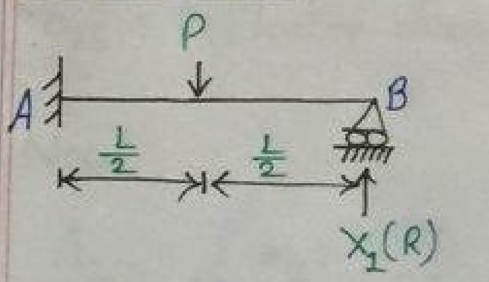
"Method of Consistent Deformation or Deflection Supervision or Force or Flexibility Method"

Advantages of Flexibility Method :

- Accurate method for analysis of Indeterminate structures
- Suitable for analysis of any structure
- Suitable for computerized approach

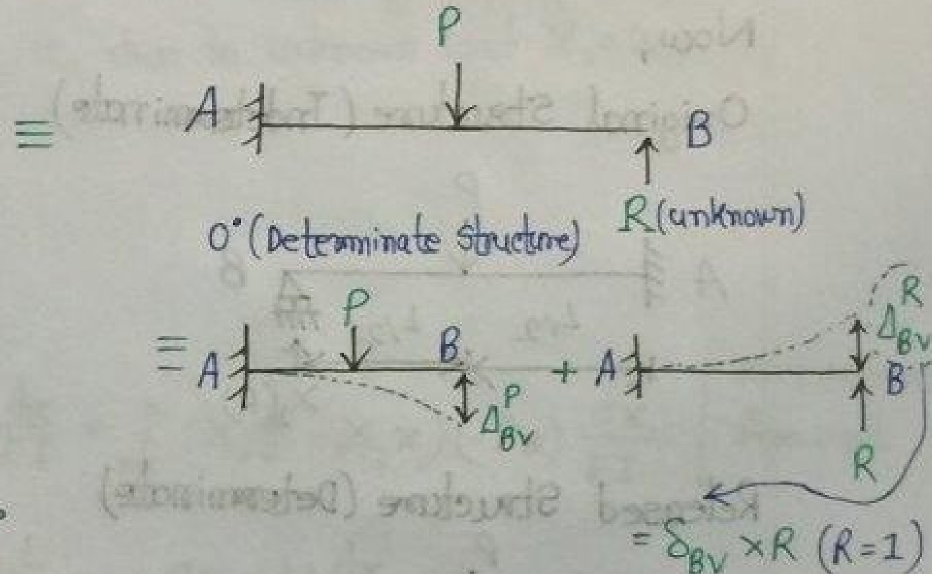
Basic Idea :



No. of Reactions = 3+1=4

No. of Equations = 3

Degree of Indeterminancy = 1°



Here, Δ_{BV}^{P} = Vertical deflection due to External Load in Released Structure

δ_{BV}^{R} = Vertical deflection due to $R=1$ in Released or Determinate structure

By unit load method,

$$\Delta_{BV}^{P} = \int_0^L m_0 m_1 \frac{dx}{EI}$$

$$\delta_{BV}^{R} = \int_0^L m_1 m_1 \frac{dx}{EI}$$

In this example,

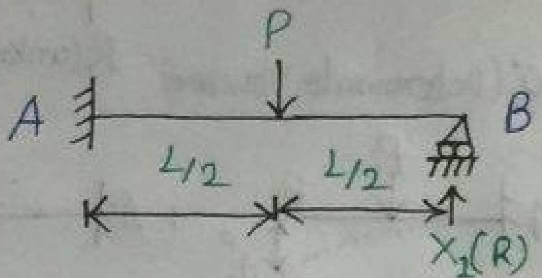
$$\Delta_{BV} = \Delta_{BV}^{P} + \Delta_{BV}^{R}$$

$$\Rightarrow 0 = \Delta_{BV}^{P} + \delta_{BV}^{R} \times R \qquad \left[\because \text{Deflection at B (roller support) is } 0\right]$$

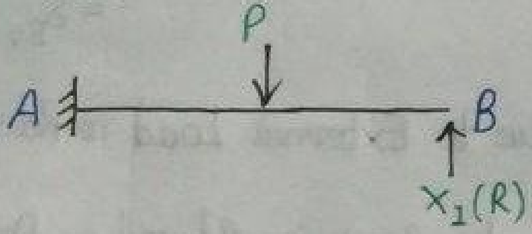
$$\therefore R = -\frac{\Delta_{BV}^{P}}{\delta_{BV}^{R}}$$

Now,

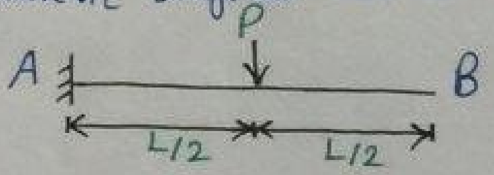
Original Structure (Indeterminate)

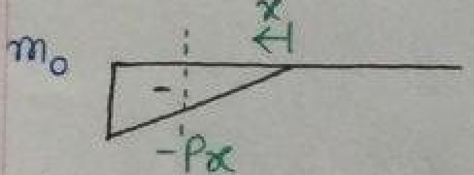


Released Structure (Determinate)

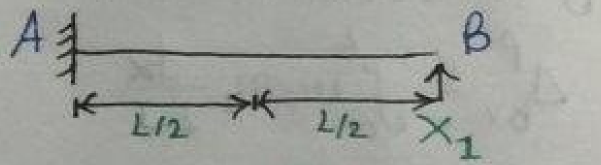


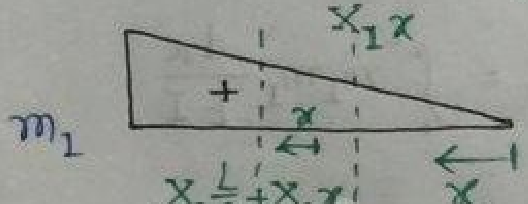
Moment Diagram Due to P





Moment Diagram Due to X_1





At B,

$$\Delta_{BV} = \Delta_{BV}^{P} + \delta_{BV}^{R} \times R$$

or, $\Delta_1 = \Delta_{10} + \delta_{11} \times X_1$

where,

Δ_{10} = Deflection along X_1 due to original load X_0

δ_{11} = Deflection along X_1 due to unknown load $X_1 = 1$

Now,

$$\Delta_{10} = \int_0^L m_1 m_0 \frac{dx}{EI}$$

$$= \int_0^{L/2} (X_1 . x) \times 0 \frac{dx}{EI} + \int_0^{L/2} \left(X_1 \frac{L}{2} + X_1 x\right)(-Px) \frac{dx}{EI}$$

[x is measured from right

$$= 0 + \int_0^{L/2} \left(-\frac{PX_1 L}{2} x - PX_1 x^2\right) \frac{dx}{EI}$$

$$= \left[-\frac{PL}{2} \cdot \frac{x^2}{2} - P \cdot \frac{x^3}{3}\right]_0^{L/2} \frac{1}{EI} \quad [\because X_1 = 1]$$

$$= \left[-\frac{PL}{4} \cdot \left(\frac{L}{2}\right)^2 - \frac{P}{3} \cdot \left(\frac{L}{2}\right)^3 + 0 + 0\right] \frac{1}{EI}$$

$$= \left[-\frac{PL^3}{16} - \frac{PL^3}{24}\right] \frac{1}{EI}$$

$$= -\frac{5PL^3}{48EI}$$

$$\delta_{11} = \int_0^L m_1 m_1 \frac{dx}{EI}$$

$$= \int_0^L (X_1 . x)(X_1 . x)\frac{dx}{EI}$$

$$= \int_0^L X_1^2 x^2 \frac{dx}{EI}$$

$$= \left[\frac{x^3}{3}\right]_0^L \frac{1}{EI} \qquad \left[\because X_1 = 1\right]$$

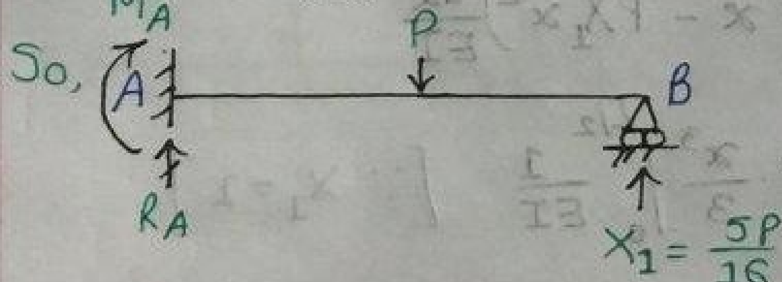
$$= \frac{L^3}{3EI}$$

Now, $X_1 = -\dfrac{\Delta_{10}}{\delta_{11}}$

$$= -\frac{\left(-\frac{5PL^3}{48EI}\right)}{\frac{L^3}{3EI}}$$

$$= \frac{5}{16}P$$

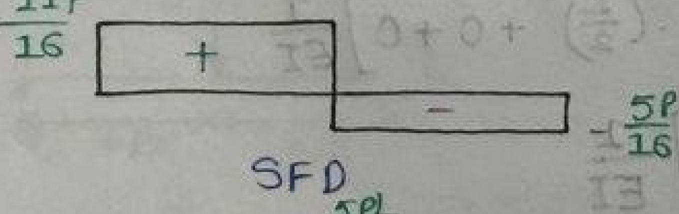
So,

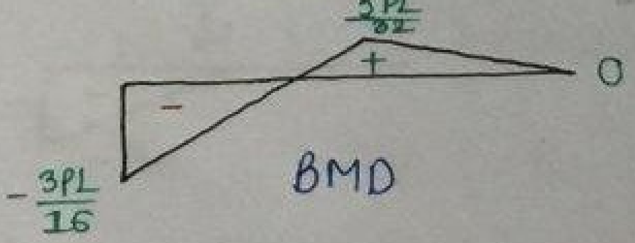


$$R_A = P - \frac{5P}{16} = \frac{11P}{16}$$

$$M_A = \frac{5P}{16} \times L - P \cdot \frac{L}{2}$$

$$= -\frac{3PL}{16}$$





Solution Steps :

① Find Degree of Indeterminancy = D°

② (a) Select Redundant Forces equal to a number of D° and release the structure

② (b) Find equations.

General Equation,

$$(\Delta_i) = (0) = (\Delta_{io}) + (\delta_{ij})(X_j) \quad \text{where } i = j = D^\circ$$

Find Δ & δ terms by unit load method

Example : For 1°

$$(\Delta_1) = (\Delta_{10}) + (\delta_{11})(X_1)$$

For 2°

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = \begin{pmatrix} \Delta_{10} \\ \Delta_{20} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

Remember : $\delta_{12} = \delta_{21}$; $\delta_{13} = \delta_{31}$ . . . . . .

For a Beam or Frame (Bending Members):

③ Draw m_0 diagram (Bending Moment diagram of Determinate structure for external load)

④ Draw $m_1, m_2, m_3 \ldots\ldots$ diagrams

(m_i diagram is the Bending Moment diagram of Determinate structure due to $X_i = 1$)

⑤ Find $\Delta_{i0} = \int m_0 m_i \frac{dx}{EI}$

⑥ Find $\delta_{ij} = \int m_i m_j \frac{dx}{EI}$

⑦ Solve for X_1, X_2, X_3 etc

⑧ Draw SFD, BMD for actual structure

For a Truss (Axial Members):

③ Find n_0 forces due to external loads

④ Find $n_1, n_2, n_3 \ldots\ldots$ etc forces due to $X_1 = 1, X_2 = 1, X_3 = 1 \ldots\ldots$ etc loads.

⑤ Find $\Delta_{io} = \sum n_o n_i \frac{L}{AE}$

⑥ Find $\delta_{ij} = \sum n_i n_j \frac{L}{AE}$

⑦ Solve for X_1, X_2, X_3 . . . . . etc

⑧ Find Bar Forces in actual truss

Summary of Solution Steps : With No Support Settlement

PART - 1 (Common for all structures)

Degree of Indeterminancy,

For Beams = No. of Reactions - No. of Eq^n s - No. of Internal Hinges

For Frames = No. of Reactions + 3 × No. of Members - 3 × No. of Joints - No. of Internal Hinges

For Trusses = No. of Bars + No. of Reactions - 2 × No. of Joints

General Equation,

$(\Delta_i) = (0) = (\Delta_{io}) + (\delta_{ij})(X_j)$ for i = j = No. of Indeterminancy

PART-2:

ⓐ For Beam & Frame (Bending Only)

$$\Delta_{io} = \int_0^L \frac{m_o m_i \, dx}{EI} \quad \ldots\ldots (A1)$$

$$\delta_{ij} = \int_0^L \frac{m_i m_j \, dx}{EI} = \delta_{ji} \quad \ldots\ldots (A2)$$

ⓑ For Truss (Axial Only)

$$\Delta_{io} = \underbrace{\sum \frac{n_o n_i L}{AE}}_{\text{External Load}} + \underbrace{\sum n_i \alpha (\Delta T) L}_{\text{Temperature Change}} + \underbrace{\sum n_i (dL)}_{\text{Misfit}} \quad \ldots\ldots (B1)$$

$$\delta_{ij} = \sum \frac{n_i n_j L}{AE} = \delta_{ji} \quad \ldots\ldots (B2)$$

ⓒ For Beam & Frame (Bending & Axial)

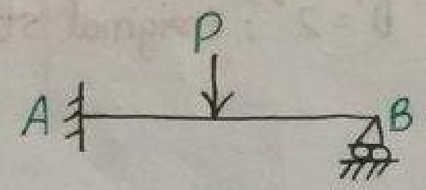
$$\Delta_{io} = A_1 + B_1$$

$$\delta_{ij} = A_2 + B_2$$

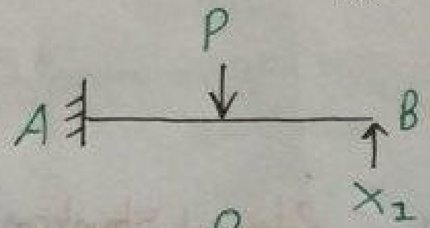
- Misfit of truss means the use of short or long members due to construction difficulties.

"Different Forms of Released (Primary) Structures & Redundants"

(1)

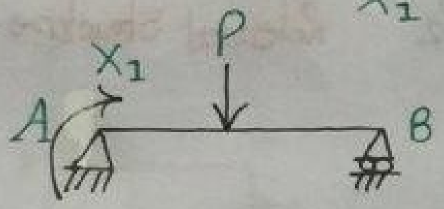


D° = 1° : original structure



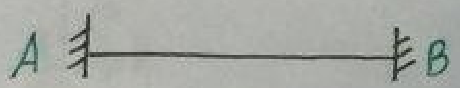
Released Structure - 1

or

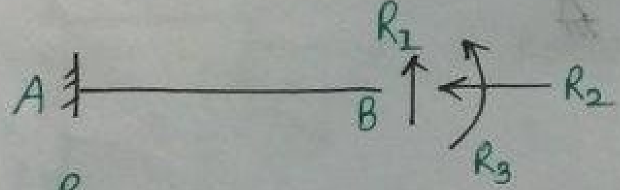


Released Structure - 2

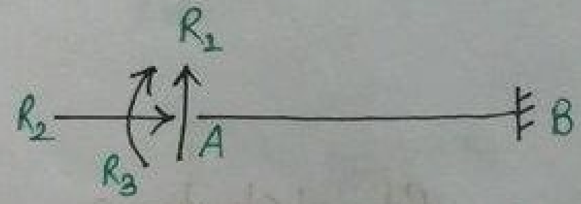
(2)



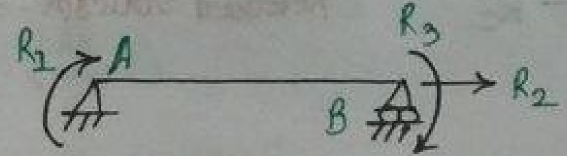
D° = 3° ; original structure



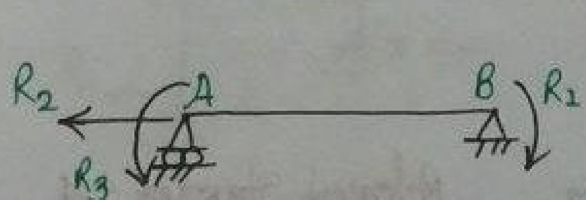
Released Structure - 1



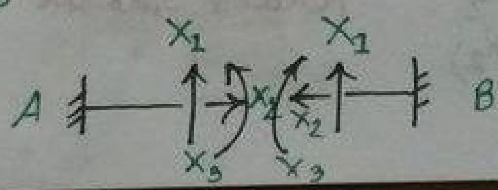
Released Structure - 2



Released Structure - 3

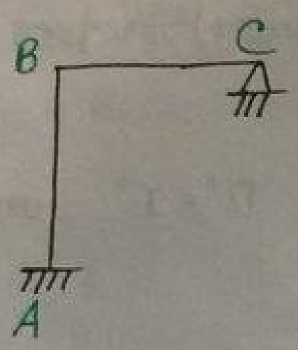


Released Structure - 4

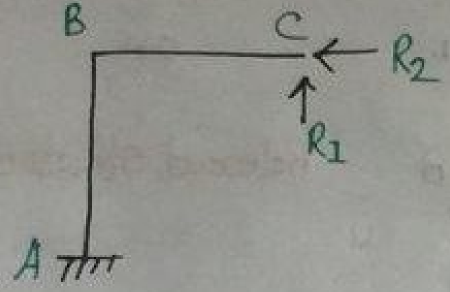


Released Structure - 5

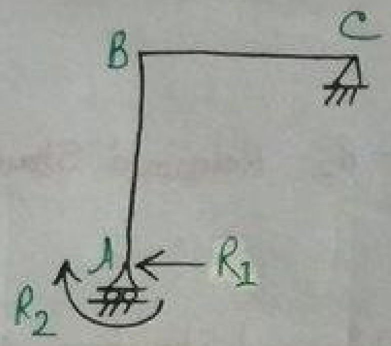
(3)



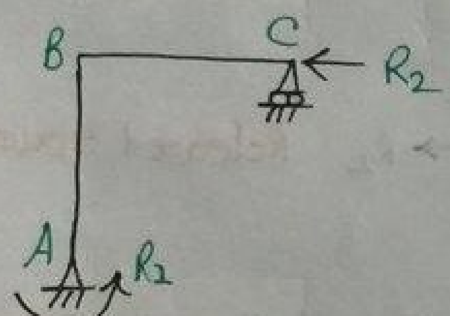
$D^\circ = 2^\circ$; original structure



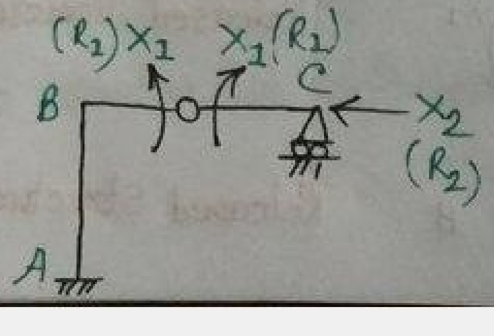
Released Structure - 1



Released Structure - 2

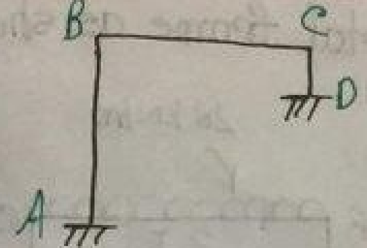


Released Structure - 3

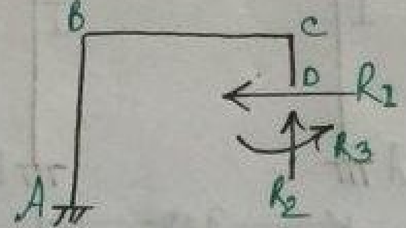


Released Structure - 4

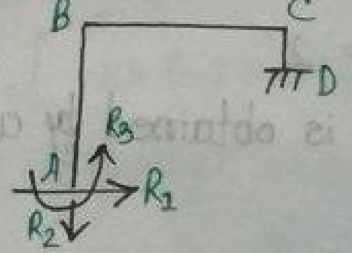
4



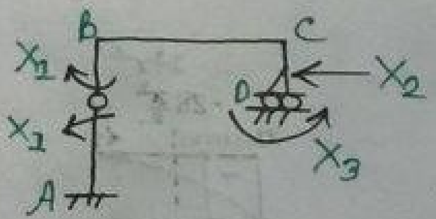
D° = 3° ; original structure



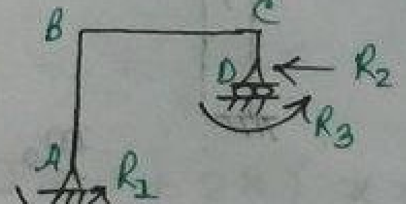
Released Structure-1



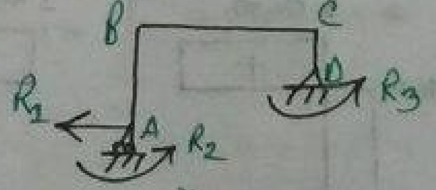
Released Structure-2



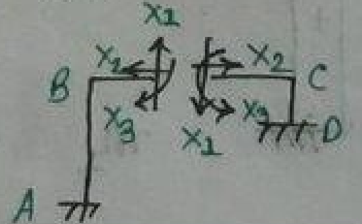
Released Structure-3



Released Structure-4

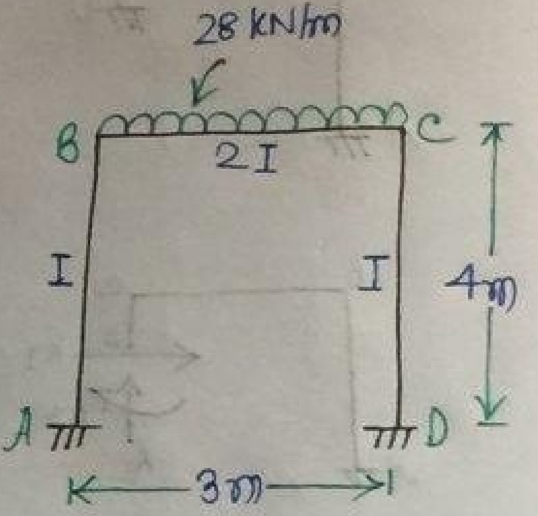


Released structure-5



Released Structure-6

FRAME-1: Analyze the portal frame as shown by the force method.



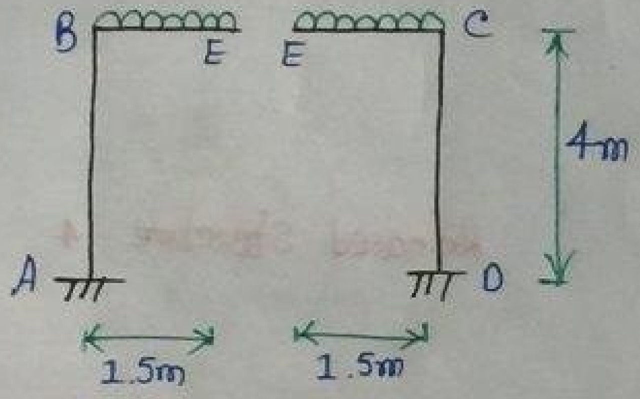
Sol^n: Due to symmetry of the frame and loading, the shear force at the middle point of BC is zero.

