of strain.

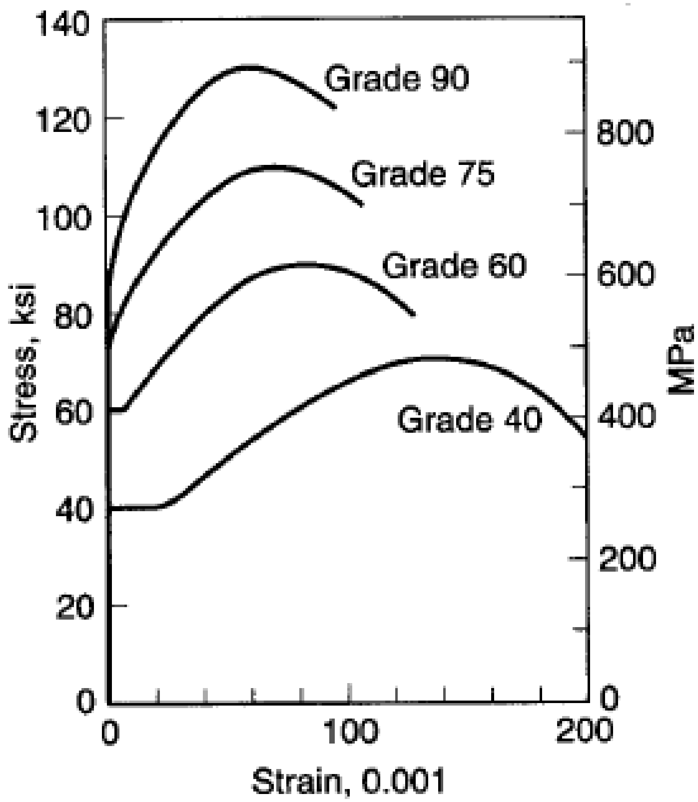
1. If the member has been loaded at a fast rate, curve *b* holds at the instant when the entire load is applied. The stress for $\epsilon = 0.001$ can be read as $f_c = 3200$ psi. Consequently, the total load can be obtained from

$$P = f_c A_c + f_s A_{st} \qquad (1.9)$$

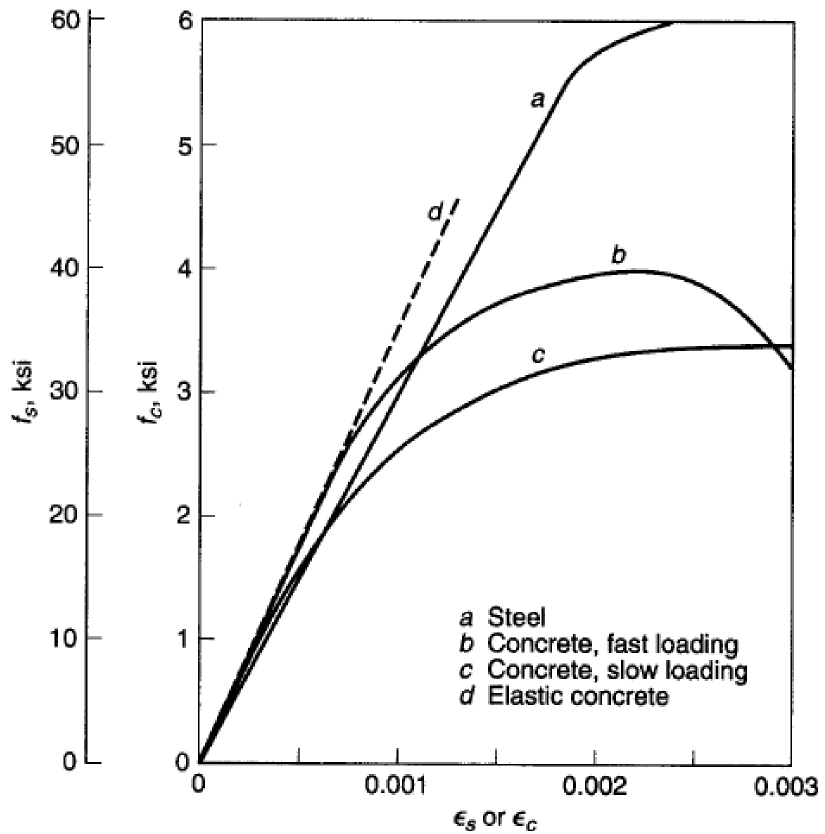
which applies in the inelastic as well as in the elastic range. Hence, $P = 3200(320 - 6) + 29{,}000 \times 6 = 1{,}005{,}000 + 174{,}000 = 1{,}179{,}000$ lb. Of this total load, the steel is seen to carry 174,000 lb, or 14.7 percent.

2. For slowly applied or sustained loading, curve *c* represents the behavior of the concrete. Its stress at a strain of 0.001 can be read as $f_c = 2400$ psi. Then $P = 2400 \times 314 + 29{,}000 \times 6 = 754{,}000 + 174{,}000 = 928{,}000$ lb. Of this total load, the steel is seen to carry 18.8 percent.

Strength



140
120
100
80
60
40
20
0
Stress, ksi
Grade 90
Grade 75
Grade 60
Grade 40
800
600
400
200
0
MPa
0
100
200
Strain, 0.001



60
50
40
30
20
10
0
f_s, ksi
6
5
4
3
2
1
0
f_c, ksi
a
b
c
d
0
0.001
0.002
0.003
ϵ_s or ϵ_c
a Steel
b Concrete, fast loading
c Concrete, slow loading
d Elastic concrete

Strength

If the small knee prior to yielding of the steel is disregarded, i.e., if the steel is assumed to be sharp-yielding, the strain at which it yields is

$$\epsilon_y = \frac{f_y}{E_s} \tag{1.10}$$

or

$$\epsilon_y = \frac{60{,}000}{29{,}000{,}000} = 0.00207$$

At this strain, curve c of Fig. 1.16 indicates a stress of 3200 psi in the concrete; therefore, by Eq. (1.9), the load in the member when the steel starts yielding is $P_y = 3200 \times 314 + 60{,}000 \times 6 = 1{,}365{,}000$ lb. At this load the concrete has not yet reached its full strength, which, as mentioned before, can be assumed as $0.85f_c' = 3400$ psi for slow or sustained loading, and therefore the load on the member can be further increased. During this stage of loading, the steel keeps yielding at constant stress. Finally, the nominal capacity† of the member is reached when the concrete crushes while the steel yields, i.e.,

$$P_n = 0.85f_c'A_c + f_yA_{st} \tag{1.11}$$

Numerous careful tests have shown the reliability of Eq. (1.11) in predicting the ultimate strength of a concentrically loaded reinforced concrete column, provided its slenderness ratio is small so that buckling will not reduce its strength.

For the particular numerical example, $P_n = 3400 \times 314 + 60{,}000 \times 6 = 1{,}068{,}000 + 360{,}000 = 1{,}428{,}000$ lb. At this stage the steel carries 25.2 percent of the load.

Nominal Strength

Nominal strength refers to strength of member calculated by accepted analysis methods. It is intended to convey that actual strength is bound to deviate to some extent from its calculated nominal value because of inevitable variations of dimensions, material properties and other parameters. Design in all cases is based on this nominal strength which represents the best available estimate of actual member strength.

Axial Tension

- If tension is small, both steel and concrete are elastic

$$P = f_{ct}(A_c + nA_{st}) \qquad (1.12)$$

where f_{ct} is the tensile stress in the concrete.

- Larger load than that cracks concrete

$$P = f_s A_{st} \qquad (1.13)$$

- At steel yields

$$P_{nt} = f_y A_{st} \qquad (1.14)$$

Materials

Chapter 2

Reinforced Concrete

- Concrete
  - Cement
  - Aggregate
    - Coarse
    - Fine
  - Water
  - Admixture

Cement

- Ordinary Portland Cement- Type I
- Chiefly calcium and aluminum silicate
- CaO-Limestone
- SiO_2 Al_2O_3 -Clay or shale
- Ground, blended and fused to clinker in kiln
- Gypsum is added

- Comes in bag of 50kg

Cement

- Water-cement ratio- 0.4 to 0.6
- High strength concrete- as low as 0.21
- Hydration process- calcium silicate Hydrate
- Heat of hydration

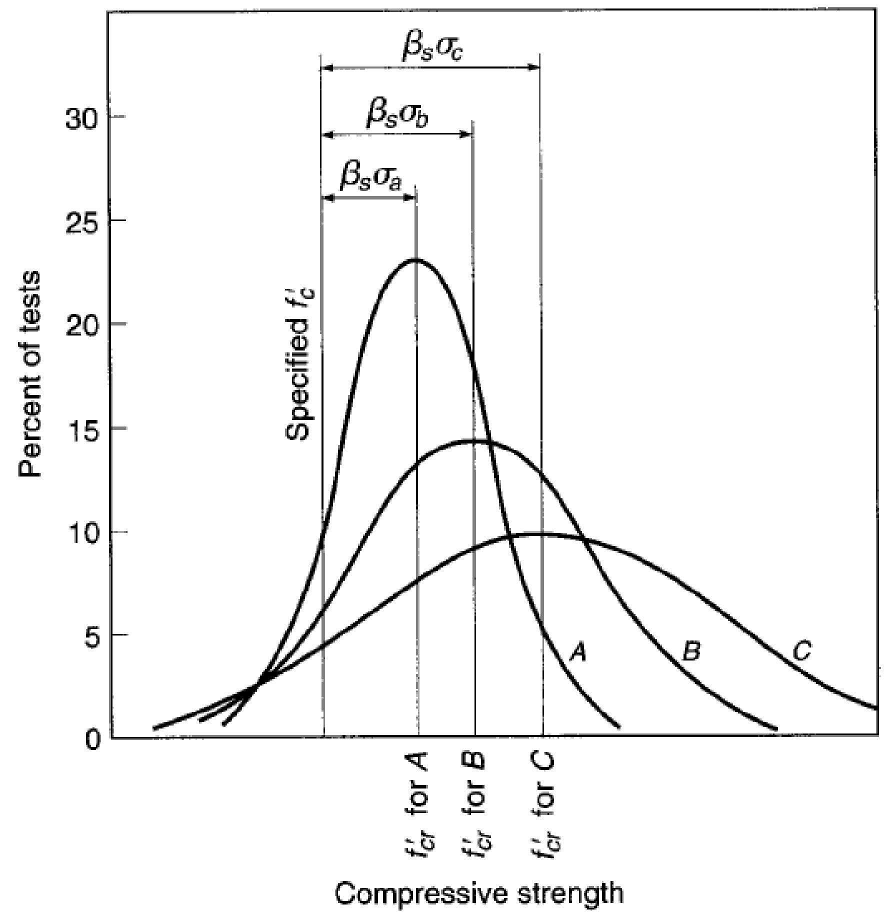
Aggregate

- 70 to 75% volume
- Coarse aggregate- stone chips, brick chips, gravel
- Fine aggregate- sand
- Concrete unit weight -140 to 152 pcf, 145pcf
- Lightweight aggregate-shale, slate, slags, brick chips
- Heavy weight aggregate
- Recycled concrete

- Proportioning and mixing
- Conveying, Placing, compacting and curing

Quality Control

FIGURE 2.2
Frequency curves and average strengths for various degrees of control of concretes with specified design strength f'_c. (*Adapted from Ref. 2.12.*)



Admixtures

- To improve concrete performance
- Air-entraining
- Accelerating
- Set Retarding
- Plasticizer, super plasticizers
- Viscosity modifying-self compacting

Properties in Compression

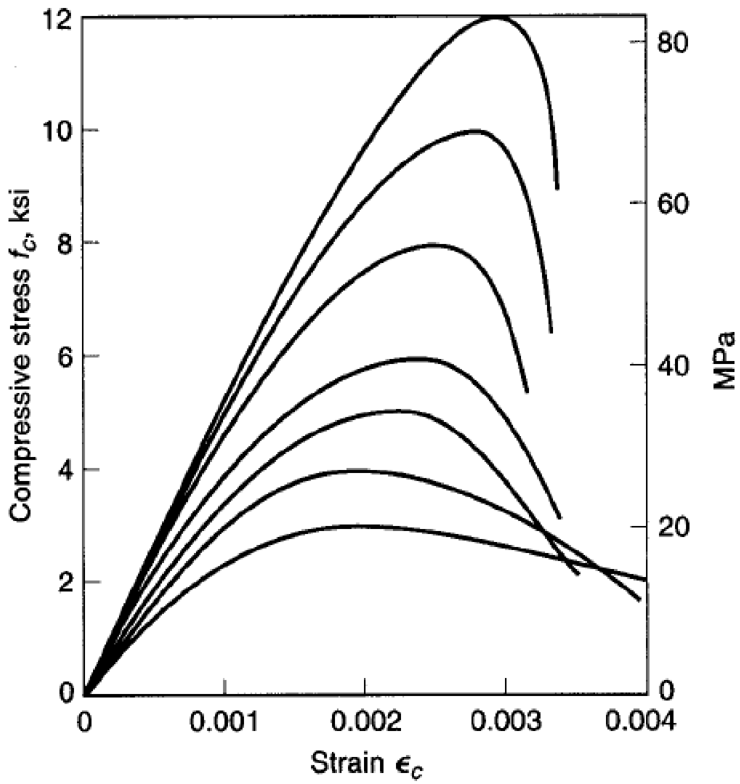


FIGURE 2.3
Typical compressive stress-strain curves for normal-density concrete with w_c = 145 pcf. (*Adapted from Refs. 2.23 and 2.24.*)

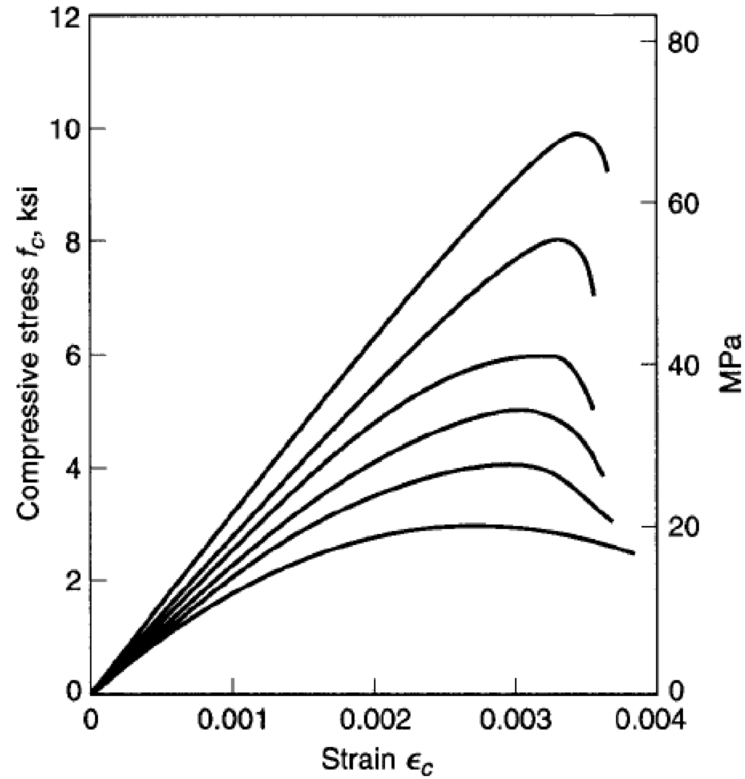


FIGURE 2.4
Typical compressive stress-strain curves for lightweight concrete with w_c = 100 pcf. (*Adapted from Refs. 2.23 and 2.24.*)

The *modulus of elasticity* E_c (in psi units), i.e., the slope of the initial straight portion of the stress-strain curve, is seen to be larger as the strength of the concrete increases. For concretes in the strength range to about 6000 psi, it can be computed with reasonable accuracy from the empirical equation found in the ACI Code

$$E_c = 33w_c^{1.5}\sqrt{f'_c} \tag{2.3}$$

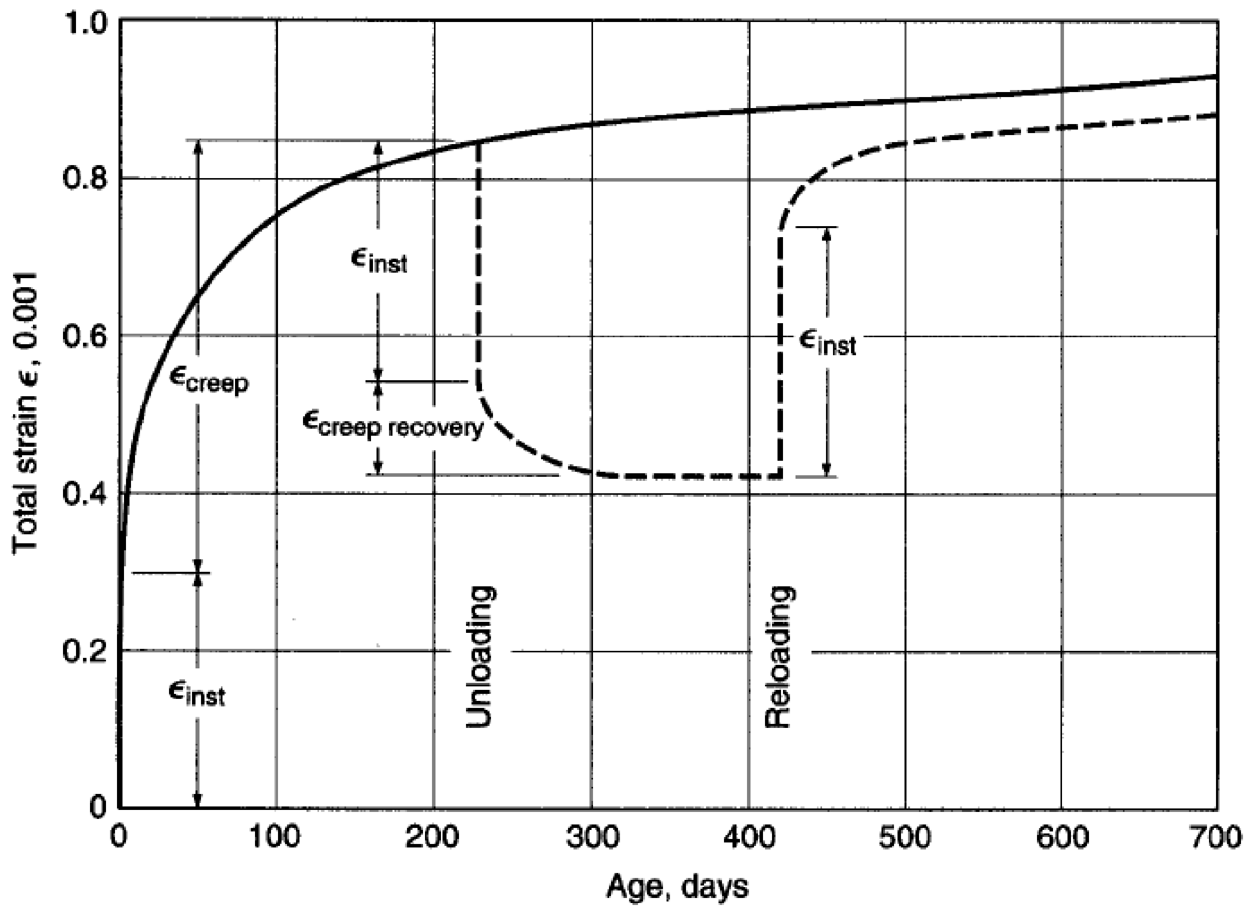
where w_c is the unit weight of the hardened concrete in pcf and f'_c is its strength in psi. Equation (2.3) was obtained by testing structural concretes with values of w_c from 90 to 155 pcf. For normal sand-and-stone concretes, with w_c = 145 pcf, E_c may be taken as

$$E_c = 57{,}000\sqrt{f'_c} \tag{2.4}$$

Poisson's ratio 0.15 to 0.20

Creep

FIGURE 2.7
Typical creep curve (concrete loaded to 600 psi at age 28 days).



Properties in tension

TABLE 2.3
Approximate range of tensile strengths of concrete

	Normalweight Concrete, psi	Lightweight Concrete, psi
Direct tensile strength f_t'	3 to $5\sqrt{f_c'}$	2 to $3\sqrt{f_c'}$
Split-cylinder strength f_{ct}	6 to $8\sqrt{f_c'}$	4 to $6\sqrt{f_c'}$
Modulus of rupture f_r	8 to $12\sqrt{f_c'}$	6 to $8\sqrt{f_c'}$

High strength concrete

- 8 to 20 ksi or higher
- High performance concrete

Reinforcement

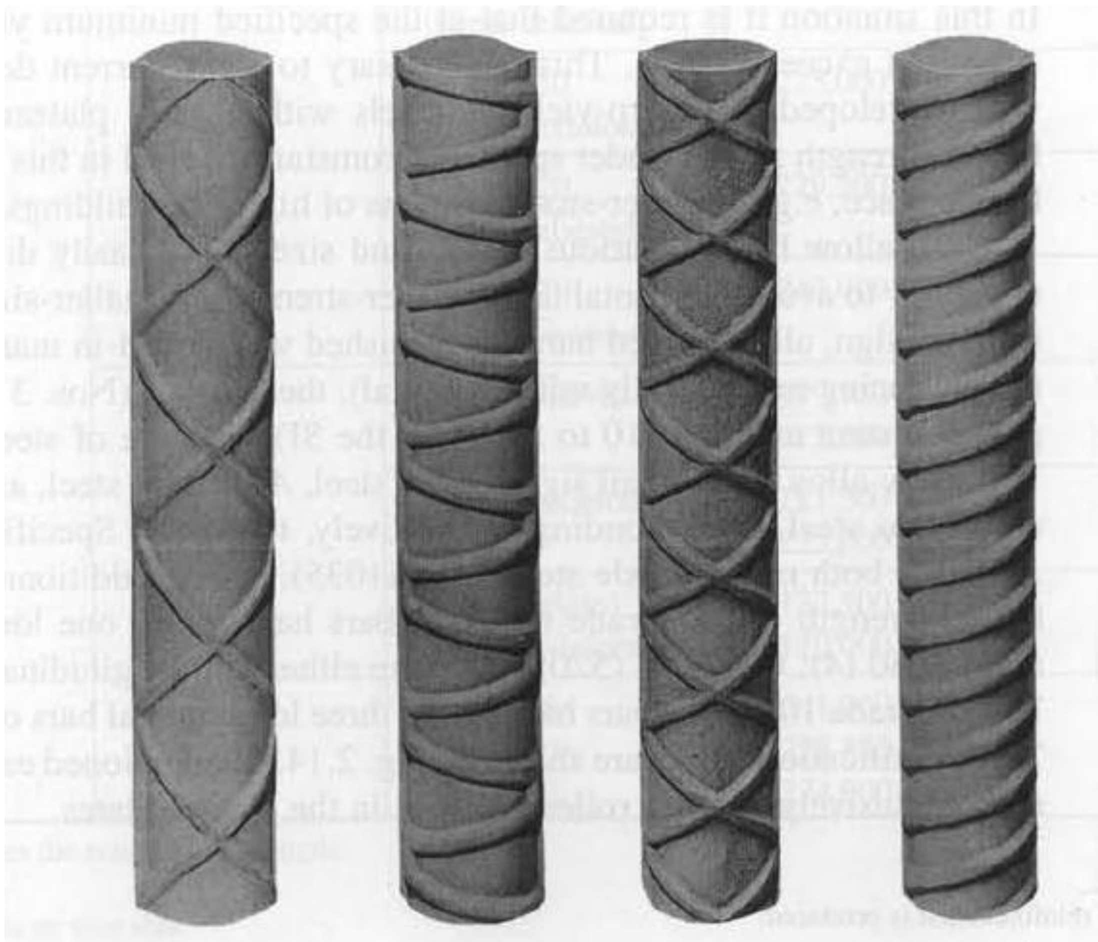
Strong Bond-no slip

two materials.

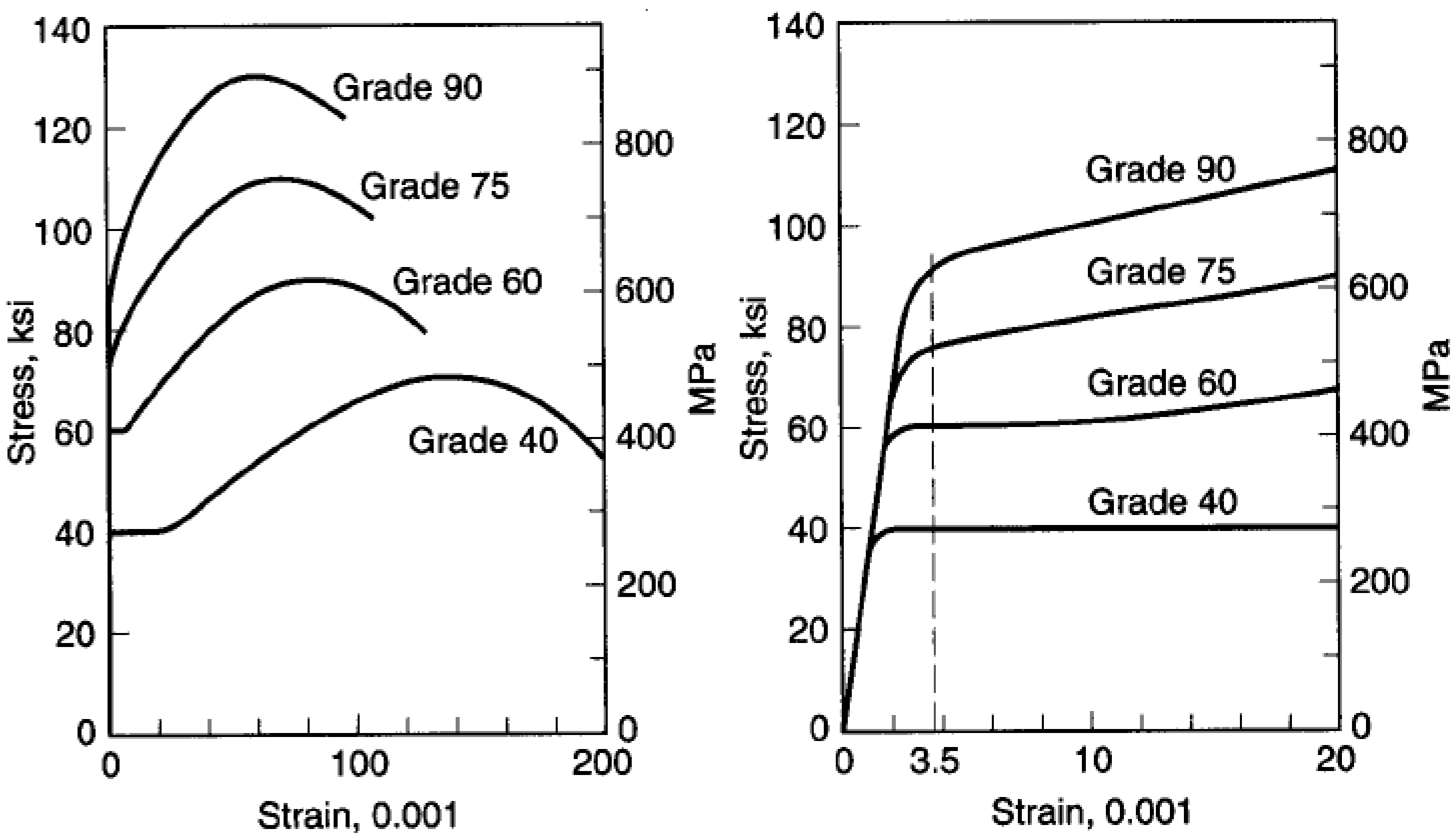
Additional features that make for the satisfactory joint performance of steel and concrete are the following:

1. The *thermal expansion coefficients* of the two materials, about 6.5×10^{-6} for steel vs. an average of 5.5×10^{-6} for concrete, are sufficiently close to forestall cracking and other undesirable effects of differential thermal deformations.
2. While the *corrosion resistance* of bare steel is poor, the concrete that surrounds the steel reinforcement provides excellent corrosion protection, minimizing corrosion problems and corresponding maintenance costs.
3. The *fire resistance* of unprotected steel is impaired by its high thermal conductivity and by the fact that its strength decreases sizably at high temperatures. Conversely, the thermal conductivity of concrete is relatively low. Thus, damage caused by even prolonged fire exposure, if any, is generally limited to the outer layer of concrete, and a moderate amount of concrete cover provides sufficient thermal insulation for the embedded reinforcement.

Reinforcing Bars

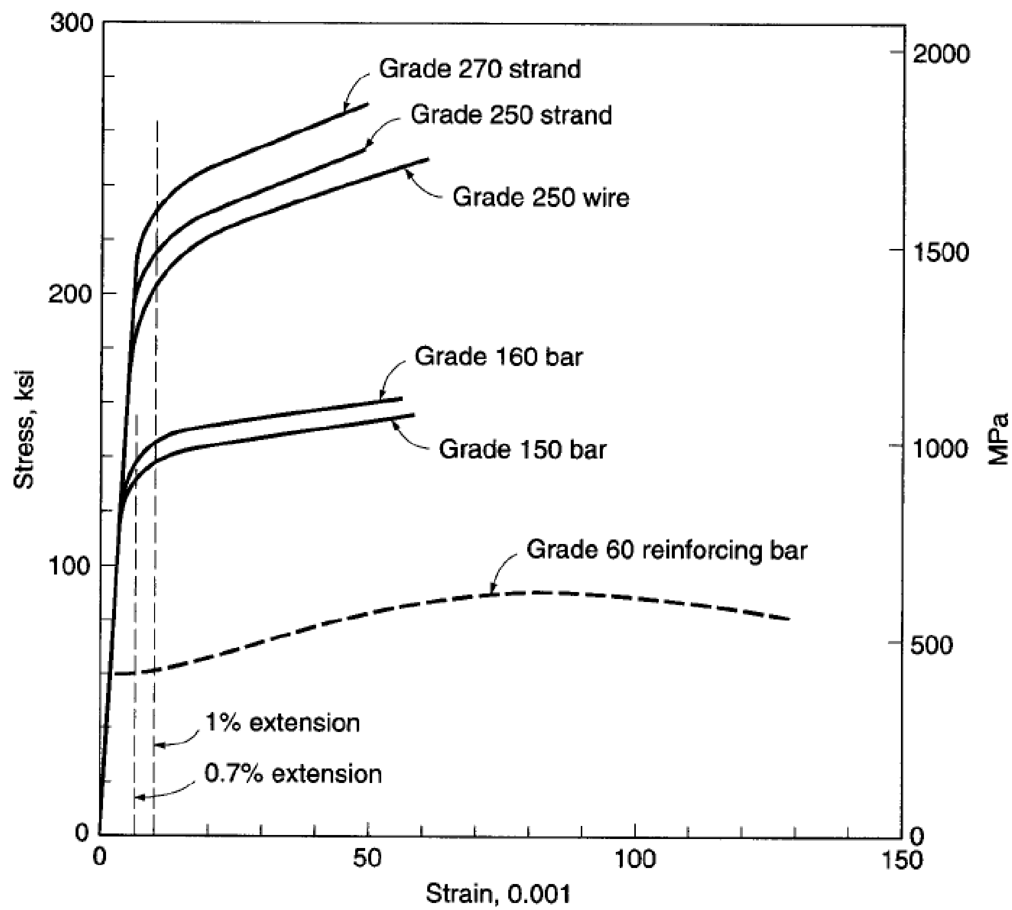


Stress strain curves



TMT bars

Prestressing steel



Flexural Analysis and Design of Beams

Chapter 3

Introduction

- Fundamental Assumptions
- Simple case of axial loading
- Same assumptions and ideal concept apply

- This chapter includes analysis and design for flexure, dimensioning cross section and reinforcement
- Shear design, bond anchorage, serviceability in chapters 4, 5, 6.

Bending of Homogeneous beam

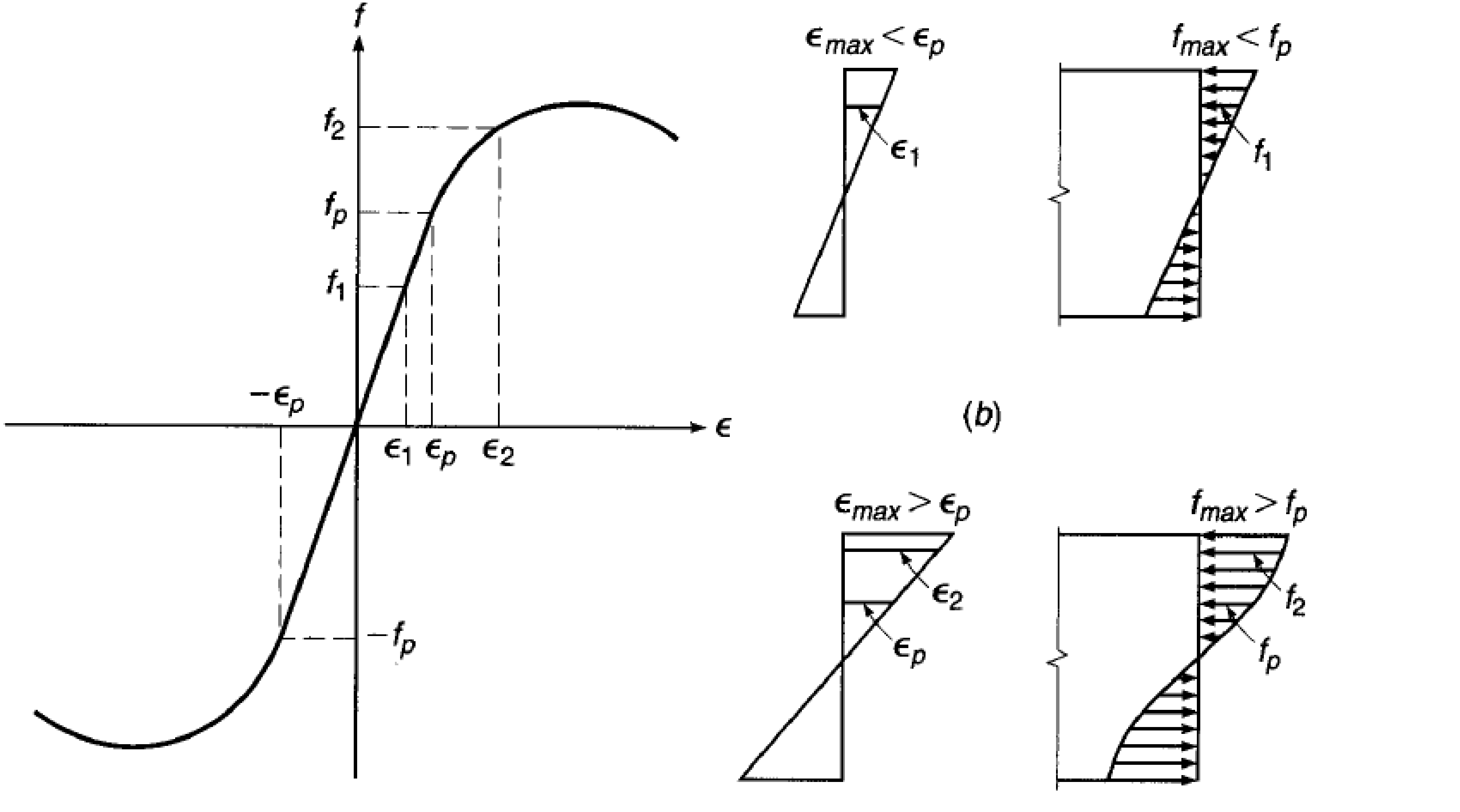
- Steel, timber
- Internal forces-normal and tangential
- Normal-bending/flexural stress-bending moment
- Tangential-shear stress-shear force

Fundamental assumptions relating to flexure and shear

1. Plane cross section remain plane
2. Bending stress *f* at any point depends on the strain at that point
3. Shear stress also depends on cross section and stress-strain diagram. Maximum at neutral axis and zero at extreme fibre. Same horizontal and vertical.
4. The intensity of principal stresses

$$t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + \nu^2}$$

where f = intensity of normal fiber stress
ν = intensity of tangential shearing stress

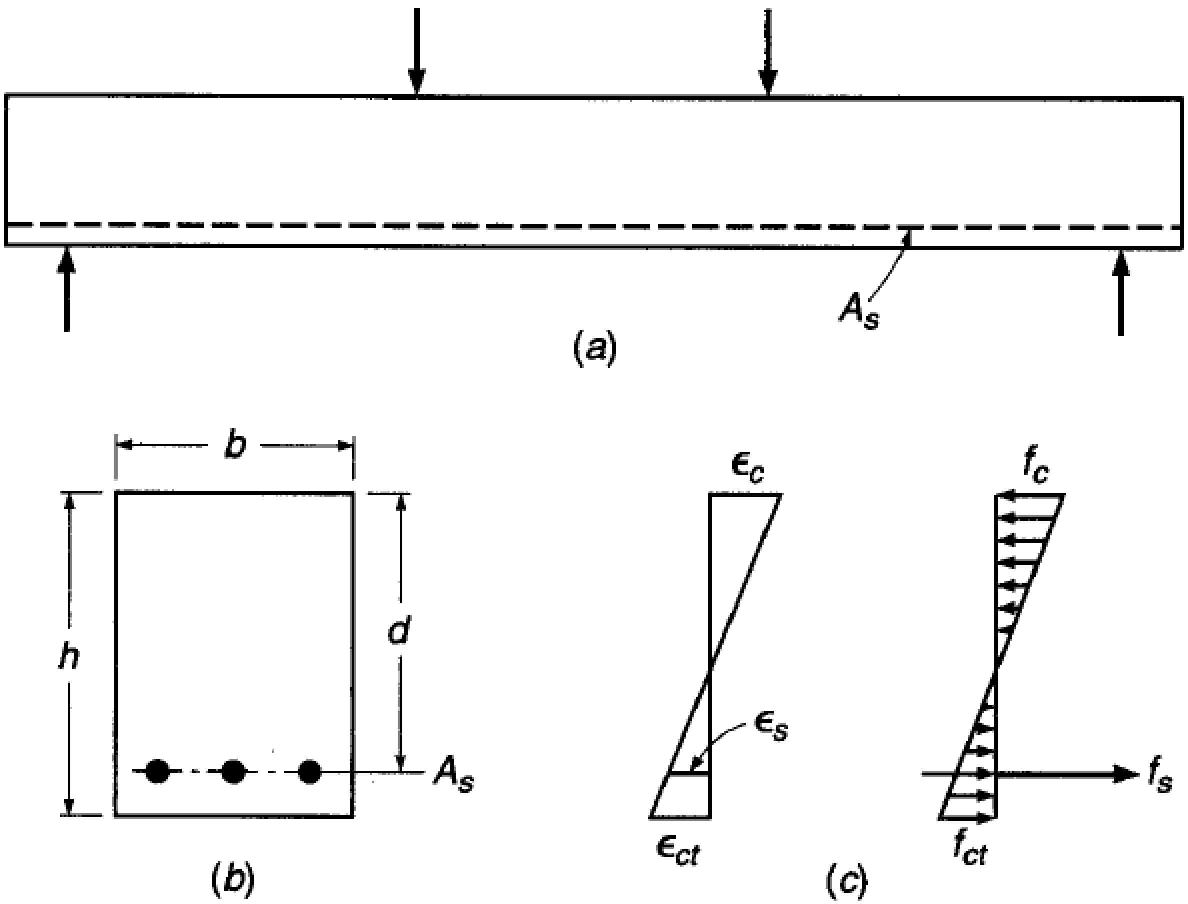


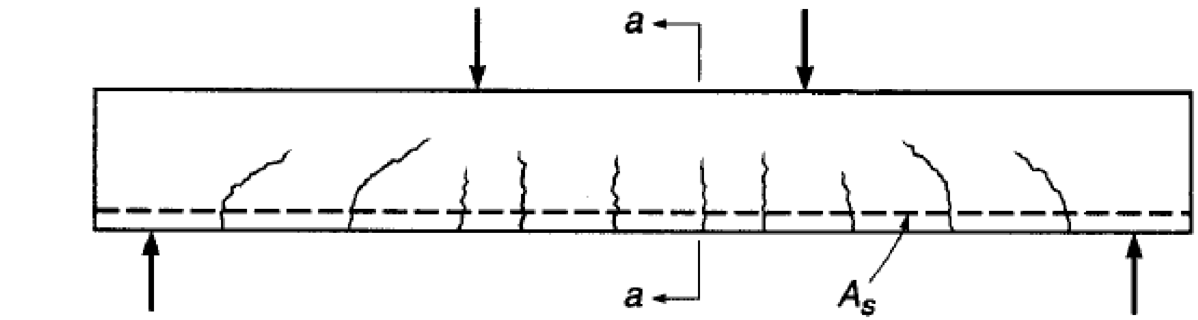
f
f_2
f_p
f_1
$-\epsilon_p$
ϵ
ϵ_1 ϵ_p ϵ_2
$-f_p$
$\epsilon_{max} < \epsilon_p$
ϵ_1
$f_{max} < f_p$
f_1
$\epsilon_{max} > \epsilon_p$
ϵ_2
ϵ_p
$f_{max} > f_p$
f_2
f_p

(a) (b) (c)

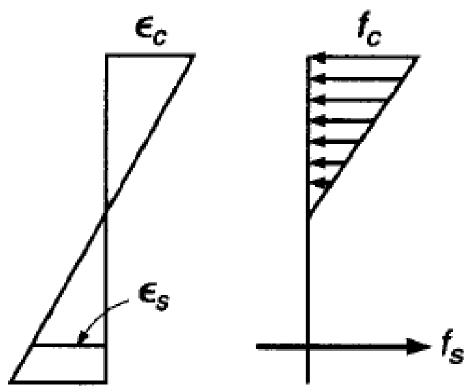
5. At neutral axis, only horizontal and vertical shear present-pure shear condition
6. When stress are smaller than proportional limit
   a. Neutral axis = cg
   b. f=My/I
   c. v=VQ/It
   d. Shear distribution parabolic, max at na, zero at outer fibre. For rectangular max=1.5V/bh

Reinforced Concrete Beam Behaviour

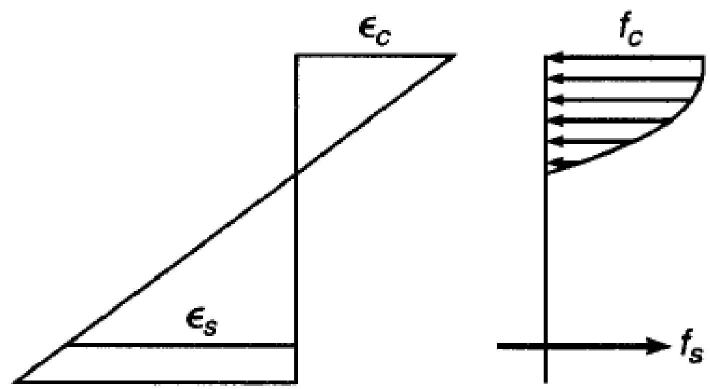




(d)



(e)

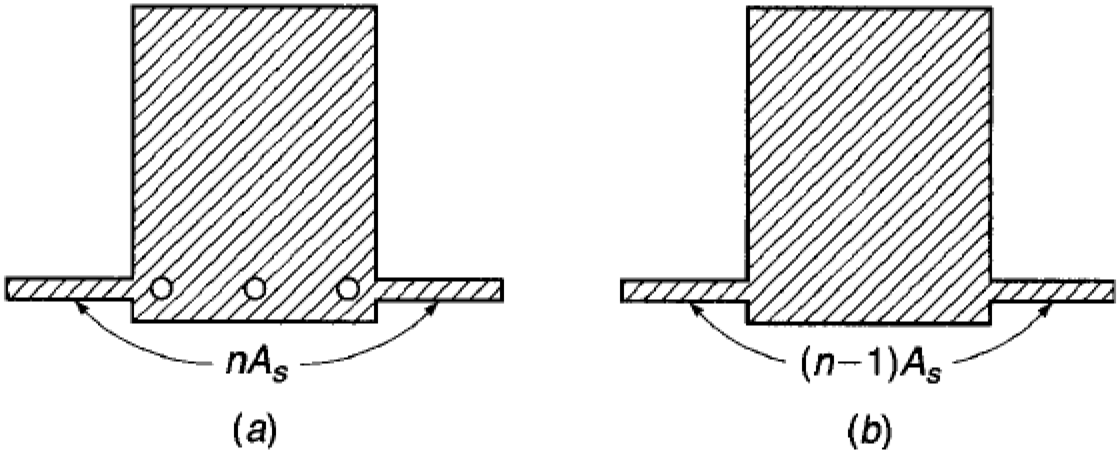


(f)

Video

- See video clips

Stresses elastic, section uncracked



- Tensile stress in concrete is smaller than modulus of rupture
  Transformed section can be used

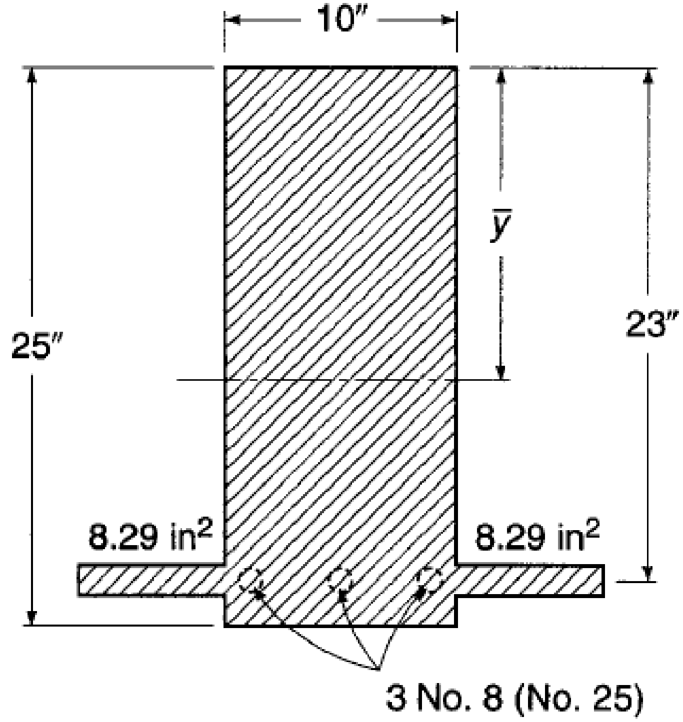
EXAMPLE 3.1 A rectangular beam has the dimensions (see Fig. 3.2*b*) $b = 10$ in., $h = 25$ in., and $d = 23$ in. and is reinforced with three No. 8 (No. 25) bars so that $A_s = 2.37$ in^2. The concrete cylinder strength f'_c is 4000 psi, and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y is 60,000 psi, the stress-strain curves of the materials being those of Fig. 1.16. Determine the stresses caused by a bending moment $M = 45$ ft-kips.

