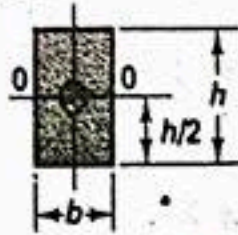


$$I_x = I_0 + Ad^2$$

TABLE 2. USEFUL PROPERTIES OF AREAS

Areas and moments of inertia of areas around centroidal axes

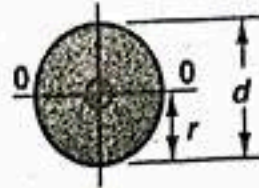
RECTANGLE



$$A = bh$$

$$I_o = bh^3/12$$

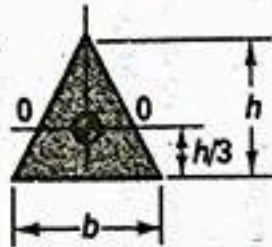
CIRCLE



$$A = \pi R^2$$

$$I_o = I_p/2 = \pi R^4/4$$

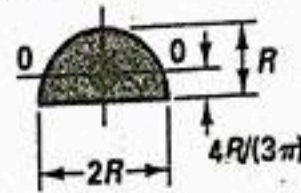
TRIANGLE



$$A = bh/2$$

$$I_o = bh^3/36$$

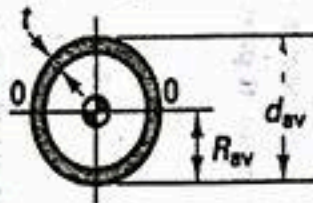
SEMICIRCLE



$$A = \pi R^2/2$$

$$I_o = 0.110R^4$$

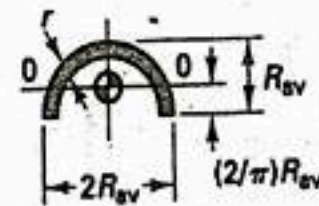
THIN TUBE



$$A = 2\pi R_{sv}t$$

$$I_o = I_p/2 \approx \pi R_{sv}^3t$$

HALF OF THIN TUBE

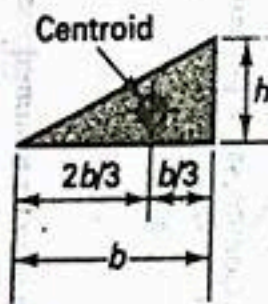


$$A = \pi R_{sv}t$$

$$I_o = 0.095\pi R_{sv}^3t$$

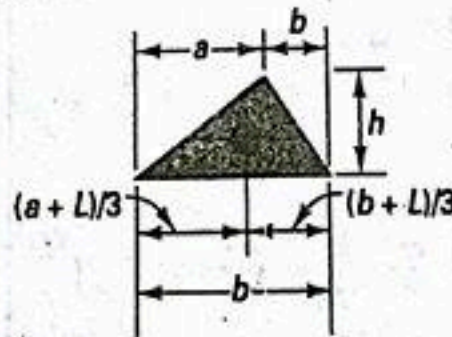
Areas and Centroids of areas

TRIANGLE



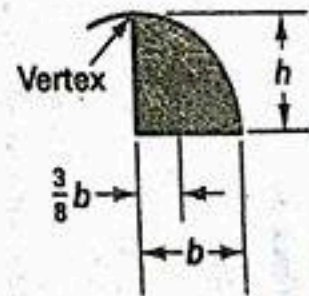
$$A = bh/2$$

TRIANGLE



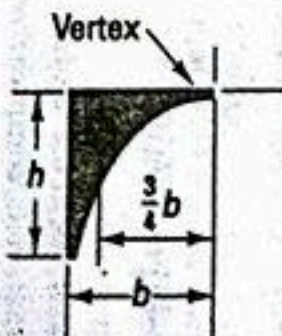
$$A = hL/2$$

PARABOLA



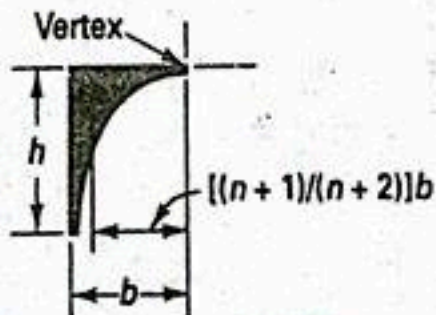
$$A = \frac{2}{3}bh$$

PARABOLA:  $y = -ax^2$



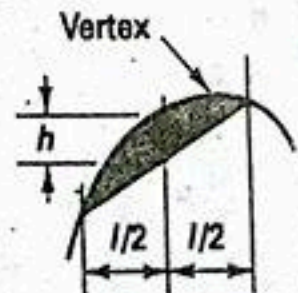
$$A = bh/3$$

$y = -ax^n$

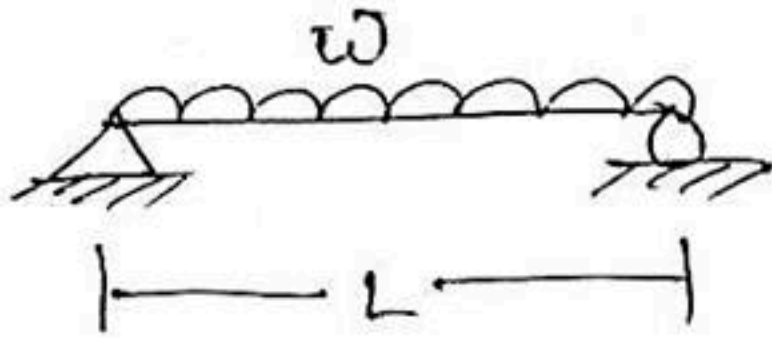


$$A = bh/(n+1)$$

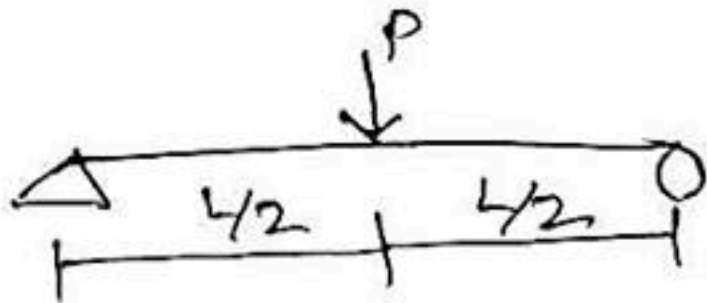
PARABOLA



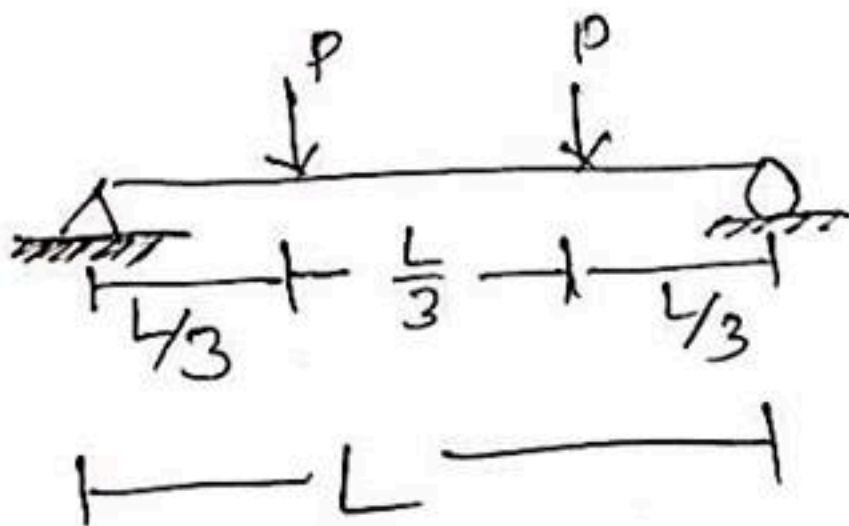
The area for any segment of a parabola is  $A = \frac{2}{3}hl$



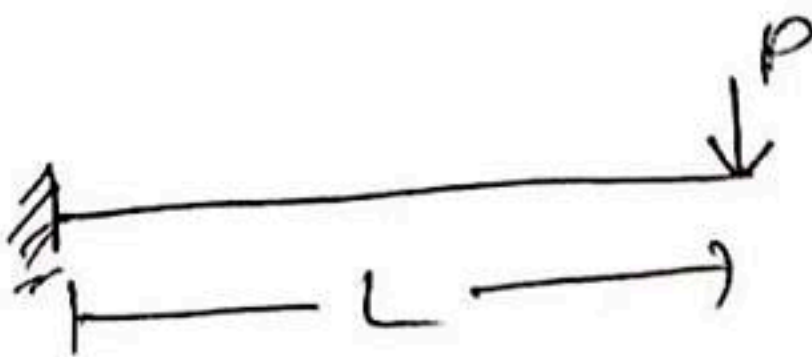
$$\Delta_{max} = \frac{5wL^4}{384EI}$$



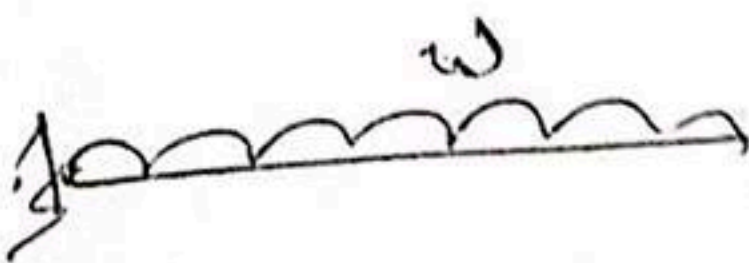
$$= \frac{PL^3}{48EI}$$



$$= \frac{23PL^3}{648EI}$$



$$\frac{PL^3}{3EI}$$



$$= \frac{wL^4}{8EI}$$

2.5.6

**Equivalent Static Force Method**

This method may be used for calculation of seismic lateral forces for all structures specified in Sec 2.5.5.1(a)

**2.5.6.1 Design Base Shear :** The total design base shear in a given direction shall be determined from the following relation :

$$V = \frac{ZIC}{R} W \quad (2.5.1)$$

- where, Z = Seismic zone coefficient given in Table 6.2.22  
 I = Structure importance coefficient given in Table 6.2.23  
 R = Response modification coefficient for structural systems given in Table 6.2.24  
 W = The total seismic dead load defined in Sec 2.5.5.2  
 C = Numerical coefficient given by the relation :

$$C = \frac{1.25S}{T^{2/3}} \quad (2.5.2)$$

- S = Site coefficient for soil characteristics as provided in Table 6.2.25  
 T = Fundamental period of vibration in seconds, of the structure for the direction under consideration as determined by the provisions of Sec 2.5.6.2.

The value of C need not exceed 2.75 and this value may be used for any structure without regard to soil type or structure period. Except for those requirements where Code prescribed forces are scaled up by 0.375R, the minimum value of the ratio C/R shall be 0.075.

**Table 6.2.22**  
Seismic Zone Coefficients, Z

Seismic Zone (see Fig 6.2.10)	Zone Coefficient
1	0.075
2	0.15
3	0.25

**Table 6.2.23**  
Structure Importance Coefficients I, I'

Structure Importance Category (see Table 6.1.1 for occupancy)	Structure Importance Coefficient	
	I	I'
I Essential facilities	1.25	1.50
II Hazardous facilities	1.25	1.50
III Special occupancy structures	1.00	1.00
IV Standard occupancy structures	1.00	1.00
V Low-risk Structures	1.00	1.00

**2.5.6.2 Structure Period :** The value of the fundamental period, T of the structure shall be determined from one of the following methods :

a) **Method A :** For all buildings the value of T may be approximated by the following formula :

$$T = C_t (h_n)^{3/4} \quad (2.5.3)$$

- where,  $C_t$  = 0.083 for steel moment resisting frames  
 = 0.073 for reinforced concrete moment resisting frames, and eccentric braced steel frames  
 = 0.049 for all other structural systems  
 $h_n$  = Height in metres above the base to level n.

Alternatively, the value of  $C_t$  for buildings with concrete or masonry shear walls may be taken as  $0.031/\sqrt{A_c}$ . The value of  $A_c$  shall be obtained from the relation :

$$A_c = \sum A_e [0.2 + (D_e/h_n)^2] \quad (2.5.4)$$

- where,  $A_c$  = The combined effective area, in square metres, of the shear walls in the first storey of the structure.  
 $A_e$  = The effective horizontal cross-sectional area, in square metres of a shear wall in the first storey of the structure.  
 $D_e$  = The length, in metre of a shear wall element in the first storey in the direction parallel to the applied forces.

The value of  $D_e/h_n$  for use in Eq (2.5.4) shall not exceed 0.9.

Table 6.2.25  
Site Coefficient  $S$  for Seismic Lateral Forces (1)

Site Soil Characteristics		Coefficient, $S$
Type	Description	
$S_1$	A soil profile with either : a) A rock-like material characterized by a shear-wave velocity greater than 762 m/s or by other suitable means of classification, or b) Stiff or dense soil condition where the soil depth is less than 61 metres	1.0
$S_2$	A soil profile with dense or stiff soil conditions, where the soil depth exceeds 61 metres	1.2
$S_3$	A soil profile 21 metres or more in depth and containing more than 6 metres of soft to medium stiff clay but not more than 12 metres of soft clay	1.5
$S_4$	A soil profile containing more than 12 metres of soft clay characterized by a shear wave velocity less than 152 m/s	2.0
Note : (1) The site coefficient shall be established from properly substantiated geotechnical data. In locations where the soil properties are not known in sufficient detail to determine the soil profile type, soil profile $S_3$ shall be used. Soil profile $S_4$ need not be assumed unless the building official determines that soil profile $S_4$ may be present at the site, or in the event that soil profile $S_4$ is established by geotechnical data.		

- b) **Method B** The fundamental period  $T$  may be calculated using the structural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. This requirement may be satisfied by using the following formula :

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^n w_i \delta_i^2}{g \sum_{i=1}^n f_i \delta_i}} \quad (2.5.5)$$

The values of  $f_i$  represent any lateral force distributed approximately in accordance with the principles of Eq (2.5.6), (2.5.7) and (2.5.8) or any other rational distribution. The elastic deflections,  $\delta_i$  shall be calculated using the applied lateral forces,  $f_i$ . The value of  $T$  determined from Eq (2.5.5) shall not exceed that calculated using Eq (2.5.3) by more than 40%.

2.5.6.3 **Vertical Distribution of Lateral Forces** : In the absence of a more rigorous procedure, the total lateral force, which is the base shear  $V$ , shall be distributed along the height of the structure in accordance with Eq (2.5.6), (2.5.7) and (2.5.8):

$$V = F_t + \sum_{i=1}^n F_i \quad (2.5.6)$$

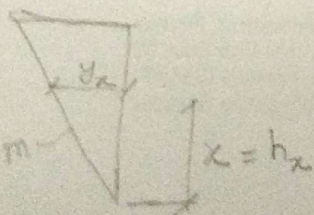
where,  $F_i$  = Lateral force applied at storey level  $-i$  and  
 $F_t$  = Concentrated lateral force considered at the top of the building in addition to the force  $F_n$ .

