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Dr. Raquib Ahsan

Lec-1

Flexibility Method:

Approximate analysis → 1) portal
2) cantilever

Exact " → 1) stiffness (সব কিছু Force এ)
2) moment distribution
3) flexibility (সব কিছু deformation এ)

$2D \rightarrow \Sigma F_x, \Sigma F_y, \Sigma M = 0$ (3টি Linearly independent Eqn)

$3D \rightarrow \Sigma F_x, \Sigma F_y, \Sigma F_z, \Sigma M_x, \Sigma M_y, \Sigma M_z = 0$ (6টি " " " ")

- approximate → forcefully একটা condition এনে eqn আনা হয়।
- exact → original deformation থেকে eqn develop করা হয়।

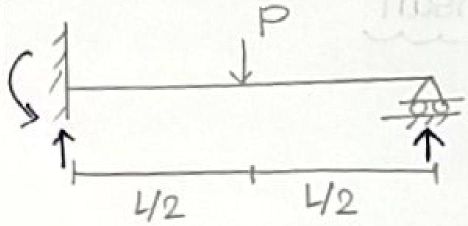
Equilibrium এ → সব Term হল Force

indeterminate এ → 1) 1 set of eqn → in terms of Force
2) new set " " → " " " deformation

compatibility eqns or conditions.

- Exact method এ → হয় সব Force or সব deformation based eqn এ convert করা হয়।

- Stiffness method more user friendly.



axial Force ignore করে,

UK = 3

Equilibrium $eq^n = 2$

Degree of Statical Indeterminacy

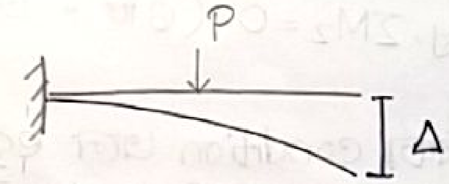
DOSI = (3-2) = 1

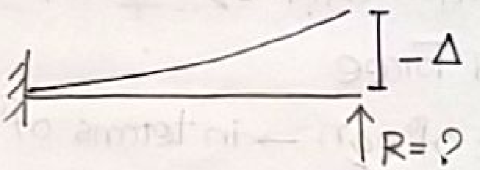
Static = related to force equilibrium

Kinematic = something related to deformation

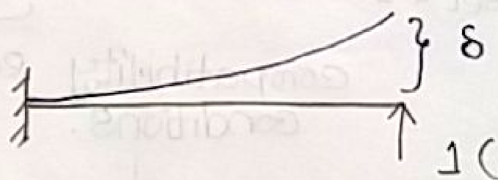
Kinetic = " " " speed / velocity

Dynamic = " " " acceleration





R = ? যাতে Δ deformation opposite দিকে হয়।



1 (Load, say)

$\boxed{\delta = \text{Flexibility}}$

1 load এর জন্য δ deformation

$\therefore$ R " " " Rδ "

$\therefore R\delta = -\Delta$

$\therefore R = -\dfrac{\Delta}{\delta}$ ← Deformation per unit load

• Stiffness = Force per unit deformation

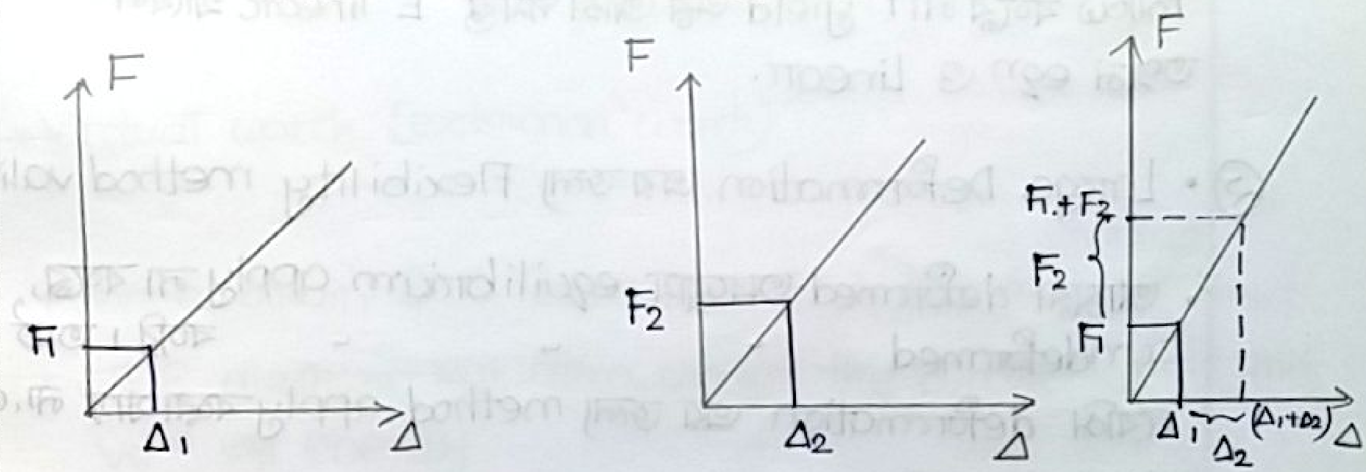
• unit amount force দিলে যদি বেশি deform করে → more flexible

" " " " " কম " " → less flexible

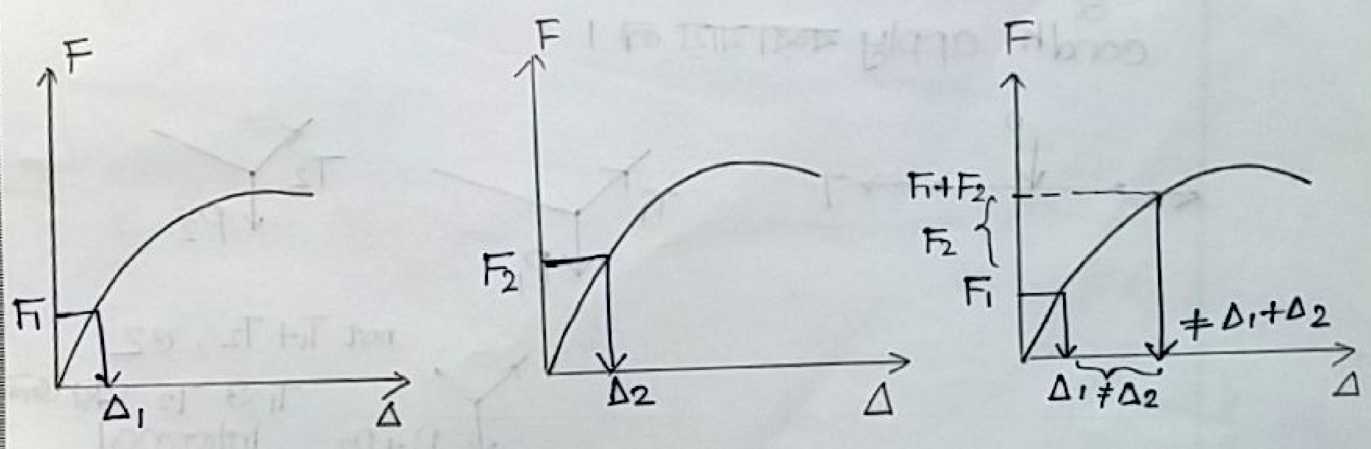
• Principle of superposition :

② → valid : যখন structure এর force-deformation relationship linear (straight line).

এদেরকে Linear structure বলে।



যখন Relationship linear



যখন Relationship linear না

• F-Δ Relationship Linear হবে, যখন,

$$\underbrace{EI \frac{d^4v}{dx^4} = p(x)}$$

Beam eqn. এটা linear eqn.

এটা non linear হবে যখন,
E or I is a function of v.
এটা অসম্ভব, যখন E = Hook's Law Follow করে না। yield এর আগ পর্যন্ত E linear থাকে। তখন eqn ও Linear.

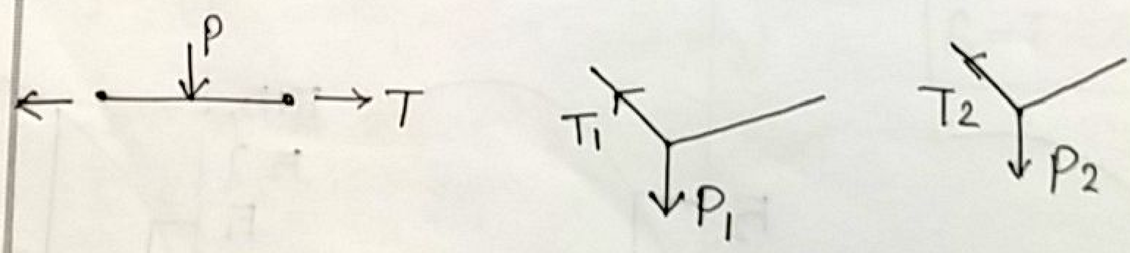
• dependent variable eq. ফর্মে থাকলে,

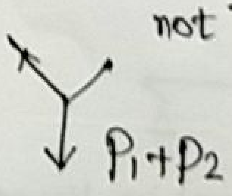
$$\left.\begin{array}{l} \rightarrow v^2 \\ \rightarrow \left(\frac{dv}{dx}\right)^2 \\ \rightarrow v \cdot \frac{dv}{dx} \end{array}\right\} \text{non Linear}$$

② • Large Deformation এর জন্য Flexibility method valid না।

• আমরা deformed অবস্থায় equilibrium apply না করে, undeformed অবস্থায় equilibrium apply করি। তাই বেশি deformation এর জন্য method apply করা যায় না, cz error আসে।

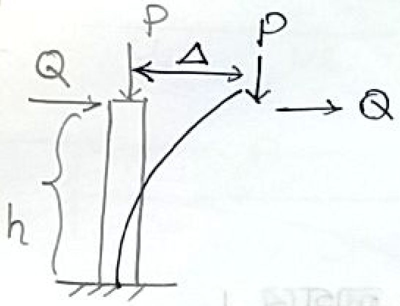
• কিন্তু cable এ undeformed state এ Equilibrium ~~sta~~ condn. apply করা যায় না।





not T_1+T_2, cz T_1 ও T_2 এর জন্য internal angle আলাদা।

• So, যেখানে deformed shape এ Equilibrium condition apply হয়, সেখানে principal of super position applicable না।



Moment Amplification

P-Δ effect আসলে deformation Moment চিন্তা করে equilibrium eqn বের করতে হয়। তাই এক্ষেত্রেও principle of superposition applicable না।

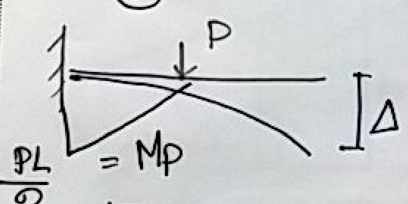
→ Virtual work (external work)

P Force strain & Q Force stress → internal virtual work

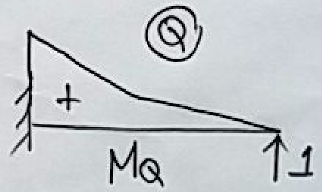
→ σ-ε diagram এর area under the curve → per unit volm এর energy

P (actual load) ; Q (Virtual load)

(P)



(Q)



External work = internal work

$$1 \times \Delta = \int_V \epsilon_P \, \sigma_Q \, dV \qquad \epsilon = \sigma/E$$

$$= \int_L \int_A \frac{M_P y}{EI} \, \frac{M_Q y}{I} \, dA \, dx$$

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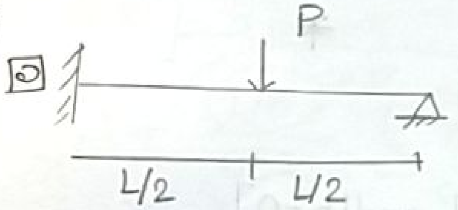
$$= \int_L \frac{M_P M_Q}{EI^2} \int_A y^2 \, dA \; dx$$

$$\Delta = \int_L \frac{M_P M_Q}{EI} dx$$

$\therefore$ P ফোর্সের BMD $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ লাগবে
and Q " BMD

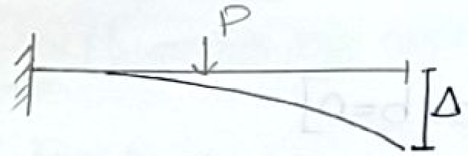
M_P & M_Q কে x এ convert করা লাগবে।

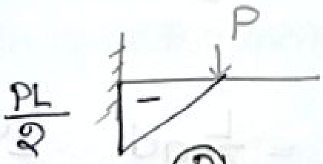
Lec-2

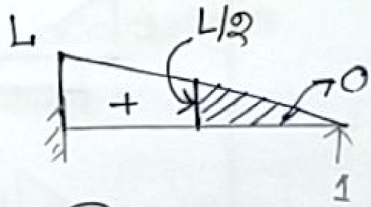


$$R\delta = -\Delta$$
$$\therefore R = -\frac{\Delta}{\delta}$$

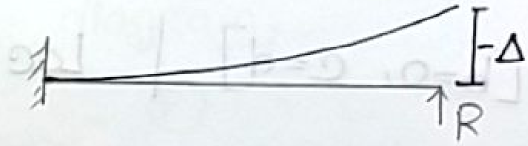
$\boxed{\Delta}$

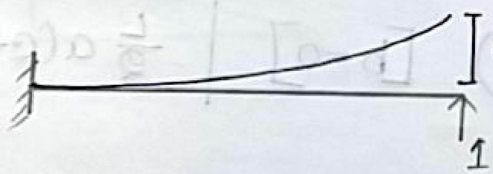




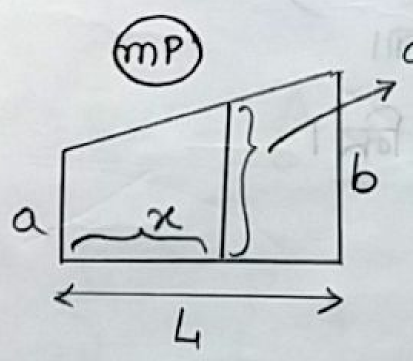


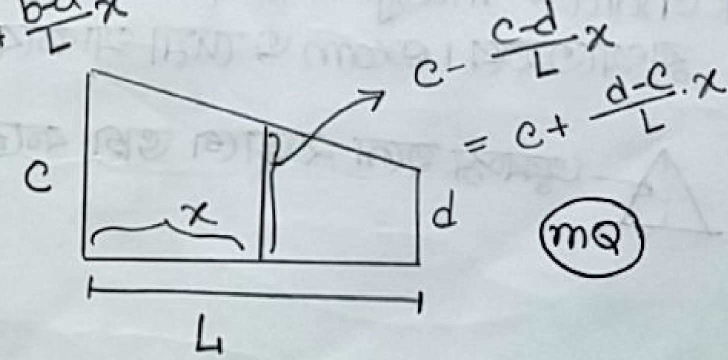
Q যেখানে deformation চাই সেখানে Q force দিব