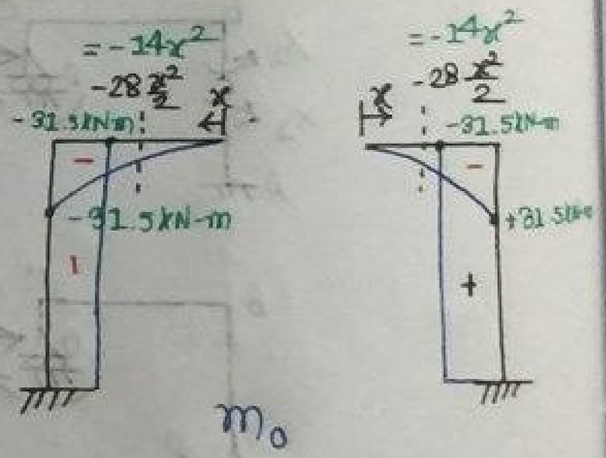
Here, $D^\circ = (3+3) - 3$

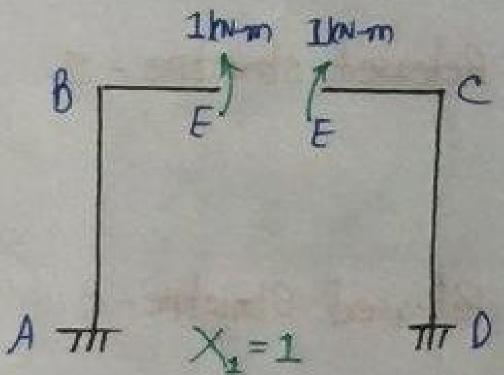
$= 3^\circ$

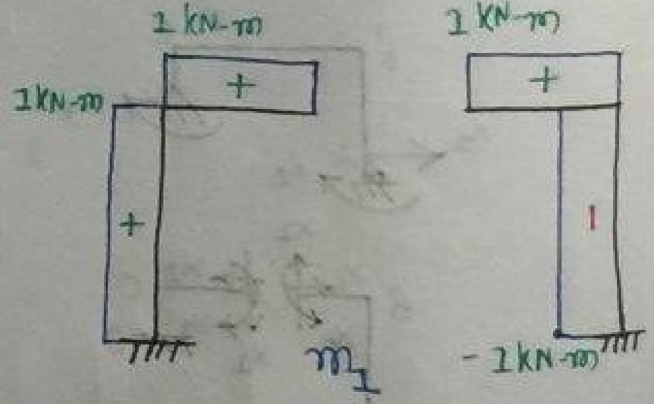
but due to symmetry, $D^\circ = 3 - 1 = 2^\circ$

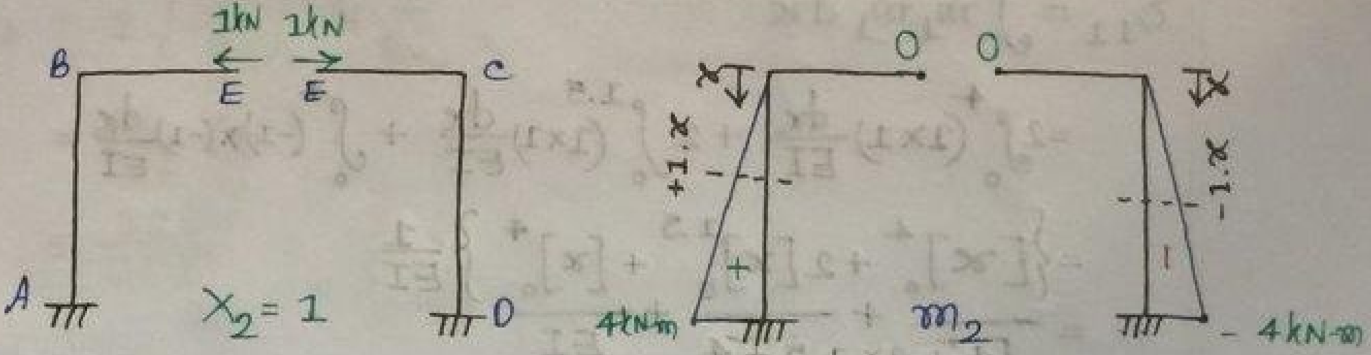
The basic determinate structure is obtained by applying a cut at the middle of BC.











Here,

$$\Delta_{10} = \int m_1 m_0 \frac{dx}{EI}$$

$$= 2\int_0^{1.5} 1 \times (-14x^2)\frac{dx}{EI} + \int_0^4 1 \times (-31.5)\frac{dx}{EI} + \int_0^4 (-1) \times (31.5)\frac{dx}{EI}$$

$$= 2 \times \left[\left(\frac{-14x^3}{3 \times 2EI}\right)\right]_0^{1.5} + 2 \times \left[-\frac{31.5x}{EI}\right]_0^4$$

$$= 2 \times \left[\frac{-14(1.5)^3}{3 \times 2EI}\right] + 2 \times \left[\frac{-31.5 \times 4}{EI}\right]$$

$$= -\frac{267.75}{EI}$$

$$\Delta_{20} = \int m_2 m_0 \frac{dx}{EI}$$

$$= 0 + 0 + 2 \times \int_0^4 x \times (-31.5)\frac{dx}{EI}$$

$$= 2 \times \left[-\frac{31.5x^2}{2EI}\right]_0^4$$

$$= 2 \times \left[\frac{-31.5 \times (4)^2}{2EI}\right]$$

$$= -\frac{504}{EI}$$

$$\delta_{11} = \int m_1 m_1 \, dx$$

$$= \int_0^4 (1\times 1)\frac{dx}{EI} + 2\int_0^{1.5} (1\times 1)\frac{dx}{EI} + \int_0^4 (-1)\times(-1)\frac{dx}{EI}$$

$$= \frac{[x]_0^4}{EI} + \frac{2[x]_0^{1.5}}{2EI} + \frac{[x]_0^4}{EI}$$

$$= \frac{4}{EI} + \frac{2\times 1.5}{2EI} + \frac{4}{EI}$$

$$= \frac{9.5}{EI}$$

$$\delta_{12} = \delta_{21} = \int m_1 m_2 \, dx$$

$$= \int_0^4 1\times(x)\frac{dx}{EI} + 0 + 0 + \int_0^4 (-1)(-x)\frac{dx}{EI}$$

$$= 2\times\int_0^4 x \frac{dx}{EI}$$

$$= 2\times\left[\frac{x^2}{2}\right]_0^4 \times\frac{1}{EI}$$

$$= 2\times\frac{(4)^2}{2}\times\frac{1}{EI}$$

$$= \frac{16}{EI}$$

$$\delta_{22} = \int m_2 m_2 \frac{dx}{EI}$$

$$= \int_0^4 (x\times x)\frac{dx}{EI} + 0 + 0 + \int_0^4 (-x)\times(-x)\frac{dx}{EI}$$

$$= 2\int_0^4 x^2 \frac{dx}{EI}$$

$$\delta_{22} = 2\left[\frac{x^3}{3}\right]_0^4 \times \frac{1}{EI}$$

$$= \frac{2}{3EI} \times (4)^3$$

$$= \frac{128}{3EI}$$

Now,

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = \begin{pmatrix} \Delta_{10} \\ \Delta_{20} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

or,

$$\Delta_1 = \Delta_{10} + \delta_{11} X_1 + \delta_{12} X_2$$

$$\Rightarrow 0 = -\frac{267.75}{EI} + \frac{9.5}{EI} X_1 + \frac{16}{EI} X_2$$

$$\therefore\ 9.5 X_1 + 16 X_2 = 267.75 \quad \ldots\ldots (1)$$

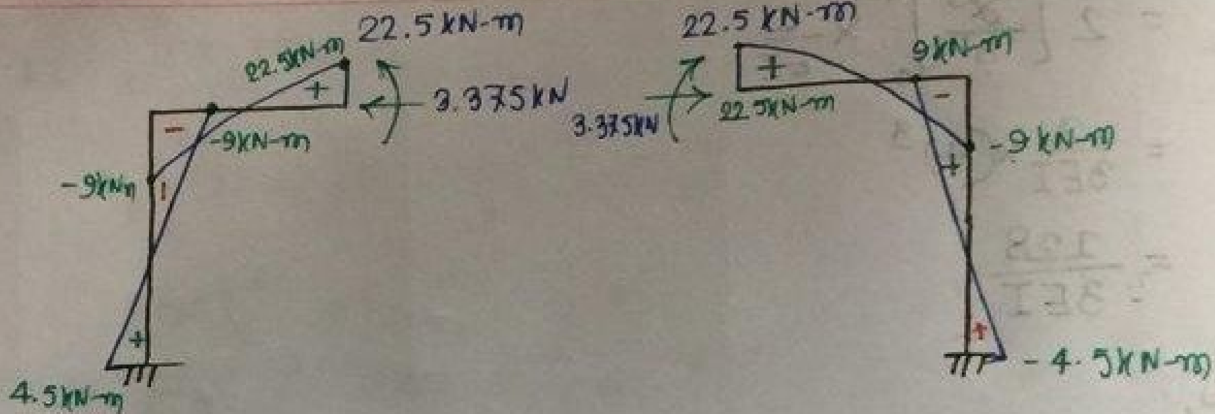
$$\Delta_2 = \Delta_{20} + \delta_{21} X_1 + \delta_{22} X_2$$

$$\Rightarrow 0 = -\frac{504}{EI} + \frac{16}{EI} X_1 + \frac{128}{3EI} X_2$$

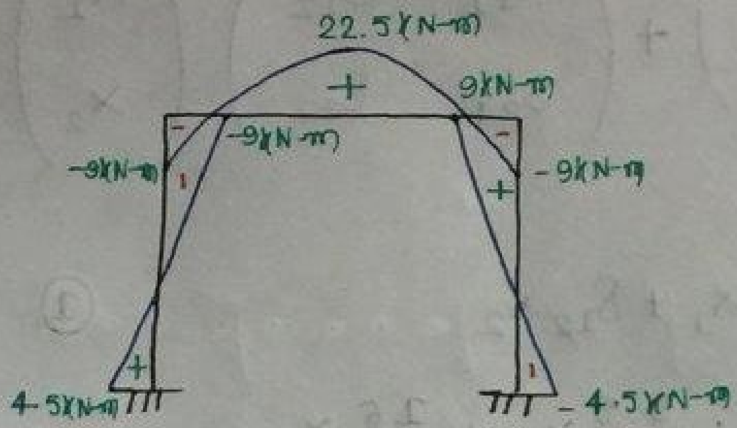
$$\Rightarrow 16 X_1 + \frac{128}{3} X_2 = 504 \quad \ldots\ldots (2)$$

By solving eq^n (1) & (2) we get,

$X_1 = 22.5$ kN & $X_2 = 3.375$ kN (Ans:)



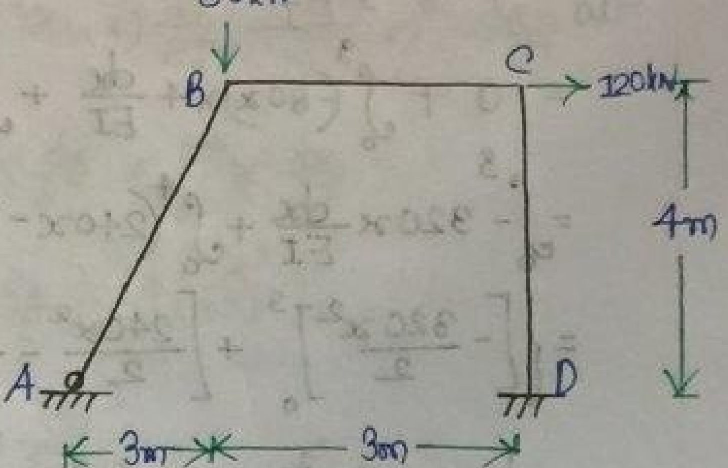
22.5 kN-m
22.5 kN-m
3.375 kN
-9 kN-m
-9 kN-m
4.5 kN-m
22.5 kN-m
9 kN-m
3.375 kN
22.5 kN-m
-9 kN-m
-4.5 kN-m



22.5 kN-m
9 kN-m
-9 kN-m
-9 kN-m
-9 kN-m
4.5 kN-m
-4.5 kN-m

Final BMD

FRAME-2: Analyze the inclined leg portal fram as shown by the force method.

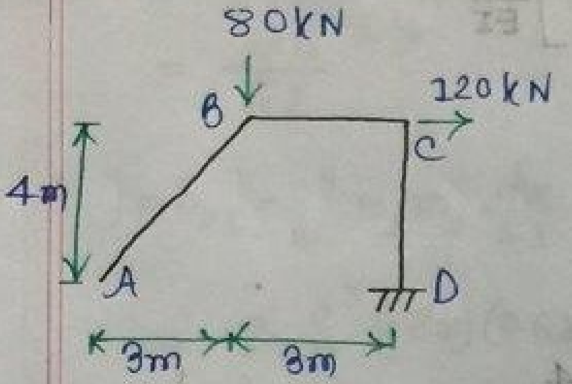


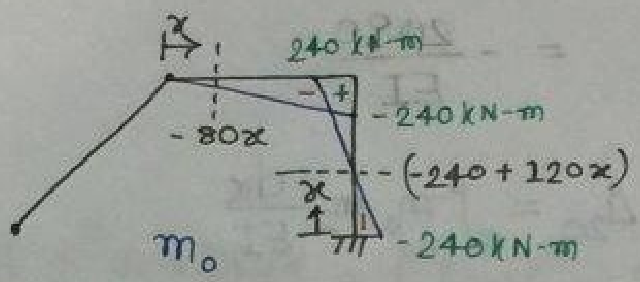
Soln: Here,

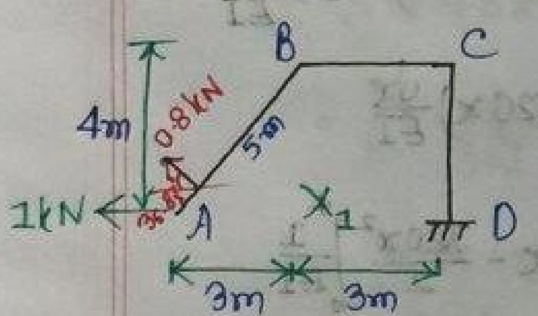
$D^\circ = (2+3)+(3\times3)-(4\times3)$

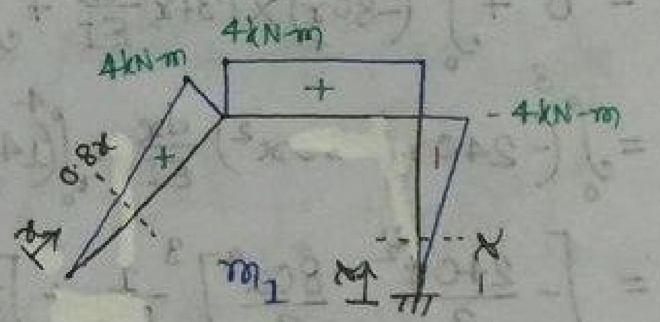
$= 2^\circ$

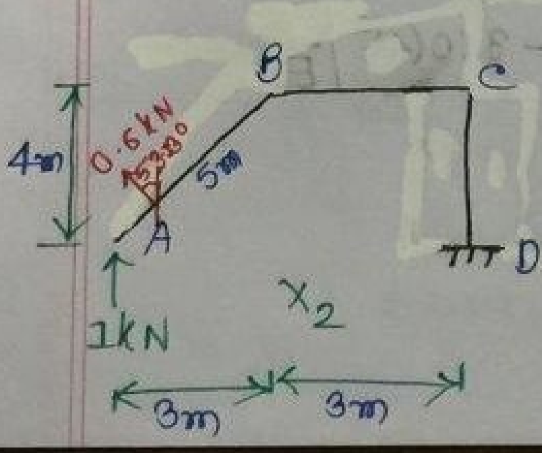
Release the Hinge support

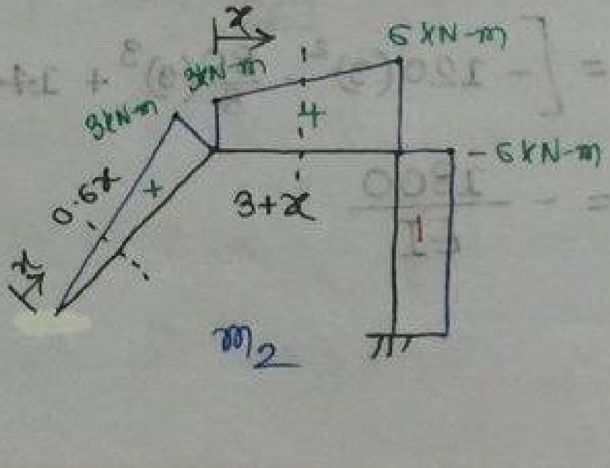












Here,

$$\Delta_{10} = \int m_0 m_I \frac{dx}{EI}$$

$$= 0 + \int_0^3 (-80x) \times 4 \frac{dx}{EI} + \int_0^4 (-240+120x) \times (-x) \frac{dx}{EI}$$

$$= \int_0^3 -320x \frac{dx}{EI} + \int_0^4 (240x - 120x^2) \frac{dx}{EI}$$

$$= \left\{ \left[-\frac{320x^2}{2} \right]_0^3 + \left[\frac{240x^2}{2} - \frac{120x^3}{3} \right]_0^4 \right\} \frac{1}{EI}$$

$$= \left[-160(3)^2 + 120(4)^2 - 40(4)^3 \right] \frac{1}{EI}$$

$$= -\frac{2080}{EI}$$

$$\Delta_{20} = \int m_0 m_2 \frac{dx}{EI}$$

$$= 0 + \int_0^3 (-80x) \times (3+x) \frac{dx}{EI} + \int_0^4 (-240+120x) \times (-6) \frac{dx}{EI}$$

$$= \int_0^3 (-240x - 80x^2) \frac{dx}{EI} + \int_0^4 (1440 - 720x) \frac{dx}{EI}$$

$$= \left[-\frac{240x^2}{2} - \frac{80x^3}{3} \right]_0^3 \frac{1}{EI} + \left[1440x - \frac{720x^2}{2} \right]_0^4 \frac{1}{EI}$$

$$= \left[-120(3)^2 - \frac{80}{3}(3)^3 + 1440(4) - 360(4)^2 \right] \frac{1}{EI}$$

$$= -\frac{1800}{EI}$$

$$\delta_{11} = \int m_1 m_1 \frac{dx}{EI}$$

$$= \int_0^5 (0.8x)(0.8x)\frac{dx}{EI} + \int_0^3 (4 \times 4)\frac{dx}{EI} + \int_0^4 (-x)(-x)\frac{dx}{EI}$$

$$= \int_0^5 0.64x^2 \frac{dx}{EI} + \int_0^3 16\frac{dx}{EI} + \int_0^4 x^2 \frac{dx}{EI}$$

$$= \left[0.64\frac{x^3}{3}\right]_0^5 \frac{1}{EI} + \left[\frac{16x}{EI}\right]_0^3 + \left[\frac{x^3}{3EI}\right]_0^4$$

$$= \frac{0.64(5)^3}{3EI} + \frac{16(3)}{EI} + \frac{(4)^3}{3EI}$$

$$= \frac{96}{EI}$$

$$\delta_{12} = \delta_{21} = \int m_1 m_2 \frac{dx}{EI}$$

$$= \int_0^5 (0.8x)(0.6x)\frac{dx}{EI} + \int_0^3 4(3+x)\frac{dx}{EI} + \int_0^4 (-x)(-6)\frac{dx}{EI}$$

$$= \int_0^5 0.48x^2 \frac{dx}{EI} + \int_0^3 (12+4x)\frac{dx}{EI} + \int_0^4 6x\frac{dx}{EI}$$

$$= \left[\frac{0.48x^3}{3EI}\right]_0^5 + \left[\frac{12x + \frac{4x^2}{2}}{EI}\right]_0^3 + \left[\frac{6x^2}{2EI}\right]_0^4$$

$$= \frac{0.48(5)^3}{3EI} + \frac{12(3) + 2(3)^2}{EI} + \frac{6(4)^2}{2EI}$$

$$= \frac{122}{EI}$$

$$\delta_{22} = \int m_2 m_2 \frac{dx}{EI}$$

$$= \int_0^5 (0.6x)(0.6x)\frac{dx}{EI} + \int_0^3 (3+x)(3+x)\frac{dx}{EI} + \int_0^4 (-6)(-6)\frac{dx}{EI}$$

$$= \int_0^5 0.36x^2 \frac{dx}{EI} + \int_0^3 (9+6x+x^2)\frac{dx}{EI} + \int_0^4 36\frac{dx}{EI}$$

$$= \left[\frac{0.36x^3}{3EI}\right]_0^5 + \left[\frac{9x+\frac{6x^2}{2}+\frac{x^3}{3}}{EI}\right]_0^3 + \left[\frac{36x}{EI}\right]_0^4$$

$$= \frac{0.36(5)^3}{3EI} + \frac{9(3)+3(3)^2+\frac{(3)^3}{3}}{EI} + \frac{36(4)}{EI}$$

$$= \frac{222}{EI}$$

Now,

$$\Delta_1 = \Delta_{10} + \delta_{11}X_1 + \delta_{12}X_2$$

$$\Rightarrow 0 = -\frac{2080}{EI} + \frac{96}{EI}X_1 + \frac{122}{EI}X_2$$

$$\Rightarrow 96X_1 + 122X_2 = 2080 \quad \text{(1)}$$

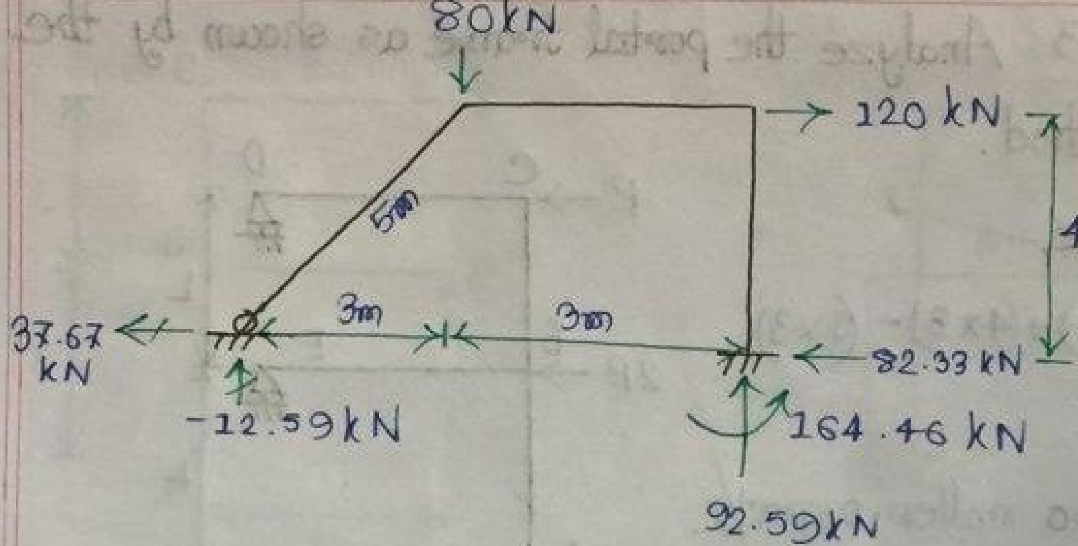
$$\Delta_2 = \Delta_{20} + \delta_{21}X_1 + \delta_{22}X_2$$

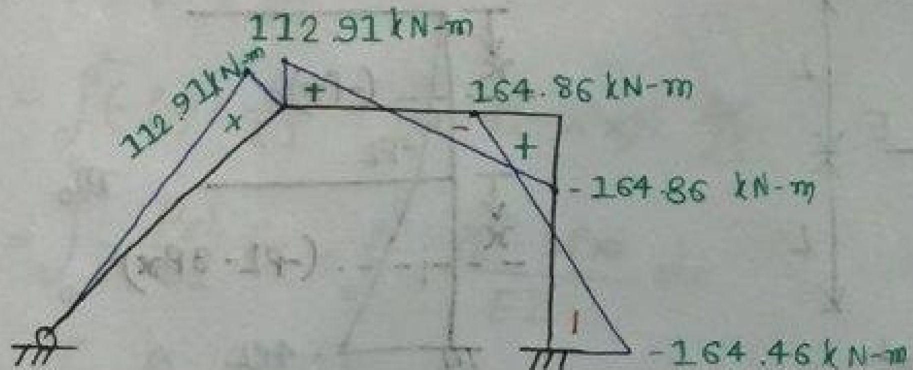
$$\Rightarrow 0 = -\frac{1800}{EI} + \frac{}{EI}X_1 + \frac{222}{EI}X_2$$

$$\Rightarrow \quad X_1 + 222X_2 = 1800 \quad \text{(2)}$$

By solving eq^n (1) & (2) we get,

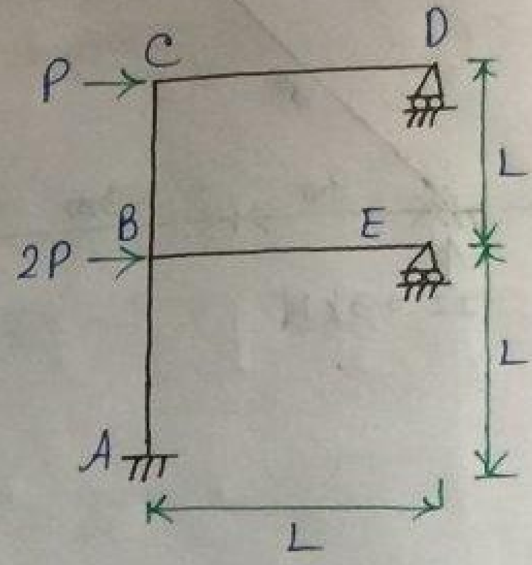
$X_1 = 37.67\,kN$ & $X_2 = -12.59\,kN$ (Ans:)


80kN
120 kN
5m
4m
3m
3m
37.67 kN
82.33 kN
-12.59kN
164.46 kN
92.59kN


112.91kN-m
112.91kN-m
164.86 kN-m
-164.86 kN-m
-164.46 kN-m
+
+
+
-

Final BMD

FRAME-3: Analyze the portal frame as shown by the force method.