The concentrated force,  $F_t$  acting at the top of the building shall be determined as follows:

$$\begin{cases} F_t = 0.07 TV \leq 0.25 V & \text{when } T > 0.7 \text{ second} & (2.5.7a) \\ F_t = 0.0 & \text{when } T \leq 0.7 \text{ second} & (2.5.7b) \end{cases}$$

The remaining portion of the base shear  $(V - F_t)$ , shall be distributed over the height of the building, including level- $n$ , according to the relation :

$$F_x = \frac{(V - F_t) w_x h_x}{\sum_{i=1}^n w_i h_i} \quad (2.5.8)$$



Handwritten notes for Eq (2.5.8):

$$\int dx = \text{Force} = (V - F_t) \quad (2.5.8)$$

$$\frac{y}{h_x} = \frac{(V - F_t)}{\sum w_i h_i}$$

$$y = \frac{(V - F_t) * h_x}{\sum w_i h_i}$$

CE 313

PROBLEM ON EARTHQUAKE

0804056  
R-004

Kouf Ska.  
Sheet-3  
(CE-08(A))

Example : Eq. Static EQ Force

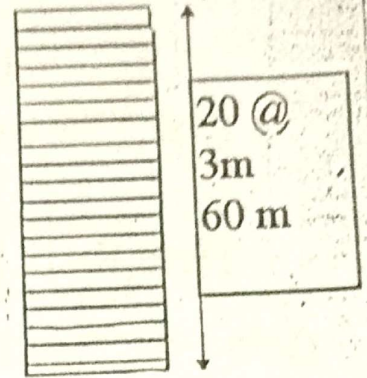
Calculate the distribution of EQ forces on the 20 storied office building shown in figure.

Location : Dhaka

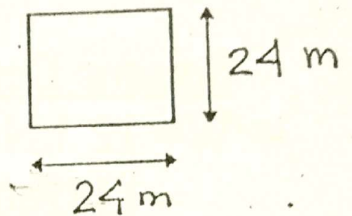
Soil Type : Soft to medium stiff clay

DL including partition = 12 kN/sq.m

Structure type : SMRF in concrete Elevation



Plan



6-2-24  
BNBL

Solution:

Zone 2:  $Z=0.15$  (Dhaka)  $I=1.0$  (Table 6.2.23)

$$C = \frac{1.25S}{T^{2/3}} \quad (\text{Eqn. 2.5.2})$$

Soil Type  $S_3 = 1.5$  (Table 6.2.25)

$$T = C_t (h_n)^{3/4} \quad (\text{Eqn 2.5.3})$$

$$h_n = 60 \text{ m}$$

$$C_t = 0.073$$

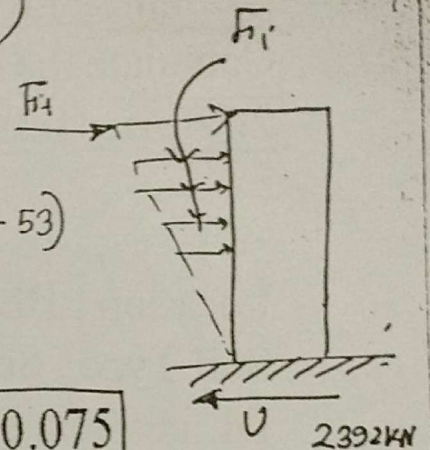
$$T = 0.073 \times (60)^{3/4}$$

$$= 1.57 \text{ Sec}$$

$$C = \frac{1.25 \times 1.5}{(1.57)^{2/3}} \quad (\text{Eqm } 2.5.2)$$

$$C^* = 1.386 < 2.75 \quad (\text{page } 6-53)$$

$C \neq 2.75$  and  $(C/R) \neq 0.075$



Here  $R=12$  (Table 6.2.24) hence:  $C/R=0.115 > 0.075$

given ←

Therefore,  $C=1.386$  O.K

Seismic Dead Load (Art. 2.5.5.2)

$$W = 12 \text{ kN/sq. m} \times (24 \times 24 \text{ sq. m}) \times 20 \text{ floors}$$

$$= 138240 \text{ kN}$$

$$V = (ZIC) \times W/R \quad (\text{Eqm. } 2.5.1)$$

$$= (0.15 \times 1 \times 1.386 / 12) \times W$$

$$= 0.0173 W$$

$$= \boxed{2392 \text{ kN}}$$

Short ton, 1 ton = 2000 lb  
long ton, 1 ton = 2240 lb

$$\frac{\text{kN}}{10} = \text{ton}$$

1 metric ton = 1000 kg

Vertical Distribution of Forces: (Eqm. 2.5.6)

The concentrated lateral force  $F_t$  at the top of the building:

$$F_t = 0.07TV \leq 0.25 V \quad \text{when } T > 0.7 \text{ second} \quad \left. \begin{array}{l} \text{Eqm.} \\ 25.7a, b \end{array} \right\}$$

$$F_t = 0.0 \quad \text{when } T \leq 0.7 \text{ second}$$

$$F_t = 0.07 \times 1.57 \times 2392 = \boxed{264 \text{ kN}} \quad (\text{Eqm } 2.5.7a)$$

← in each floor

here :  $F_t \leq 0.25 V$

i.e.  $F_t = 264 \text{ kN}$  O.K.

$$F_x = \frac{(V - F_t) w_x h_x}{\sum_{i=1}^n w_i h_i} \quad (\text{Eqn. 2.5.8})$$

$W_x = 12 \times 576$  kN for each floor.

In this example  $w$  is same for all the floors.

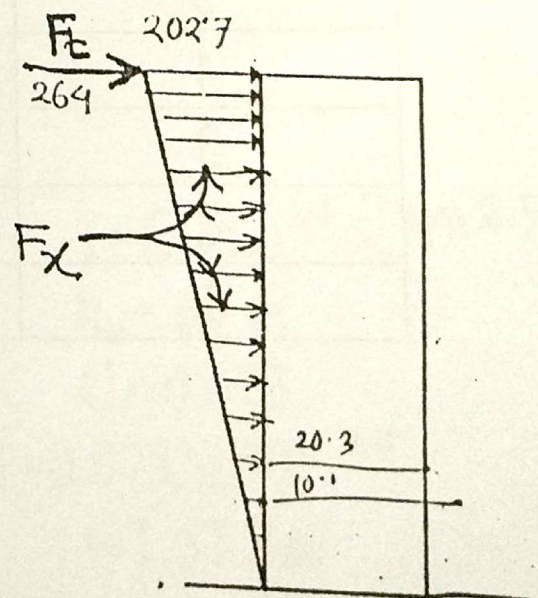
For any building for which  $w$  is same for each floor:

$$F_x = \frac{w_x (V - F_t) h_x}{w_i \sum_{i=1}^n h_i}$$

$$F_x = \frac{(2392 - 264) h_x}{(3 + 6 + 9 + \dots + 60)}$$

$$F_x = \frac{2128 h_x}{3 \times \frac{(20 \times 21)}{2}}$$

$$F_x = 3.38 h_x$$



# EQ Forces at various levels of the Building.

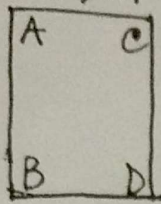
Floor Level	$h_x, m$	$F_x, kN$
20	60	202.7+264
19	57	192.5
18	54	182.4
17		
16		
15	45	152.1
14		
13		
12		
11		
10	30	101.4
9		
8		
7		
6	18	60.8
5		
4		
3		
2	6	20.3
1	3	10.1

# PROBLEM ON WIND LOAD

OF (CE 311)

P1/2

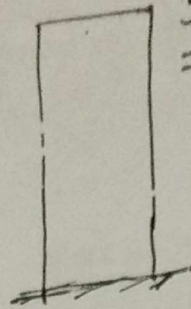
L = 33m



B = 66m

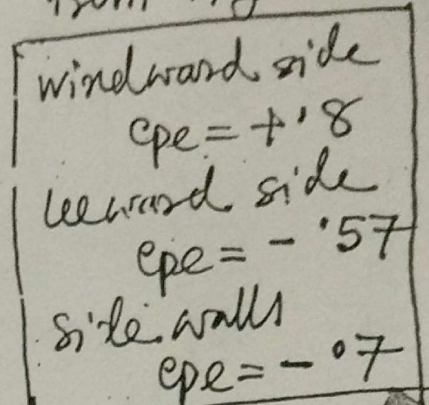
$$\frac{L}{B} = 15$$

$$\frac{h}{B} = .76$$



50m = h

From Fig. 6.2.5



Plan

Site: Dhaka ELV =

$V_b = 210 \text{ km/h}$

Exposure A

$$q_z = C_e C_i C_z V_b^2$$

$$= (47.2 \times 10^{-6}) (1.0) \times C_z (210)^2 = 2.082 C_z \text{ kN/m}^2$$

( $C_z$  in Table 6.2.10)

$$P_z = C_g C_p q_z = C_g C_p 2.082 C_z \text{ (kN/m}^2)$$

$C_g = G_h \rightarrow$  in Table 6.2.11 (= 1.238 at 50m)

$C_p = c_{pe} \rightarrow$  Fig. 6.2.5

$$= 2.082 (1.238) C_p C_z \text{ (kN/m}^2)$$

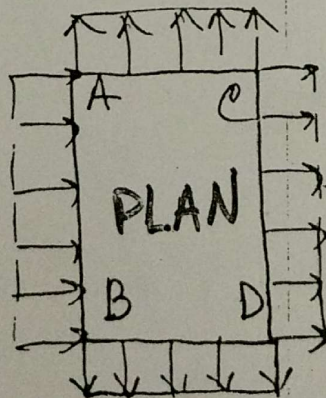
$2.578 (.8) C_z$  for windward wall (AB)

$2.578 (-.57) C_z$  for leeward wall (CD)

$2.578 (-.7) C_z$  for side walls (AC, BD)

Method 1

$$F_1 = \sum p A_z \Rightarrow$$



$P_z$

Pressure Diagram

H 6E 21

Static EO Force

P2/2

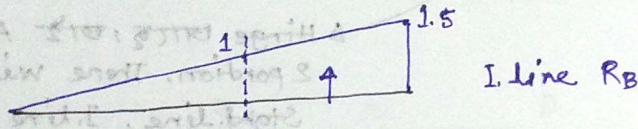
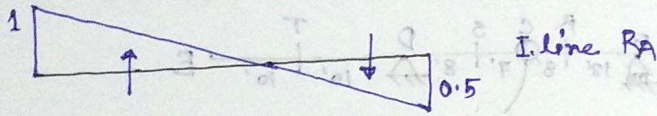
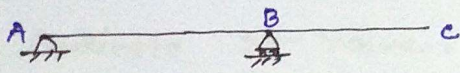
By Method 1

WIND PRESSURE ON (KN/m<sup>2</sup>)

Ht(m)	Cz	Windward (W)	Leeward (L)	Total Side wall
0-4.5	.368	.76	.54	.66
6	.415	.86	.61	.75
9	.497			
12	.565	1.17	.83	1.02
15	.624			
18	.677			
21	.725			
24	.769	1.59	1.13	1.39
27	.810			
30	.849			
35	.909			
40	.965	1.99	1.42	1.74
45	1.017			
50	1.065	2.20	1.56	1.92

(.08) | (-.57) | (-.7)  
 2.578





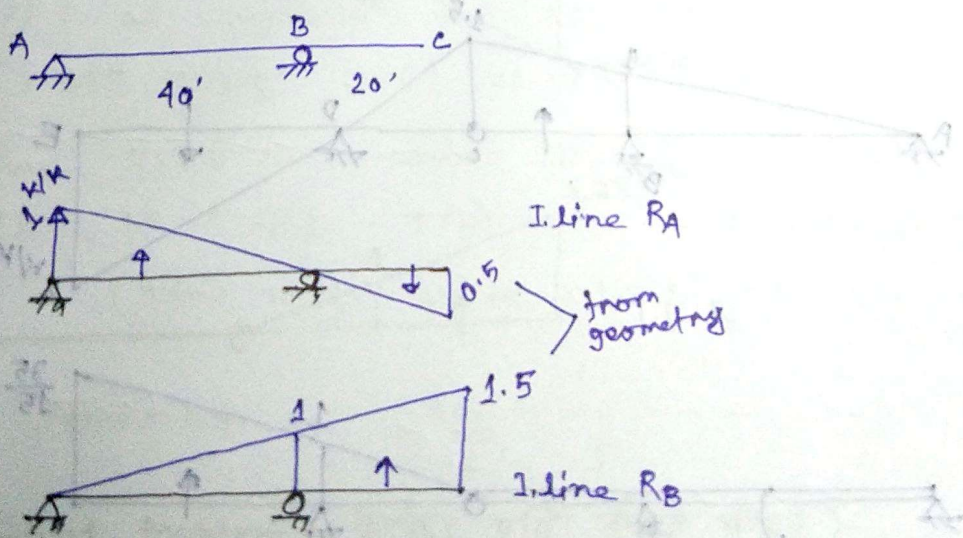
\*\* I.line is a curve which shows the variation of a function under the action of moving unit load.

# Drawing of I.line for R's in Beams:-

- Steps:
- 1) Push the support up by 1.
  - 2) Draw the  $\delta$ -shape (deflected)  $\Rightarrow$  is the I.line
  - 3) Complete the coordinates & direction (sign)

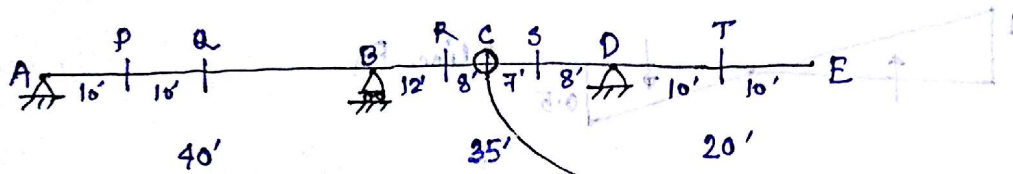
[ If support is i.line maka (if support is push up by 1: ]

# Beam 1:



+ I direction always up because load is down.

# Beam 2:



Hinge,  $\Delta$   $\Rightarrow$  ABC, CDE  
 2 portion. There will be 2  
 Stght. line. 1. line will be  
 combination of 2 Stght. lines.

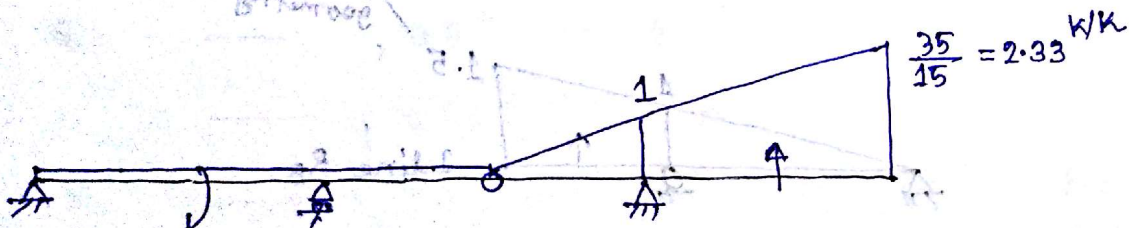
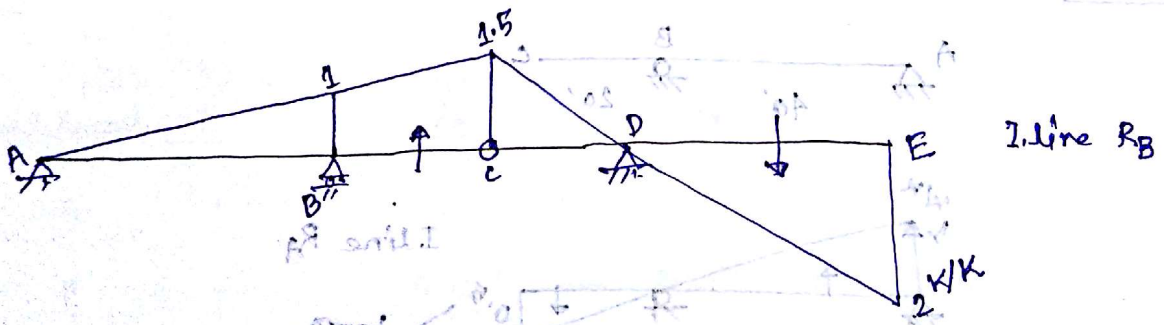
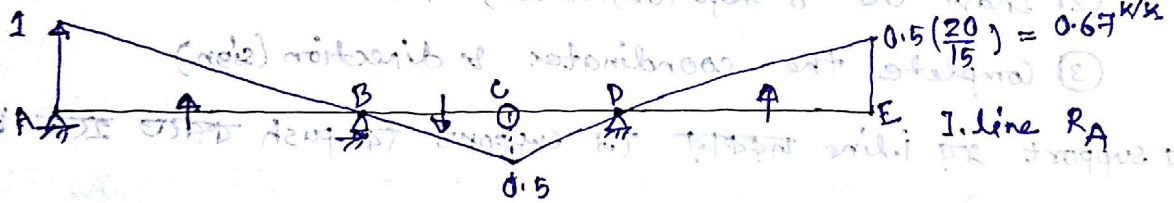
[\*\* Original beam  $\Rightarrow$  यद्युत्तम Stght. line, 1. line  $\Rightarrow$  उद्युत्तम Stght. line

C hinge  $\Rightarrow$  कटिले, CDE शब्दे माहा  $\Rightarrow$  do not cut  
 ABC  $\Rightarrow$  self stable portion, CDE  $\Rightarrow$  stable नट

असमस ABC portion आहत draw करत शब्द for deflection shape.]