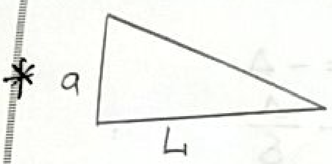
$$\Delta = \int_L \frac{m_P\, m_Q}{EI}\, dx$$



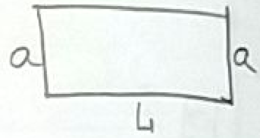


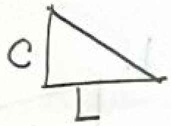
$$I = \int_L m_P\, m_Q\, dx = \int_0^L \left[\left\{a + (b-a)\frac{x}{L}\right\}\left\{c + (d-c)\frac{x}{L}\right\}\right] dx$$

$$= \frac{L}{6}\left\{c(2a+b) + d(a+2b)\right\} = \frac{L}{6}\left\{a(2c+d) + b(c+2d)\right\}$$

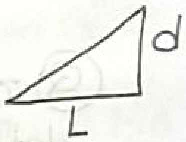
* 

$b=0$

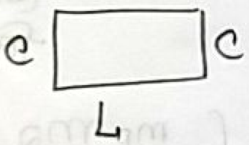
* 

1) 

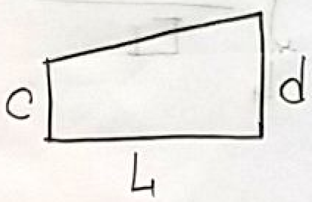
$= \frac{L}{3} ac$ [$d=0$] and [$b=0$]

2) 

$= \frac{L}{6} ad$ [$c=0$, $b=0$]

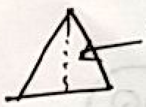
3) 

$= \frac{L}{2} ac$ [$b=0$, $c=d$] | Lac

4) 

$= \frac{L}{6} a(2c+d)$ [$b=0$] | $\frac{L}{2} a(c+d)$

* chart → integral of product (chart). এটা মনে রাখতে হবে। exam এ দেয়া থাকবে না।


এরকম হলে ২ ভাগে ভাগ করে নিব।

এরপর,

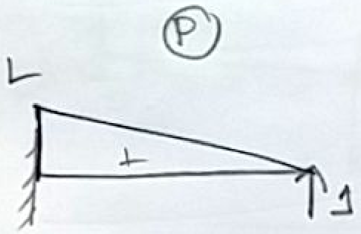
$\Delta \times$ ▭ $\quad \Delta = \frac{1}{6} a (2c+d)$ $\quad \Delta = \int_L^0 \frac{m_p m_Q}{EI} dx$

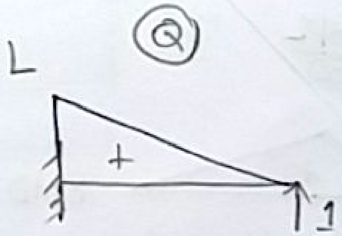
$$\Delta = \frac{L/2}{6EI} \underbrace{\left(-\frac{PL}{2}\right)}_{a} \times \left(2\underset{c}{l} + \underset{d}{\frac{L}{2}}\right) = -\frac{PL^2}{24EI} \cdot \frac{5L}{2} = -\frac{5PL^3}{48EI}$$

-ve sign indicate করে, আমরা 1 unit load যেদিকে apply করেছি, তার opposite এ deformation হবে।

Q চিত্র ৪

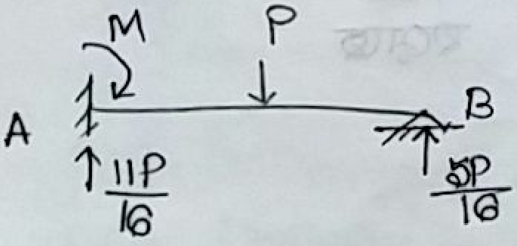
diagram,





$$\delta = \frac{L}{3EI} L^2 = \frac{L^3}{3EI}$$

$$\therefore R = -\frac{\Delta}{\delta} = -\frac{-\frac{5PL^3}{48EI}}{\frac{L^3}{3EI}} = \frac{5P}{16}$$

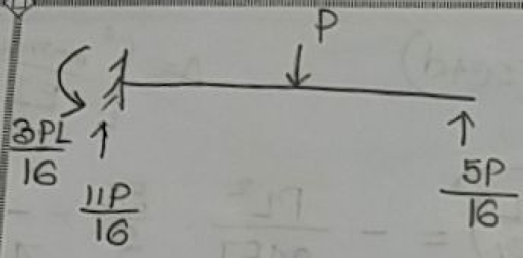


$\Sigma M_A = 0$

$\Rightarrow P \times \frac{L}{2} - \frac{5P}{16} \times L + M = 0$

$\Rightarrow M = \frac{5PL}{16} - \frac{PL}{2} = -\frac{3}{16} PL$

$\therefore M = \frac{3}{16} PL$ (↺)

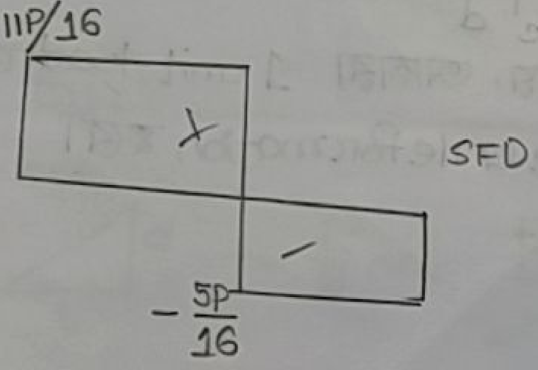


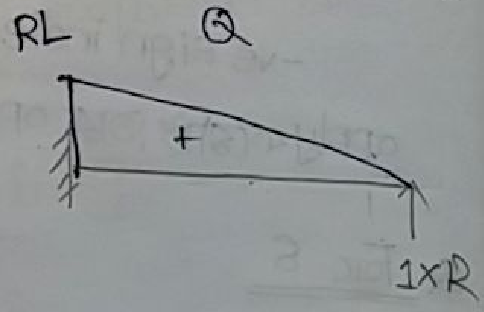
superposition

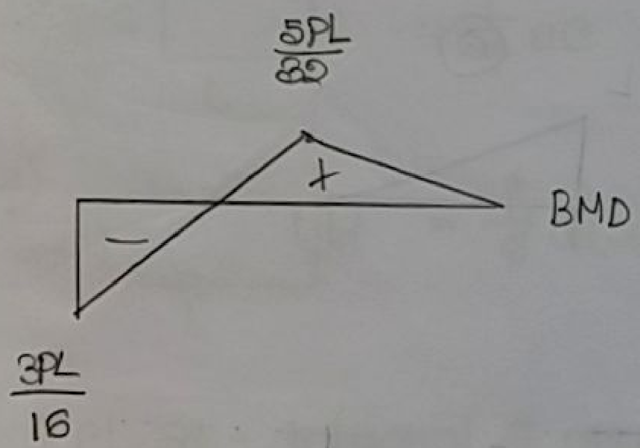
$$\overbrace{M = m_0 + m_1 R}$$

Redundent

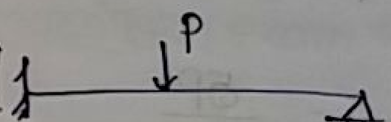
unit load এর জন্য



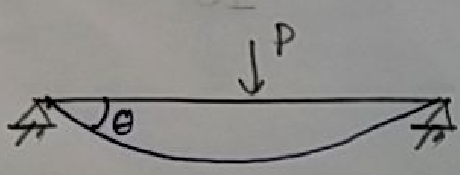


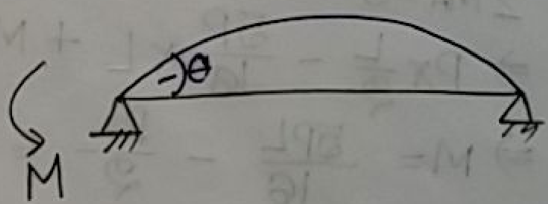


$\boxed{A\rightarrow}$



আগের Math, fixed support কে redundent করা হয়েছে





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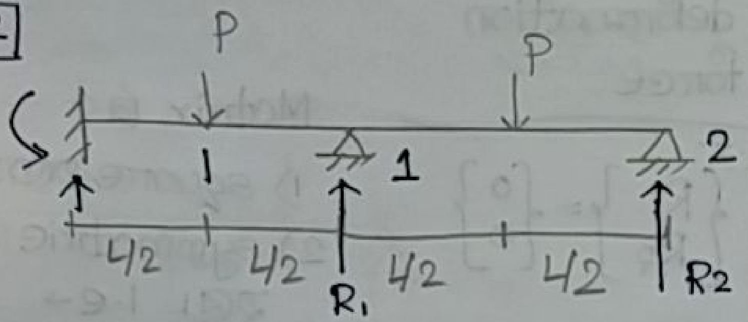


DOSI = 2 (No axial Load)

$\Delta_{01} + \Delta_{11} + \Delta_{12} = 0$

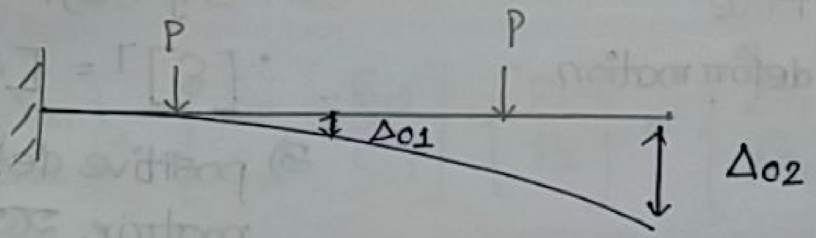
$\Delta_{02} + \Delta_{21} + \Delta_{22} = 0$

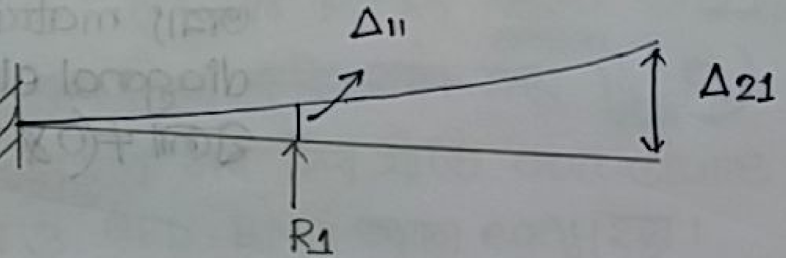
$\Delta_{11} = \delta_{11} R_1$

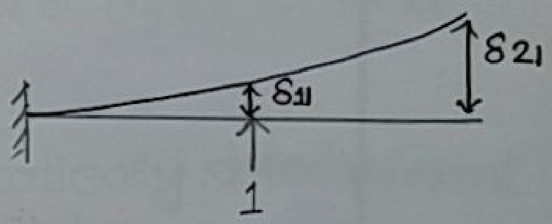
$\Delta_{21} = \delta_{21} R_1$

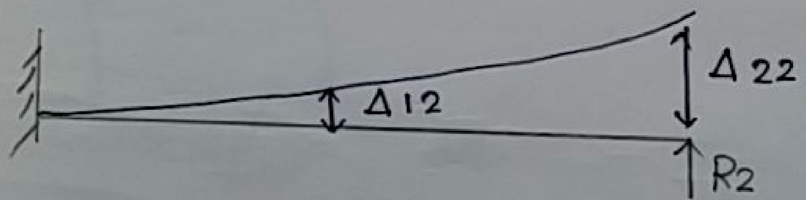
$\Delta_{12} = \delta_{12} R_2$

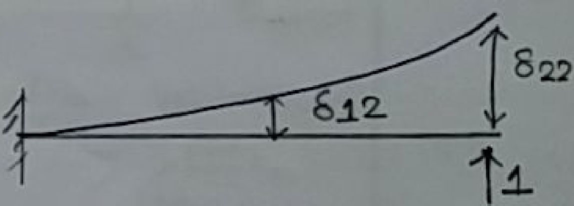
$\Delta_{22} = \delta_{22} R_2$











Now, $\Delta_{01} + \delta_{11} R_1 + \delta_{12} R_2 = 0$

$\Delta_{02} + \delta_{21} R_1 + \delta_{22} R_2 = 0$

$\delta = \frac{\Delta}{R}$

= Deformation per unit force

= flexibility

$$\boxed{\{\Delta_0\} + [\delta]\{R\} = \{\Delta_s\}}$$ ← Flexibility eqn

$\{\Delta_0\}$ = Deformation vector of the released structure

$[\delta]$ = Flexibility matrix

$\{R\}$ = Redundent vector

$\{\Delta_s\}$ = Support deformation vector

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$\delta_{ij} \rightarrow$ i = কোন deformation
j = " force

$$\begin{Bmatrix}\Delta_{01}\\ \Delta_{02}\end{Bmatrix} + \begin{bmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{bmatrix}\begin{Bmatrix}R_1\\ R_2\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

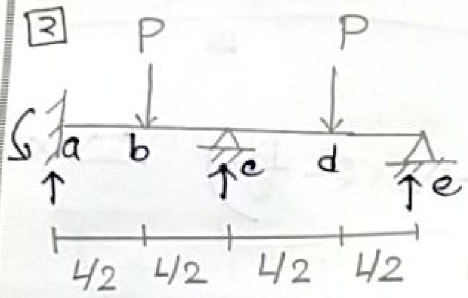
Matrix টা

1) square হতে হবে
2) symmetric হতে হবে। i.e.→
- $\delta_{ij} = \delta_{ji}$
- $[\delta]^T = [\delta]$

3) positive definite matrix হতে হবে। অর্থাৎ matrix এর diagonal element গুলো ≠ (0 & neg value).