Solution. With a value $n = E_s/E_c = 29{,}000{,}000/3{,}600{,}000 = 8$, one has to add to the rectangular outline an area $(n - 1)A_s = 7 \times 2.37 = 16.59$ in^2, disposed as shown on Fig. 3.4, to obtain the uncracked, transformed section. Conventional calculations show that the location of the neutral axis of this section is given by $\bar{y} = 13.2$ in. from the top of the section, and its moment of inertia about this axis is 14,740 in^4. For $M = 45$ ft-kips $= 540{,}000$ in-lb, the concrete compression stress at the top fiber is, from Eq. (3.3),

$$f_c = \frac{M\bar{y}}{I} = \frac{540{,}000 \times 13.2}{14{,}740} = 484 \text{ psi}$$

and, similarly, the concrete tension stress at the bottom fiber, 11.8 in. from the neutral axis, is

$$f_{ct} = \frac{540{,}000 \times 11.8}{14{,}740} = 432 \text{ psi}$$



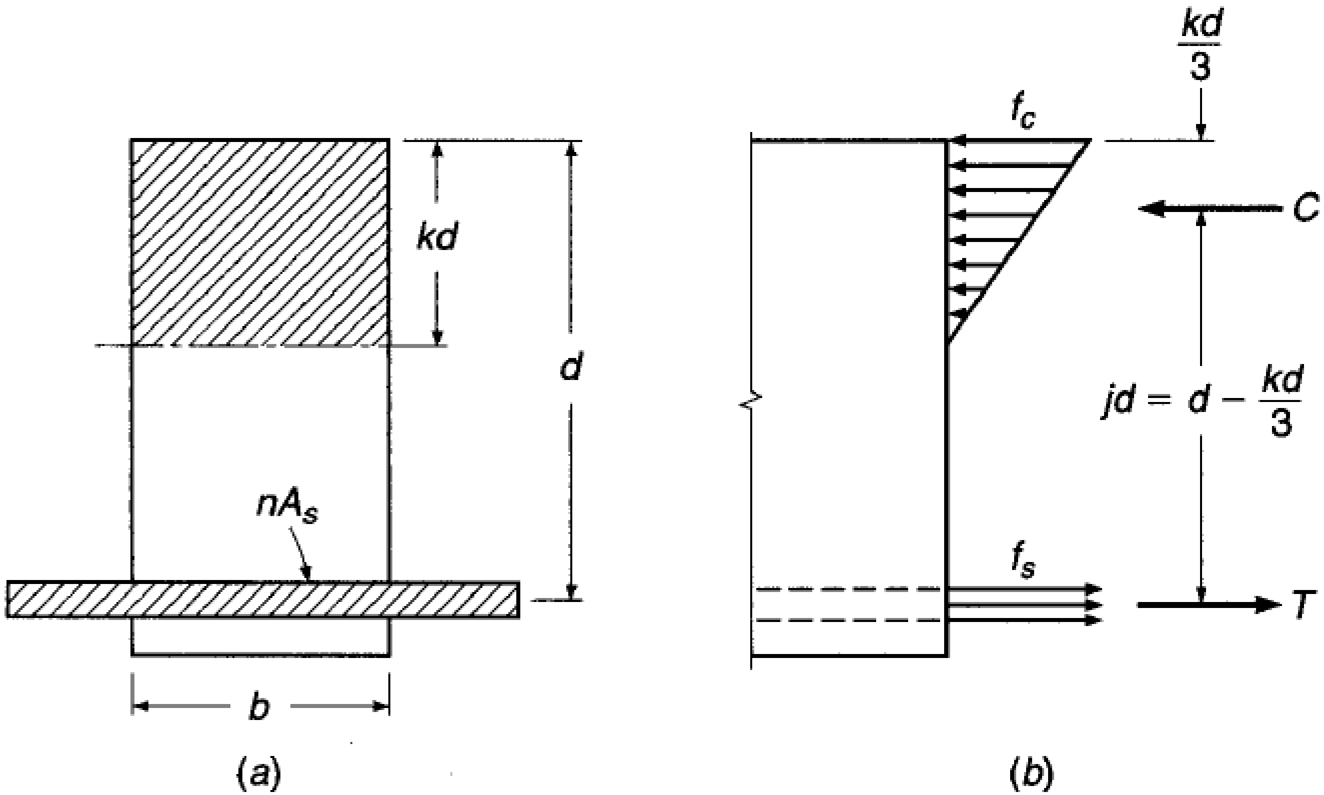
Since this value is below the given tensile bending strength of the concrete, 475 psi, no tension cracks will form, and calculation by the uncracked, transformed section is justified. The stress in the steel, from Eqs. (1.6) and (3.2), is

$$f_s = n\frac{My}{I} = 8\left(\frac{540{,}000 \times 9.8}{14{,}740}\right) = 2870 \text{ psi}$$

By comparing f_c and f_s with the concrete cylinder strength and the yield point, respectively, it is seen that at this stage the actual stresses are quite small compared with the available strengths of the two materials.

Stresses Elastic, Section cracked

- Concrete tensile stress exceeds mod of rupture
- Concrete compressive stress is less than $f_c'/2$
- Steel stress less than yield
- Assume tension crack up to neutral axis
- Transformed section can still be used


kd
d
nA_s
b
(a)
kd/3
f_c
C
jd = d − kd/3
f_s
T
(b)

neutral axis, the moment of the tension area about the axis is set equal to the moment of the compression area, which gives

$$b\,\frac{(kd)^2}{2} - nA_s(d - kd) = 0 \tag{3.5}$$

Having obtained kd by solving this quadratic equation, one can determine the moment of inertia and other properties of the transformed section as in the preceding case. Alternatively, one can proceed from basic principles by accounting directly for the forces that act on the cross section. These are shown in Fig. 3.5*b*. The concrete stress, with maximum value f_c at the outer edge, is distributed linearly as shown. The entire steel area A_s is subject to the stress f_s. Correspondingly, the total compression force C and the total tension force T are

$$C = \frac{f_c}{2}\,bkd \qquad \text{and} \qquad T = A_s f_s \tag{3.6}$$

The requirement that these two forces be equal numerically has been taken care of by the manner in which the location of the neutral axis has been determined.

Equilibrium requires that the couple constituted by the two forces C and T be equal numerically to the external bending moment M. Hence, taking moments about C gives

$$M = Tjd = A_s f_s jd \tag{3.7}$$

where jd is the internal lever arm between C and T. From Eq. (3.7), the steel stress is

$$f_s = \frac{M}{A_s jd} \tag{3.8}$$

Conversely, taking moments about T gives

$$M = Cjd = \frac{f_c}{2} bkdjd = \frac{f_c}{2} kjbd^2 \tag{3.9}$$

from which the concrete stress is

$$f_c = \frac{2M}{kjbd^2} \tag{3.10}$$

In using Eqs. (3.6) through (3.10), it is convenient to have equations by which k and j may be found directly, to establish the neutral axis distance kd and the internal lever arm jd. First defining the *reinforcement ratio* as

$$\rho = \frac{A_s}{bd} \tag{3.11}$$

then substituting $A_s = \rho bd$ into Eq. (3.5) and solving for k, one obtains

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \tag{3.12}$$

From Fig. 3.5*b* it is seen that $jd = d - kd/3$, or

$$j = 1 - \frac{k}{3} \tag{3.13}$$

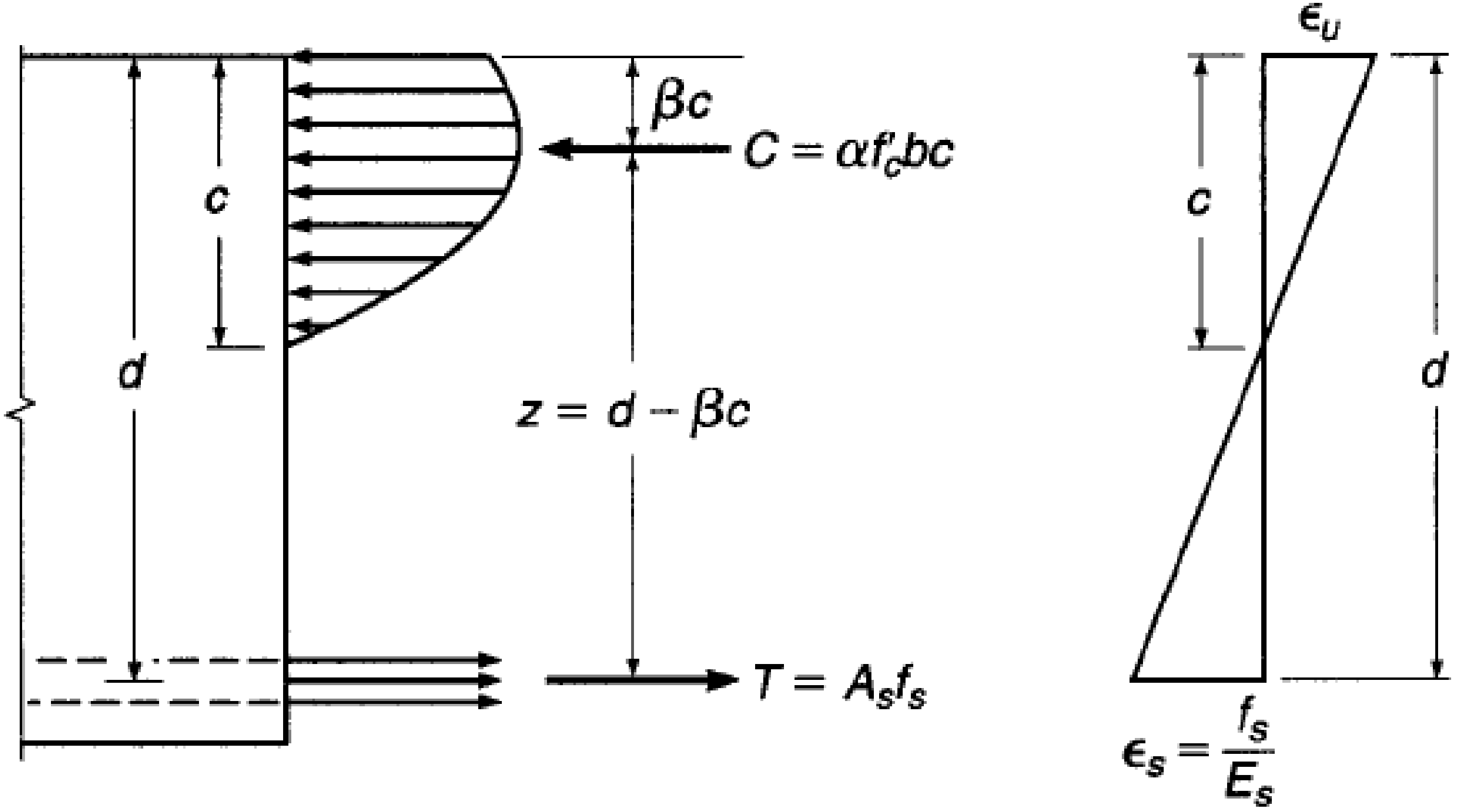
Values of k and j for elastic cracked section analysis, for common reinforcement ratios and modular ratios, are found in Table A.6 of Appendix A.

The beam of Example 3.1 is subject to a bending moment $M = 90$ ft-kips (rather than 45 ft-kips as previously). Calculate the relevant properties and stresses.

Solution. If the section were to remain uncracked, the tensile stress in the concrete would now be twice its previous value, that is, 864 psi. Since this exceeds by far the modulus of rupture of the given concrete (475 psi), cracks will have formed and the analysis must be adapted consistent with Fig. 3.5. Equation (3.5), with the known quantities b, n, and A_s inserted, gives the distance to the neutral axis $kd = 7.6$ in., or $k = 7.6/23 = 0.33$. From Eq. (3.13), $j = 1 - 0.33/3 = 0.89$. With these values the steel stress is obtained from Eq. (3.8) as $f_s =$ 22,300 psi, and the maximum concrete stress from Eq. (3.10) as $f_c = 1390$ psi.

Comparing the results with the pertinent values for the same beam when subject to one-half the moment, as previously calculated, one notices that (1) the neutral plane has migrated upward so that its distance from the top fiber has changed from 13.2 to 7.6 in.; (2) even though the bending moment has only been doubled, the steel stress has increased from 2870 to 22,300 psi, or about 7.8 times, and the concrete compression stress has increased from 484 to 1390 psi, or 2.9 times; (3) the moment of inertia of the cracked transformed section is easily computed to be 5910 in^4, compared with 14,740 in^4 for the uncracked section. This affects the magnitude of the deflection, as discussed in Chapter 6. Thus, it is seen how radical is the influence of the formation of tension cracks on the behavior of reinforced concrete beams.

Flexural Strength



- Yielding of steel $f_s = f_y$
- Crushing of concrete ε_u= 0.003-0.004
- Either can reach first
- Exact shape not necessary
- Necessary –Total compressive force and location
- βc- location from comp face

$$\alpha = \frac{f_{av}}{f'_c} \tag{3.14}$$

Then

$$C = \alpha f'_c bc \tag{3.15}$$

α equals 0.72 for $f_c' \leq 4000$ psi and decreases by 0.04 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, $\alpha = 0.56$.

β equals 0.425 for $f_c' \leq 4000$ psi and decreases by 0.025 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, $\beta = 0.325$.

$$C = T \qquad \text{or} \qquad \alpha f_c' bc = A_s f_s \tag{3.16}$$

Also, the bending moment, being the couple of the forces C and T, can be written as either

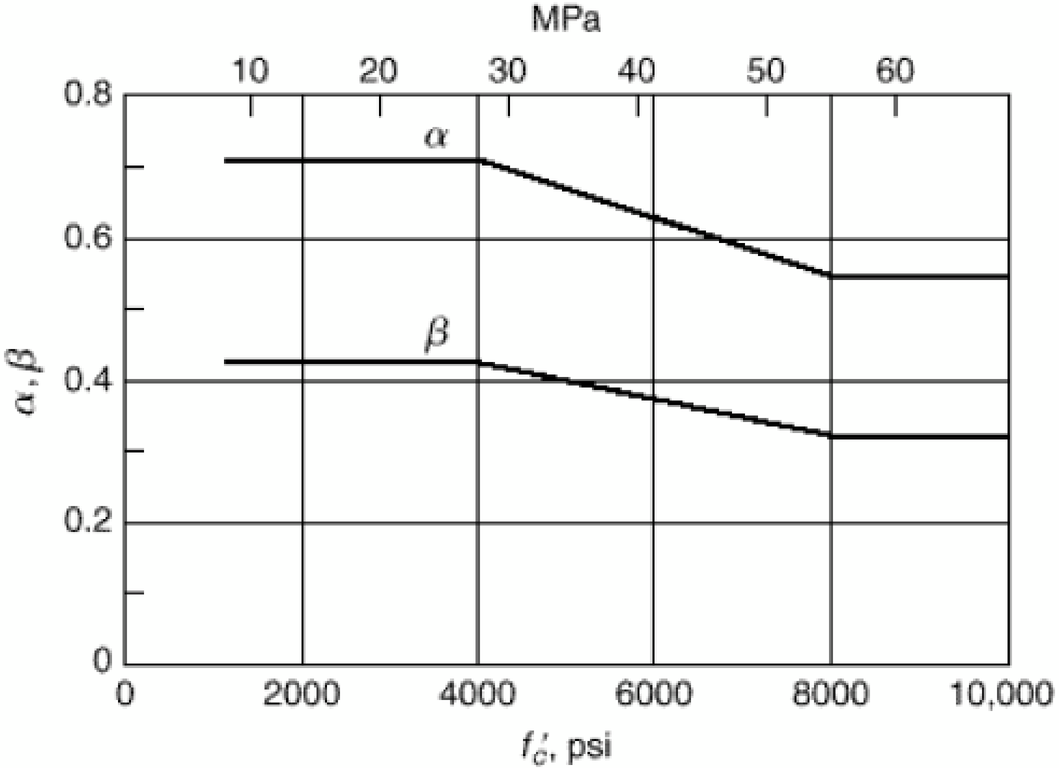
$$M = Tz = A_s f_s (d - \beta c) \tag{3.17}$$

or

$$M = Cz = \alpha f_c' bc (d - \beta c) \tag{3.18}$$

For failure initiated by yielding of the tension steel, $f_s = f_y$. Substituting this value in Eq. (3.16), one obtains the distance to the neutral axis

$$c = \frac{A_s f_y}{\alpha f_c' b} \tag{3.19a}$$



α equals 0.72 for $f_c' \leq 4000$ psi and decreases by 0.04 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, $\alpha = 0.56$.

β equals 0.425 for $f_c' \leq 4000$ psi and decreases by 0.025 for every 1000 psi above 4000 up to 8000 psi. For $f_c' > 8000$ psi, $\beta = 0.325$.

Failure initiated by yielding

For failure initiated by yielding of the tension steel, $f_s = f_y$. Substituting this value in Eq. (3.16), one obtains the distance to the neutral axis

$$c = \frac{A_s f_y}{\alpha f_c' b} \tag{3.19a}$$

Alternatively, using $A_s = \rho bd$, the neutral axis distance is

$$c = \frac{\rho f_y d}{\alpha f_c'} \tag{3.19b}$$

giving the distance to the neutral axis when tension failure occurs. The nominal moment M_n is then obtained from Eq. (3.17) with the value for c just determined, and $f_s = f_y$; that is,

$$M_n = \rho f_y b d^2 \left(1 - \frac{\beta f_y \rho}{\alpha f_c'}\right) \tag{3.20a}$$

With the specific, experimentally obtained values for α and β given previously, this becomes

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) \tag{3.20b}$$

Failure by concrete crushing

If, for larger reinforcement ratios, the steel does not reach yield at failure, then the strain in the concrete becomes $\epsilon_u = 0.003$, as previously discussed. The steel stress f_s, not having reached the yield point, is proportional to the steel strain ϵ_s; i.e., according to Hooke's law,

$$f_s = \epsilon_s E_s$$

From the strain distribution of Fig. 3.6, the steel strain ϵ_s can be expressed in terms of the distance c by evaluating similar triangles, after which it is seen that

$$f_s = \epsilon_u E_s \frac{d - c}{c} \tag{3.21}$$

Then, from Eq. (3.16),

$$\alpha f_c' bc = A_s \epsilon_u E_s \frac{d - c}{c} \tag{3.22}$$

Quadratic equation for c

Balanced reinforcement ratio ρ_b

$$c = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d \tag{3.23}$$

Substituting that value of c into Eq. (3.16), with $A_s f_s = \rho b d f_y$, one obtains for the balanced reinforcement ratio

$$\rho_b = \frac{\alpha f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y} \tag{3.24}$$

Example 3.3

Determine the nominal moment M_n at which the beam of Examples 3.1 and 3.2 will fail.

Solution. For this beam the reinforcement ratio $\rho = A_s/(bd) = 2.37/(10 \times 23) = 0.0103$. The balanced reinforcement ratio is found from Eq. (3.24) to be 0.0284. Since the amount of steel in the beam is less than that which would cause failure by crushing of the concrete, the beam will fail in tension by yielding of the steel. Its nominal moment, from Eq. (3.20*b*), is

$$M_n = 0.0103 \times 60{,}000 \times 10 \times 23^2 \left(1 - 0.59 \frac{0.0103 \times 60{,}000}{4000}\right)$$

$$= 2{,}970{,}000 \text{ in-lb} = 248 \text{ ft-kips}$$

When the beam reaches M_n, the distance to its neutral axis, from Eq. (3.19*b*), is

$$c = \frac{0.0103 \times 60{,}000 \times 23}{0.72 \times 4000} = 4.94$$

It is informative to compare this result with those of Examples 3.1 and 3.2. In the previous calculations, it was found that at low loads, when the concrete had not yet cracked in tension, the neutral axis was located at a distance of 13.2 in. from the compression edge; at higher loads, when the tension concrete was cracked but stresses were still sufficiently small to be elastic, this distance was 7.6 in. Immediately before the beam fails, as has just been shown, this distance has further decreased to 4.9 in. For these same stages of loading, the stress in the steel increased from 2870 psi in the uncracked section, to 22,300 psi in the cracked elastic section, and to 60,000 psi at the nominal moment capacity. This migration of the neutral axis toward the compression edge and the increase in steel stress as load is increased is a graphic illustration of the differences between the various stages of behavior through which a reinforced concrete beam passes as its load is increased from zero to the value that causes it to fail. The examples also illustrate the fact that nominal moments cannot be determined accurately by elastic calculations.

Design of Tension-reinforced Rectangular Beams

- Demand < Capacity

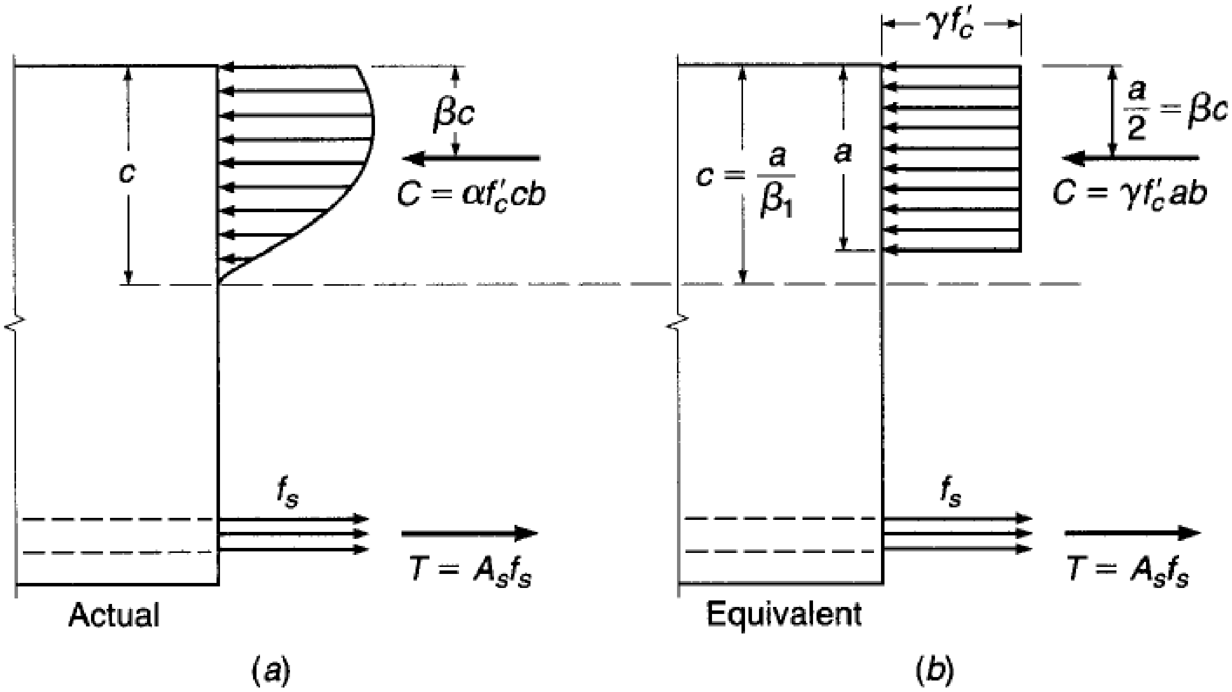
$$M_u \leq \phi M_n$$

$$P_u \leq \phi P_n$$

$$V_u \leq \phi V_n$$

Equivalent Rectangular Stress Distribution

FIGURE 3.8
Actual and equivalent rectangular stress distributions at ultimate load.



$$C = \alpha f'_c cb = \gamma f'_c ab \qquad \text{from which} \qquad \gamma = \alpha \frac{c}{a}$$

C S Whitney

TABLE 3.1
Concrete stress block parameters

	f'_c, psi				
	≤4000	5000	6000	7000	≥8000
α	0.72	0.68	0.64	0.60	0.56
β	0.425	0.400	0.375	0.350	0.325
$\beta_1 = 2\beta$	0.85	0.80	0.75	0.70	0.65
$\gamma = \alpha/\beta_1$	0.85	0.85	0.85	0.86	0.86

With $a = \beta_1 c$, this gives $\gamma = \alpha/\beta_1$. The second condition simply requires that in the equivalent rectangular stress block, the force C be located at the same distance βc from the top fiber as in the actual distribution. It follows that $\beta_1 = 2\beta$.

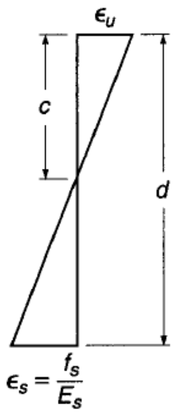
To supply the details, the upper two lines of Table 3.1 present the experimental evidence of Fig. 3.7 in tabular form. The lower two lines give the just-derived parameters β_1 and γ for the rectangular stress block. It is seen that the stress intensity factor γ is essentially independent of f'_c and can be taken as 0.85 throughout. Hence, regardless of f'_c, the concrete compression force at failure in a rectangular beam of width b is

$$C = 0.85 f'_c ab \qquad (3.25)$$

Also, for the common concretes with $f'_c \leq 4000$ psi, the depth of the rectangular stress block is $a = 0.85c$, with c being the distance to the neutral axis. For higher-strength concretes, this distance is $a = \beta_1 c$, with the β_1 values shown in Table 3.1. This is expressed in ACI Code 10.2.7.3 as follows: For f'_c between 2500 and 4000 psi, β_1 shall be taken as 0.85; for f'_c above 4000 psi, β_1 shall be reduced linearly at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken as less than 0.65. In mathematical terms, the relationship between β_1 and f'_c can be expressed as

$$\beta_1 = 0.85 - 0.05\,\frac{f'_c - 4000}{1000} \qquad \text{and} \qquad 0.65 \leq \beta_1 \leq 0.85 \tag{3.26}$$

Balanced Strain condition



$$c = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d \qquad (3.27)$$

which is seen to be identical to Eq. (3.23). Then from the equilibrium requirement that $C = T$

$$\rho_b f_y bd = 0.85 f_c' ab = 0.85 \beta_1 f_c' bc$$

from which

$$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y} \qquad (3.28)$$

This is easily shown to be equivalent to Eq. (3.24).

Underreinforced beam

- Compression failure is abrupt
- Tensile failure gradual
- ρ should be less than ρ_b

Read points why?

In actual practice, the upper limit on ρ should be below ρ_b for the following reasons: (1) for a beam with ρ exactly equal to ρ_b, the compressive strain limit of the concrete would be reached, theoretically, at precisely the same moment that the steel reaches its yield stress, without significant yielding before failure; (2) material properties are never known precisely; (3) strain-hardening of the reinforcing steel, not accounted for in design, may lead to a brittle concrete compression failure even though ρ may be somewhat less than ρ_b; (4) the actual steel area provided, considering standard reinforcing bar sizes, will always be equal to or larger than required, based on selected reinforcement ratio ρ, tending toward overreinforcement; and (5) the extra ductility provided by beams with lower values of ρ increases the deflection capability substantially and thus provides warning prior to failure.

ACI provisions for underreinforced beam

- ACI establishes some safe limits
- Net tensile strain ϵ_t at farthest from comp face
- Strength reduction factor φ

the reinforcement d. Substituting d_t for d and ϵ_t for ϵ_y in Eq. (3.27), the net tensile strain may be represented as

$$\epsilon_t = \epsilon_u \frac{d_t - c}{c} \tag{3.29}$$

Then based on Eq. (3.28), the reinforcement ratio to produce a selected value of net tensile strain is

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d_t}{d} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \tag{3.30a}$$

or somewhat conservatively

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \tag{3.30b}$$

To ensure underreinforced behavior, ACI Code 10.3.5 establishes a minimum net tensile strain ϵ_t at the nominal member strength of 0.004 for members subjected to axial loads less than $0.10 f'_c A_g$, where A_g is the gross area of the cross section. By way of comparison ϵ_y, the steel strain at the balanced condition, is 0.00207 for $f_y = 60{,}000$ psi and 0.00259 for $f_y = 75{,}000$ psi.

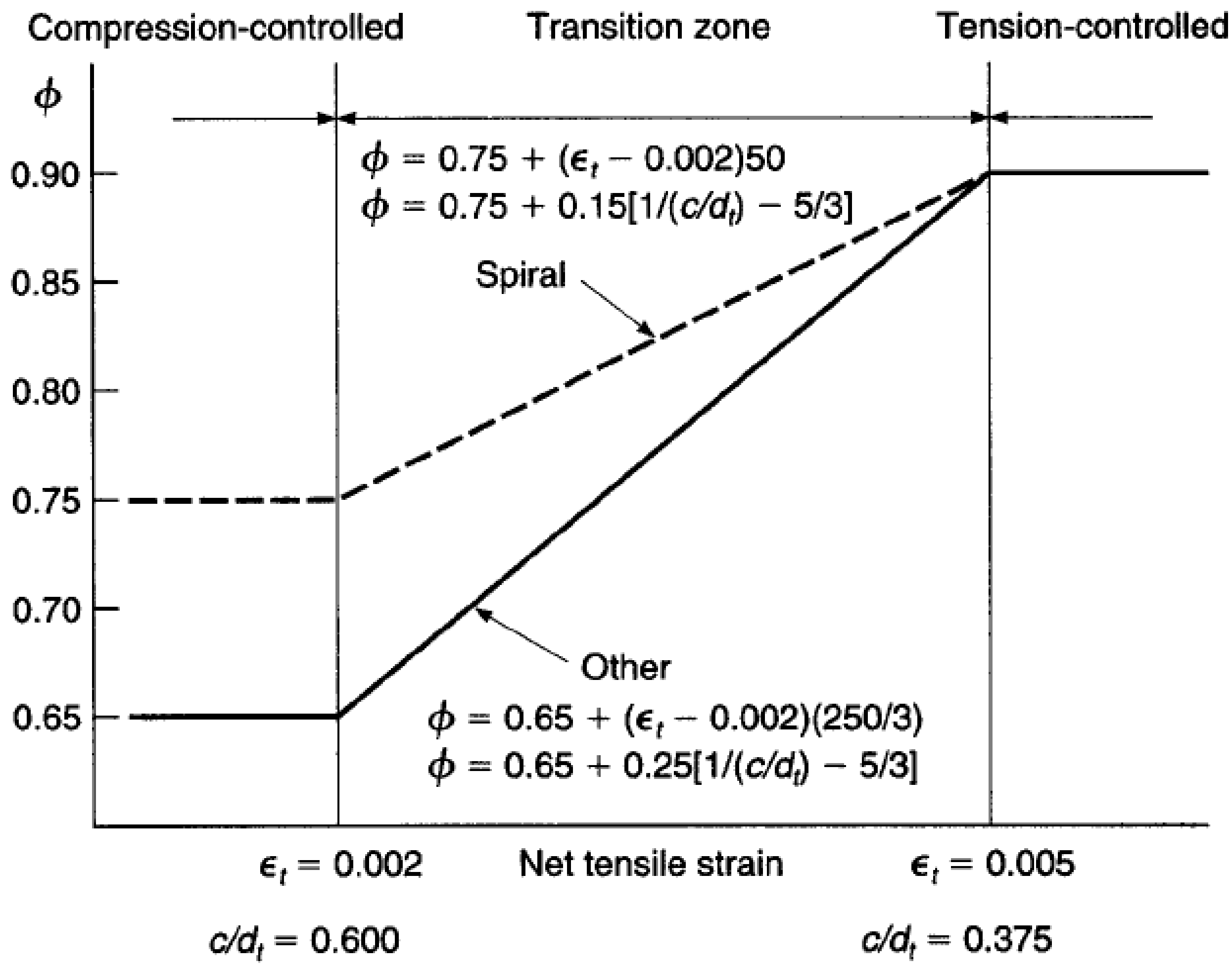
Using $\epsilon_t = 0.004$ in Eq. (3.30b) provides the maximum reinforcement ratio allowed by the ACI Code for beams

$$\rho_{\max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \tag{3.30c}$$

$$\rho_{0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} \tag{3.30d}$$

A comparison of Eqs. (3.30*c*) and (3.30*d*) shows that, for a given concrete cross section, using $\epsilon_t = 0.004$ will result in a higher reinforcement ratio, and thus a higher nominal flexural strength, than using $\epsilon_t = 0.005$. This higher strength, however, cannot be used to full advantage in design because the increase in flexural strength is canceled by the drop in ϕ as ϵ_t decreases from 0.005 to 0.004. As a result, the maximum practical reinforcement ratio for beams is attained at a net tensile strain of 0.005. Values of ϵ_t below 0.005 are not recommended for the design of members with low axial loads.

FIGURE 3.9
Variation of strength reduction factor with net tensile strain in the steel.



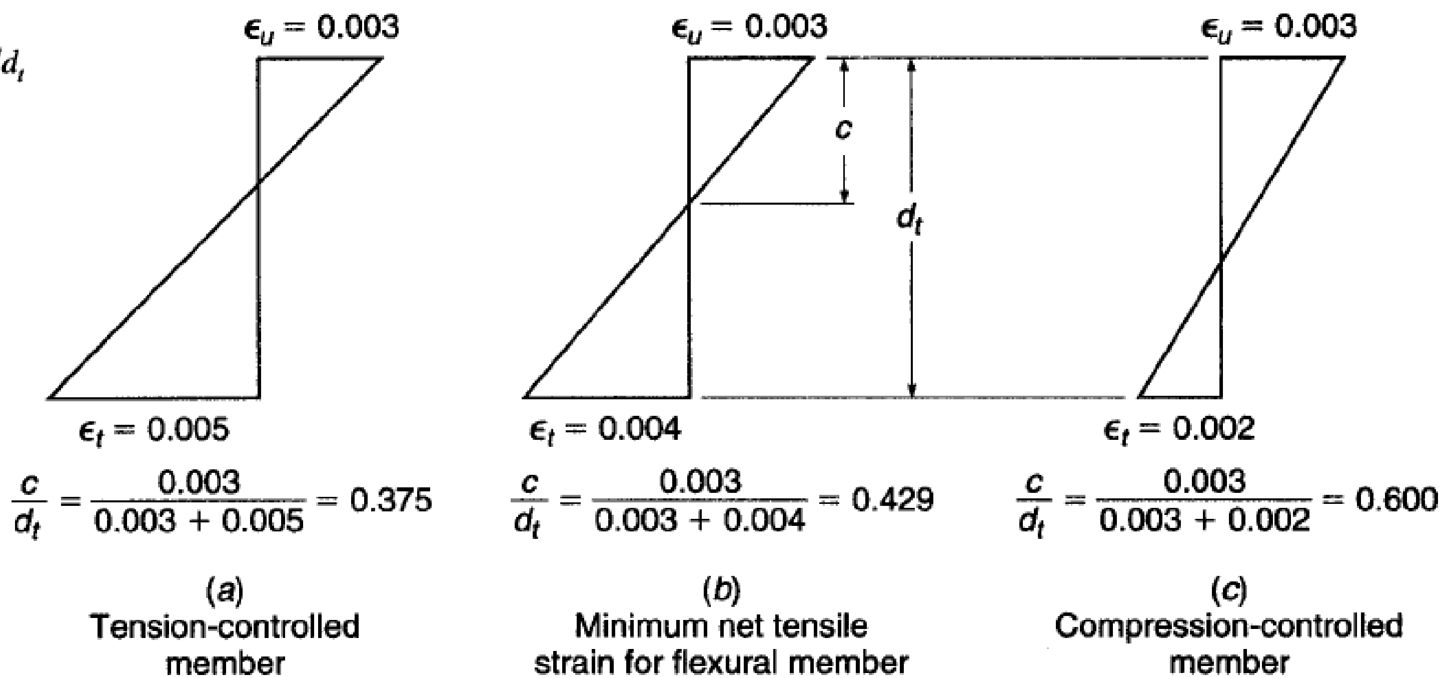
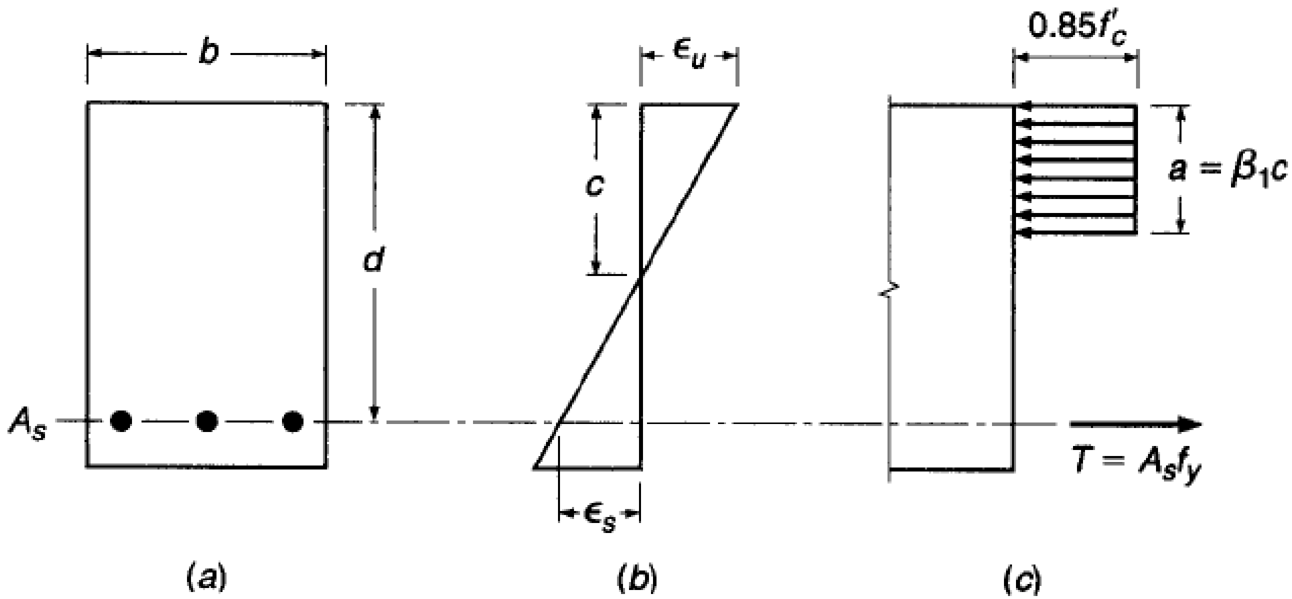


FIGURE 3.10
Net tensile strain and c/d_t ratios.

FIGURE 3.11
Singly reinforced rectangular beam.



that steel is yielding in tension, $f_s = f_y$ at failure, and the nominal flexural strength (referring to Fig. 3.11) is given by

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{3.31}$$

where

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{3.32}$$

It is convenient for everyday design to combine Eqs. (3.31) and (3.32) as follows. Noting that $A_s = \rho bd$, Eq. (3.32) can be rewritten as

$$a = \frac{\rho f_y d}{0.85 f'_c} \tag{3.33}$$

This is then substituted into Eq. (3.31) to obtain

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c} \right) \tag{3.34}$$

which is identical to Eq. (3.20*b*) derived in Section 3.3c. This basic equation can be simplified further as follows:

$$M_n = R b d^2 \tag{3.35}$$

in which

$$R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f'_c} \right) \tag{3.36}$$

The *flexural resistance factor* R depends only on the reinforcement ratio and the

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right) \tag{3.37}$$

or, alternatively,

$$\phi M_n = \phi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c} \right) \tag{3.38}$$

or

$$\phi M_n = \phi R b d^2 \tag{3.39}$$

EXAMPLE 3.4 Using the equivalent rectangular stress distribution, directly calculate the nominal strength of the beam previously analyzed in Example 3.3. Recall that $b = 10$ in., $d = 23$ in., $A_s = 2.37$ in^2., $f'_c = 4000$ psi, $f_y = 60{,}000$ psi, and $\beta_1 = 0.85$.

Solution. The distribution of stresses, internal forces, and strains is shown in Fig. 3.11. The maximum practical reinforcement ratio is calculated from Eq. (3.30*d*) as

$$\rho_{0.005} = 0.85 \times 0.85 \frac{4000}{60{,}000} \frac{0.003}{0.003 + 0.005} = 0.0181$$

and comparison with the actual reinforcement ratio of 0.0103 confirms that the member is underreinforced and will fail by yielding of the steel. Alternatively, recalling that $c = 4.94$ in.,

$$\frac{c}{d_t} = \frac{c}{d} = \frac{4.94}{23} = 0.215$$

which is less than 0.375, the value of c/d_t corresponding to $\epsilon_t = 0.005$, also confirming that the member is underreinforced. The depth of the equivalent stress block is found from the equilibrium condition that $C = T$. Hence $0.85f'_c ab = A_s f_y$, or $a = 2.37 \times 60{,}000/(0.85 \times 4000 \times 10) = 4.18$. The nominal moment is

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 2.37 \times 60{,}000(23 - 2.09) = 2{,}970{,}000 \text{ in-lb} = 248 \text{ ft-kips}$$

EXAMPLE 3.4 ***(continued)***

Calculate the design moment capacity ϕM_n for the beam analyzed earlier in Example 3.4.

Solution. Comparing ρ with $\rho_{0.005}$ or c/d_t for the beam with the value of c/d_t corresponding to $\epsilon_t = 0.005$ demonstrates that $\epsilon_t > 0.005$. Therefore, $\phi = 0.90$ and the design capacity is

$$\phi M_n = 0.9 \times 248 = 223 \text{ ft-kips}$$

Minimum Reinforcement Ratio

- If the flexural strength (of cracked section) is less than the moment that produced cracking of the previously uncracked section, the beam fails immediately upon formation of first flexural crack.
- To ensure against this type of failure, a minimum amount of reinforcement is provided

For a rectangular section having width b, total depth h, and effective depth d (see Fig. 3.2b), the section modulus with respect to the tension fiber is $bh^2/6$. For typical cross sections, it is satisfactory to assume that $h/d = 1.1$ and that the internal lever arm at flexural failure is $0.95d$. If the modulus of rupture is taken as $f_r = 7.5\sqrt{f'_c}$, as usual, then an analysis equating the cracking moment to the flexural strength results in

$$A_{s,\min} = \frac{1.6\sqrt{f'_c}}{f_y} bd \tag{3.40a}$$

This development can be generalized to apply to beams having a T cross section (see Section 3.8 and Fig. 3.16). The corresponding equations depend on the proportions of the cross section and on whether the beam is bent with the flange (slab) in tension or in compression. For T beams of typical proportions that are bent with the flange in compression, analysis will confirm that the minimum steel area should be

$$A_{s,\min} = \frac{2.7\sqrt{f_c'}}{f_y} b_w d \tag{3.40b}$$

where b_w is the width of the web, or stem, projecting below the slab. For T beams that are bent with the flange in tension, from a similar analysis, the minimum steel area is

$$A_{s,\min} = \frac{6.2\sqrt{f_c'}}{f_y} b_w d \tag{3.40c}$$

The ACI Code requirements for minimum steel area are based on the results just discussed, but there are some differences. According to ACI Code 10.5, at any section where tensile reinforcement is required by analysis, with some exceptions as noted below, the area A_s provided must not be less than

$$A_{s,\min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200 b_w d}{f_y} \tag{3.41}$$

According to ACI Code 10.5, the requirements of Eq. (3.41) need not be imposed if, at every section, the area of tensile reinforcement provided is at least one-third greater than that required by analysis. This provides sufficient reinforcement for large members such as grade beams, where the usual equations would require excessive amounts of steel.

For structural slabs and footings of uniform thickness, the minimum area of tensile reinforcement in the direction of the span is that required for shrinkage and temperature steel (see Section 13.3 and Table 13.2), and the above minimums need not be imposed. The maximum spacing of such steel is the smaller of 3 times the total slab thickness or 18 in.

Review problem

EXAMPLE 3.5 **Flexural strength of a given member.** A rectangular beam has width 12 in. and effective depth 17.5 in. It is reinforced with four No. 9 (No. 29) bars in one row. If $f_y = 60{,}000$ psi and $f'_c = 4000$ psi, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to the ACI Code?

Solution. From Table A.2 of Appendix A, the area of four No. 9 (No. 29) bars is 4.00 in². Assuming that the beam is underreinforced and using Eq. (3.32),

$$a = \frac{4.00 \times 60}{0.85 \times 4 \times 12} = 5.88 \text{ in.}$$

The depth of the neutral axis is $c = a/\beta_1 = 5.88/0.85 = 6.92$, giving

$$\frac{c}{d_t} = \frac{6.92}{17.5} = 0.395$$

which is between 0.429 and 0.375, the values corresponding, respectively, to $\epsilon_t = 0.004$ and $\epsilon_t = 0.005$, as shown in Fig. 3.10. Thus, the beam is, as assumed, underreinforced, and from Eq. (3.31)

$$M_n = 4.00 \times 60\left(17.5 - \frac{5.88}{2}\right) = 3490 \text{ in-kips}$$

The fact that the beam is unreinforced could also have been established by calculating $\rho = 4.00/(12 \times 17.5) = 0.190$, which just exceeds $\rho_{0.005}$, which is calculated using Eq. (3.30*d*).

$$\rho_{0.005} = 0.85 \times 0.85\left(\frac{4}{60}\right)\left(\frac{0.003}{0.003 + 0.005}\right) = 0.0181$$

Because the net tensile strain ϵ_t is between 0.004 and 0.005, ϕ must be calculated: $\epsilon_t = \epsilon_u(d - c)/c = 0.003 \times 17.5 - 6.92/6.92 = 0.00458$. Using linear interpolation from Fig. 3.9, $\phi = 0.87$, and the design strength is taken as

$$\phi M_n = 0.87 \times 3490 = 3040 \text{ in-kips}$$

The ACI Code limits on the reinforcement ratio

$$\rho_{max} = 0.0206$$

$$\rho_{min} = \frac{3\sqrt{4000}}{60,000} \geq \frac{200}{60,000} = 0.0033$$

are satisfied for this beam.

Design Problem

EXAMPLE 3.6 **Concrete dimensions and steel area to resist a given moment.** Find the concrete cross section and the steel area required for a simply supported rectangular beam with a span of 15 ft that is to carry a computed dead load of 1.27 kips/ft and a service live load of 2.15 kips/ft, as shown in Fig. 3.12. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. Load factors are first applied to the given service loads to obtain the factored load for which the beam is to be designed, and the corresponding moment:

$$w_u = 1.2 \times 1.27 + 1.6 \times 2.15 = 4.96 \text{ kips/ft}$$

$$M_u = \frac{1}{8} \times 4.96 \times 15^2 \times 12 = 1670 \text{ in-kips}$$

The concrete dimensions will depend on the designer's choice of reinforcement ratio. To minimize the concrete section, it is desirable to select the maximum permissible reinforcement ratio. To maintain $\phi = 0.9$, the maximum reinforcement ratio corresponding to a net tensile strain of 0.005 will be selected (see Fig. 3.9). Then, from Eq. (3.30*d*)

$$\rho_{0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.85 \times 0.85 \left(\frac{4}{60}\right)\left(\frac{0.003}{0.003 + 0.005}\right) = 0.0181$$

Using Eq. (3.30*c*) gives $\rho_{max} = 0.0206$, but would require a lower strength reduction factor. Setting the required flexural strength equal to the design strength from Eq. (3.38), and substituting the selected values for ρ and material strengths,

$$M_u = \phi M_n$$

$$1670 = 0.90 \times 0.0181 \times 60bd^2\left(1 - 0.59\,\frac{0.0181 \times 60}{4}\right)$$

from which

$$bd^2 = 2040 \text{ in}^3$$

A beam with width $b = 10$ in. and $d = 14.3$ in. will satisfy this requirement. The required steel area is found by applying the chosen reinforcement ratio to the required concrete dimensions:

$$A_s = 0.0181 \times 10 \times 14.3 = 2.59 \text{ in}^2$$

Two No. 10 (No. 32) bars provide 2.54 in^2, which is very close to the required area.

Assuming 2.5 in. concrete cover from the centroid of the bars, the required total depth is $h = 16.8$ in. In actual practice, however, the concrete dimensions b and h are always rounded up to the nearest inch, and often to the nearest multiple of 2 in. (see Section 3.5). The actual d is then found by subtracting the required concrete cover dimension from h. For the present example, $b = 10$ in. and $h = 18$ in. will be selected, resulting in effective depth $d = 15.5$ in. Improved economy then may be possible, refining the steel area based on the actual, larger, effective depth. One can obtain the revised steel requirement directly by solving Eq. (3.38) for ρ, with $\phi M_n = M_u$. A quicker solution can be obtained by iteration. First a reasonable value of a is assumed, and A_s is found from Eq. (3.37). From Eq. (3.32) a revised estimate of a is obtained, and A_s is revised. This method converges very rapidly. For example, assume $a = 5$ in. Then

$$A_s = \frac{1670}{0.90 \times 60(15.5 - 2.5)} = 2.38 \text{ in}^2$$

Checking the assumed a gives

$$a = \frac{2.38 \times 60}{0.85 \times 4 \times 10} = 4.20 \text{ in.}$$

This is close enough to the assumed value that no further calculation is required. The required steel area of 2.38 in^2 could be provided using three No. 8 (No. 25) bars, but for simplicity of construction, two No. 10 (No. 32) bars will be used as before.

A somewhat larger beam cross section using less steel may be more economical, and will tend to reduce deflections. As an alternative solution, the beam will be redesigned with a lower reinforcement ratio of $\rho = 0.60\rho_{max} = 0.60 \times 0.0206 = 0.0124$. Setting the required strength equal to the design strength [Eq. (3.38)] as before,

$$1670 = 0.90 \times 0.0124 \times 60bd^2\left(1 - 0.59\,\frac{0.0124 \times 60}{4}\right)$$

and

$$bd^2 = 2800 \text{ in}^3$$

A beam with $b = 10$ in. and $d = 16.7$ in. will meet the requirement, for which

$$A_s = 0.0124 \times 10 \times 16.7 = 2.07 \text{ in}^2$$

Two No. 9 (No. 29) bars are almost sufficient, providing an area of 2.00 in^2. If the total concrete height is rounded up to 20 in., a 17.5 in. effective depth results, reducing the required steel area to 1.96 in^2. Two No. 9 (No. 29) bars remain the best choice.

- Infinite number of solution is possible

- Economic $0.5\rho_{0.005}$ to $0.75\rho_{0.005}$

Determination of steel area

EXAMPLE 3.7 **Determination of steel area.** Using the same concrete dimensions as were used for the second solution of Example 3.6 ($b = 10$ in., $d = 17.5$ in., and $h = 20$ in.) and the same material strengths, find the steel area required to resist a moment M_u of 1300 in-kips.

Solution. Assume $a = 4.0$ in. Then

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 2.0)} = 1.55 \text{ in}^2$$

Checking the assumed a gives

$$a = \frac{1.55 \times 60}{0.85 \times 4 \times 10} = 2.74 \text{ in.}$$

Next assume $a = 2.6$ in. and recalculate A_s:

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 1.3)} = 1.49 \text{ in}^2$$

No further iteration is required. Use $A_s = 1.49$ in^2. Two No. 8 (No. 25) bars, $A_s = 1.58$ in.^{2}, will be used. A check of the reinforcement ratio shows $\rho < \rho_{0.005}$ and $\phi = 0.9$.