Unit:

K/K  
 or #/#  
 or kg/kg  
 (Reaction  
 म unit  $\rightarrow$   
 माहा)

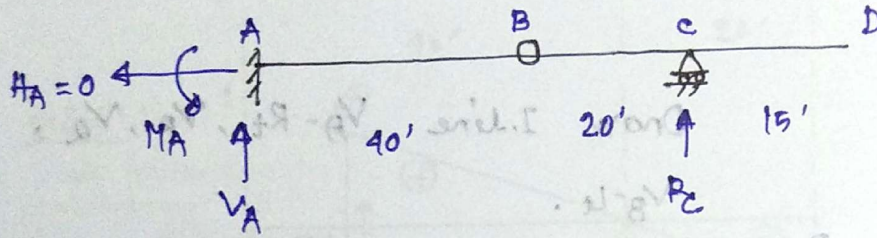


(कोन movement नाई, zero line)

[Design sheet  $\Rightarrow$  Draw करले कोन वृत्ता याय]

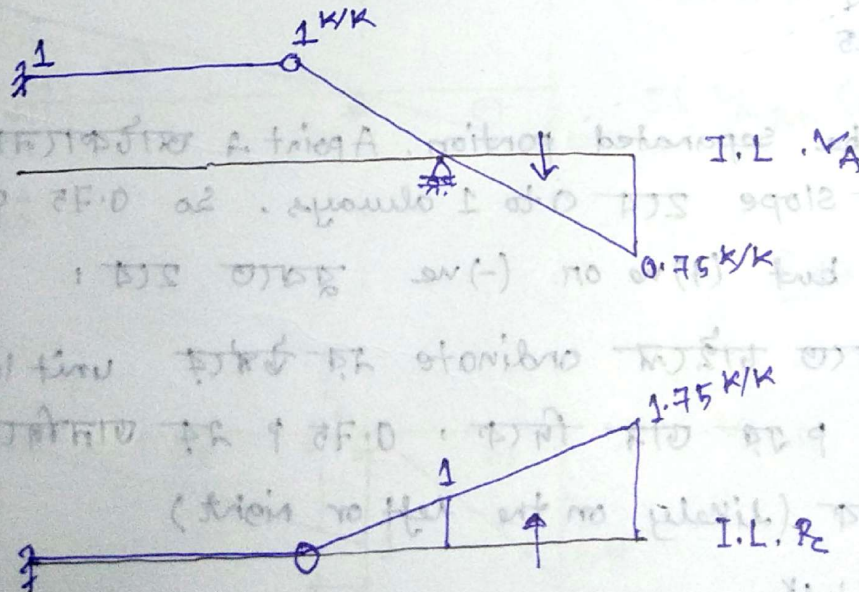
\*\* Shape, Ordinate & value, direction & unit influence line A.

# Beam 3:



\*  $M_A$  moment reaction. Stable part AB.  $M_A$  must be maintained. Line straight upward, it is straight.

\* Hinge push  $90^\circ$  must be maintained. Line straight upward, it is straight.



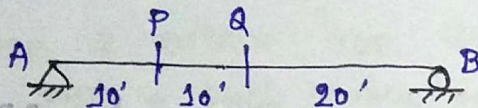
\* Practice Solved Problem (উন্নীত ছবি দিতে পারেন)

## # I.line for SF in Beams

### \*Steps:

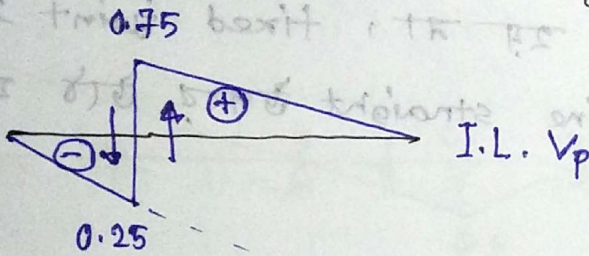
- ① cut at the section X.
- ② Push the left part down & right part up (by 2 sides of X)
- ③ Draw S-shape  $\rightarrow$  is the I.line
- ④ Put the ordinates & sign.

### # Beam 1:



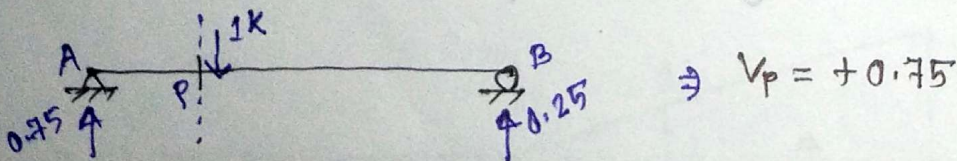
Draw I.line  $V_A - R_L, V_P, V_Q, V_B - L$ .

[\* support এর উভয় side shear force (V) calculate করা যায় না, তাই আমরা left or right দিকের করে। ]



\* AP & BP will be separated portion. A point A আটকানো, P তে free. zeta line এর slope এর 0 to 1 always. So 0.75 & 0.25 will be get but (+)ve or (-)ve বুঝতে হবে।

\* sign বের করতে চাইলে ordinate এর উভয় side unit load বসাতে হবে exactly P এর ডান দিকে, 0.75 P এর বামদিকে, 0.25 P এর ডানদিকে (likely on the left or right)



### \*Steps:

- ① প্রথমে reaction বের করতে হবে,
- ② section নিতে হবে,
- ③  $\sum F_y = 0$

I-line for SF in beams:

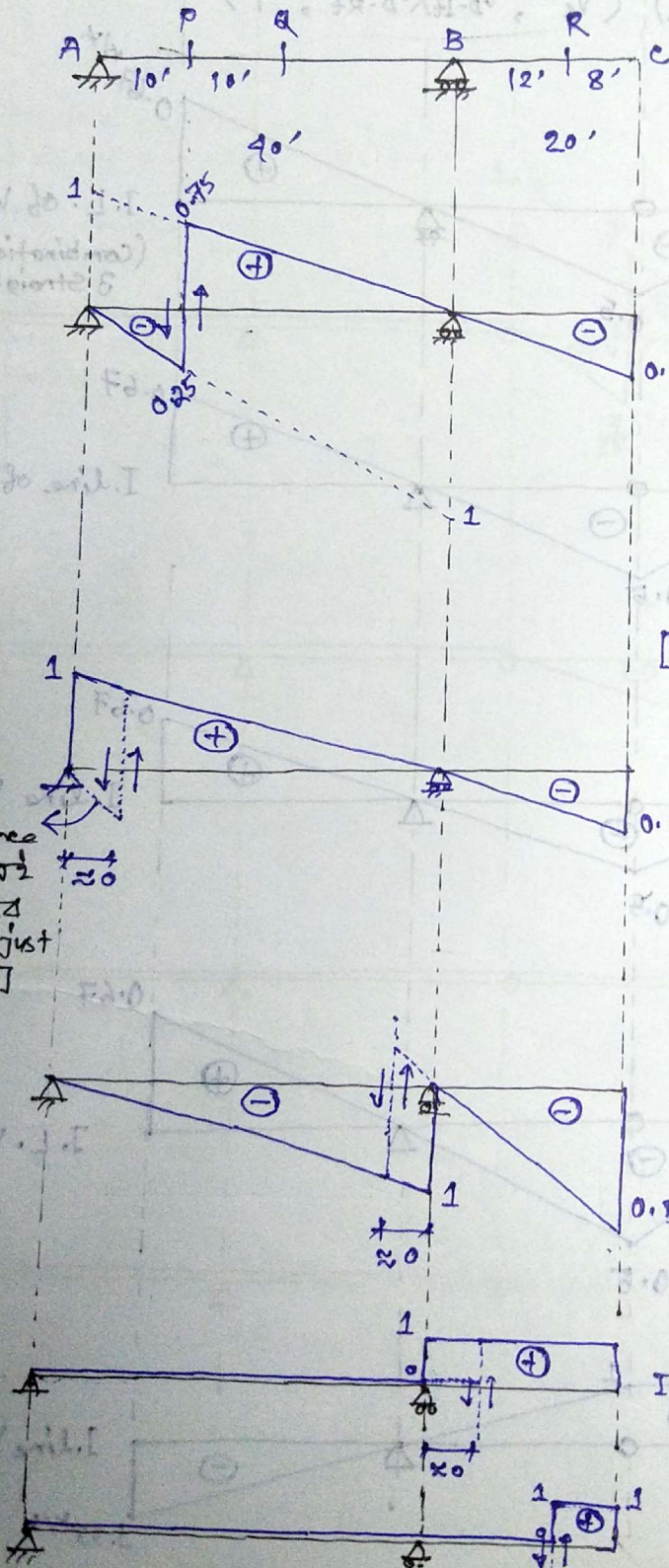
# Beam 2:

Draw I-line for

$V_{A-Rt}$ ,  $V_p$ ,  $V_q$ ,  
 $V_{B-L}$ ,  $V_{B-Rt}$ ,  $V_R$

[Total section এর পর্যন্ত infinity]

\*Soln:



IL  $V_p$   
→ SF at P  
[P এর কাটিলে 2টা straight line AP & PBC]

\* Shear force এর I-line 0 & 1 slope এর দুইটি parallel থাকবে

[\*+ upper portion is (+)ve lower portion is (-)ve]

I.L.  $V_{A-Rt}$  (just right of A)

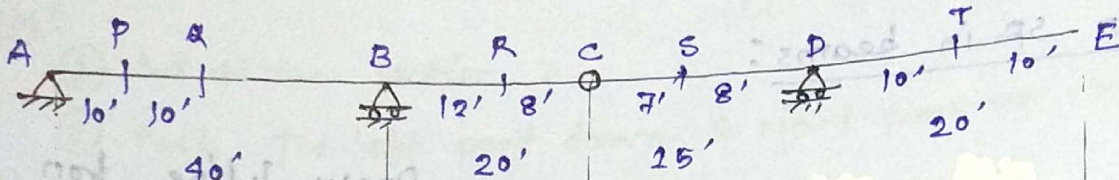
[যেহেতু distance ০, রেফারেন্স হইছে জেরণ থাকবে না। তাই adjust করতে হবে]

[\*\*\* টম section একটিনাত্র ৩ section এর

- ① দুইটা parallel slope will be same
- ② দুইটা ordinate এর absolute value এর sum will be 1.

# # Beam 3 (Beam 1 & 2)

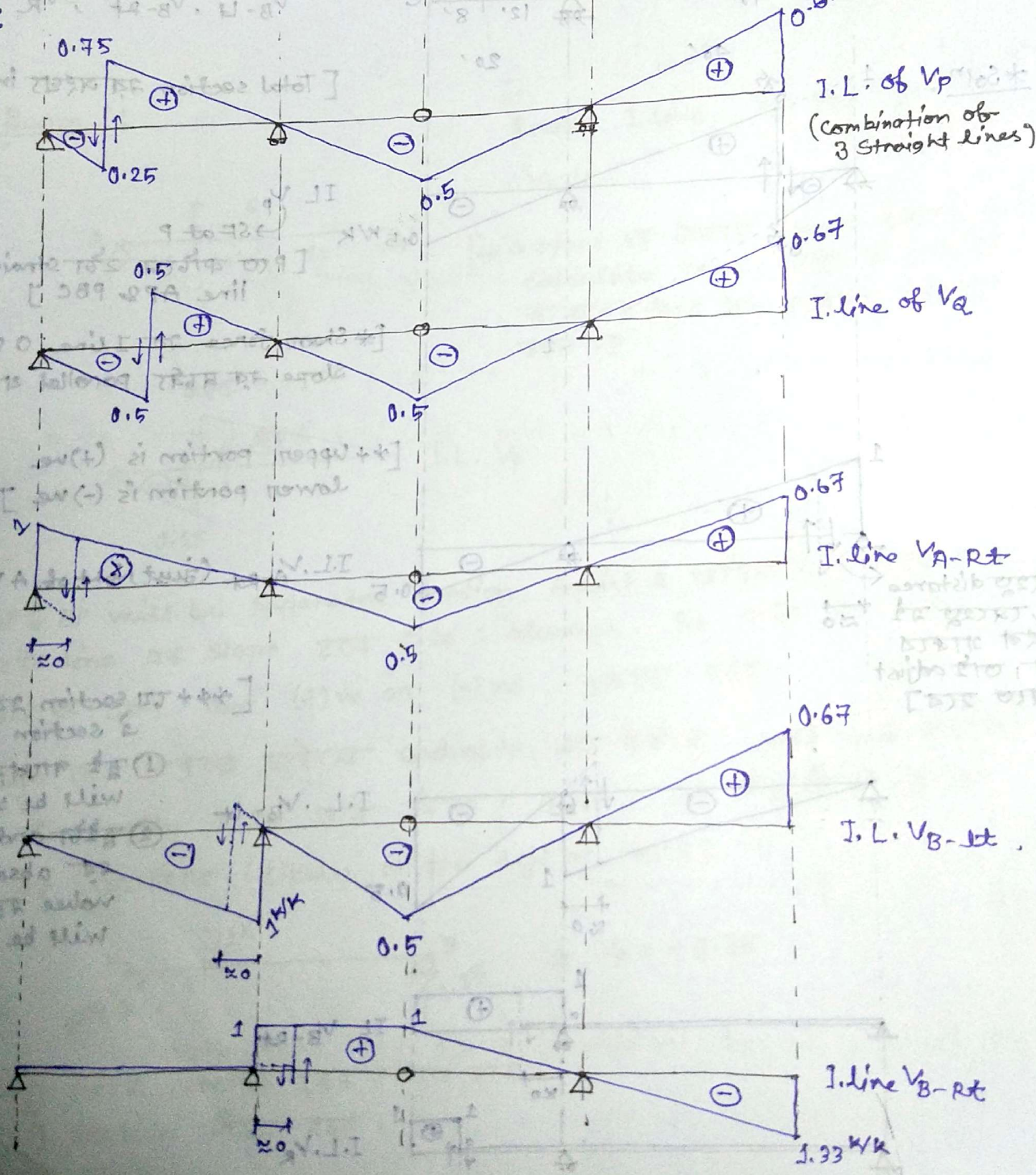
CE 311

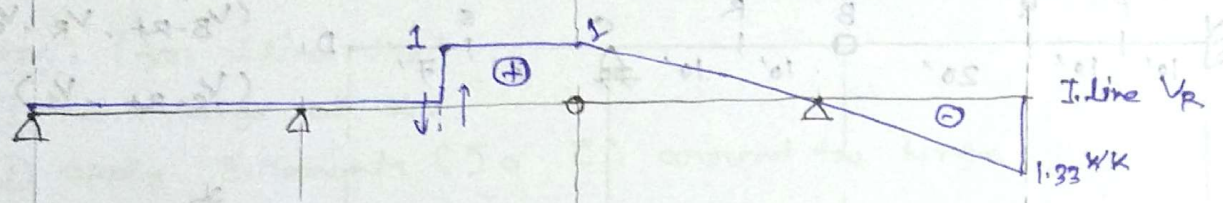
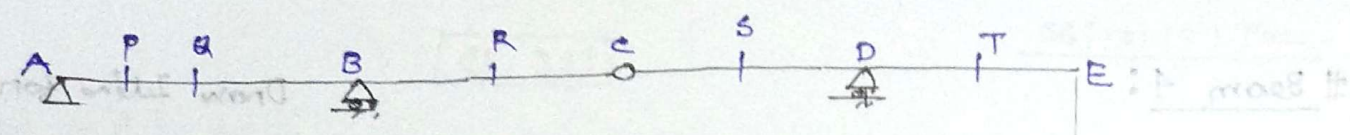