$\underbrace{K_{ij}}_{\text{stiffness}} \rightarrow$ i → force
j → deformation

Lec-3



Flexibility eq^n.

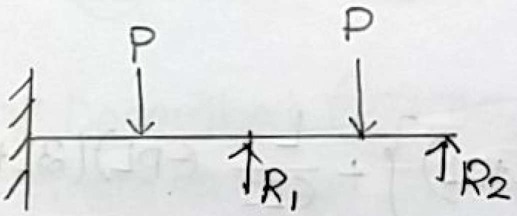
$$\{\Delta_o\} + [\delta]\{R\} = \{\Delta s\}$$

$DOSI = 2 = (4-2)$

order ও হবে 2.

$$\begin{Bmatrix} \Delta_{o1} \\ \Delta_{o2} \end{Bmatrix} + \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} \Delta s_1 \\ \Delta s_2 \end{Bmatrix}$$

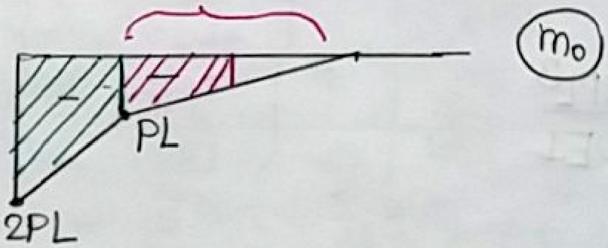
- যে কোন দুইটাকে R_1, R_2 ধরতে পারি। (৪টা থেকে যে কোন ২টা বাদ দিলে ~~বাকি ২টা হবে~~ ওখানে R_1, R_2 ধরবে)
- এমনভাবে বাদ দিব যাতে structure statically determinant হয় ও SFD, BMD আঁকা easy হয়।

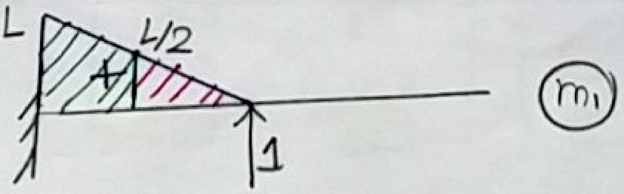


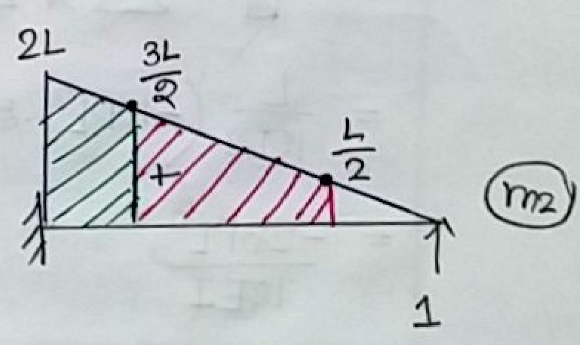
$$\Delta_{o1} = \int_L \frac{m_o m_1}{EI} dx$$

m_o = released structure এর M.

for $\Delta o2$







$$\underbrace{\Delta_{01} = \int \frac{m_0 m_1}{EI}\,dx}_{\text{trapezeum} \times \text{trapezeum}} = \frac{L/2}{6EI}\left\{(-2PL)\left(2L+\frac{L}{2}\right) - PL\left(2\times\frac{L}{2}+L\right)\right\}$$

$$+ \frac{L/2}{6EI} \times \frac{L}{2}\left(-PL\times 2 - \frac{PL}{2}\right)$$

$$= \frac{L}{12EI}\left\{-5PL^2 - 2PL^2\right\} + \frac{L^2}{24EI}\left(-\frac{5PL}{2}\right)$$

$$= -\frac{7PL^3}{12EI} - \frac{5PL^3}{48EI} = -\frac{33PL^3}{48EI} = (-)\frac{11PL^3}{16EI}$$

virtual load (1 unit) ↑
but deformation ↓
তাই (-ve) এসেছে।

$$\Delta_{02} = \int \frac{m_0 m_2}{EI}\,dx$$

$$= \frac{L/2}{6EI}\left\{(-2PL)\left(4L+\frac{3L}{2}\right) - PL(3L+2L)\right\} + \frac{L}{6EI}(-PL)\left(3L+\frac{L}{2}\right)$$

$$= \frac{L}{12EI}\left(-11PL^2 - 5PL^2\right) - \frac{7PL^3}{12EI}$$

$$= \frac{-23PL^3}{12EI}$$

Sub: $\delta_{11}, \delta_{22} \rightarrow$ must be(+ve)

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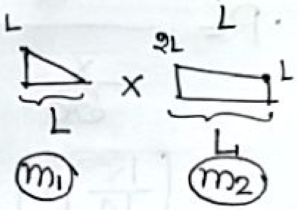
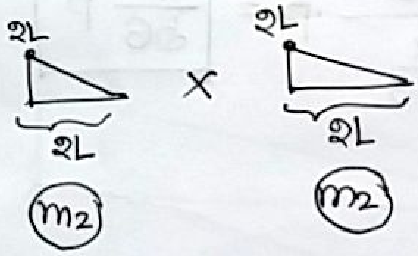
$$\delta_{11} = \int_L \frac{m_1 m_1}{EI} dx = \frac{L}{3EI} \times L \times L = \frac{L^3}{3EI}$$ (L length এর ২টা triangle হলে)

$$\delta_{12} = \delta_{21} = \int_L \frac{m_1 m_2}{EI} dx = \frac{L}{6EI} \times L\,(2 \times 2L + L) = \frac{5L^3}{6EI}$$



$$\delta_{22} = \int_L \frac{m_2 m_2}{EI} dx = \frac{2L}{3EI} \times 2L \times 2L = \frac{8L^3}{3EI}$$



* Described (বলে দেয়া) support deformation (কতটুকু নামবে বা উঠবে) না থাকলে $\Delta s_1, \Delta s_2 = 0$.

$$\begin{Bmatrix} -\frac{11PL^3}{16EI} \\ -\frac{23PL^3}{12EI} \end{Bmatrix} + \begin{bmatrix} \frac{L^3}{3EI} & \frac{5L^3}{6EI} \\ \frac{5L^3}{6EI} & \frac{8L^3}{3EI} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{array}{c} \begin{array}{cc} R_1 & R_2 \end{array} \\ \begin{bmatrix} \frac{L^3}{3EI} & \frac{5L^3}{6EI} \\ \frac{5L^3}{6EI} & \frac{8L^3}{3EI} \end{bmatrix} \end{array} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} \frac{11PL^3}{16EI} \\ \frac{23PL^3}{12EI} \end{Bmatrix}$$

using cramer's Rule:

$$R_1 = \frac{\begin{vmatrix} \frac{11PL^3}{16EI} & \frac{5L^3}{6EI} \\ \frac{23PL^3}{12EI} & \frac{8L^3}{3EI} \end{vmatrix}}{\begin{vmatrix} \frac{L^3}{3EI} & \frac{5L^3}{6EI} \\ \frac{5L^3}{6EI} & \frac{8L^3}{3EI} \end{vmatrix}} = \frac{\begin{vmatrix} \frac{11}{16} & \frac{5}{6} \\ \frac{23}{12} & \frac{8}{3} \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} & \frac{5}{6} \\ \frac{5}{6} & \frac{8}{3} \end{vmatrix}} \cdot P = \frac{\frac{17}{72}}{\frac{7}{36}} \cdot P$$

$$= \boxed{\frac{17}{14} P}$$

$$R_2 = \frac{\begin{vmatrix} \frac{1}{3} & \frac{11}{16} \\ \frac{5}{6} & \frac{23}{12} \end{vmatrix}}{\frac{7}{36}} \cdot P = \frac{\frac{19}{288}}{\frac{7}{36}} \cdot P = \boxed{\frac{19}{56} P}$$

$$M = m_0 + m_1 R_1 + m_2 R_2$$

$$M_a = -2PL + L \times \frac{17}{14} P + 2L \times \frac{19}{56} P$$

$$= -2PL + \frac{17}{14} PL + \frac{19}{28} PL$$

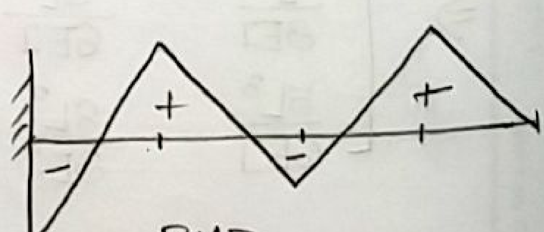
$$\boxed{M_a = -\frac{3}{28} PL}$$

- point load থাকলে st. line
- UDL হলে curve হবে।

$$M_b = -PL + \frac{L}{2} \times \frac{17}{14} P + \frac{3L}{2} \times \frac{19}{56} P$$

$$\boxed{M_b = +\frac{13}{112} PL}$$

* SFD এর জন্য V_0, V_1, V_2 diagram এঁকে math start করব।



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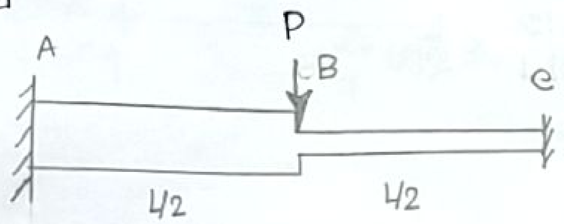


$AB = 2EI$

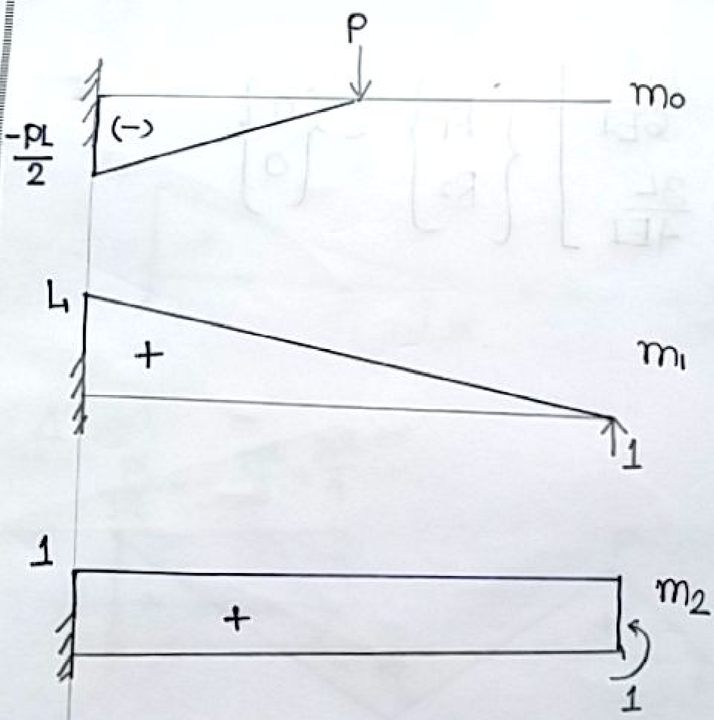
$BC = EI$

$DOSI = 4-2 = 2$

$$\{\Delta_0\} + [\delta]\{R\} = \{\Delta_S\}$$

$$\begin{Bmatrix}\Delta_{01}\\ \Delta_{02}\end{Bmatrix} + \begin{bmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{bmatrix}\begin{Bmatrix}R_1\\ R_2\end{Bmatrix} = \begin{Bmatrix}\Delta_{S1}\\ \Delta_{S2}\end{Bmatrix}$$

Released str:



$$\Delta_{01} = \int_L \frac{m_0 m_1}{EI}dx$$

$$= \frac{L/2}{2\times 6}\times\left(-\frac{PL}{2}\right)\left(2L+\frac{L}{2}\right)$$

$$= -\frac{5PL^3}{96EI} \quad \text{(only deformation)}$$

$$\Delta_{02} = \int_L \frac{m_0 m_2}{EI}dx$$

$$= \frac{L/2}{2\times 2EI}\times\left(-\frac{PL}{2}\right)\times 1$$

$$= -\frac{PL^2}{16EI} \quad \text{(only rotation)}$$

$$\delta_{11} = \int_L \frac{m_1 m_1}{EI}dx$$

$$= \frac{L/2}{6\cdot 2EI}\left[L\times(2L+L/2) + \frac{L}{2}\left(L+2\times\frac{L}{2}\right)\right] + \frac{L/2}{3EI}\times\frac{L}{2}\times\frac{L}{2}$$

$$= \frac{L}{24EI}\left(\frac{5L^2}{2}+\frac{L^2}{2}+\frac{L^2}{4}\right) = \frac{3L^3}{16EI}$$

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Sub:

$$\delta_{12} = \delta_{21} = \int_L \frac{m_1 m_2}{EI} dx$$

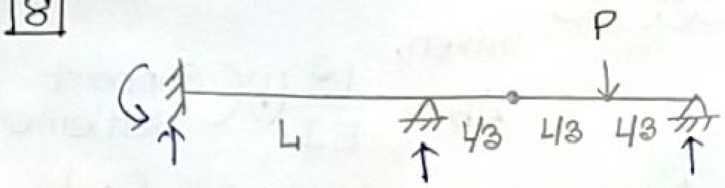
$$= \frac{L/2}{2\times2EI} \times 1 \times \left(L + \frac{L}{2}\right) + \frac{L/2}{2EI} \times \frac{L}{2}$$

$$= \frac{5L^2}{16EI}$$

$$\delta_{22} = \int_L \frac{m_2 m_2}{EI} dx = \frac{L/2}{2EI} \times 1 \times 1 + \frac{L/2}{EI} \times 1 \times 1 = \frac{3L}{4EI}$$

$$\begin{Bmatrix} -\frac{5PL^3}{96EI} \\ -\frac{PL^2}{16EI} \end{Bmatrix} + \begin{bmatrix} \frac{9L^3}{48EI} & \frac{5L^2}{6EI} \\ \frac{5L^2}{6EI} & \frac{3L}{4EI} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