EXAMPLE 3.8 **Determination of steel area and variable strength reduction factor.** Architectural considerations limit the height of a 20 ft long simple span beam to 16 in. and the width to 12 in. The following loads and material properties are given: $w_d = 0.79$ kips/ft, $w_l = 1.65$ kips/ft, $f'_c = 5000$ psi, and $f_y = 60{,}000$ psi. Determine the reinforcement for the beam.

Solution. Calculating the factored loads gives

$$w_u = 1.2 \times 0.79 + 1.6 \times 1.65 = 3.59 \text{ kips/ft}$$

$$M_u = 3.59 \times \frac{20^2}{8} = 179 \text{ ft-kips} = 2150 \text{ in-kips}$$

Assume $a = 4.0$ in. and $\phi = 0.90$. The structural depth is (16 − 2.5) in. = 13.5 in. Calculating A_s gives

$$A_s = \frac{M_u/\phi}{f_y(d - a/2)} = \frac{2150/0.90}{60(13.5 - 2.0)} = 3.46 \text{ in}^2$$

Try two No. 10 (No. 32) and one No. 9 (No. 29) bar, $A_s = 3.54$ in^2.

Check $a = 3.54 \times 60/(0.85 \times 5 \times 12) = 4.16$ in. from Eq. (3.32). This is more than assumed; therefore, continue to check the moment capacity.

$$M_n = 3.54 \times 60(13.5 - 4.16/2) = 2426 \text{ in-kips}$$

Using a ϕ of 0.90 gives $\phi M_n = 2183$ in-kips, which is adequate; however, the net tensile strai must be checked to validate the selection of $\phi = 0.9$. In this case $c = a/\beta_1 = 4.16/0.80$ = 5.20 in. The c/d ratio is $0.385 > 0.375$, so $\epsilon_t > 0.005$ is not satisfied. The corresponding n tensile strain is

$$\epsilon_t = 0.003\,\frac{13.5 - 5.2}{5.2} = 0.00479$$

A value of $\epsilon_t = 0.00479$ is allowed by the ACI Code, but only if the strength reduction factor adjusted. A linear interpolation from Fig. 3.9 gives $\phi = 0.88$ and $M_u = \phi M_n = 2140$ in-kip which is less than the required capacity. Try increasing the reinforcement to three No. 10 (No. 3 bars, $A_s = 3.81\ \text{in}^2$. Repeating the calculations,

$$a = \frac{3.81 \times 60}{0.85 \times 5 \times 12} = 4.48\ \text{in.}$$

$$c = \frac{4.48}{0.80} = 5.60\ \text{in.}$$

$$M_n = 3.81 \times 60\left(13.5 - \frac{4.48}{2}\right) = 2574\ \text{in-kips}$$

$$\epsilon_t = \frac{0.003(13.5 - 5.60)}{5.60} = 0.00423$$

$$\phi = 0.483 + 83.3 \times 0.00423 = 0.835$$

$$M_u = \phi M_n = 0.835 \times 2574 = 2150\ \text{in-kips}$$

which meets the design requirements.

In actuality, the first solution deviates less than 1 percent from the desired value and would likely be acceptable. The remaining portion of the example demonstrates the design implications of requiring a variable strength reduction factor when the net tensile strain falls between 0.005 and 0.004. In this example, the reinforcement increased nearly 8 percent, yet the design moment capacity ϕM_n only increased 0.5 percent due to the decreasing strength reduction factor. For this reason, designs with $\rho < \rho_{0.005}$ are desirable.

Overreinforced beam

According to the ACI Code, all beams are to be designed for yielding of the tension steel with ϵ_t not less than 0.004 and thus $\rho \leq \rho_{max}$. Occasionally, however, such as when analyzing the capacity of existing construction, it may be necessary to calculate the flexural strength of an overreinforced compression-controlled member, for which f_s is less than f_y at flexural failure.

In this case, the steel strain, in Fig. 3.11*b*, will be less than the yield strain, but can be expressed in terms of the concrete strain ϵ_u and the still-unknown distance c to the neutral axis:

$$\epsilon_s = \epsilon_u \frac{d - c}{c} \tag{3.42}$$

From the equilibrium requirement that $C = T$, one can write

$$0.85\beta_1 f_c' bc = \rho\epsilon_s E_s bd$$

Substituting the steel strain from Eq. (3.42) in the last equation, and defining $k_u = c/d$, one obtains a quadratic equation in k_u as follows:

$$k_u^2 + m\rho k_u - m\rho = 0$$

Here, $\rho = A_s/bd$ as usual, and m is a material parameter given by

$$m = \frac{E_s\epsilon_u}{0.85\beta_1 f_c'} \tag{3.43}$$

Solving the quadratic equation for k_u,

$$k_u = \sqrt{m\rho + \left(\frac{m\rho}{2}\right)^2} - \frac{m\rho}{2} \tag{3.44}$$

The neutral axis depth for the overreinforced beam can then easily be found from $c = k_u d$, after which the stress-block depth $a = \beta_1 c$. With steel strain ϵ_s then computed from Eq. (3.42), and with $f_s = E_s\epsilon_s$, the nominal flexural strength is

$$M_n = A_s f_s\left(d - \frac{a}{2}\right) \tag{3.45}$$

The strength reduction factor ϕ will equal 0.65 for beams in this range.

Design Aids: Find M_n

EXAMPLE 3.9 **Flexural strength of a given member.** Find the nominal flexural strength and design strength of the beam in Example 3.5, which has $b = 12$ in. and $d = 17.5$ in. and is reinforced with four No. 9 (No. 29) bars. Make use of the design aids of Appendix A. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. From Table A.2, four No. 9 (No. 29) bars provide $A_s = 4.00$ in^2, and with $b =$ 12 in. and $d = 17.5$ in., the reinforcement ratio is $\rho = 4.00/(12 \times 17.5) = 0.0190$. According to Table A.4, this is below $\rho_{max} = 0.0206$ and above $\rho_{min} = 0.0033$. Then from Table A.5b, with

$f'_c = 4000$ psi, $f_y = 60{,}000$ psi, and $\rho = 0.019$, the value $R = 949$ psi is found. The nominal and design strengths are (with $\phi = 0.87$ from Example 3.5), respectively,

$$M_n = Rbd^2 = 949 \times 12 \times \frac{17.5^2}{1000} = 3490 \text{ in-kips}$$

$$\phi M_n = 0.87 \times 3490 = 3040 \text{ in-kips}$$

as before.

TABLE A.4
Limiting steel reinforcement ratios for tension-controlled members

f_y, psi	f'_c, psi	β_1	$\rho_{0.005}$[a] $\epsilon_t = 0.005$[b]	ρ_{max}[a] $\epsilon_t = 0.004$[c]	$\rho_{min} = \frac{200}{f_y}$	$\rho_{min} = \frac{3\sqrt{f'_c}}{f_y}$
40,000	3000	0.85	0.0203	0.0232	0.0050	0.0041
	4000	0.85	0.0271	0.0310	0.0050	0.0047
	5000	0.80	0.0319	0.0364	0.0050	0.0053
	6000	0.75	0.0359	0.0410	0.0050	0.0058
	7000	0.70	0.0390	0.0446	0.0050	0.0063
	8000	0.65	0.0414	0.0474	0.0050	0.0067
	9000	0.65	0.0466	0.0533	0.0050	0.0071
50,000	3000	0.85	0.0163	0.0186	0.0040	0.0033
	4000	0.85	0.0217	0.0248	0.0040	0.0038
	5000	0.80	0.0255	0.0291	0.0040	0.0042
	6000	0.75	0.0287	0.0328	0.0040	0.0046
	7000	0.70	0.0312	0.0357	0.0040	0.0050
	8000	0.65	0.0332	0.0379	0.0040	0.0054
	9000	0.65	0.0373	0.0426	0.0040	0.0057
60,000	3000	0.85	0.0135	0.0155	0.0033	0.0027
	4000	0.85	0.0181	0.0206	0.0033	0.0032
	5000	0.80	0.0213	0.0243	0.0033	0.0035
	6000	0.75	0.0239	0.0273	0.0033	0.0039
	7000	0.70	0.0260	0.0298	0.0033	0.0042
	8000	0.65	0.0276	0.0316	0.0033	0.0045
	9000	0.65	0.0311	0.0355	0.0033	0.0047
75,000	3000	0.85	0.0108	0.0124	0.0027	0.0022
	4000	0.85	0.0145	0.0165	0.0027	0.0025
	5000	0.80	0.0170	0.0194	0.0027	0.0028
	6000	0.75	0.0191	0.0219	0.0027	0.0031
	7000	0.70	0.0208	0.0238	0.0027	0.0033
	8000	0.65	0.0221	0.0253	0.0027	0.0036
	9000	0.65	0.0249	0.0284	0.0027	0.0038

[a] $\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003 + \epsilon_t}$

[b] $\frac{c}{d_t} = 0.375, \frac{a}{d_t} = 0.375\beta_1$

[c] $\frac{c}{d_t} = 0.429, \frac{a}{d_t} = 0.429\beta_1$

TABLE A.5a
Flexural resistance factor: $R = \rho f_y\left(1 - 0.588\dfrac{\rho f_y}{f'_c}\right)$ psi

	f_y = 40,000 psi				f_y = 60,000 psi			
	f'_c, psi				f'_c, psi			
ρ	3000	4000	5000	6000	3000	4000	5000	6000
0.0005	20	20	20	20	30	30	30	30
0.0010	40	40	40	40	59	59	60	60
0.0015	59	59	60	60	88	89	89	89
0.0020	79	79	79	79	117	118	118	119
0.0025	98	99	99	99	146	147	147	148
0.0030	117	118	118	119	174	175	176	177
0.0035	136	137	138	138	201	204	205	206
0.0040	155	156	157	157	229	232	233	234
0.0045	174	175	176	177	256	259	261	263
0.0050	192	194	195	196	282	287	289	291
0.0055	211	213	214	215	309	314	317	319
0.0060	229	232	233	234	335	341	345	347
0.0065	247	250	252	253	360	368	372	375
0.0070	265	268	271	272	385	394	399	403
0.0075	282	287	289	291	410	420	426	430
0.0080	300	305	308	310	435	446	453	457
0.0085	317	323	326	329	459	472	479	485
0.0090	335	341	345	347	483	497	506	511
0.0095	352	359	363	366	506	522	532	538
0.0100	369	376	381	384	529	547	558	565
0.0105	385	394	399	403	552	572	583	591
0.0110	402	412	417	421	575	596	609	617
0.0115	419	429	435	439	597	620	634	643
0.0120	435	446	453	457	618	644	659	669
0.0125	451	463	471	476	640	667	684	695
0.0130	467	480	488	494	661	691	708	720
0.0135	483	497	506	511	681	714	733	746
0.0140	499	514	523	529	702	736	757	771
0.0145	514	531	540	547	722	759	781	796
0.0150	529	547	558	565	741	781	805	821
0.0155	545	563	575	582	760	803	828	845
0.0160	560	580	592	600		825	852	870
0.0165	575	596	609	617		846	875	894
0.0170	589	612	626	635		867	898	918
0.0175	604	628	642	652		888	920	942
0.0180	618	644	659	669		909	943	966
0.0185	633	660	676	686		929	965	989
0.0190	647	675	692	703		949	987	1013
0.0195	661	691	708	720		969	1009	1036
0.0200	675	706	725	737		988	1031	1059

TABLE A.5b
Flexural resistance factor: $R = \rho f_y\left(1 - 0.588\,\frac{\rho f_y}{f'_c}\right)$ **psi**

ρ	f_y = 40,000 psi				f_y = 60,000 psi			
	f'_c, psi				f'_c, psi			
	3000	4000	5000	6000	3000	4000	5000	6000
0.003	117	118	118	119	174	175	176	177
0.004	155	156	157	157	229	232	233	234
0.005	192	194	195	196	282	287	289	291
0.006	229	232	233	234	335	341	345	347
0.007	265	268	271	272	385	394	399	403
0.008	300	305	308	310	435	446	453	457
0.009	335	341	345	347	483	497	506	511
0.010	369	376	381	384	529	547	558	565
0.011	402	412	417	421	575	596	609	617
0.012	435	446	453	457	618	644	659	669
0.013	467	480	488	494	661	691	708	720
0.014	499	514	523	529	702	736	757	771
0.015	529	547	558	565	741	781	805	821
0.016	560	580	592	600	779	825	852	870
0.017	589	612	626	635		867	898	918
0.018	618	644	659	669		909	943	966
0.019	647	675	692	703		949	987	1013
0.020	675	706	725	737		988	1031	1059
0.021	702	736	757	771			1073	1104
0.022	728	766	789	804			1115	1149
0.023	754	796	820	837			1156	1193
0.024		825	852	870			1196	1237
0.025		853	882	902				1280
0.026		881	913	934				1322
0.027		909	943	966				1363
0.028		936	972	997				
0.029		962	1002	1028				
0.030		988	1031	1059				
0.031		1014	1059	1089				
0.032			1087	1119				
0.033			1115	1149				
0.034			1142	1179				
0.035			1170	1208				
0.036			1196	1237				
0.037				1265				
0.038				1294				
0.039				1322				
0.040				1349				
0.041				1376				

Design Aids: Concrete dimensions and steel

EXAMPLE 3.10 **Concrete dimensions and steel area to resist a given moment.** Find the cross section of concrete and the area of steel required for the beam in Example 3.6, making use of the design aids of Appendix A. $M_u = 1670$ in-kips, $f'_c = 4000$ psi, and $f_y = 60{,}000$ psi. Use a reinforcement ratio of $0.60\rho_{max}$.

Solution. From Table A.4, the maximum reinforcement ratio is $\rho_{max} = 0.0206$. For economy, a value of $\rho = 0.60\rho_{max} = 0.0124$ will be used. For that value, by interpolation from Table A.5a, the required value of R is 663. Then

$$bd^2 = \frac{M_u}{\phi R} = \frac{1670 \times 1000}{0.90 \times 663} = 2800 \text{ in}^3$$

Concrete dimensions $b = 10$ in. and $d = 16.7$ in. will satisfy this, but the depth will be rounded to 17.5 in. to provide a total beam depth of 20.0 in. It follows that

$$R = \frac{M_u}{\phi bd^2} = \frac{1670 \times 1000}{0.90 \times 10 \times 17.5^2} = 606 \text{ psi}$$

and from Table A.5a, by interpolation, $\rho = 0.0112$. This leads to a steel requirement of $A_s = 0.0112 \times 10 \times 17.5 = 1.96$ in^2 as before.

Design Aids: find steel area

EXAMPLE 3.11 **Determination of steel area.** Find the steel area required for the beam in Example 3.7, with concrete dimensions $b = 10$ in. and $d = 17.5$ in. known to be adequate to carry the factored load moment of 1300 in-lb. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Solution. Note that in cases in which the concrete dimensions are known to be adequate and only the reinforcement must be found, the iterative method used earlier is not required. The necessary flexural resistance factor is

$$R = \frac{M_u}{\phi b d^2} = \frac{1300 \times 1000}{0.90 \times 10 \times 17.5^2} = 472 \text{ psi}$$

According to Table A.5a, with the specified material strengths, this corresponds to a reinforcement ratio of $\rho = 0.0085$, giving a steel area of

$$A_s = 0.0085 \times 10 \times 17.5 = 1.49 \text{ in}^2$$

as before. Two No. 8 (No. 25) bars will be used.

Practical considerations in the design of Beams: Concrete Protection for reinforcement

- Protection for steel against fire and corrosion
- Concrete cover depends on member and exposure
- Surfaces not exposed to ground or weather
  - Not less than ¾ in for slab
  - Not less than 1.5 in for beams and columns
- Surfaces exposed to weather or in contact with ground
  - At least 2in
- Cast against ground with no form work
  - Min 3 in cover

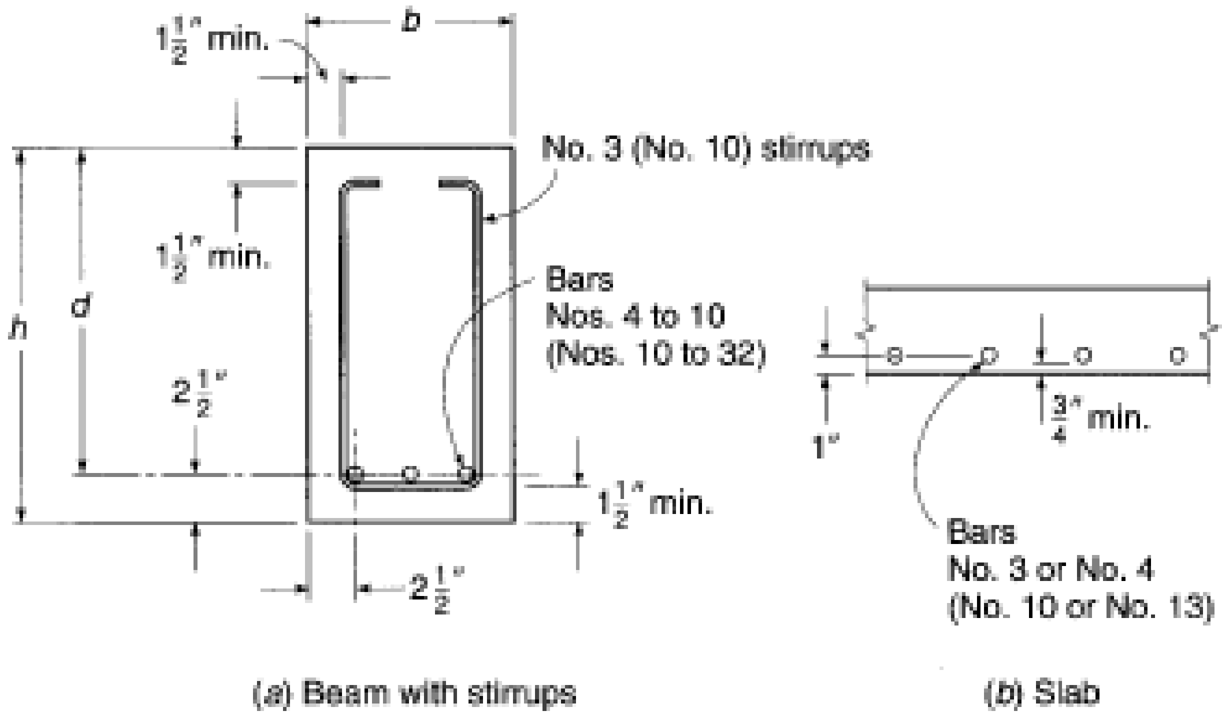


FIGURE 3.13
Requirements for concrete cover in beams and slabs.

- b and h are rounded to 1 or 2 inch
- Slab rounded to ¼ or ½ inch (greater than 6 inch)

- Proportions- d 2-3 times of b

Selection of bar and spacing

- No 3 to No 11 for beams
- No 14 and No 18 for columns

- Mixing of sizes allowed with 2 bar sizes

Gap between bars

- Clear distance between bars not less than bar dia or 1 inch
- Two or more layers- min 1 inch
- Upper bar directly above

DOUBLY REINFORCED BEAM

- Beams with tension and compression reinforcement
- Cross section is limited
- Compression steel is used for other reasons- long term deflection, reversal of moment, hanger bar for stirrup

Tension and compression steel both at yields

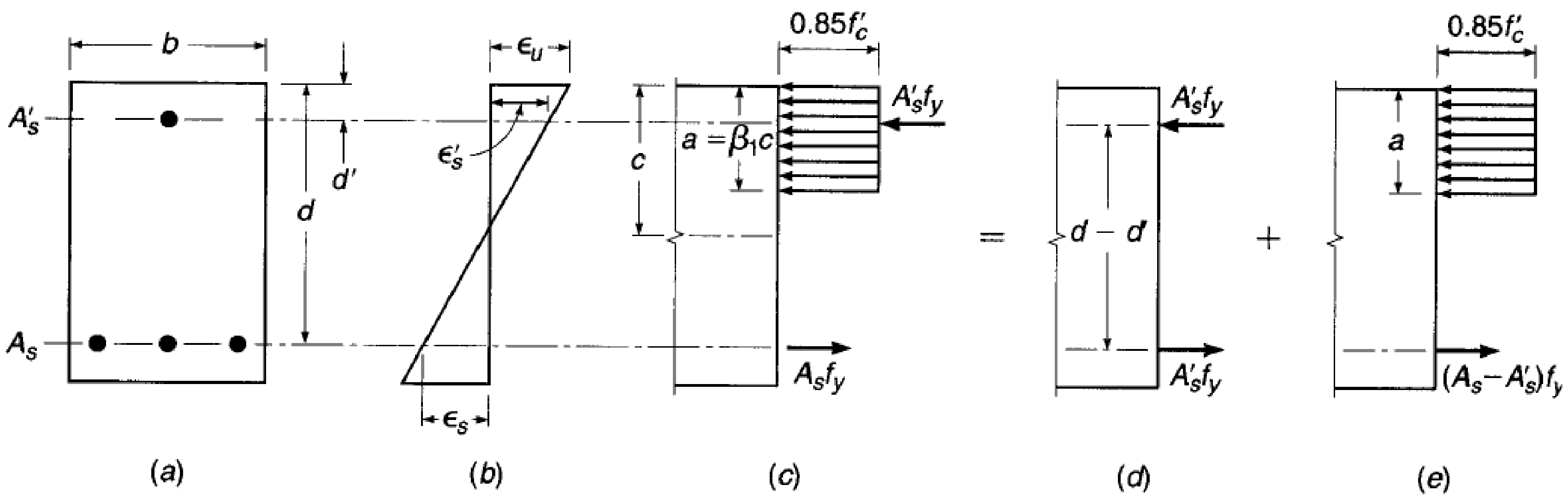


FIGURE 3.14
Doubly reinforced rectangular beam.

$$M_{n1} = A'_s f_y(d - d') \tag{3.46a}$$

as shown in Fig. 3.14*d*. The second part, M_{n2}, is the contribution of the remaining tension steel $A_s - A'_s$ acting with the compression concrete:

$$M_{n2} = (A_s - A'_s)f_y\left(d - \frac{a}{2}\right) \tag{3.46b}$$

as shown in Fig. 3.14*e*, where the depth of the stress block is

$$a = \frac{(A_s - A'_s)f_y}{0.85f'_c b} \tag{3.47a}$$

With the definitions $\rho = A_s/bd$ and $\rho' = A'_s/bd$, this can be written

$$a = \frac{(\rho - \rho')f_y d}{0.85f'_c} \tag{3.47b}$$

The total nominal resisting moment is then

$$M_n = M_{n1} + M_{n2} = A'_s f_y(d - d') + (A_s - A'_s)f_y\left(d - \frac{a}{2}\right) \tag{3.48}$$

$$\bar{\rho}_b = \rho_b + \rho' \tag{3.49}$$

where ρ_b is the balanced reinforcement ratio for the corresponding singly reinforced beam and is calculated from Eq. (3.28). The ACI Code limits the net tensile strain, not the reinforcement ratio. To provide the same margin against brittle failure as for singly reinforced beams, the maximum reinforcement ratio should be limited to

$$\bar{\rho}_{\max} = \rho_{\max} + \rho' \tag{3.50a}$$

Because $\rho_{\max}$ establishes the location of the neutral axis, the limitation in Eq. (3.50*a*) will provide acceptable net tensile strains. A check of ϵ_t is required to determine the strength reduction factor ϕ and to verify net tensile strain requirements are satisfied. Substituting $\rho_{0.005}$ for $\rho_{\max}$ in Eq. (3.50*a*) will give the maximum reinforcement ratio for $\phi = 0.90$.

$$\bar{\rho}_{0.005} = \rho_{0.005} + \rho' \tag{3.50b}$$

Compression steel below yield stress

The preceding equations, through which the fundamental analysis of doubly reinforced beams is developed clearly and concisely, are valid *only* if the compression steel has yielded when the beam reached its nominal capacity. In many cases, such as for wide, shallow beams, beams with more than the usual concrete cover over the compression bars, beams with high yield strength steel, or beams with relatively small amounts of tensile reinforcement, the compression bars will be below the yield stress at failure. It is necessary, therefore, to develop more generally applicable equations to account for the possibility that the compression reinforcement has not yielded when the doubly reinforced beam fails in flexure.

Whether or not the compression steel will have yielded at failure can be determined as follows. Referring to Fig. 3.14*b*, and taking as the limiting case $\epsilon_s' = \epsilon_y$, one obtains, from geometry,

$$\frac{c}{d'} = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \qquad \text{or} \qquad c = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} d'$$

Summing forces in the horizontal direction (Fig. 3.14*c*) gives the *minimum* tensile reinforcement ratio $\bar{\rho}_{cy}$ that will ensure yielding of the compression steel at failure:

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' \tag{3.51}$$

If the *tensile reinforcement ratio* is *less than* this limiting value, the neutral axis is sufficiently high that the compression steel stress at failure is less than the yield stress. In this case, it can easily be shown on the basis of Fig. 3.14*b* and *c* that the balanced reinforcement ratio is

$$\bar{\rho}_b = \rho_b + \rho' \frac{f'_s}{f_y} \tag{3.52}$$

where

$$f'_s = E_s \epsilon'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + \epsilon_y) \right] \leq f_y \tag{3.53a}$$

To determine $\rho_{\max}$, $\epsilon_t = 0.004$ is substituted for ϵ_y in Eq. (3.53*a*), giving

$$f'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004) \right] \leq f_y \tag{3.53b}$$

Likewise, for $\epsilon_t = 0.005$,

$$f'_s = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.005) \right] \leq f_y \tag{3.53c}$$

Hence, the maximum reinforcement ratio permitted by the ACI Code is

$$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y} \tag{3.54a}$$

and the maximum reinforcement ratio for $\phi = 0.90$ is

$$\bar{\rho}_{0.005} = \rho_{0.005} + \rho' \frac{f'_s}{f_y} \tag{3.54b}$$

If the tensile reinforcement ratio is less than $\bar{\rho}_b$, as given by Eq. (3.52), and less than $\bar{\rho}_{cy}$, as given by Eq. (3.51), then the tensile steel is at the yield stress at failure but the compression steel is not, and new equations must be developed for compression steel stress and flexural strength. The compression steel stress can be expressed in terms of the still-unknown neutral axis depth as

$$f'_s = \epsilon_u E_s \frac{c - d'}{c} \tag{3.55}$$

Consideration of horizontal force equilibrium (Fig. 3.14*c* with compression steel stress equal to f'_s) then gives

$$A_s f_y = 0.85\beta_1 f'_c bc + A'_s \epsilon_u E_s \frac{c - d'}{c} \tag{3.56}$$

This is a quadratic equation in c, the only unknown, and is easily solved for c. The nominal flexural strength is found using the value of f'_s from Eq. (3.55), and $a = \beta_1 c$ in the expression

$$M_n = 0.85 f'_c ab\left(d - \frac{a}{2}\right) + A'_s f'_s (d - d') \tag{3.57}$$

TABLE 3.2
Minimum beam depths for compression reinforcement to yield

	$\epsilon_t = 0.004$		$\epsilon_t = 0.005$	
f_y, psi	Maximum d'/d	Minimum d for $d' = 2.5$ in., in.	Maximum d'/d	Minimum d for $d' = 2.5$ in., in.
40,000	0.23	10.8	0.20	12.3
60,000	0.13	18.8	0.12	21.5
75,000	0.06	42.7	0.05	48.8

Example 3.12

Flexural strength of a given member. A rectangular beam, shown in Fig. 3.15, has a width of 12 in. and an effective depth to the centroid of the tension reinforcement of 24 in. The tension reinforcement consists of six No. 10 (No. 32) bars in two rows. Compression reinforcement consisting of two No. 8 (No. 25) bars is placed 2.5 in. from the compression face of the beam. If $f_y = 60{,}000$ psi and $f'_c = 5000$ psi, what is the design moment capacity of the beam?

Solution. The steel areas and ratios are

$$A_s = 7.62 \text{ in}^2 \qquad \rho = \frac{7.62}{12 \times 24} = 0.0265$$

$$A'_s = 1.58 \text{ in}^2 \qquad \rho' = \frac{1.58}{12 \times 24} = 0.0055$$

Check the beam first as a singly reinforced beam to see if the compression bars can be disregarded,

$$\rho_{max} = 0.0243 \qquad \text{from Table A.4 or Eq. (3.30}c\text{)}$$

The actual $\rho = 0.0265$ is larger than ρ_{max}, so the beam must be analyzed as doubly reinforced. From Eq. (3.51), with $\beta_1 = 0.80$,

$$\bar{\rho}_{cy} = 0.85 \times 0.80 \times \frac{5}{60} \times \frac{2.5}{24} \times \frac{0.003}{0.003 - 0.00207} + 0.0055 = 0.0245$$

The tensile reinforcement ratio is greater than this, so the compression bars will yield when the beam fails. The maximum reinforcement ratio thus can be found from Eq. (3.50),

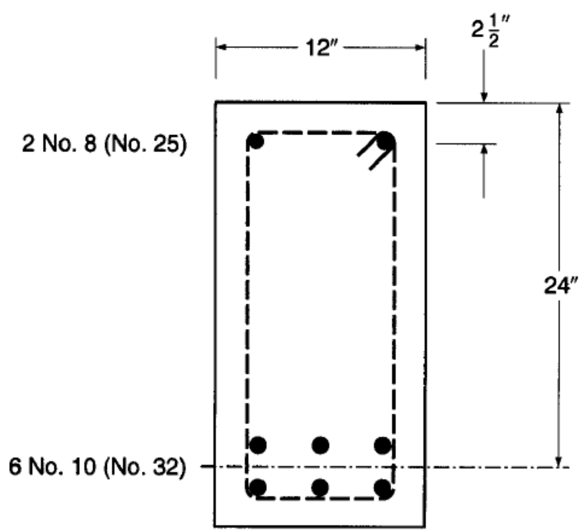
$$\bar{\rho}_{max} = 0.0243 + 0.0055 = 0.0298$$

The actual tensile reinforcement ratio is below the maximum value, as required. Then, from Eq. (3.47*a*),

$$a = \frac{(7.62 - 1.58)60}{0.85 \times 5 \times 12} = 7.11 \text{ in.}$$

$$c = a/\beta_1 = \frac{7.11}{0.80} = 8.89 \text{ in.}$$

$$\epsilon_t = 0.003\left(\frac{24 - 8.89}{8.89}\right) = 0.0051$$



and

$$\phi = 0.90$$

and from Eq. (3.48),

$$M_n = 1.58 \times 60(24 - 2.5) + 6.04 \times 60\left(24 - \frac{7.11}{2}\right) = 9450 \text{ in-kips}$$

The design strength is

$$\phi M_n = 0.90 \times 9450 = 8500 \text{ in-kips}$$

Nadim Hassoun

Doubly Reinforced beam

Nadim Hassoun

1. Calculate ρ, ρ' and $(\rho - \rho')$
   ρ_{max}, ρ_{min}

2. Calculate
$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

3. If $\rho \geq \bar{\rho}_{cy}$ compression steel yields, $f_s' = f_y$
   $\rho < \bar{\rho}_{cy}$ " " does not yield $f_s' < f_y$

4. If comp steel yields, then
   a. Check that $\rho_{max} \geq (\rho - \rho') \geq \rho_{min}$ $\phi = 0.9$
      or check $\varepsilon_t \geq 0.005$

   b. Calculate
$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b}$$

   c. Calculate
$$\phi M_n = \phi\left[(A_s - A_s')f_y\left(d - \frac{a}{2}\right) + A_s' f_y (d - d')\right]$$

5. If comp. steel does not yield, then

$$T = C$$

a. $\Rightarrow A_s f_y = 0.85 f_c' * b * \beta_1 c + A_s' f_s'$

$$A_s f_y = 0.85 \beta_1 b c f_c' + A_s' . E_s \varepsilon_u \frac{c - d'}{c}$$

$\Rightarrow$ solve this quadratic equation to find c.

b. Find $f_s' = E_s \varepsilon_u \frac{c-d'}{c}$

$\phi = 0.9$

$\varepsilon_t =$

c. Check $\rho_{max} \geq \left(\rho - \rho' . \frac{f_s'}{f_y}\right) \geq \rho_{min}$

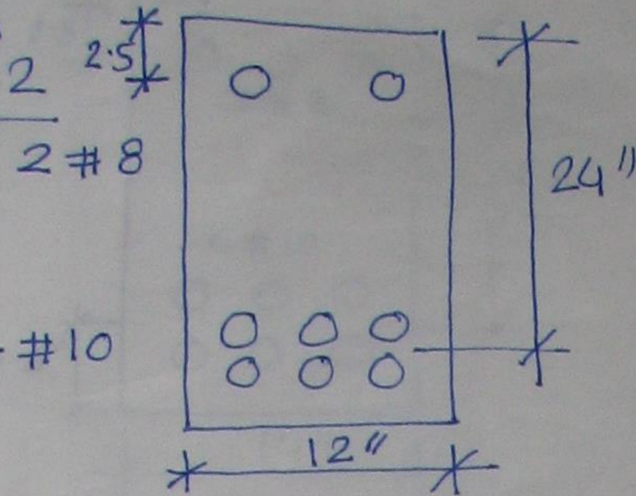
d. Calculate $a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b}$ or $a = \beta_1 c$ (check)

e. Calculate

$$\phi M_n = \phi\left[(A_s f_y - A_s' f_s')\left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')\right]$$

Find φM_n

Nilson
Ex 3.12



$f_c' = 5000$ psi

$f_y = 60{,}000$ psi

Design moment capacity = ?

$\phi M_n = ?$

Sol

$A_s = 6 \times 1.27 = 7.62 \text{ in}^2$

$\rho = \frac{A_s}{bd} = 0.0265$

$A_s' = 2 * 0.79 = 1.58 \text{ in}^2$

$\rho' = \frac{A_s'}{bd} = 0.0055$

$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + .004}$

$\rho_{max} = 0.0243$

$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$

$\rho_{min} = 0.00354$

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

$$= 0.85 * .8 \frac{5}{60} \frac{2.5}{24} \frac{.003}{.003 - .00207} + .0055 = 0.0245$$

$\rho > \bar{\rho}_{cy} \Rightarrow$ Comp bars yield when beam fails.

$\rho_{max} = .0243$ OK.

$\rho - \rho' = 0.021$

$\rho_{min} = 0.00354$

$$a = \frac{A_s - A_s'}{0.85 f_c' b} f_y = 7.11 \qquad c = \frac{a}{\beta_1} = 8.88$$

$$\varepsilon_t = 0.003 \frac{24 - 8.88}{8.88} = 0.0051 \Rightarrow \phi = 0.9$$

(d_t)

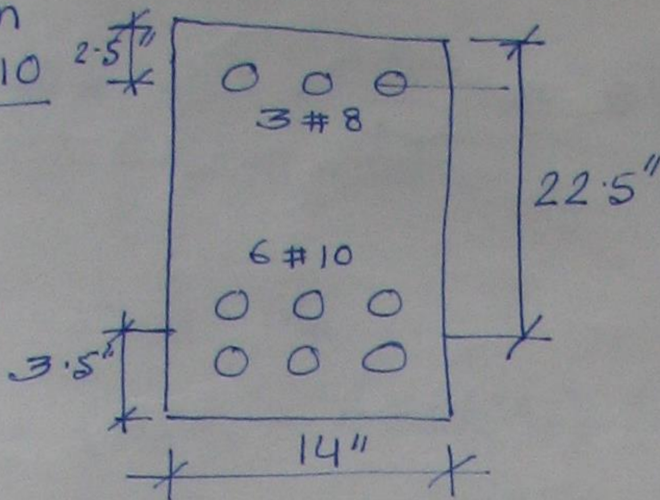
$$M_n = A_s' f_y (d - d') + (A_s - A_s') f_y (d - a/2)$$

$$= 1.58 * 60 (24 - 2.5) + 6.04 * 60 \left(24 - \frac{7.11}{2}\right) = 9447 \text{ kin}$$

$\phi M_n = 8503$ k-in

Nadim

Ex 3.10



Find ϕM_n

$f_c' = 5$ ksi $f_y = 60$ ksi.

$A_s' = 3 * 0.79 = 2.37 in^2$ $\rho' = 0.007524$

$A_s = 6 * 1.27 = 7.62 in^2$ $\rho = 0.0242$

$\rho - \rho' = 0.01667$.

$\rho_{max} = 0.0243$ $\leftarrow (\rho - \rho')$ is ok.

$\rho_{min} = 0.00354$

2. $$\bar{\rho}_{cy} = 0.85 * 0.8 * \frac{5}{60} \frac{2.5}{22.5} \frac{.003}{.003 - .00207} + 0.007524 = 0.02783$$

$\rho < \bar{\rho}_{cy} \Rightarrow$ Comp bar does not yield

to be more correct

$\rho < \rho_{cy} \Rightarrow$ Comp ...

to be more correct

3. $T = C = C_s + C_c$

$\Rightarrow A_s f_y = 0.85 f_c' \beta_1 c.b + A_s' E_s \varepsilon_u \frac{c-d'}{c} - 0.85 f_c' A_s'$

$\Rightarrow 7.62 * 60 = 0.85 * 5 * .8 * c * 14 + 2.37 * 29 * 10^3 * .003 \frac{c-2.5}{c} - .85 * 5 * 2.37$

$\Rightarrow 457.2 = 47.6c + 206.19 \frac{c-2.5}{c} - 10.07$

$\Rightarrow 47.6c^2 - 261.08c - 515.475 = 0$

$c = \frac{261.08 \pm \sqrt{261.08^2 + 4 * 47.6 * 515.475}}{2 * 47.6}$

$= 7.026$ in

$a = \beta_1 c = 5.62$

$f_s' = E_s \varepsilon_u \frac{c-d'}{c}$

$= 56.04$ ksi.

$$\rho - \rho' \frac{f_s'}{f_y} = 0.0243 - 0.007524 * \frac{56.04}{60}$$
$$= 0.01724 < \rho_{max} \quad \text{ok.}$$

5. Find M_n

$M_n = (A_s' f_s' - 0.85 f_c' A_s')(d - d') + (A_s f_y - A_s' f_s')(d - \frac{a}{2})$

$= (2.37 * 56 - 0.85 * 5 * 2.37)(22.5 - 2.5)$

$+ (7.62 * 60 - 2.37 * 56.04 + .85 * 5 * 2.37)(22.5 - \frac{5.62}{2})$

$= 122.7 * 20.0 + 334.5 * 19.69$

$= 9040.3$ k.in.

Comp. in concrete $= 0.85 f_c' ab = C_c = 334.5^k$

Comp. in steel $= A_s' f_s'$ - force in displaced concrete

$= C_s = A_s'(f_s' - 0.85 f_c') = 122.7^k$.

Tension in steel $= A_s f_y = 457.2^k$

$T = C$ ok.

6. $\frac{\varepsilon_t}{d_t - c} = \frac{\varepsilon_u}{c}$

$\varepsilon_t = \frac{.003}{7.026}(23 - 7.026)$ (23 ← 22.5 + .5)

$= 0.00682 > .005 \quad \phi = 0.9$

7. $\phi M_n = 8136$ k.in

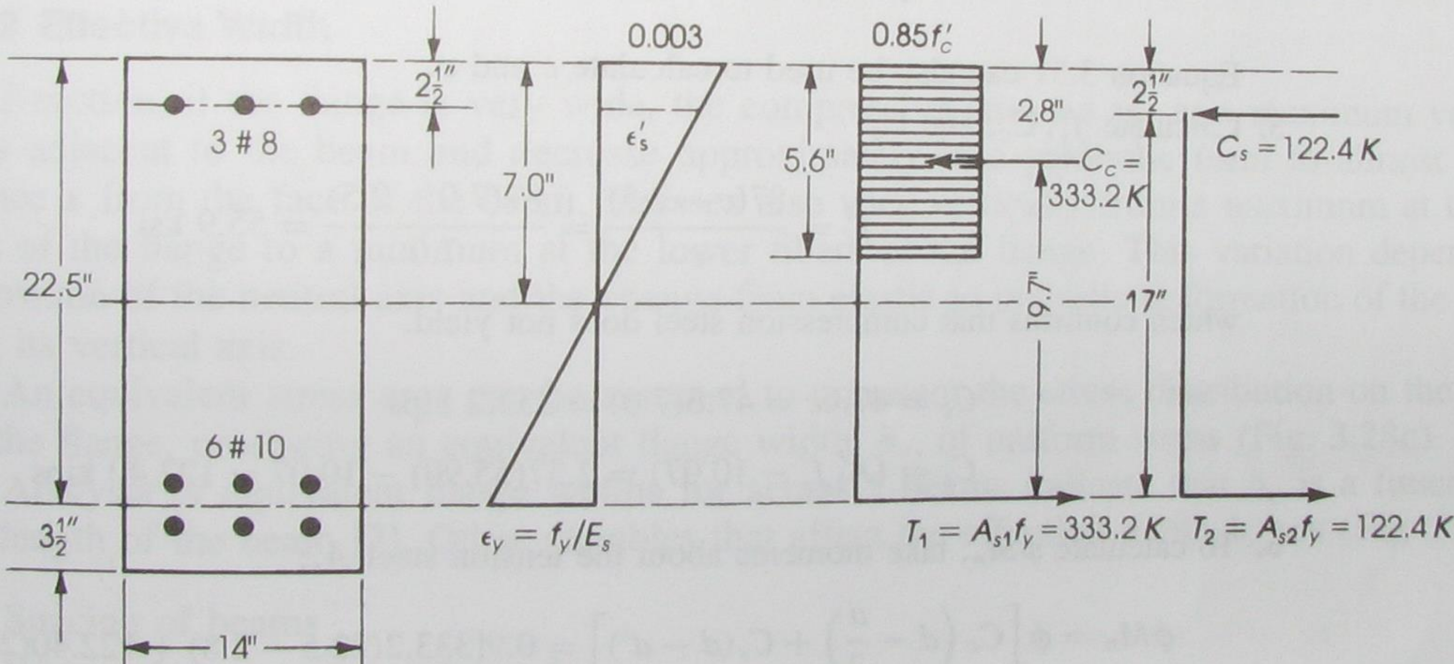


Figure 3.27 Example 3.10 analysis solution.

Design of Doubly Reinforced Beam

Design problem from Nadim

Ex 4.5 A beam section is limited to a width $b = 10 in$ and total depth of $h = 22 in$ and has to resist a factored moment of 226.5 k-ft. Calculate the required reinforcement. Given $f_c' = 3 ksi$ and $f_y = 50 ksi$.

Sol

$$\rho_{0.005} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005} = 0.01625$$

$d = 22 - 3.5 = 18.5$ two layer

$A_s = 0.01625 * 10 * 18.5 = 3 in^2$

$a = \frac{A_s f_y}{0.85 f_c' b} = 5.88 in.$

$M_u = 226.5 \times 12 = 2718$ k-"

$M_n = A_s f_y (d - \frac{a}{2}) = 2333.8$ kin.

$\phi M_n = 0.9 * 2333.8 = 2100.4$ k-in.

Doubly Reinforced beam reqd.

$\phi M_{n1} = 2100.4$

$\phi M_{n2} = 2718 - 2100.4 = 617.6 \text{ k''}$

Assuming comp steel yield

$$A_{s2} = \frac{617.6}{\phi f_y (d - d')} = \frac{617.6}{0.9 * 50 (18.5 - 2.5)} = 0.86 \text{ in}^2$$

Total tension steel $= A_s = 3.0 + 0.86 = 3.86 \text{ in}^2$

Comp. steel $A_s' = 0.86 \text{ in}^2$.

$\rho' = 0.00465$

$\rho = 0.02086$

$\rho - \rho' = 0.01621$

Use actual area provided

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

$$= 0.85 * 0.85 \frac{3}{50}\frac{2.5}{18.5}\frac{.003}{.003 - 0.001724} + 0.00465$$

$$= 0.01842$$

$\rho > \bar{\rho}_{cy}$ comp steel yields.

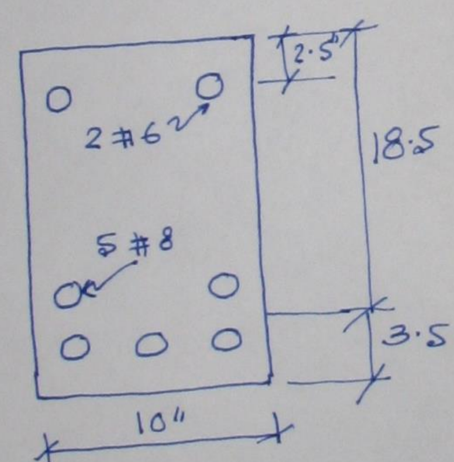
$$a = \frac{A_s - A_s'}{0.85 f_c' b} f_y = \frac{3.86 - 0.86}{0.85 * 3 * 10} \times 50 = 5.88 \text{ in.}$$

$$c = \frac{a}{\beta_1} = 6.92 \text{ in.}$$

outer layer

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \frac{19.5 - 6.92}{6.92} = 0.00545$$

$\phi = 0.9$ ok.



Nilson

Ex 3.13

Design

$LL = 2.47^{k/'}$

$DL = 1.05^{k/'}$

simply supported span = 18'

Beam section ⇒ 10" × 20"

$f_c' = 4000$ psi $f_y = 60{,}000$ psi

find reinforcement.

Sol

$w_u = 1.2 * 1.05 + 1.6 * 2.47 = 5.212^{k/'}$

$M_u = \frac{1}{8} w_u l^2 = \frac{1}{8} \times 5.212 * 18^2 = 211.09^{k'} = 2533^{k''}$

$d' = 2.5''$ (if needed)

$d = 20 - 4 = 16''$ (two layer)

First check if possible to design singly reinforced.

$\varepsilon_t = 0.005$ $\rho_{0.005} = 0.0181$

$A_s = \rho b d = 0.0181 * 10 * 16 = 2.89 in^2$

a ... $= 6''$

$A_s = \rho b d = 0.0181 * 10 * 16 = \ldots$

$a = \frac{A_s f_y}{.85 f'_c b} = 5.1$ in $c = \frac{a}{\beta_1} = 6''$

$M_n = A_s f_y (d - \frac{a}{2}) = 2.89 * 60 (16 - \frac{5.1}{2}) = 2332^{k''}$

$\phi M_n = 0.9 * 2332 = 2099^{k''} < M_u$

Doubly Reinf reqd.

Remaining moment $\phi M_n = 2533 - 2099 = 434^{k''}$

$M_n = 482.4^{k''}$

Assuming comp bars yield

$A_{s2} = \frac{434}{0.9 * 60 * (16 - 2.5)} = 0.6\ in^2$.

Total tension steel $= 2.89 + 0.6 = 3.49\ in^2$

Comp steel $= 0.6\ in^2$

4 #9
$4\ in^2$.

2 #6
$0.88\ in^2$.

$$\rho = \frac{4}{10\times16} = 0.025 \qquad \rho' = \frac{0.88}{10\times16} = 0.0055$$

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} + \rho'$$

$$= 0.85 * 0.85 \frac{4}{60} \frac{2.5}{16} \frac{.003}{.003 - .00207} + 0.0055$$

$$= 0.0298$$

$\rho < \bar{\rho}_{cy} \Rightarrow$ Comp steel does not yield

$$A_s f_y = 0.85 f_c' b \beta_1 c + A_s' f_s' - A_s' * 0.85 f_c'$$

$$\Rightarrow 4 * 60 = 0.85 * 4 * 10 * .85c + A_s' * \left[E_s \varepsilon_u \frac{c - d'}{c} - 0.85 f_c'\right]$$

$$\Rightarrow 240c = 28.9c^2 - 2.992c + (76.56c - 191.4)$$

$$\Rightarrow 28.9c^2 - 166.43c - 191.4 = 0$$

$$c = 6.74 \text{ in.} \qquad a = 5.73 \text{ in.}$$

$$f_s' = E_s \varepsilon_u \frac{c - d'}{c} = 54.7 \text{ ksi.}$$

$$C_c = 0.85 f_c' . b . \beta_1 c = 194.8^k$$

$$C_s = A_s' \left[f_s' - 0.85 f_c' \right] = 45.14^k$$

checked.

$$T = A_s f_y = 4 * 60 = 240^k$$

$$M_n = C_c \left(d - \frac{a}{2} \right) + C_s \left(d - d' \right)$$

$$= 194.8 \left(16 - \frac{5.73}{2} \right) + 45.14 \left(16 - 2.5 \right)$$

$$= 2558.5 + 609.44 = 3167.9^{k''}$$

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \, \frac{17.5 - 6.74}{6.74}$$

$$= 0.00479$$

$$\phi = 0.483 + 83.3\,\varepsilon_t = 0.882$$

$$\phi M_n = 2793.9^{k} > M_u = 2533^{k''} \quad \underline{\text{ok}}.$$

T-beam

- RC beam and slab are monolithically cast
- Beam stirrups and bent bars extend into the slab
- A part of slab act along with beam top to take longitudinal compression
- Slab forms the beam flange
- Part of beam below slab is called web/stem

Effective flange width

The criteria for effective width given in ACI Code 8.12 are as follows:

1. For symmetric T beams, the effective width *b* shall not exceed one-fourth the span length of the beam. The overhanging slab width on either side of the beam web shall not exceed 8 times the thickness of the slab or go beyond one-half the clear distance to the next beam.
2. For beams having a slab on one side only, the effective overhanging slab width shall not exceed one-twelfth the span length of the beam, 6 times the slab thickness, or one-half the clear distance to the next beam.
3. For isolated beams in which the flange is used only for the purpose of providing additional compressive area, the flange thickness shall not be less than one-half the width of the web, and the total flange width shall not be more than 4 times the web width.