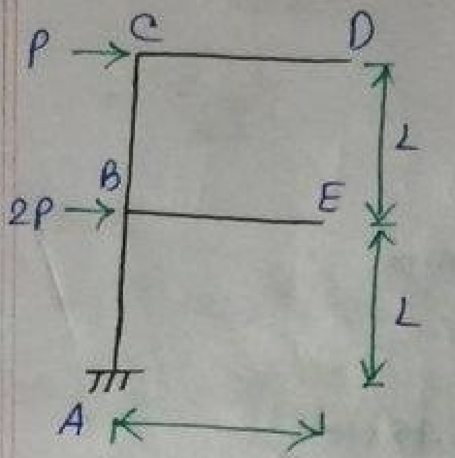


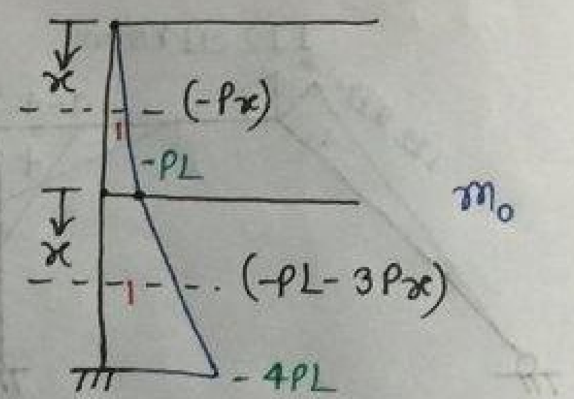
Sol^n: Here,

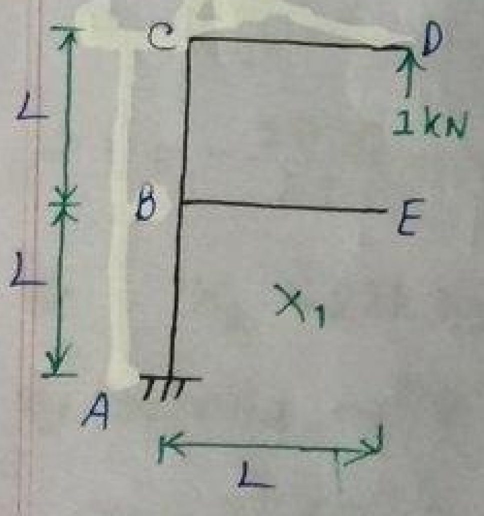
$D^\circ = (3+1+1) + (4\times 3) - (5\times 3)$

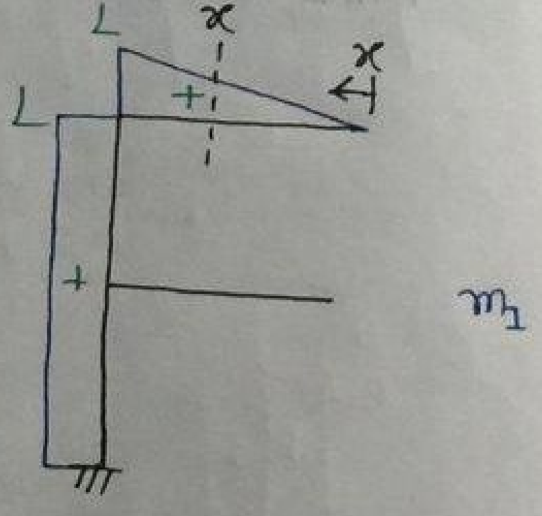
$= 2^\circ$

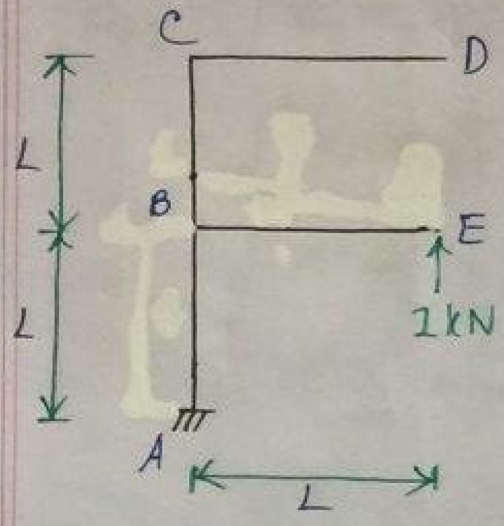
Release two roller supports.

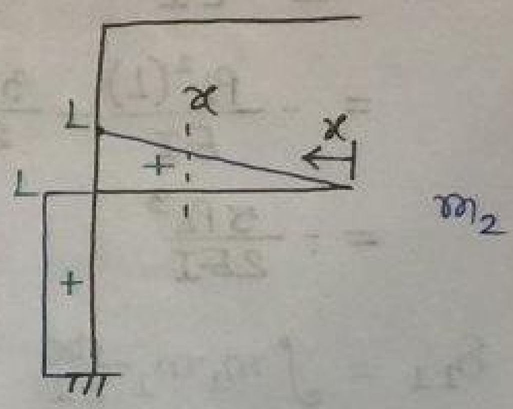












Here,

$$\Delta_{10} = \int m_0 m_1 \frac{dx}{EI}$$

$$= \int_0^L (-Px)L\frac{dx}{EI} + \int_0^L (-PL - 3Px)L\frac{dx}{EI}$$

$$= \int_0^L -\frac{PxL}{EI}\,dx + \int_0^L \frac{-PL^2 - 3PxL}{EI}\,dx$$

$$= \left[-\frac{PL}{EI}\cdot\frac{x^2}{2}\right]_0^L + \left[\frac{-PL^2x - \frac{3PLx^2}{2}}{EI}\right]_0^L$$

$$= -\frac{PL(L)^2}{2EI} + \frac{-PL^2(L) - \frac{3PL}{2}(L)^2}{EI}$$

$$= -\frac{3PL^3}{EI}$$

$$\Delta_{20} = \int m_0 m_2 \frac{dx}{EI}$$

$$= 0 + \int_0^L (-PL - 3Px)L\frac{dx}{EI}$$

$$= \int_0^L (-PL^2 - 3PLx)\frac{dx}{EI}$$

$$\Delta_{20} = \left[-\frac{PL^2 x}{EI} - \frac{3PLx^2}{2EI} \right]_0^L$$

$$= -\frac{PL^2(L)}{EI} - \frac{3PL(L)^2}{2EI}$$

$$= -\frac{5PL^3}{2EI}$$

$$\delta_{11} = \int m_1 m_1 \frac{dx}{EI}$$

$$= \int_0^L (x)(x) \frac{dx}{EI} + \int_0^{2L} (L)(L) \frac{dx}{EI}$$

$$= \int_0^L x^2 \frac{dx}{EI} + \int_0^{2L} L^2 \frac{dx}{EI}$$

$$= \left\{ \left[\frac{x^3}{3} \right]_0^L + \left[L^2 x \right]_0^{2L} \right\} \frac{1}{EI}$$

$$= \frac{(L)^3}{3EI} + \frac{L^2(2L)}{EI}$$

$$= \frac{7L^3}{3EI}$$

$$\delta_{12} = \delta_{21} = \int m_1 m_2 \frac{dx}{EI}$$

$$= 0 + 0 + \int_0^L (L)(L) \frac{dx}{EI}$$

$$= \left[L^2 x \right]_0^L \frac{1}{EI}$$

$$= \frac{L^2(L)}{EI}$$

$$= \frac{L^3}{EI}$$

$$\delta_{22} = \int m_2 m_2 \frac{dx}{EI}$$

$$= \int_0^L (x)(x)\frac{dx}{EI} + \int_0^L (L)(L)\frac{dx}{EI}$$

$$= \int_0^L x^2 \frac{dx}{EI} + \int_0^L L^2 \frac{dx}{EI}$$

$$= \left\{ \left[\frac{x^3}{3}\right]_0^L + \left[L^2 x\right]_0^L \right\} \frac{1}{EI}$$

$$= \frac{(L)^3}{3EI} + \frac{L^2(L)}{EI}$$

$$= \frac{4L^3}{3EI}$$

Now,

$$\Delta_1 = \Delta_{10} + \delta_{11} X_1 + \delta_{12} X_2$$

$$\Rightarrow 0 = -\frac{3PL^3}{EI} + \frac{7L^3}{3EI} X_1 + \frac{L^3}{EI} X_2$$

$$\therefore \quad 7X_1 + 3X_2 = 9P \quad \ldots\ldots\ldots (1)$$

$$\Delta_2 = \Delta_{20} + \delta_{21} X_1 + \delta_{22} X_2$$

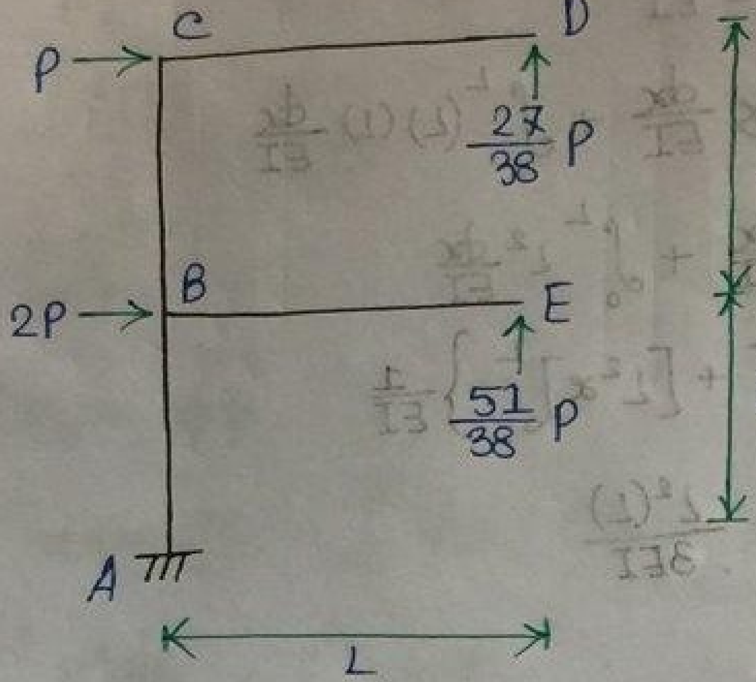
$$\Rightarrow 0 = -\frac{5PL^3}{2EI} + \frac{L^3}{EI} X_1 + \frac{4L^3}{3EI} X_2$$

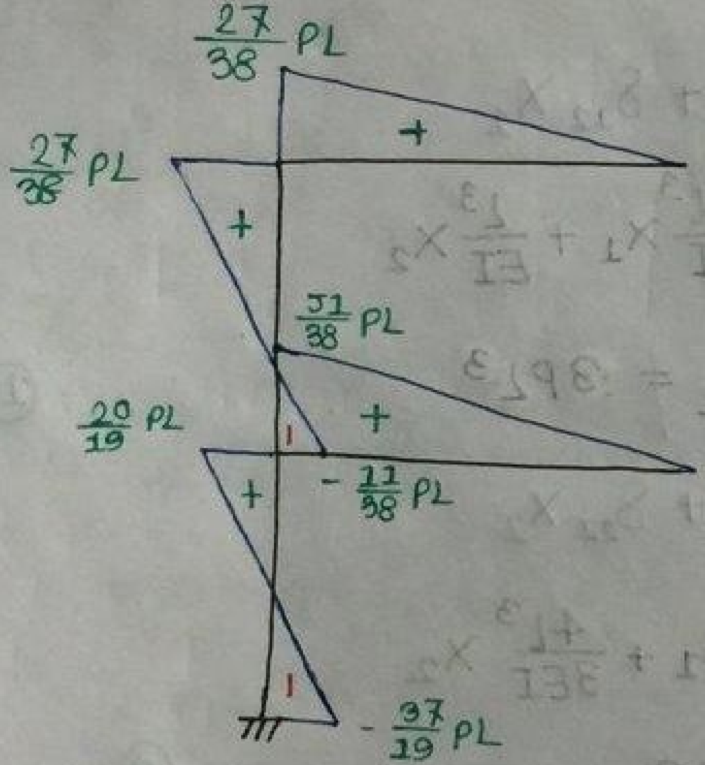
$$\therefore 6X_1 + 8X_2 = 15P \quad \ldots\ldots\ldots (2)$$

By solving eq^n (1) & (2) we get,

$$X_1 = \frac{27}{38} P \quad \& \quad X_2 = \frac{51}{38} P$$

(Ans:)


C
D
P
$\frac{27}{38}P$
L
B
E
2P
$\frac{51}{38}P$
L
A
L


$\frac{27}{38}PL$
+
$\frac{27}{38}PL$
+
$\frac{51}{38}PL$
$\frac{20}{19}PL$
+
$-\frac{11}{38}PL$
+
$-\frac{37}{19}PL$

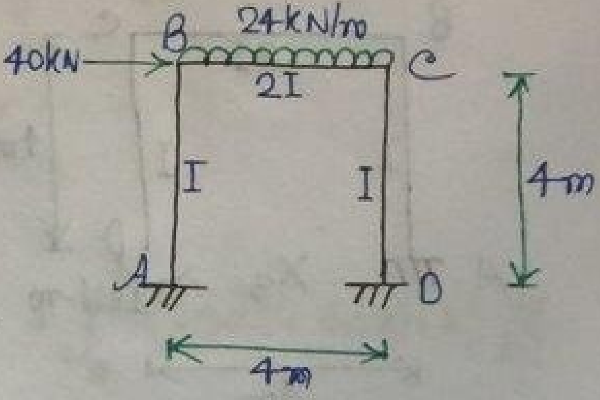
Final BMD

FRAME - 4 : Analyze the portal frame as shown by the force method.

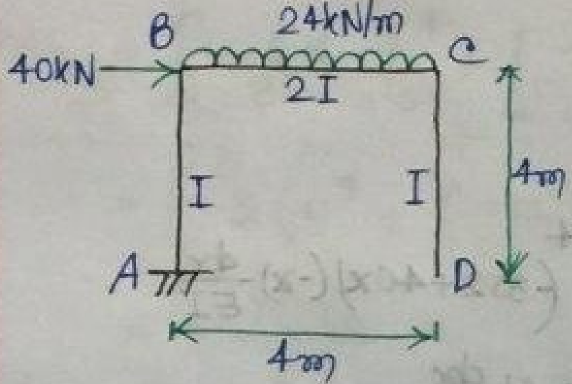
Sol^n: Here,

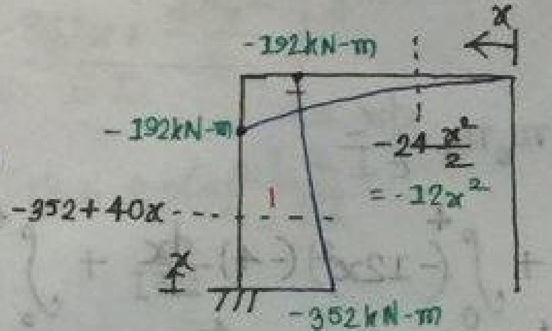
$D^\circ = (3+3) + (3\times3) - (4\times3)$

$= 3^\circ$

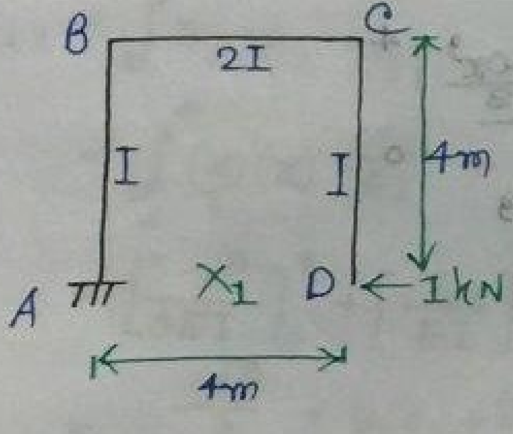


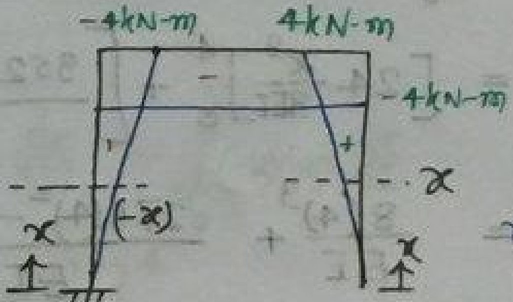
Release the right fixed support.



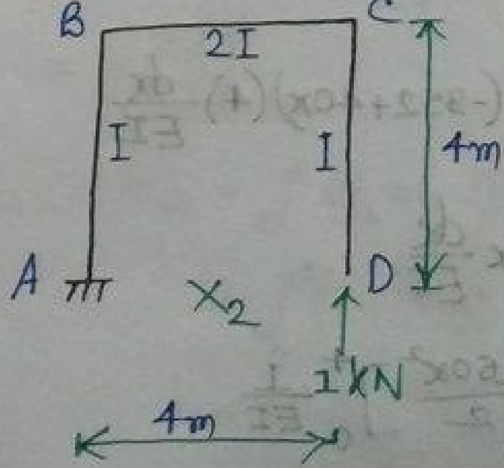


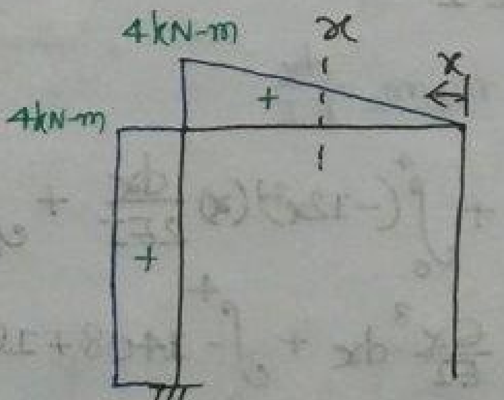
m_0



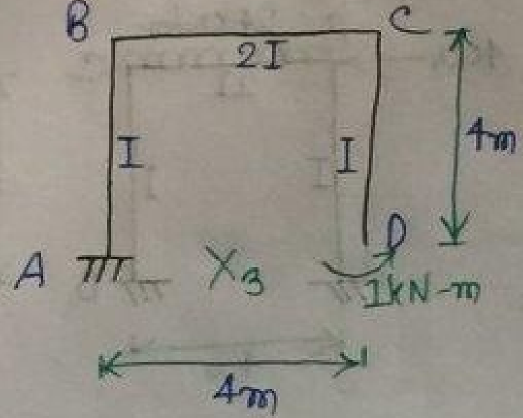


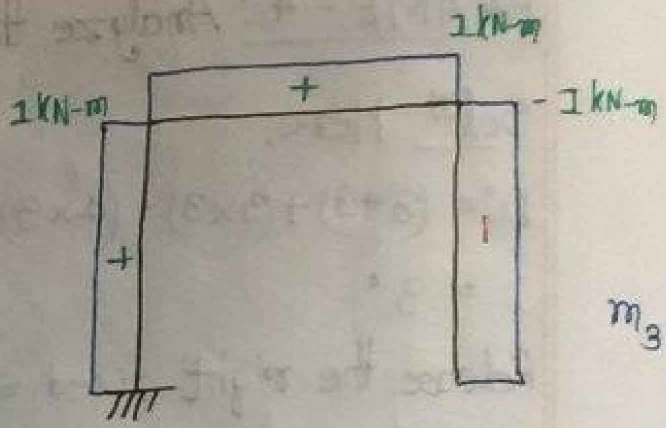
m_1





m_2





Now,

$$\Delta_{10} = \int m_0 m_3 \frac{dx}{EI}$$

$$= 0 + \int_0^4 (-12x^2)(-4)\frac{dx}{2EI} + \int_0^4 (-352+40x)(-x)\frac{dx}{EI}$$

$$= \int_0^4 48x^2 \frac{dx}{2EI} + \int_0^4 (352x - 40x^2)\frac{dx}{EI}$$

$$= \left[24\frac{x^3}{3EI}\right]_0^4 + \left[\frac{352\frac{x^2}{2} - \frac{40x^3}{3}}{EI}\right]_0^4$$

$$= \frac{8(4)^3}{EI} + \frac{176(4)^2 - \frac{40}{3}(4)^3}{EI}$$

$$= \frac{7424}{3EI}$$

$$\Delta_{20} = \int m_0 m_2 \frac{dx}{EI}$$

$$= 0 + \int_0^4 (-12x^2)(x)\frac{dx}{2EI} + \int_0^4 (-352+40x)(4)\frac{dx}{EI}$$

$$= \int_0^4 -\frac{6x^3}{EI}dx + \int_0^4 -1408 + 160x\frac{dx}{EI}$$

$$= \left[-\frac{6x^4}{4EI}\right]_0^4 + \left[-1408x + \frac{160x^2}{2}\right]_0^4 \frac{1}{EI}$$

$$\Delta_{20} = -\frac{6(4)^4}{4EI} - \frac{1408(4)}{EI} + \frac{80(4)^2}{EI}$$

$$= -\frac{4736}{EI}$$

$$\Delta_{30} = \int m_0 m_3 \frac{dx}{EI}$$

$$= 0 + \int_0^4 (-12x^2)(1)\frac{dx}{2EI} + \int_0^4 (-352+40x)(1)\frac{dx}{EI}$$

$$= \left[-\frac{6x^3}{3EI}\right]_0^4 + \left[\frac{-352x + \frac{40x^2}{2}}{EI}\right]_0^4$$

$$= \left[-2(4)^3 + (-352)(4) + 20(4)^2\right]\frac{1}{EI}$$

$$= -\frac{1216}{EI}$$

$$\delta_{11} = \int m_1 m_1 \frac{dx}{EI}$$

$$= \int_0^4 (x)(x)\frac{dx}{EI} + \int_0^4 (-4)(-4)\frac{dx}{2EI} + \int_0^4 (-x)(-x)\frac{dx}{EI}$$

$$= \left[\frac{x^3}{3EI}\right]_0^4 + \left[\frac{8x}{EI}\right]_0^4 + \left[\frac{x^3}{3EI}\right]_0^4$$

$$= \frac{\frac{(4)^3}{3} + 8(4) + \frac{(4)^3}{3}}{EI}$$

$$= \frac{224}{3EI}$$

$$\delta_{12} = \delta_{21} = \int m_1 m_2 \frac{dx}{EI}$$

$$= \int_0^4 (+x)0\frac{dx}{EI} + \int_0^4 (-4)(x)\frac{dx}{2EI} + \int_0^4 (-x)(4)\frac{dx}{EI}$$

$$= 0 + \int_0^4 (-2x)\frac{dx}{EI} + \int_0^4 (-4x)\frac{dx}{EI}$$

$$\delta_{12} = \delta_{21} = \left[-\frac{2x^2}{2EI}\right]_0^4 + \left[\frac{-4x^2}{2EI}\right]_0^4$$

$$= -\frac{(4)^2}{EI} - \frac{2(4)^2}{EI}$$

$$= -\frac{48}{EI}$$

$$\delta_{13} = \delta_{31} = \int m_1 m_3 \frac{dx}{EI}$$

$$= \int_0^4 (+x)(-1)\frac{dx}{EI} + \int_0^4 (-4)(1)\frac{dx}{2EI} + \int_0^4 (-x)(1)\frac{dx}{EI}$$

$$= 2\int_0^4 (-x)\frac{dx}{EI} + \int_0^4 (-2)\frac{dx}{EI}$$

$$= 2\left[-\frac{x^2}{2EI}\right]_0^4 + \left[\frac{-2x}{EI}\right]_0^4$$

$$= -\frac{(4)^2}{EI} - \frac{2(4)}{EI}$$

$$= -\frac{24}{EI}$$

$$\delta_{22} = \int m_2 m_2 \frac{dx}{EI}$$

$$= 0 + \int_0^4 (x)(x)\frac{dx}{2EI} + \int_0^4 (4)(4)\frac{dx}{EI}$$

$$= \int_0^4 \frac{x^2}{2EI}dx + \int_0^4 16\frac{dx}{EI}$$

$$= \left[\frac{x^3}{3\times 2EI}\right]_0^4 + \left[\frac{16x}{EI}\right]_0^4$$

$$= \frac{(4)^3}{6EI} + \frac{16\times 4}{EI}$$

$$= \frac{224}{3EI}$$

$$\delta_{23} = \delta_{32} = \int m_2 m_3 \frac{dx}{EI}$$

$$= 0 + \int_0^4 (x)(1)\frac{dx}{2EI} + \int_0^4 (4)(1)\frac{dx}{EI}$$

$$= \left[\frac{x^2}{2\times 2EI}\right]_0^4 + \left[\frac{4x}{EI}\right]_0^4$$

$$= \frac{(4)^2}{4EI} + \frac{4\times 4}{EI}$$

$$= \frac{20}{EI}$$

$$\delta_{33} = \int m_3 m_3 \frac{dx}{EI}$$

$$= \int_0^4 (-1)(-1)\frac{dx}{EI} + \int_0^4 (1)(1)\frac{dx}{2EI} + \int_0^4 (1)(1)\frac{dx}{EI}$$

$$= 2.5\left[\frac{x}{EI}\right]_0^4$$

$$= \frac{2.5(4)}{EI}$$

$$= \frac{10}{EI}$$

Now,

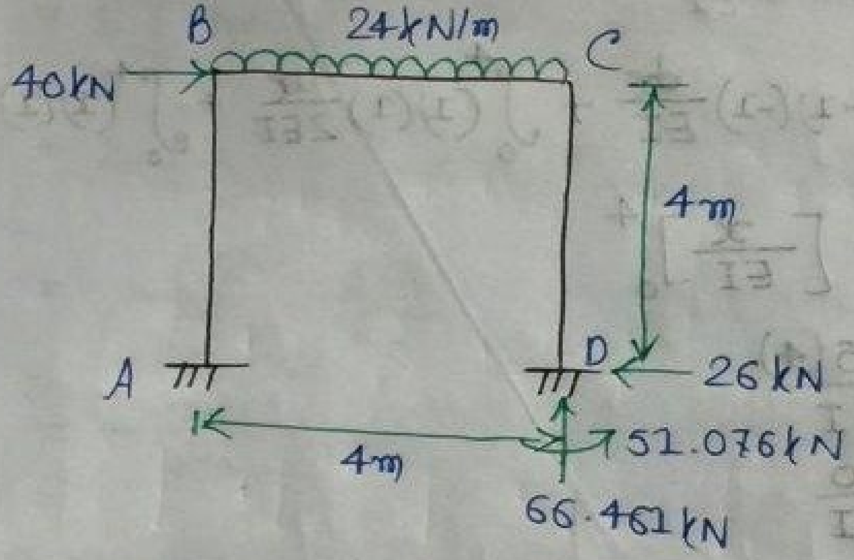
$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} + \begin{pmatrix} \Delta_{10} \\ \Delta_{20} \\ \Delta_{30} \end{pmatrix}$$