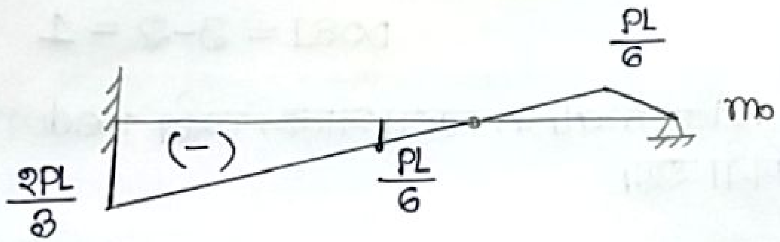
$\boxed{8}$



DOSI = 1

[No. of Reaction = 4
eqn of equilibrium = 2
from internal hinge → = 1 ($\Sigma M = 0$ at hinge)

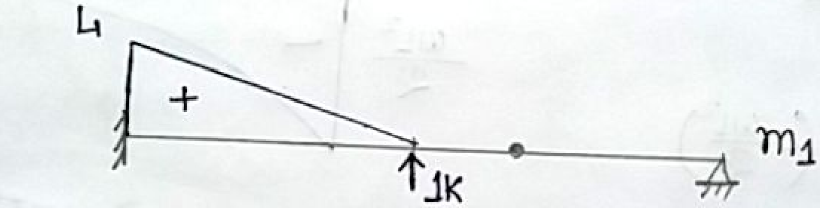
So, DOSI = 4 − 2 − 1
= 1]



$$\Delta_0 = \int_L \frac{m_0 m_1}{EI} dx$$

$$= \frac{L}{6EI} \times L \times \left(-\frac{4PL}{3} - \frac{PL}{6}\right)$$

$$= -\frac{PL^3}{4EI}$$



$$\delta = \int_L \frac{m_1 m_1}{EI} dx$$

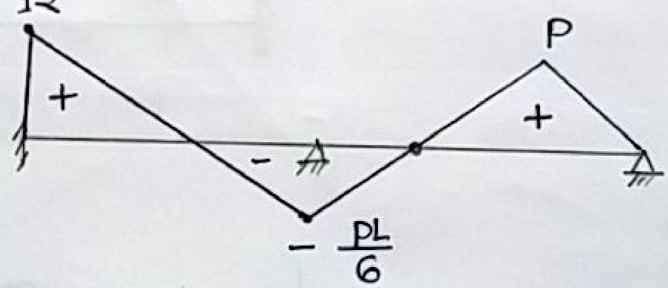
$$= \frac{L}{3EI} \times L \times L = \frac{L^3}{3EI}$$

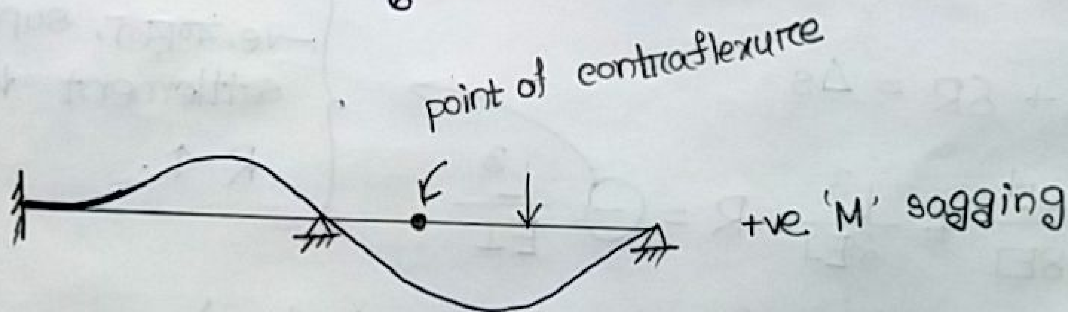
$$\therefore -\frac{PL^3}{4EI} + \frac{L^3}{3EI} \times R = 0$$

$$\therefore R = \frac{PL^3}{4EI} \times \frac{3EI}{L^3} = \frac{3P}{4}$$

Δ

$\frac{PL}{12} = -\frac{2PL}{3} + \frac{3PL}{4}$





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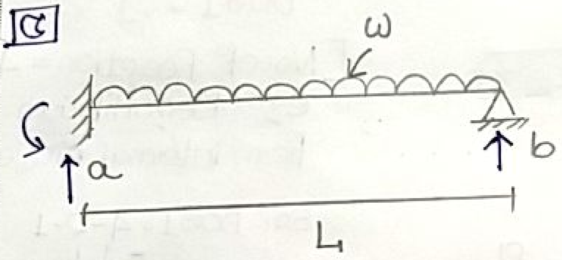


Given,

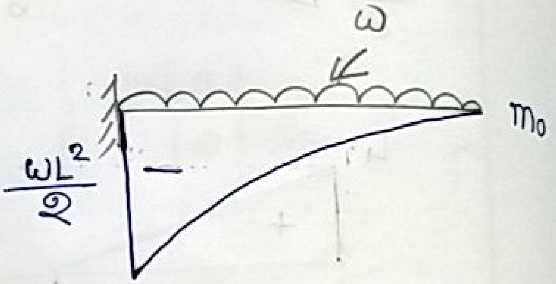
$$\Delta_b = \frac{L^3}{EI}(\downarrow) \text{ (Support settlement)}$$

$$\{\Delta_0\} + [\delta]\{R\} = \{\Delta_s\}$$

$$DOSI = 3-2 = 1$$

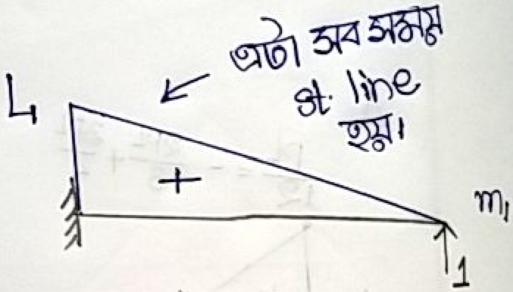
- যেই support এর deformation দেয়া থাকে, সেটা redundent ধরলে soln এ সুবিধা হয়।

$$\Delta_0 = \int_L \frac{m_0 m_1}{EI} dx$$



$$= \frac{L}{12EI}(3L+0)\left(-\frac{\omega L^2}{2}\right)$$

$$= -\frac{\omega L^4}{8EI}$$



$$\delta = \int_L \frac{m_1 m_1}{EI} dx$$

$$= \frac{L}{3EI} \times L \times L = \frac{L^3}{3EI}$$

$$\Delta_0 + \delta R = \Delta_s$$

$$\therefore -\frac{\omega L^4}{8EI} + \frac{L^3}{3EI} \cdot R = -\frac{L^3}{EI}$$

{ −ve, কারণ, support settlement ↓, but R ↑. }

$$\Rightarrow R = \left(\frac{\omega L^4}{8EI} - \frac{L^3}{EI}\right) \times \frac{3EI}{L^3} = \left(\frac{\omega L}{8} - 1\right) \times 3$$

অর্থাৎ, settlement নিচে হওয়াতে Reaction কমে গেছে।

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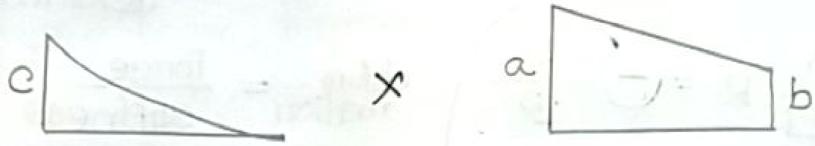
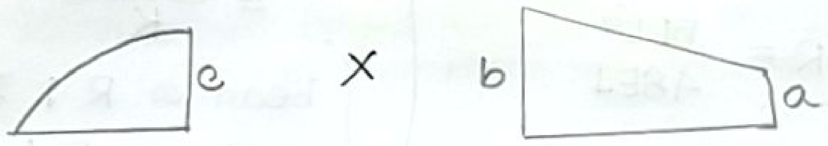
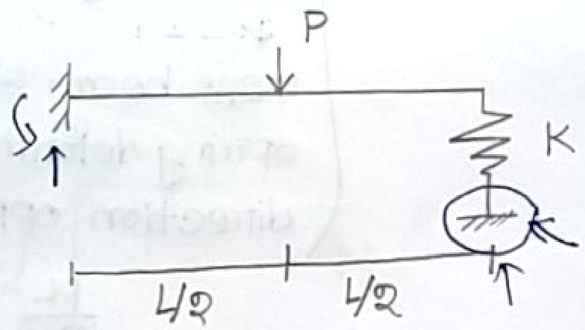


$= \frac{L}{12}(3a+b)c$



$= \frac{L}{12}(5a+3b)c$

ঙ

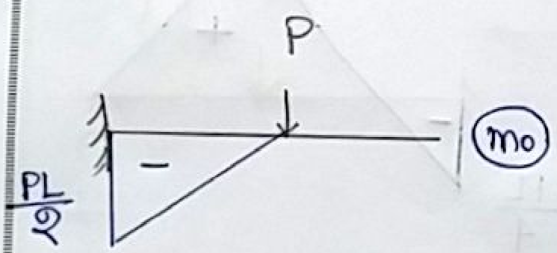


$K = \frac{EI}{12L^3}$

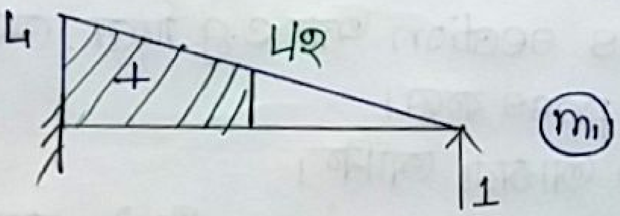
এটা fixed support না, just spring বুঝায়। ১টা reaction আসবে spring এর ↑ বরাবর।

$DOKI = 3-2 = 1$

$\Delta_0 + \delta R = \Delta_S$



$\Delta_0 = \int_L \frac{m_0 m_1}{EI}\,dx$



$= \frac{L/2}{6EI}\left(-\frac{PL}{2}\right)\left(2L + \frac{L}{2}\right)$

$= -\frac{5PL^3}{48EI}$

$$\delta = \int_L \frac{m_1 m_1}{EI} dx = \frac{L^3}{3EI}$$

Δ_s = spring deformation

$$\therefore -\frac{5PL^3}{48EI} + \frac{L^3}{3EI}\cdot R = -\frac{R}{\frac{L^3}{12EI}}$$

deformation = $\frac{\text{Force}}{\text{stiffness}}$

$= \frac{R}{K}$

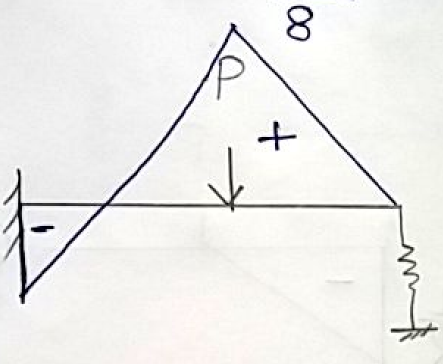
$$\Rightarrow \left(\frac{1}{3}+\frac{1}{12}\right)\frac{L^3}{EI}\cdot R = \frac{5PL^3}{48EI}$$

$$\therefore R = \frac{5P}{48}\times\frac{12}{5} = \boxed{\frac{P}{4}}$$

Beam এ R ↑ হলে, spring এ R ↓.
so, spring deformation ↓.
অর্থাৎ beam এর R ও spring deformation এর direction opposite.

$$M = m_0 + m_1 R$$

$$M_a = -\frac{PL}{4}$$



- Ordinary DE দিয়ে যেগুলো solve করা যায়, সেগুলো 1D structure. f(x).
- Partial DE দিয়ে যেগুলো solve করা যায় সেগুলো 2D, 3D structure.
- 1D structure কে skeleton structure ও বলা হয়।

Longitudinal axis → cross section এর c.g দিয়ে যেই line pass করে। এটাই আমরা আঁকি।

- we assume all the (parameters and sectional properties, loads, deformation) are only f(x). কারণ,

$$\boxed{EI\frac{d^4y}{dx^4} = p(x)} \leftarrow \text{ODE}$$

$\boxed{x}$

1D – beam, frame

2D – slab, truss

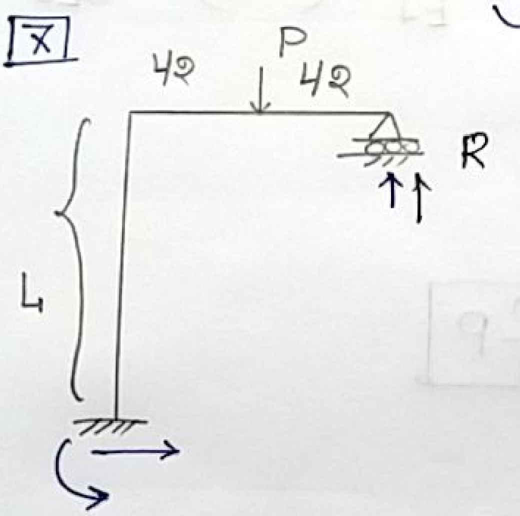
3D – Dome, shell

- Idealization: beam 3D বস্তু। But analysis সহজ করার জন্য 1D ধরে নিচ্ছি। একে dimensional idealization বলে।

- আরেকটা হল material idealization.