1. Symmetrical T beams

$b < 16h_f+b_w$

$b <$ Span/4

$b <$ c/c beam spacing

2. Beam having slab on one side

$b <$ span/12+b_w

$b < 6h_f+b_w$

$b <$ Half the clear span +b_w

3. Isolated T beam

$h_f > b_w/2$

$b < 4b_w$

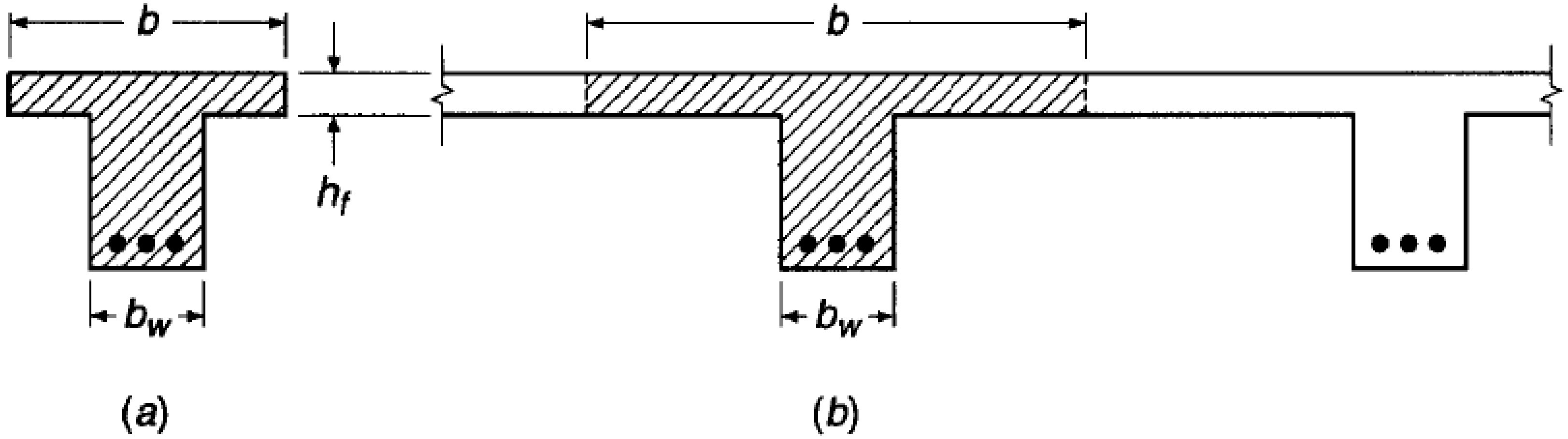
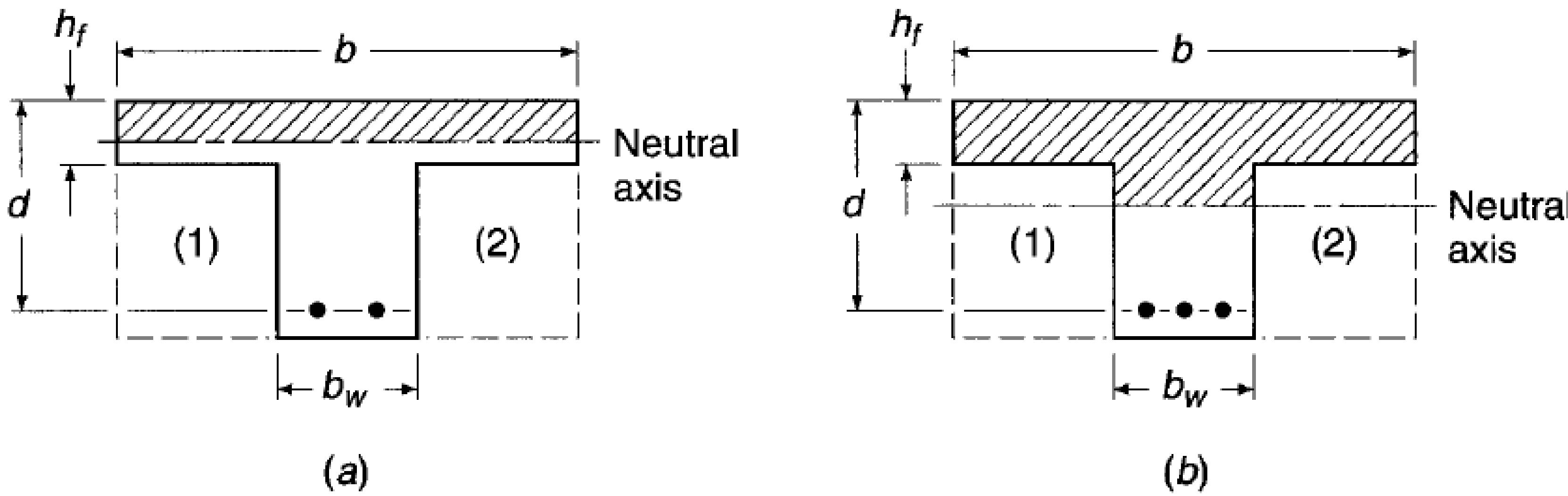
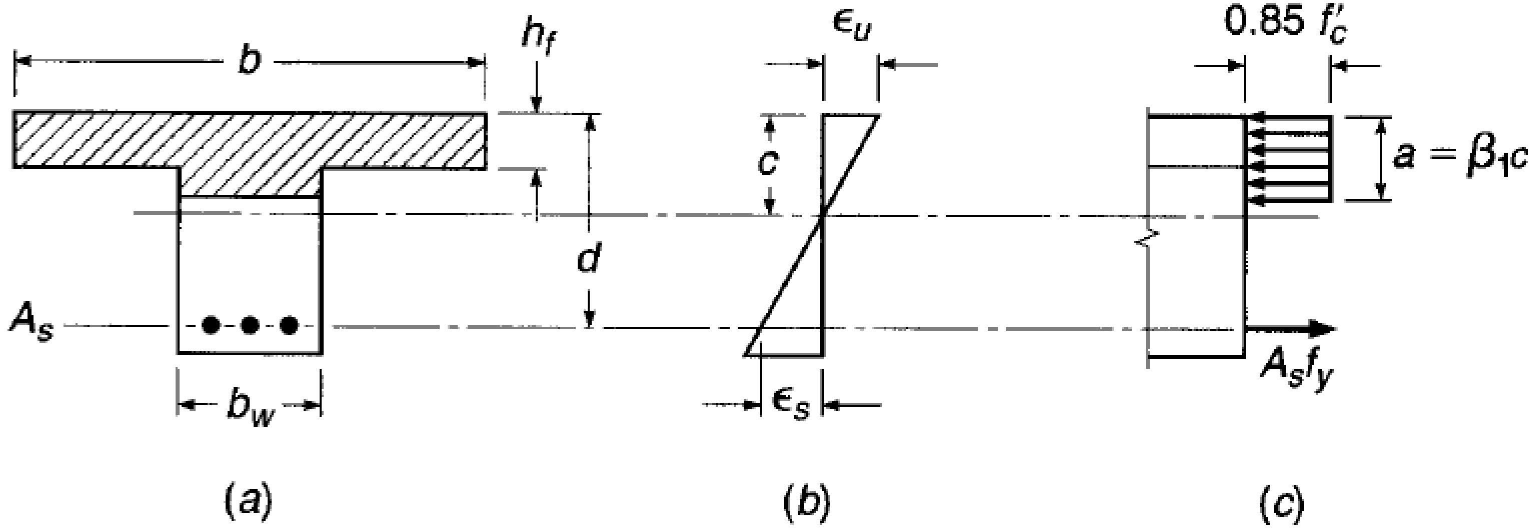


FIGURE 3.17
Effective flange width of T beams.

Strength Analysis

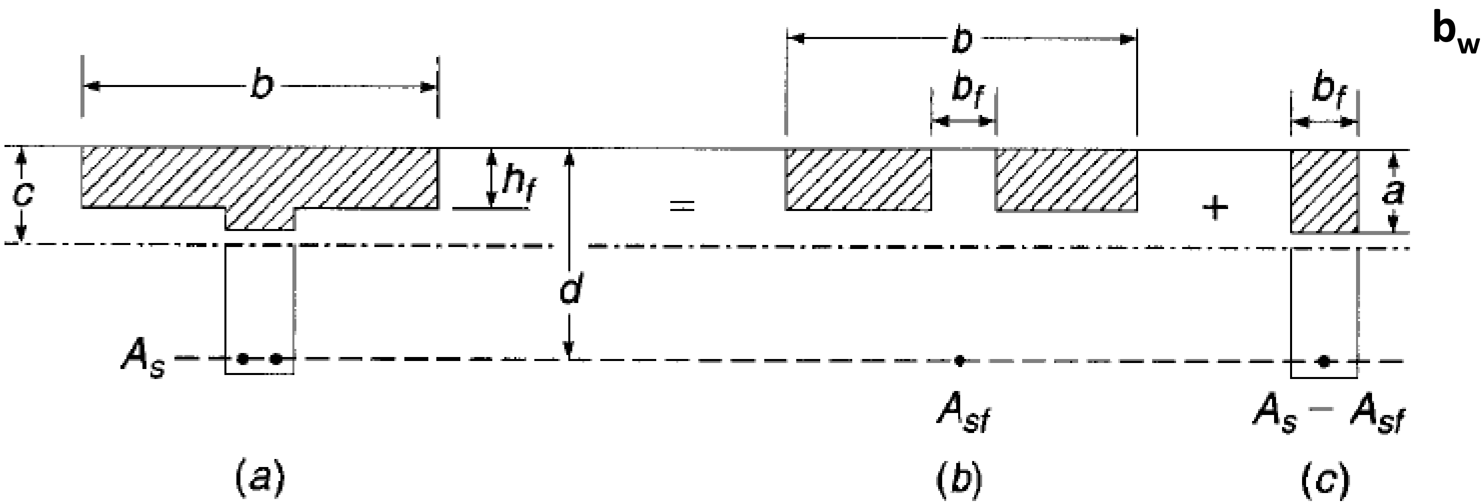


Two possibilities
- Just like rectangular beam
- T-beam analysis required



$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho f_y d}{0.85 f'_c} \qquad (3.58)$$

If $a > h_f$ T-beam



$$A_{sf} = \frac{0.85f'_c\,(b - b_w)h_f}{f_y} \tag{3.59}$$

$$M_{n1} = A_{sf}f_y\left(d - \frac{h_f}{2}\right) \tag{3.60}$$

The remaining steel area $A_s - A_{sf}$.

$$a = \frac{(A_s - A_{sf})f_y}{0.85f'_c\,b_w} \tag{3.61}$$

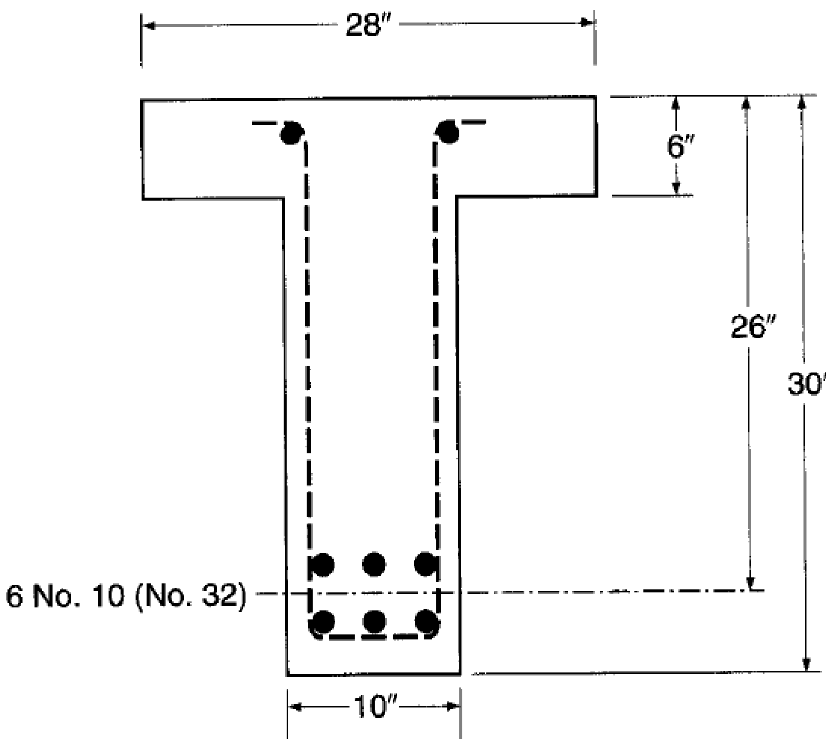
$$M_{n2} = (A_s - A_{sf})f_y\left(d - \frac{a}{2}\right) \tag{3.62}$$

and the total nominal resisting moment is the sum of the parts:

$$M_n = M_{n1} + M_{n2} = A_{sf} f_y \left(d - \frac{h_f}{2} \right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2} \right) \tag{3.63}$$

EXAMPLE 3.14

Moment capacity of a given section. The isolated T beam shown in Fig. 3.21 is composed of a flange 28 in. wide and 6 in. deep cast monolithically with a web of 10 in. width that extends 24 in. below the bottom surface of the flange to produce a beam of 30 in. total depth. Tensile reinforcement consists of six No. 10 (No. 32) bars placed in two horizontal rows separated by 1 in. clear spacing. The centroid of the bar group is 26 in. from the top of the beam. The concrete has a strength of 3000 psi, and the yield strength of the steel is 60,000 psi. What is the design moment capacity of the beam?



Solution. It is easily confirmed that the flange dimensions are satisfactory according to the ACI Code for an isolated beam. The entire flange can be considered effective. For six No. 10 (No. 32) bars, $A_s = 7.62 \text{ in}^2$. First check the location of the neutral axis, on the assumption that rectangular beam equations may be applied. Using Eq. (3.32)

$$a = \frac{7.62 \times 60}{0.85 \times 3 \times 28} = 6.40 \text{ in.}$$

This exceeds the flange thickness, and so a T beam analysis is required. From Eq. (3.59) and Fig. 3.19*b*,

$$A_{sf} = 0.85 \times \frac{3}{60}(28 - 10) \times 6 = 4.59 \text{ in}^2$$

Then, from Eq. (3.60),

$$M_{n1} = 4.59 \times 60(26 - 3) = 6330 \text{ in-kips}$$

Then, from Fig. 3.19*c*,

$$A_s - A_{sf} = 7.62 - 4.59 = 3.03 \text{ in}^2$$

and from Eqs. (3.58) and (3.59)

$$a = \frac{3.03 \times 60}{0.85 \times 3 \times 10} = 7.13 \text{ in.}$$

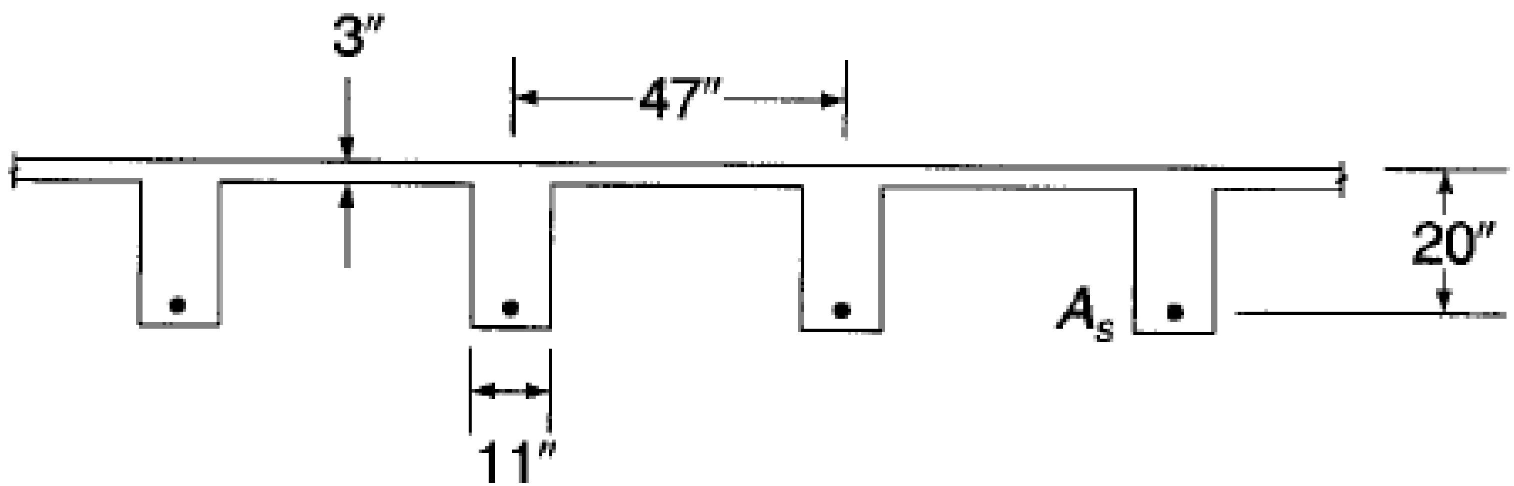
$$M_{n2} = 3.03 \times 60(26 - 3.56) = 4080 \text{ in-kips}$$

The depth to the neutral axis is $c = a/\beta_1 = 7.13/0.85 = 8.39$ and $d_t = 27.5$ in. to the lowest bar. The c/d_t ratio is $8.39/27.5 = 0.305 < 0.375$, so the $\epsilon_t > 0.005$ requirement is met and $\phi = 0.90$. When the ACI strength reduction factor is incorporated, the design strength is

$$\phi M_n = 0.90(6330 + 4080) = 9370 \text{ in-kips}$$

EXAMPLE 3.15

Determination of steel area for a given moment. A floor system, shown in Fig. 3.22, consists of a 3 in. concrete slab supported by continuous T beams with a 24 ft span, 47 in. on centers. Web dimensions, as determined by negative-moment requirements at the supports, are b_w = 11 in. and d = 20 in. What tensile steel area is required at midspan to resist a factored moment of 6400 in-kips if f_y = 60,000 psi and f'_c = 3000 psi?



SOLUTION. First determining the effective flange width,

$$16h_f + b_w = 16 \times 3 + 11 = 59 \text{ in.}$$

$$\frac{\text{Span}}{4} = 24 \times \frac{12}{4} = 72 \text{ in.}$$

$$\text{Centerline beam spacing} = 47 \text{ in.}$$

The centerline T beam spacing controls in this case, and $b = 47$ in. The concrete dimensions b_w and d are known to be adequate in this case, since they have been selected for the larger negative support moment applied to the effective rectangular section $b_w d$. The tensile steel at midspan is most conveniently found by trial. Assuming the stress-block depth a is equal to the flange thickness of $h_f = 3$ in., one gets

$$d - \frac{a}{2} = 20 - 1.50 = 18.50 \text{ in.}$$

Trial:

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{6400}{0.90 \times 60 \times 18.50} = 6.41 \text{ in}^2$$

Checking the assumed value for a,

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{6.41 \times 60}{0.85 \times 3 \times 47} = 3.21 \text{ in.}$$

Since a is greater than h_f, a T beam design is required and $\phi = 0.90$ is assumed.

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y} = \frac{0.85 \times 3 \times 36 \times 3}{60} = 4.59 \text{ in}^2$$

$$\phi M_{n1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2} \right) = 0.90 \times 4.59 \times 60 \times 18.50 = 4590 \text{ in-kips}$$

$$\phi M_{n2} = M_u - \phi M_{n1} = 6400 - 4590 = 1810 \text{ in-kips}$$

Assume $a = 4.00$ in.:

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y (d - a/2)} = \frac{1810}{0.90 \times 60 \times (20 - 4.0/2)} = 1.86 \text{ in}^2$$

Check:

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{1.86 \times 60}{0.85 \times 3 \times 11} = 3.98 \text{ in.}$$

This is satisfactorily close to the assumed value of 4 in. Then

$$A_s = A_{sf} + A_s - A_{sf} = 4.59 + 1.86 = 6.45 \text{ in}^2$$

Checking to ensure that the net tensile strain of 0.005 is met to allow $\phi = 0.90$,

$$c = \frac{a}{\beta_1} = \frac{3.98}{0.85} = 4.68$$

$$\frac{c}{d_t} = \frac{4.68}{20} = 0.23 < 0.325$$

indicating that the design is satisfactory.

The close agreement should be noted between the approximate tensile steel area of 6.41 in^2 found by assuming the stress-block depth equal to the flange thickness and the more exact value of 6.45 in^2 found by T beam analysis. The approximate solution would be satisfactory in most cases.

Shear and Diagonal Tension in Beams

Chapter 4

Introduction

- Chapter 3 deals with flexure
- Beams must have adequate safety margin against other types of failure
- They may be more dangerous-more uncertain and more catastrophic
- Shear failure is one such failure
- It is not fully understood and sudden without warning

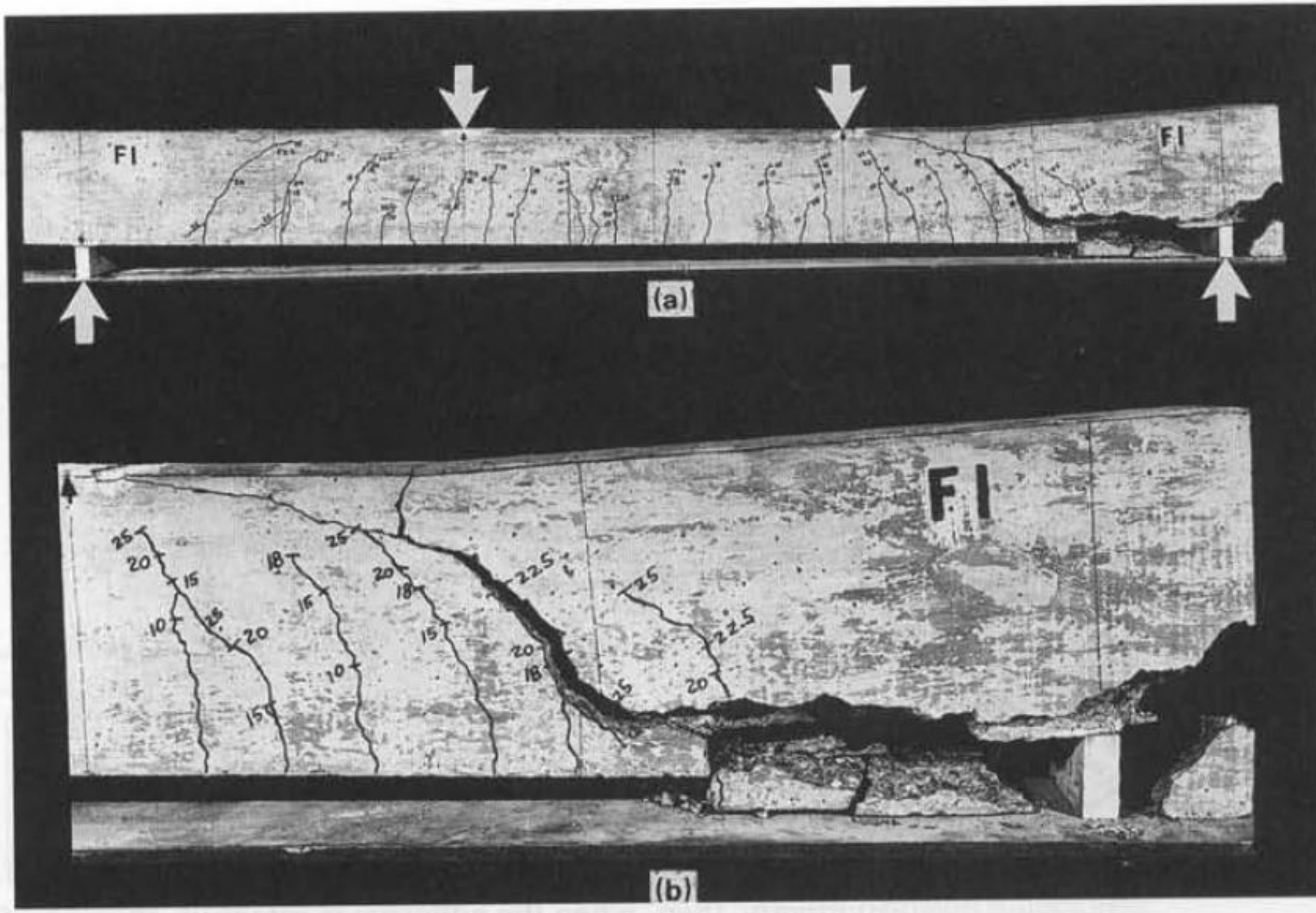
- Special shear reinforcement are provided to ensure flexural failure would occur before shear failure if overloading happens.

DIAGONAL TENSION

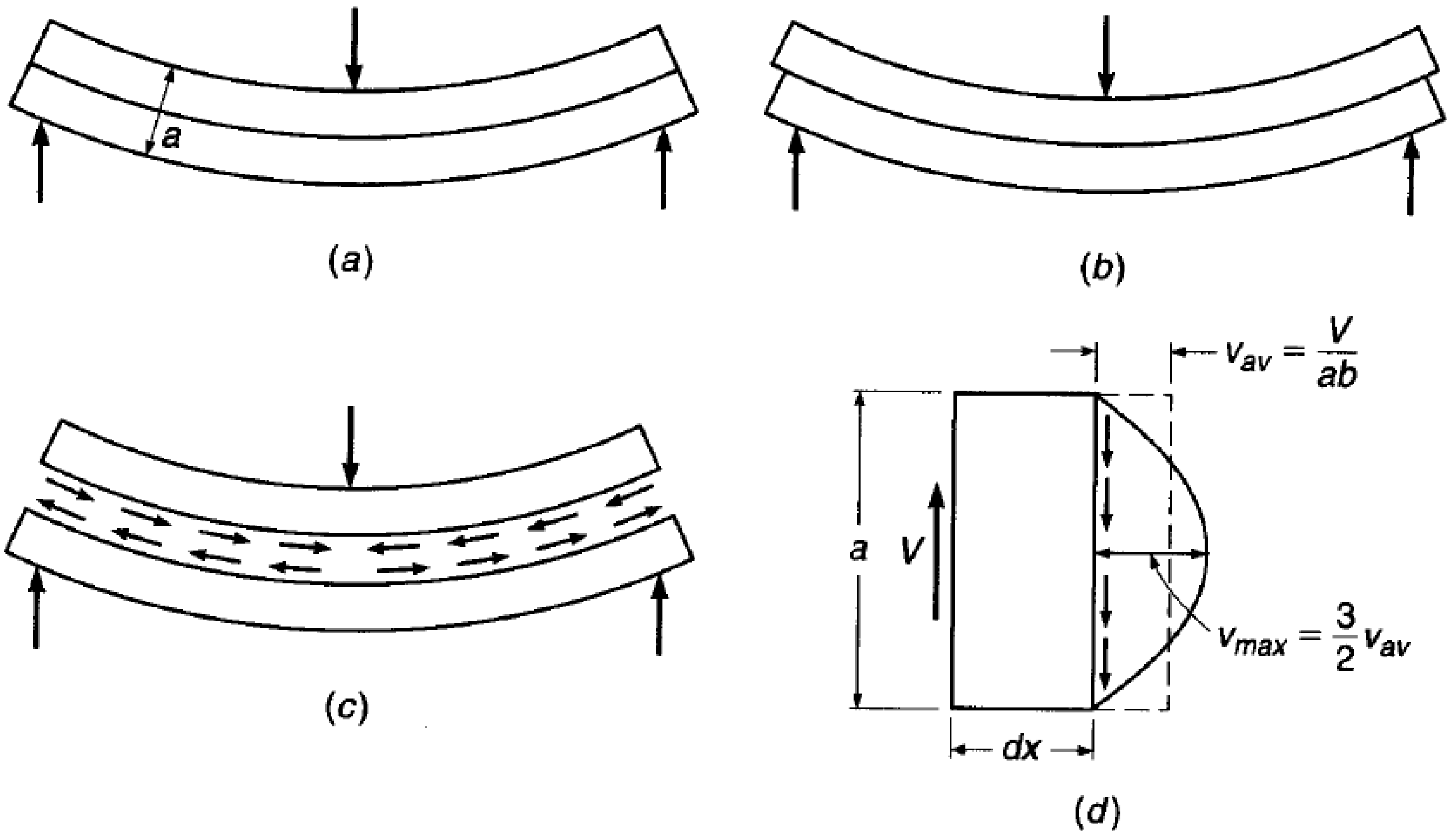
- Shear analysis and design are not really concerned about shear as such.
- The real concern is *DIAGONAL TENSION*

DIAGONAL TENSION: video

FIGURE 4.1
Shear failure of reinforced concrete beam: (*a*) overall view, (*b*) detail near right support.



Diagonal Tension in homogeneous beams



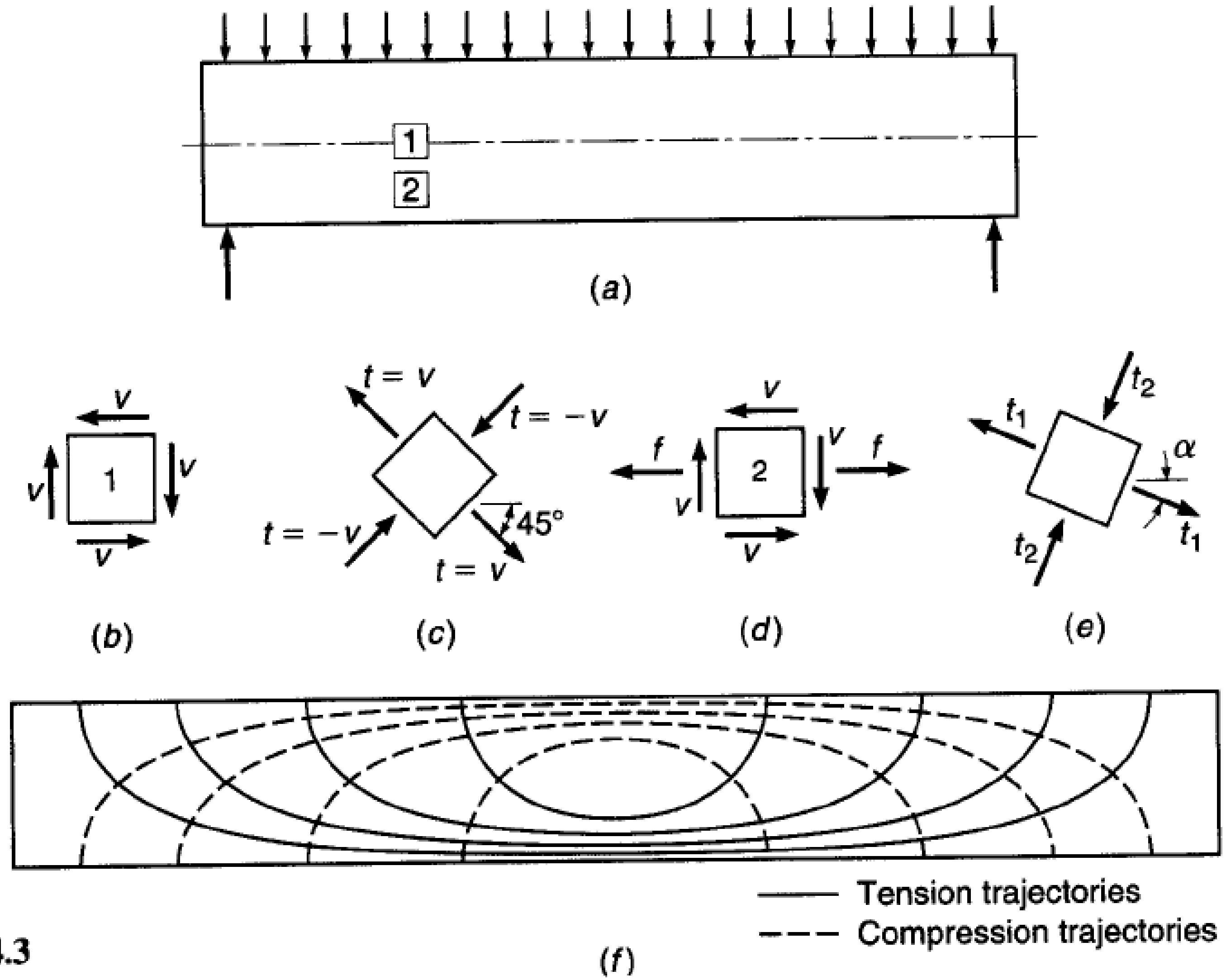


FIGURE 4.3
Stress trajectories in homogeneous rectangular beam.

Principal Stresses

compressive stresses and a pair of inclined tensile stresses that act at right angles to each other. They are known as *principal stresses* (Fig. 4.3*e*). Their value, as mentioned in Section 3.2, is given by

$$t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2} \tag{3.1}$$

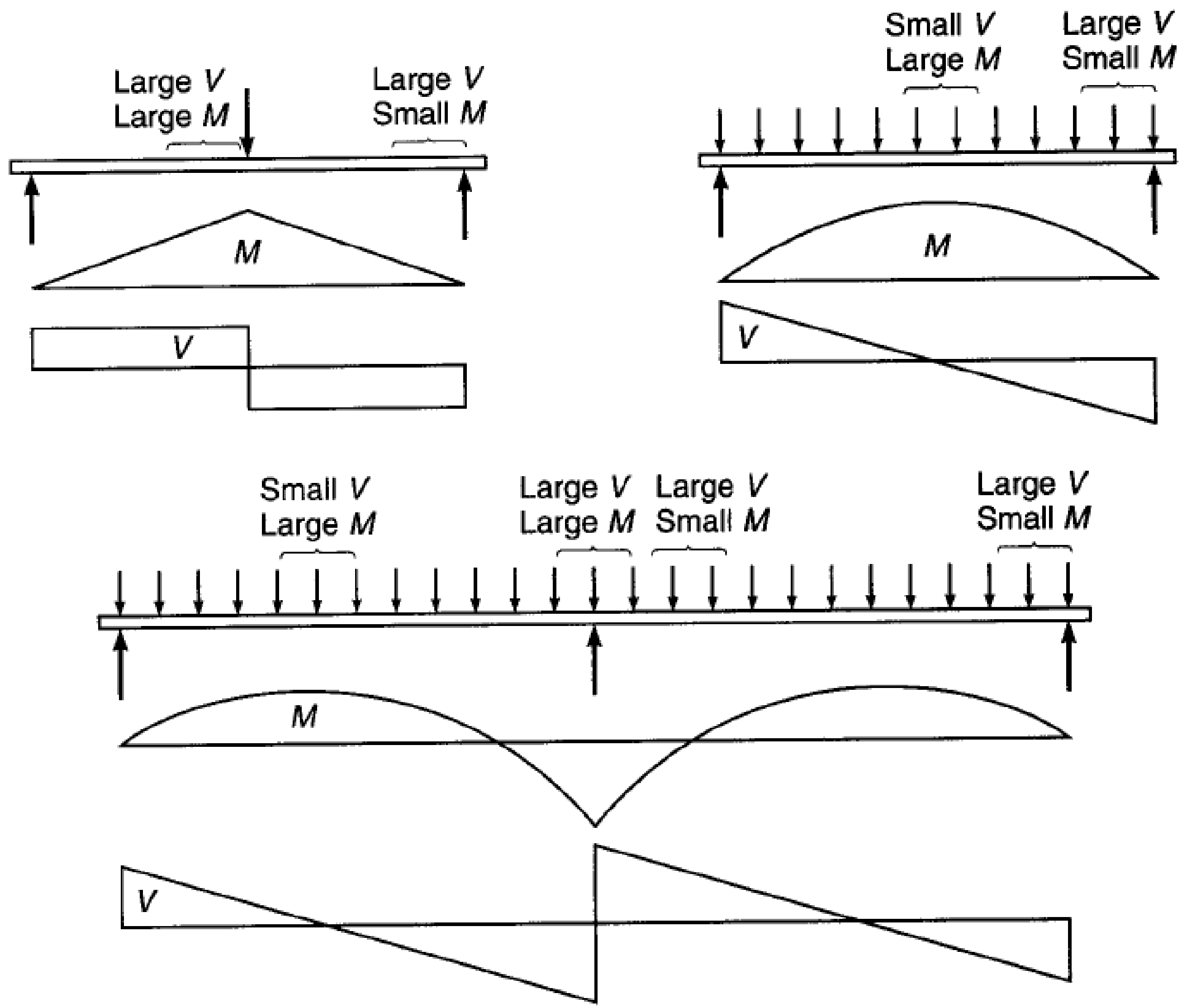
and their inclination α by $\tan 2\alpha = 2v/f$.

RC BEAMS WITHOUT SHEAR REINFORCEMENTS

RC Beams without shear reinforcements

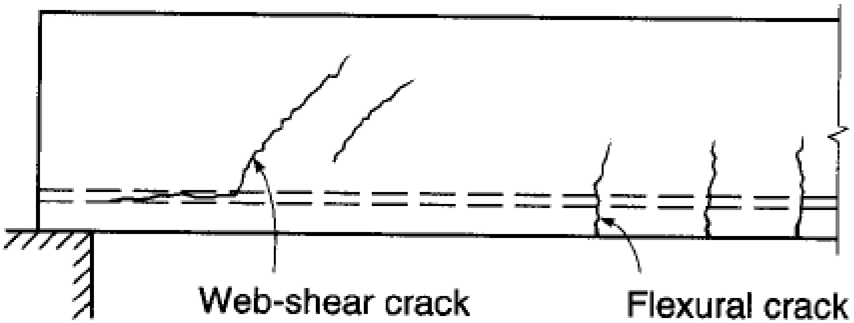
- No flexural failure unlike plain concrete beam
- Diagonal tension failure occurs elsewhere
- Caused by shear alone or combined action of shear and flexure

FIGURE 4.4
Typical locations of critical combinations of shear and moment.



Web-shear crack

- when flexure small
- Near neutral axis
- Shear stress equals tensile strength

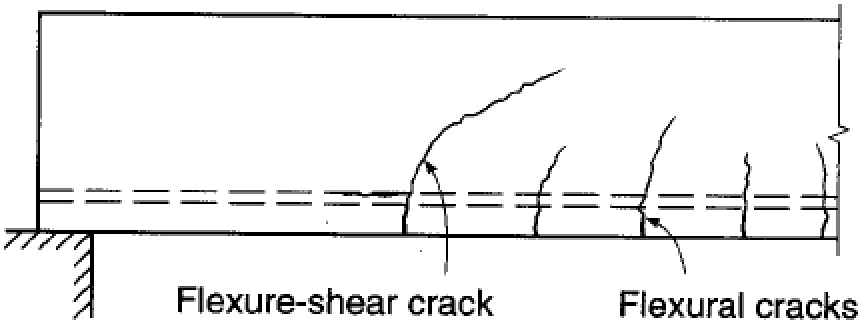


(*a*) Web-shear cracking

$$v_{cr} = \frac{V_{cr}}{bd} = 3.5\sqrt{f'_c} \qquad (4.2a)$$

Flexure shear cracking

- Both moment and shear significant
- Fails at a lower stress due to preexisting cracks due to flexure



(*b*) Flexure-shear cracking

$$v_{cr} = \frac{V_{cr}}{bd} = 1.9\sqrt{f_c'} \quad (4.2b)$$

- The shear at which diagonal cracks develop depends on the ratio of shear force to bending moment
- more precisely on the *v* to *f* near the top of flexural crack

$$v_{cr} = \frac{V_{cr}}{bd} = 1.9\sqrt{f'_c} + 2500\,\frac{\rho V d}{M} \leq 3.5\sqrt{f'_c} \tag{4.3a}$$

where

$$V_{cr} = v_{cr}\,bd$$

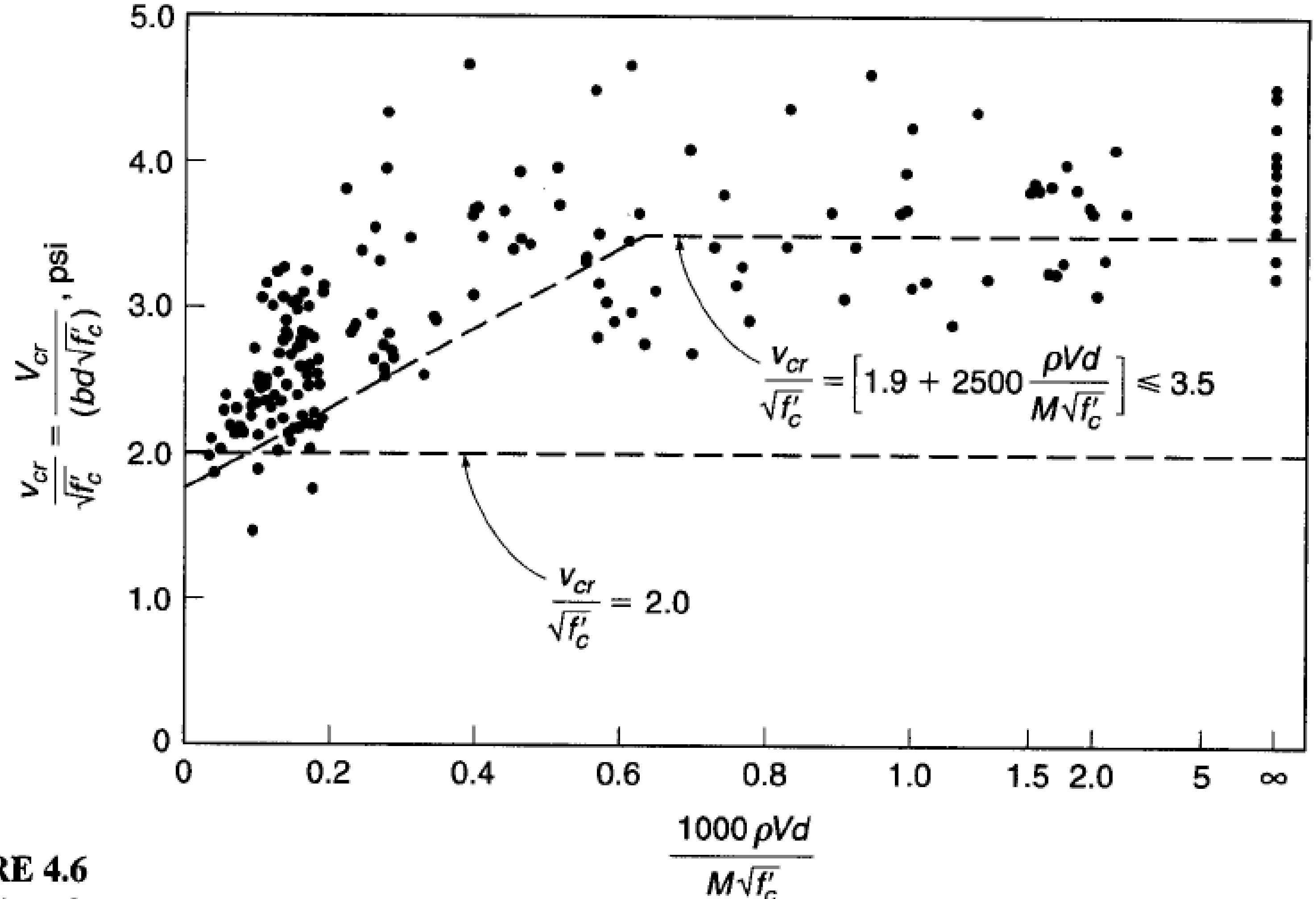


FIGURE 4.6
Correlation of Eq. (4.3*a*) with test results.

Behaviour of diagonally cracked beam

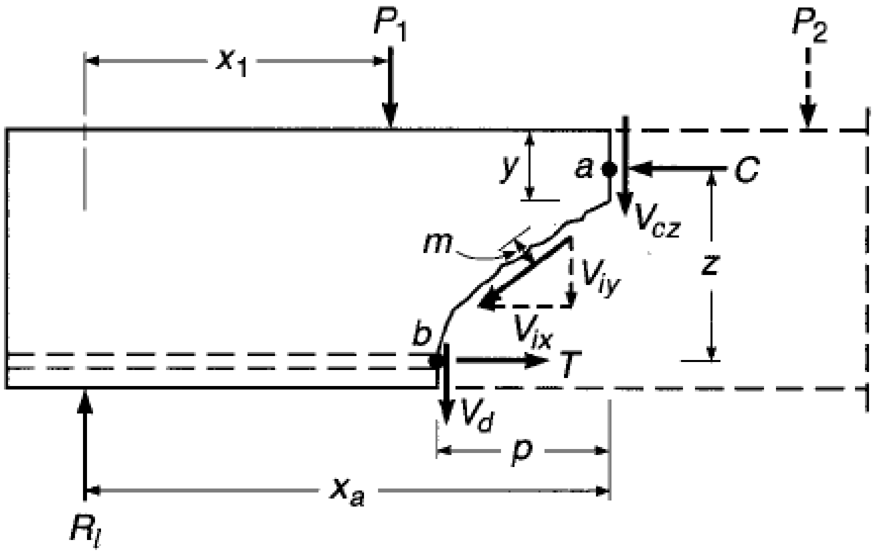
- Flexural cracks are harmless as longitudinal steel is present
- If shear reinforcement is not present, diagonal cracks is critical and determines the strength

•Two types of behaviour as diagonal cracks formed:

- The diagonal cracks spread immediately from tension face to compression, splitting it in to two and failing- for shallow beams (span to depth 8 or more)
- For deeper beams, the crack spreads partially in to compression zone. Failure load is significantly higher

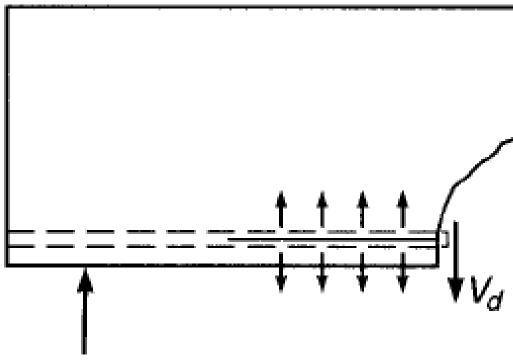
Once crack is formed

$$V_{int} = V_{cz} + V_d + V_{iy}$$



(a)

Dowel V is small and can cause splitting



(b)

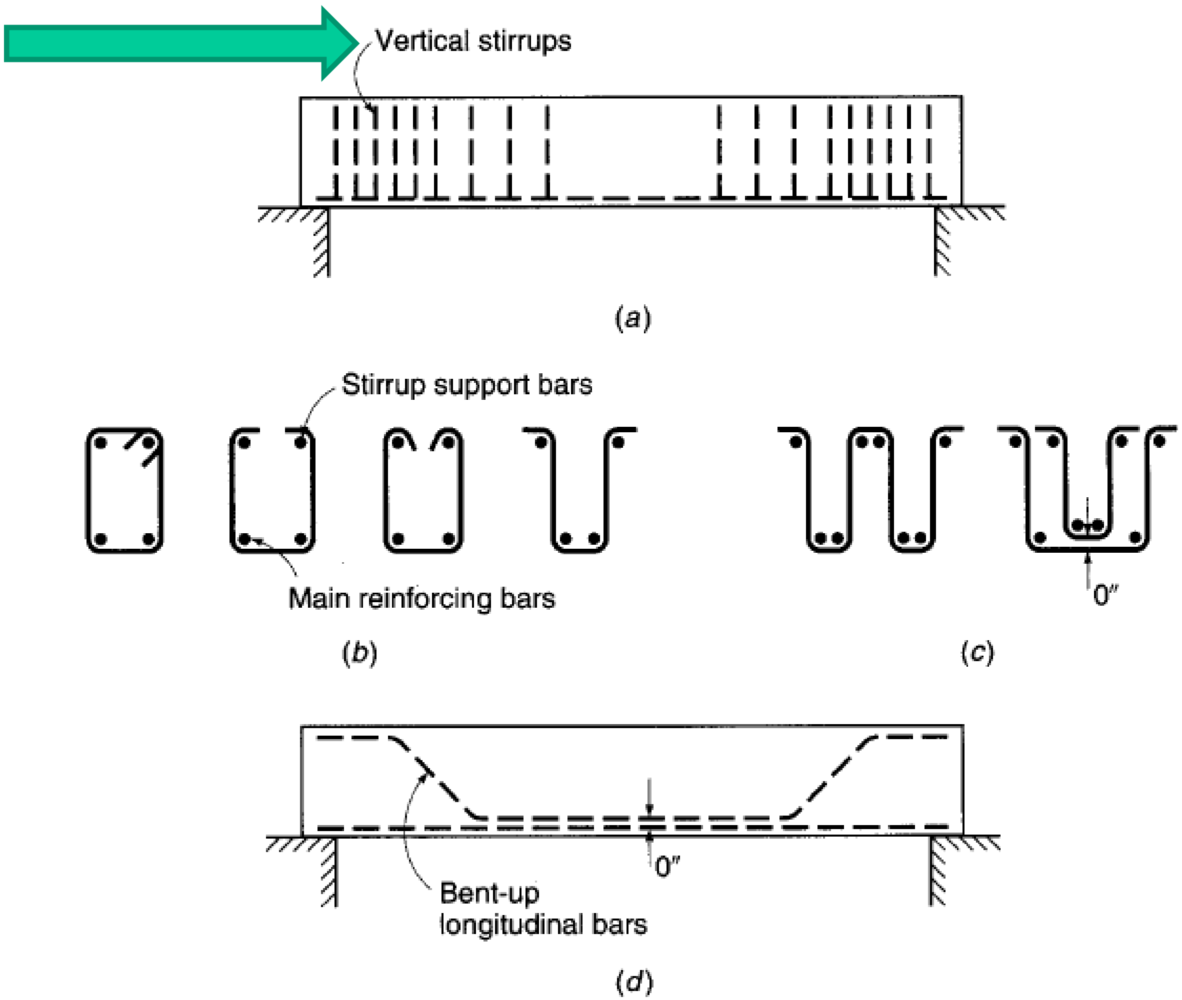
Formation of diagonal crack produce following redistribution of stress

- Shear stress increases on the uncracked area
- Compression force increases
- Tension in steel increases

- Failure occurs in various ways-yielding, crushing, splitting, pull out
- Relatively deep beam can take significant load after formation of diagonal crack, but this reserve strength is erratic and is ignored

RC BEAM WITH WEB REINFORCEMENT

Types of web reinforcement



Behaviour of web reinforced concrete beams

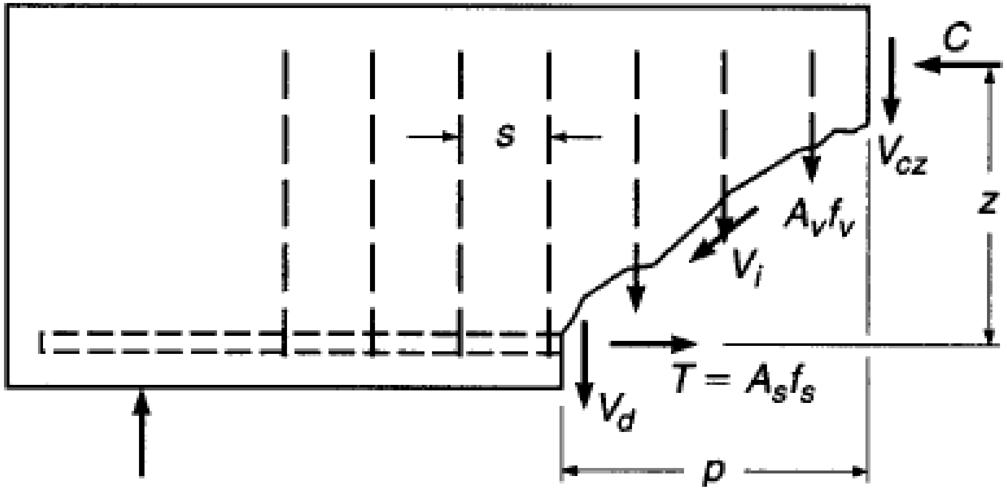
formation. After diagonal cracks have developed, web reinforcement augments the shear resistance of a beam in four separate ways:

1. Part of the shear force is resisted by the bars that traverse a particular crack. The mechanism of this added resistance is discussed below.
2. The presence of these same bars restricts the growth of diagonal cracks and reduces their penetration into the compression zone. This leaves more uncracked concrete available at the head of the crack for resisting the combined action of shear and compression, already discussed.
3. The stirrups also counteract the widening of the cracks, so that the two crack faces stay in close contact. This makes for a significant and reliable interface force V_i (see Fig. 4.7).
4. As shown in Fig. 4.8, the stirrups are arranged so that they tie the longitudinal reinforcement into the main bulk of the concrete. This provides some measure of restraint against the splitting of concrete along the longitudinal reinforcement, shown in Figs. 4.1 and 4.7*b*, and increases the share of the shear force resisted by dowel action.