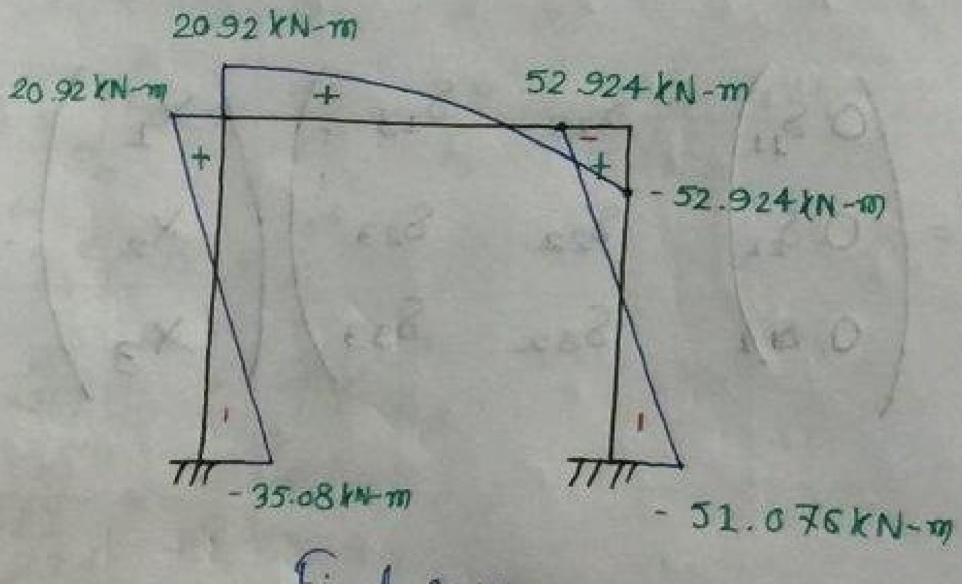
or,

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{224}{3EI} & -\frac{48}{EI} & -\frac{24}{EI} \\ -\frac{48}{EI} & \frac{224}{3EI} & \frac{20}{EI} \\ -\frac{24}{EI} & \frac{20}{EI} & \frac{10}{EI} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \begin{pmatrix} \frac{7424}{3EI} \\ -\frac{4736}{EI} \\ -\frac{1216}{EI} \end{pmatrix}$$

By solving the matrix, we get,

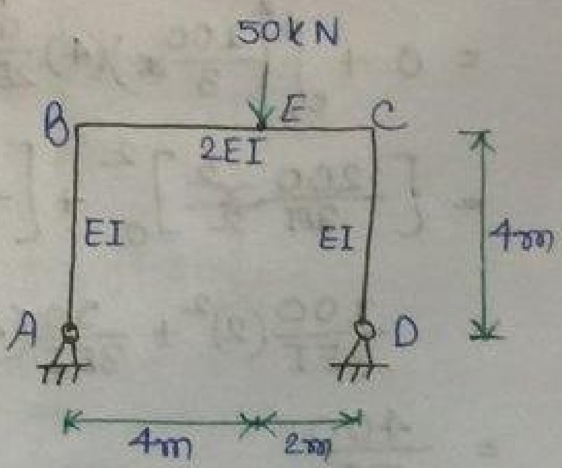
$X_1 = 26\ kN$; $X_2 = 66.461\ kN$; $X_3 = 51.076\ kN$





Final BMD

FRAME-5: It is required to determine the horizontal reaction component for the frame. The EI value for the beam is twice the value for the columns.

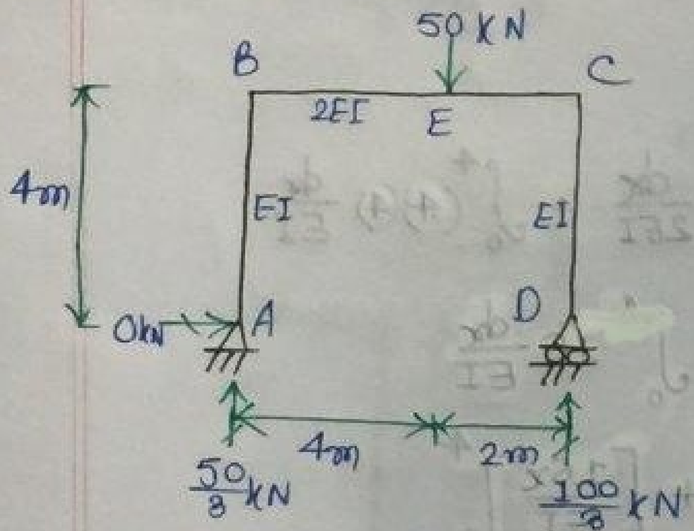


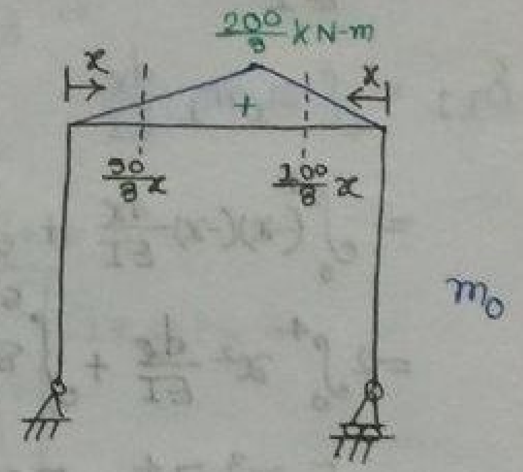
Sol^n: Here,

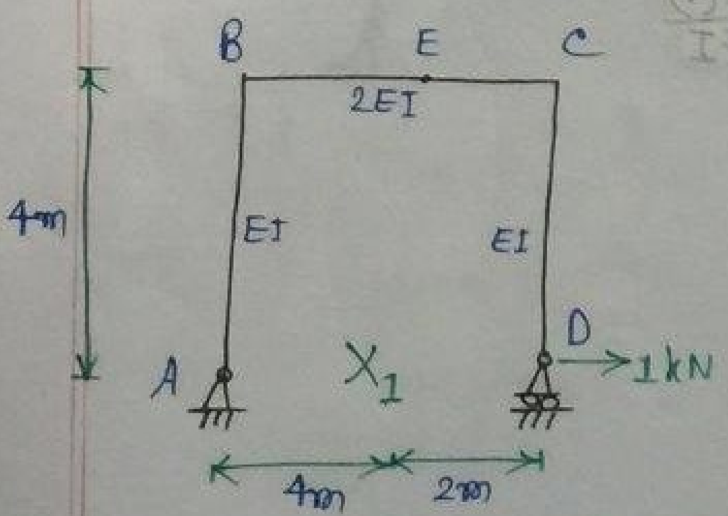
$D^\circ = (2+2) + (3\times3) - (4\times3)$

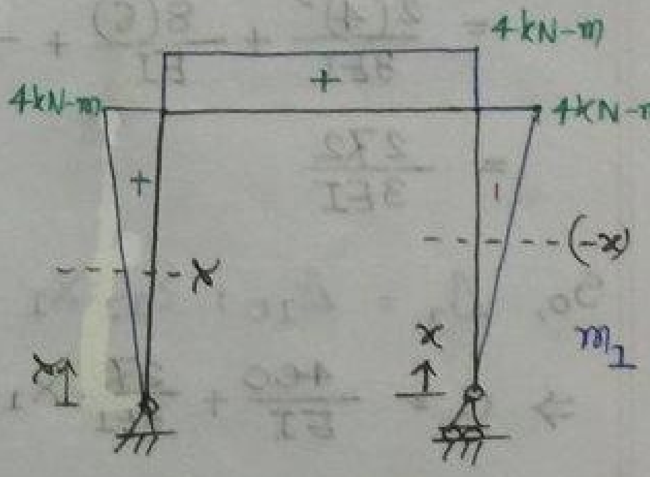
$= 1^\circ$

Release horizontal reaction from the right support.









Now,

$$\Delta_{10} = \int m_0 m_1 \frac{dx}{EI}$$

$$= 0 + \int_0^2 \left(\frac{100}{3}x\right)(4)\frac{dx}{2EI} + \int_0^4 \left(\frac{50}{3}x\right)(4)\frac{dx}{2EI} + 0$$

$$= \left[\frac{200}{3EI}\frac{x^2}{2}\right]_0^2 + \left[\frac{100x^2}{3EI \times 2}\right]_0^4$$

$$= \frac{100}{3EI}(2)^2 + \frac{50}{3EI}(4)^2$$

$$= \frac{400}{EI}$$

$$\delta_{11} = \int m_1 m_1 \frac{dx}{EI}$$

$$= \int_0^4 (-x)(-x)\frac{dx}{EI} + \int_0^6 (4)(4)\frac{dx}{2EI} + \int_0^4 (x)(x)\frac{dx}{EI}$$

$$= 2\int_0^4 x^2 \frac{dx}{EI} + \int_0^6 8\frac{dx}{EI}$$

$$= 2\left[\frac{x^3}{3EI}\right]_0^4 + \left[\frac{8x}{EI}\right]_0^6$$

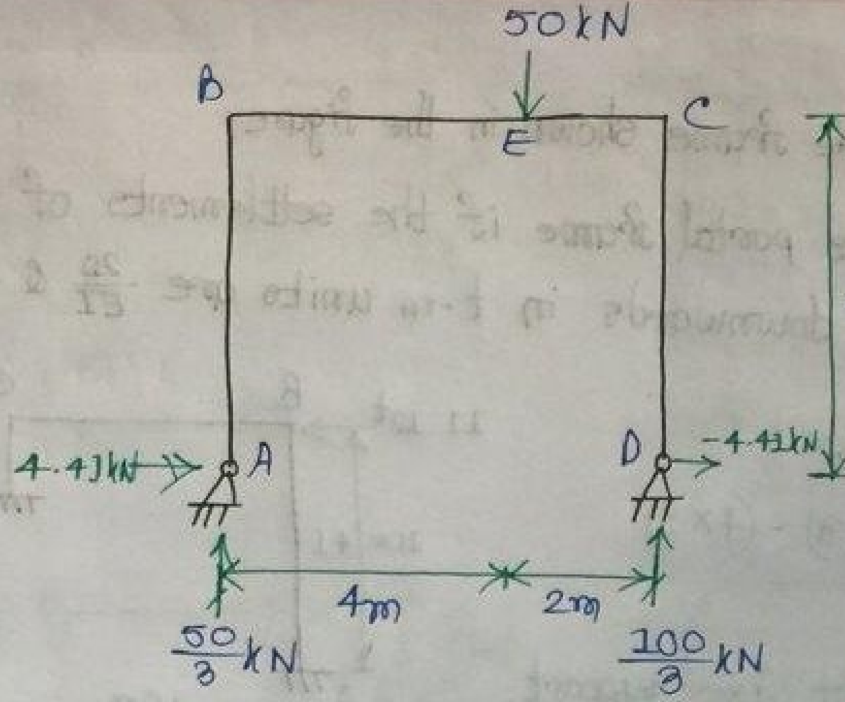
$$= \frac{2(4)^3}{3EI} + \frac{8(6)}{EI} +$$

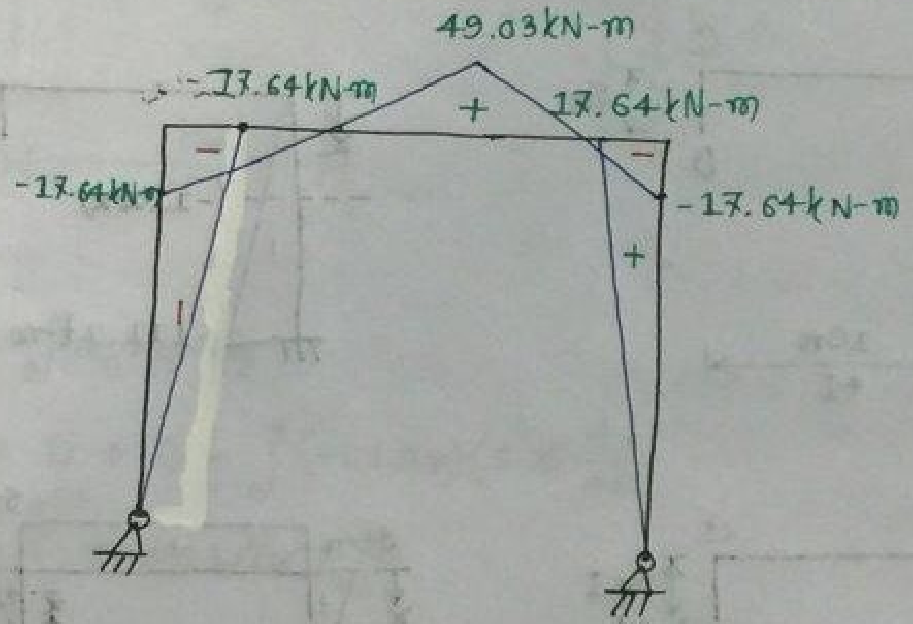
$$= \frac{272}{3EI}$$

So, $\Delta_1 = \Delta_{10} + \delta_{11} X_1$

$$\Rightarrow 0 = \frac{400}{EI} + \frac{272}{3EI}X_1$$

$\therefore X_1 = -4.41$ kN (Ans:)


50kN
B
C
E
4m
4.43kN
A
D
-4.43kN
4m
2m
50/3 kN
100/3 kN

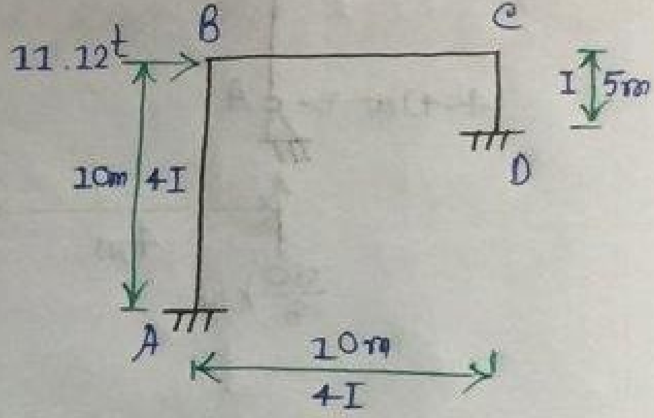

49.03kN-m
-17.64kN-m
17.64kN-m
-17.64kN-m
-17.64kN-m
+
−
−
+

FRAME - 6 :

(a) Analyze the frame shown in the figure.

(b) Analyze the portal frame if the settlements of support D to the right and downwards in t-m units are $\frac{20}{EI}$ & $\frac{50}{EI}$ respectively.

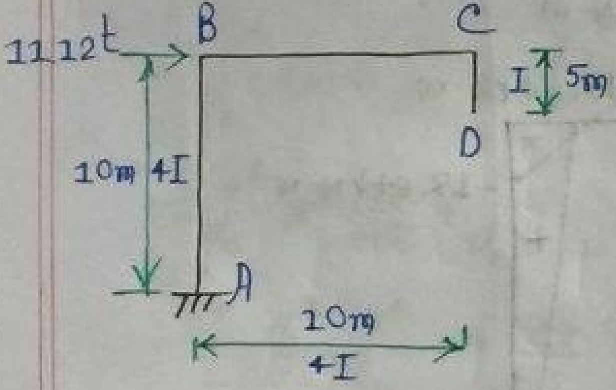
Solⁿ: (a) Here,

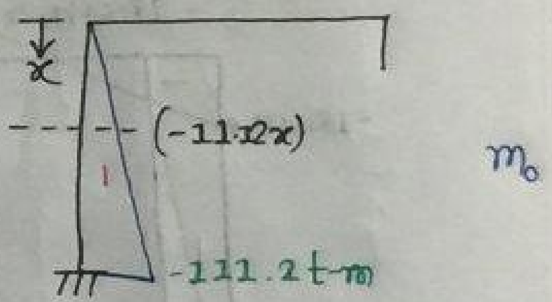


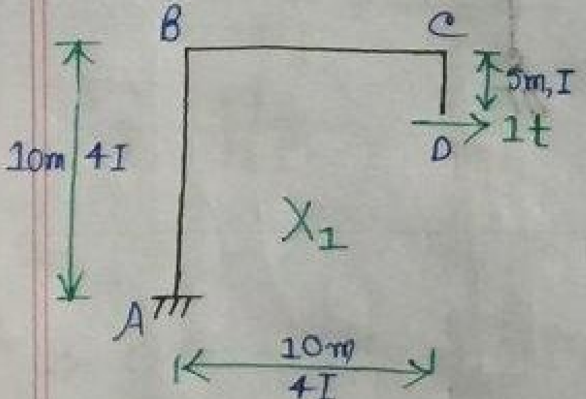
$D^\circ = (3+3) + (3 \times 3) - (4 \times 3)$

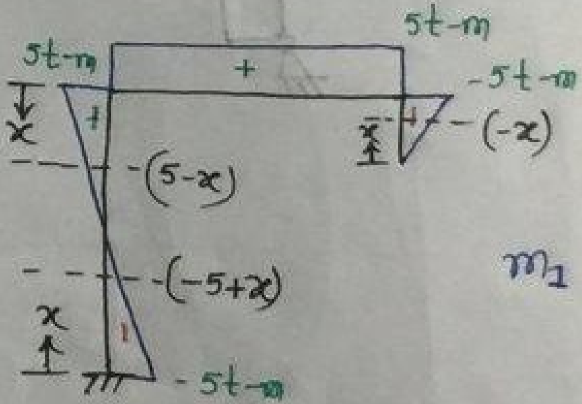
$= 3^\circ$

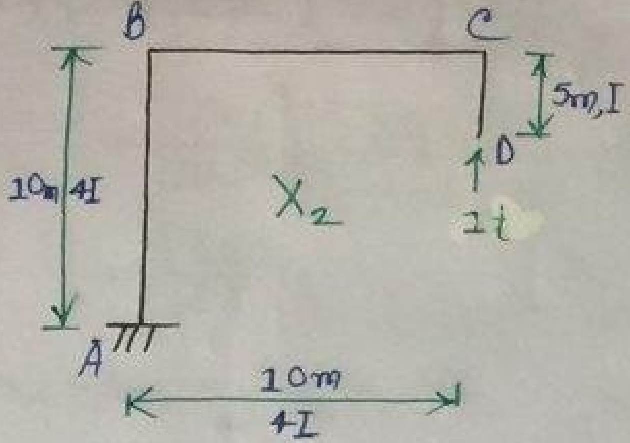
Release the right fixed support.

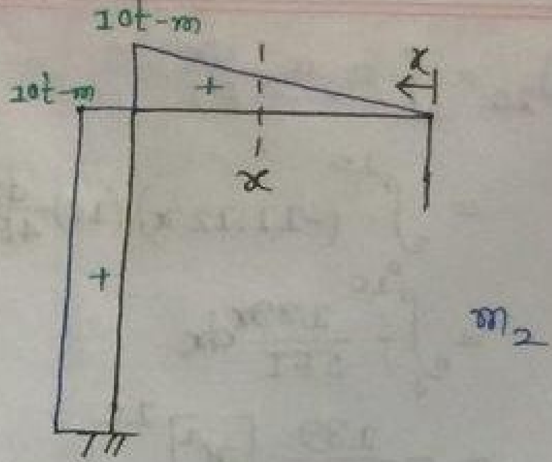


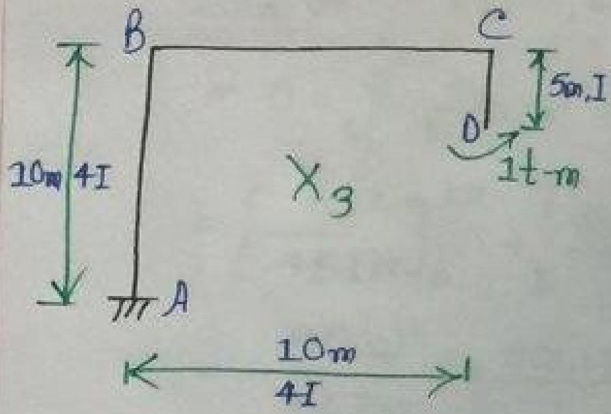


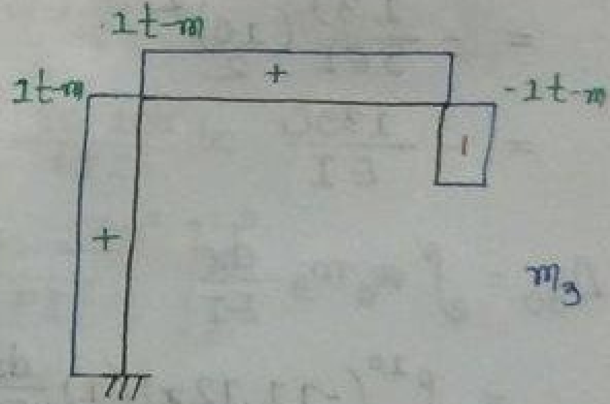












Now,

$$\Delta_{10} = \int m_0 m_1 \frac{dx}{EI}$$

$$= 0 + 0 + \int (-11.12x)(5-x)\frac{dx}{4EI}$$

$$= \int_0^{10} (-55.6x + 11.12x^2)\frac{dx}{4EI}$$

$$= \frac{1}{4EI}\left[-\frac{55.6x^2}{2} + \frac{11.12x^3}{3}\right]_0^{10}$$

$$= \frac{1}{4EI}\left[-\frac{55.6(10)^2}{2} + \frac{11.12(10)^3}{3}\right]$$

$$= \frac{695}{3EI}$$

$$\Delta_{20} = \int m_0 m_2 \frac{dx}{EI}$$

$$= \int_0^{10} (-11.12x)(10)\frac{dx}{4EI}$$

$$= \int_0^{10} -\frac{139x}{5EI} dx$$

$$= -\frac{139}{5EI}\left[\frac{x^2}{2}\right]_0^{10}$$

$$= -\frac{139}{5EI}\frac{(10)^2}{2}$$

$$= -\frac{1390}{EI}$$

$$\Delta_{30} = \int m_0 m_3 \frac{dx}{EI}$$

$$= \int_0^{10} (-11.12x)(1)\frac{dx}{4EI}$$

$$= \frac{1}{4EI}\left[-11.12\frac{x^2}{2}\right]_0^{10}$$

$$= \frac{1}{4EI}\left[-\frac{11.12(10)^2}{2}\right]$$

$$= -\frac{139}{EI}$$

$$\delta_{11} = \int m_1 m_1 \frac{dx}{EI}$$

$$= \int_0^5 (-x)(-x)\frac{dx}{EI} + \int_0^{10} (5)(5)\frac{dx}{4EI} + \int_0^{10} (5-x)^2 \frac{dx}{4EI}$$