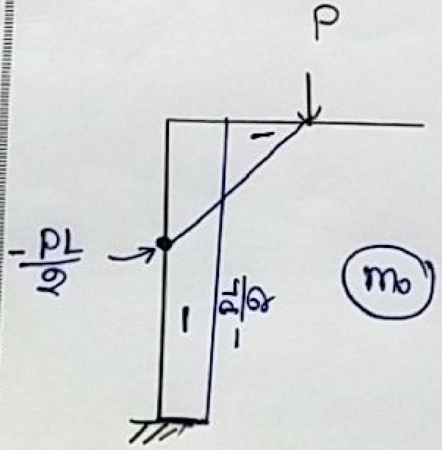
Frame

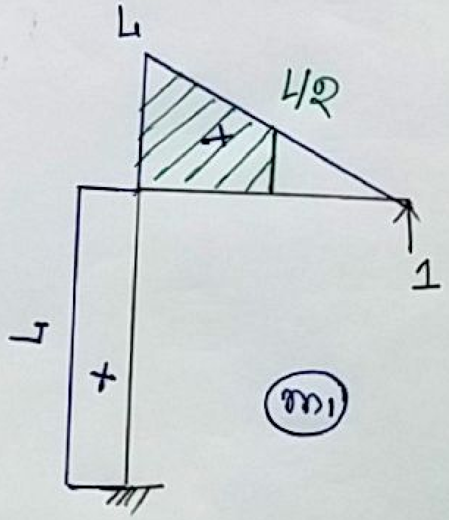
$\boxed{x}$



E, I, A

DOSI = 3 − 2 = 1





$$\Delta_0 = \int_L \frac{m_0 m_1}{EI}\,dx$$

$$= \frac{L}{EI}\left(-\frac{PL}{2}\right)L + \frac{L/2}{6EI}\left(-\frac{PL}{2}\right)\left(2L+\frac{L}{2}\right)$$

$$= -\frac{PL^3}{2EI} - \frac{5PL^3}{48EI} = -\frac{29PL^3}{48EI}$$

$$\delta = \int_L \frac{m_1 m_1}{EI}\,dx$$

$$= \frac{L}{EI} \times L \times L + \frac{L}{3EI} \times L \times L = \frac{L^3}{EI}\left(1+\frac{1}{3}\right) = \frac{4L^3}{3EI}$$

$$\therefore\ -\frac{29PL^3}{48EI} + \frac{4L^3}{3EI}R = 0$$

$$\therefore\ R = \frac{29}{48} \times \frac{3}{4} \times P = \boxed{\frac{29}{64}P}$$

Lec-6

$\boxed{8}$

axial force consider করে।

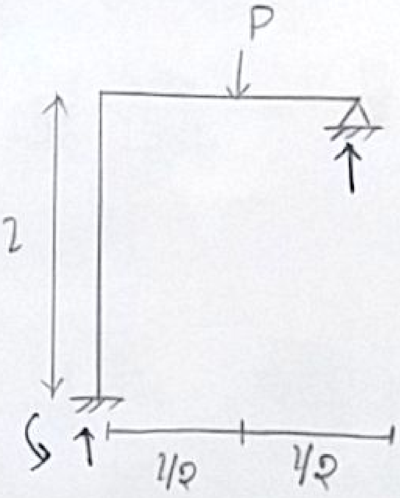
DOSI = 5−3 = 2

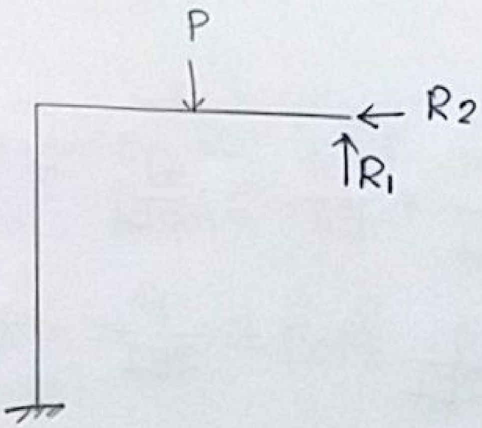
$$\{\Delta_0\} + [\delta]\{R\} = \{\Delta_S\}$$

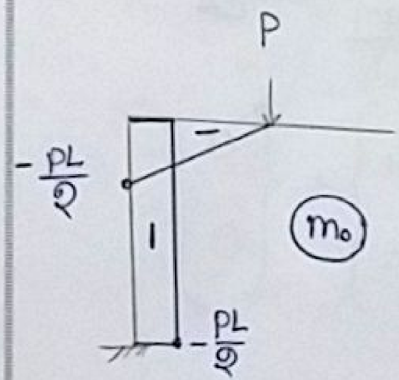
$$\begin{Bmatrix}\Delta_{01}\\ \Delta_{02}\end{Bmatrix} + \begin{bmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{bmatrix}\begin{Bmatrix}R_1\\ R_2\end{Bmatrix} = \begin{Bmatrix}\Delta_{S1}\\ \Delta_{S2}\end{Bmatrix}$$

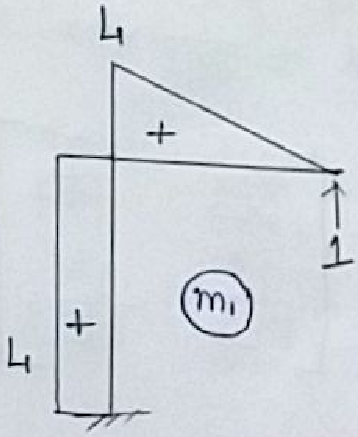
$\Delta_{S1} = \Delta_{S2} = 0$

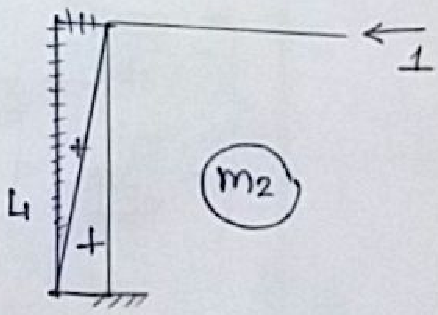
· compression side এ diagram আঁকি।











$$\Delta_{01} = \int_l \frac{m_0 m_1}{EI} dx$$

$$= \frac{l/2}{6EI} \times \left(-\frac{PL}{2}\right)\left(2l + \frac{l}{2}\right) + \frac{l}{EI}\left(-\frac{Pl}{2}\right)l = -\frac{5Pl^3}{48EI} - \frac{PL^3}{2EI} = -\frac{29Pl^3}{48EI} - \frac{PL}{EA}$$

$$\Delta_{02} = \int_l \frac{m_0 m_2}{EI} dx$$

$$= \frac{l}{2EI}\left(-\frac{PL}{2}\right)l = -\frac{Pl^3}{4EI}$$

$$\delta_{11} = \int_l \frac{m_1 m_1}{EI} dx$$

$$= \frac{l}{3EI} \times l \times l + \frac{l}{EI} \times l \times l = \frac{l^3}{3EI} + \frac{l^3}{EI} = \frac{4l^3}{3EI} + \frac{L}{EA}$$

$$\delta_{12} = \delta_{21} = \int_l \frac{m_1 m_2}{EI} dx = \frac{l}{2EI} \times l \times l = \frac{l^3}{2EI}$$

$$\delta_{22} = \int \frac{m_2 m_2}{EI} dx = \frac{l}{3EI} \times l \times l = \frac{l^3}{3EI} + \frac{L}{EA}$$

$$\begin{Bmatrix} -\frac{29PL^3}{48EI} \\ -\frac{Pl^3}{4EI} \end{Bmatrix} + \begin{bmatrix} \frac{4l^3}{3EI} & \frac{l^3}{2EI} \\ \frac{l^3}{2EI} & \frac{l^3}{3EI} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$R_1 = \frac{\begin{vmatrix} \frac{29PL^3}{48EI} & \frac{l^3}{2EI} \\ \frac{PL^3}{4EI} & \frac{l^3}{3EI} \end{vmatrix}}{\begin{vmatrix} \frac{4l^3}{3EI} & \frac{l^3}{2EI} \\ \frac{l^3}{2EI} & \frac{l^3}{3EI} \end{vmatrix}} = \frac{\frac{29}{48}\times\frac{1}{3}-\frac{1}{2}\times\frac{1}{4}}{\frac{4}{3}\times\frac{1}{3}-\frac{1}{2}\cdot\frac{1}{2}}P = \frac{\frac{11}{144}}{\frac{7}{36}}P$$

$$= \frac{11}{28}P$$

$$= 0.4P$$

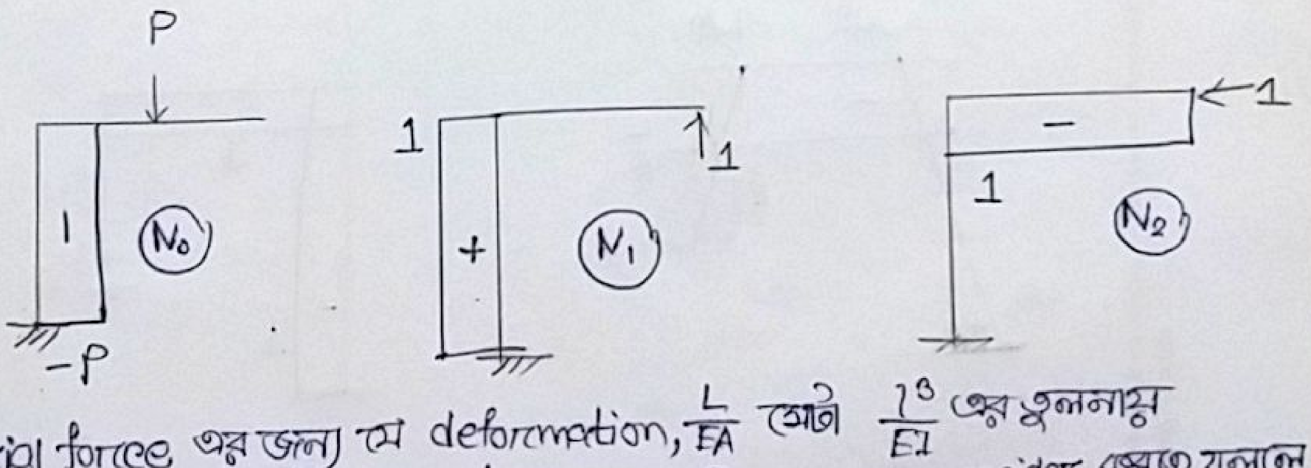
$$R_2 = \frac{\begin{vmatrix} \frac{4l^3}{3EI} & \frac{29PL^3}{48EI} \\ \frac{l^3}{2EI} & \frac{PL^3}{4EI} \end{vmatrix}}{7/36}$$

ঐ support এ $R_1 = 0.45P$

A " " $R_1 = 0.4P$; horizontal Force R_2 থাকায় কমে গেছে R_1.

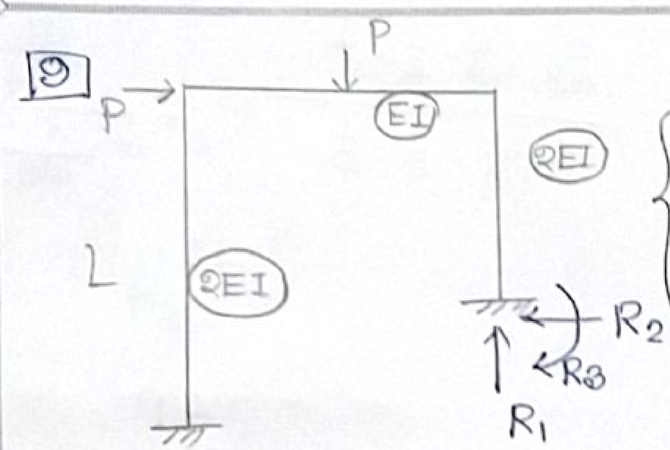
তাই Beam এ axial force neglect করলেও Frame এ neglect করা যাবে না।

তখন, $\Delta_{01} = \int_L \frac{m_0 m_1}{EI}dx + \sum \frac{N_0 N_1}{EA} L$



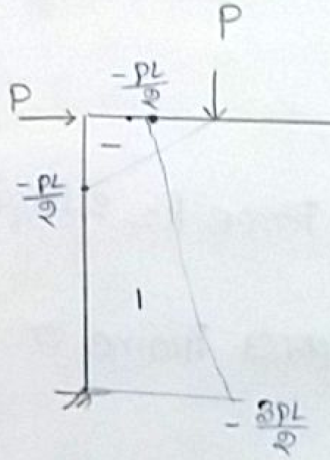
• axial force এর জন্য যে deformation, $\frac{L}{EA}$ যেটা $\frac{l^3}{EI}$ এর তুলনায় অনেক কম, তাই সেটা neglect করতে পারি। exam এ consider করতে বললে consider (A. Defor.) করব।

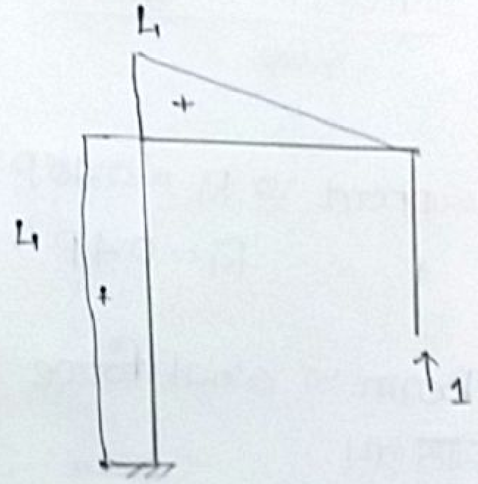
$\boxed{3}$

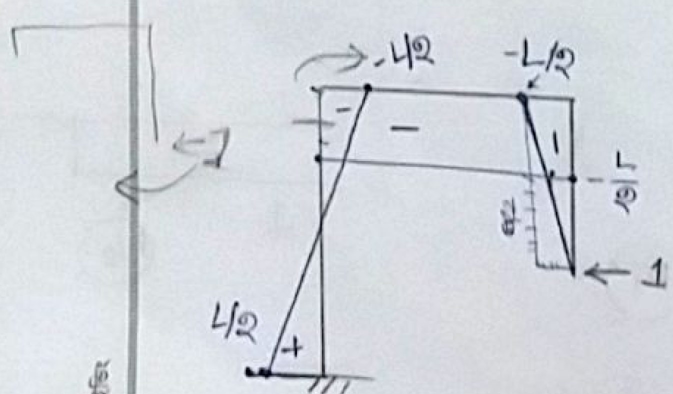


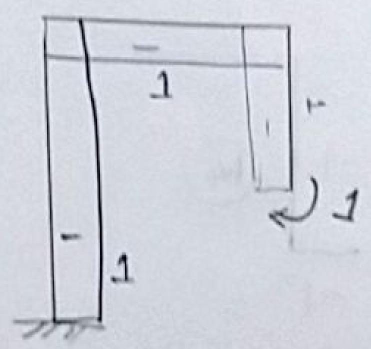
DOSI = 3

$$\begin{Bmatrix} \Delta_{01} \\ \Delta_{02} \\ \Delta_{03} \end{Bmatrix} + \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{12} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$