From this it is clear that failure will be imminent when the stirrups start yielding. This not only exhausts their own resistance but also permits a wider crack opening with consequent reduction of the beneficial restraining effects, points 2 to 4, above.

Beams with vertical stirrups

FIGURE 4.9
Forces at a diagonal crack in a beam with vertical stirrups.



$$V_{ext} = V_{cz} + V_d + V_{iy} + V_s \qquad (a)$$

$$V_c = V_{cz} + V_d + V_{iy} \qquad (b)$$

The number of stirrups n spaced a distance s apart was seen to depend on the length p of the horizontal projection of the diagonal crack. This length is conservatively assumed to be equal to the effective depth of the beam; thus $n = d/s$, implying a crack somewhat flatter than 45°. Then, at failure, when $V_{ext} = V_n$, Eqs. (a) and (b) yield for the nominal shear strength

$$V_n = V_c + \frac{A_v f_{yt} d}{s} \tag{4.7a}$$

where V_c is taken equal to the cracking shear V_{cr} given by Eq. (4.3a); that is,

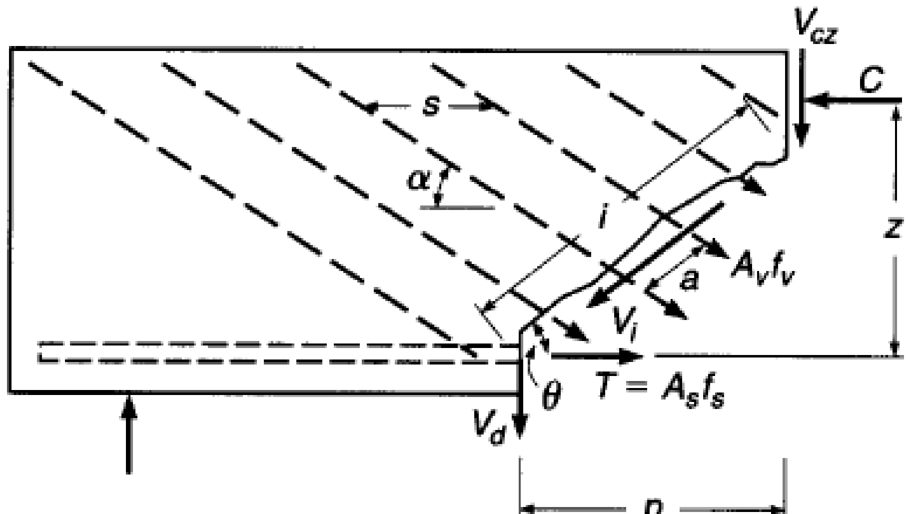
$$V_c = \left(1.9\sqrt{f_c'} + 2500\,\frac{\rho V d}{M}\right) bd \leq 3.5\sqrt{f_c'}\, bd \tag{4.3a}$$

Dividing both sides of Eq. (4.7a) by bd, the same relation is expressed in terms of the nominal shear stress:

$$v_n = \frac{V_n}{bd} = v_c + \frac{A_v f_{yt}}{bs} \tag{4.7b}$$

Beams with inclined bars

FIGURE 4.11
Forces at a diagonal crack in a beam with inclined web reinforcement.



$$V_n = V_c + \frac{A_v f_{yt} d(\sin\alpha + \cos\alpha)}{s} \qquad (4.9)$$

ACI CODE PROVISIONS

According to ACI Code 11.1.1, the design of beams for shear is to be based on the relation

$$V_u \leq \phi V_n \tag{4.10}$$

where V_u is the total shear force applied at a given section of the beam due to factored loads and $V_n = V_c + V_s$ is the nominal shear strength, equal to the sum of the contributions of the concrete and the web steel if present. Thus for vertical stirrups

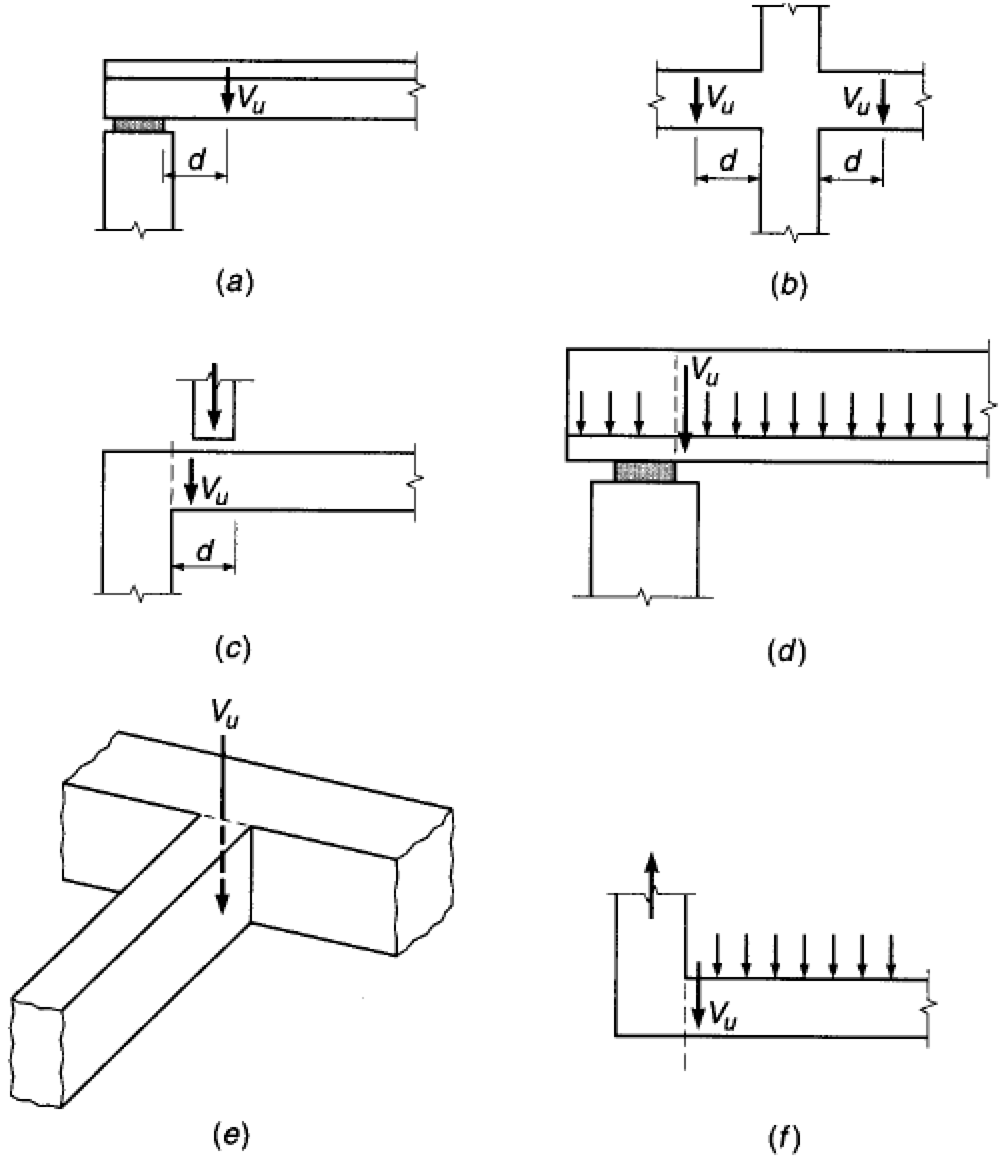
$$V_u \leq \phi V_c + \frac{\phi A_v f_{yt} d}{s} \tag{4.11a}$$

and for inclined bars

$$V_u \leq \phi V_c + \frac{\phi A_v f_{yt} d(\sin \alpha + \cos \alpha)}{s} \tag{4.11b}$$

where all terms are as previously defined. The strength reduction factor ϕ is to be taken equal to 0.75 for shear. The additional conservatism, compared with the value of $\phi = 0.90$ for bending for typical beam designs, reflects both the sudden nature of diagonal tension failure and the large scatter of test results.

FIGURE 4.12
Location of critical section for shear design: (*a*) end-supported beam; (*b*) beam supported by columns; (*c*) concentrated load within *d* of the face of the support; (*d*) member loaded near the bottom; (*e*) beam supported by girder of similar depth; (*f*) beam supported by monolithic vertical element.



V_c

$$V_c = \left(1.9\lambda\sqrt{f'_c} + 2500\,\frac{\rho_w V_u d}{M_u}\right) b_w d \le 3.5\lambda\sqrt{f'_c}\, b_w d \qquad (4.12a)$$

where ρ_w = longitudinal reinforcement ratio $A_s/b_w d$ or A_s/bd. With the section dimension b_w and d in inches and $V_u d$ and M_u in consistent units, V_c is expressed in pounds. In Eq. (4.12*a*), the quantity $V_u d/M_u$ is not to be taken greater than 1.0.

$$V_c = 2\lambda\sqrt{f'_c}\, b_w d \qquad (4.12b)$$

- Conservative where shear-moment ratio is high

The term λ in Eq. (4.12*a*) is a modification factor reflecting the lower tensile strength of lightweight concrete compared with normalweight concrete of the same compressive strength (see Table 2.2 and Ref. 4.13). Lightweight aggregate concretes having densities from 90 to 120 pcf are used widely, particularly for precast elements. In accordance with ACI Code 8.6.1, $\lambda = 0.85$ for "sand-lightweight" concrete and 0.75 for "all-lightweight" concrete. Linear interpolation between 0.75 and 0.85, based on volumetric fractions, is permitted when a portion of the lightweight fine aggregate is replaced by normalweight fine aggregate. Linear interpolation between 0.85 and 1.0 is also permitted for concretes containing normalweight fine aggregate and a blend of lightweight and normalweight coarse aggregate. If the average split-cylinder strength of lightweight concrete (a good measure of its direct tensile strength) is specified, $\lambda = f_{ct}/(6.7\sqrt{f_c'}) \leq 1.0$. For normalweight concert, $\lambda = 1.0$.

The tests on which Eqs. (4.12*a*) and (4.12*b*) are based used beams with concrete compressive strength mostly in the range of 3000 to 5000 psi. More recent experimental results (Refs. 4.14 to 4.17) have shown that in beams constructed using high-strength concrete (see Section 2.12) with f_c' above 6000 psi, the concrete contribution to shear strength V_c is less than predicted by those equations. Differences become increasingly significant, the higher the concrete strength. For this reason, ACI Code 11.1.2 places an upper limit of 100 psi on the value of $\sqrt{f_c'}$ to be used in Eqs. (4.12*a*) and (4.12*b*), *as well as in all other ACI Code shear provisions*. However, values of $\sqrt{f_c'}$ greater than 100 psi may be used in computing V_c if a minimum amount of web reinforcement is used (see Section 4.5b).

Minimum web reinforcement

If V_u, the shear force at factored loads, is no larger than ϕV_c, calculated by Eq. (4.12a) or alternatively by Eq. (4.12b), then theoretically no web reinforcement is required. Even in such a case, however, ACI Code 11.4.6 requires provision of at least a minimum area of web reinforcement equal to

$$A_{v,\min} = 0.75\sqrt{f_c'}\,\frac{b_w s}{f_{yt}} \geq 50\,\frac{b_w s}{f_{yt}} \tag{4.13}$$

where s = longitudinal spacing of web reinforcement, in.
f_{yt} = yield strength of web steel, psi
$A_{v,\min}$ = total cross-sectional area of web steel within distance s, in^2

This provision holds unless V_u is one-half or less of the design shear strength provided by the concrete ϕV_c. Specific exceptions to this requirement for minimum

Read from book

EXAMPLE 4.1 **Beam without web reinforcement.** A rectangular beam is to be designed to carry a shear force V_u of 27 kips. No web reinforcement is to be used, and f'_c is 4000 psi. What is the minimum cross section if controlled by shear?

SOLUTION. If no web reinforcement is to be used, the cross-sectional dimensions must be selected so that the applied shear V_u is no larger than one-half the design shear strength ϕV_c. The calculations will be based on Eq. (4.12*b*). Thus,

$$V_u = \frac{1}{2}\phi(2\lambda\sqrt{f'_c}b_w d)$$

$$b_w d = \frac{27{,}000}{0.75 \times 1.0\sqrt{4000}} = 569 \text{ in}^2$$

A beam with $b_w = 18$ in. and $d = 32$ in. is required. Alternately, if the minimum amount of web reinforcement given by Eq. (4.13) is used, the concrete shear resistance may be taken at its full value ϕV_c, and it is easily confirmed that a beam with $b_w = 12$ in. and $d = 24$ in. will be sufficient.

c. Region in Which Web Reinforcement Is Required

If the required shear strength V_u is greater than the design shear strength ϕV_c provided by the concrete in any portion of a beam, there is a theoretical requirement for web reinforcement. Elsewhere in the span, web steel at least equal to the amount given by Eq. (4.13) must be provided, unless the factored shear force is less than $\frac{1}{2}\phi V_c$.

The portion of any span through which web reinforcement is theoretically necessary can be found from the shear diagram for the span, superimposing a plot of the shear strength of the concrete. Where the shear force V_u exceeds ϕV_c, shear reinforcement must provide for the excess. The additional length through which at least the minimum web steel is needed can be found by superimposing a plot of $\phi V_c/2$.

EXAMPLE 4.2

Limits of web reinforcement. A simply supported rectangular beam 16 in. wide having an effective depth of 22 in. carries a total factored load of 9.4 kips/ft on a 20 ft clear span. It is reinforced with 7.62 in^2 of tensile steel, which continues uninterrupted into the supports. If $f_c' = 4000$ psi, throughout what part of the beam is web reinforcement required?

Solution. The maximum external shear force occurs at the ends of the span, where $V_u = 9.4 \times 20/2 = 94$ kips. At the critical section for shear, a distance d from the support, $V_u = 94 - 9.4 \times 1.83 = 76.8$ kips. The shear force varies linearly to zero at midspan. The variation of V_u is shown in Fig. 4.13*a*. Adopting Eq. (4.12*b*) gives

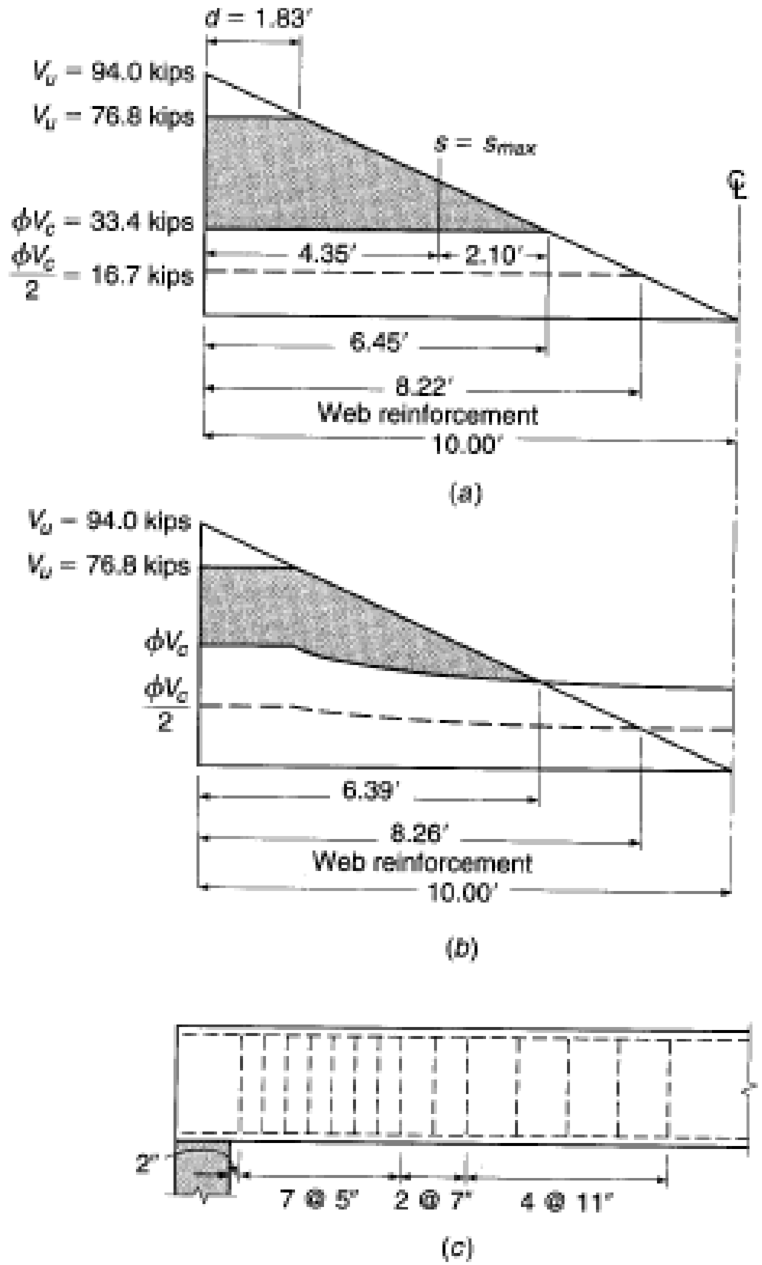
$$V_c = 2\lambda\sqrt{f_c'}b_w d = 2 \times 1.0\sqrt{4000} \times 16 \times 22 = 44{,}500 \text{ lb}$$

Hence $\phi V_c = 0.75 \times 44.5 = 33.4$ kips. This value is superimposed on the shear diagram, and, from geometry, the point at which web reinforcement theoretically is no longer required is

$$10\left(\frac{94.0 - 33.4}{94.0}\right) = 6.45 \text{ ft}$$

from the support face. However, according to the ACI Code, at least a minimum amount of web reinforcement is required wherever the shear force exceeds $\phi V_c/2$, or 16.7 kips in this case. As seen from Fig. 4.13*a*, this applies to a distance

$$10\left(\frac{94.0 - 16.7}{94.0}\right) = 8.22 \text{ ft}$$



d = 1.83′
V_u = 94.0 kips
V_u = 76.8 kips
s = s_max
φV_c = 33.4 kips
φV_c/2 = 16.7 kips
4.35′
2.10′
6.45′
8.22′
Web reinforcement
10.00′
(a)
V_u = 94.0 kips
V_u = 76.8 kips
φV_c
φV_c/2
6.39′
8.26′
Web reinforcement
10.00′
(b)
2″
7 @ 5″
2 @ 7″
4 @ 11″
(c)

from the support face. To summarize, at least the minimum web steel must be provided within a distance of 8.22 ft from the supports, and within 6.45 ft the web steel must provide for the shear force corresponding to the shaded area.

If the alternative Eq. (4.12*a*) is used, the variation along the span of ρ_w, V_u, and M_u must be known so that V_c can be calculated. This is shown in tabular form in Table 4.1.

The factored shear V_u and the design shear capacity ϕV_c are plotted in Fig. 4.13*b*. From the graph it is found that stirrups are theoretically no longer required 6.39 ft from the support face. However, from the plot of $\phi V_c/2$ it is found that at least the minimum web steel is to be provided within a distance of 8.26 ft.

When Figs. 4.13*a* and *b* are compared, it is evident that the length over which web reinforcement is needed is nearly the same for this example whether Eq. (4.12*a*) or (4.12*b*) is used. However, the smaller shaded area of Fig. 4.13*b* indicates that substantially less web-steel area would be needed within that required distance if the more accurate Eq. (4.12*a*) were adopted.

TABLE 4.1
Shear design example

Distance from Support, ft	M_u, ft-kips	V_u, kips	V_c[a]	ϕV_c
0	0	94.0	61.3	46.0
1	89	84.6	61.3	46.0
2	169	75.2	57.8	43.4
3	240	65.8	51.9	38.9
4	301	56.4	48.8	36.6
5	353	47.0	47.0	35.2
6	395	37.6	45.6	34.2
7	428	28.2	44.6	33.5
8	451	18.8	43.8	32.8
9	465	9.4	43.0	32.3
10	470	0	42.3	31.7

[a] $V_c = (1.9\lambda\sqrt{f_c'} + 2500\rho_w V_u d/M_u)b_w d \leq 3.5\lambda\sqrt{f_c'}\, b_w d$ and $V_u d/M_u \leq 1.0$

Design of web reinforcement

The design of web reinforcement, under the provisions of the ACI Code, is based on Eq. (4.11*a*) for vertical stirrups and Eq. (4.11*b*) for inclined stirrups or bent bars. In design, it is usually convenient to select a trial web-steel area A_v based on standard stirrup sizes [usually in the range from No. 3 to 5 (No. 10 to 16) for stirrups, and according to the longitudinal bar size for bent-up bars], for which the required spacing s can be found. Equating the design strength ϕV_n to the required strength V_u and transposing Eqs. (4.11*a*) and (4.11*b*) accordingly, one finds that the required spacing of web reinforcement is, for vertical stirrups,

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} \qquad (4.14a)$$

and for bent bars

$$s = \frac{\phi A_v f_{yt} d(\sin\alpha + \cos\alpha)}{V_u - \phi V_c} \qquad (4.14b)$$

Where web reinforcement is needed, the Code requires it to be spaced so that every 45° line, representing a potential diagonal crack and extending from the middepth $d/2$ of the member to the longitudinal tension bars, is crossed by at least one line of web reinforcement; in addition, the Code specifies a maximum spacing of 24 in. When V_s exceeds $4\sqrt{f'_c}b_w d$, these maximum spacings are halved. These limitations are shown in Fig. 4.14 for both vertical stirrups and inclined bars, for situations in which the excess shear does not exceed the stated limit.

For design purposes, Eq. (4.13) giving the minimum web-steel area A_v is more conveniently inverted to permit calculation of maximum spacing s for the selected A_v. Thus, for the usual case of vertical stirrups, with $V_s \leq 4\sqrt{f'_c}b_w d$, the maximum spacing of stirrups is the smallest of

$$s_{max} = \frac{A_v f_{yt}}{0.75\sqrt{f'_c}\, b_w} \leq \frac{A_v f_{yt}}{50 b_w} \tag{4.15a}$$

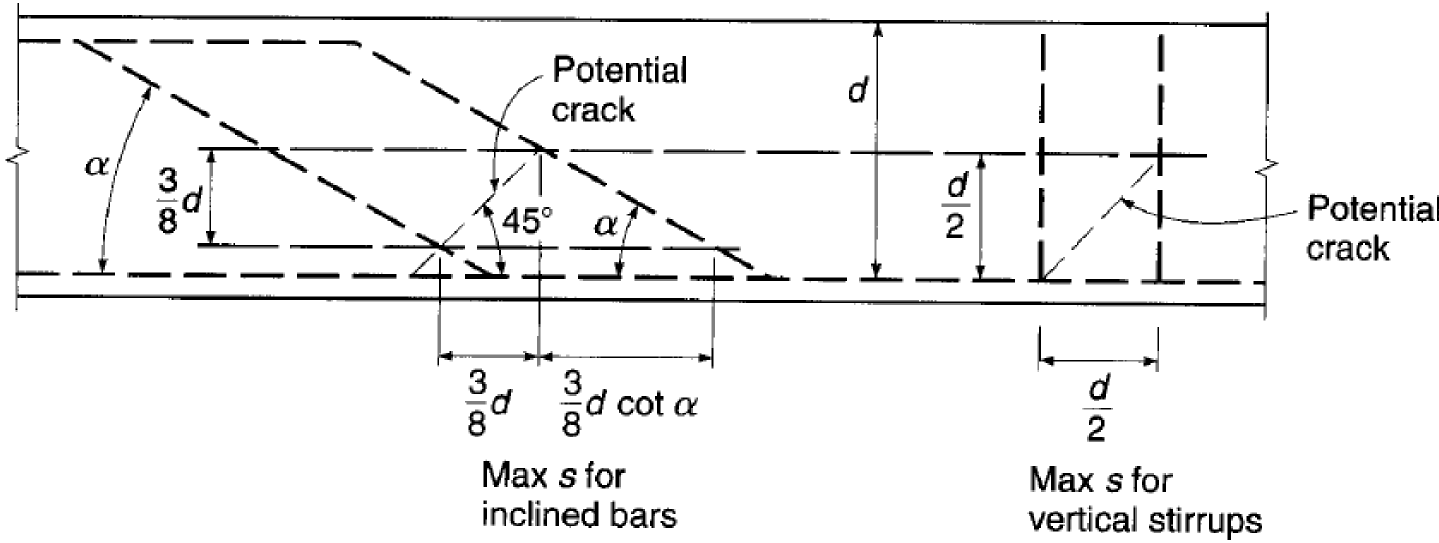
$$s_{max} = \frac{d}{2} \tag{4.15b}$$

$$s_{max} = 24 \text{ in.} \tag{4.15c}$$

For longitudinal bars bent at 45°, Eq. (4.15*b*) is replaced by $s_{max} = 3d/4$, as confirmed by Fig. 4.14.

To avoid excessive crack width in beam webs, the ACI Code limits the yield strength of the reinforcement to $f_{yt} = 60{,}000$ psi or less for reinforcing bars and 80,000 psi or less for welded wire reinforcement. In no case, according to the ACI Code, is V_s to exceed $8\sqrt{f'_c}\, b_w d$, regardless of the amount of web steel used.

FIGURE 4.14
Maximum spacing of web reinforcement as governed by diagonal crack interception.



EXAMPLE 4.3

Design of web reinforcement. Using vertical U stirrups with $f_{yt} = 60{,}000$ psi, design the web reinforcement for the beam in Example 4.2.

Solution. The solution will be based on the shear diagram in Fig. 4.13*a*. The stirrups must be designed to resist that part of the shear shown shaded. With No. 3 (No. 10) stirrups used for trial, the three maximum spacing criteria are first applied. For $\phi V_s = V_u - \phi V_c = 43{,}400$ lb, which is less than $4\phi\sqrt{f'_c}\, b_w d = 66{,}800$ lb, the maximum spacing must exceed neither $d/2 =$ 11 in. nor 24 in. Also, from Eq. (4.15*a*),

$$s_{max} = \frac{A_v f_{yt}}{0.75\sqrt{f'_c}\ b_w} = \frac{0.22 \times 60{,}000}{0.75\sqrt{4000} \times 16} = 17.4 \text{ in.}$$

$$\leq \frac{A_v f_{yt}}{50 b_w} = \frac{0.22 \times 60{,}000}{50 \times 16} = 16.5 \text{ in.}$$

The first criterion controls in this case, and a maximum spacing of 11 in. is imposed. From the support to a distance d from the support, the excess shear $V_u - \phi V_c$ is 43,400 lb. In this region, the required spacing is

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times 0.22 \times 60{,}000 \times 22}{43{,}400} = 5.0 \text{ in.}$$

This is neither so small that placement problems would result nor so large that maximum spacing criteria would control, and the choice of No. 3 (No. 10) stirrups is confirmed. Solving Eq. (4.14*a*) for the excess shear at which the maximum spacing can be used gives

$$V_u - \phi V_c = \frac{\phi A_v f_{yt} d}{s} = \frac{0.75 \times 0.22 \times 60{,}000 \times 22}{11} = 19{,}800 \text{ lb}$$

With reference to Fig. 4.13*a*, this is attained at a distance x_1 from the point of zero excess shear, where $x_1 = 6.45 \times 19{,}800/60{,}600 = 2.10$ ft. This is 4.35 ft from the support face. With this information, a satisfactory spacing pattern can be selected. The first stirrup is usually placed at a distance $s/2$ from the support. The following spacing pattern is satisfactory:

$$\begin{aligned}
1 \text{ space at } 2 \text{ in.} &= 2 \text{ in.} \\
7 \text{ spaces at } 5 \text{ in.} &= 35 \text{ in.} \\
2 \text{ spaces at } 7 \text{ in.} &= 14 \text{ in.} \\
4 \text{ spaces at } 11 \text{ in.} &= \underline{44 \text{ in.}} \\
\text{Total} &= 95 \text{ in.} = 7 \text{ ft } 11 \text{ in.}
\end{aligned}$$

Graphically

The resulting stirrup pattern is shown in Fig. 4.13*c*. As an alternative solution, it is possible to plot a curve showing required spacing as a function of distance from the support. Once the required spacing at some reference section, say at the support, is determined,

$$s_0 = \frac{0.75 \times 0.22 \times 60{,}000 \times 22}{94{,}000 - 33{,}400} = 3.59 \text{ in.}$$

it is easy to obtain the required spacings elsewhere. In Eq. (4.14*a*), only $V_u - \phi V_c$ changes with distance from the support. For uniform load, this quantity is a linear function of distance from the point of zero excess shear, 6.45 ft from the support face. Hence, at 1 ft intervals,

$$s_1 = 3.59 \times 6.45/5.45 = 4.25 \text{ in.}$$

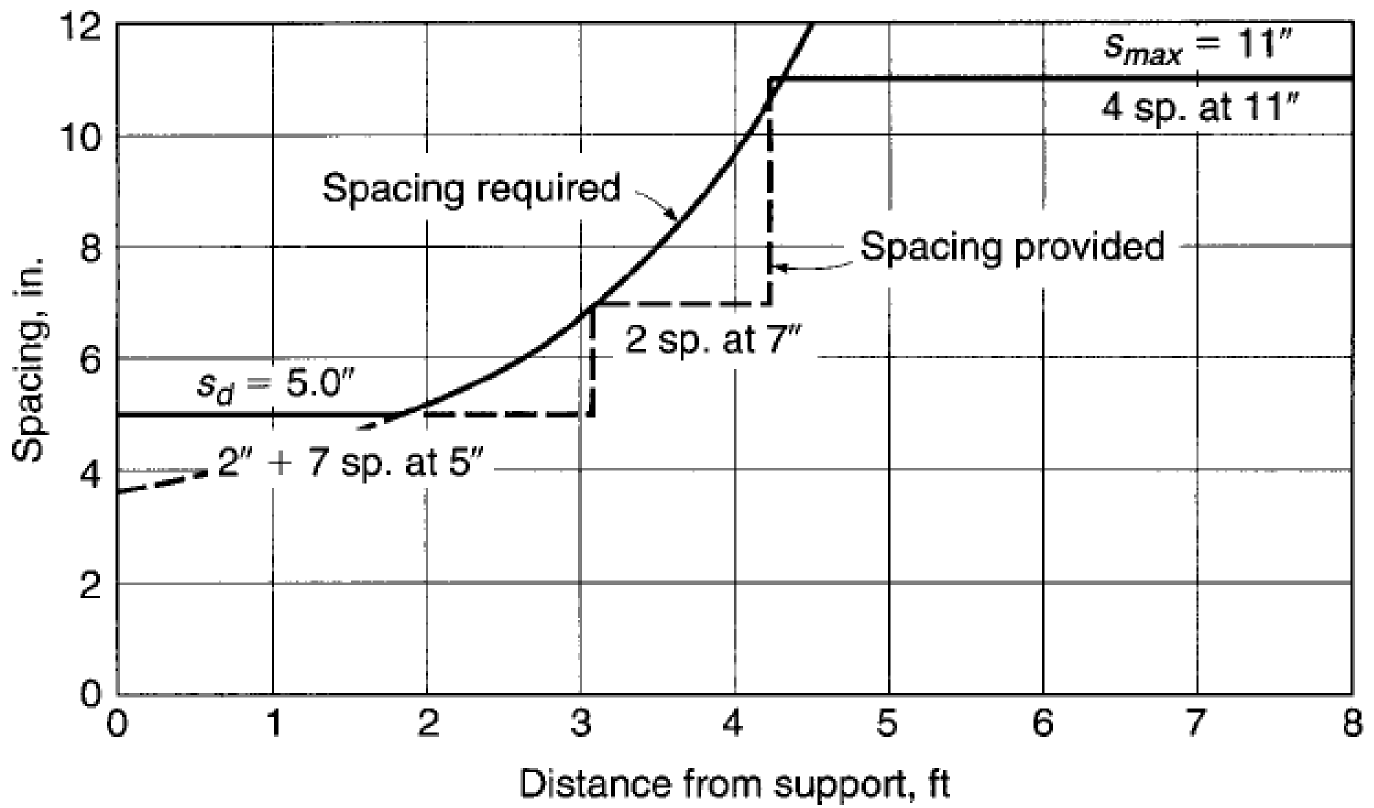
$$s_2 = 3.59 \times 6.45/4.45 = 5.20 \text{ in.}$$

$$s_3 = 3.59 \times 6.45/3.45 = 6.70 \text{ in.}$$

$$s_4 = 3.59 \times 6.45/2.45 = 9.45 \text{ in.}$$

$$s_5 = 3.59 \times 6.45/1.45 = 15.97 \text{ in.}$$

This is plotted in Fig. 4.15 together with the maximum spacing of 11 in., and a practical spacing pattern is selected. The spacing at a distance *d* from the support face is selected as the minimum



requirement, in accordance with the ACI Code. The pattern of No. 3 (No. 10) U-shaped stirrups selected (shown on the graph) is identical with the previous solution. In most cases, the experienced designer would find it unnecessary actually to plot the spacing diagram of Fig. 4.15 and would select a spacing pattern directly after calculating the required spacing at intervals along the beam.

More rigorous formula

If the web steel were to be designed on the basis of the excess-shear diagram in Fig. 4.13*b*, the second approach illustrated above would necessarily be selected, and spacings would be calculated at intervals along the span. In this particular case, a spacing of 7.07 in. is calculated up to 20 in. from the face of the support. The calculated spacing drops to 6.76 in. at d from the face of the support, and then increases to 11 in., the maximum permissible spacing, 4 ft from the support. The following practical spacing could be used:

$$
\begin{aligned}
1 \text{ space at } 3 \text{ in.} &= 3 \text{ in.} \\
6 \text{ spaces at } 7 \text{ in.} &= 42 \text{ in.} \\
4 \text{ spaces at } 11 \text{ in.} &= \underline{44 \text{ in.}} \\
\text{Total} &= 89 \text{ in.} = 7 \text{ ft } 5 \text{ in.}
\end{aligned}
$$

Thus, 11 No. 3 (No. 10) stirrups would be used, rather than the 14 previously calculated, in each half of the span.

Provide stirrups even if not required

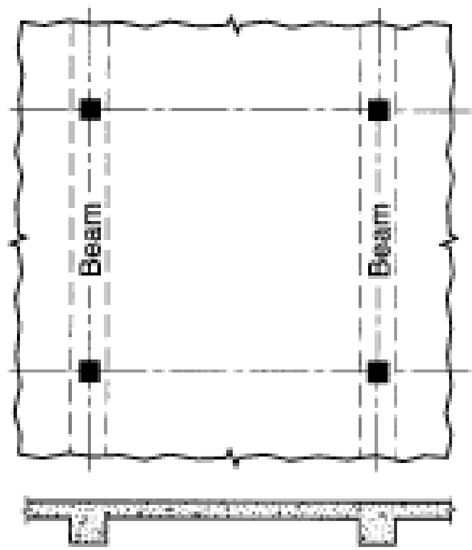
The number of stirrups just calculated represents the minimum for each of the two expressions for V_c. Although not required by the ACI Code, it is good design practice to continue the stirrups (at maximum spacing) through the middle region of the beam, even though the calculated shear is low. Doing so satisfies the dual purposes of providing continuing support for the top longitudinal reinforcement that is required wherever stirrups are used and providing additional shear capacity in the region to handle load cases not considered in developing the shear diagram. If this were done, the number of stirrups would increase from 14 and 11 to $16\frac{1}{2}$ and $13\frac{1}{2}$ per half-span (i.e., one stirrup at midspan), respectively.

Analysis and Design of Slabs

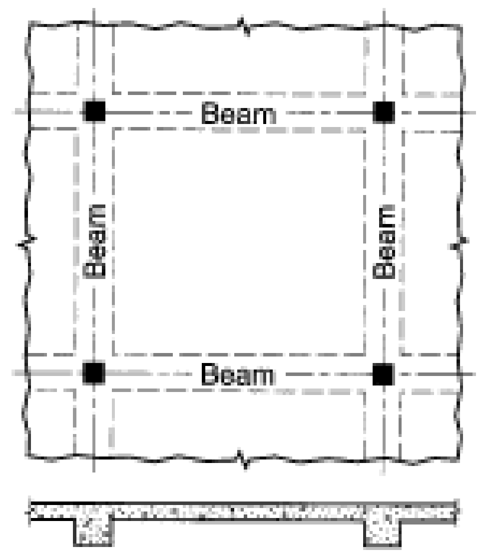
Chapter 13

Types of Slabs

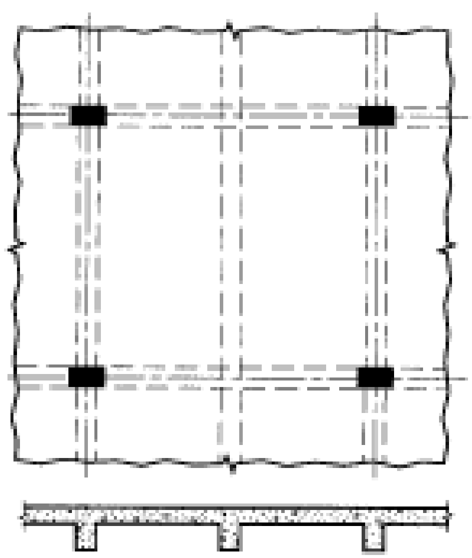
- Useful surface
- Supported on monolithic beams, steel joist, masonry or RC wall, directly on column, continuously on ground
-



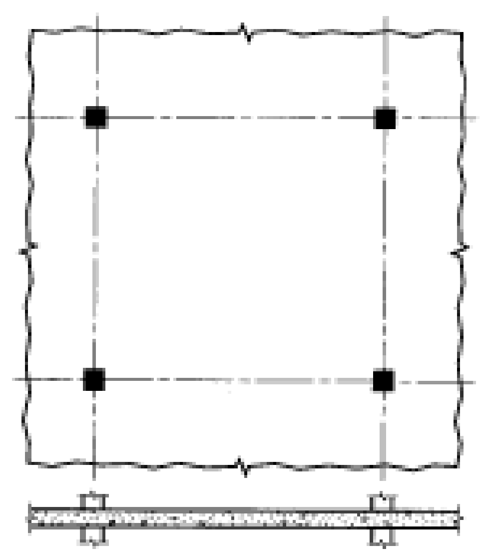
(*a*) One-way slab



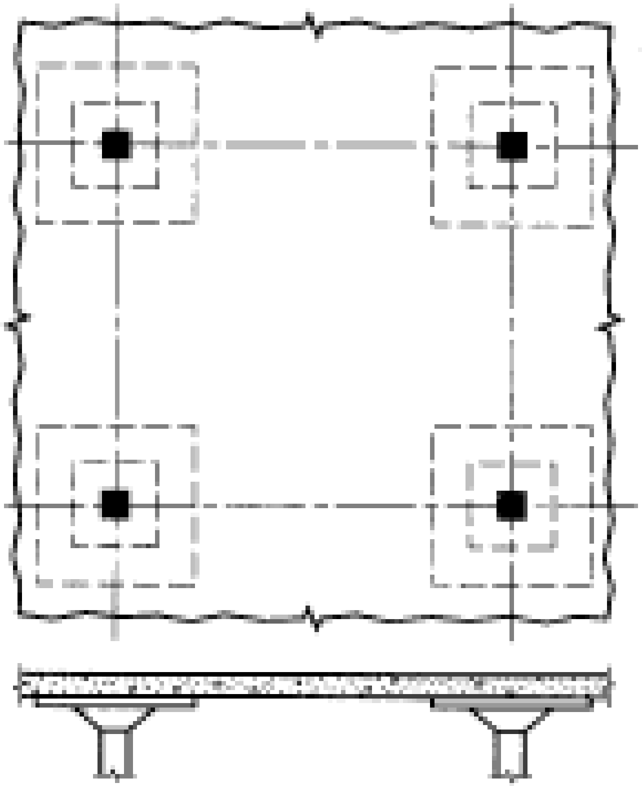
(*b*) Two-way slab



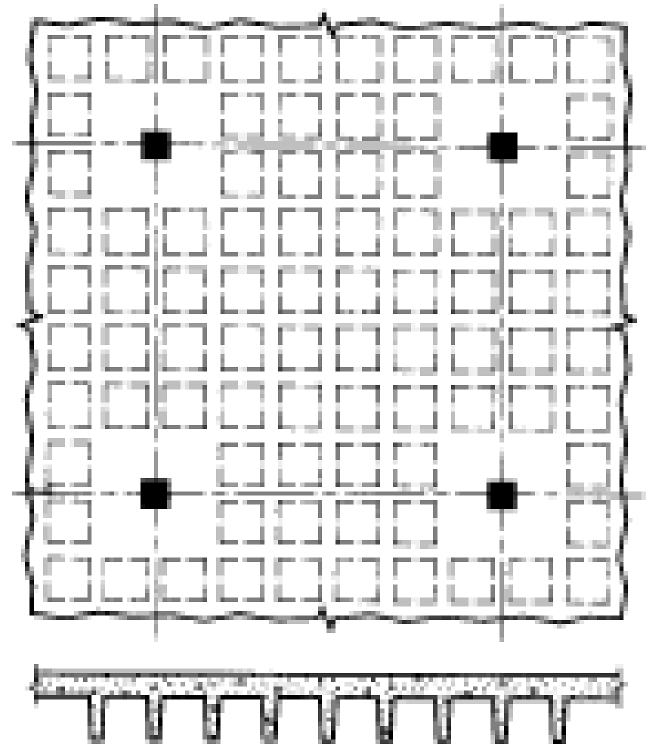
(*c*) One-way slab



(*d*) Flat plate



(*e*) Flat slab



(*f*) Grid or waffle slab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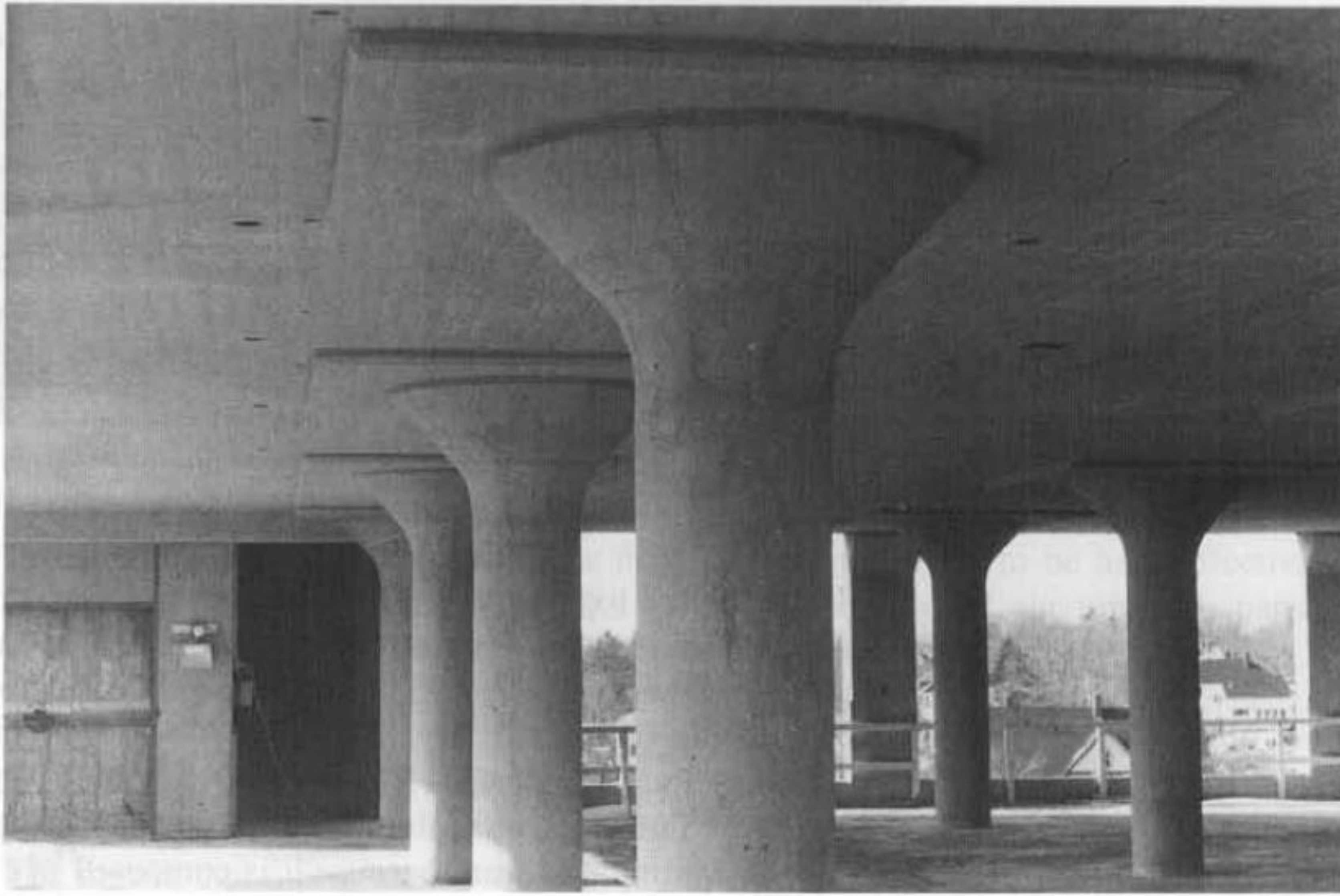
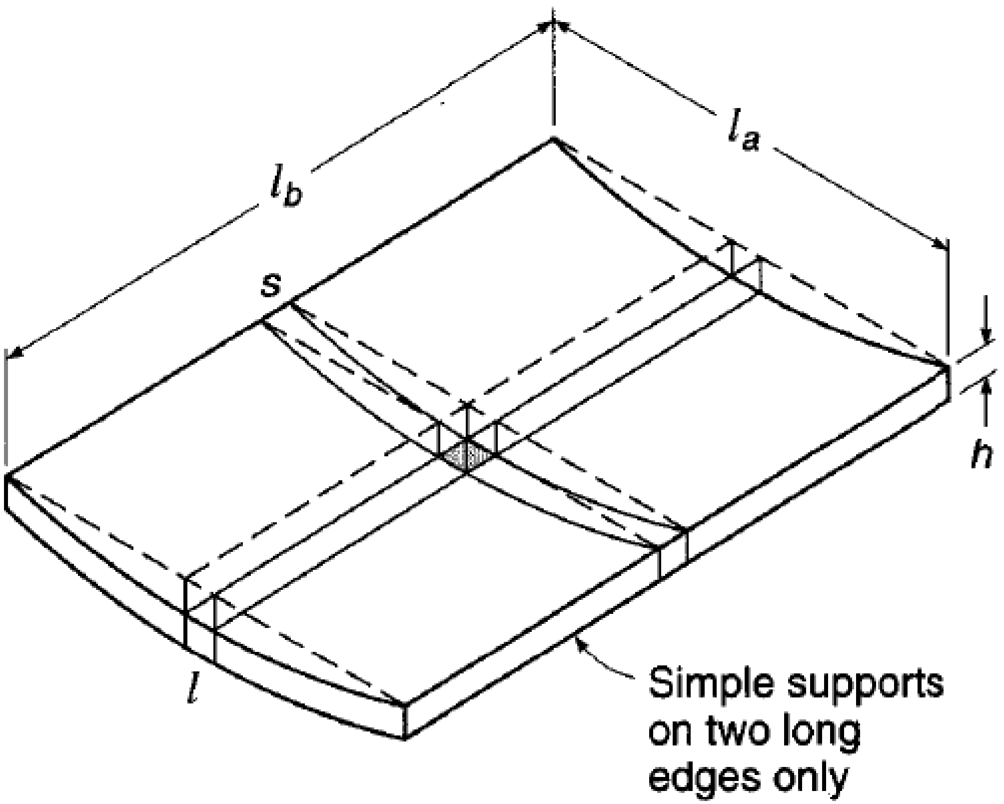
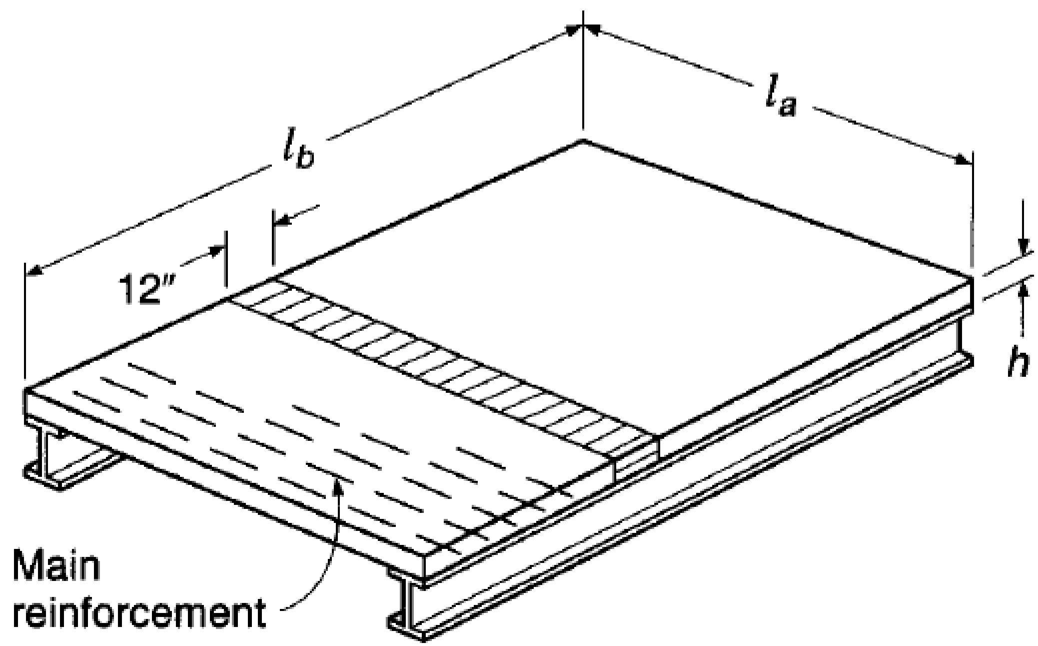


Design of One-way slab





l_b
l_a
12″
h
Main
reinforcement

Thickness of one-way slab

TABLE 13.1
Minimum thickness *h* of nonprestressed one-way slabs

Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

ACI Code 9.5.2 specifies the minimum thickness in Table 13.1 for nonprestressed slabs of normalweight concrete (w_c = 145 pcf) using Grade 60 reinforcement, provided that the slab is not supporting or attached to construction that is likely to be damaged by large deflections. Lesser thicknesses may be used if calculation of deflections indicates no adverse effects. For concretes having unit weight w_c in the range from 90 to 115 pcf, the tabulated values should be multiplied by $1.65 - 0.005w_c$, but not less than 1.09. For reinforcement having a yield stress f_y other than 60,000 psi, the tabulated values should be multiplied by $0.4 + f_y/100{,}000$. Slab deflections may be calculated, if required, by the same methods as for beams (see Section 6.7).