Draw I. line for

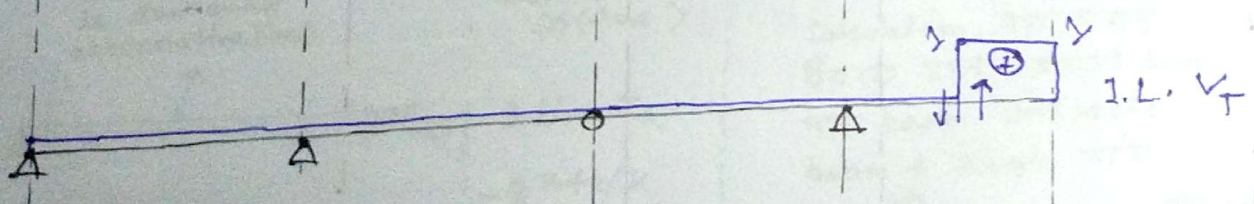
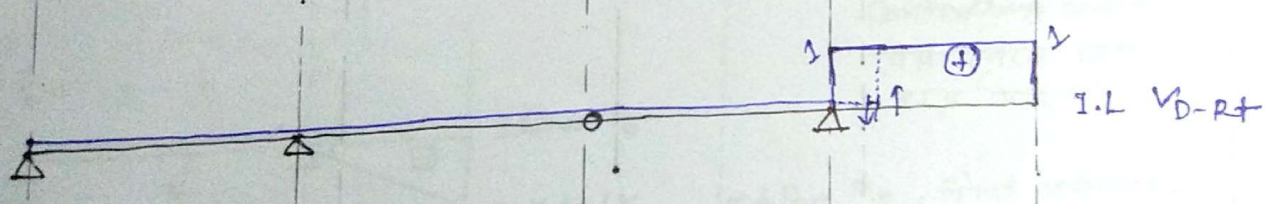
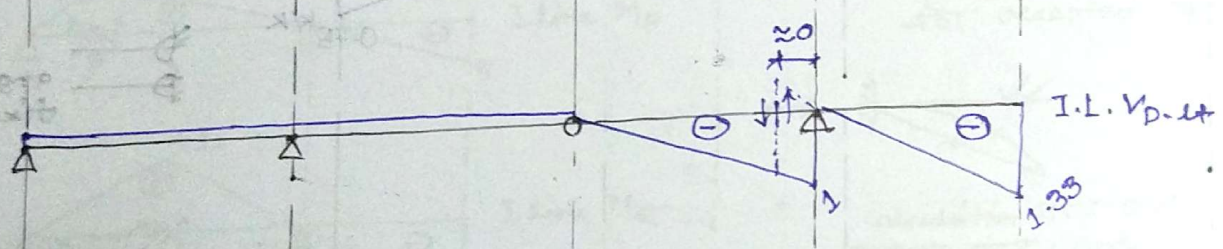
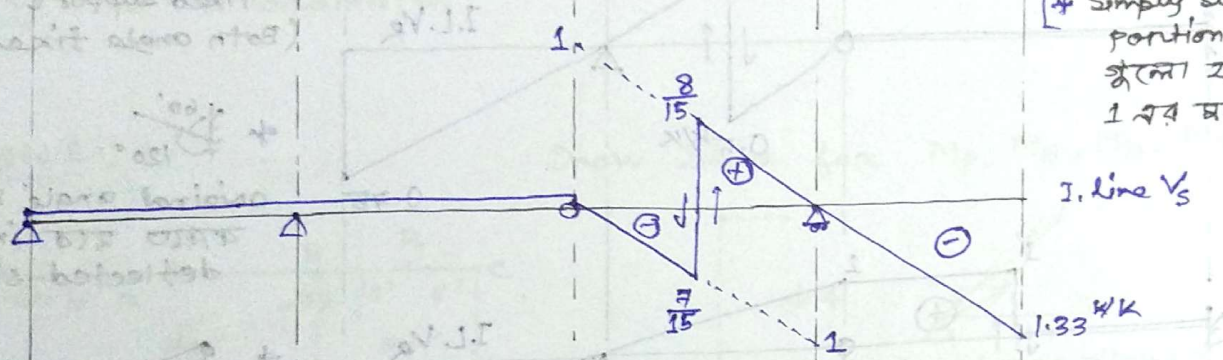
$(V_{A-Rt}, V_p, V_q, V_{B-Lt}, V_{B-Rt}, V_R)$   $(V_s, V_{D-Lt}, V_{D-Rt}, V_T)$

\* Soln:



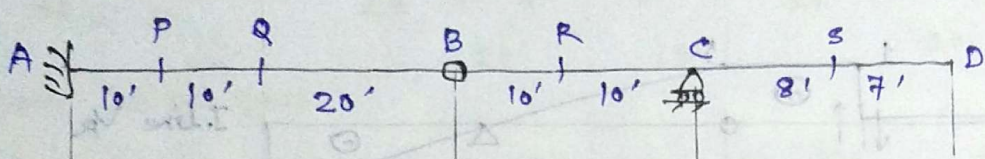


[+ Simply supported position of line के लिये 2 रक 0 and 1 रक घबरेल ]

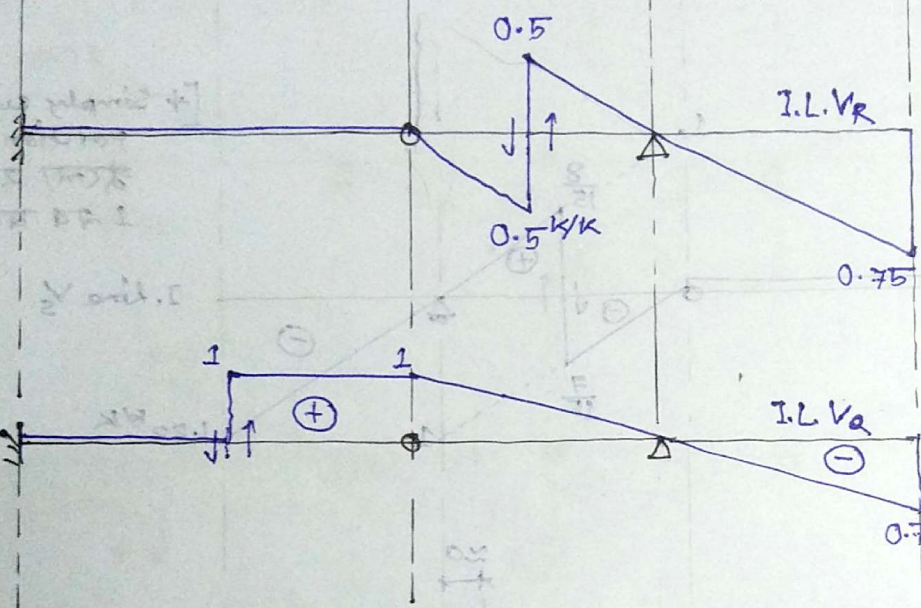


# Beam 4:

Draw I.L. for  
 $(V_{A-R}, V_P, V_Q, V_{B-L})$   
 $(V_{B-R}, V_R, V_{C-L})$   
 $(V_{C-R}, V_S)$



\* Soln :



- \* fixed support (Both angle fixed)
- \* original angle maintain  $60^\circ$  &  $120^\circ$  in deflected shape
- \* Hinge  $\Rightarrow$  angle fixed at
- \* angle fixed fixed support

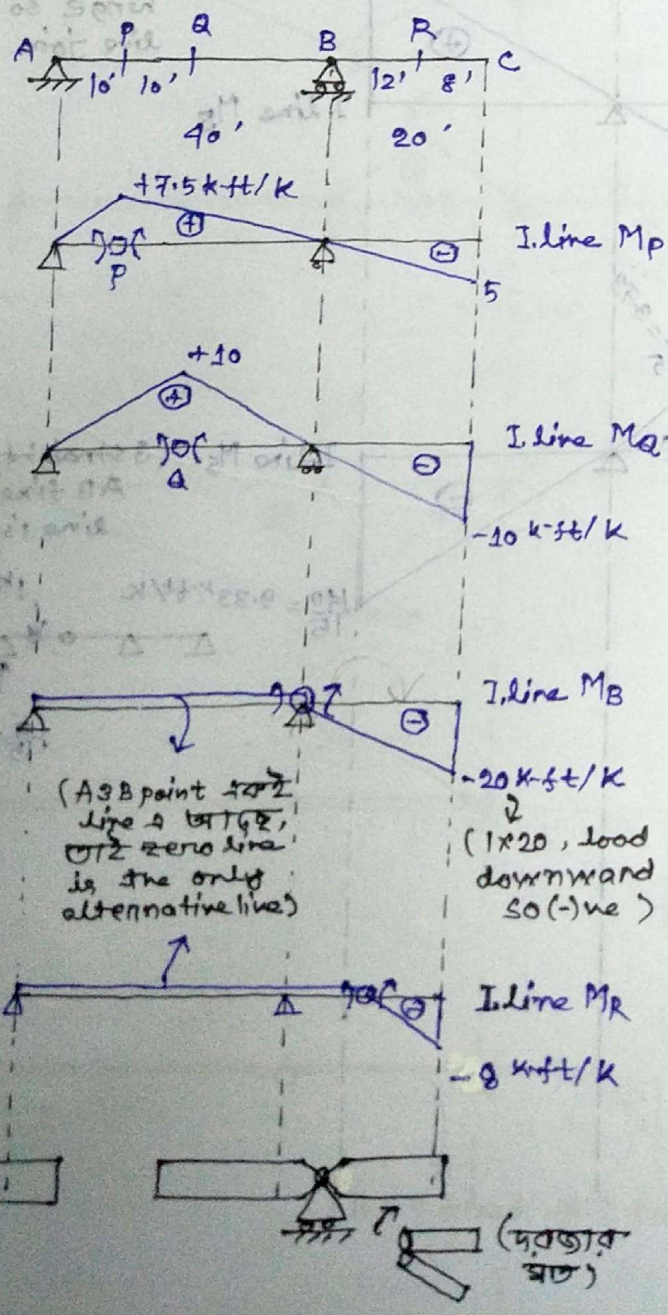
Influence line for Bending moment in Beams:

# Steps: (for I line  $M_x$ )

- ① Put a hinge at  $x$ .
- ② apply B.Moments ( $50 \text{ } \tau$ ) around the hinge
- ③ Draw  $\delta$  shape
- ④ Find the ordinates

# Beam 2:

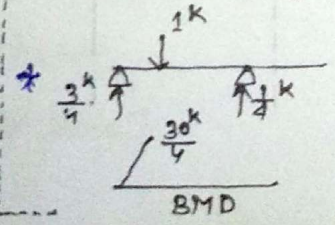
Draw I.line for  $M_p, M_a, M_B, M_R$



angular change allowed at hinge actually curve line  $\delta$  shape. एका accepted न.

Calculation कर ordinates खर करणे खर. ordinate at P on C खर करणे खर. Controlling ordinates (एकटा खर खरने आरेखणा खर खर यार)

\*\* For  $M_p$ , Find ordinate at P. Put  $1 \text{ k}$  load at P in the original beam & Find  $M_p$ . Calculation खर प्रत्यु original beam खर खर, कारण hinge यारात, खर beam unstable किनु नोल beam  $\rightarrow$  hinge नाई.



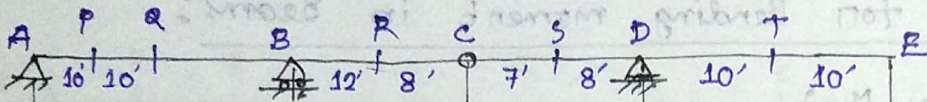
- ① Section
  - ② Reaction
  - ③  $\Sigma M = 0$
- For  $M_a$  find ordinate at c.

# Beam 3:

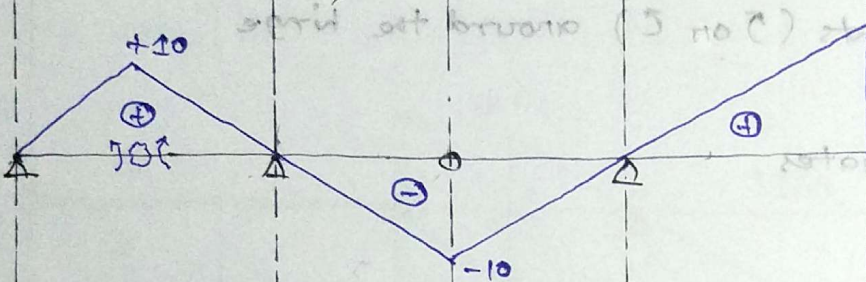
118 10

Draw I. line for

$M_p, M_a, M_B, M_R, M_s, M_D, M_T$

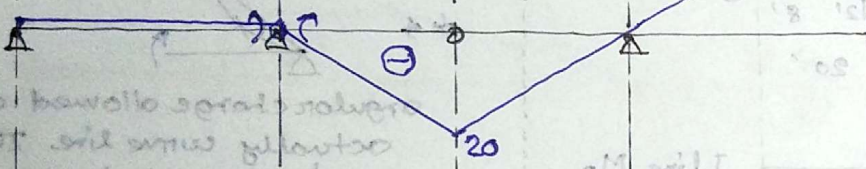


Soln:



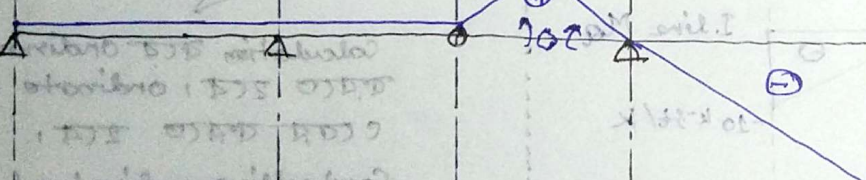
13.33 k-ft/k

I. line  $M_a$



26.67

I. line  $M_B$

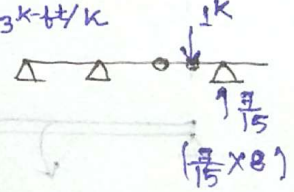


I. line  $M_s$

[+ CDE portion is connected by hinge so straight line join shd]

(3 straight line - AB fixed, only line is zero line)