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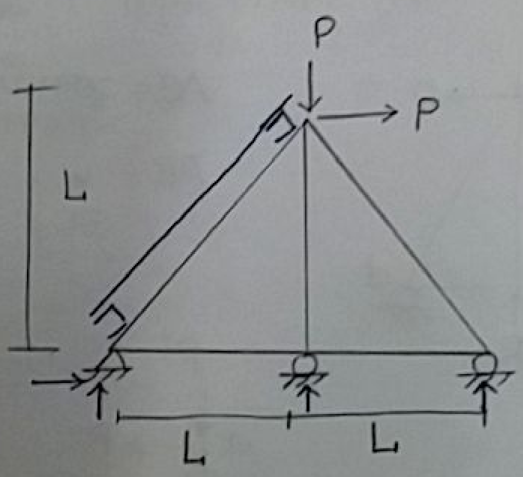
Truss Problem:

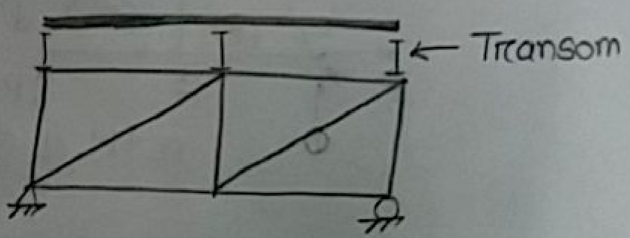


Truss Characteristics

• সবগুলো member pin connected.

• সবগুলো Force joint এ কাজ করবে।

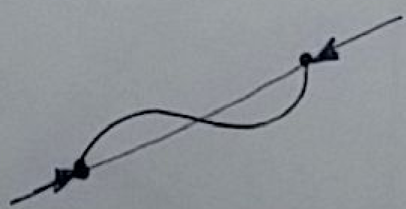
• perlin গুলো joint এ connected থাকে, so load joint এ আছে।



• co-planner con-current হতে হবে Moment যাতে develop করতে না পারে, সে জন্য। Line of action of resultant force at joints same হতে হবে।

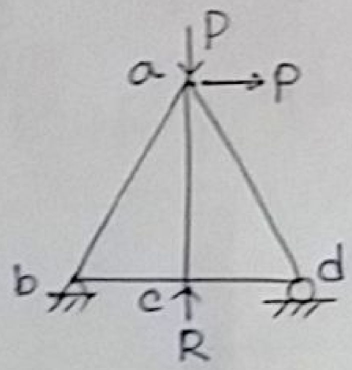
• Truss এর Member গুলো straight হলে শুধু axial force develop করে।

• Curve হলে V, M ও develop করে।



• Axial force develop করলে material optimization হয়, X-section এ uniform stress develop করে।

কিন্তু V, M develop করলে কোথাও কম, কোথাও বেশি material লাগবে।



AC = zero force member

AB = " " "

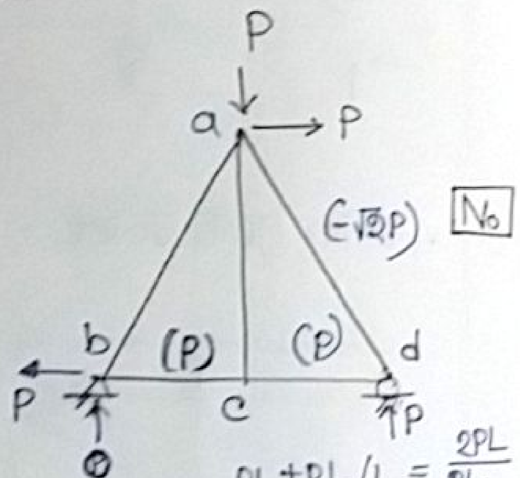
$DOSI = 4-3 = 1$

$\Delta_0 + \delta R = \Delta_s$

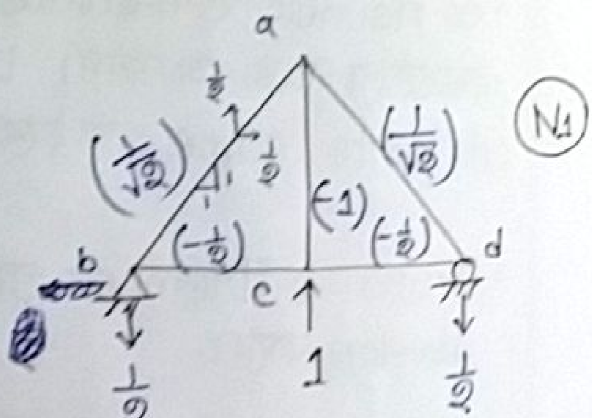
$$\Delta_0 = \sum \frac{N_0 N_1 L}{AE}$$

$$\delta = \sum \frac{N_1 N_1 L}{AE}$$

$\Delta_s = 0$



$PL + PL/L = \frac{2PL}{2L}$
$= P$



• ← (C)

• → (T)

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	Bars	L	N_0	N_1	$N_0 N_1 L$	$N_1 N_1 L$
diagonal	ab	$\sqrt{2}L$	0	$1/\sqrt{2}$	0	$L/\sqrt{2}$
	ad	$\sqrt{2}L$	$-\sqrt{2}P$	$1/\sqrt{2}$	$-\sqrt{2}PL$	$+L/\sqrt{2}$
bottom chord	bc	L	P	$-1/2$	$-\frac{1}{2}PL$	$L/4$
	cd	L	P	$-1/2$	$-\frac{1}{2}PL$	$L/4$
vertical	ac	L	0	-1	0	L
					$\Sigma = -PL(1+\sqrt{2})$	$\Sigma = L\left(\frac{3}{2}+\sqrt{2}\right)$

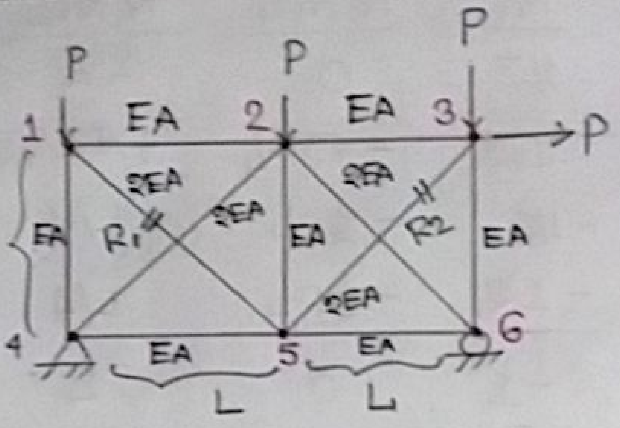
$$R = -\frac{\Delta_0}{\delta} = -\frac{\frac{-PL(1+\sqrt{2})}{EA}}{\frac{L\left(\frac{3}{2}+\sqrt{2}\right)}{EA}} = \frac{1+\sqrt{2}}{\frac{3}{2}+\sqrt{2}} \cdot P$$

$$\boxed{N = N_0 + N_1 R}$$

$$N_{ab} = 0 + \frac{1}{\sqrt{2}} \cdot \left(\frac{1+\sqrt{2}}{\frac{3}{2}+\sqrt{2}}\right) \cdot P =$$

$$N_{ad} = -\sqrt{2}P + \frac{1}{\sqrt{2}} \left(\frac{1+\sqrt{2}}{3/2+\sqrt{2}}\right) \cdot P =$$

11



Internal indeterminancy

Unknowns:

$b + r = \underbrace{11}_{\text{member force}} + \underbrace{3}_{\text{support reaction}} = 14$

concurrent force system এর জন্য equi. eqⁿ = ২টা,
Moment zero. F_x, F_y.

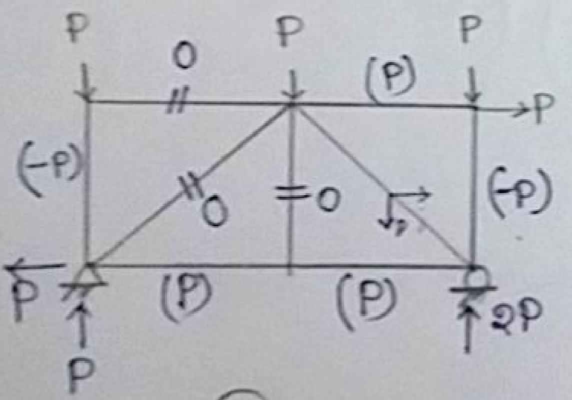
Known = $2j = 2 \times 6 = 12$
↖ no. of joint

∴ DOSI = 14 − 12 = 2 (internal)

Support force কে redundant করতে পারব না, unbalance হয়ে যাবে।

$$\begin{Bmatrix} \Delta_{01} \\ \Delta_{02} \end{Bmatrix} + \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} \Delta_{j1} \\ \Delta_{j2} \end{Bmatrix}$$

$$\Delta_{01} = \sum \frac{N_0 N_1 L}{EA}$$



N_0

Tp বা external force বা অন্য কিছুর effect Δ_0 তে আসে।

Redundent এর misfit এর effect Δ_j তে আসে।

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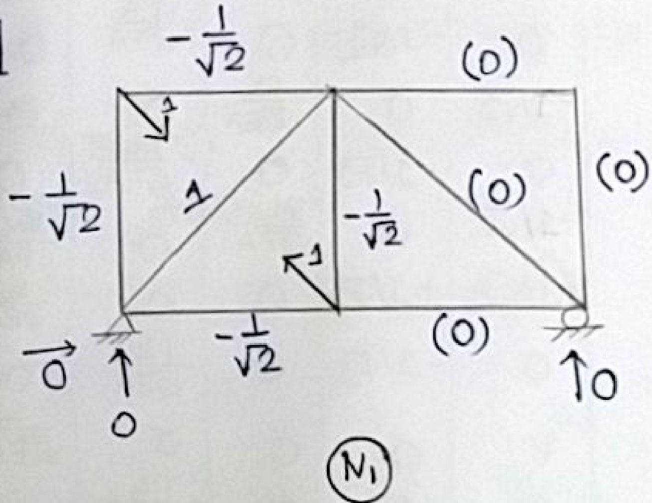
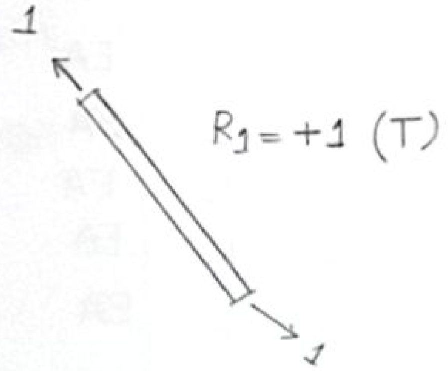
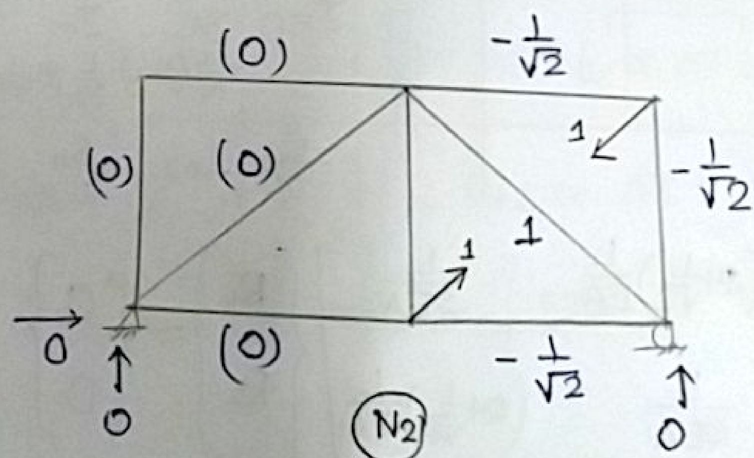
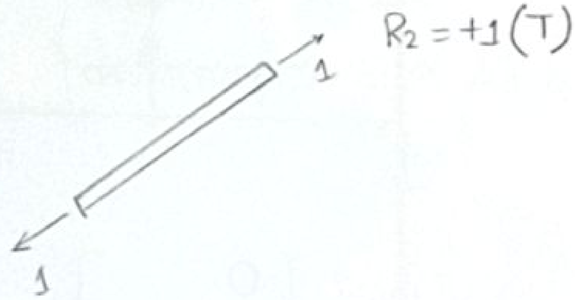
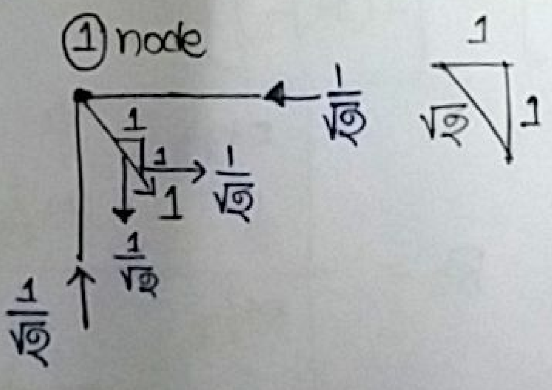








Global structure চিন্তা করলে 1, 1 দিকে ২টি colinear, balance হয়ে যাবে। তাই support reaction গুলো zero.