Design as a beam

- Same as beam of unit width

One-way slabs are normally designed with tensile reinforcement ratios well below the maximum practical value of $\rho_{0.005}$. Typical reinforcement ratios range from about 0.004 to 0.008. This is partially for reasons of economy, because the saving in steel associated with increasing the effective depth more than compensates for the cost of the additional concrete, and partially because very thin slabs with high reinforcement ratios would be likely to permit large deflections. Thus, flexural design may start with selecting a relatively low reinforcement ratio, say about $0.3\rho_{0.005}$, setting $M_u = \phi M_n$ in Eq. (3.38), and solving for the required effective depth d, given that $b = 12$ in. for the unit strip. Alternatively, Table A.5 or Graph A.1 of Appendix A may be used. Table A.9 is also useful. The required steel area per 12 in. strip $A_s = \rho bd$ is then easily found.

Clear Cover

The total slab thickness h is usually rounded to the next higher $\frac{1}{4}$ in. for slabs up to 6 in. thickness, and to the next higher $\frac{1}{2}$ in. for thicker slabs. Best economy is often achieved when the slab thickness is selected to match nominal lumber dimensions. The concrete protection below the reinforcement should follow the requirements of ACI Code 7.7.1, calling for $\frac{3}{4}$ in. below the bottom of the steel (see Fig. 3.13*b*). In a typical slab, 1 in. below the center of the steel may be assumed. The lateral spacing of the bars, except those used only to control shrinkage and temperature cracks (see Section 13.3), should not exceed 3 times the thickness h or 18 in., whichever is less, according to ACI Code 7.6.5. Generally, bar size should be selected so that the actual spacing is not less than about 1.5 times the slab thickness, to avoid excessive cost for bar fabrication and handling. Also, to reduce cost, straight bars are usually used for slab reinforcement, cut off where permitted as described for beams in Section 5.10.

Temperature and Shrinkage Reinforcement

- Why?

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement should be provided in a structural slab in which the principal reinforcement extends in one direction only. ACI Code 7.12.2 specifies the minimum ratios of reinforcement area to *gross concrete area* (i.e., based on the total depth of the slab) shown in Table 13.2, but in no case may such reinforcing bars be placed farther apart than 5 times the slab thickness or more than 18 in. In no case is the reinforcement ratio to be less than 0.0014.

TABLE 13.2
Minimum ratios of temperature and shrinkage reinforcement in slabs based on gross concrete area

Slabs where Grade 40 or 50 deformed bars are used	0.0020
Slabs where Grade 60 deformed bars or welded wire fabric (smooth or deformed) is used	0.0018
Slabs where reinforcement with yield strength exceeding 60,000 psi measured at yield strain of 0.35 percent is used	$\frac{0.0018 \times 60{,}000}{f_y}$

EXAMPLE 13.1

One-way slab design. A reinforced concrete slab is built integrally with its supports and consists of two equal spans, each with a clear span of 15 ft. The service live load is 100 psf, and 4000 psi concrete is specified for use with steel with a yield stress equal to 60,000 psi. Design the slab, following the provisions of the ACI Code.

Solution. The thickness of the slab is first estimated, based on the minimum thickness from Table 13.1: $l/28 = 15 \times 12/28 = 6.43$ in. A trial thickness of 6.50 in. will be used, for which the weight is $150 \times 6.50/12 = 81$ psf. The specified live load and computed dead load are multiplied by the ACI load factors:

$$\begin{aligned} \text{Dead load} &= 81 \times 1.2 = 97 \text{ psf} \\ \text{Live load} &= 100 \times 1.6 = \underline{160 \text{ psf}} \\ \text{Total} &= 257 \text{ psf} \end{aligned}$$

For this case, factored moments at critical sections may be found using the ACI moment coefficients (see Table 12.1):

At interior support: $-M = \frac{1}{9} \times 0.257 \times 15^2 = 6.43$ ft-kips

At midspan: $+M = \frac{1}{14} \times 0.257 \times 15^2 = 4.13$ ft-kips

At exterior support: $-M = \frac{1}{24} \times 0.257 \times 15^2 = 2.41$ ft-kips

The maximum practical reinforcement ratio is, according to Eq. (3.30*d*),

$$\rho_{0.005} = (0.85^2)\,\frac{4}{60}\,\frac{0.003}{0.003 + 0.005} = 0.0181$$

TABLE 12.1
Moment and shear values using ACI coefficients†

Positive moment	
End spans	
If discontinuous end is unrestrained	$\frac{1}{11} w_u l_n^2$
If discontinuous end is integral with the support	$\frac{1}{14} w_u l_n^2$
Interior spans	$\frac{1}{16} w_u l_n^2$
Negative moment at exterior face of first interior support	
Two spans	$\frac{1}{9} w_u l_n^2$
More than two spans	$\frac{1}{10} w_u l_n^2$
Negative moment at other faces of interior supports	$\frac{1}{11} w_u l_n^2$
Negative moment at face of all supports for (1) slabs with spans not exceeding 10 ft and (2) beams and girders where ratio of sum of column stiffness to beam stiffness exceeds 8 at each end of the span	$\frac{1}{12} w_u l_n^2$
Negative moment at interior faces of exterior supports for members built integrally with their supports	
Where the support is a spandrel beam or girder	$\frac{1}{24} w_u l_n^2$
Where the support is a column	$\frac{1}{16} w_u l_n^2$
Shear in end members at first interior support	$1.15 \frac{w_u l_n}{2}$
Shear at all other supports	$\frac{w_u l_n}{2}$

† w_u = total factored load per unit length of beam or per unit area of slab.
l_n = clear span for positive moment and shear and the average of the two adjacent clear spans for negative moment.

If this value of ρ were actually used, the minimum required effective depth, controlled by negative moment at the interior support, would be found from Eq. (3.38) to be

$$d^2 = \frac{M_u}{\phi \rho f_y b(1 - 0.59\rho f_y/f'_c)}$$

$$= \frac{6.43 \times 12}{0.90 \times 0.0181 \times 60 \times 12[1 - 0.59 \times 0.0181 \times (60/4)]} = 7.83 \text{ in}^2$$

$$d = 2.80 \text{ in.}^{\dagger}$$

This is less than the effective depth of 6.50 − 1.00 = 5.50 in. resulting from application of Code restrictions, and the latter figure will be adopted. At the interior support, if the stress-block depth $a = 1.00$ in., the area of steel required per foot of width in the top of the slab is [Eq. (3.37)]

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{6.43 \times 12}{0.90 \times 60 \times (5.50 - 1.00/2)} = 0.29 \text{ in}^2$$

Checking the assumed depth a by Eq. (3.32), one gets

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.29 \times 60}{0.85 \times 4 \times 12} = 0.43 \text{ in.}$$

A second trial will be made with $a = 0.43$ in. Then

$$A_s = \frac{6.43 \times 12}{0.90 \times 60 \times (5.50 - 0.43/2)} = 0.27 \text{ in}^2$$

for which $a = 0.43 \times 0.27/0.29 = 0.40$ in. No further revision is necessary. At other critical-moment sections, it will be satisfactory to use the same lever arm to determine steel areas, and

At midspan: $$A_s = \frac{4.13 \times 12}{0.90 \times 60 \times (5.50 - 0.40/2)} = 0.17 \text{ in}^2$$

At exterior support: $$A_s = \frac{2.41 \times 12}{0.90 \times 60 \times (5.50 - 0.40/2)} = 0.10 \text{ in}^2$$

The minimum reinforcement is that required for control of shrinkage and temperature cracking. This is

$$A_s = 0.0018 \times 12 \times 6.50 = 0.14 \text{ in}^2$$

per 12 in. strip. This requires a small increase in the amount of steel used at the exterior support.

The factored shear force at a distance d from the face of the interior support is

$$V_u = 1.15 \times \frac{257 \times 15}{2} - 257 \times \frac{5.50}{12} = 2100 \text{ lb}$$

By Eq. (4.12b), the nominal shear strength of the concrete slab is

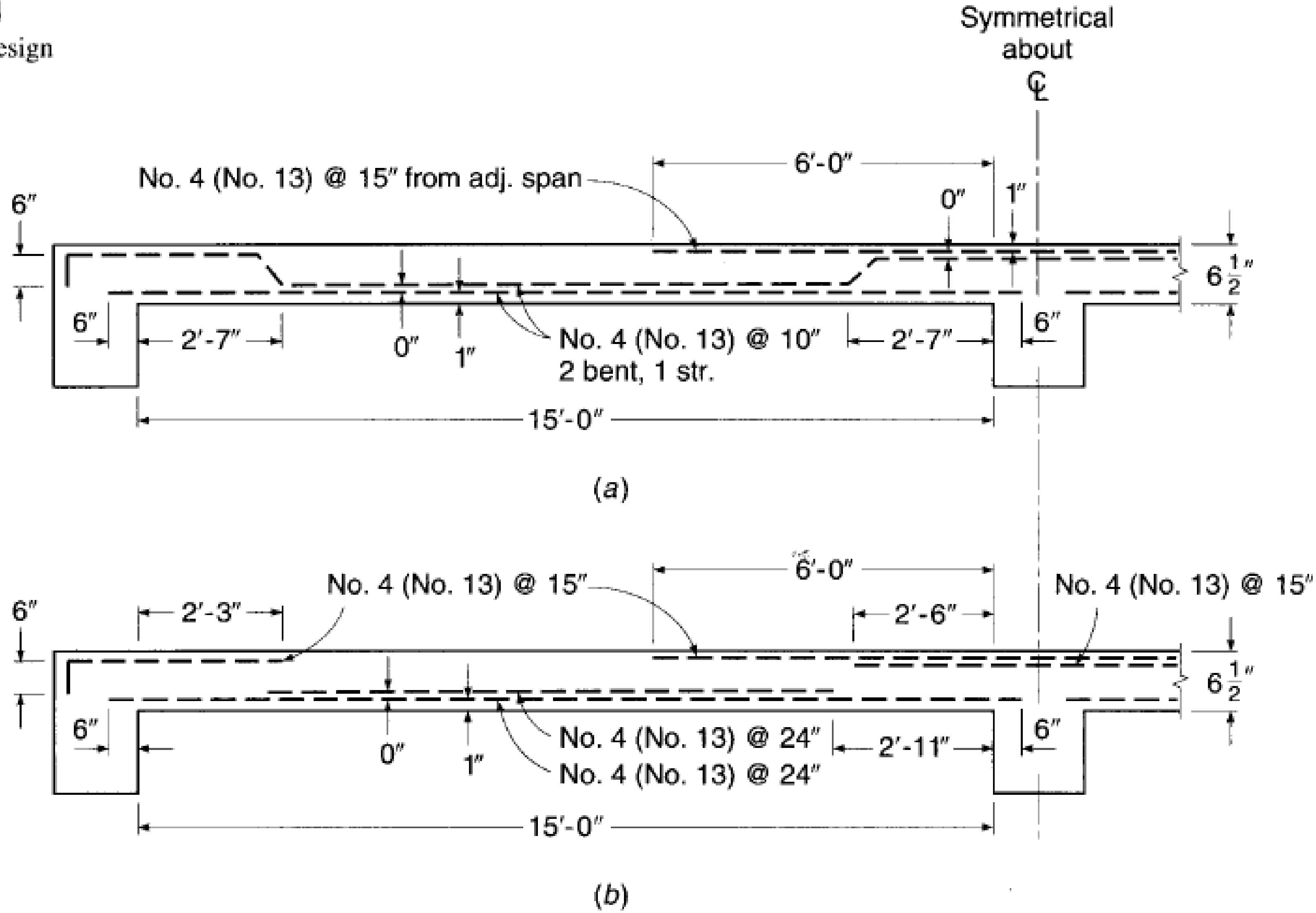
$$V_n = V_c = 2\lambda\sqrt{f_c'}\,bd = 2 \times 1\sqrt{4000} \times 12 \times 5.50 = 8350 \text{ lb}$$

Thus, the design strength of the concrete slab $\phi V_c = 0.75 \times 8350 = 6260$ lb is well above the required strength in shear of $V_u = 2100$.

The required tensile steel areas may be provided in a variety of ways, but whatever the selection, due consideration must be given to the actual placing of the steel during construction. The arrangement should be such that the steel can be placed rapidly with the minimum of labor costs even though some excess steel is necessary to achieve this end.

Two possible arrangements are shown in Fig. 13.4. In Fig. 13.4a, bent bars are used, while in Fig. 13.4b all bars are straight.

FIGURE 13.4
One-way slab design example.



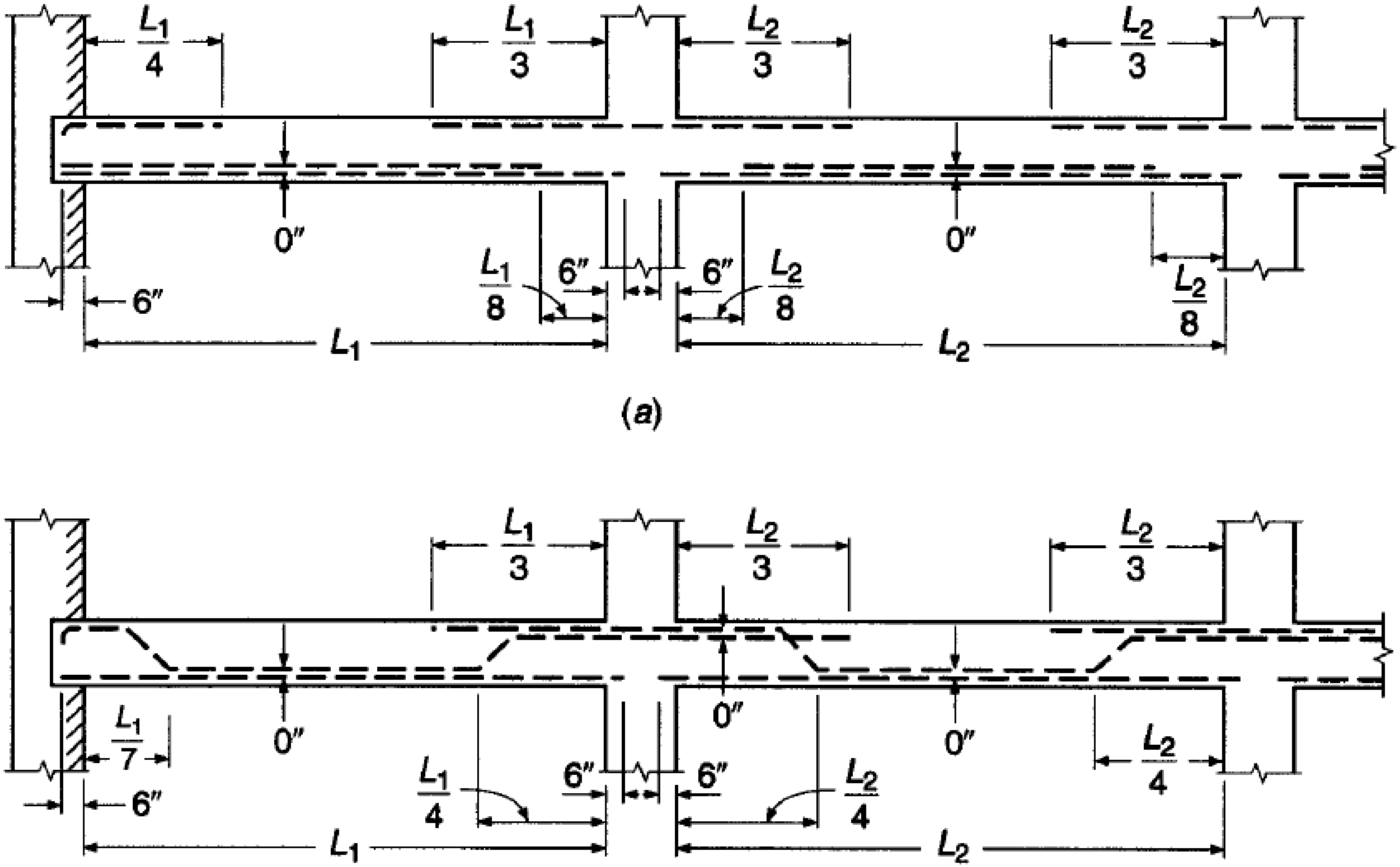


FIGURE 5.20
Cutoff or bend points for bars in approximately equal spans with uniformly distributed loads.

Two-way edge supported slab

Two-way slab

- Load transmits in two direction
- Dished shape
- Types
  - Supported on walls or stiff beams on all sides
  - Flat plate
  - Flat slab
  - Waffle slab

Slab supported on edges

- By wall
- By stiff RC beam
- By steel joist

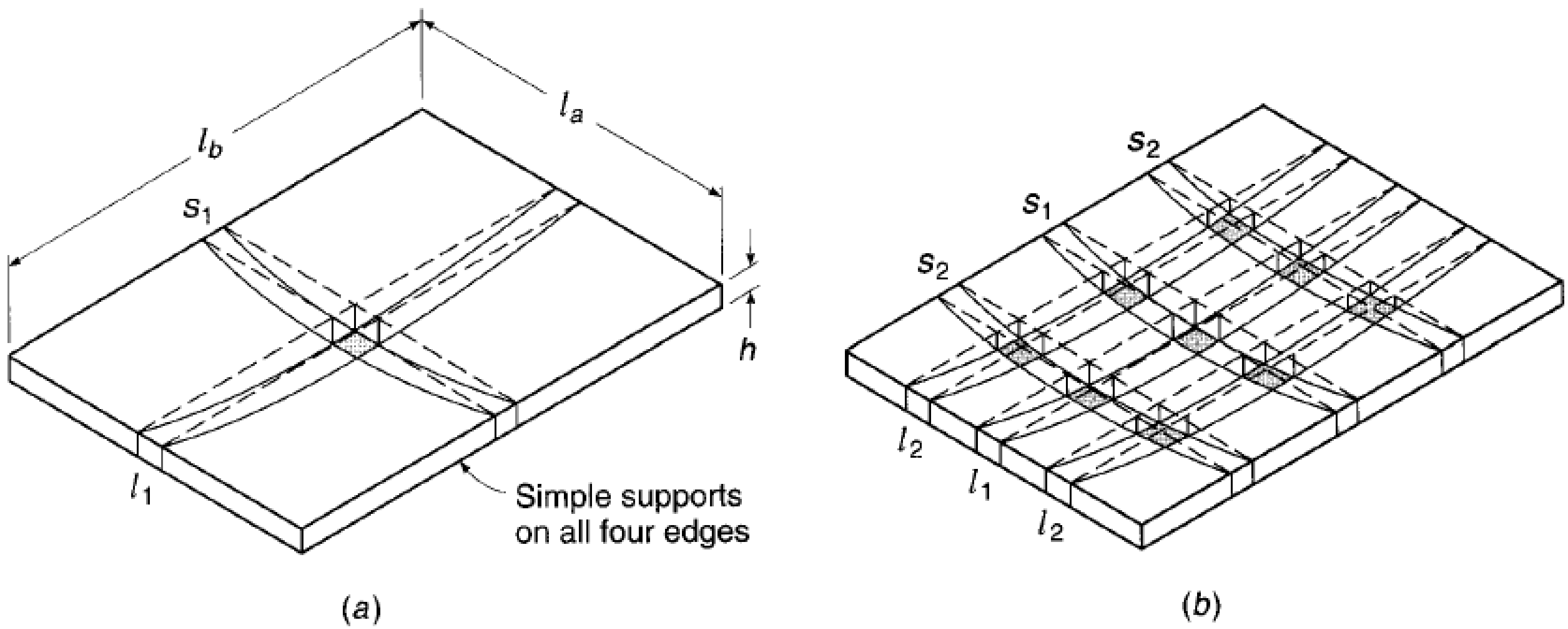


FIGURE 13.5
Two-way slab on simple edge supports: (*a*) bending of center strips of slab; (*b*) grid model of slab.

$$\frac{5q_a l_a^4}{384EI} = \frac{5q_b l_b^4}{384EI}$$

$$\frac{q_a}{q_b} = \frac{l_b^4}{l_a^4}$$

$$\frac{(q/2)l^2}{8} = 0.0625ql^2$$

$$0.048ql^2$$

$$0.036ql^2$$

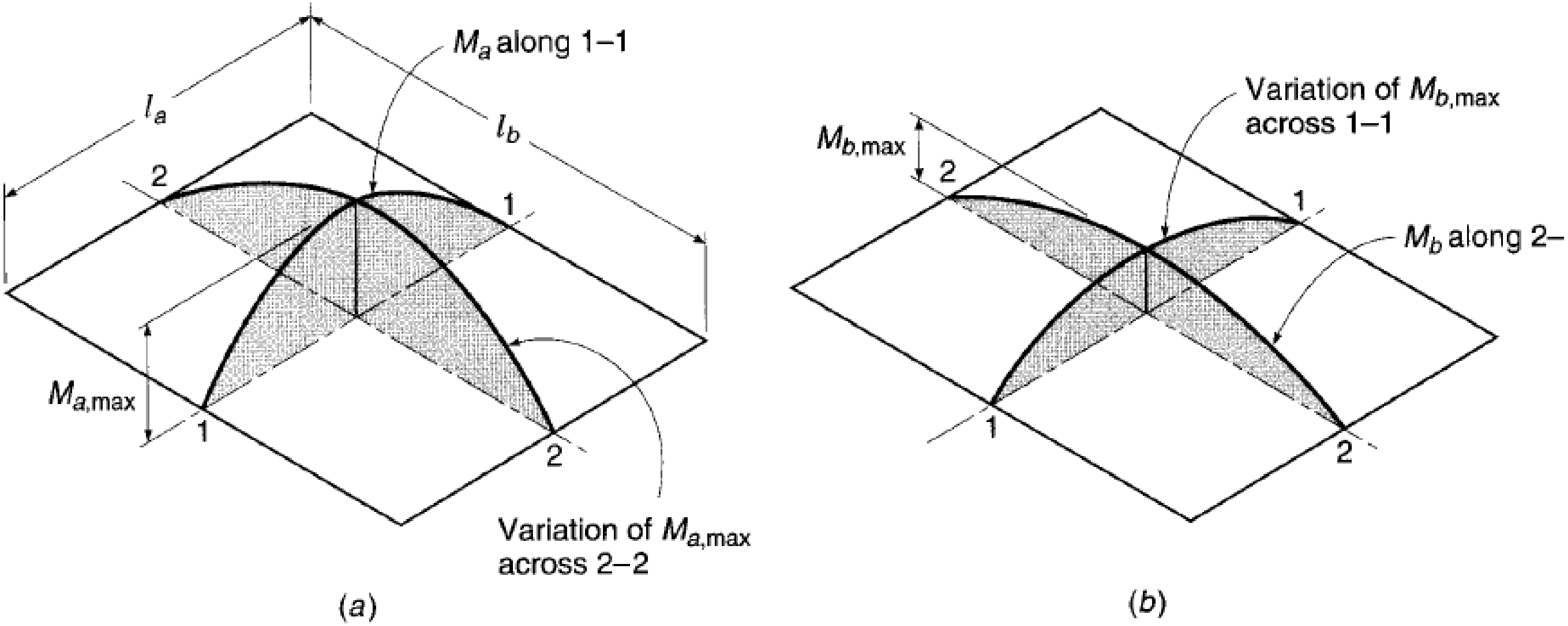


FIGURE 13.6
Moments and moment variations in a uniformly loaded slab with simple supports on four sides.

Moment Coefficient method

- Simplified method of calculating moments and shear
- 1963 ACI Code- later discontinued from 1977
- Still part of BNBC
- For two-way slabs supported on all sides by stiff beams (not less than 3 time slab thickness)

Moment Coefficient method

- Use tables of moment coefficient for various end conditions
- Based on elastic plate analysis and considers stress redistribution
- Still valid

tion. The moments in the middle strips in the two directions are computed from

$$M_a = C_a w l_a^2 \tag{12.1}$$

and

$$M_b = C_b w l_b^2 \tag{12.2}$$

where C_a, C_b = tabulated moment coefficients
w = uniform load, psf
l_a, l_b = length of clear span in short and long directions respectively

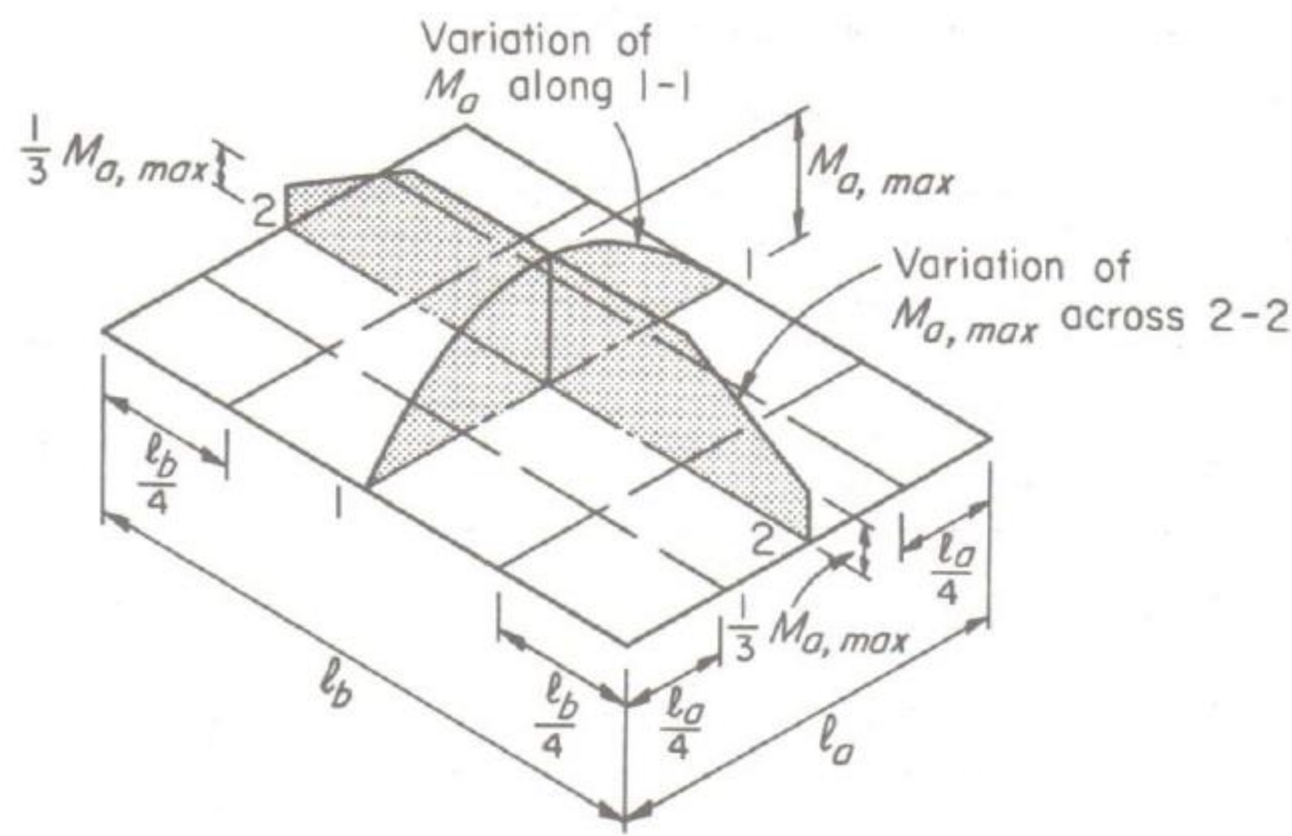


FIGURE 12.7
Variation of moments across the width of critical sections assumed for design.

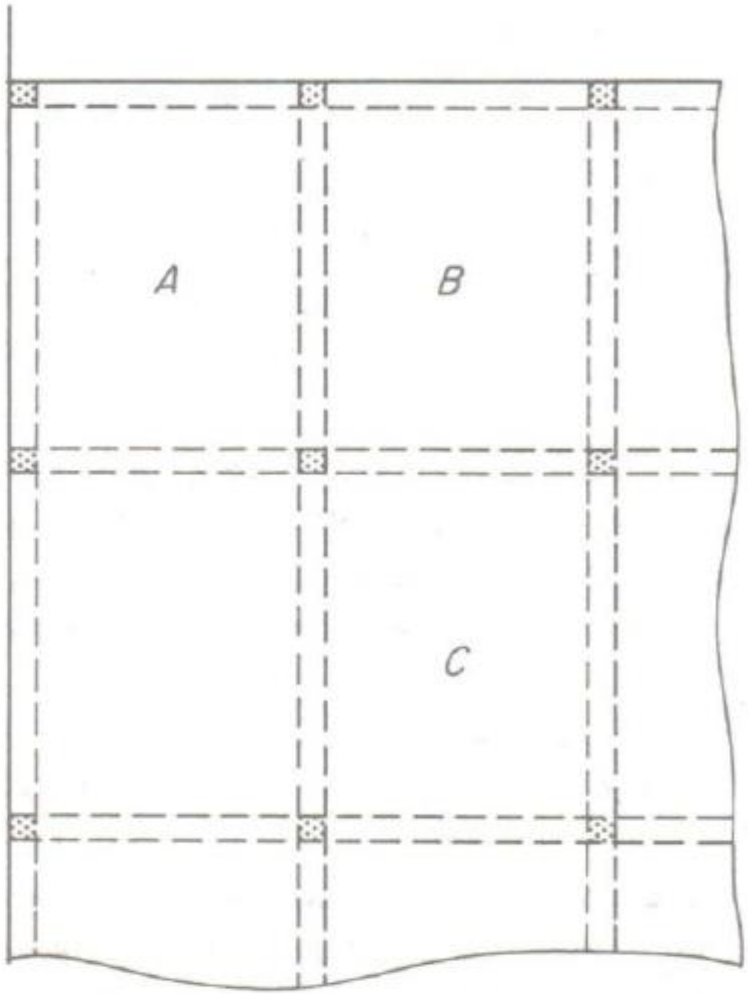
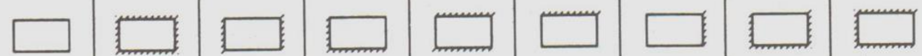


FIGURE 12.8
Plan of a typical two-way slab floor with beams on column lines.

Table 12.3 Coefficients for negative moments in slabs[a]

$M_{a,\text{neg}} = C_{a,\text{neg}} w l_a^2$
$M_{b,\text{neg}} = C_{b,\text{neg}} w l_b^2$ where $w =$ total uniform dead plus live load



Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,\text{neg}}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{b,\text{neg}}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{a,\text{neg}}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{b,\text{neg}}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{a,\text{neg}}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{b,\text{neg}}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{a,\text{neg}}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{b,\text{neg}}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{a,\text{neg}}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{b,\text{neg}}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{a,\text{neg}}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{b,\text{neg}}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{a,\text{neg}}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{b,\text{neg}}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{a,\text{neg}}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{b,\text{neg}}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{a,\text{neg}}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{b,\text{neg}}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{a,\text{neg}}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{b,\text{neg}}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{a,\text{neg}}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{b,\text{neg}}$		0.006	0.022	0.006			0.014	0.010	0.003

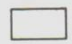
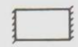
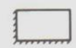
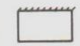
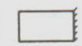
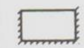
[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

1

2

Table 12.4 Coefficients for dead load positive moments in slabs[a]

$M_{a,\text{pos},dl} = C_{a,dl} w l_a^2$
where w = total uniform dead load
$M_{b,\text{pos},dl} = C_{b,dl} w l_b^2$

Ratio		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
$m = \frac{l_a}{l_b}$										
1.00	$C_{a,dl}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{b,dl}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{a,dl}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{b,dl}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{a,dl}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{b,dl}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{a,dl}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{b,dl}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{a,dl}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{b,dl}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{a,dl}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{b,dl}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{a,dl}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{b,dl}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{a,dl}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{b,dl}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{a,dl}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{b,dl}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{a,dl}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{b,dl}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{a,dl}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{b,dl}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

3

Table 12.5 Coefficients for live load positive moments in slabs[a]

$M_{a,\mathrm{pos},ll} = C_{a,ll} w l_a^2$
$M_{b,\mathrm{pos},ll} = C_{b,ll} w l_b^2$ where w = total uniform live load

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,ll}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{b,ll}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{a,ll}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{b,ll}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{a,ll}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{b,ll}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{a,ll}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{b,ll}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{a,ll}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{b,ll}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{a,ll}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{b,ll}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{a,ll}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{b,ll}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{a,ll}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{b,ll}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{a,ll}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{b,ll}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{a,ll}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{b,ll}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{a,ll}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{b,ll}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

4

Table 12.6 Ratio of load W in l_a and l_b directions for shear in slab and load on supports[a]

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	W_a	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_b	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_a	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_b	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_a	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_b	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_a	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_b	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_a	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_b	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_a	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_b	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_a	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_b	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_a	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_b	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_a	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_b	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_a	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_b	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_a	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_b	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

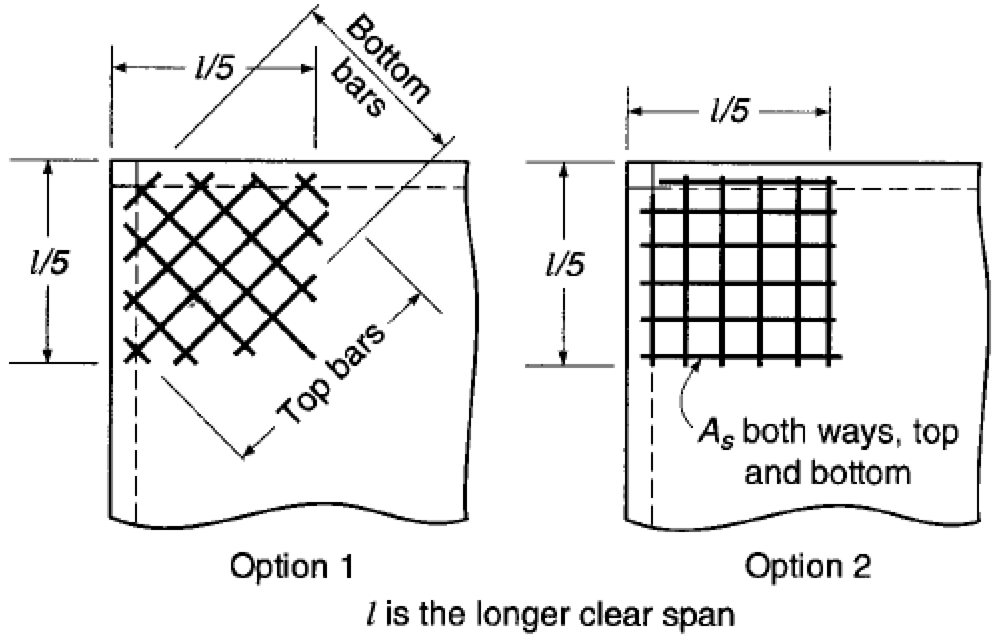
- Negative moment at discontinous edge =1/3 positive moment in the span
- d smaller by 1d in midspan
- Short direction reinf placed below
- Minimum reinforcement
- Spacing at critical section less than 2h

Corner reinforcement

The twisting moments discussed in Sec. 12.4 are usually of consequence only at exterior corners of a two-way slab system, where they tend to crack the slab at the bottom along the panel diagonal, and at the top perpendicular to the panel diagonal. Special reinforcement should be provided at exterior corners in both the bottom and top of the slab, for a distance in each direction from the corner equal to one-fifth the longer span of the corner panel, as shown in Fig. 12.9. The reinforcement at the top of the slab should be parallel to the diagonal from the corner, while that at the bottom should be perpendicular to the diagonal. Alternatively, either layer of steel may be placed in two bands parallel to the sides of the slab. The positive and negative reinforcement, in any case, should be of a size and spacing equivalent to that required for the maximum positive moment in the panel, according to ACI Code 13.4.6.

Corner reinforcement

FIGURE 13.7
Special reinforcement at exterior corners of a beam-supported two-way slab.



Example 12.2 Design of two-way edge-supported slab. A monolithic reinforced concrete floor is to be composed of rectangular bays measuring 21×26 ft, as shown in Fig. 12.10. Beams of width 12 in. and depth 24 in. are provided on all column lines; thus the clear-span dimensions for the two-way slab panels are 20×25 ft. The floor is to be designed to carry a service live load of 137 psf uniformly distributed over its surface, in addition to its own weight, using concrete of strength $f_c' = 3000$ psi and reinforcement having $f_y = 60{,}000$ psi. Find the required slab thickness and reinforcement for the corner panel shown.

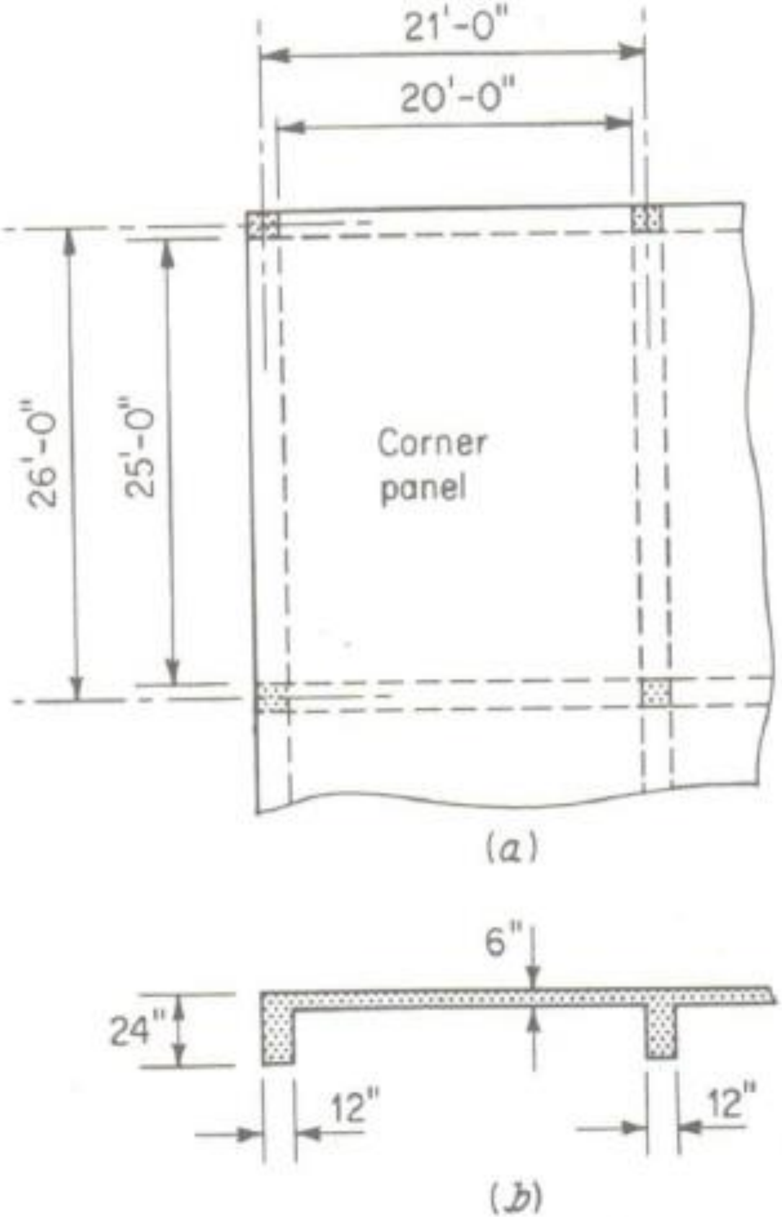


FIGURE 12.10
Two-way edge-supported slab example: (*a*) partial floor plan; (*b*) typical cross section.

This will be selected for a trial depth. The corresponding dead load is $\frac{1}{2} \times 150 = 75$ psf. Thus, the factored loads on which the design is to be based are

$$\begin{aligned} \text{Live load} &= 1.7 \times 137 = 233 \text{ psf} \\ \text{Dead load} &= 1.4 \times \ \ 75 = \underline{105} \text{ psf} \\ \text{Total load} &= 338 \text{ psf} \end{aligned}$$

With the ratio of panel sides $m = l_a/l_b = 20/25 = 0.8$, the moment calculations for the slab *middle strips* are as follows.

Negative moments at continuous edges (Table 12.3)

$$M_{a,\text{neg}} = 0.071 \times 338 \times 20^2 = 9600 \text{ ft-lb} = 115{,}000 \text{ in-lb}$$
$$M_{b,\text{neg}} = 0.029 \times 338 \times 25^2 = 6130 \text{ ft-lb} = 73{,}400 \text{ in-lb}$$

Positive moments (Tables 12.4 and 12.5)

$$\begin{aligned} M_{a,\text{pos},dl} &= 0.039 \times 105 \times 20^2 = 1638 \text{ ft-lb} = 19{,}700 \text{ in-lb} \\ M_{a,\text{pos},ll} &= 0.048 \times 233 \times 20^2 = 4470 \text{ ft-lb} = \underline{53{,}700} \text{ in-lb} \\ M_{a,\text{pos,tot}} & \qquad\qquad\qquad\qquad\qquad\qquad\quad = 73{,}400 \text{ in-lb} \\ M_{b,\text{pos},dl} &= 0.016 \times 105 \times 25^2 = 1050 \text{ ft-lb} = 12{,}600 \text{ in-lb} \\ M_{b,\text{pos},ll} &= 0.020 \times 233 \times 25^2 = 2910 \text{ ft-lb} = \underline{35{,}000} \text{ in-lb} \\ M_{b,\text{pos,tot}} & \qquad\qquad\qquad\qquad\qquad\qquad\quad = 47{,}600 \text{ in-lb} \end{aligned}$$

Negative moments at discontinuous edges ($\frac{1}{3}$ × positive moments)

$$M_{a,\text{neg}} = \tfrac{1}{3}(73{,}400) = 24{,}500 \text{ in-lb}$$

$$M_{b,\text{neg}} = \tfrac{1}{3}(47{,}600) = 15{,}900 \text{ in-lb}$$

The required reinforcement in the *middle strips* will be selected with the help of Graph A.1 of App. A.

Short direction

1. Midspan

$$\frac{M_u}{\phi b d^2} = \frac{73{,}400}{0.90 \times 12 \times 5^2} = 272 \qquad \rho = 0.0048$$

   $A_s = 0.0048 \times 12 \times 5 = 0.288$ in²/ft. From Table A.4 of App. A, No. 4 bars at 7 in. spacing are selected, giving $A_s = 0.34$ in²/ft.

2. Continuous edge

$$\frac{M_u}{\phi b d^2} = \frac{115{,}000}{0.90 \times 12 \times 5^2} = 426 \qquad \rho = 0.0078\dagger$$

   $A_s = 0.0078 \times 12 \times 5 = 0.468$ in²/ft. If two of every three positive bars are bent up, and likewise for the adjacent panel, the negative-moment steel area furnished at the continuous edge will be $\frac{4}{3}$ times the positive-moment steel in the span, or $A_s = \frac{4}{3} \times 0.34 = 0.453$ in²/ft. It is seen that this is 3 percent less than the required amount of 0.468. On the other hand, the positive-moment steel furnished, 0.34 in²/ft, represents about 15 percent more than the required amount. As discussed in Sec. 11.9*e*, the ACI Code permits a certain amount of inelastic redistribution, within strictly specified limits. In the case at hand, the negative steel furnished suffices for only 97 percent of the calculated moment, but the positive steel permits about 115 percent of the calculated moment to be resisted. This more than satisfies the conditions for inelastic moment redistribution set by the ACI Code. This situation illustrates how such moment redistribution can be utilized to obtain a simpler and more economical distribution of steel.