$\frac{140}{15} = 9.33 \text{ k-ft/k}$

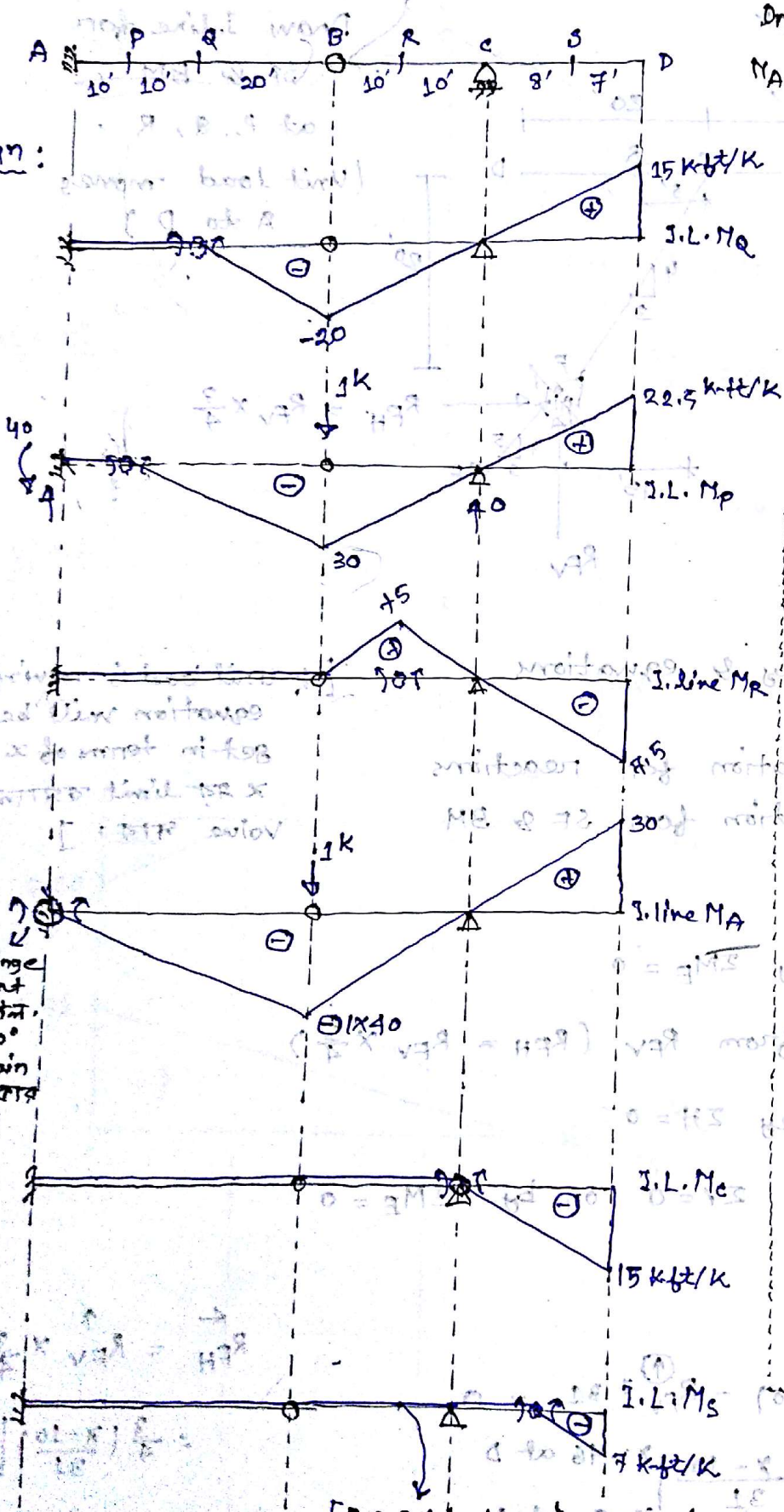


(Add joint for 2nd line and area for 1st line)

# Beam A:

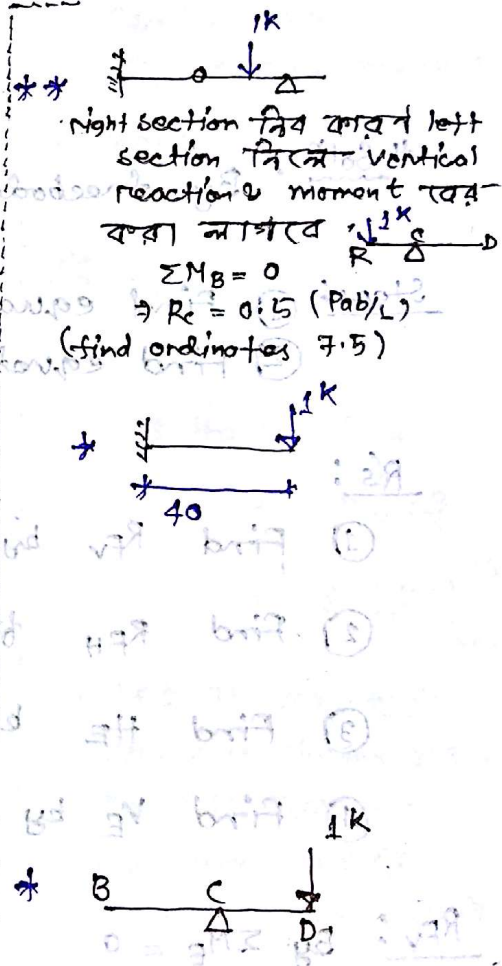
Draw I. line for  
 $M_A, M_P, M_Q, M_R, M_C, M_S$

\*Soln:

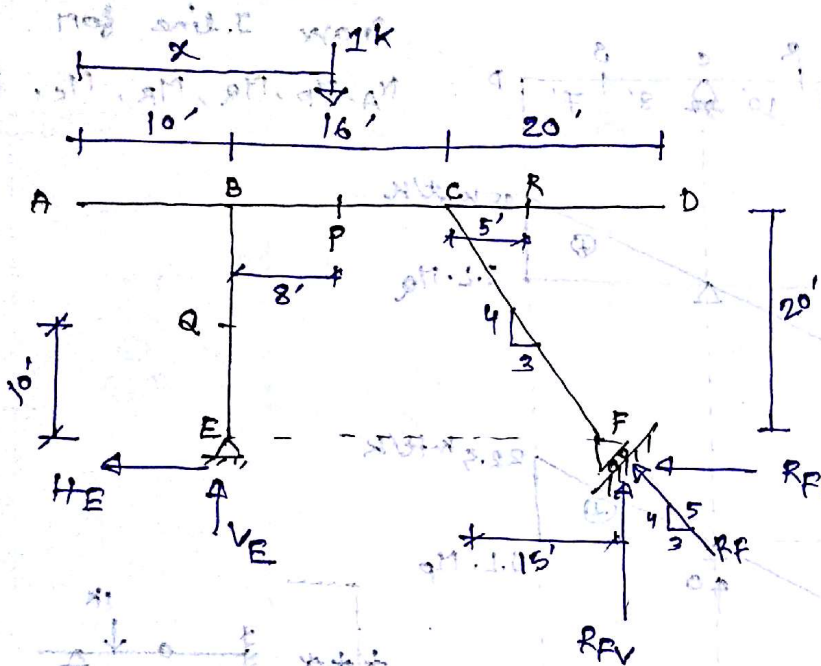


[Supporting support  
 হয়ে সেন,  
 তাই 90°  
 maintain  
 এর দরকার  
 নাই]

[B is fixed, তাই zero line]



I. line for frames :



Draw I. line for SF & BM at P, Q, R.

(Unit load moves A to D)

$$R_{FH} = R_{FV} \times \frac{3}{4}$$

By freebody & equations

- Steps:
- ① Find equation for reactions
  - ② Find equation for SF & BM

[\* unit load is moving. equation will be get in terms of x. x ka limit ke liye value prate.]

R's :

- ① Find  $R_{FV}$  by  $\sum M_E = 0$
- ② Find  $R_{FH}$  from  $R_{FV}$  ( $R_{FH} = R_{FV} \times \frac{3}{4}$ )
- ③ Find  $H_E$  by  $\sum H = 0$
- ④ Find  $V_E$  by  $\sum V = 0$  or by  $\sum M_F = 0$

$R_{FV}$  : By  $\sum M_E = 0$

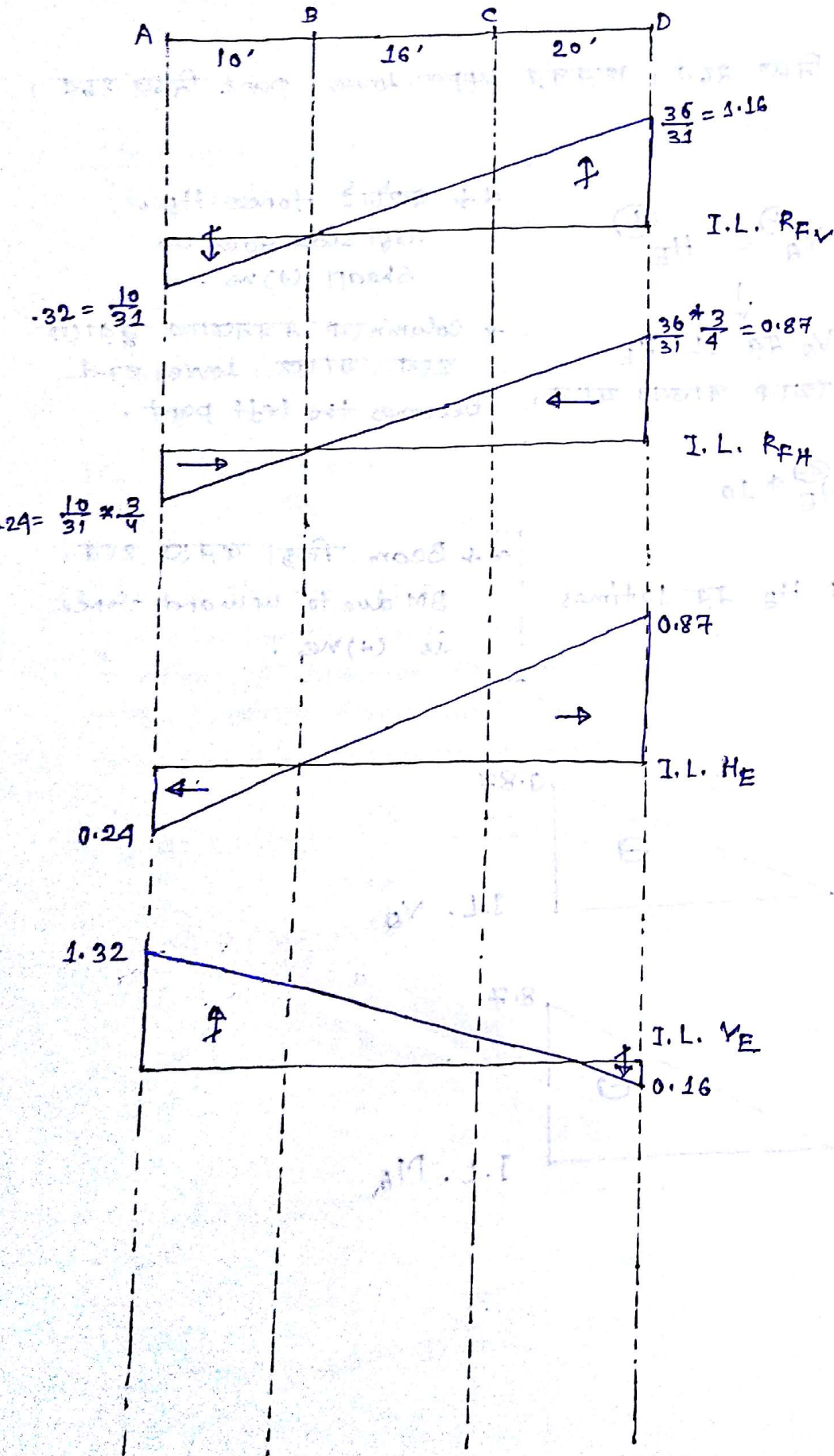
$$\Rightarrow \int + (x-10) - R_{FV} \cdot 31 = 0$$

$$\therefore R_{FV} = \frac{x-10}{31} \quad \left. \begin{array}{l} x=46 \text{ at D} \\ x=0 \text{ at A} \end{array} \right\}$$

$$= -\frac{10}{31} \text{ at A} \quad \& \quad \frac{36}{31} \text{ at D}$$

$$R_{FH} = R_{FV} \times \frac{3}{4}$$

$$= \frac{3}{4} \left( \frac{x-10}{31} \right) \Big|_{x=0}^{46}$$



\* Vertical up रतन horizontal left.  
 So, vertical down रतन horz. right रतन.  
 $R_{FH}$  is a factor of  $R_{FV}$ . So, I. line will be same, ordinate will be different.

\* Reaction नरु गुनरु sign नरु, direction important  
 \* coordinate always decimal नरु निरुत रतन.

$H_E$ : By  $\sum H = 0$

$$\Rightarrow \overleftarrow{H_E} + \overleftarrow{R_{FH}} = 0$$

$$\Rightarrow \overleftarrow{H_E} = -\overleftarrow{R_{FH}}$$

$$= -R_{FV} \times \frac{3}{4}$$

$$= -\frac{3}{4} \left( \frac{7-10}{31} \right) \Big|_{x=0}^{46}$$

\* रतन IL रतन, रतन direction change रतन,  $R_{FH}$  left रतन  $H_E$  right रतन as रतन रतन horz, reaction नरु

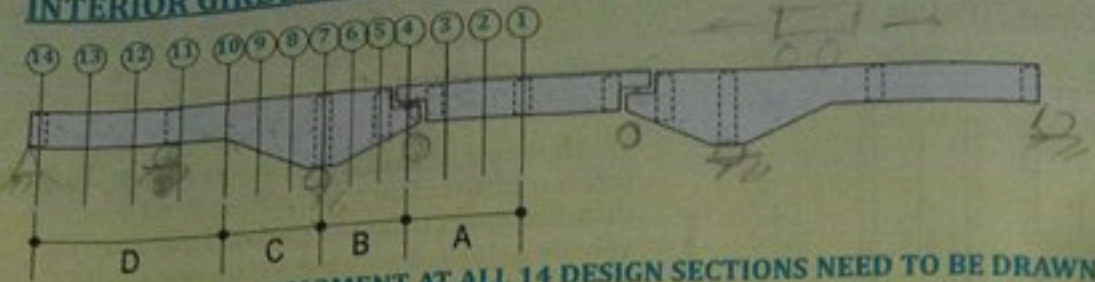
\*  $\sum Y = 0 = 1 - V_E - R_{FV}$

$$V_E = 1 - R_{FV}$$

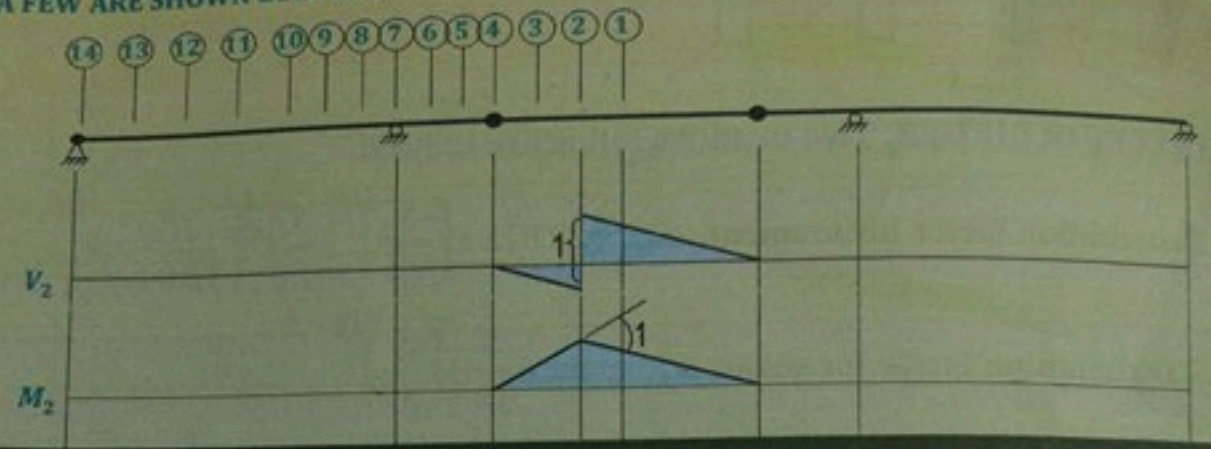
$$= 1 - \left( \frac{7-10}{31} \right) \Big|_{x=0}^{46}$$