member	L	EA	N_0	N_1	N_2	$\frac{N_0N_1L}{EA}$	$\frac{N_0N_2L}{EA}$	$\frac{N_1^2L}{EA}$	$\frac{N_2^2L}{EA}$	$\frac{N_1N_2L}{EA}$
12	L	EA	0	$-1/\sqrt{2}$	0	0	0	$\frac{L}{2EA}$	0	0
23	L	EA	P	0	$-1/\sqrt{2}$	0	$-\frac{PL}{\sqrt{2}EA}$	0	$\frac{L}{2EA}$	0
45	L	EA	P	$-1/\sqrt{2}$	0	$-\frac{PL}{\sqrt{2}EA}$	0	$\frac{L}{2EA}$	0	0
56	L	EA	P	0	$-1/\sqrt{2}$	0	$-\frac{PL}{\sqrt{2}EA}$	0	$\frac{L}{2EA}$	0
14	L	EA	$-P$	$-1/\sqrt{2}$	0	$\frac{PL}{\sqrt{2}EA}$	0	$\frac{L}{2EA}$	0	0
25	L	EA	0	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	0	$\frac{L}{2EA}$	$\frac{L}{2EA}$	$\frac{L}{2EA}$
36	L	EA	$-P$	0	$-1/\sqrt{2}$	0	$\frac{PL}{\sqrt{2}EA}$	0	$\frac{L}{2EA}$	0
24	$\sqrt{2}L$	$2EA$	0	1	0	0	0	$\frac{L}{\sqrt{2}EA}$	0	0
26	$\sqrt{2}L$	$2EA$	$-\sqrt{2}P$	0	1	0	$-\frac{PL}{\sqrt{2}EA}$	0	$\frac{L}{\sqrt{2}EA}$	0
						$\Sigma=0$	$\Sigma=-\frac{PL}{AE}(1+\frac{1}{\sqrt{2}})$	$\Sigma=\frac{L}{EA}(2+\frac{1}{\sqrt{2}})$	$\Sigma=\frac{L}{EA}(2+\frac{1}{\sqrt{2}})$	$\Sigma=\frac{L}{2EA}$
						Δ_{01}	Δ_{02}	δ_{11}	δ_{22}	δ_{12}

$$\therefore \begin{Bmatrix} 0 \\ \frac{PL}{AE}(1+\frac{1}{\sqrt{2}}) \end{Bmatrix} + \begin{bmatrix} (2+\frac{1}{\sqrt{2}})\frac{L}{EA} & \frac{L}{2EA} \\ \frac{L}{2EA} & (2+\frac{1}{\sqrt{2}})\frac{L}{EA} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 2+\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{2} & 2+\frac{1}{\sqrt{2}} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -(1+\frac{1}{\sqrt{2}}) \end{Bmatrix} P$$

$$R_1 = \frac{\begin{vmatrix} 0 & \frac{1}{2} \\ -(1+\frac{1}{\sqrt{2}}) & 2+\frac{1}{\sqrt{2}} \end{vmatrix}}{\begin{vmatrix} 2+\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{2} & 2+\frac{1}{\sqrt{2}} \end{vmatrix}}, \quad R_2 = \frac{\begin{vmatrix} 2+\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -(1+\frac{1}{\sqrt{2}}) \end{vmatrix}}{\begin{vmatrix} 2+\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{2} & 2+\frac{1}{\sqrt{2}} \end{vmatrix}}$$

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$$\boxed{N = N_0 + N_1R_1 + N_2R_2}$$

Redundent এর missfit

- Redundent এর দিকে shortening হলে R.H.S. এ Δ (+ve) হিসেবে বসবে।
- " elongated হলে " " Δ (−ve) হিসেবে বসবে।

For Tp change

$$\Delta_{01} = \sum\left(\frac{N_0N_1L}{EA} + N_1 \cdot \overbrace{\alpha \cdot \Delta T \cdot L}^{\text{For Tp.}}\right)$$

strain $= \alpha \cdot \Delta T$
deformation $= \alpha \cdot \Delta T \cdot L$

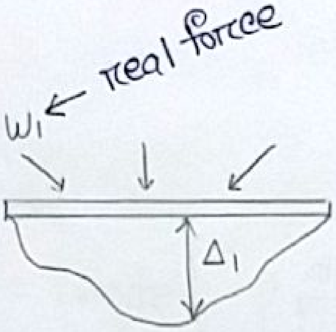
$$\Delta_{02} = \sum\left(\frac{N_0N_2L}{EA} + N_2 \cdot \alpha \cdot \Delta T \cdot L\right)$$

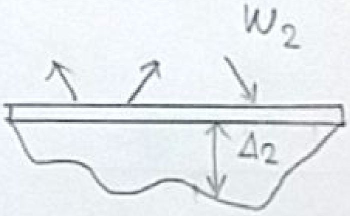
$N_0 \leftarrow$ For external Force

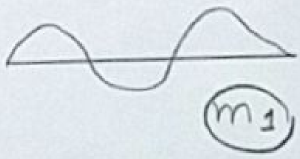
শনিবার ৮:০০টায় রাকিব স্যারের class
মঙ্গলবার ৯:৩০ এ C.T → Truss

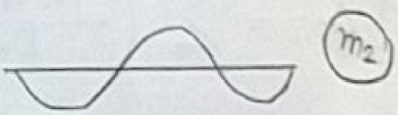
Influence Lines of Indeterminate Structure

· Betty's Law :









virtual load x real deformation = virtual work

Here, $P = W_1 =$ real force

$Q = W_2 =$ virtual "

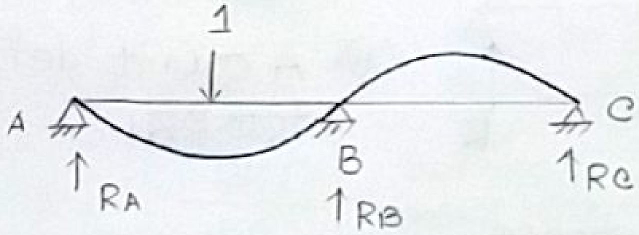
$$\underbrace{\sum W_1 \Delta_2 = \int \frac{m_1 m_2}{EI} dx}_{\text{internal work done}}$$

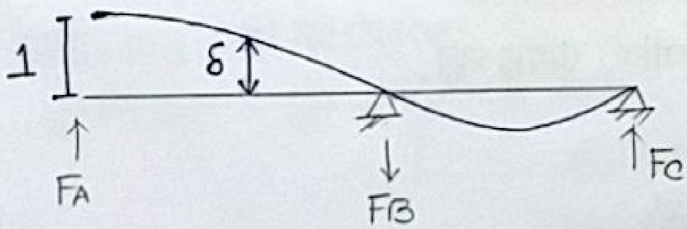
Again, if, $P = W_2$, $Q = W_1$

then, $\sum W_2 \Delta_1 = \int_L \frac{m_2 m_1}{EI} dx$

$$\therefore \boxed{\sum W_1 \Delta_2 = \sum W_2 \Delta_1}$$

Muller-Breslau's Principle





A তে unit deformation দিলে F_A পরিমান load দিতে হবে। ফলে B তে F_B ও F_C force develop করবে।

$R_A \times 1 - 1 \times \delta = 0$

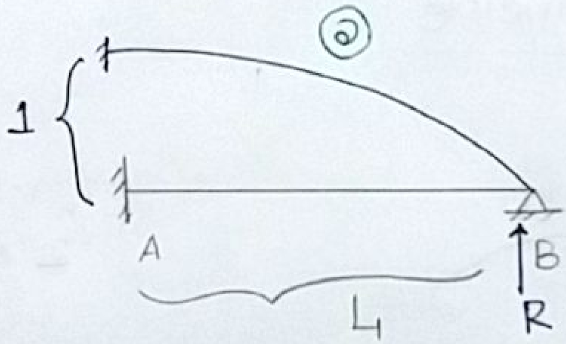
$\therefore \boxed{R_A = \delta}$

যেখানকার I.L চাই ওখানে unit deformation দিলে যেই deformed shape পাব সেটাই ঐ point এ reaction.

- determinate structure এ rigid body motion হয়, যখন strain হয় না। So, I.L সবসময় straight line হবে।

- Indeterminate structure এ rigid body motion হয় না, strain হয়। তাই I.L curve হবে।

$\boxed{1}$

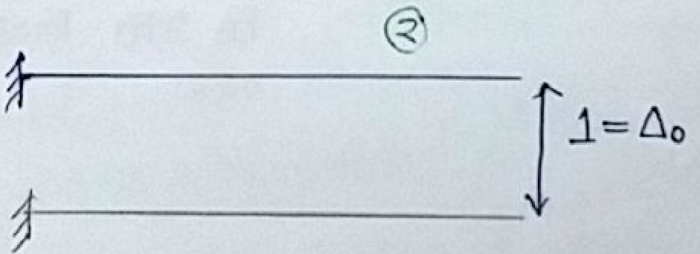


- Draw I.L of R_A.

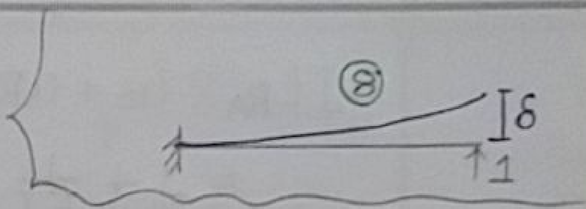
(i) • A তে unit deformation দিতে হবে।

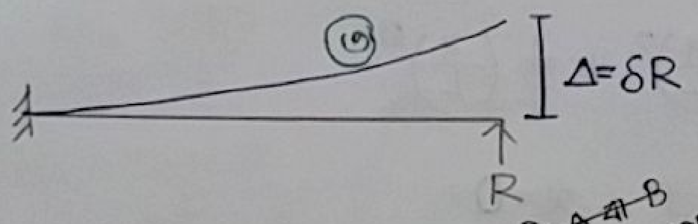
(ii) • যদি B কে redundant choose করি, তবে released structure হবে, A তে unit deformation দেবার পর,

≡



যেহেতু unit deformation দিয়েছি, তাই $\Delta_0 = 1$.





R এর জন্য deform mation Δ.

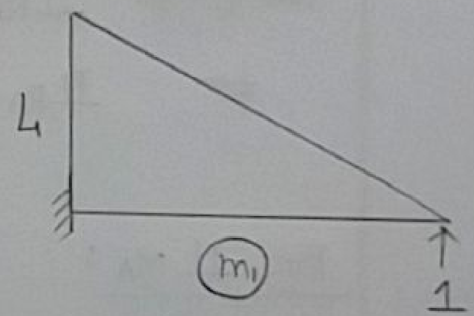
So, $\Delta_0 + \Delta = \Delta_j$ → ? ~~যদি A বা B point এ support settlement হয় তবে কি হবে?~~

$\Rightarrow \Delta_0 + \delta R = \Delta_j$

$\Rightarrow 1 + \frac{L^3}{3EI} R = 0$

$$\boxed{\therefore R = -\frac{3EI}{L^3}}$$

Δ_j → হল ১ নং Diagram এ যে point কে redundent choose করছি সেই point এর Δ. এখানে Δ_j for ১ নং diagram হল $\Delta_B = 0$.



$$\delta = \int_L \frac{m \cdot m_1}{EI} dx$$

- আমাদের ১ নং curve এর eqn লাগবে।
- ১ নং eqn = ২ + ৩ এর eqn

৩ এর eqn পাওয়া যাবে ৪ নং eqn হতে।

$$\frac{d^2u}{dx^2} = \frac{M}{EI}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{EI}\int_0^x M\,dx = \frac{1}{EI}\int_0^x (L-x)\,dx = \frac{1}{EI}\left(Lx - \frac{x^2}{2}\right)$$

$$\therefore u(x) = \frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)$$

$\circlearrowright$ +ve
$\circlearrowleft$ -ve

$$IL_{RA} = u_D + u_R$$

$$= 1 + \frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)\left(-\frac{3EI}{L^3}\right)$$

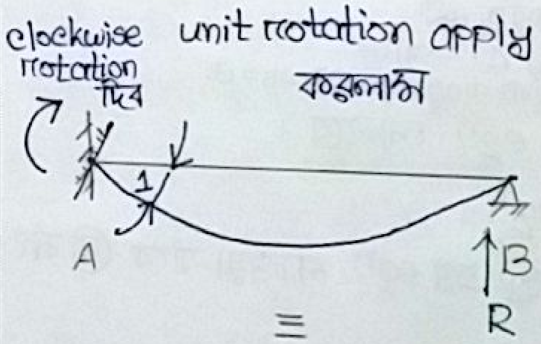
$$= 1 - \frac{3}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{2}\left(\frac{x}{L}\right)^3$$

~~For L = 0~~: $IL_{RA} = 1$, $x = 0$

~~L =~~ $IL_{RA} = 0$, $x = L$

For IL_{M_A}:

- Reaction এর জন্য deformation Δ.
- Moment এর জন্য rotation apply করব।



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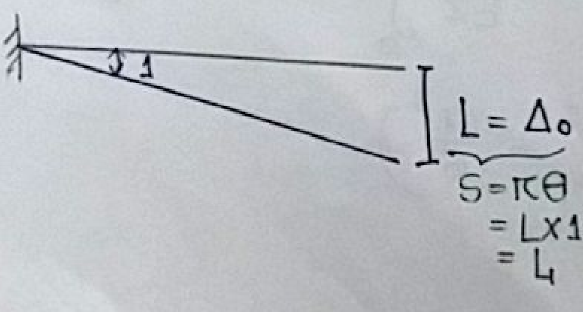

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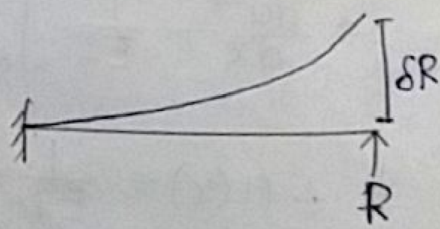

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$\Delta_0 + \delta R = \Delta_j$