3. Discontinuous edge. The negative moment at the discontinuous edge is one-third the positive moment in the span; it would be adequate to bend up every third bar from the bottom to provide negative-moment steel at the discontinuous edge. However, this would result in a 21 in. spacing, which is larger than the maximum spacing of $2h = 12$ in. permitted by the ACI Code. Hence, for the discontinuous edge, two of every three bars will be bent up from the bottom steel.

Long direction

1. Midspan

$$\frac{M_u}{\phi b d^2} = \frac{47{,}600}{0.90 \times 12 \times 4.5^2} = 218 \qquad \rho = 0.0038$$

---

†Note that this value of ρ, which is the maximum required anywhere in the slab, is about half the permitted maximum value of $0.75\rho_b = 0.0160$, indicating that a thinner slab might be used. However, use of the minimum possible thickness would require an increase in the tensile steel area and would be less economical for this reason. In addition, a thinner slab may produce undesirably large deflections. The trial depth of 6 in. will be retained for the final design.

(The positive-moment steel in the long direction is placed on top of that for the short direction. This is the reason for using $d = 4.5$ in. for the positive-moment steel in the long direction and $d = 5$ in. in all other locations.) $A_s = 0.0038 \times 12 \times 4.5 = 0.205$ in^2/ft. From Table A.4 of App. A, No. 3 bars at 6 in. spacing are selected, giving $A_s = 0.22$ in^2/ft.

2. Continuous edge

$$\frac{M_u}{\phi b d^2} = \frac{73{,}400}{0.90 \times 12 \times 5^2} = 272 \qquad \rho = 0.0048$$

$A_s = 0.0048 \times 12 \times 5 = 0.288$ in^2/ft. Again bending up two of every three bottom bars from both panels adjacent to the continuous edge, one has, at that edge, $A_s = \frac{4}{3} \times 0.22 = 0.29$ in^2/ft.

3. Discontinuous edge. For the reasons discussed in connection with the short direction, two out of every three bottom bars will, likewise, be bent up at this edge.

The preceding steel selections refer to the *middle strips* in both directions. For the *column strips*, the moments are assumed to decrease linearly from the full calculated value at the inner edge of the column strip to one-third of this value at the edge of the supporting beam. To simplify steel placement, a uniform spacing will be used in the column strips. The average moments in the column strips being two-thirds of the corresponding moments in the middle strips, adequate column steel will be furnished if the spacing of this steel is $\frac{3}{2}$ times that in the middle strip. Maximum spacing limitations should be checked.

Bend points for rebars will be located as suggested in Fig. 5.15, i.e., $l/4$ from should be checked.

Bend points for rebars will be located as suggested in Fig. 5.15, i.e., $l/4$ from the face of the supporting beam at the continuous ends and $l/7$ from the beam face at the discontinuous ends. The corresponding distances from the beam face to bend point are 5 ft 0 in. and 2 ft 10 in. for the short-direction positive bars, and 6 ft 3 in. and 3 ft 7 in. for the long-direction positive bars, at the continuous end and discontinuous end, respectively. Negative bars carried over from the adjacent panels will be cut off at $l/3$ from the support face, at 6 ft 8 in. for the short-direction negative bars and 8 ft 4 in. for the long-direction negative bars. At the exterior edges, negative bars will be extended as far as possible into the supporting beams, then bent downward in a 90° hook to provide anchorage.

At the exterior corner of the panel, No. 4 bars at 8 in. spacing will be used, parallel to the slab diagonal at the top, and perpendicular to the diagonal at the bottom, according to Fig. 12.9. This will provide an area of 0.29 in^2/ft each way, equal to that required for the maximum positive bending moment in the panel. This reinforcement will be carried to a point $25/5 = 5$ ft from the corner, with lengths varying as indicated in Fig. 12.9.

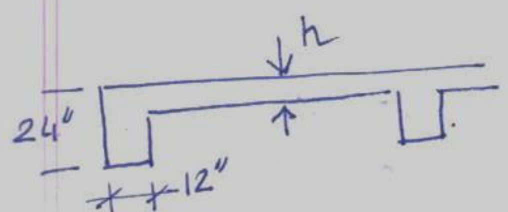
The reactions of the slab are calculated from Table 12.6, which indicates that 71 percent of the load is transmitted in the short direction and 29 percent in the long direction. The total load on the panel being $20 \times 25 \times 338 = 169{,}000$ lb, the load per foot on the long beam is $(0.71 \times 169{,}000)/(2 \times 25) = 2400$ lb/ft and on the short beam is $(0.29 \times 169{,}000)/(2 \times 20) = 1220$ lb/ft. The shear to be transmitted by the slab to these beams is numerically equal to these beam loads, reduced to a critical section a distance d from the beam face. The shear strength of the slab is

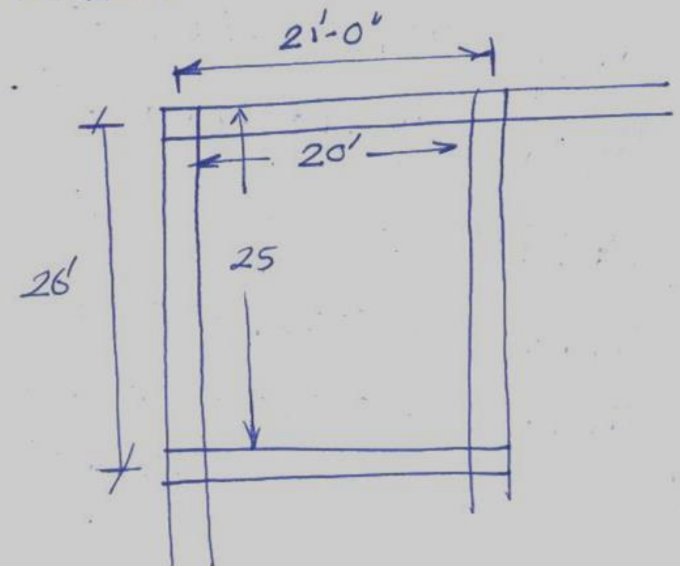
$$\phi V_c = 0.85 \times 2\sqrt{3000} \times 12 \times 5 = 5590 \text{ lb}$$

well above the required shear strength at factored loads.

Edge supported slab · Problem

A monolithic RC floor is to be composed of rectangular bays measuring 21 × 26 ft, as shown in fig. Beams of width 12 in. and depth 24 in. are provided on all column lines; thus the clear span dimensions for the two-way slab panels are 20×25 ft. The floor is to be designed to carry LL = 60 psf, FF = 25 psf in addition to radom PW = 40 psf and its own weight, using $f_c' = 3$ ksi and $f_y = 60$ ksi. Find the reqd slab thickness and reinforcement for the slab system shown.





Old method

$$h = \frac{\text{perimeter}}{180}$$

$$= \frac{2(20+25)\times 12}{180} \doteq 6 \text{ in.}$$

New formula $\alpha_m > 2.0$

$$h = \frac{l_n(0.8 + f_y/200{,}000)}{36 + 9\beta}$$

$$= \frac{25\times 12\left(0.8 + \frac{60{,}000}{200{,}000}\right)}{36 + 9\frac{25}{20}} = 6.98 \text{ in} \simeq 7 \text{ in.}$$

$\text{self wt} = \frac{7}{12} \times 150 = 87.5 \text{ psf.}$

$DL = 1.2(87.5 + 40 + 25) = 1.2 \times 152.5 = 183 \text{ psf}$

$LL = 1.6 * 60 \qquad\qquad\qquad\qquad = 96$

$w_u = 279 \text{ psf.}$

$m = \frac{l_a}{l_b} = \frac{20}{25} = 0.8$ **Case 4**

Moments

Negative moments at cont edge

$M_{a,neg} = 0.071 * 279 * 20^2 = 7923.6 \text{ lb-ft.}$

$M_{b,neg} = 0.029 * 279 * 25^2 = 5057 \text{ lb-ft.}$

Positive moments

Positive moments

$M_{a,pos,dL} = 0.039 * 183 * 20^2 = 2854.8$

$M_{a,pos,LL} = 0.048 * 96 * 20^2 = 1843.2$

$M_{a,pos,tot} = 4698$ lb-ft.

$M_{b,pos,dL} = 0.016 * 183 * 25^2 = 1830$ lb-ft

$M_{b,pos,LL} = 0.020 * 96 * 25^2 = 1200$

$M_{b,pos,tot} = 3030$ lb-ft

Negative moment at discontinuous end

$= \frac{1}{3}$ positive mom

$M_{a,neg} = \frac{1}{3} \times 4698 = 1566$ lb-ft.

$M_{b,neg} = \frac{1}{3} \times 3030 = 1010$ "

d check $\varepsilon_t = 0.005$

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000}{1000} \qquad 0.65 \leq \beta_1 \leq 0.85$$

$$\Rightarrow \rho = 0.85 * 0.85 * \frac{3}{60} \frac{.003}{.003 + .005}$$

$$= 0.0135$$

$$M_u = \phi M_n = \phi \times \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

$$\Rightarrow 7923.6 * 12 = 0.9 * 0.0135 * 60{,}000 * 12 * d^2 \left(1 - 0.59 * .0135 * \frac{60}{3}\right)$$

$$d = 3.59 \text{ in} \qquad d_{prov} = 7 - 1 = 6 \text{ in.}$$

$\underline{A_s}$

$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$

$\Rightarrow \quad 79236 * 12 = 0.9 * A_s * 60{,}000 \left(6 - \frac{1}{2}\right)$

$A_s = 0.32 \text{ in}^2/\text{ft}.$

$a = \frac{A_s f_y}{0.85 f_c' b} = \quad 0.628$

$A_s = 0.31$

$a = 0.61$

$A_s = \underline{0.31 \text{ in}^2/\text{ft}}$

$a = 0.61$

$c = \frac{0.61}{0.85} = 0.7176$

$\varepsilon_t = \varepsilon_u \frac{d-c}{c}$

$= 0.02.$

7924 #'/' $\Rightarrow$ 0.31 in^2

4698 lb-ft/ft

$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$

$\Rightarrow 4698 * 12 = 0.9 * A_s * 60{,}000 \left(6 - \frac{1}{2}\right)$

$A_s = 0.1898$

$a = \frac{A_s f_y}{0.85 f'_c b} = 0.372$

$A_s = 0.1796$

$a = 0.352$

$A_s = \underline{0.179}$ in^2/ft.

$* 60{,}000 \left(6 - \frac{1}{2}\right.$

A_s = 0.1...

1566

$1566 * 12 = 0.9 * A_s * 60{,}000 (6 - \frac{1}{2}$

$A_s = 0.063$ ✓

$a = 0.124$ ✓

$A_s = 0.059$

$a = 0.1149$

$A_s = \underline{0.059}$

Long span

5057 lb-ft/ft $d = 6$ in.

$5057 \times 12 = 0.9 * A_s * 60{,}000 \left(6 - \frac{1}{2}\right)$

$A_s = 0.204$

$a = 0.4$

$A_s = 0.194$

$a = 0.38$

$A_s = \underline{0.194}$

3030 $d = 6 - \frac{1}{2} = 5.5$ in. ⟸

$3030 \times 12 = 0.9 * A_s * 60{,}000 \left(5.5 - \frac{1}{2}\right)$

$A_s = 0.135$

$a = 0.264$

$A_s = 0.125$

$a = 0.246$

$A_s = \underline{0.125}$ in²/ft.

$\left(6 - \frac{1}{2}\right)$

<u>1010</u>

$$1010 \times 12 = 0.9 * A_s * 60,000 \left(6 - \frac{1}{2}\right)$$

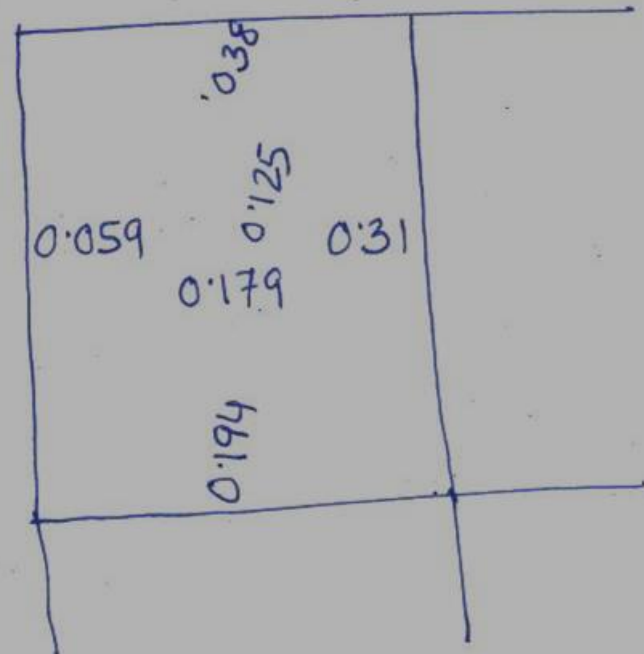
$$A_s = 0.0408$$

$$a = 0.080$$

$$A_s = 0.038$$

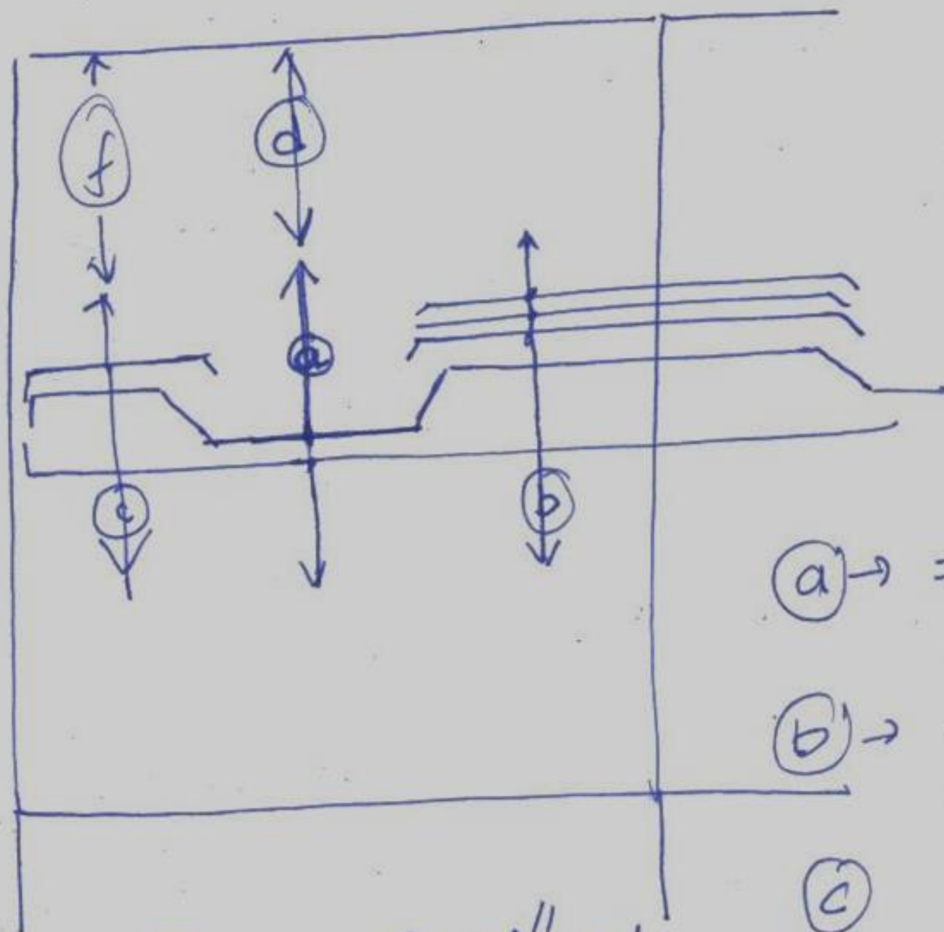
$$a = 0.074$$

$$A_s = \underline{0.0376} \text{ in}^2/\text{ft}.$$



Min $A_s = 0.0018\,bh = 0.0018 \times 12 \times 7 = 0.1512$ in^2/ft.

spacing, $s = \frac{A_{bar}}{A_{req}} \times 12$



(a) → #3 @ 7" c/c alt ckd.

(b) → 3 #3 Extra betⁿ alt ckd.

(c) 1 # 3 Extra bet alt. ckd.

0.179 ⇒ #3 @ 7" c/c

0·31

12 in reqd ⇒ 0·31

⇒ 14 in reqd ⇒ $\frac{0.31}{12} \times 14 =$ 0·362 in²/14 in

Extra reqd = 0·362 − 0·11 = 0·251

\# 3 reqd = $\frac{0.251}{0.11}$ = 3.

0·059 Alt ckd ⇒ # 3 @ 14" c/c (2h is ok)

temp & shrinkage = 0·1512

0·1512 × $\frac{14}{12}$ − 0·11 = 0·0664

1 # 3 Extra.

Col strip

temp

(a) ⇒ (d) 7" × 1.5 = 10.5 in c/c. × 8.5" c/c.

(2)

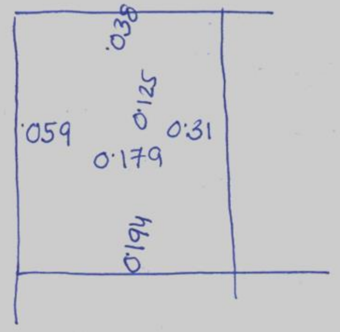
(b) ⇒ (e) ⇒ 3 # 3 Extra top. (e) 2 Extra top

(c) ⇒ (f) = alt ckd. ⇒ 21"

temp & shr = $0.1512 + \frac{21}{12} - 0.11 = 0.1546$

$\div 11$

= 2 # 3.



(b) ⇒ $A = \frac{A_{bar}}{s} \times 12 = 0.18857$

$0.31 - .18857/2 = .215$

$s = \frac{.11}{.215} \times 12 = 6.1$

$\simeq$ 6" c/c

(c) $A_{s\,reqd} = 0.1512 - \frac{.18857}{2} = .0561$

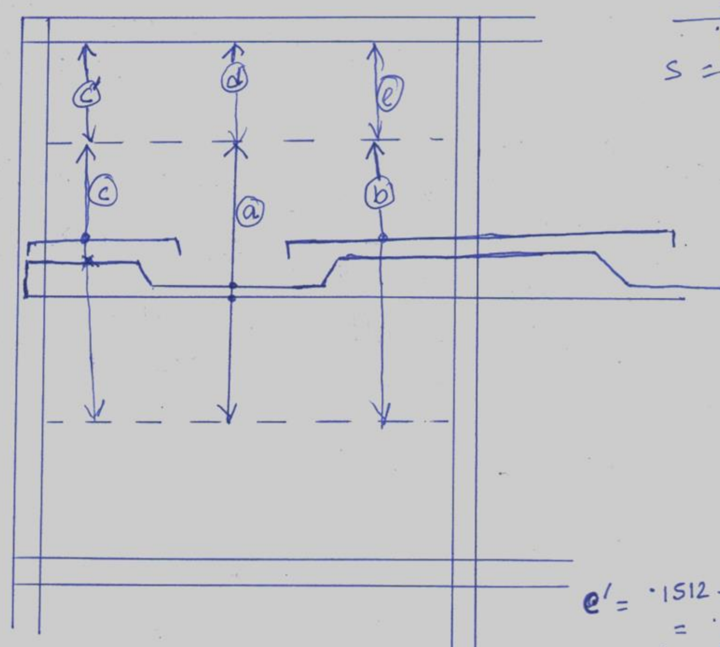
$s = 23"$ c/c

(e) ⇒ $0.31 \times \frac{2}{3} = 0.207$

$- \frac{.11}{21} \times 12$

$.1438$

$s = 9"$



$e' = .1512 - \frac{.11}{17} \times 12$

$= .088$

$s = 17"$ c/c.

8.4

a. #3 @ 7" c/c alt ckd

b. #3 @ 6" c/c extra top.

c. #3 @ 23" c/c extra top.

d. #3 @ 10.5" c/c alt ckd. ← #3 @ 8.5" c/c

e. #3 @ 9" c/c extra top ← #3 @ 10" c/c

(b) → $A = \frac{A_{bar}}{s} \times 12 = 0.18857$

$0.31 - .18857/2 = .215$

$s = \frac{.11}{.2157} \times 12 = 6.1$

$\simeq 6''$ c/c

(c) $A_{s\,reqd} = 0.1512 - \frac{.18857}{2} = .0561$

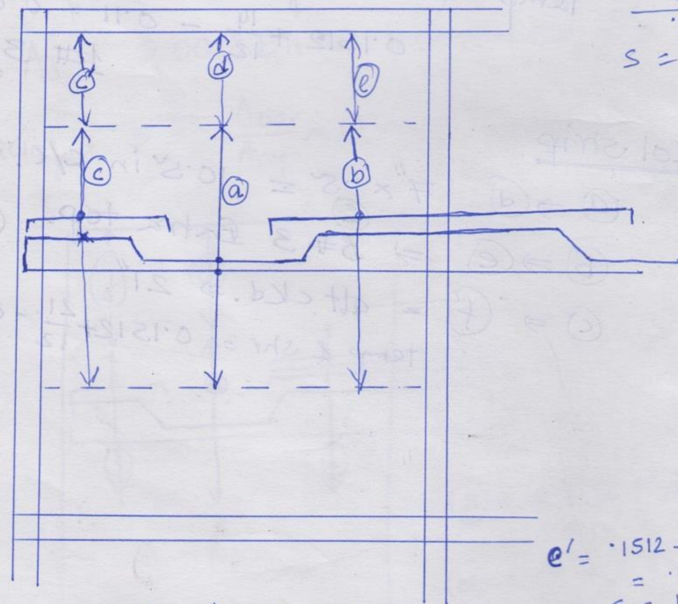
$s = 23''$ c/c

(e) → $0.31 \times \frac{2}{3} = 0.207$

$- \frac{.11}{21} \times 12$

$.1438$

$s = 9''$



$e' = .1512 - \frac{.11}{17} \times 12$

$= .088$

$s = 17''$ c/c.

8.4

(a) a. #3 @ 7" c/c alt ckd

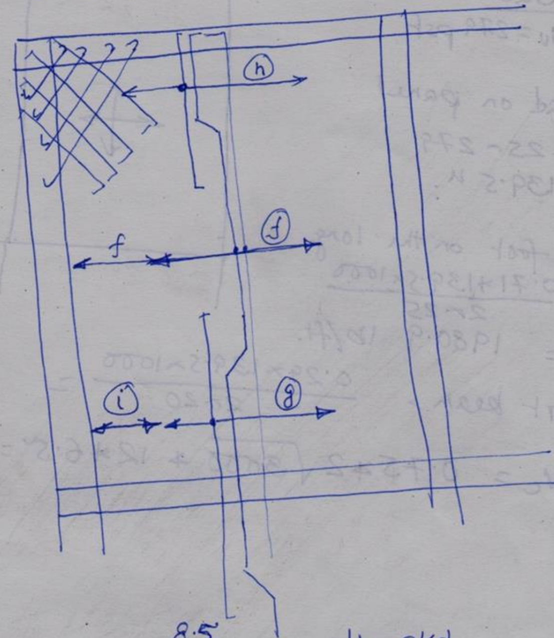
b. #3 @ 6" c/c extra top.

c. #3 @ 23" c/c extra top.

d. #3 @ 10.5" c/c alt ckd. ← #3 @ 8.5" c/c

e. #3 @ 9" c/c extra top ← #3 @ 10" c/c

Corner Reinf. #3@7" c/c both top & bottom



$.194 - \frac{.11}{17} \times 12 = 0.116$

$s = 11"$

f. #3 @ ~~9~~ 8.5" c/c alt. ckd.

$s = 6.6$

g. #3 @ 11" c/c ~~alt. ckd~~ extra top

$0.1512 - \frac{.11}{17} \times 12 = .07$

$s = 17"$ c/c

h. #3 @ 17" c/c extra top

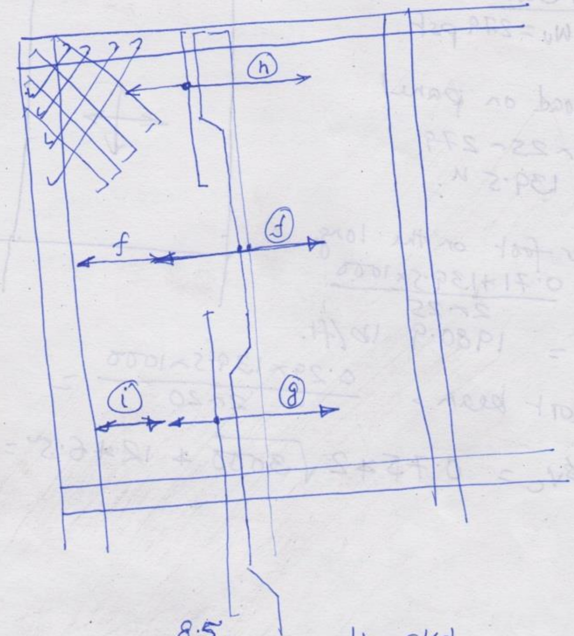
i. #3 @ 17" c/c " "

g. অনুসারে ⇒ 0.194 in²/ft ⇒ $17" = \frac{.194}{12} \times 17 = 0.275$ in²/17"

Provided :, 0.11 Reqr = 0.1648

2 - #3 Extra betn alt. ckd.

Corner Reinf. #3@7" c/c both top & bottom



$\cdot 194 - \frac{\cdot 11}{17} \times 12 = 0\cdot 116$

s = 11"

f. # 3 @ ~~8~~ 8.5" c/c alt. ckd.

\# 3 @ 11" c/c ~~alt.~~ ~~chair~~ extra top

g. # 3 @ 17" c/c extra top

h. # 3 @ 17" c/c " "

i. # 3 @ 17" c/c

s = 6.6

$0\cdot 1512 - \frac{\cdot 11}{17} \times 12 = \cdot 07$

s = 17" c/c

g. প্রয়োজন ⇒ 0.194 in²/ft ⇒ $17" = \frac{\cdot 194}{12} \times 17 = 0\cdot 275$ in²/ 17"

Provided = 0.11 Rest = 0.1648

2 - # 3 Extra betn alt. ckd.

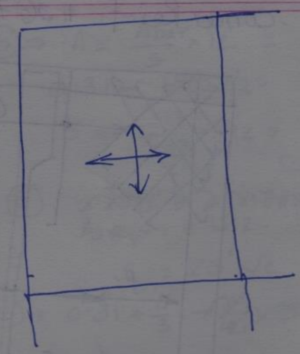
Shear Check

$w_u = 279$ psf.

Total load on panel

$= 20 \times 25 \times 279$

$= 139.5$ k.



Load per foot on the long beam $= \frac{0.71 \times 139.5 \times 1000}{2 \times 25}$

$= 1980.9$ lb/ft.

On short beam $= \frac{0.29 \times 139.5 \times 1000}{2 \times 20} = 1011.3$ lb/ft.

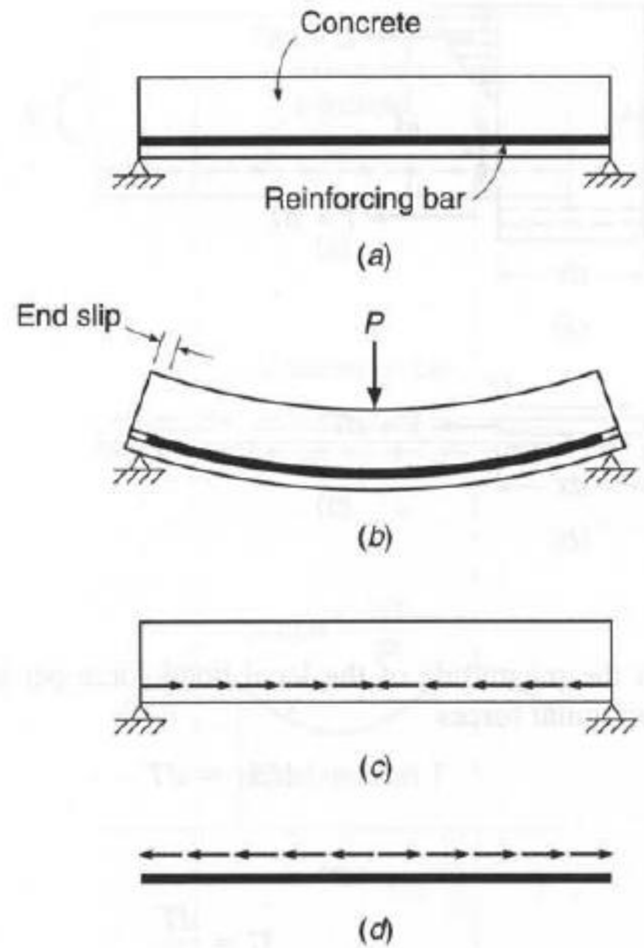
$\phi V_c = 0.75 \times 2 \sqrt{3000} \times 12 \times 6.5 = 6408$ lb/ft.

Bond, Anchorage and Development Length

Chapter 5

Fundamentals of flexural bond

FIGURE 5.1
Bond forces due to flexure: (*a*) beam before loading; (*b*) unrestrained slip between concrete and steel; (*c*) bond forces acting on concrete; (*d*) bond forces acting on steel.

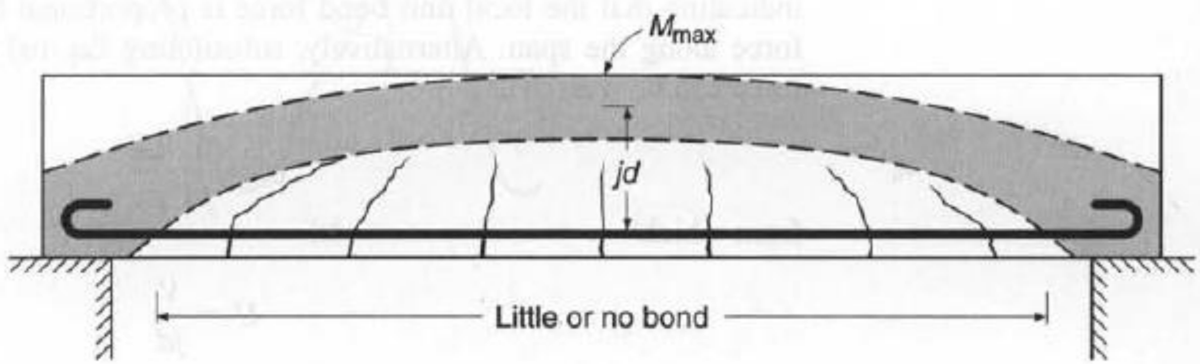


In a RC beam, if plain bars are used and greased, it will behave like figure.

Strain compatibility will not be valid

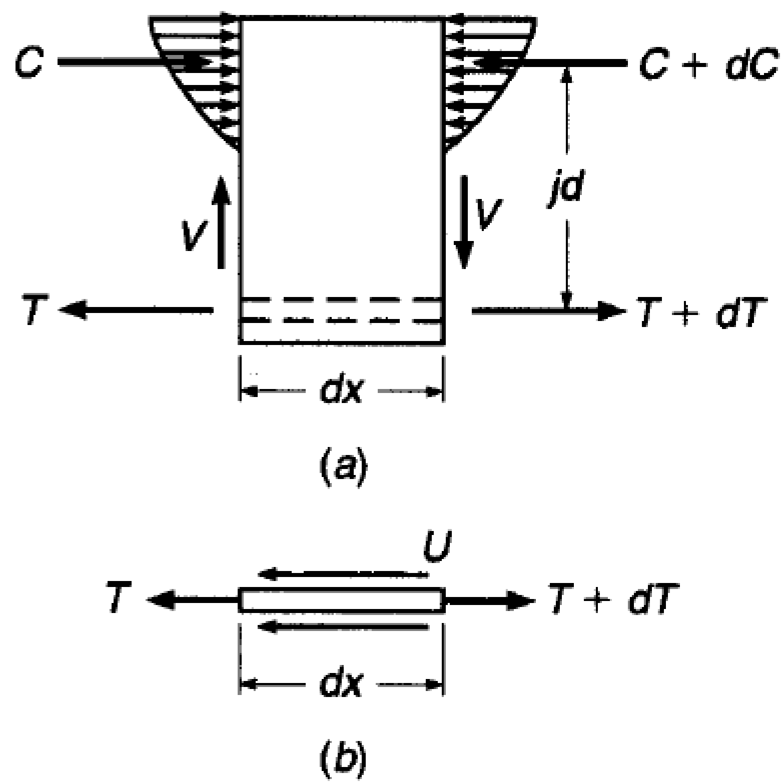
To reinforced concrete to behave as intended, sufficient bond stress must develop at interface of concrete and steel

FIGURE 5.2
Tied-arch action in a beam with little or no bond.



- Plain bars- bond provided by chemical adhesion and mechanical friction between concrete and steel.
- This bond is rather small and easily slipped.
- End anchorage are provided with hooks.
- Steel stress is almost constant along length, elongation and deflection, cracking are higher.
- Deformed bars overcome this problem

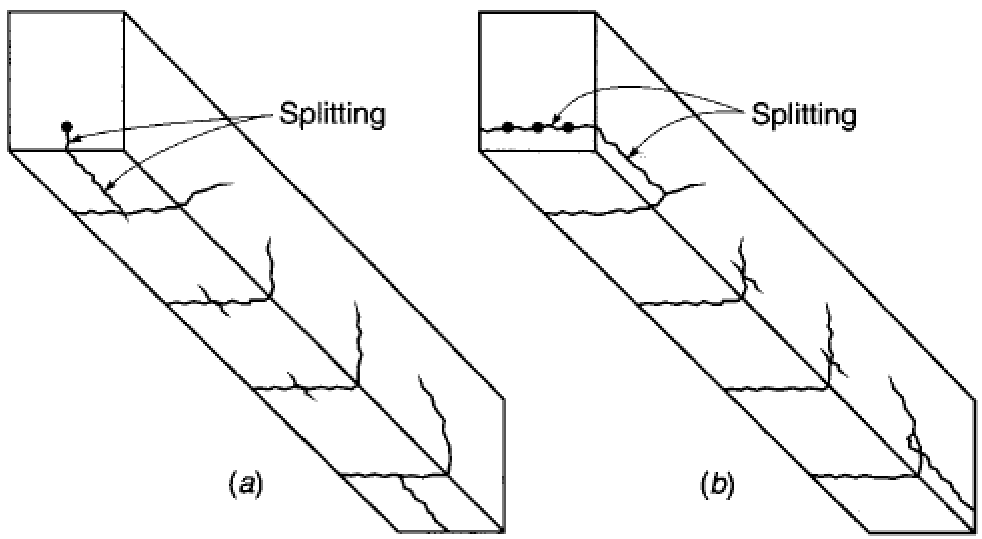
FIGURE 5.3
Forces acting on elemental length of beam: (*a*) free-body sketch of reinforced concrete element; (*b*) free-body sketch of steel element.



BOND STRENGTH AND DEVELOPMENT LENGTH

Bond Strength

FIGURE 5.6
Splitting of concrete along reinforcement.



For reinforcing bars in Tension, two types of bond failure occur:

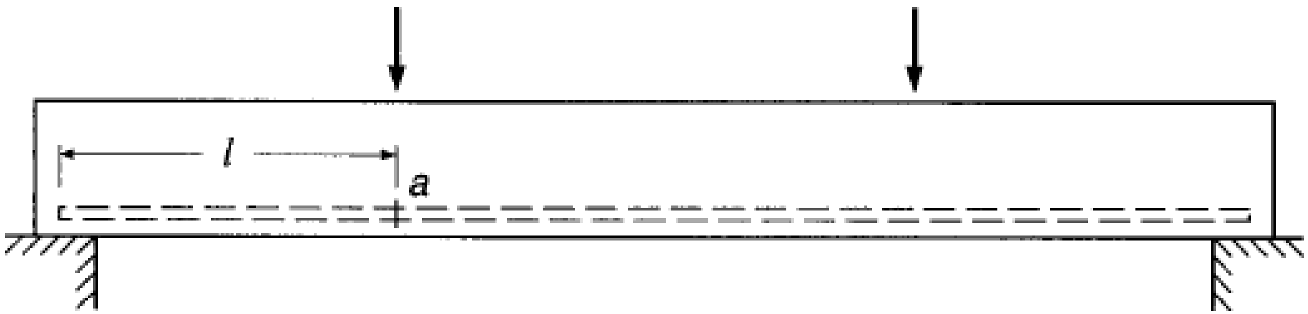
- Direct pull-out- where ample confinement is provided
- Splitting of concrete-where cover, confinement, bar spacing is insufficient

- Both types of failures need to be considered.

- When there is sufficient cover, as the tensile force in reinforcement increase, the adhesive bond and friction are overcome, concrete eventually crushes in front of the bar deformations, pull-out occurs.
- Bond failure from splitting is more common- either in vertical or horizontal plain. Horizontal failure frequently occurs at diagonal crack.
- Shear and bond failures are intricately related.

Development Length

FIGURE 5.7
Development length.



Development length is defined as the length of embedment necessary to develop full tensile strength of bar, controlled either by pull-out or splitting.

Factors Influencing Development Length

- Tensile Strength of Concrete
- Cover distance
- Bar Spacing
- Transverse reinforcement
- Vertical location of bar
- Epoxy coating
- Excess reinforcement
- Diameter of bar

ACI Code Provisions For Development of Tension Reinforcement

a. Basic Equation for Development of Tension Bars

According to ACI Code 12.2.3, for deformed bars or deformed wires,

$$l_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\left[\dfrac{(c_b + K_{tr})}{d_b} \right]} \right) d_b \tag{5.4}$$

in which the term $(c + K_{tr})/d_b$ shall not be taken greater than 2.5. In Eq. (5.4), terms are defined and values established as follows.

ψ_t = reinforcement location factor

Horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice:	1.3
Other situations:	1.0

ψ_e = coating factor

Epoxy-coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$: 1.5

All other epoxy-coated bars or wires: 1.2

Uncoated and zinc-coated (galvanized) reinforcement: 1.0

However, the product of $\psi_t\psi_e$ need not be taken greater than 1.7.

ψ_s = reinforcement size factor

No. 6 (No. 19) and smaller bars and deformed wires: 0.8†

No. 7 (No. 22) and larger bars: 1.0

λ = lightweight aggregate concrete factor

When lightweight aggregate concrete is used: 0.75

However, when f_{ct} is specified, $\lambda = f_{ct}/(6.7\sqrt{f'_c}) \leq 1.0$.

When normalweight concrete is used: 1.0

c = spacing or cover dimension, in.

Use the smaller of either the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of the bars being developed.

K_{tr} = transverse reinforcement index: $40A_{tr}/sn$

where A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and that crosses the potential plane of splitting through the reinforcement being developed, in^2

s = maximum spacing of transverse reinforcement within l_d center to center, in.

n = number of bars or wires being developed along the plane of splitting

Simplified Equations for Development Length

TABLE 5.1
Simplified tension development length in bar diameters according to the ACI Code

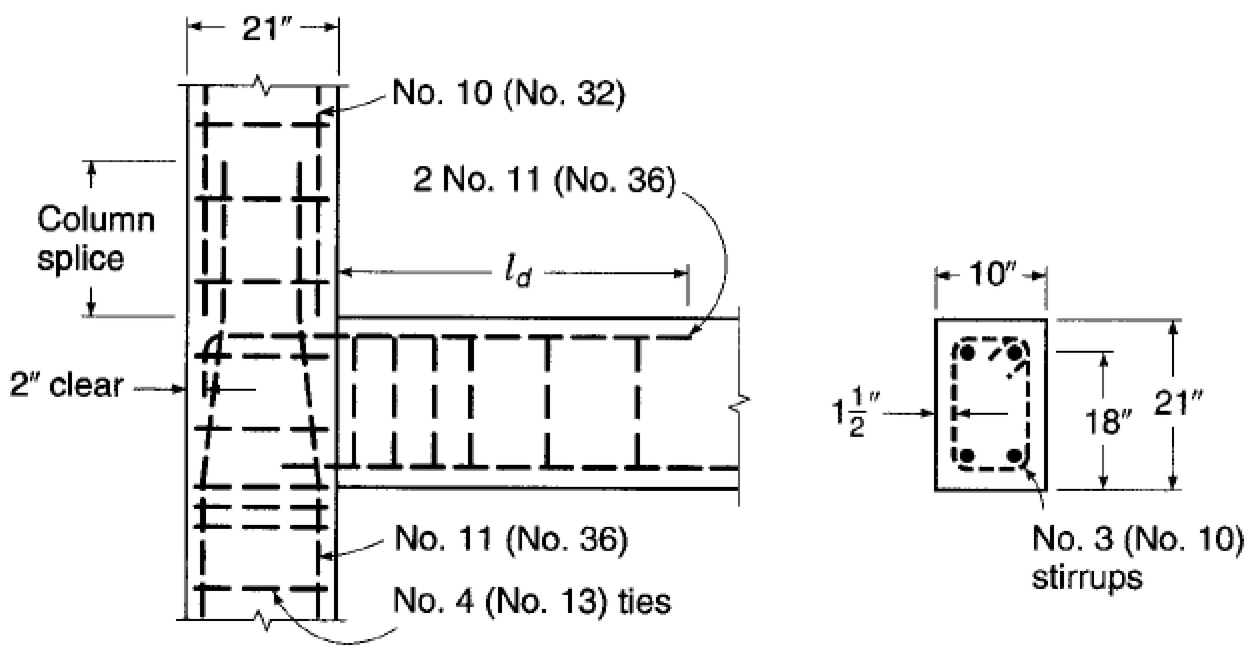
	No. 6 (No. 19) and Smaller Bars and Deformed Wires†	No. 7 (No. 22) and Larger Bars
Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout l_d not less than the Code minimum	$l_d = \left(\frac{f_y \psi_t \psi_e}{25\lambda\sqrt{f'_c}}\right) d_b$	$l_d = \left(\frac{f_y \psi_t \psi_e}{20\lambda\sqrt{f'_c}}\right) d_b$
Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover $\geq d_b$	Same as above	Same as above
Other cases	$l_d = \left(\frac{3f_y \psi_t \psi_e}{50\lambda\sqrt{f'_c}}\right) d_b$	$l_d = \left(\frac{3f_y \psi_t \psi_e}{40\lambda\sqrt{f'_c}}\right) d_b$

† For reasons discussed in Section 5.3a, ACI Committee 408 recommends that l_d for No. 7 (No. 22) and larger bars be used for all bar sizes.

Appendix A.10

Example 5.1

Development length in tension. Figure 5.8 shows a beam-column joint in a continuous building frame. Based on frame analysis, the negative steel required at the end of the beam is 2.90 in^2; two No. 11 (No. 36) bars are used, providing A_s = 3.12 in^2. Beam dimensions are b = 10 in., d = 18 in., and h = 21 in. The design will include No. 3 (No. 10) stirrups spaced four at 3 in., followed by a constant 5 in. spacing in the region of the support, with 1.5 in. clear cover. Normalweight concrete is to be used, with f'_c = 4000 psi, and reinforcing bars have f_y = 60,000 psi. Find the minimum distance l_d at which the negative bars can be cut off, based on development of the required steel area at the face of the column, (a) using the simplified equations of Table 5.1, (b) using Table A.10, of Appendix A, and (c) using the basic Eq. (5.4).



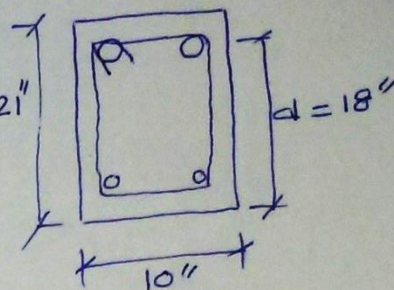
(a) (b)

Example 5.1 $A_{s,reqd} = 2.9\ in^2$ $A_{s,prov} = 3.12\ in^2$

$b = 10''$, $d = 18''$, $h = 21''$ stirrup ⇒ #3 @ 3″ 4 Nos + @ 5″ c/c

cover = 1.5″ $f_c' = 4\,ksi$, $f_y = 60\,ksi$

Find l_d → negative bar cutoff



(a) Simplified using Table 5.1

$$\text{Clear distance} = 10 - 2(1.5 + 0.375 + 1.41)$$
$$= 3.43'' = 2.43\,d_b$$

$$\text{Clear cover side} = 1.5 + 0.375 = 1.88'' = 1.33\,d_b$$

$$\text{Clear cover top} = 3.0 - \frac{1.41}{2} = 2.30'' = 1.63\,d_b$$

Table 5.1 ⇒ 2nd row

$\psi_t = 1.3$, $\psi_e = 1.0$ $\lambda = 1.0$

$$l_d = \frac{f_y \psi_t \psi_e}{20 \lambda \sqrt{f_c'}} d_b$$

$$= \frac{60{,}000 * 1.3 * 1}{20 * 1 * \sqrt{4000}} * 1.41$$

$$= 62 * 1.41 = 87 \text{ in.}$$

$$\text{Can be reduced} = 87 * \frac{2.9}{3.12} = 81 \text{ in.}$$

(b) Using Table A.10

$$\frac{l_d}{d_b} = 62 \quad \text{as before}$$

(c) Using the basic Eq. (5.4)

Centre to centre spacing $= 10 - 2\left(1.5 + 0.375 + \frac{1.41}{2}\right)$

$= 4.84''$

Half $= 4.84 \div 2 = 2.42''$

Centre to side $= 1.5 + 0.375 + \frac{1.41}{2} = 2.58''$

centre to top $= 3.0$

$c =$ smallest $= 2.42''$ (Horizontal plane splitting)
↳ A_{tr} = two times A_v

$K_{tr} = \frac{40 A_{tr}}{sn} = \frac{40 * 0.11 * 2}{5 * 2} = 0.88$

if top cover controls, splitting plane vertical ⇒ A_{tr} = One time A_v

$\frac{c + K_{tr}}{d_b} = \frac{2.42 + 0.88}{1.41} = 2.34$ < 2.5 to avoid pull-out
1.5 in approx eqn.

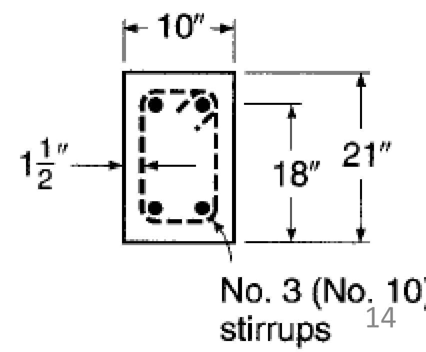
$$l_d = \frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s}{\frac{c_b + K_{tr}}{d_b}} d_b$$

$$= \frac{3}{40} \frac{60{,}000}{1 * \sqrt{4000}} \frac{1.3 * 1 * 1}{2.34} * 1.41$$

$$= 40 * 1.41 = 55.7''$$

Reqd. development length $= 55.7 * \frac{2.90}{3.12}$

$= 52''$ (much smaller than simplified eqn → 81")



5.4 Anchorage Of Tension Bars By Hooks

a. Standard Dimensions

In the event that the desired tensile stress in a bar cannot be developed by bond alone, it is necessary to provide special anchorage at the ends of the bar, usually by means of a 90° or a 180° hook or a headed bar (the latter is discussed in Section 5.5). The dimensions and bend radii for hooks have been standardized in ACI Code 7.1 as follows (see Fig. 5.9):

1. A 180° bend plus an extension of at least 4 bar diameters, but not less than $2\frac{1}{2}$ in. at the free end of the bar, or
2. A 90° bend plus an extension of at least 12 bar diameters at the free end of the bar, or
3. For stirrup and tie anchorage only:
   (*a*) For No. 5 (No. 16) bars and smaller, a 90° bend plus an extension of at least 6 bar diameters at the free end of the bar, or
   (*b*) For Nos. 6, 7, and 8 (Nos. 19, 22, and 25) bars, a 90° bend plus an extension of at least 12 bar diameters at the free end of the bar, or
   (*c*) For No. 8 (No. 25) bars and smaller, a 135° bend plus an extension of at least 6 bar diameters at the free end of the bar.

FIGURE 5.9
Standard bar hooks: (*a*) main reinforcement; (*b*) stirrups and ties.