$$= \int_0^5 \frac{x^2}{EI} dx + \int_0^{10} \frac{25}{4EI} dx + \int_0^{10} \frac{25 - 10x + x^2}{4EI} dx$$

$$\delta_{11} = \left[\frac{x^3}{3EI}\right]_0^5 + \left[\frac{25x}{4EI}\right]_0^{10} + \left[\frac{25x - \frac{10x^2}{2} + \frac{x^3}{3}}{4EI}\right]_0^{10}$$

$$= \frac{(5)^3}{3EI} + \frac{25(10)}{4EI} + \frac{25(10) - 5(10)^2 + \frac{(10)^3}{3}}{4EI}$$

$$= \frac{125}{EI}$$

$$\delta_{12} = \delta_{21} = \int m_1 m_2 \frac{dx}{EI}$$

$$= \int_0^5 (-x)\, 0 \frac{dx}{EI} + \int_0^{10} (5)\, x \frac{dx}{4EI} + \int_0^{10} (5-x)(10) \frac{dx}{4EI}$$

$$= 0 + \int_0^{10} \frac{5x}{4EI}\, dx + \int_0^{10} \frac{50-10x}{4EI}\, dx$$

$$= \left[\frac{5x^2}{4EI \times 2}\right]_0^{10} + \left[\frac{50x - \frac{10x^2}{2}}{4EI}\right]_0^{10}$$

$$= \frac{5(10)^2}{4EI \times 2} + \frac{50(10) - 5(10)^2}{4EI}$$

$$= \frac{62.5}{EI}$$

$$\delta_{13} = \delta_{31} = \int m_1 m_3 \frac{dx}{EI}$$

$$= \int_0^5 (-x)(-1) \frac{dx}{EI} + \int_0^{10} (5)(1) \frac{dx}{4EI} + \int_0^{10} (5-x)(1) \frac{dx}{4EI}$$

$$= \left[\frac{x^2}{2EI}\right]_0^5 + \left[\frac{5x}{4EI}\right]_0^{10} + \left[\frac{5x - \frac{x^2}{2}}{4EI}\right]_0^{10}$$

$$= \frac{(5)^2}{2EI} + \frac{5(10)}{4EI} + \frac{5(10) - \frac{(10)^2}{2}}{4EI}$$

$$= \frac{25}{EI}$$

$$\delta_{22} = \int m_2 m_2 \frac{dx}{EI}$$

$$= 0 + \int_0^{10} (x)(x) \frac{dx}{4EI} + \int_0^{10} (10)(10) \frac{dx}{4EI}$$

$$= \left[\frac{x^3}{3 \times 4EI}\right]_0^{10} + \left[\frac{100x}{4EI}\right]_0^{10}$$

$$= \frac{(10)^3}{3EI} + \frac{100(10)}{4EI}$$

$$= \frac{1000}{3EI}$$

$$\delta_{23} = \delta_{32} = \int m_2 m_3 \frac{dx}{EI}$$

$$= 0 + \int_0^{10} (x)(1) \frac{dx}{4EI} + \int_0^{10} (10)(1) \frac{dx}{4EI}$$

$$= \left[\frac{x^2}{2 \times 4EI}\right]_0^{10} + \left[\frac{10x}{4EI}\right]_0^{10}$$

$$= \frac{(10)^2}{8EI} + \frac{10(10)}{4EI}$$

$$= \frac{37.5}{EI}$$

$$\delta_{33} = \int m_3 m_3 \frac{dx}{EI}$$

$$= \int_0^5 (-1)(-1) \frac{dx}{EI} + \int_0^{10} (1)(1) \frac{dx}{4EI} + \int_0^{10} (1)(1) \frac{dx}{4EI}$$

$$= \left[\frac{x}{EI}\right]_0^5 + 2 \times \left[\frac{x}{4EI}\right]_0^{10}$$

$$= \frac{5}{EI} + 2 \times \frac{10}{4EI} = \frac{10}{EI}$$

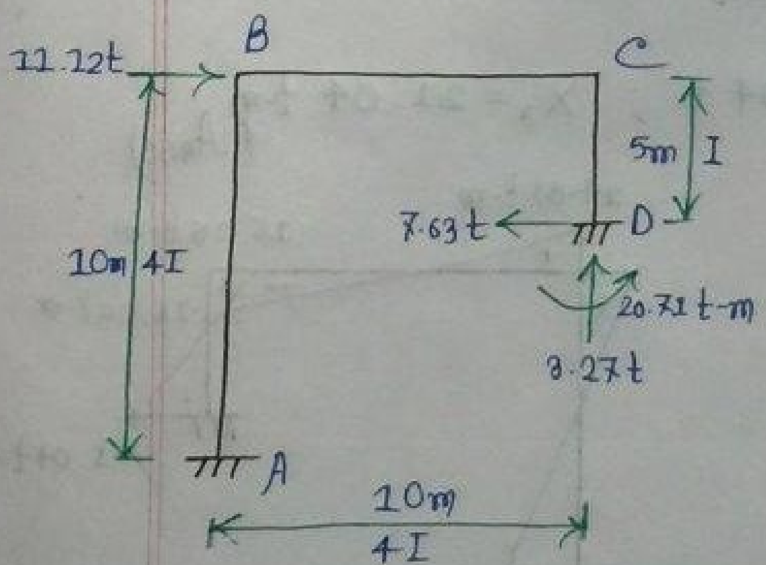
(a) For, No Support Settlements,

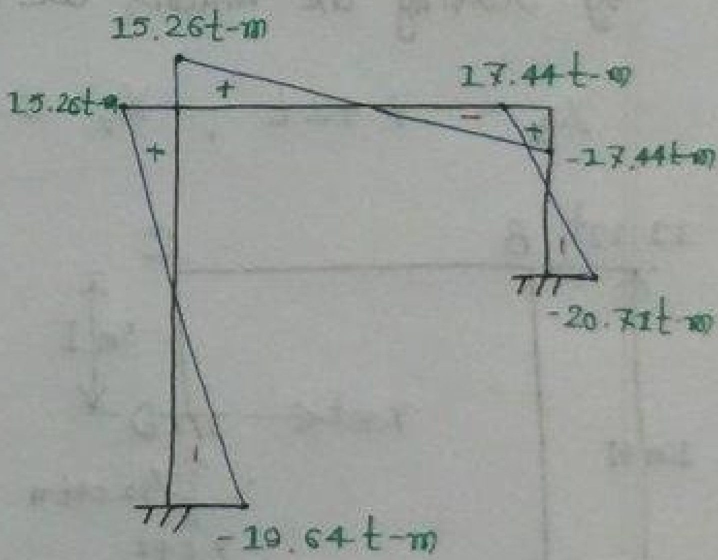
$$\begin{pmatrix}\Delta_1\\ \Delta_2\\ \Delta_3\end{pmatrix} = \begin{pmatrix}\Delta_{10}\\ \Delta_{20}\\ \Delta_{30}\end{pmatrix} + \begin{pmatrix}\delta_{11} & \delta_{12} & \delta_{13}\\ \delta_{21} & \delta_{22} & \delta_{23}\\ \delta_{31} & \delta_{32} & \delta_{33}\end{pmatrix}\begin{pmatrix}X_1\\ X_2\\ X_3\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}0\\ 0\\ 0\end{pmatrix} = \begin{pmatrix}\frac{695}{3EI}\\ -\frac{1390}{EI}\\ -\frac{139}{EI}\end{pmatrix} + \begin{pmatrix}\frac{125}{EI} & \frac{62.5}{EI} & \frac{25}{EI}\\ \frac{62.5}{EI} & \frac{1000}{3EI} & \frac{37.5}{EI}\\ \frac{25}{EI} & \frac{37.5}{EI} & \frac{10}{EI}\end{pmatrix}\begin{pmatrix}X_1\\ X_2\\ X_3\end{pmatrix}$$

By solving the matrix, we get,

$X_1 = -7.63\ t$; $X_2 = 3.27\ t$; $X_3 = 20.71\ t\text{-}m$ (Ans.)





Final BMD

(b) With support settlements, $\Delta_1 = \frac{20}{EI}$ (same direction to X_1)

& $\Delta_2 = -\frac{50}{EI}$ (opposite direction to X_2)

Hence,

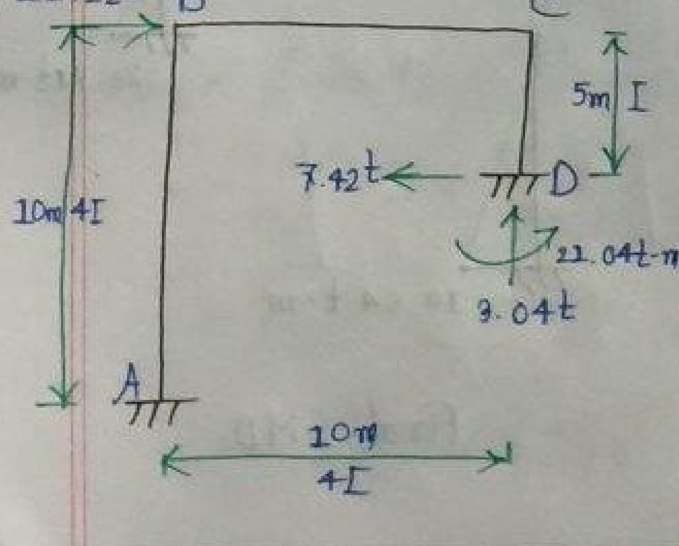
$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} \Delta_{10} \\ \Delta_{20} \\ \Delta_{30} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

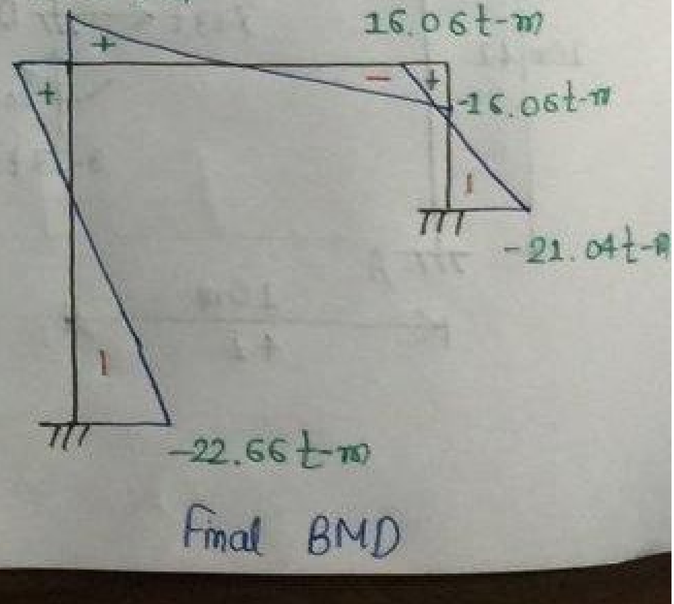
or,

$$\begin{pmatrix} \frac{20}{EI} \\ -\frac{50}{EI} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{695}{3EI} \\ -\frac{1390}{EI} \\ -\frac{139}{EI} \end{pmatrix} + \begin{pmatrix} \frac{125}{EI} & \frac{62.5}{EI} & \frac{25}{EI} \\ \frac{62.5}{EI} & \frac{1000}{3EI} & \frac{37.5}{EI} \\ \frac{25}{EI} & \frac{37.5}{EI} & \frac{10}{EI} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

By solving the matrix we get,

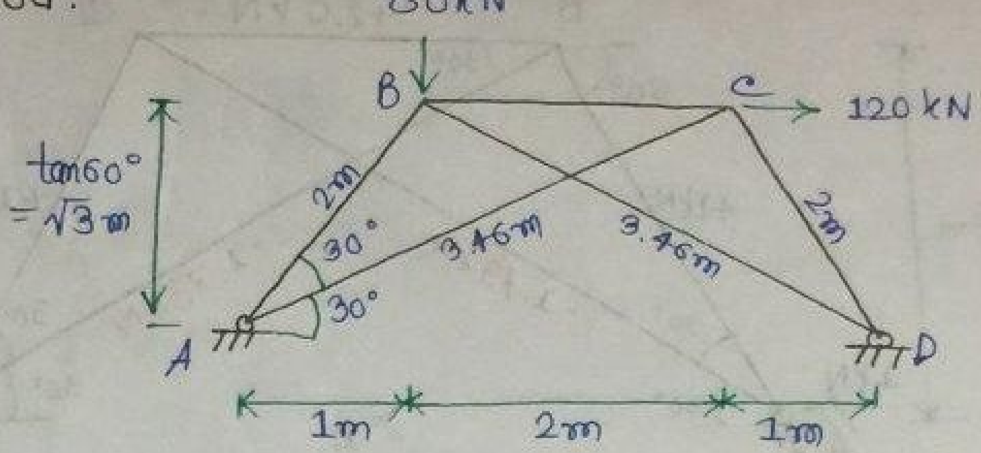
$X_1 = -7.42\,t$; $X_2 = 3.04\,t$; $X_3 = 21.04$ t-m (Ans)





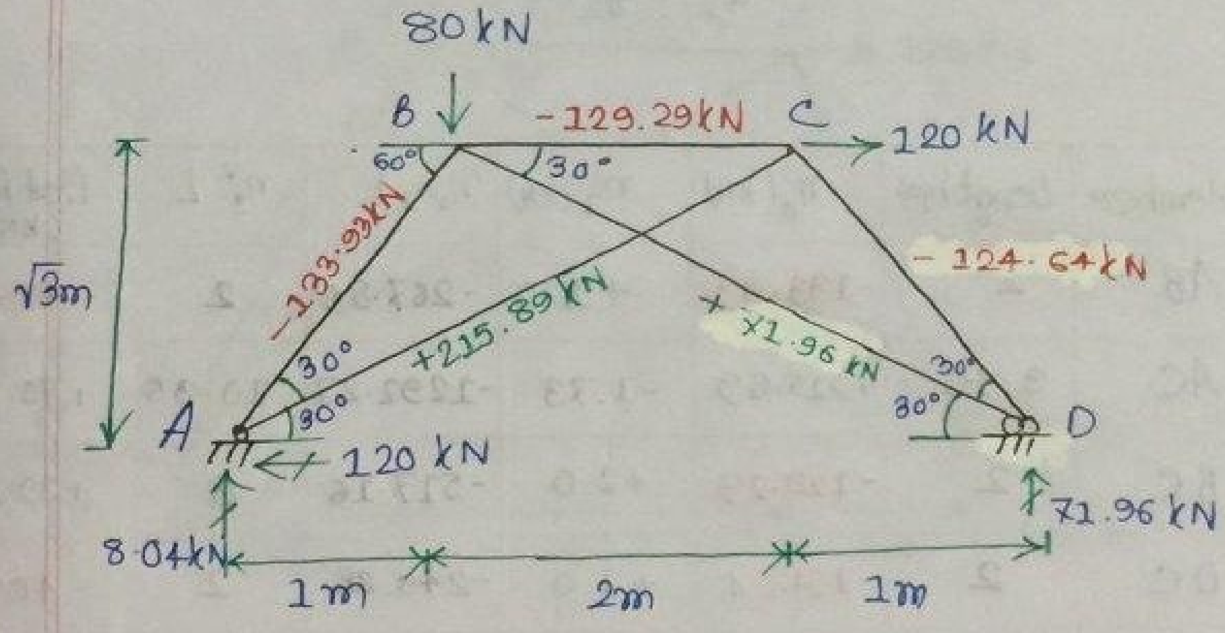
Final BMD

TRUSS-1: Analyze the indeterminate pin-jointed frame (truss) by the force method.

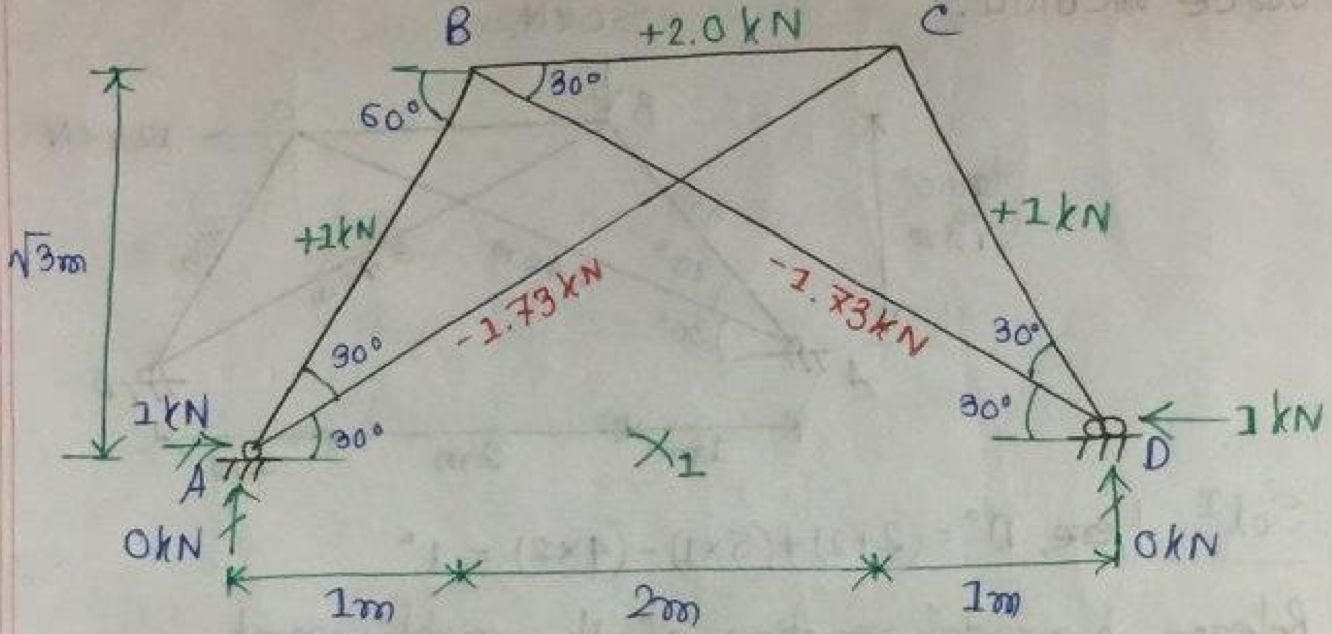


Soln: Here, $D^\circ = (2+2)+(5\times1)-(4\times2) = 1^\circ$

Release horizontal reaction of the right support.



n_0 - diagram



n_1 - diagram

Member	Length(m)	n_0(kN)	n_1(kN)	$n_0 n_1 L$	$n_1^2 L$	Final force (kN)
AB	2	-133.93	+1.0	-267.86	2	-49.66
AC	3.46	+215.89	-1.73	-1292.27	10.36	+70.1
BC	2	-129.29	+2.0	-517.16	8	+39.25
DC	2	-124.64	+1.0	-249.28	2	-40.37
DB	3.46	+71.96	-1.73	-430.74	10.36	-73.83
				-2757.28	32.72	(Using formula)

Note: Final force, $n = n_0 + \sum n_i X_i$

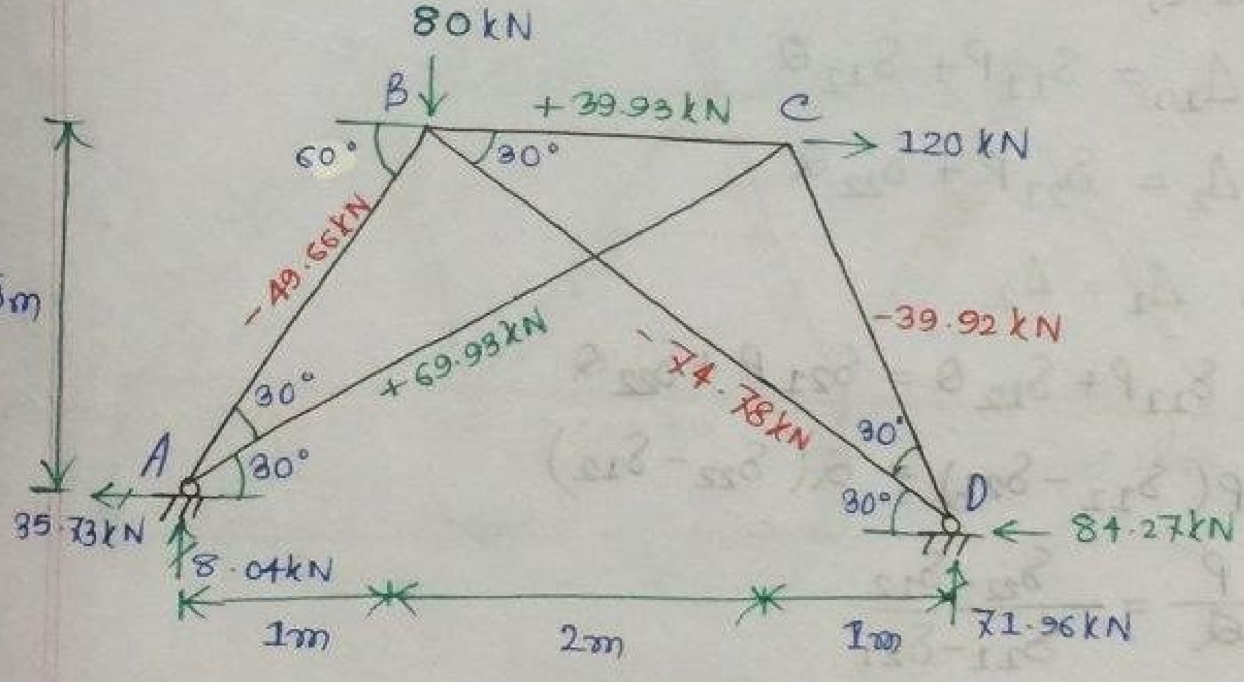
Now,

$$\Delta_1 = \Delta_{10} + \delta_{11} X_1$$

$$\Rightarrow \Delta_1 = \sum \frac{n_0 n_1 L}{AE} + \sum \frac{n_1 n_1 L}{AE} X_1$$

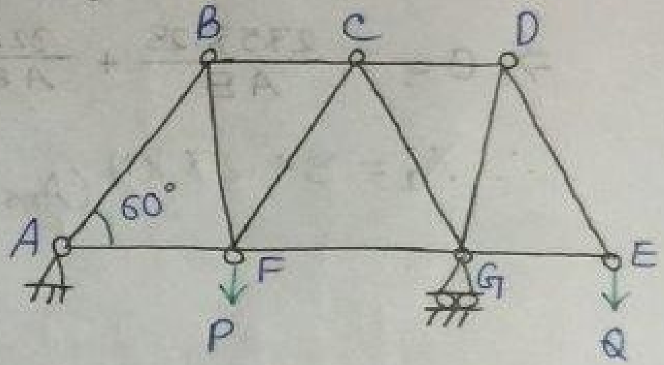
$$\Rightarrow 0 = -\frac{2757.28}{AE} + \frac{32.72}{AE} X_1$$

$$\therefore X_1 = 84.27 \text{ kN} \quad \text{(Ans.)}$$



Final Bar Forces (Using Truss Analysis)

TRUSS-2: For the pin jointed truss with loads P and Q as shown in figure, determine the ratio $\frac{P}{Q}$ so that the joint E and F have the same vertical displacement. $\frac{L}{AE}$ is constant for all members.