(+ive रतन  $V_E$ )

**INTERIOR GIRDER : LIVE LOAD ANALYSIS**

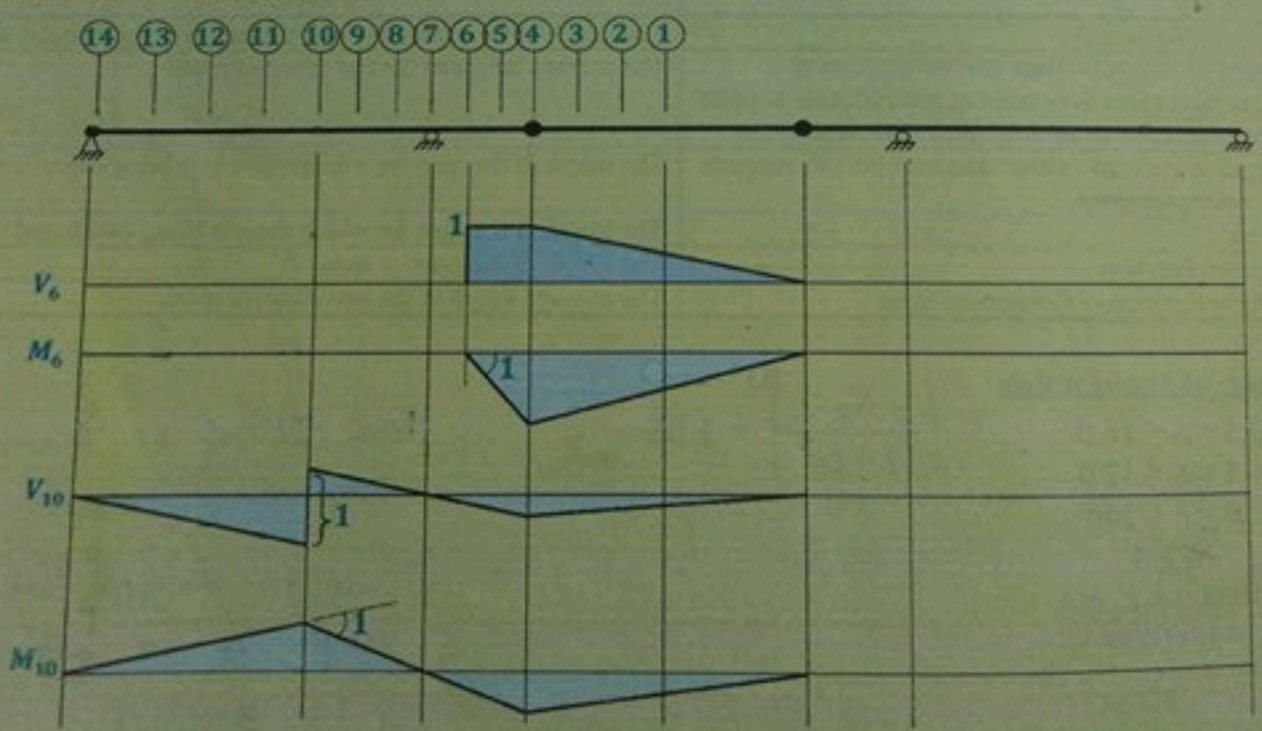


I.L. FOR SHEAR AND MOMENT AT ALL 14 DESIGN SECTIONS NEED TO BE DRAWN  
 A FEW ARE SHOWN BELOW.....



**INTERIOR GIRDER : LIVE LOAD ANALYSIS**

INFLUENCE LINES CONTD.....

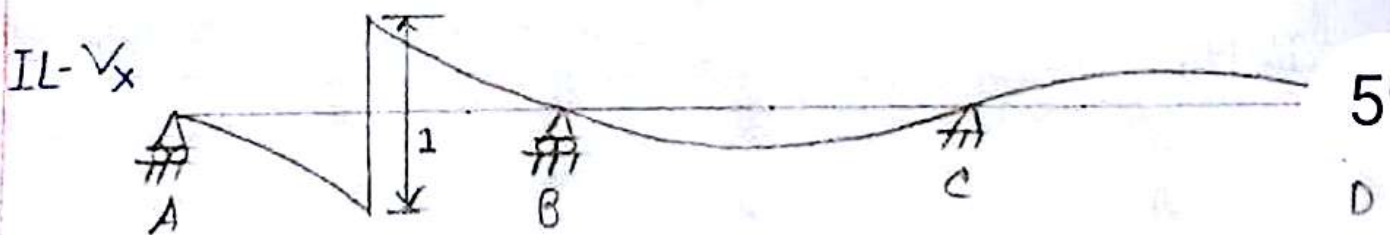
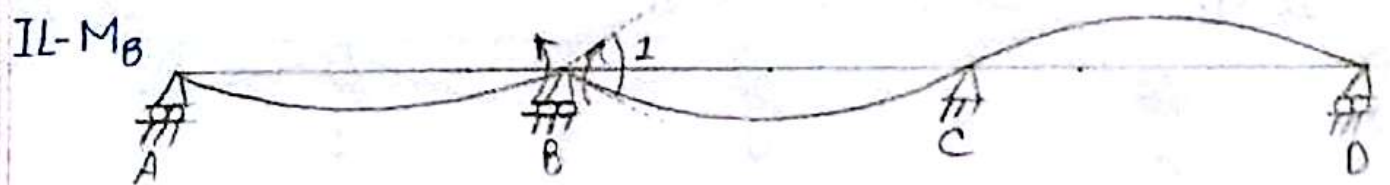
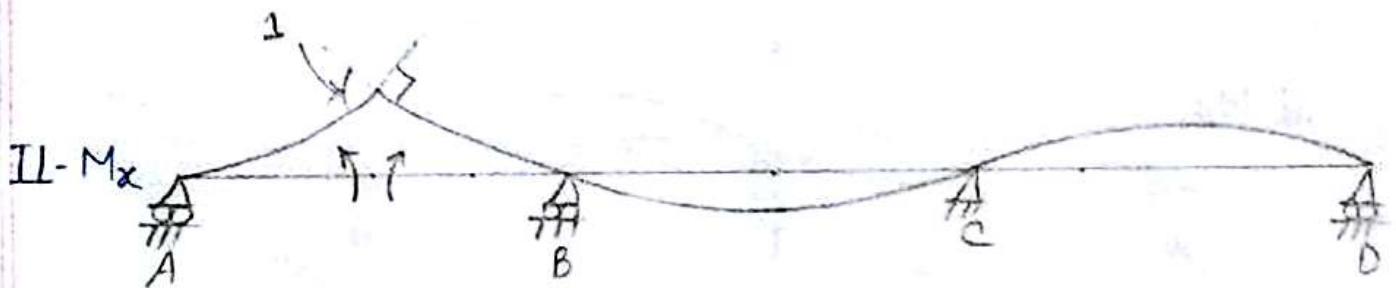
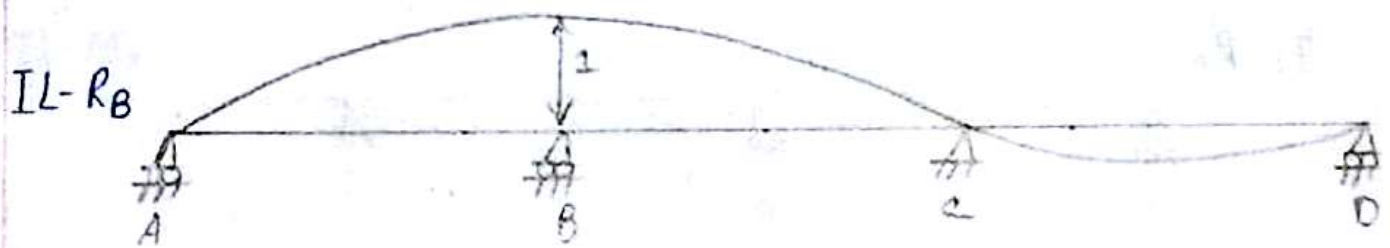
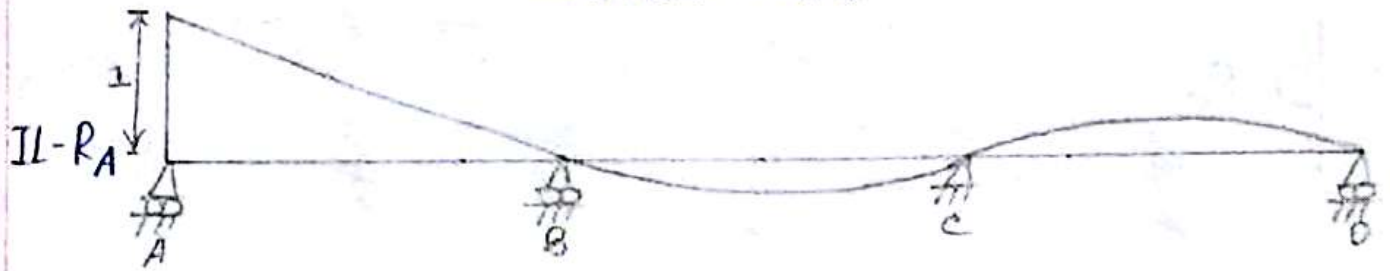


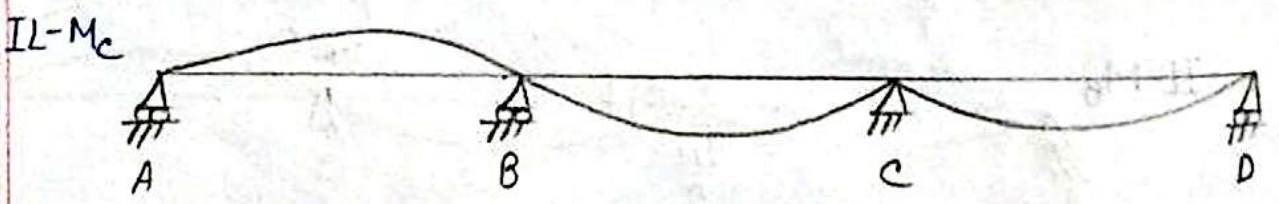
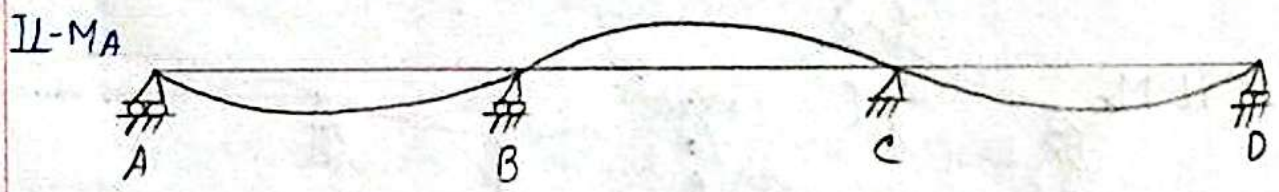
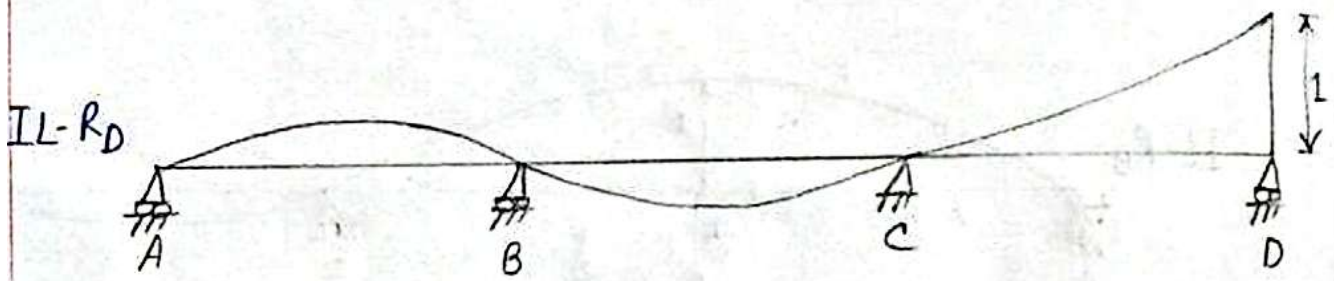
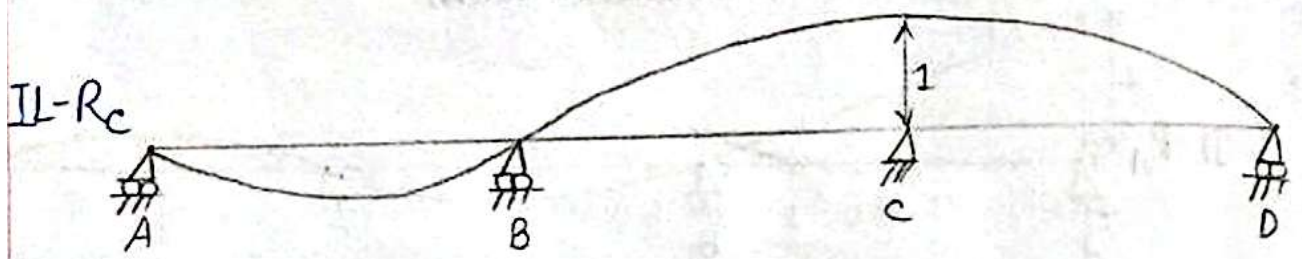
IL of  $M = 12$  diagrams [ $M = 0$  at 4 and 14], IL of  $V = 15$  diagrams [two at 7]

# Qualitative Influence Lines for Statically Indeterminate Structures

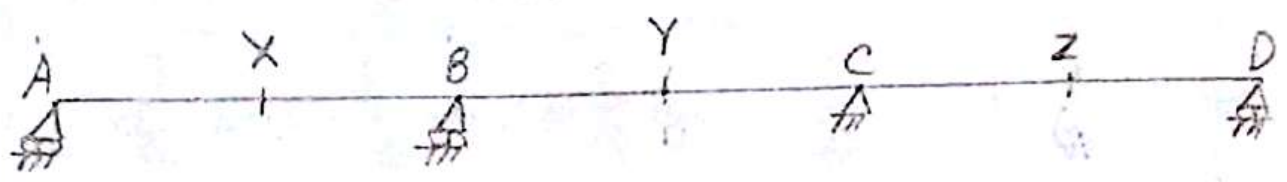


Continuous Beam

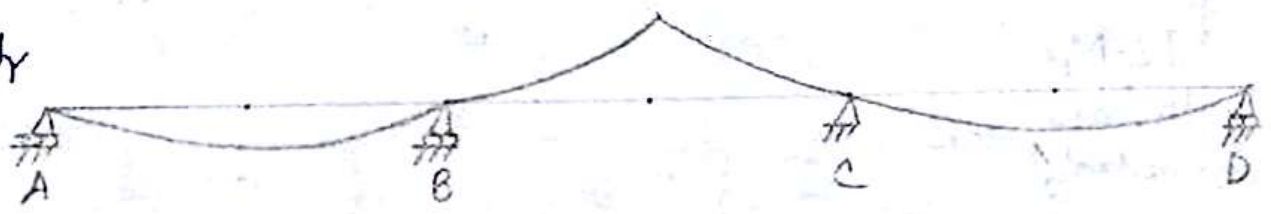




IL-M<sub>y</sub> IL-M<sub>z</sub> IL-V<sub>y</sub> IL-V<sub>z</sub> IL-V<sub>e(left)</sub>