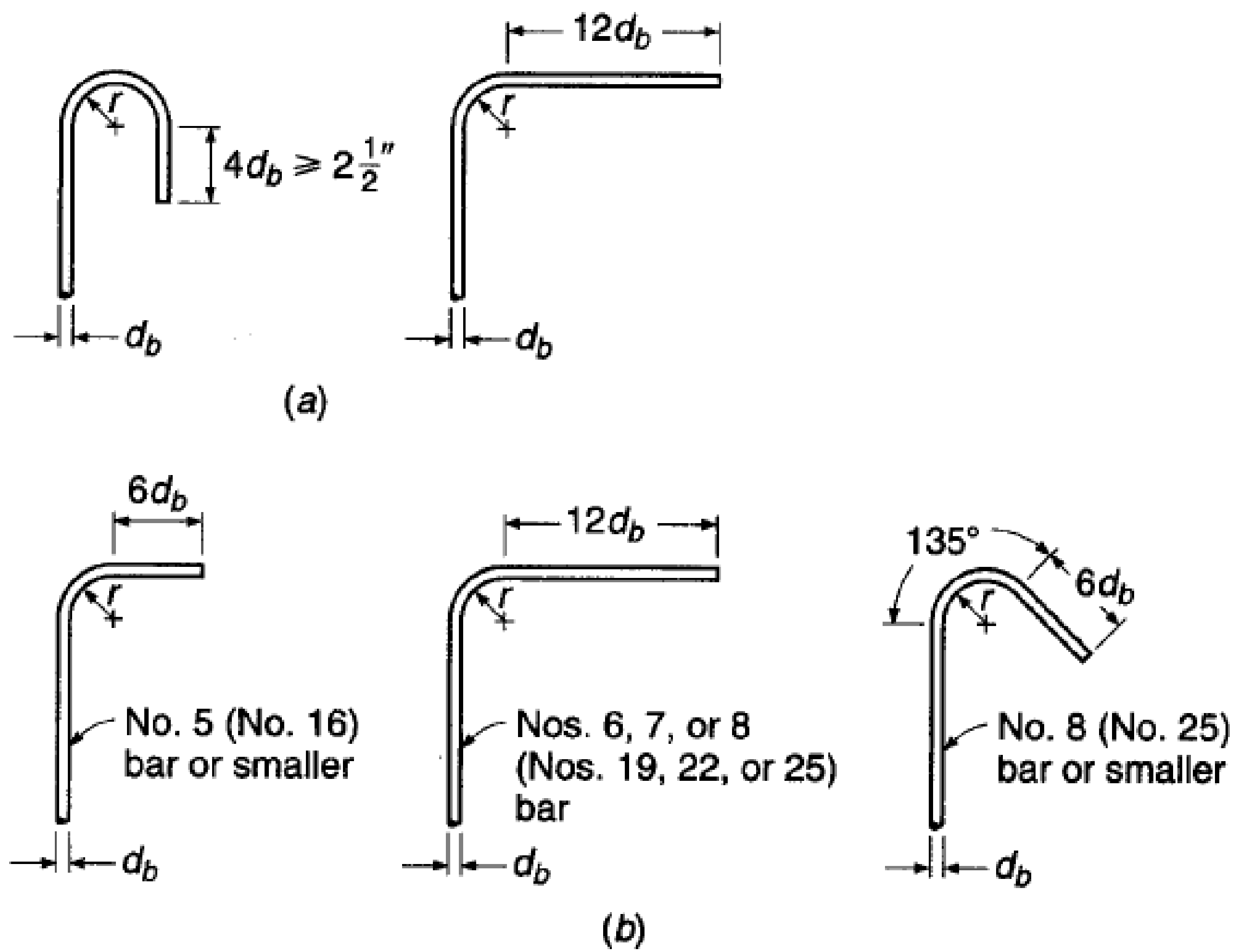


Fig 5.9: Standard bar Hooks: (a) Main reinforcement (b) stirrups and ties

TABLE 5.2
Minimum diameters of bend for standard hooks

Bar Size	Minimum Diameter
Nos. 3 through 8 (Nos. 10 through 25)	6 bar diameters
Nos. 9, 10, and 11 (Nos. 29, 32, and 36)	8 bar diameters
Nos. 14 and 18 (Nos. 43 and 57)	10 bar diameters

- For stirrup and tie hooks, for sizes No. 5 and smaller, the inside diameter of bend should not be less than 4 bar diameters, according to ACI Code
- For stirrups and tie hooks, greater than No. 5, Table 5.2 applies.

Development Length and Modification Factors for Hooked Bars

FIGURE 5.10
Bar details for development of standard hooks.

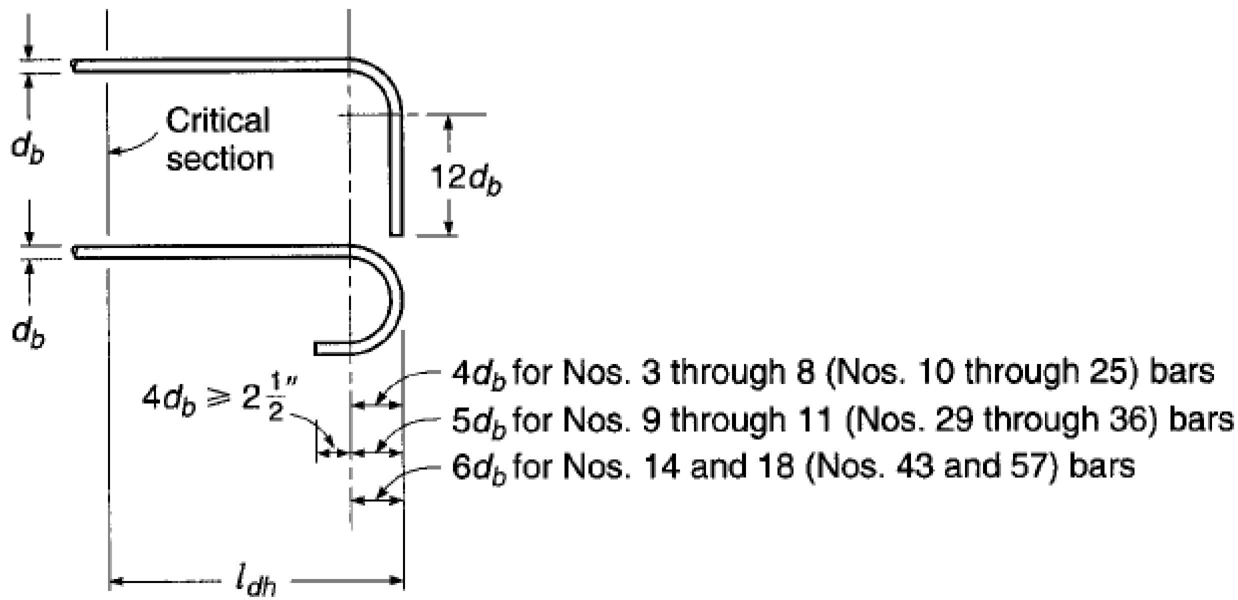


Fig5.10: Bar details for development of standard hooks

$$l_{dh} = \left(\frac{0.02\, \psi_e f_y}{\lambda \sqrt{f'_c}} \right) d_b$$

with $\psi_e = 1.2$ for epoxy-coated reinforcement and $\lambda = 0.75$ for lightweight aggregate concrete. For other cases, ψ_e and λ are taken as 1.0.

- l_{dh} should not be less than 8 bar diameter or 6 in.

TABLE 5.3

Development lengths for hooked deformed bars in tension

A. Development length l_{dh} for hooked bars	$\left(\frac{0.02\psi_e f_y}{\lambda\sqrt{f'_c}}\right) d_b$
B. Modification factors applied to l_{dh}	
For No. 11 (No. 36) and smaller bar hooks with side cover (normal to plane of hook) not less than $2\frac{1}{2}$ in., and for 90° hooks with cover on bar extension beyond hook not less than 2 in.	0.7
For 90° hooks of No. 11 (No. 36) and smaller bars that are either enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along the development length l_{dh} of the hook; or enclosed within ties or stirrups parallel to the bar being developed, spaced not greater than $3d_b$ along the length of the tail extension of the hook plus bend	0.8
For 180° hooks of No. 11 (No. 36) and smaller bars that are enclosed within ties or stirrups perpendicular to the bar being developed, spaced not greater than $3d_b$ along the development length l_{dh} of the hook	0.8
Where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis	$\frac{A_s \text{ required}}{A_s \text{ provided}}$
ψ_e:	
For epoxy-coated bars	1.2
For other bars	1.0
λ:	
For lightweight concrete	0.75
For normalweight concrete	1.0

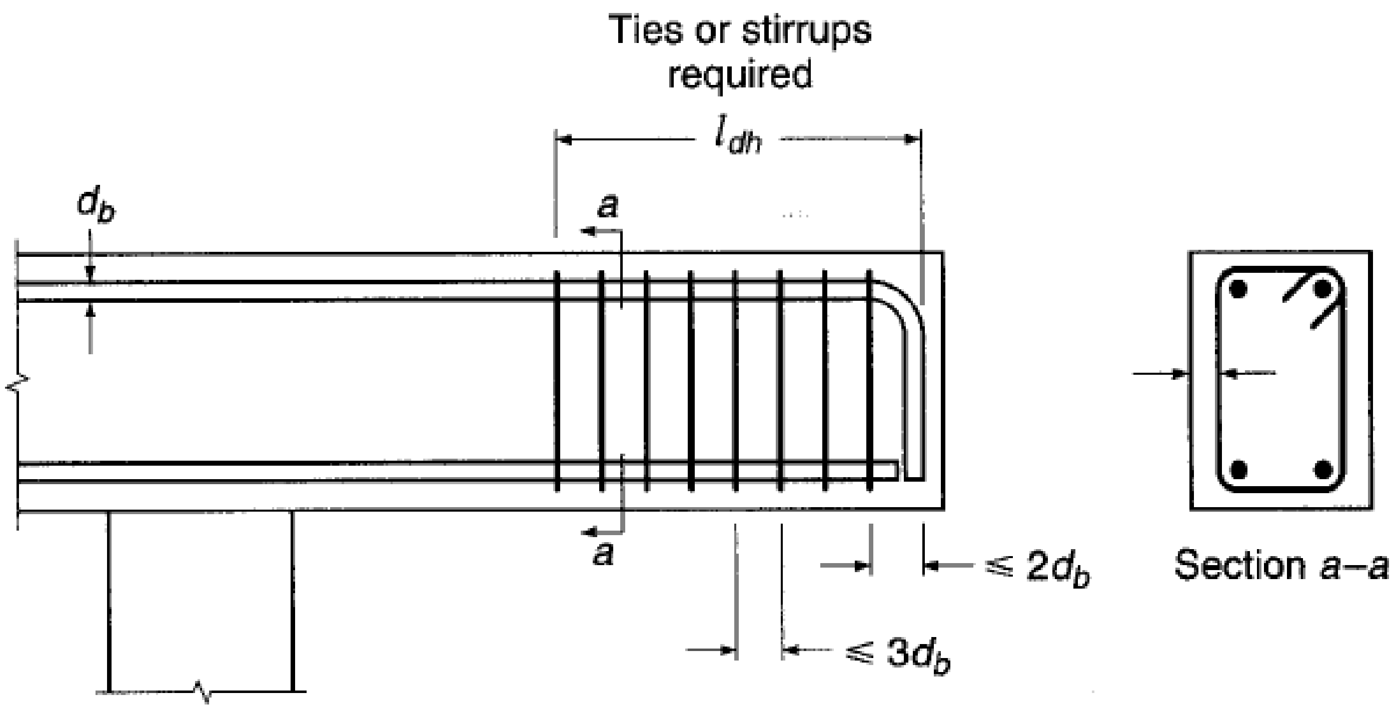


Fig. 5.11: Transverse reinforcement requirements at discontinuous ends of members with small cover distances (less than 2.5in.)

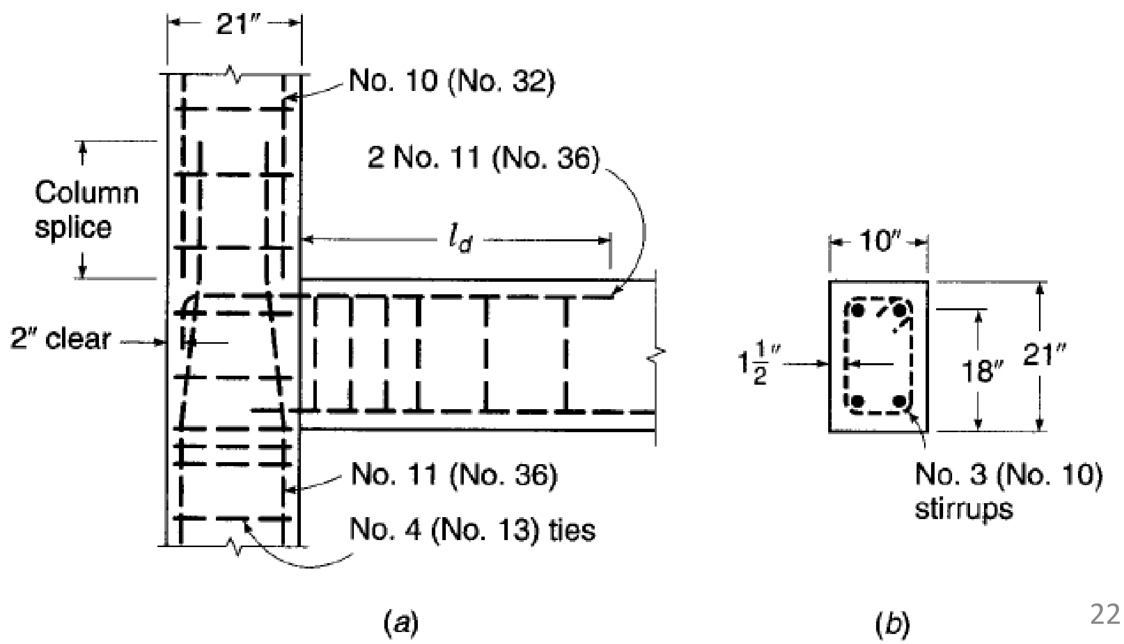
0.8 does not apply

Example 5.2

Development of hooked bars in tension. Referring to the beam-column joint shown in Fig. 5.8, the No. 11 (No. 36) negative bars are to be extended into the column and terminated in a standard 90° hook, keeping 2 in. clear to the outside face of the column. The column width in the direction of beam width is 16 in. Find the minimum length of embedment of the hook past the column face, and specify the hook details.

SOLUTION. The development length for hooked bars, measured from the critical section along the bar to the far side of the vertical hook, is given by Eq. (5.6):

$$l_{dh} = \frac{0.02 \times 1.0 \times 60{,}000}{1.0 \times \sqrt{4000}} 1.41 = 27 \text{ in.}$$



In this case, side cover for the No. 11 (No. 36) bars exceeds 2.5 in. and cover beyond the bent bar is adequate, so a modifying factor of 0.7 can be applied. The only other factor applicable is for excess reinforcement, which is 0.93 as for Example 5.1. Accordingly, the minimum development length for the hooked bars is

$$l_{dh} = 27 \times 0.7 \times 0.93 = 18 \text{ in.}$$

With 21 − 2 = 19 in. available, the required length is contained within the column. The hook will be bent to a minimum diameter of 8 × 1.41 = 11.28 in. The bar will continue for 12 bar diameters, or 17 in. past the end of the bend in the vertical direction.

Headed bar

FIGURE 5.12
Headed deformed reinforcing bar with an obstruction of the deformations that extends less than 2 bar diameters from the bearing face of the head.

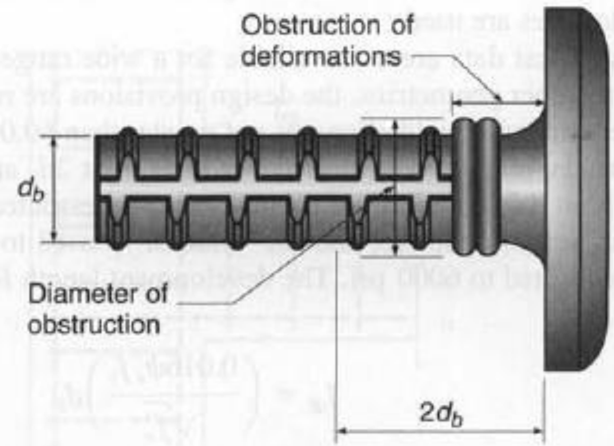
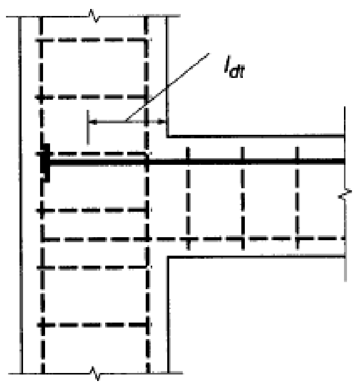


TABLE 5.4
Development lengths for headed deformed bars in tension

A. Development length l_{dt} for headed bars	$\left(\frac{0.016\psi_e f_y}{\sqrt{f'_c}}\right) d_b$
B. Modification factors applied to l_{dt}	
Where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis	$\frac{A_s \text{ required}}{A_s \text{ provided}}$
ψ_e	
For epoxy-coated bars	1.2
For other bars	1.0

FIGURE 5.15
Headed deformed bar extended to far side of column with anchorage length that exceeds l_{dt}.

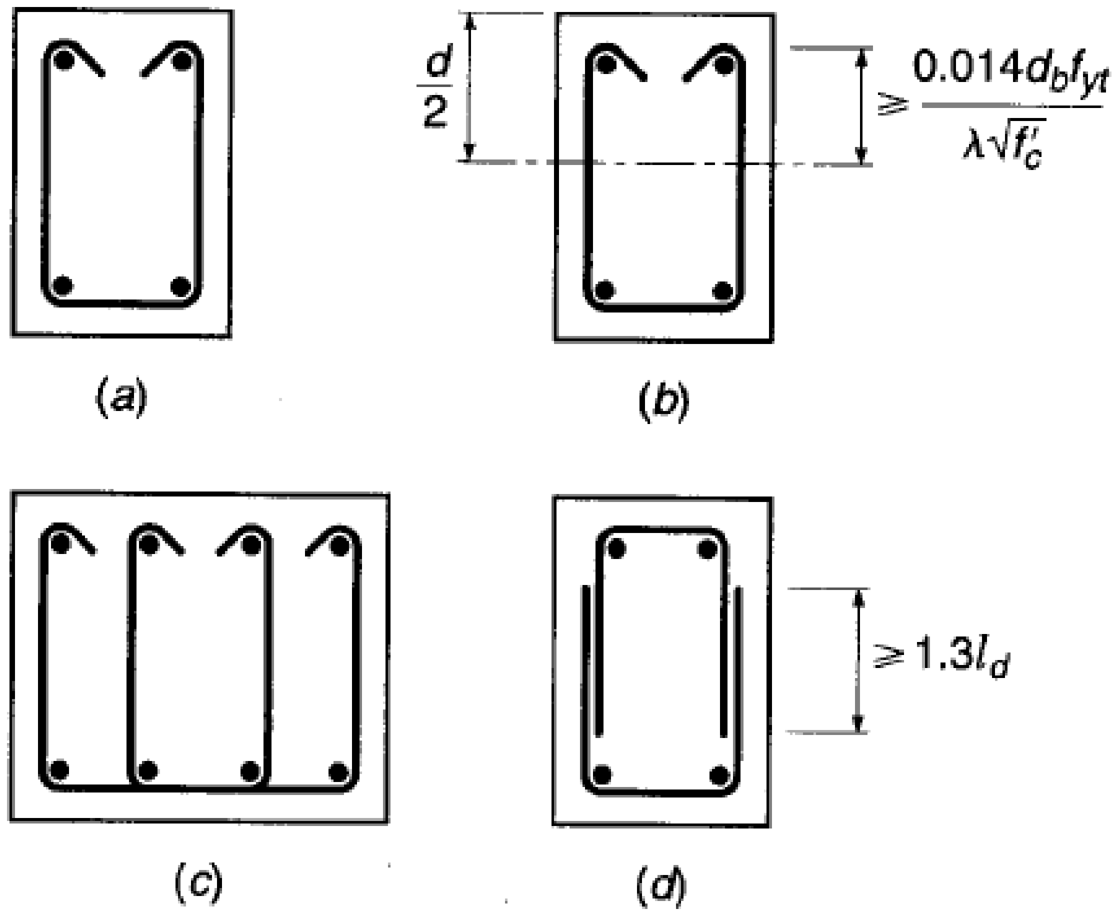


5.6 Anchorage Requirements For Web Reinforcement

ACI Code 12.13 includes special provisions for anchorage of web reinforcement. The ends of single-leg, simple-U, or multiple-U stirrups are to be anchored by one of the following means:

1. For No. 5 (No. 16) bars and smaller, and for Nos. 6, 7, and 8 (Nos. 19, 22, and 25) bars with f_{yt} of 40,000 psi or less, a standard hook around longitudinal reinforcement, as shown in Fig. 5.17*a*.
2. For Nos. 6, 7, and 8 (Nos. 19, 22, and 25) stirrups with f_{yt} greater than 40,000 psi, a standard hook around a longitudinal bar, plus an embedment between midheight of the member and the outside end of the hook equal to or greater than $0.014 d_b f_{yt} / \lambda \sqrt{f'_c}$ in., as shown in Fig. 5.17*b*.

FIGURE 5.17
ACI requirements for stirrup anchorage: (*a*) No. 5 (No. 16) stirrups and smaller, and Nos. 6, 7, and 8 (Nos. 19, 22, and 25) stirrups with yield stress not exceeding 40,000 psi; (*b*) Nos. 6, 7, and 8 (Nos. 19, 22, and 25) stirrups with yield stress exceeding 40,000 psi; (*c*) wide beam with multiple-leg U stirrups; (*d*) pairs of U stirrups forming a closed unit. See Fig. 5.9 for alternative standard hook details.



5.8 Development Bars In Compression

According to ACI Code 12.3, the development length in compression is the greater of

$$l_{dc} = \left(\frac{0.02 f_y}{\lambda\sqrt{f'_c}}\right) d_b \qquad (5.10a)$$

and

$$l_{dc} = 0.0003 f_y d_b \qquad (5.10b)$$

Modification factors summarized in part *B* of Table 5.5, as applicable, are applied to the development length in compression to obtain the value of development length l_{dc} to be used in design. In no case is l_d to be less than 8 in., according to the ACI Code. Basic and modified compressive development lengths are given in Table A.11 of Appendix A.

- **Development length should not be less than 8 in.**

TABLE 5.5

Development lengths for deformed bars in compression

A. Basic development length l_{dc}	$\begin{cases} \geq \left(\dfrac{0.02 f_y}{\lambda \sqrt{f'_c}} \right) d_b \\ \geq 0.0003 f_y d_b \end{cases}$
B. Modification factors to be applied to l_{dc}	
Reinforcement in excess of that required by analysis	$\dfrac{A_s \text{ required}}{A_s \text{ provided}}$
Reinforcement enclosed within spiral reinforcement not less than $\frac{1}{4}$ in. diameter and not more than 4 in. pitch or within No. 4 (No. 13) ties spaced at not more than 4 in. on centers	0.75

Appendix Table A11

5.9 Bundled Bars

It was pointed out in Section 3.6c that it is sometimes advantageous to "bundle" tensile reinforcement in large beams, with two, three, or four bars in contact, to provide for improved placement of concrete around and between bundles of bars. Bar bundles are typically triangular or L-shaped for three bars, and square for four. When bars are cut off in a bundled group, the cutoff points must be staggered at least 40 diameters.

According to ACI Code 12.4, the development length of individual bars within a bundle, for both tension and compression, is that of the individual bar increased by 20 percent for a three-bar bundle and by 33 percent for a four-bar bundle, to account for the probable deficiency of bond at the inside of the bar group.

For bundled bars, to determine the appropriate spacing and cover values (1) for use in Table 5.1, (2) when calculating the confinement term K_{tr} in Eq. (5.4), or (3) when selecting the epoxy coating factor ψ_e, the unit of bundled bars is treated as a single bar with a diameter derived from the equivalent total area and having a centroid that coincides with that of the bar group.

5.10 Bar cutoff and bend points in beams

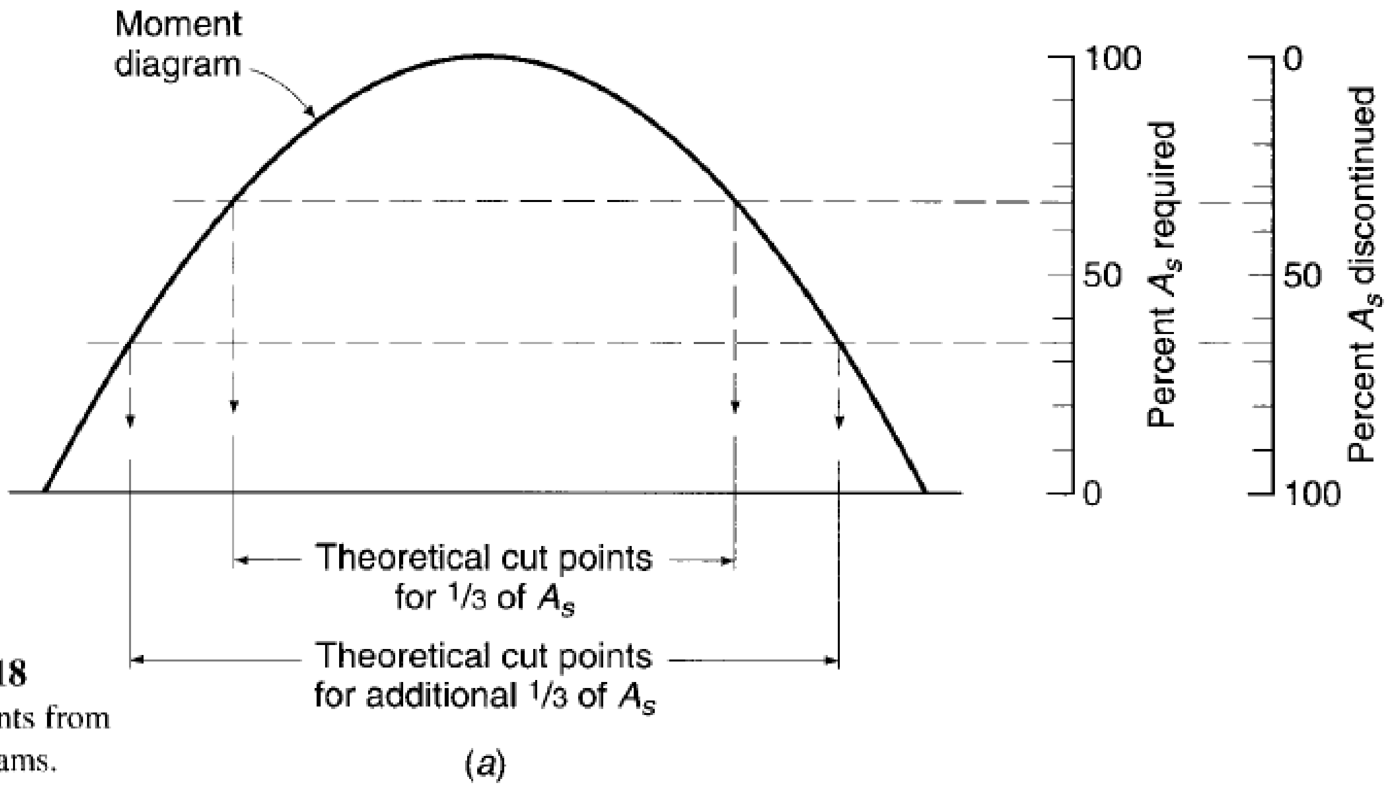


FIGURE 5.18
Bar cutoff points from moment diagrams.

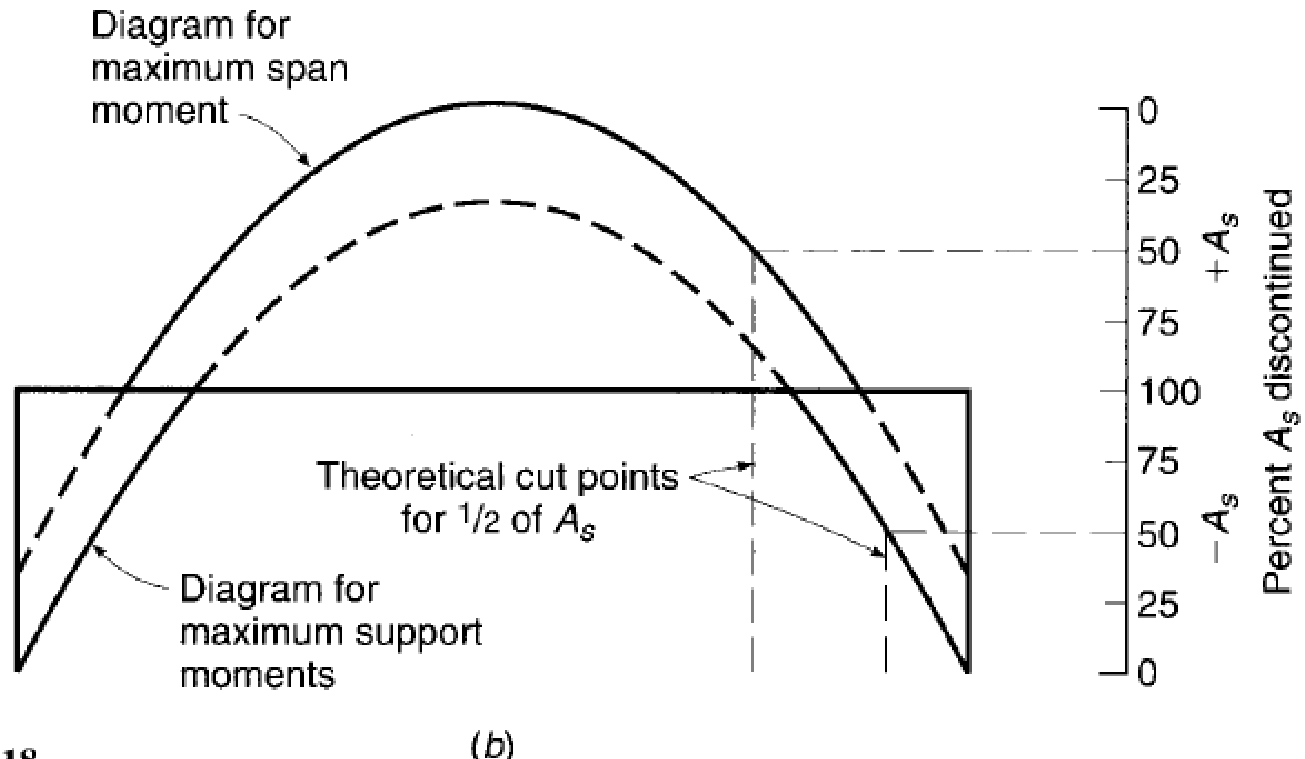


FIGURE 5.18
Bar cutoff points from moment diagrams.

Practical Considerations and ACI Code requirements

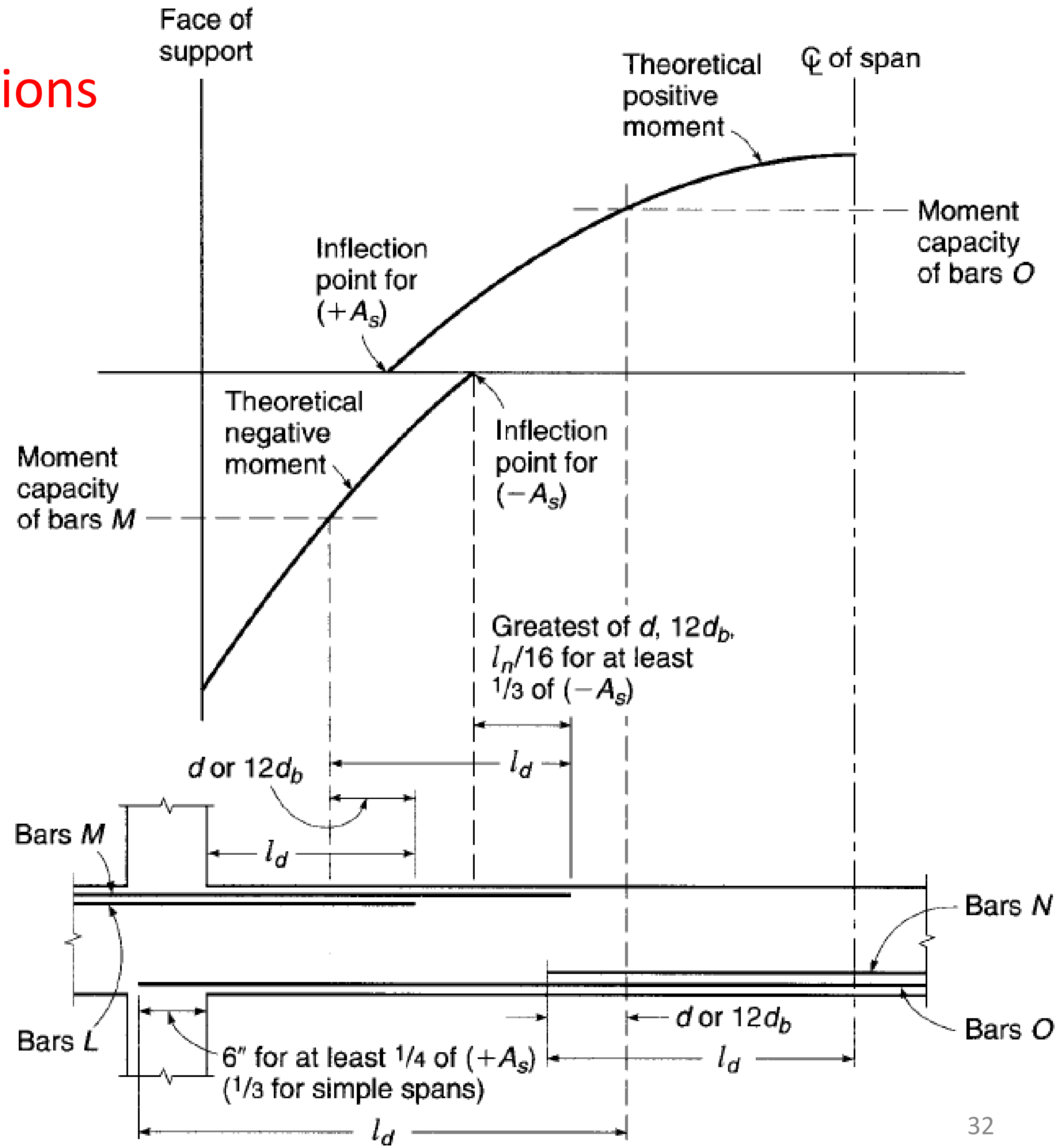


FIGURE 5.19
Bar cutoff requirements of the ACI Code.

Cutoff or bend points

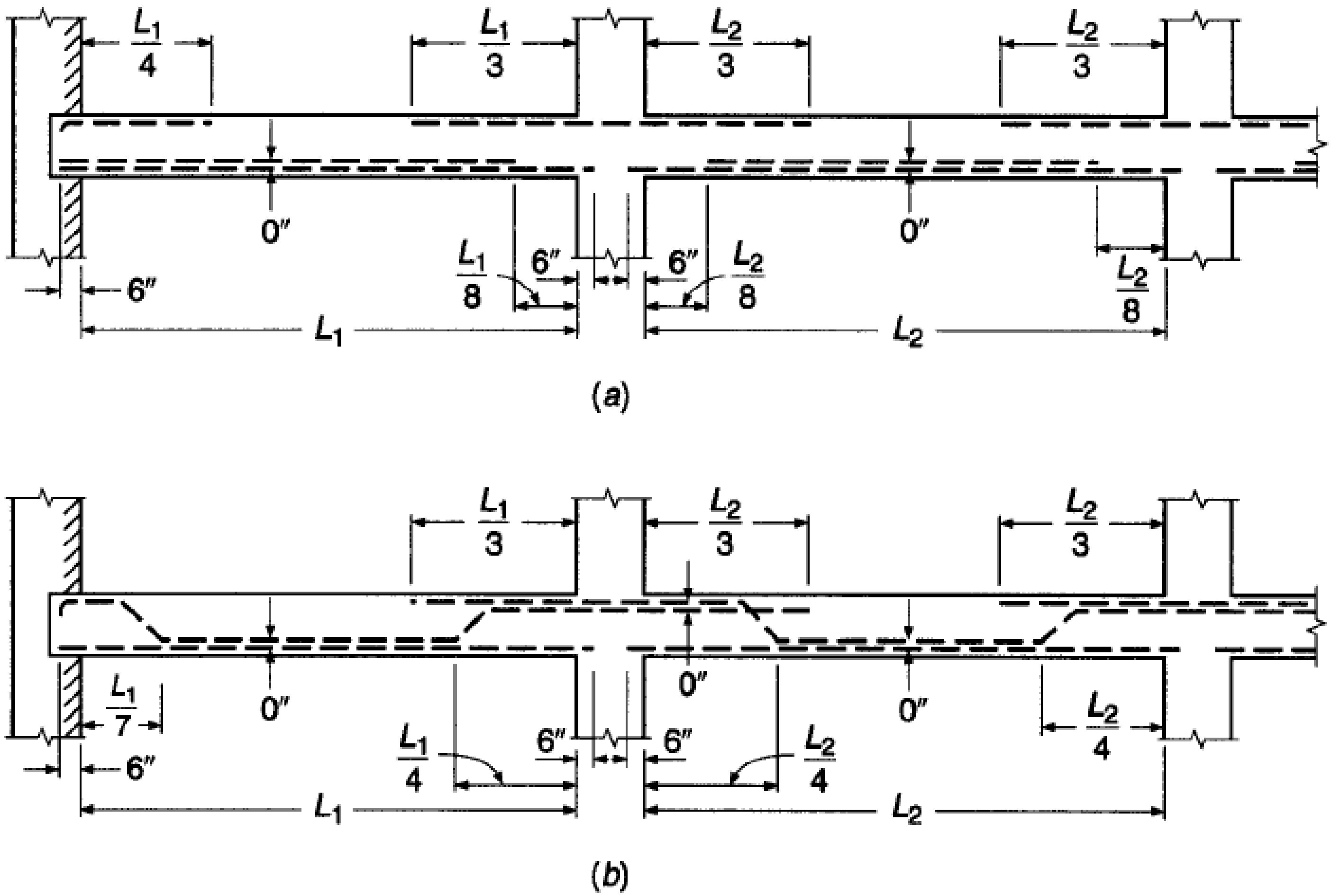


FIGURE 5.20
Cutoff or bend points for bars in approximately equal spans with uniformly distributed loads.

Structural integrity provisions

- Read article
- Integrated beam design example 5.12

Bar Splices

- Bars are supplied at 40ft lengths
- Splice at maximum stress should be avoided
- Lapping on sufficient distance to transfer stress through bond from one bar to the other.
- Tension splice, compressions splice.
- Splices for No. 11 and smaller are usually made the bars a sufficient distance to transfer stress by bond from one bar to the other. Lapped bars are usually placed in contact and lightly wired.

- Splices may also be by welding, sleeves or mechanical devices.

a. Lap Splices in Tension

The required length of lap for tension splices is stated in terms of the development length l_d. In the process of calculating l_d, the usual modification factors are applied except that the reduction factor for excess reinforcement should not be applied because that factor is already accounted for in the splice specification.

Two different classifications of lap splices are established, corresponding to the minimum length of lap required: a Class A splice requires a lap of $1.0l_d$, and a Class B splice requires a lap of $1.3l_d$. In either case, a minimum length of 12 in. applies. For Class B splices, the 12 in. minimum applies to $1.3l_d$, not to the value of l_d used to calculate the lap length. Lap splices, in general, must be Class B splices, according to ACI Code 12.15.2, except that Class A splices are allowed when the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice *and* when one-half or less of the total reinforcement is spliced within the required lap length. The effect of these requirements is to encourage designers to locate splices away from regions of maximum stress, to a location where the actual steel area is at least twice that required by analysis, and to stagger splices.

Spiral reinforcement is spliced with a lap of $48d_b$ for uncoated bars and $72d_b$ for epoxy-coated bars, in accordance with ACI Code 7.10.4.5. The lap for epoxy-coated bars is reduced to $48d_b$ if the bars are anchored with a standard stirrup or tie hook.

b. Compression Splices

Reinforcing bars in compression are spliced mainly in columns, where bars are most often terminated just above each floor or every other floor. This is done partly for construction convenience, to avoid handling and supporting very long column bars, but it is also done to permit column steel area to be reduced in steps, as loads become lighter at higher floors.

Compression bars may be spliced by lapping, by direct end bearing, or by welding or mechanical devices that provide positive connection. The minimum length of lap for compression splices is set according to ACI Code 12.16:

For bars with $f_y \le 60{,}000$ psi	$0.0005 f_y d_b$
For bars with $f_y > 60{,}000$ psi	$(0.0009 f_y - 24) d_b$

but not less than 12 in. For f'_c less than 3000 psi, the required lap is increased by one-third. When bars of different size are lap-spliced in compression, the splice length is to be the larger of the development length of the larger bar and the splice length of the smaller bar. In exception to the usual restriction on lap splices for large-diameter bars, No. 14 and No. 18 bars *may* be lap-spliced to No. 11 and smaller bars.

Direct end bearing of the bars has been found by test and experience to be an effective means for transmitting compression. In such a case, the bars must be held in proper alignment by a suitable device. The bar ends must terminate in flat surfaces within 1.5° of a right angle, and the bars must be fitted within 3° of full bearing after assembly, according to ACI Code 12.16.4. Ties, closed stirrups, or spirals must be used.

c. Column Splices

Lap splices, butt-welded splices, mechanical connections, or end-bearing splices may be used in columns, with certain restrictions. Reinforcing bars in columns may be subjected to compression or tension, or, for different load combinations, both tension and compression. Accordingly, column splices must conform in some cases to the requirements for compression splices only or tension splices only or to requirements for both. ACI Code 12.17 requires that a minimum tension capacity be provided in each face of all columns, even where analysis indicates compression only. Ordinary compressive lap splices provide sufficient tensile resistance, but end-bearing splices may require additional bars for tension, unless the splices are staggered.

For lap splices, where the bar stress due to factored loads is compression, column lap splices must conform to the requirements presented in Section 5.13b for compression splices. Where the stress is tension and does not exceed $0.5f_y$, lap splices must be Class B if more than one-half the bars are spliced at any section, or Class A if one-half or fewer are spliced and alternate lap splices are staggered by l_d. If the stress is tension and exceeds $0.5f_y$, then lap splices must be Class B, according to ACI Code.

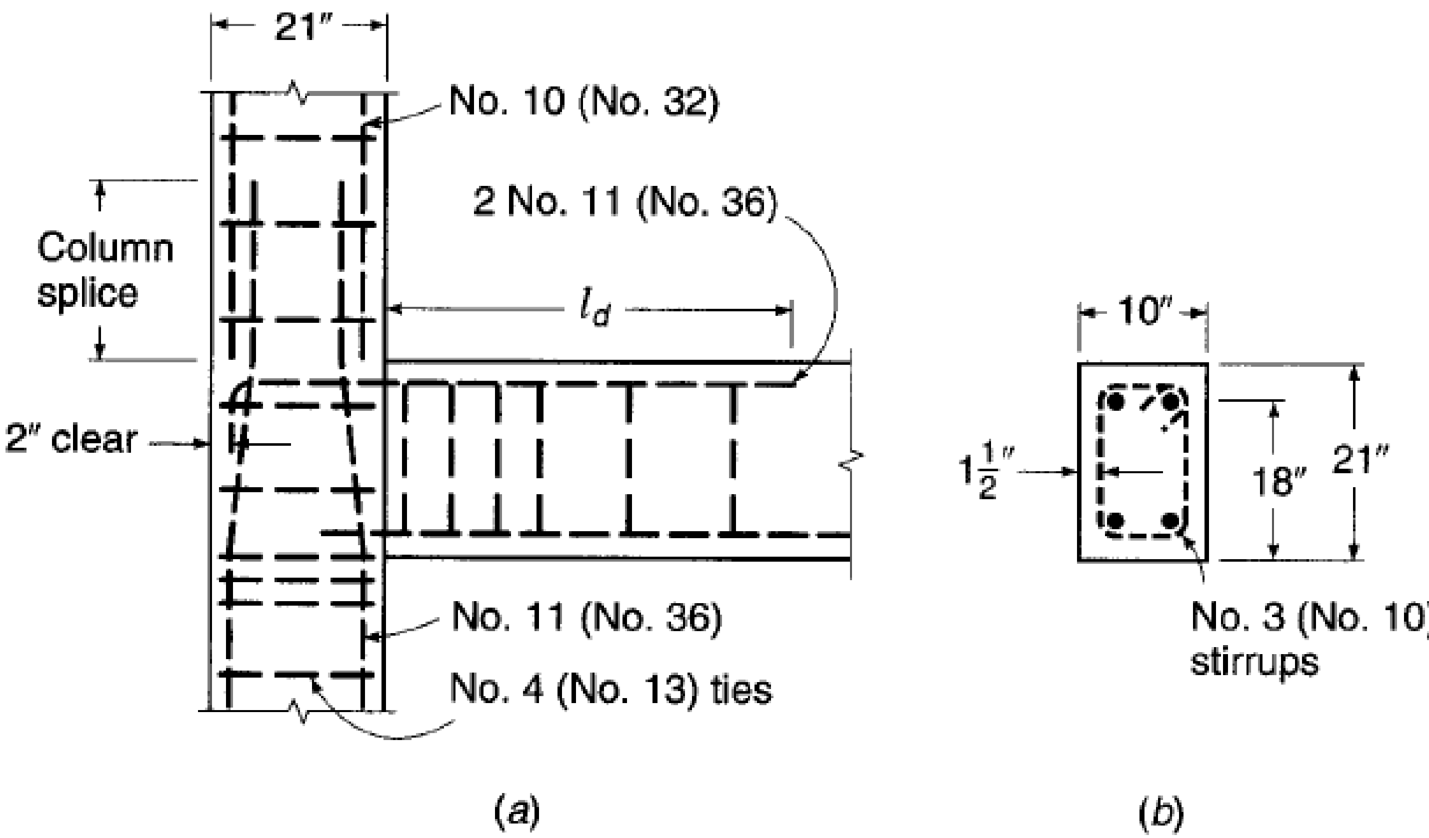
If lateral ties are used throughout the splice length having an area of at least $0.0015hs$ in both directions, where s is the spacing of ties and h is the overall thickness of the member, the required splice length may be multiplied by 0.83 but must not be less than 12 in. If spiral reinforcement confines the splice, the length required may be multiplied by 0.75 but again must not be less than 12 in.

End-bearing splices, as described above, may be used for column bars stressed in compression, if the splices are staggered or additional bars are provided at splice locations. The continuing bars in each face must have a tensile strength of not less than $0.25f_y$ times the area of reinforcement in that face.

As mentioned in Section 5.13b, column splices are commonly made just above a floor. However, for frames subjected to lateral loads, a better location is within the center half of the column height, where the moments due to lateral loads are much lower than at floor level. Such placement is mandatory for columns in "special moment frames" designed for seismic loads, as will be discussed in Chapter 20.

EXAMPLE 5.5

Compression splice of column reinforcement. In reference to Fig. 5.8, four No. 11 (No. 36) column bars from the floor below are to be lap-spliced with four No. 10 (No. 32) column bars from above, and the splice is to be made just above a construction joint at floor level. The column, measuring 12 in. × 21 in. in cross section, will be subject to compression only for all load combinations. Transverse reinforcement consists of No. 4 (No. 13) ties at 16 in. spacing. All vertical bars may be assumed to be fully stressed. Calculate the required splice length. Material strengths are f_y = 60,000 psi and f_c' = 4000 psi.



Solution. The length of the splice must be the larger of the development length of the No. 11 (No. 36) bars and the splice length of the No. 10 (No. 32) bars. For the No. 11 (No. 36) bars, the development length is equal to the larger of the values obtained with Eqs. (5.10*a*) and (5.10*b*):

$$l_{dc} = \frac{0.02 \times 60{,}000}{\sqrt{4000}} 1.41 = 27 \text{ in.}$$

$$l_{dc} = 0.0003 \times 60{,}000 \times 1.41 = 25 \text{ in.}$$

The first criterion controls. No modification factors apply. For the No. 10 (No. 32) bars, the compression splice length is $0.0005 \times 60{,}000 \times 1.27 = 38$ in. In the check for use of the modification factor for tied columns, the critical column dimension is 21 in., and the required effective tie area is thus $0.0015 \times 21 \times 16 = 0.50 \text{ in}^2$. The No. 4 (No. 13) ties provide an area of only $0.20 \times 2 = 0.40 \text{ in}^2$, so the reduction factor of 0.83 cannot be applied to the splice length. Thus the compression splice length of 38 in., which exceeds the development length of 27 in. for the No. 11 (No. 36) bars, controls here, and a lap splice of 38 in. is required. Note that if the spacing of the ties at the splice were reduced to 12.8 in. or less (say 12 in.), the required lap would be reduced to $38 \times 0.83 = 32$ in. This would save steel, and, although placement cost would increase slightly, would probably represent the more economical design.