Soln: Here,

$D^\circ = (2+1)+(11\times1)-(7\times2)$
$= 0^\circ$

But due to vertical displacement, we have,

$$\Delta_1 = \delta_{11}P + \delta_{12}Q$$
$$\Delta_2 = \delta_{21}P + \delta_{22}Q$$

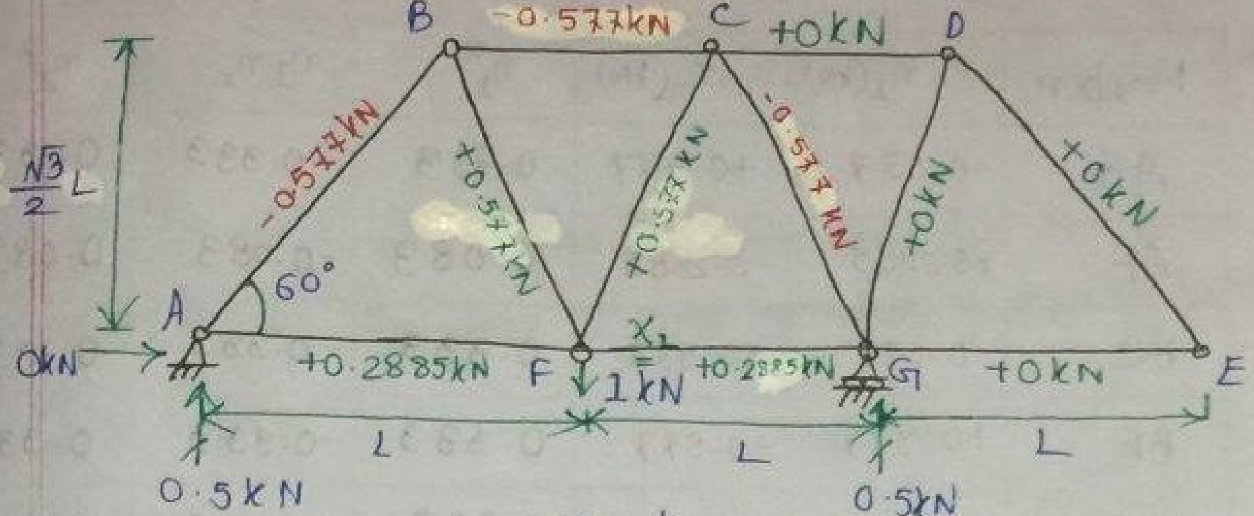
Now, $\Delta_1 = \Delta_2$

So, $\delta_{11}P + \delta_{12}Q = \delta_{21}P + \delta_{22}Q$

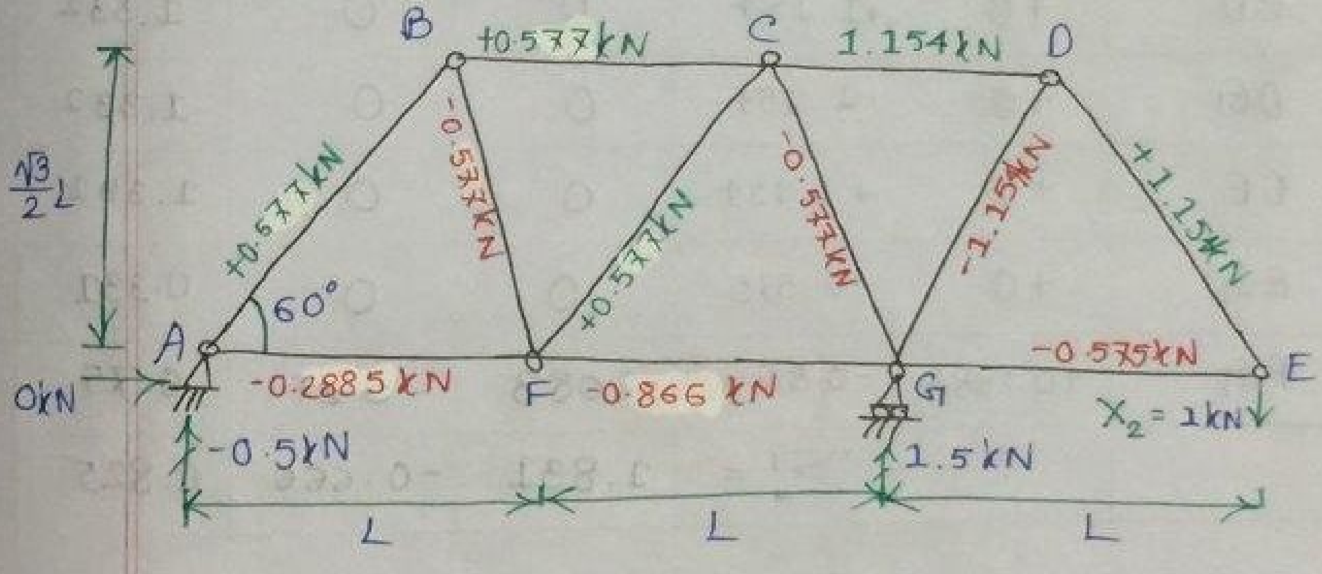
$\Rightarrow P(\delta_{11}-\delta_{21}) = Q(\delta_{22}-\delta_{12})$

$\therefore \frac{P}{Q} = \frac{\delta_{22}-\delta_{12}}{\delta_{11}-\delta_{21}}$

where, δ_{11} & δ_{21} are the displacements of points F & E respectively due to a unit vertical load at joint F. Similarly, δ_{12} & δ_{22} are the displacements of points F & E respectively due to a unit vertical load at joint E



n_1- diagram



n_2- diagram

Member	n_1(kN)	n_2(kN)	n_1^2	$n_1 n_2$	n_2^2
AB	-0.577	+0.577	0.333	-0.333	0.333
AF	+0.2885	-0.2885	0.083	-0.083	0.083
BC	-0.577	+0.577	0.333	-0.333	0.333
BF	+0.577	-0.577	0.333	-0.333	0.333
CF	+0.577	+0.577	0.333	+0.333	0.333
CG	-0.577	-0.577	0.333	+0.333	0.333
CD	+0	+1.154	0	0	1.332
DG	+0	-1.154	0	0	1.332
DE	+0	+1.154	0	0	1.332
EG	+0	-0.575	0	0	0.331
FG	+0.2885	-0.866	0.083	-0.25	0.75
		Σ =	1.831	-0.666	6.825

Now, $\delta_{11} = \Sigma \frac{n_1^2 L}{AE} = \frac{1.831 L}{AE}$

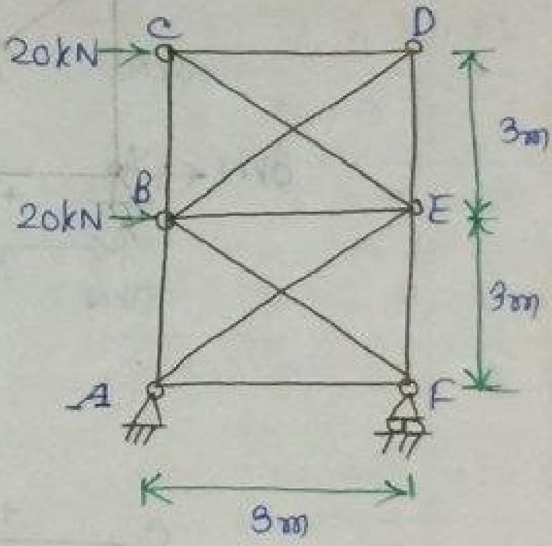
$$\delta_{12} = \delta_{21} = \Sigma \frac{n_1 n_2 L}{AE} = -\frac{0.666 L}{AE}$$

$$\delta_{22} = \Sigma \frac{n_2^2 L}{AE} = \frac{6.825 L}{AE}$$

Hence, $\frac{P}{Q} = \frac{\delta_{22} - \delta_{12}}{\delta_{11} - \delta_{21}} = \frac{\frac{5.825L}{AE} - \left(-\frac{0.666L}{AE}\right)}{\frac{1.831L}{AE} - \left(-\frac{0.666L}{AE}\right)} = 3$ (Ans)

TRUSS-3: It is required to determine the bar forces in the steel truss. The area of each member is $500 \times 10^{-6} m^2$ (500 mm^2).

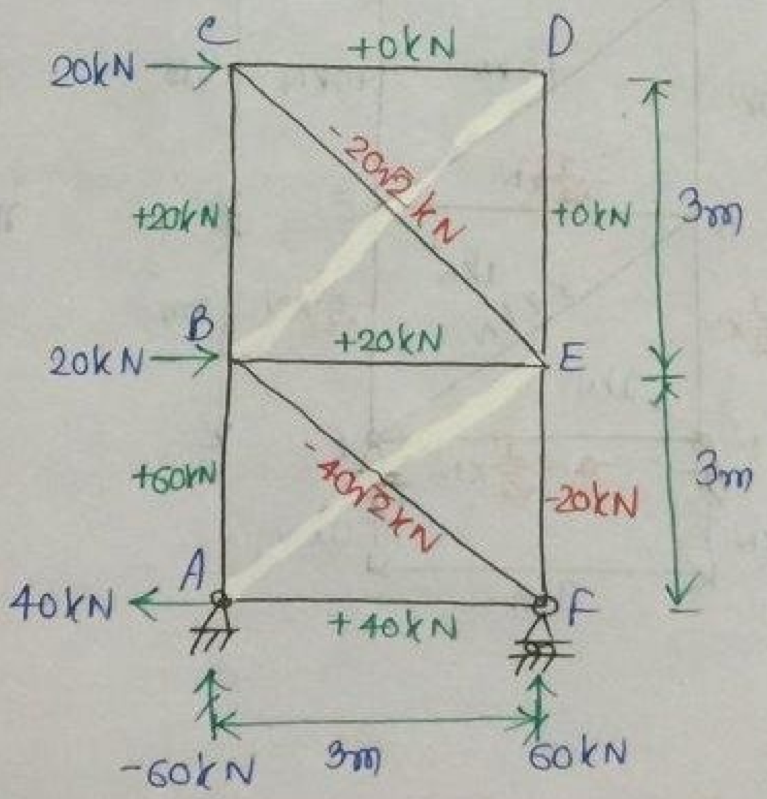
Soln: Here,



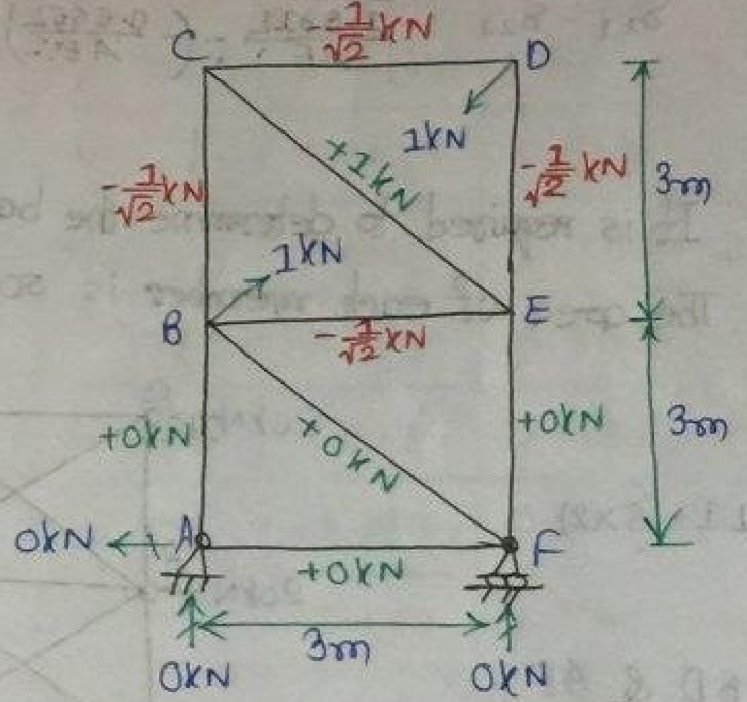
$D^\circ = (2+1) + 11 - (6 \times 2)$

$= 2^\circ$

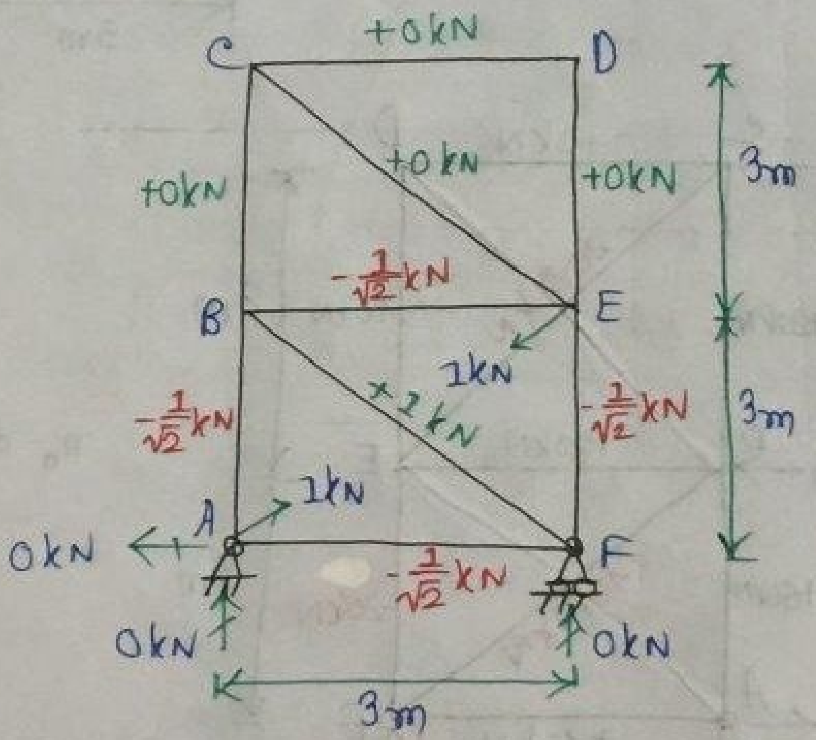
Release bar BD & AE.



n_o - diagram



n_1- diagram



n_2- diagram

Member	Length (m)	n_0 (kN)	n_1 (kN)	n_2 (kN)	$n_0 n_1 L$	$n_0 n_2 L$	$n_1^2 L$	$n_1 n_2 L$	$n_2^2 L$	Final Bar Force $= n_0 + n_1 X_1 + n_2 X_2$
AB	3	+60	+0	$-\frac{1}{\sqrt{2}}$	0	$-90\sqrt{2}$	0	0	1.5	+58.74
BC	3	+20	$-\frac{1}{\sqrt{2}}$	+0	$-30\sqrt{2}$	0	1.5	0	0	+12.2
FE	3	-20	+0	$-\frac{1}{\sqrt{2}}$	0	$30\sqrt{2}$	0	0	1.5	-41.
ED	3	+0	$-\frac{1}{\sqrt{2}}$	+0	0	0	1.5	0	0	-7.8
AF	3	+40	+0	$-\frac{1}{\sqrt{2}}$	0	$-60\sqrt{2}$	0	0	1.5	+18.74
BE	3	+20	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-30\sqrt{2}$	$-30\sqrt{2}$	1.5	1.5	1.5	-9.06
CD	3	+0	$-\frac{1}{\sqrt{2}}$	+0	0	0	1.5	0	0	-7.8
AE	$3\sqrt{2}$	+0	+0	+1	0	0	0	0	$3\sqrt{2}$	+30.071
BF	$3\sqrt{2}$	$-40\sqrt{2}$	+0	+1	0	-240	0	0	$3\sqrt{2}$	-26.50
BD	$3\sqrt{2}$	+0	+1	+0	0	0	$3\sqrt{2}$	0	0	+11.03
CE	$3\sqrt{2}$	$-20\sqrt{2}$	+1	+0	-120	0	$3\sqrt{2}$	0	0	-17.26
				Σ	$-60\sqrt{2}-120$	$-150\sqrt{2}$ -240	$6+6\sqrt{2}$	1.5	$6+6\sqrt{2}$	

Now,

$$\Delta_1 = \Delta_{10} + \delta_{11} X_1 + \delta_{12} X_2$$

$$\Delta_2 = \Delta_{20} + \delta_{21} X_1 + \delta_{22} X_2$$

Where, $\Delta_{10} = \Sigma \frac{n_0 n_1 L}{AE}$

$$\Delta_{20} = \Sigma \frac{n_0 n_2 L}{AE}$$

$$\delta_{11} = \Sigma \frac{n_1^2 L}{AE}$$

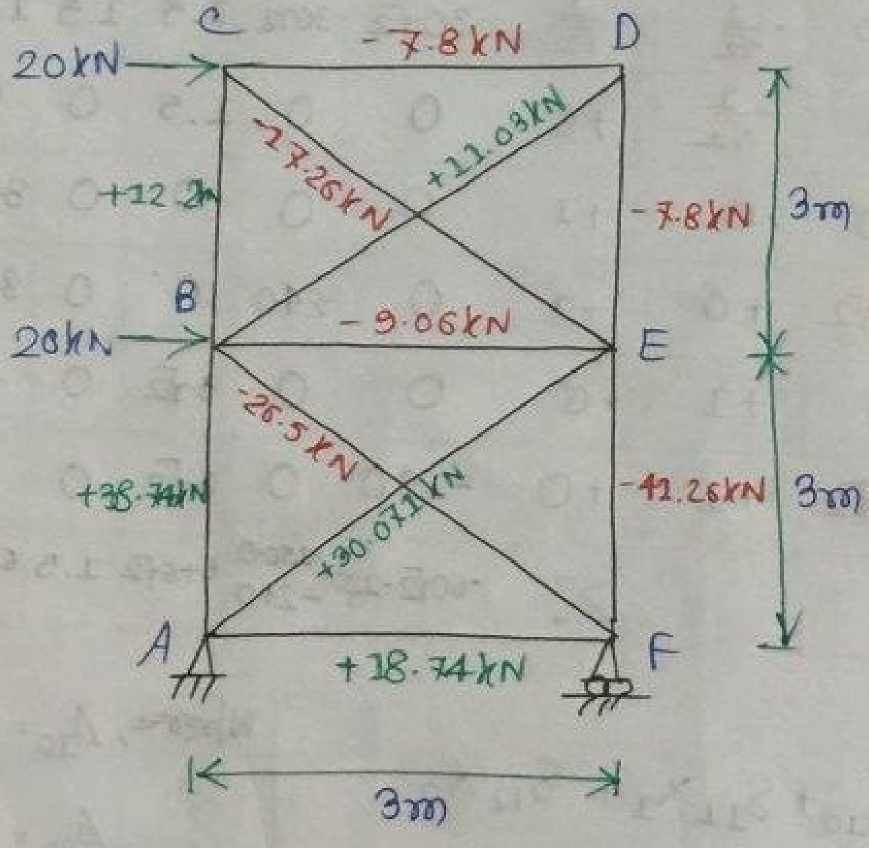
$$\delta_{12} = \delta_{21} = \Sigma \frac{n_1 n_2 L}{AE}$$

$$\delta_{22} = \Sigma \frac{n_2^2 L}{AE}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{-60\sqrt{2}-120}{AE} \\ \frac{-150\sqrt{2}-240}{AE} \end{pmatrix} + \begin{pmatrix} \frac{6+6\sqrt{2}}{AE} & \frac{1.5}{AE} \\ \frac{1.5}{AE} & \frac{6+6\sqrt{2}}{AE} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

By solving the matrix we get,

$X_1 = 11.028\, kN$ & $X_2 = 30.071\, kN$

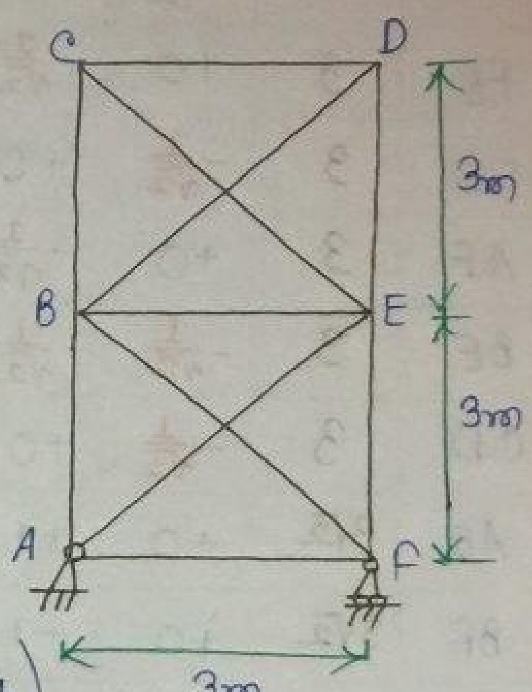


Final Bar Forces

TRUSS-4 : Consider the given truss and supposing that there is a temperature drop of 30°c on all the outer members AB, BC, CD, DE, EF, find the forces set up in the members due to temperature drop only. Take $\alpha = 1.0 \times 10^{-5}/°C$.

Sol^n: Here, same as TRUSS-3.

Instead of External loads, due to temperature drop, the outer members AB, BC, CD, DE, EF induces bar forces in those.



Here, $D° = 2°$

So,

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = \begin{pmatrix} \Delta_{10} \\ \Delta_{20} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

where, $\Delta_{i0} = \sum n_i \alpha (\Delta T) L$

$$\delta_{ij} = \sum \frac{n_i n_j L}{AE}$$

n_1 & n_2 diagram is same as TRUSS-3.

Given, $\alpha = 1.0 \times 10^{-5}/°C$

$\Delta T = -30°C$

$AE = (500 \times 10^{-6}\, m^2 \times 200 \times 10^6\, kPa) = 10^5$

	Member	Length	n_1	n_2	$n_1 L \times \alpha \Delta T$	$n_2 \alpha \Delta T L$ ($\xrightarrow{} -30°C$)	$n_1^2 L$	$n_1 n_2 L$	$n_2^2 L$	Final Bar Force $= n_1 X_1 + n_2 X_2$
ΔT	AB	3	+0	$-\frac{1}{\sqrt{2}}$	0	$+6.36 \times 10^{-4}$	0	0	1.5	+5.3
ΔT	BC	3	$-\frac{1}{\sqrt{2}}$	+0	$+6.36 \times 10^{-4}$	0	1.5	0	0	+8.76
ΔT	FE	3	+0	$-\frac{1}{\sqrt{2}}$	0	$+6.36 \times 10^{-4}$	0	0	1.5	+5.3
ΔT	ED	3	$-\frac{1}{\sqrt{2}}$	+0	$+6.36 \times 10^{-4}$	0	1.5	0	0	+8.76
	AF	3	+0	$-\frac{1}{\sqrt{2}}$	–	–	0	0	1.5	+5.3
	BE	3	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	–	–	1.5	1.5	1.5	+14.07
ΔT	CD	3	$-\frac{1}{\sqrt{2}}$	+0	$+6.36 \times 10^{-4}$	0	1.5	0	0	+8.76
	AE	$3\sqrt{2}$	+0	+1	–	–	0	0	$3\sqrt{2}$	-7.498
	BF	$3\sqrt{2}$	+0	+1	–	–	0	0	$3\sqrt{2}$	-7.498
	BD	$3\sqrt{2}$	+1	+0	–	–	$3\sqrt{2}$	0	0	-12.396
	CE	$3\sqrt{2}$	+1	+0	–	–	$3\sqrt{2}$	0	0	-12.396
				Σ	$1.908 \times 10^{-}$	1.272×10^{-3}	$6+6\sqrt{2}$	1.5	$6+6\sqrt{2}$	

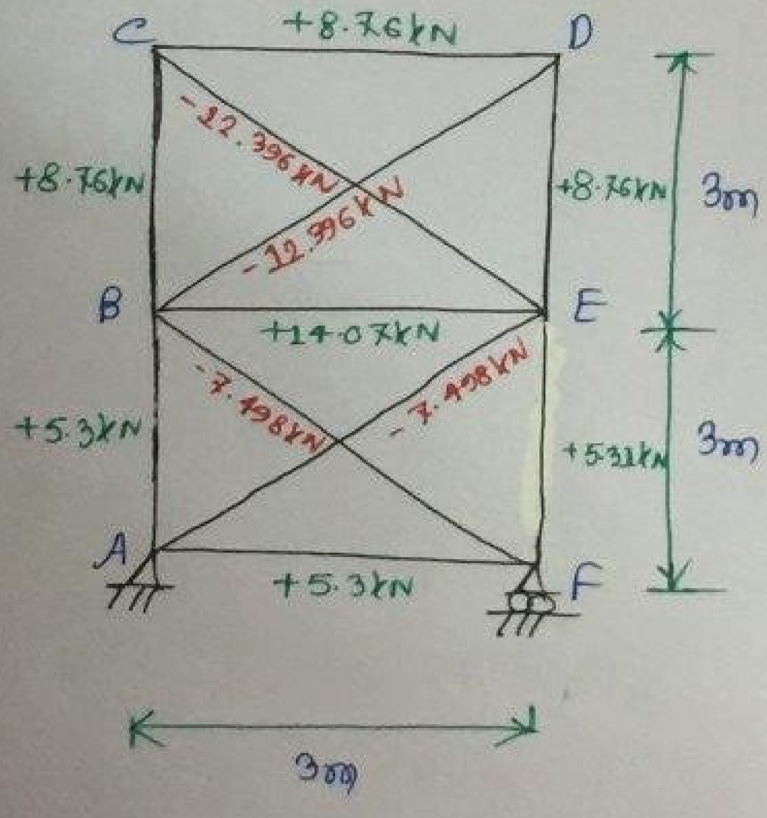
Now,

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = \begin{pmatrix} \Delta_{10} \\ \Delta_{20} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

Or, $$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1.908 \times 10^{-3} \\ 1.272 \times 10^{-3} \end{pmatrix} + \begin{pmatrix} \frac{6+6\sqrt{2}}{10^5} & \frac{1.5}{10^5} \\ \frac{1.5}{10^5} & \frac{6+6\sqrt{2}}{10^5} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

By solving the matrix, we get,

$X_1 = -12.396$ kN & $X_2 = -7.498$



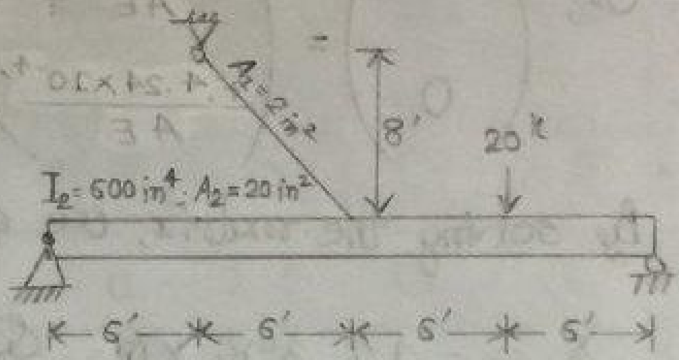
Composite Structure

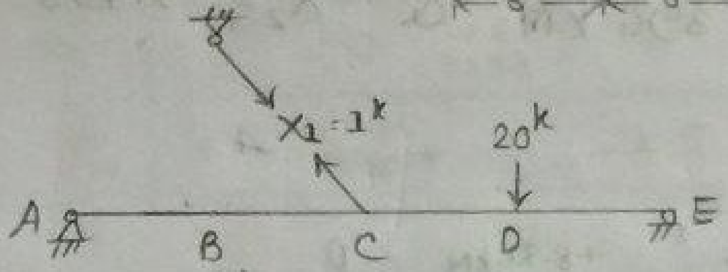
Example 9.8: Compute the force in the tie rod of this structure.