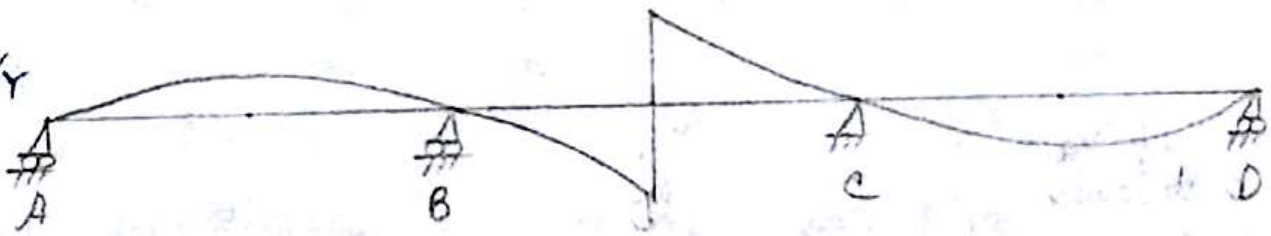
IL-M<sub>y</sub>



IL-M<sub>z</sub>



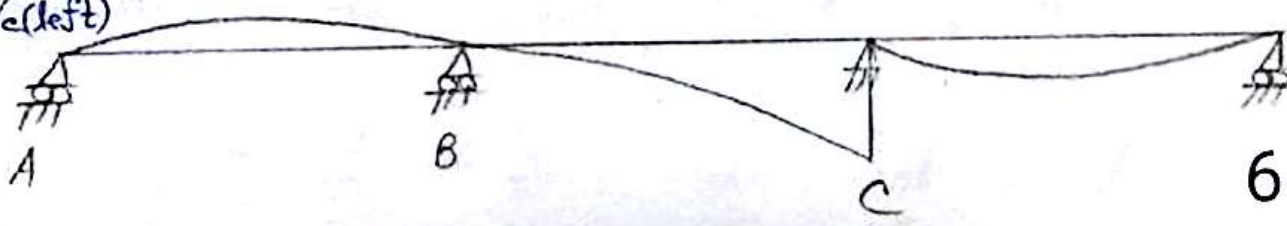
IL-V<sub>y</sub>



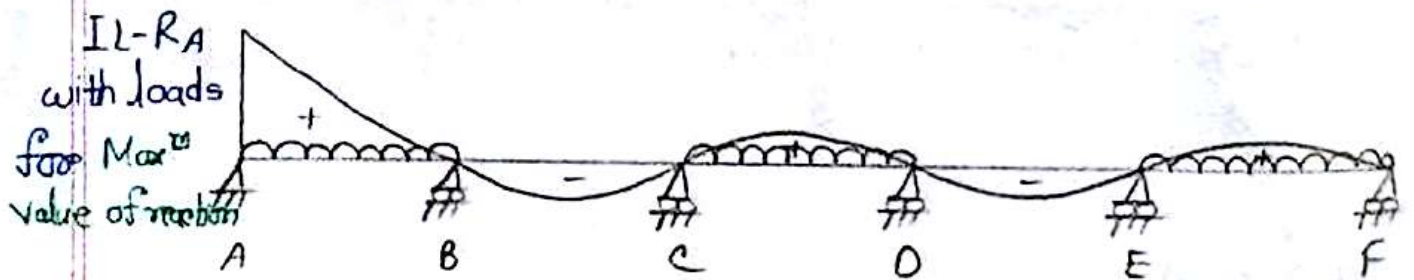
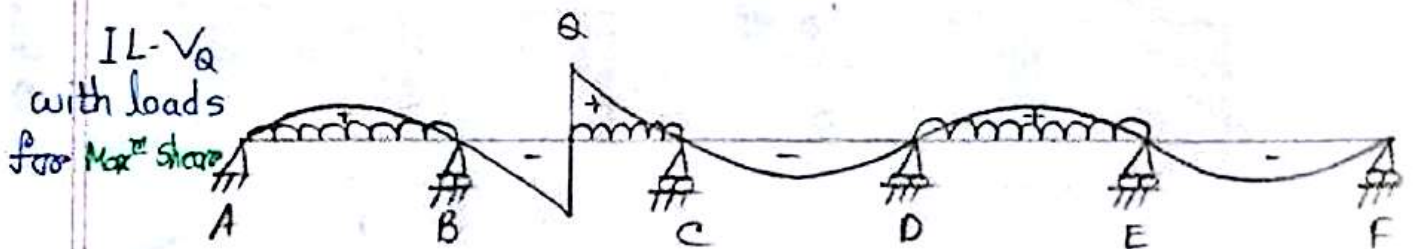
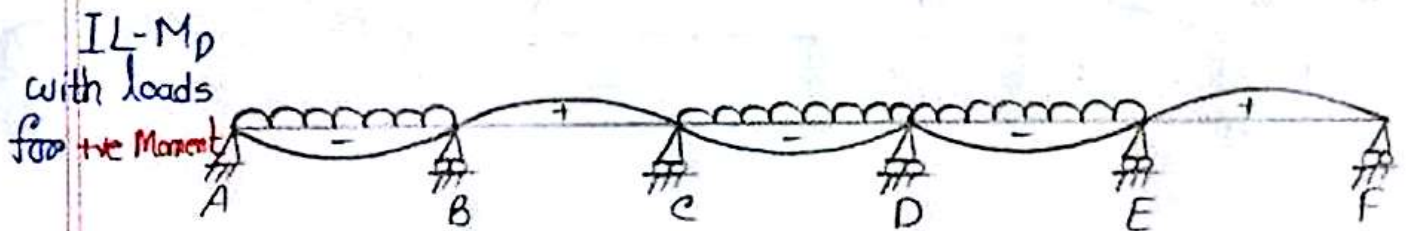
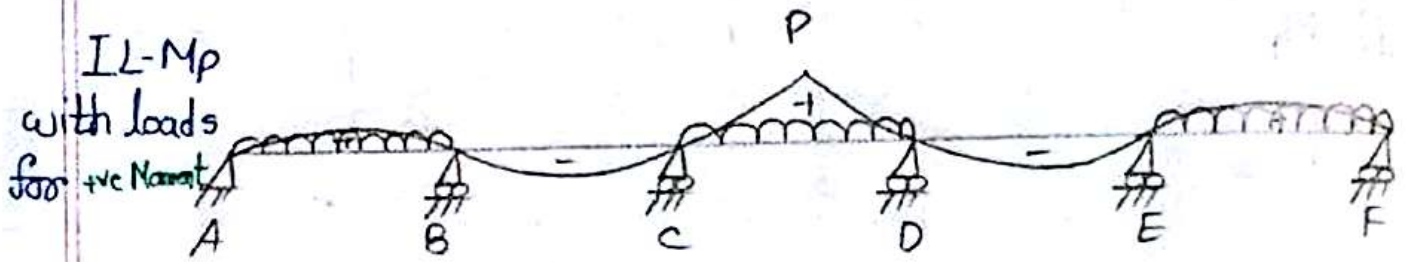
IL-V<sub>z</sub>



IL-V<sub>e(left)</sub>



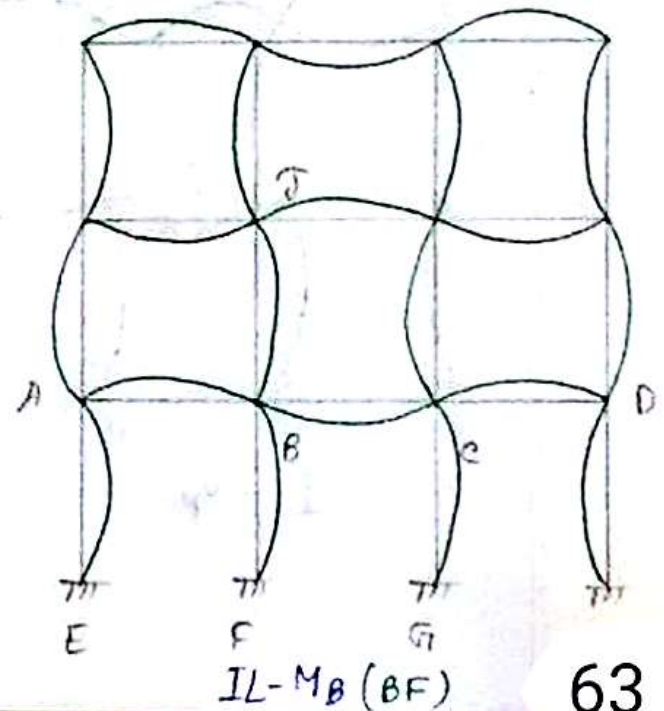
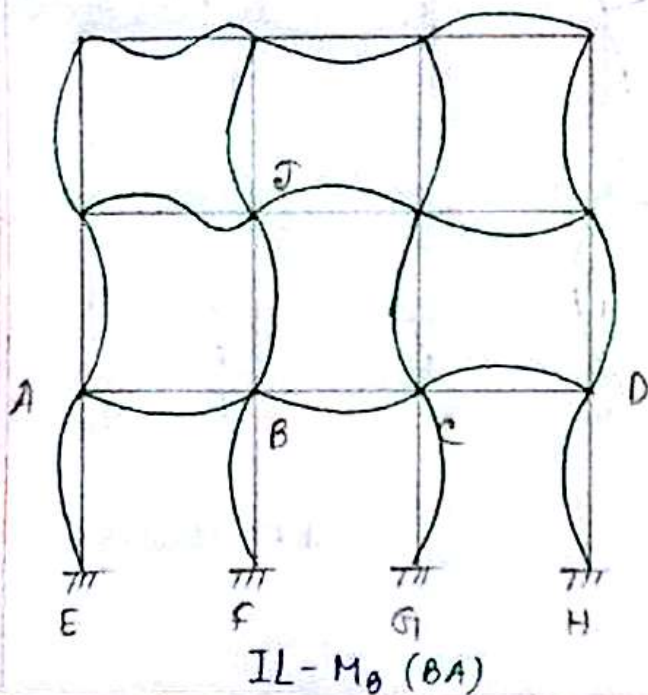
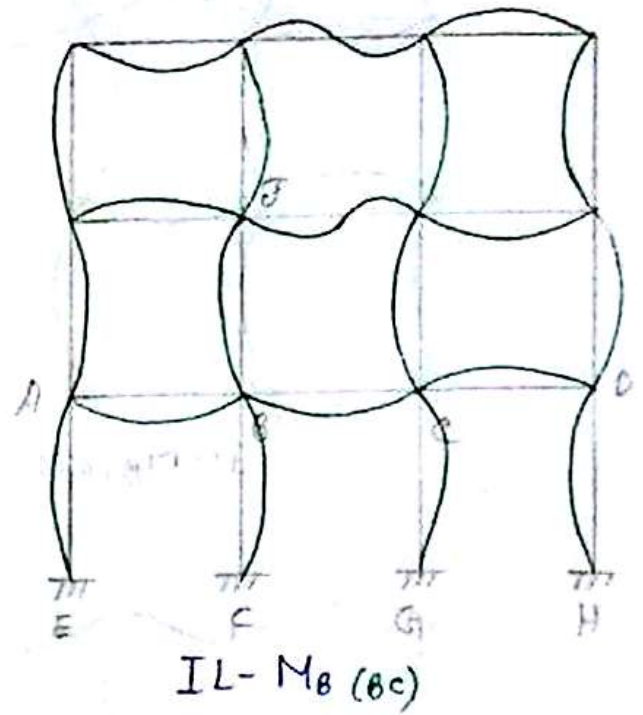
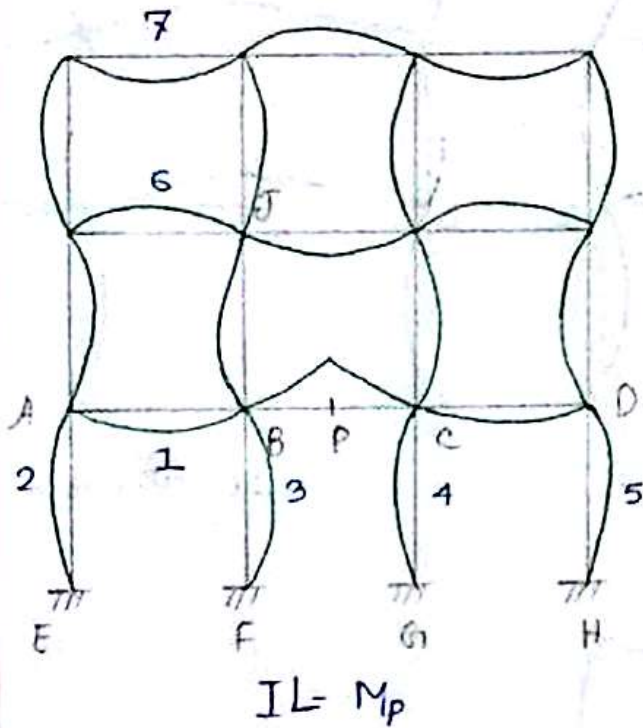
# Qualitative Influence Lines & Load Patterns