$\Rightarrow -L + \frac{L^3}{3EI} R = 0$

$\therefore R = \frac{3EI}{L^2}$

$s = r\theta$
$= x \times 1$
$= x$

$ILM_A = u_0 + uR$

$= -x + \frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right) \times \frac{3EI}{L^2}$

$= -x + \frac{3}{2}\frac{x}{L}x - \frac{1}{2}\left(\frac{x}{L}\right)^2 \cdot x$

$= -x\left\{1 - \frac{3}{2}\frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right\}$

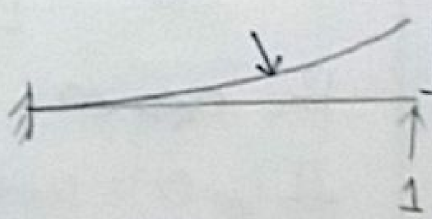
$x = 0,\ ILM_A = 0$

$x = L,\ ILM_A = 0$

unit deformation এর জন্য

Load

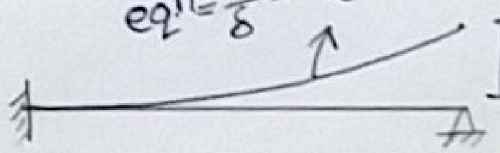
$eq^n,\ u(x) = \frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)$



$\delta = 1$ হলে, $\cancel{1 = \frac{L^3}{3EI}}$

For IL_{RB}:

$eq^n = \frac{u}{\delta} = \frac{3}{2}\left(\frac{x}{L}\right)^2 - \frac{1}{2}\left(\frac{x}{L}\right)^3$

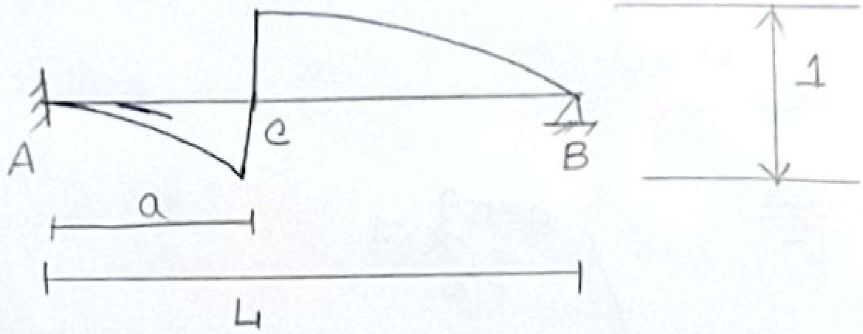


$\delta \rightarrow u$

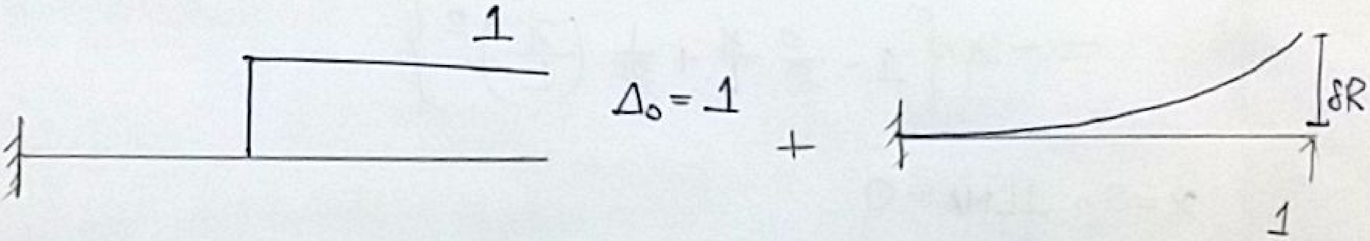
$\therefore 1 \rightarrow \frac{u}{\delta}$

$\therefore eq^n = \frac{u}{\delta} = \frac{\frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)}{\frac{L^3}{3EI}} = 3\left(\frac{x^2}{2L^2} - \frac{x^3}{6L^3}\right)$

$\boxed{2}$



$\equiv$



$\cancel{\Delta_0 = \delta R}$ $\Delta_0 + \delta R = 0$

$$R = -\frac{3EI}{L^3}$$

for $0 \le x < a$

$$IL_{Vc}(x) = u_0 + uR$$

$$= 0 + \frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)\left(-\frac{3EI}{L^3}\right)$$

$$= -\frac{3}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{2}\left(\frac{x}{L}\right)^3$$

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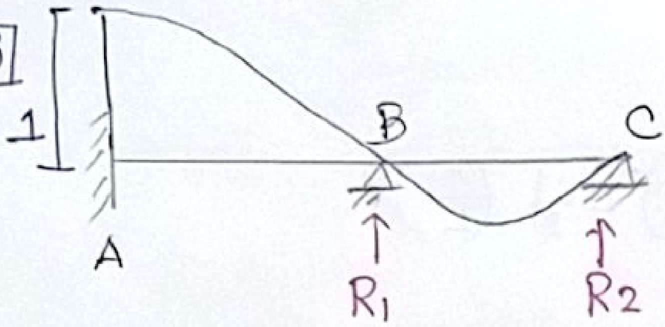
For $a < x \leq L$

$IL_{V_C}(x) = u_0 + u_R$

$$= 1 - \frac{3}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{2}\left(\frac{x}{L}\right)^3$$

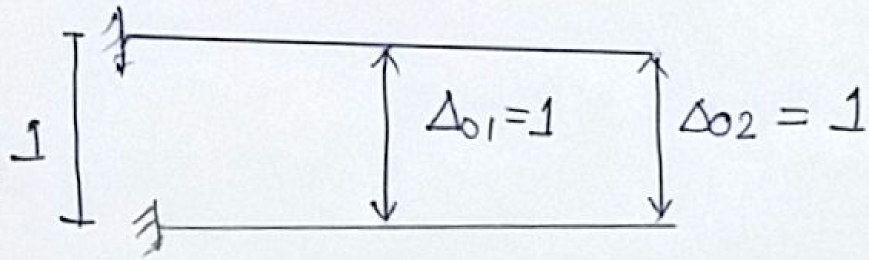
$\boxed{৩}$



$ILR_A(x) = ?$

$\{\Delta_0\} + [\delta]\{R\} = \{\Delta_j\}$

$\equiv$



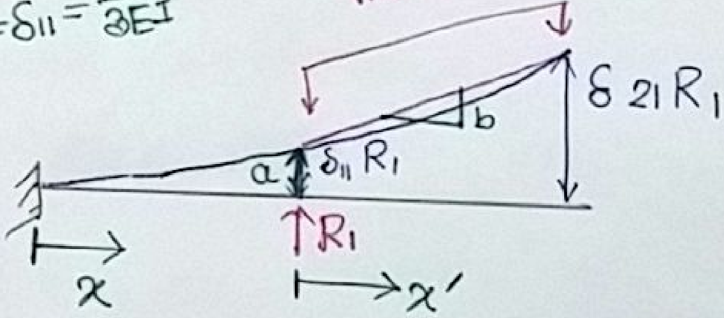
$+$

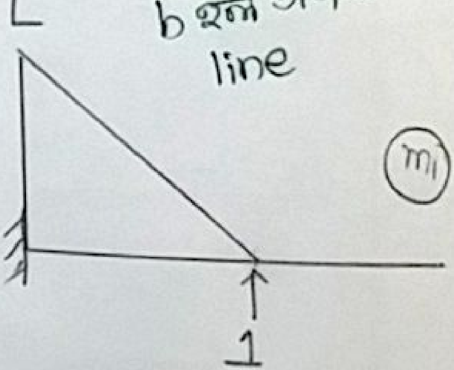
moment নাই, bend করবে না, তাই st. line হবে

eqⁿ of this st. line $= a + bx'$

b হল slope of line

$a = \delta_{11} = \frac{L^3}{3EI}$





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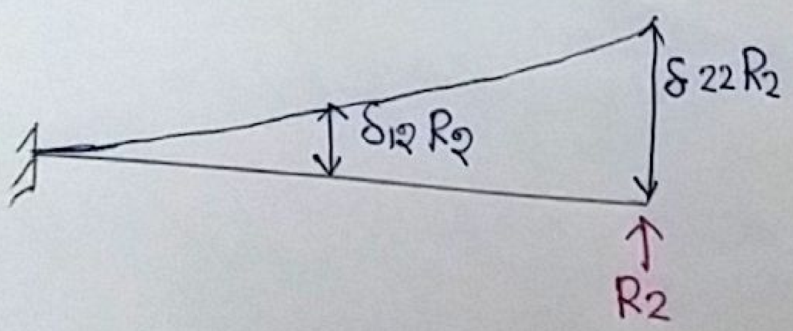

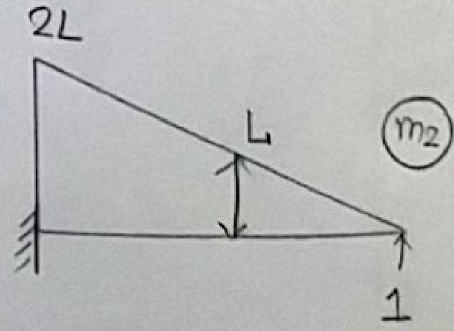

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$$\delta_{11} = \int_L \frac{m_1 m_1}{EI} dx = \frac{L}{3EI} \times L \times L = \frac{L^3}{3EI}$$

$$\delta_{12} = \delta_{21} = \int_L \frac{m_1 m_2}{EI} dx = \frac{L}{6EI} \times L \times (2 \times 2L + L) = \frac{5L^3}{6EI}$$

$$\delta_{22} = \int_L \frac{m_2 m_2}{EI} dx = \frac{2L}{3EI} \times 2L \times 2L = \frac{8L^3}{3EI}$$

$$\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{bmatrix} \frac{L^3}{3EI} & \frac{5L^3}{6EI} \\ \frac{5L^3}{6EI} & \frac{8L^3}{3EI} \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$R_1 = \frac{\begin{vmatrix} -1 & 5/6 \\ -1 & 8/3 \end{vmatrix}}{\begin{vmatrix} 1/3 & 5/6 \\ 5/6 & 8/3 \end{vmatrix}} \frac{EI}{L^3} \qquad R_2 = \frac{\begin{vmatrix} 1/3 & -1 \\ 5/6 & -1 \end{vmatrix}}{\begin{vmatrix} 1/3 & 5/6 \\ 5/6 & 8/3 \end{vmatrix}} \frac{EI}{L^3}$$

$$= -\frac{11/6}{7/36} \frac{EI}{L^3} \qquad = \frac{1/2}{7/36} \frac{EI}{L^3}$$

$$= -\frac{66}{7} \frac{EI}{L^3} \qquad = \frac{18}{7} \frac{EI}{L^3}$$

$$IL\,R_A(x) = u_0 + u_1 R_1 + u_2 R_2$$

cont..... (A)

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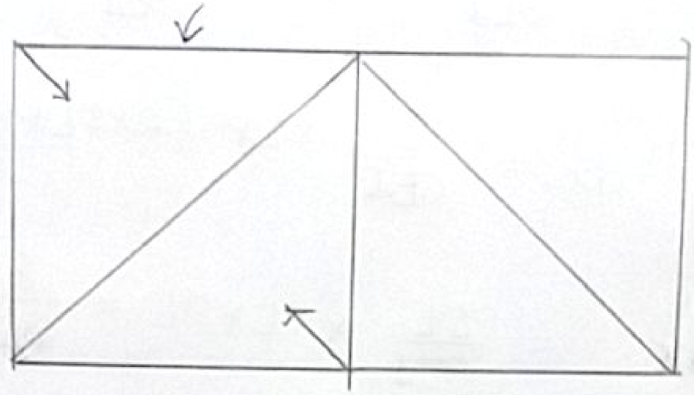

$$\boxed{\begin{gathered} -\frac{N_1^2}{EA} L \cdot R_1 = \Delta_j \\ \{\Delta_0\} + [\delta]\{R\} = \{\Delta_j\} \end{gathered}}$$

আগের math এর সাথে ঃ এটা add হবে।

~~but deform~~

but deformation force এর উল্টা, তাই (-ve)

- elongation বা shortening Redundent এর জন্যই নিবা। অর্থাৎ, কোন member এর elongation বা shortening দেয়া থাকলে তাকেই Redundent choose করব।

- But কোন member এর Tp এর effect দেয়া থাকলে ওটাকে Redundent choose করব না।

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(A) cont..... • u মানে BMD কে ২বার integrate করে।
~~অথবা~~ • u কে ২বার differentiate করলে slope b পাওয়া যাবে।

For $0 \leq x \leq L$

• $u_0 = 1$

• $u_1 = \frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)$

For $L \leq x \leq 2L$

• $u_0 = 1$

• $u_1 = \frac{L^3}{3EI} + \frac{L^2}{2EI}(x-L)$

Here, eqn of line $= a + bx'$

$a = \frac{L^3}{3EI} = \delta_{11}$

in general, $\frac{d^2u}{dx^2} = \frac{L-x}{EI}$

$\frac{du}{dx} = \frac{1}{EI}\left(Lx - \frac{x^2}{2}\right)$

$\therefore \theta(L) = \frac{L^2}{2EI}$ (slope) $= b$

$\therefore b = \frac{L^2}{2EI}$

$x' = (x - L)$

• $u_2 = \frac{1}{EI}\left(Lx^2 - \frac{x^3}{6}\right)$

$\therefore IL R_A = 1 + \frac{1}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right) * \left(-\frac{66}{7}\frac{EI}{L^3}\right) + \frac{1}{EI}\left(Lx^2 - \frac{x^3}{6}\right) \times \left(\frac{18}{7}\frac{EI}{L^3}\right)$

$\boxed{\text{for } 0 \leq x \leq L}$

• $u_2 = \frac{1}{EI}\left(Lx^2 - \frac{x^3}{6}\right)$

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- যেটার IL আঁকতে চাই সেদিকে unit deformation দিচ্ছি।

* যেহেতু এখানে Reaction হলো সবসময়ই concentrated load বা moment হবে, তাই BMD সবসময়ই constant বা st. line হবে।