Soln: Here, $D^\circ = (2+1+1)-1$

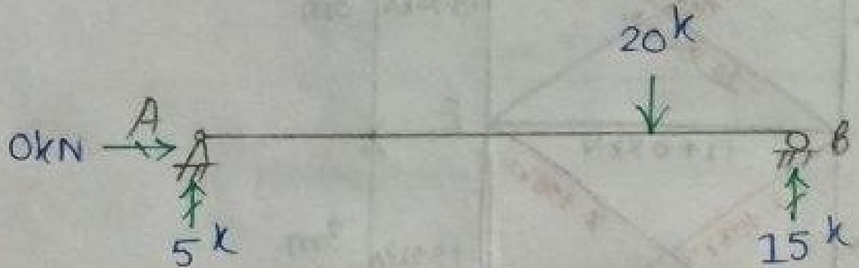
$= 1^\circ$

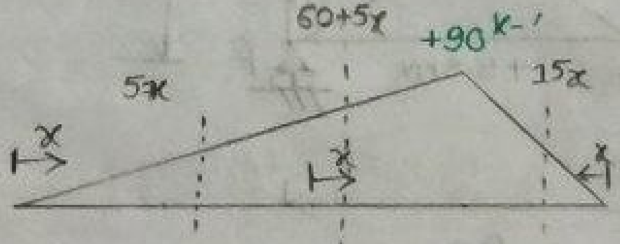
Cut the tie rod and select its bar force as redundant.





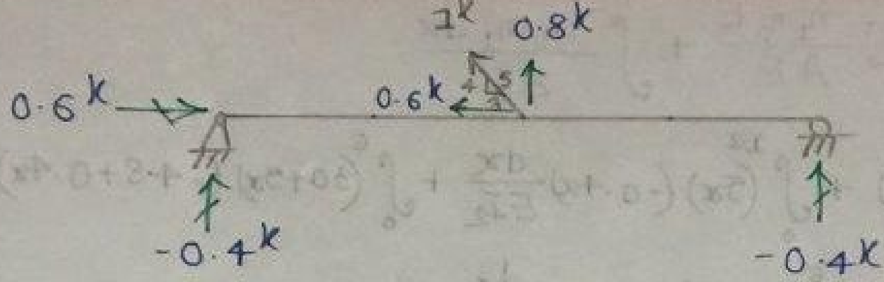
Primary Structure

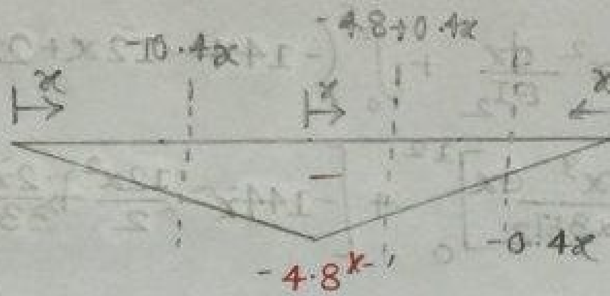




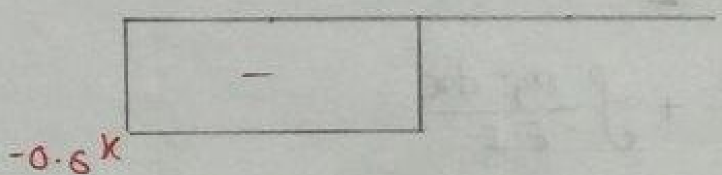
m_0

n_0



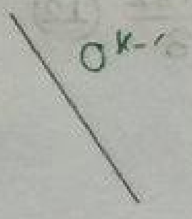


m_1



n_1

n_1 (cable)



m_1 (cable)

Now, $\Delta_{10} = \sum \frac{n_1 n_0 L}{AE} + \int \frac{m_0 m_1 \, dx}{EI}$

$$= 0 + \int_0^{12} (5x)(-0.4x)\frac{dx}{EI_2} + \int_0^{6} (60+5x)(-4.8+0.4x)\frac{dx}{EI_2}$$

$$+ \int_0^{6} (15x)(-0.4x)\frac{dx}{EI_2}$$

$$= \int_0^{12} -2x^2 \frac{dx}{EI_2} + \int_0^{6} (-288 - 0 \cdot x + 2x^2)\frac{dx}{EI_2} + \int_0^{6} (-6x^2)\frac{dx}{EI_2}$$

$$= \left[-\frac{2x^3}{3EI_2}\right]_0^{12} + \left[-288x + \frac{2x^3}{3}\right]_0^{6} \frac{1}{EI} + \left[-\frac{6x^3}{3EI_2}\right]_0^{6}$$

$$= \frac{1}{EI_2} \times \left[-\frac{2(12)^3}{3} - 288(6) + \frac{2(6)^3}{3} - 2(6)^3\right]$$

$$= -\frac{3168}{EI_2}$$

$\delta_{11} = \sum \frac{n_1^2 L}{AE} + \int \frac{m_1^2 \, dx}{EI}$

$$= \frac{(-0.6)^2 \times 12}{\frac{20}{(12)^2} \times E} + \frac{(1)^2 \times 10}{\frac{2}{(12)^2} \times E} + 2\int_0^{12} (-0.4x)^2 \frac{dx}{EI_2}$$

$$= \frac{31.104}{E} + \frac{720}{E} + 2\int_0^{12} 0.16x^2 \frac{dx}{EI_2}$$

$$= \frac{751.104}{E} + 0.32\left[\frac{x^3}{3}\right]_0^{12} \frac{1}{EI_2}$$

$$= \frac{751.104}{E} + \frac{0.32}{3}(12)^3 \cdot \frac{1}{E \times I_2}$$

$$\Rightarrow EI_2\,\delta_{11} = 751.104\, I_2 + 184.32$$

$$= 751.104 \times \frac{600}{(12)^4} + 184.32$$

$$\delta_{11} = \frac{206.0533}{EI_2}$$

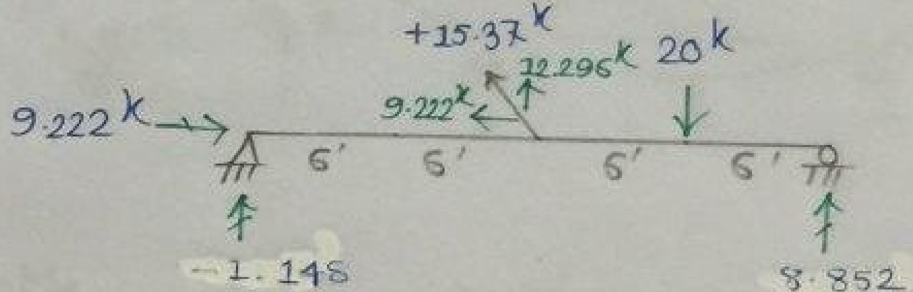
Now,

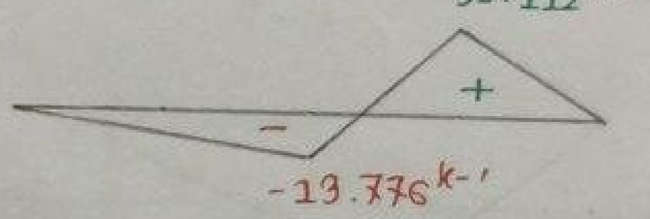
$$\Delta_1 = \Delta_{10} + \delta_{11} X_1$$

$$\Rightarrow 0 = -\frac{3168}{EI_2} + \frac{206.0533}{EI_2} X_1$$

$$\therefore X_1 = +15.37^k$$

(Ans.)



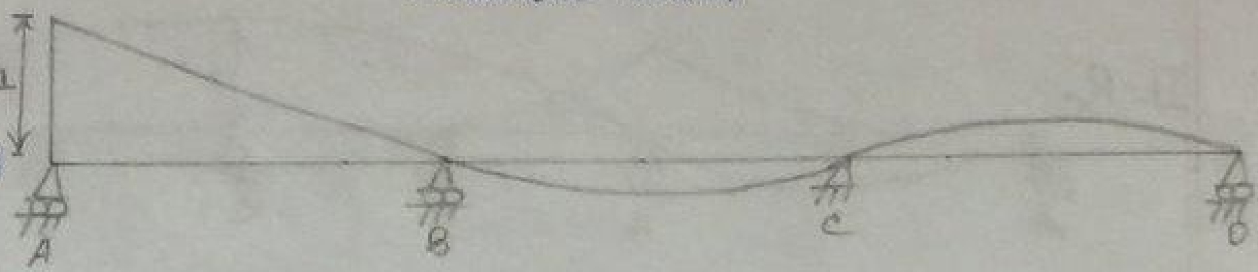


BMD

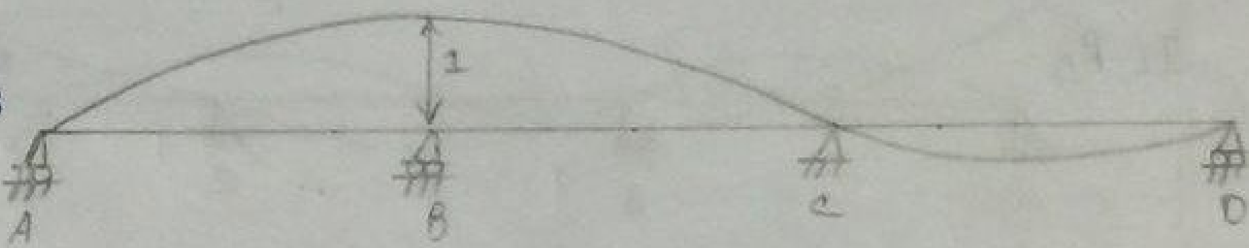
Qualitative Influence Lines for Statically Indeterminate Structures

Continuous Beam

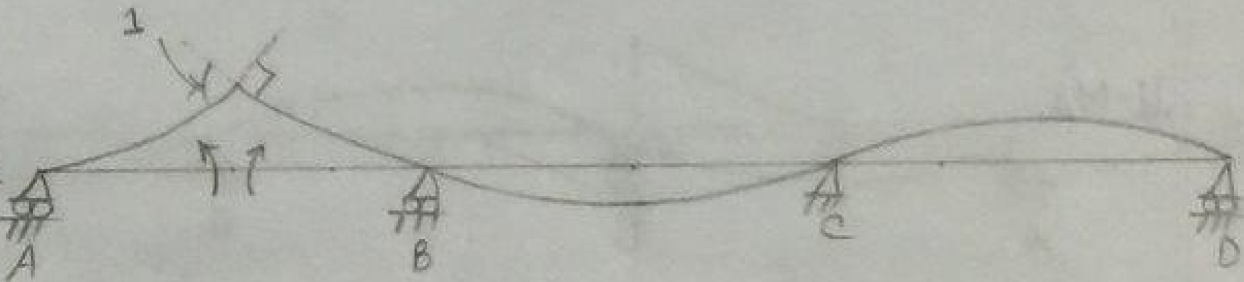
$IL-R_A$



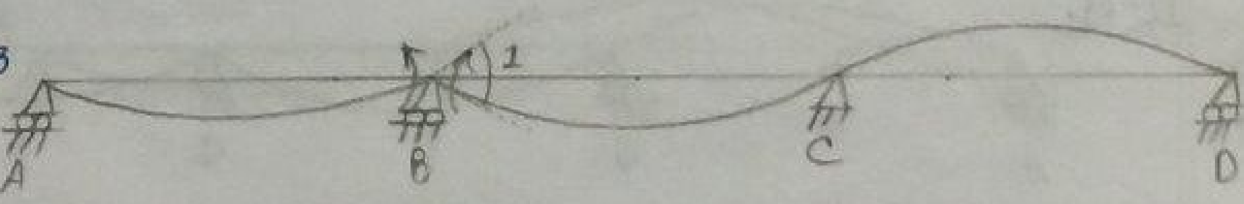
$IL-R_B$



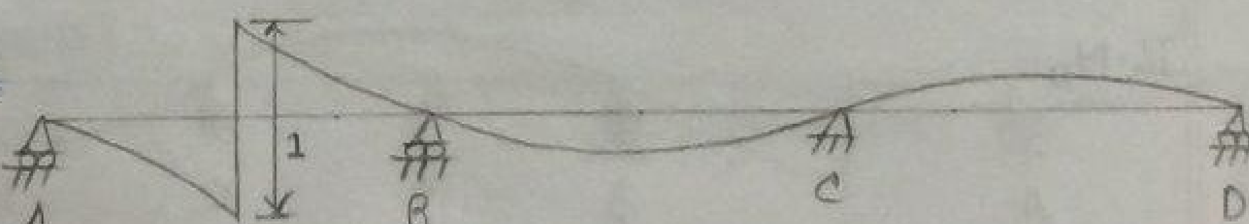
$IL-M_x$

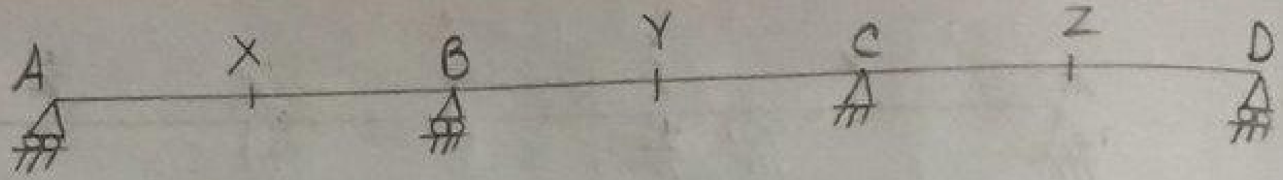


$IL-M_B$

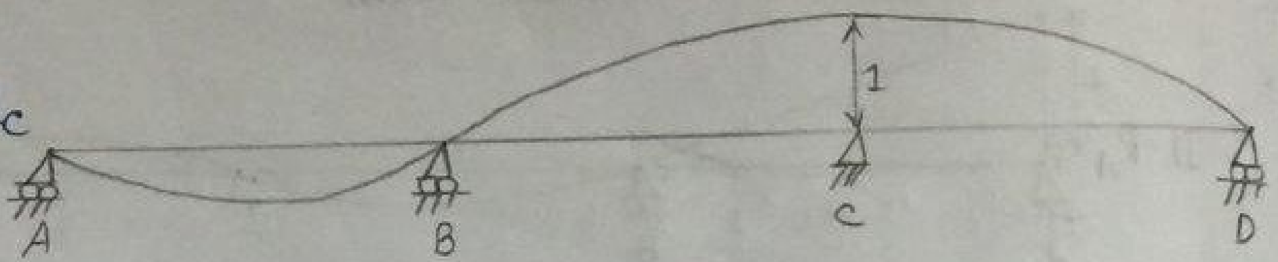


$IL-V_x$

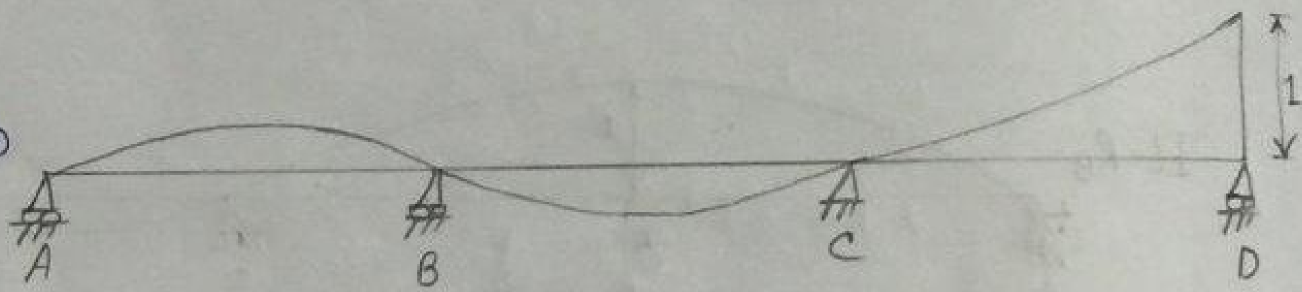




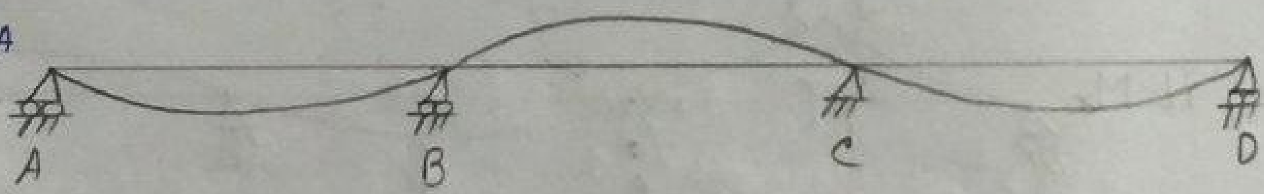
IL-R_C



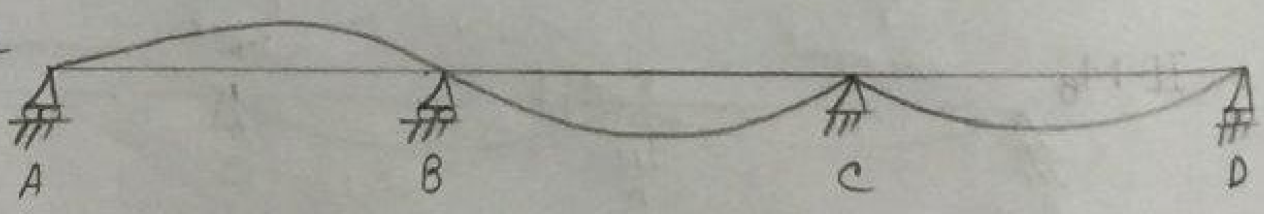
IL-R_D



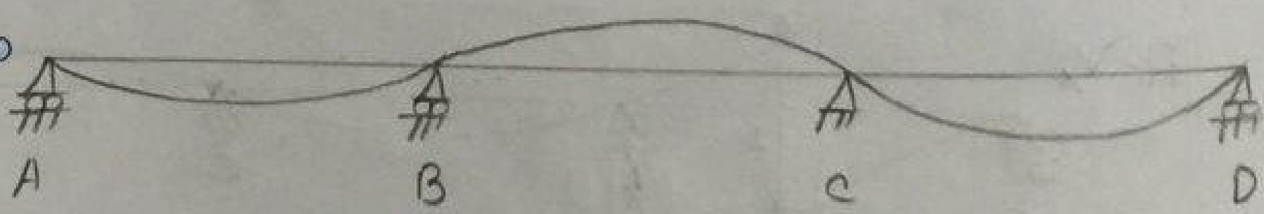
IL-M_A

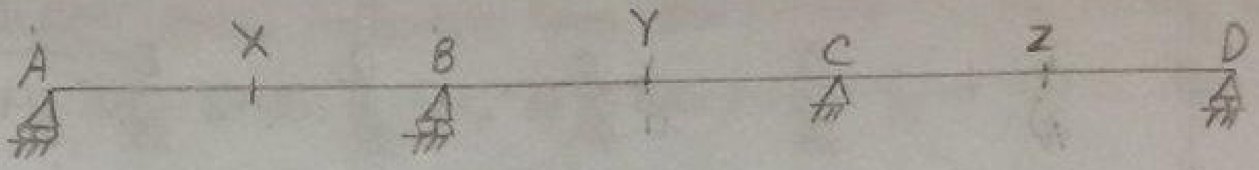


IL-M_C

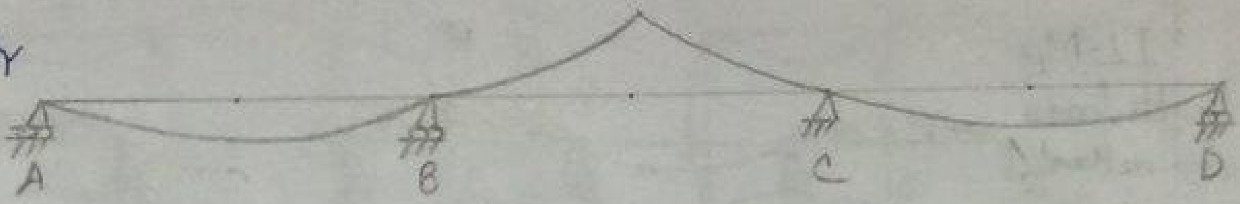


IL-M_D

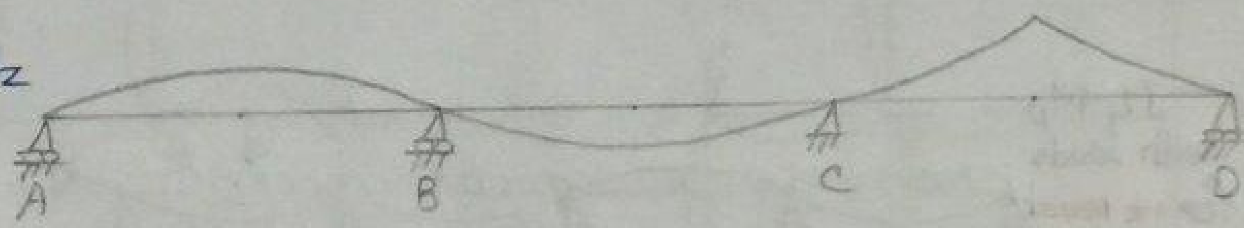




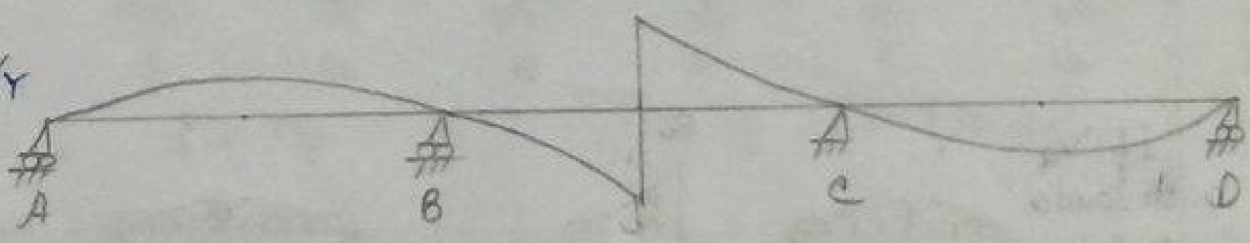
IL-M_Y



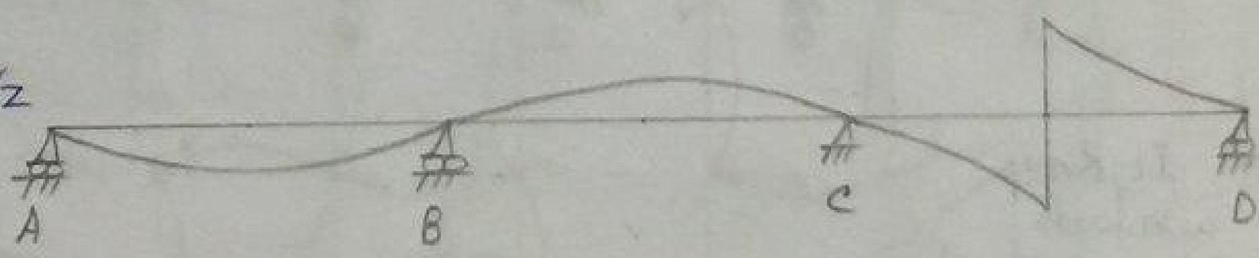
IL-M_Z



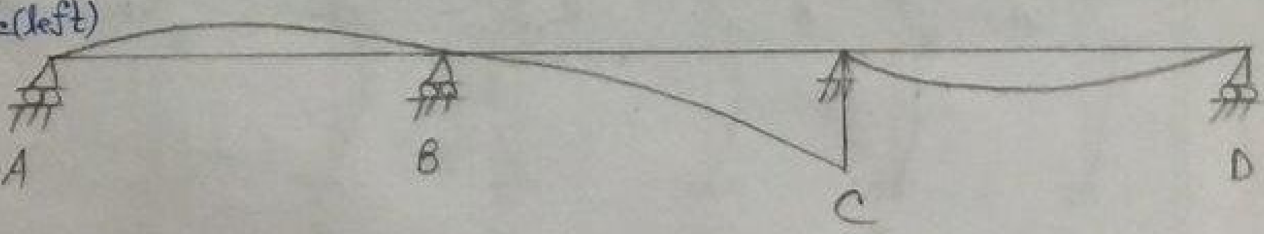
IL-V_Y



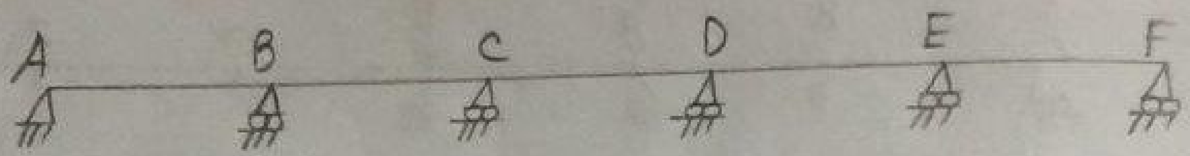
IL-V_Z



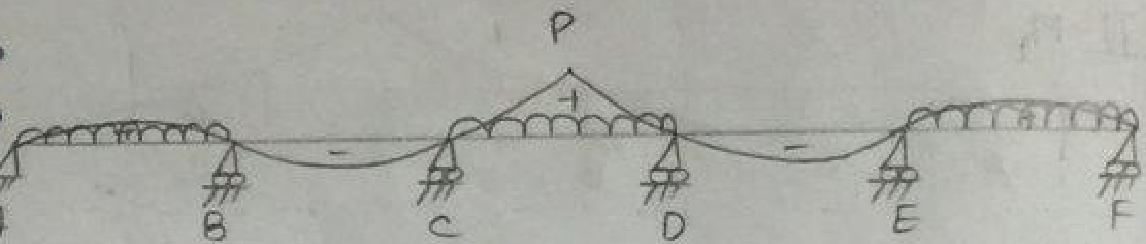
IL-$V_{C(left)}$



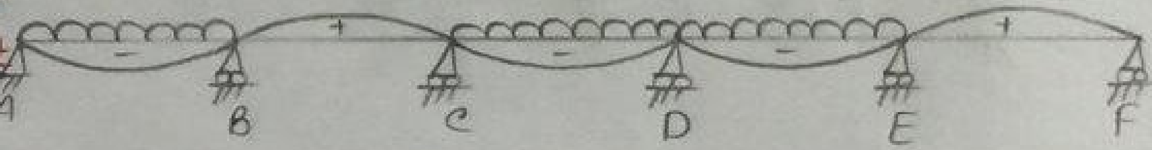
Qualitative Influence Lines & Load Patterns



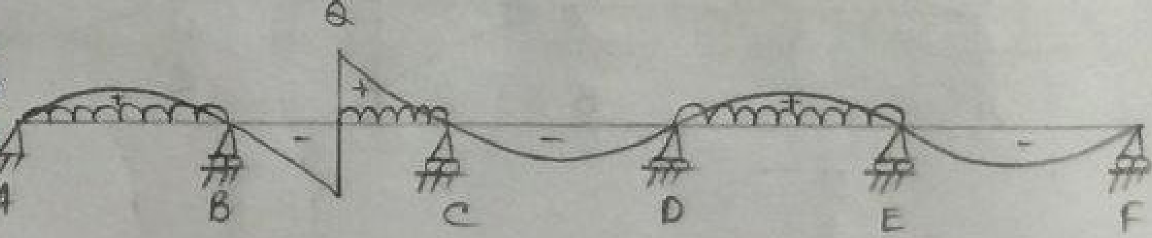
IL-M_P
with loads
for +ve Moment



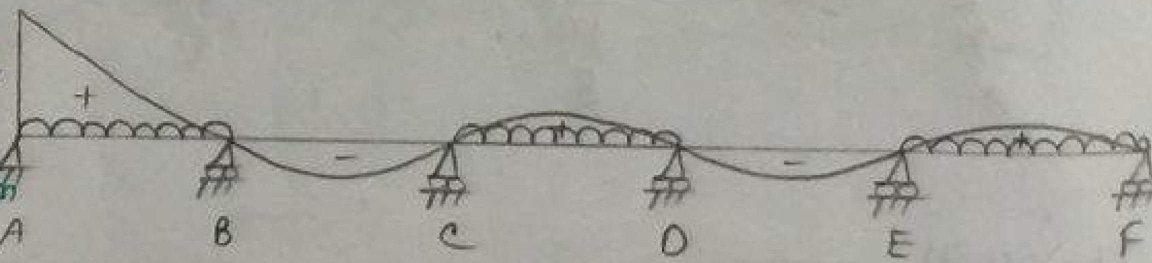
IL-M_D
with loads
for +ve Moment



IL-V_Q
with loads
for Max^m Shear



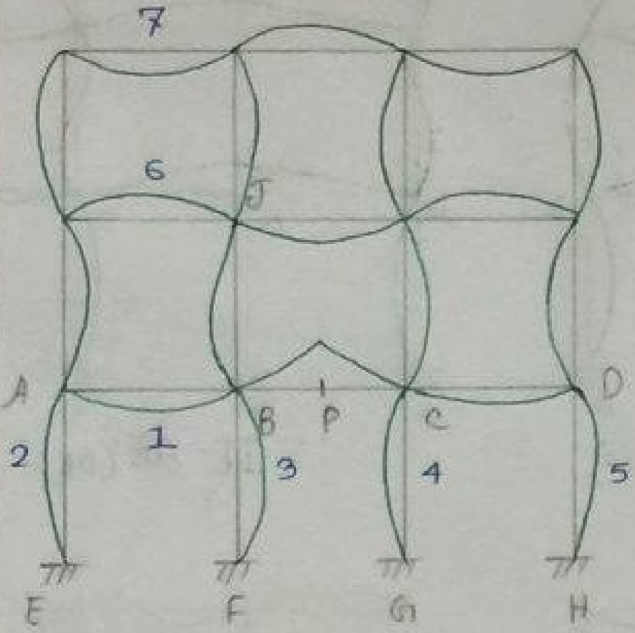
IL-R_A
with loads
for Max^m
value of reaction



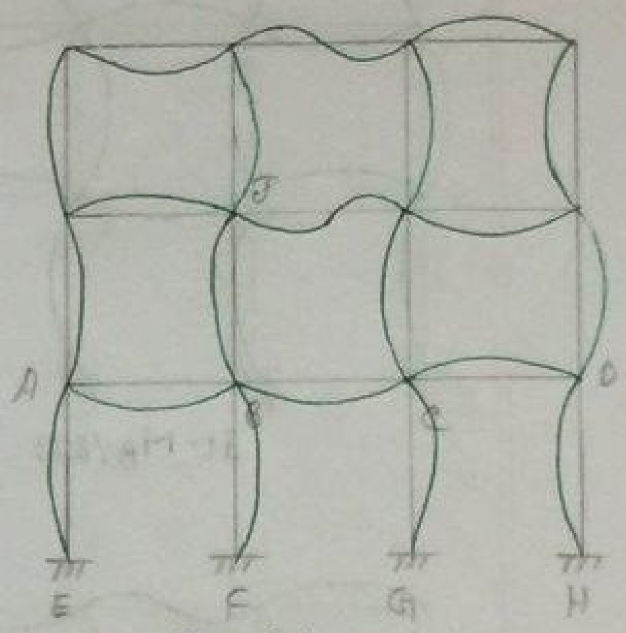
FRAMES

PROBLEM: Draw influence lines for (i) M_P, (ii) M_B of Beam BC (iii) M_B of Beam BA (iv) M_B of Column BF (v) M_B of column BJ (vi) Axial force in column BF (vii) V_P

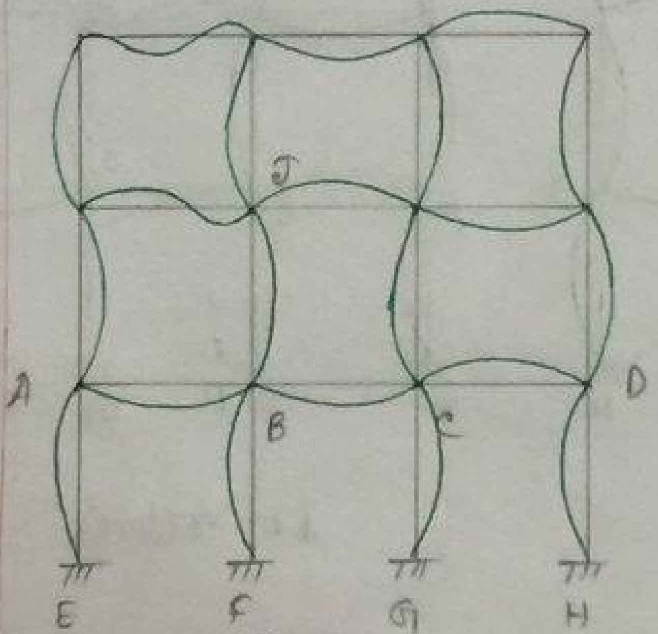
Note: Sequence for drawing IL-1-2-3-4-5-6-7



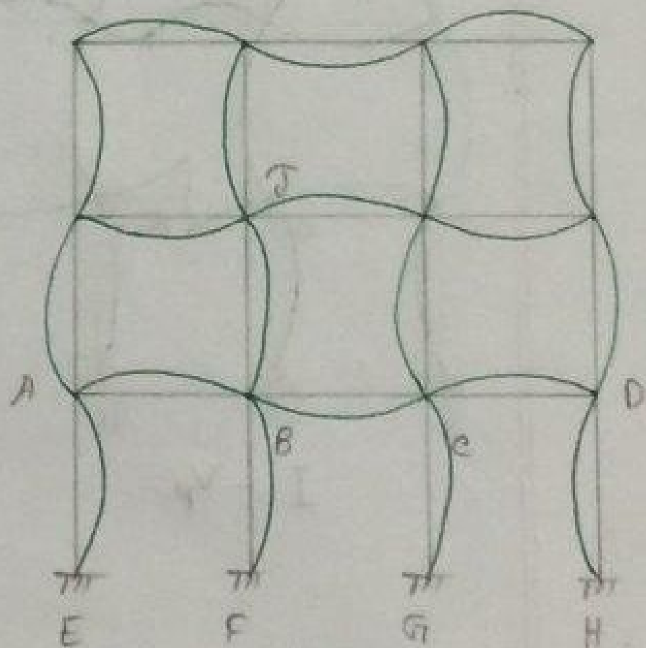
IL- M_P



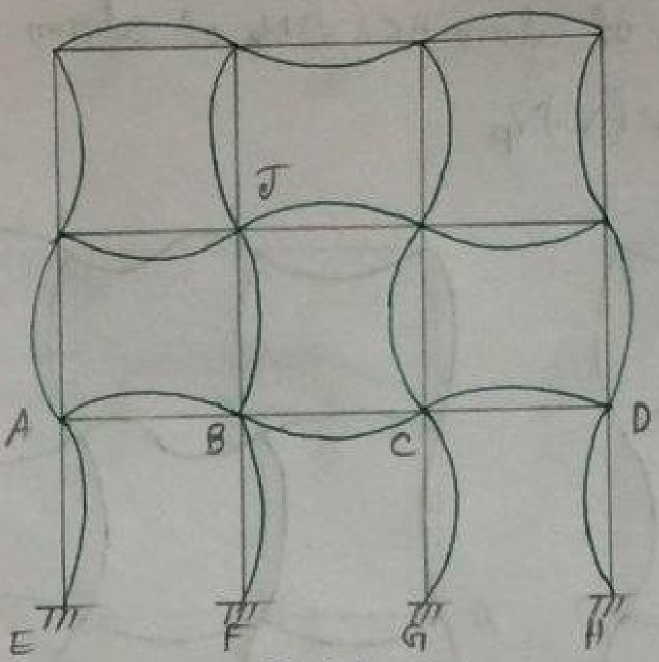
IL- M_B (BC)



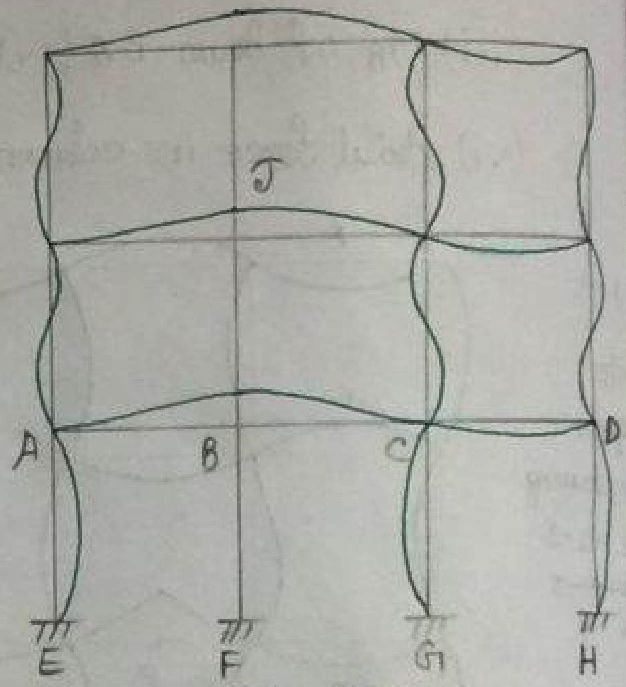
IL- M_B (BA)



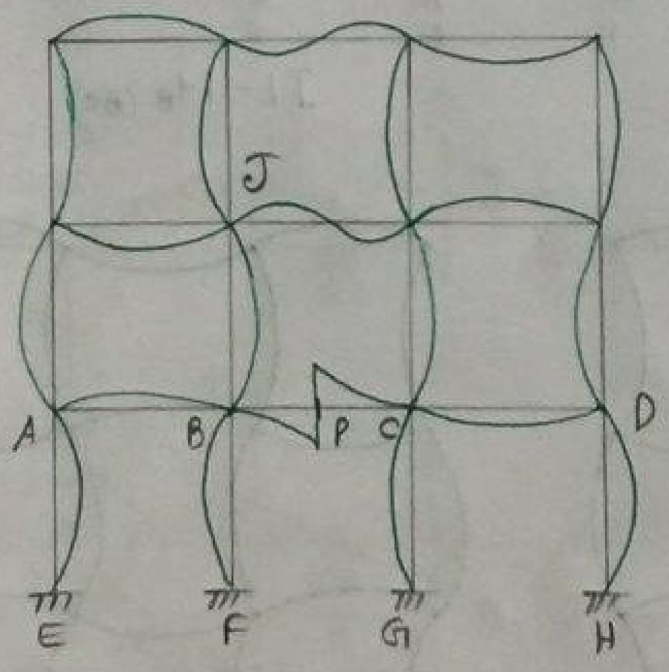
IL- M_B (BF)



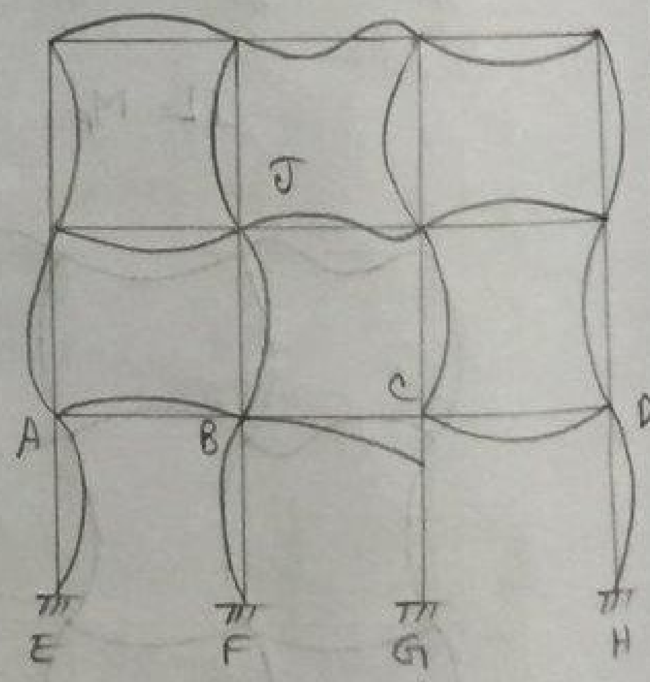
IL - M_B (BJ)



IL - A.F. (BF)



IL - V_P

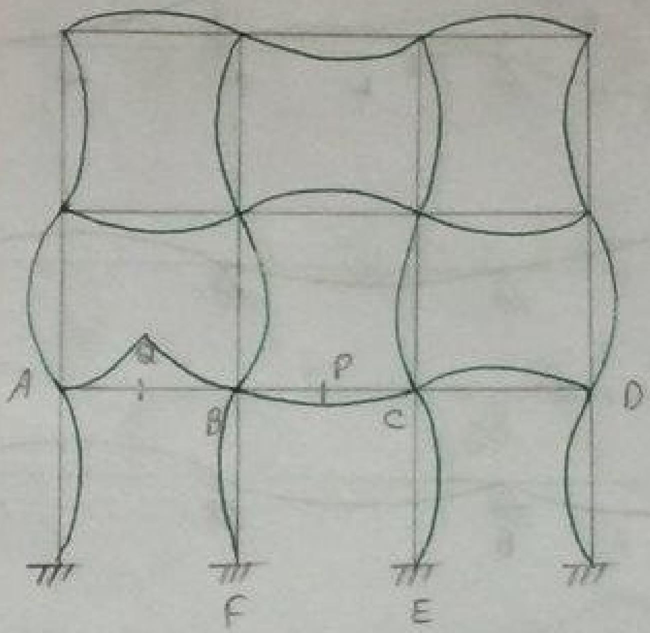


IL - V_C (left)

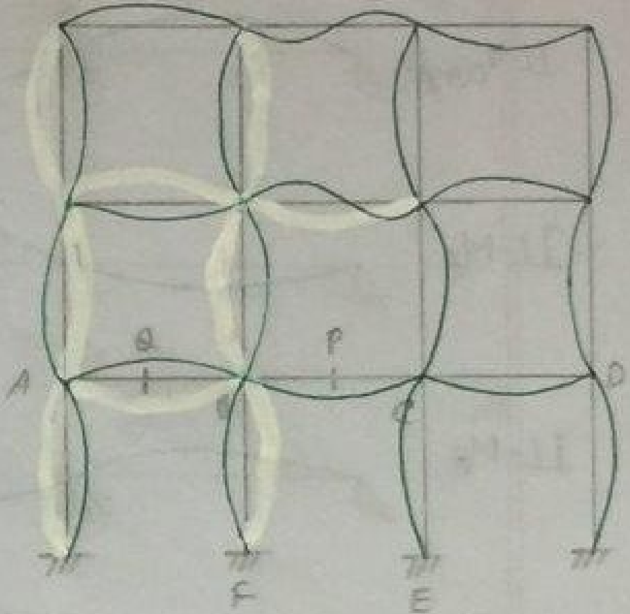
Question Paper Solutions

"2014-15"

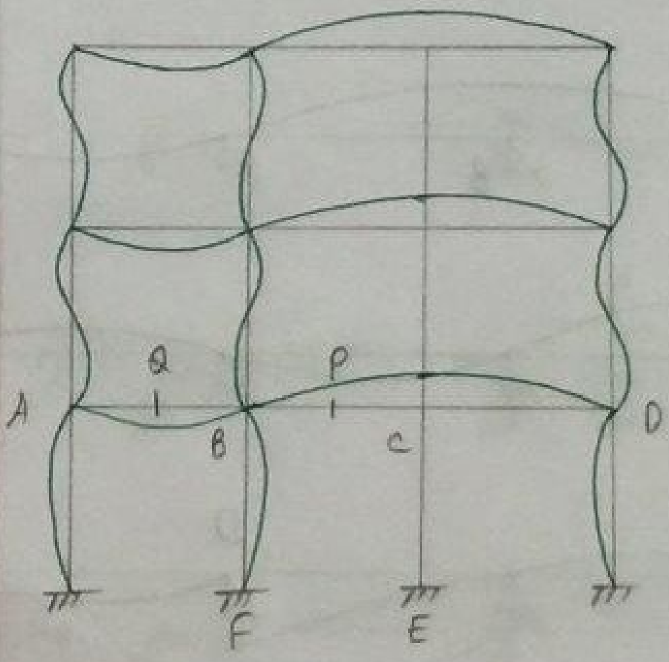
2.(b)



IL-M_Q



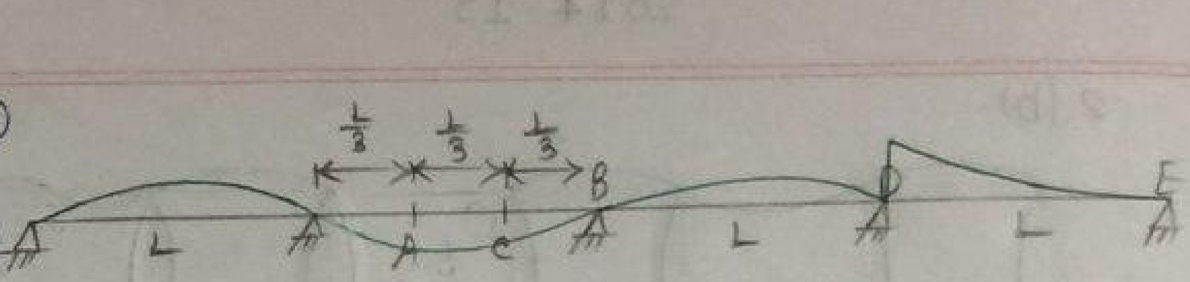
IL-$M_{C(BC)}$



IL-A.F.(CE)

7. (b)

IL-$V_{D(right)}$

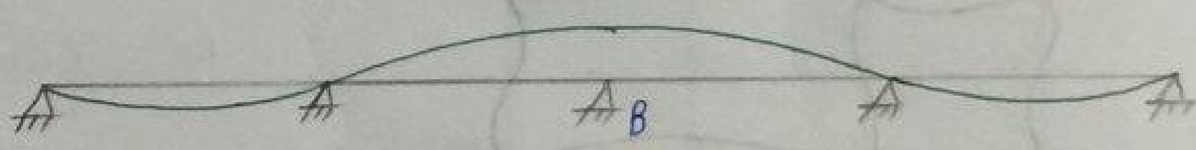


IL-M_A

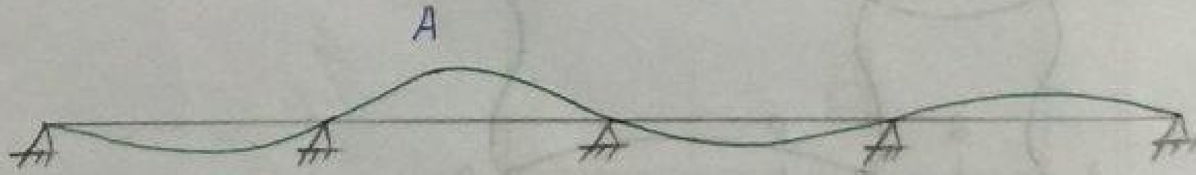
IL-M_B

IL-V_A

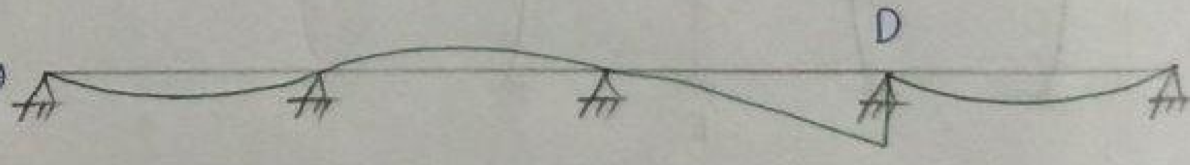
IL-R_B



IL-Δ_A



IL-$V_{D(left)}$



IL-R_E