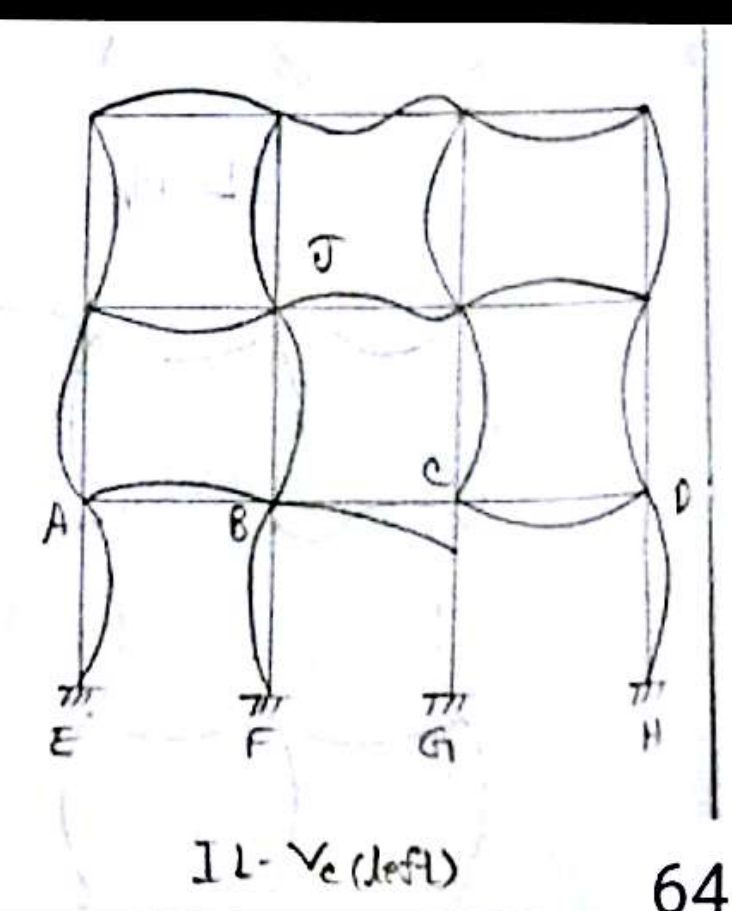
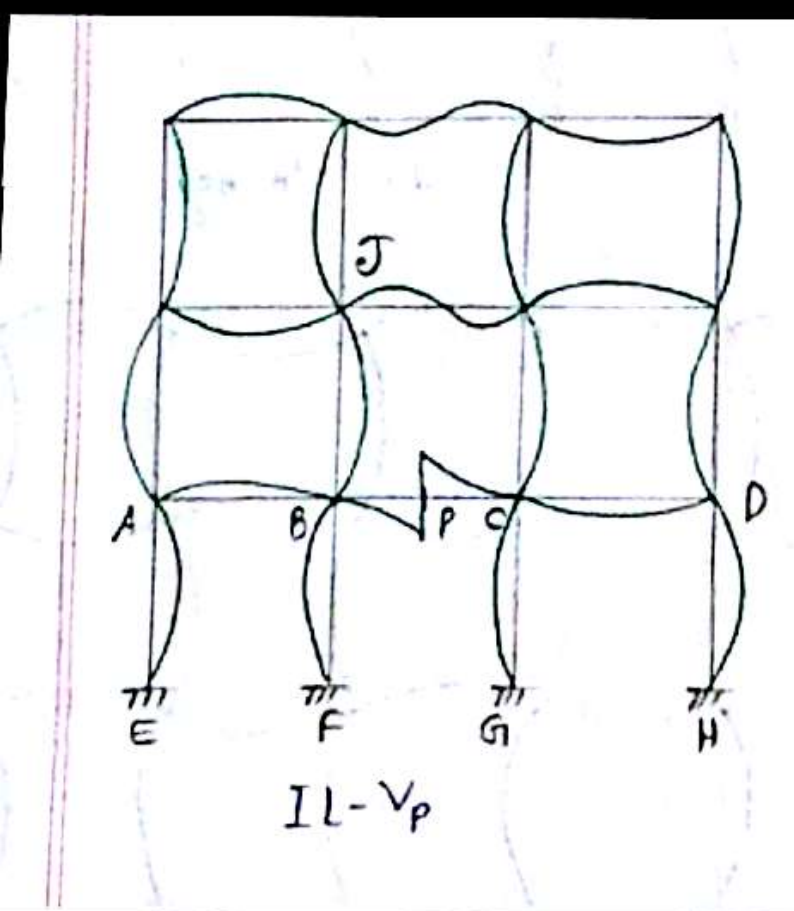
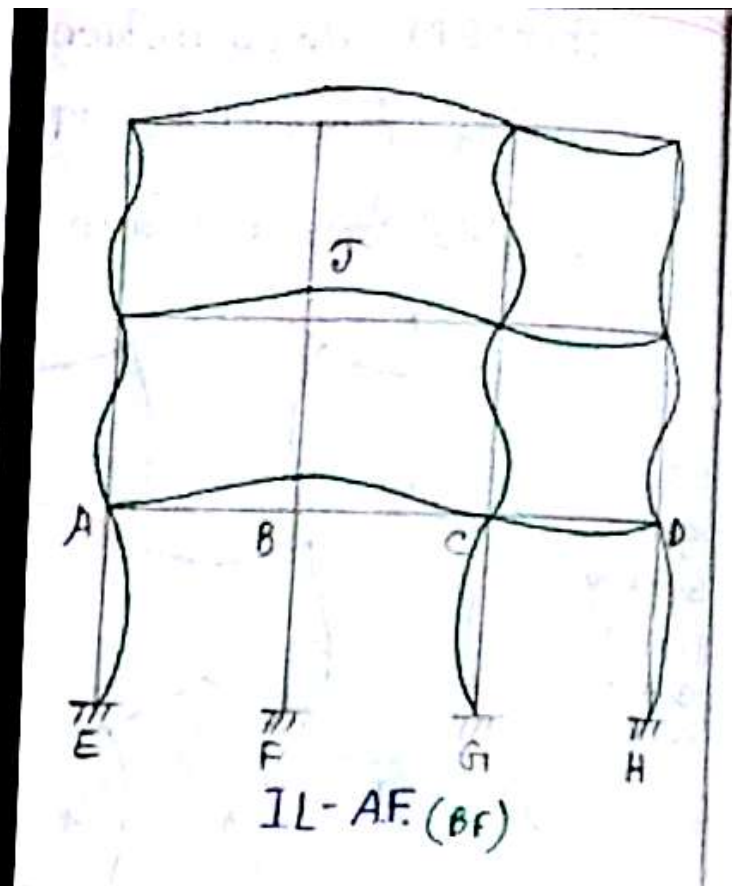
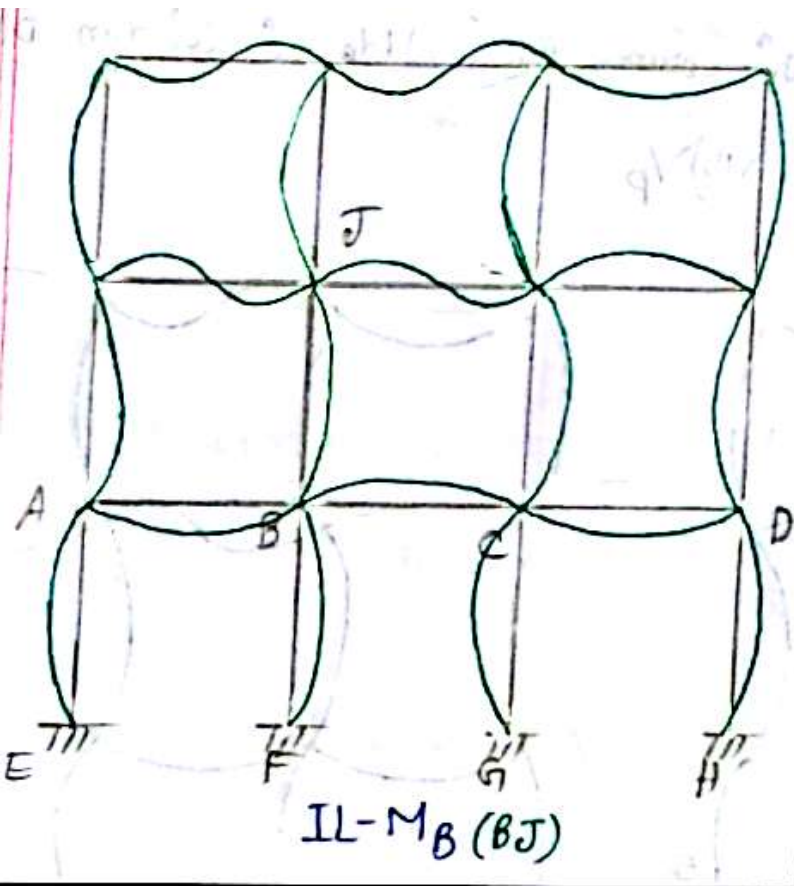


# FRAMES

PROBLEM: Draw influence lines for (i)  $M_p$ , (ii)  $M_B$  of beam BC (iii)  $M_B$  of beam BA (iv)  $M_B$  of column BF (v)  $M_B$  of column BT (vi) Axial force in column BF (vii)  $V_p$

Note:  
Sequence  
for drawing  
IL-1-2  
~3-4-5  
~6-7

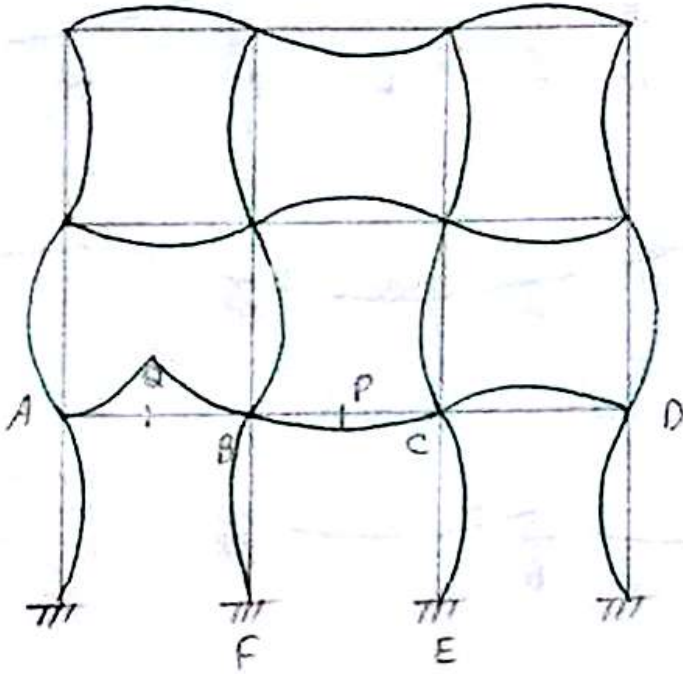




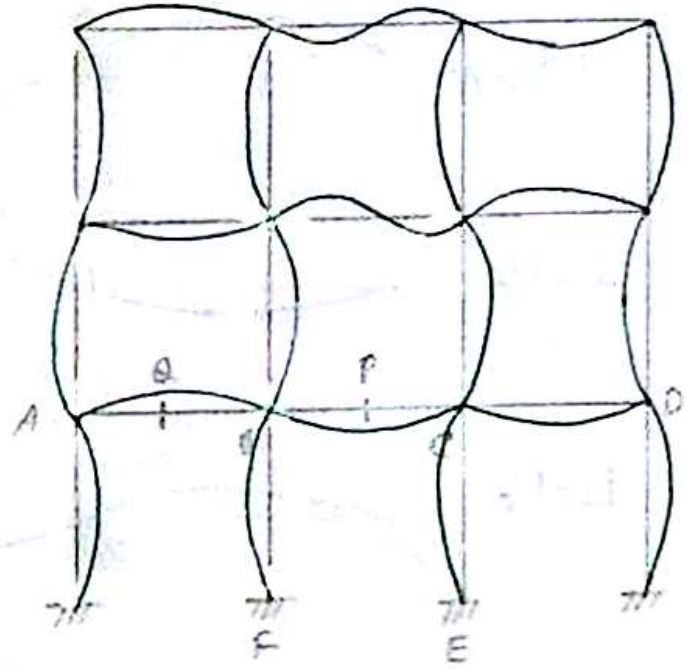
# Question Paper Solutions

"2014-15"

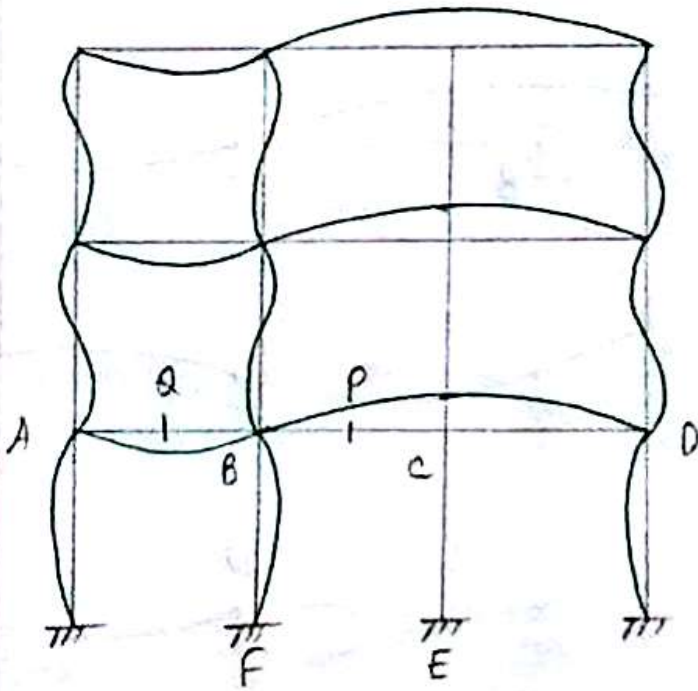
2.(b)



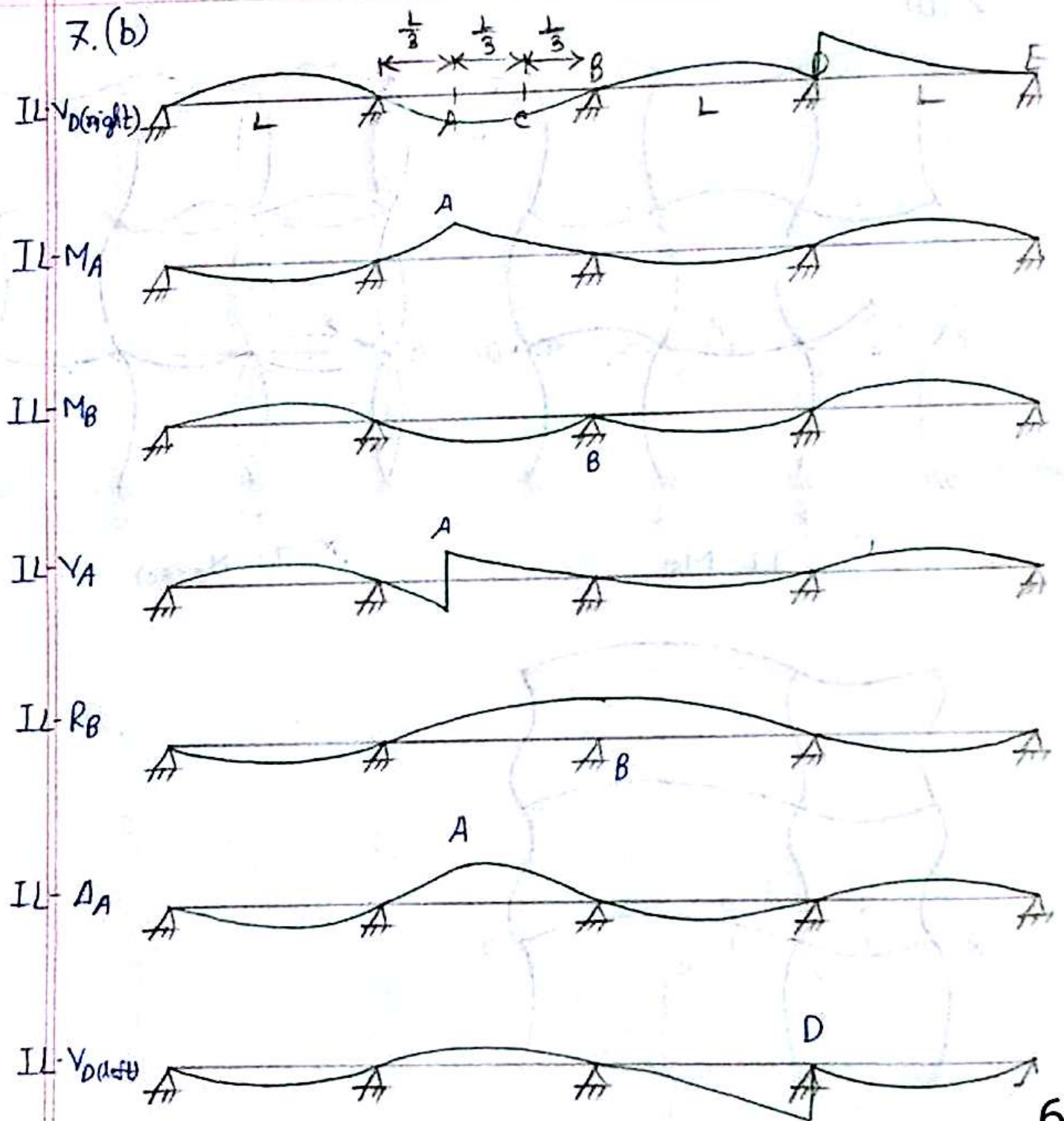
IL-M<sub>Q</sub>



IL-M<sub>c(bc)</sub>



IL-A.F.(CE)



### Example 7-4

Consider earlier Example 7-2 and determine the internal system of forces at sections  $a-a$  and  $b-b$ ; see Fig. 7-15(a).

#### SOLUTION.

A free body for the member, including reactions, is shown in Fig. 7-15(a). A free body to the left of section  $a-a$  in Fig. 7-15(b) shows the maximum ordinate for the isolated part of the applied load. From equilibrium conditions,

$$V_a = -9 + \frac{1}{2} \times 2 \times \frac{2}{3} \times 10 = -2.33 \text{ kN}$$

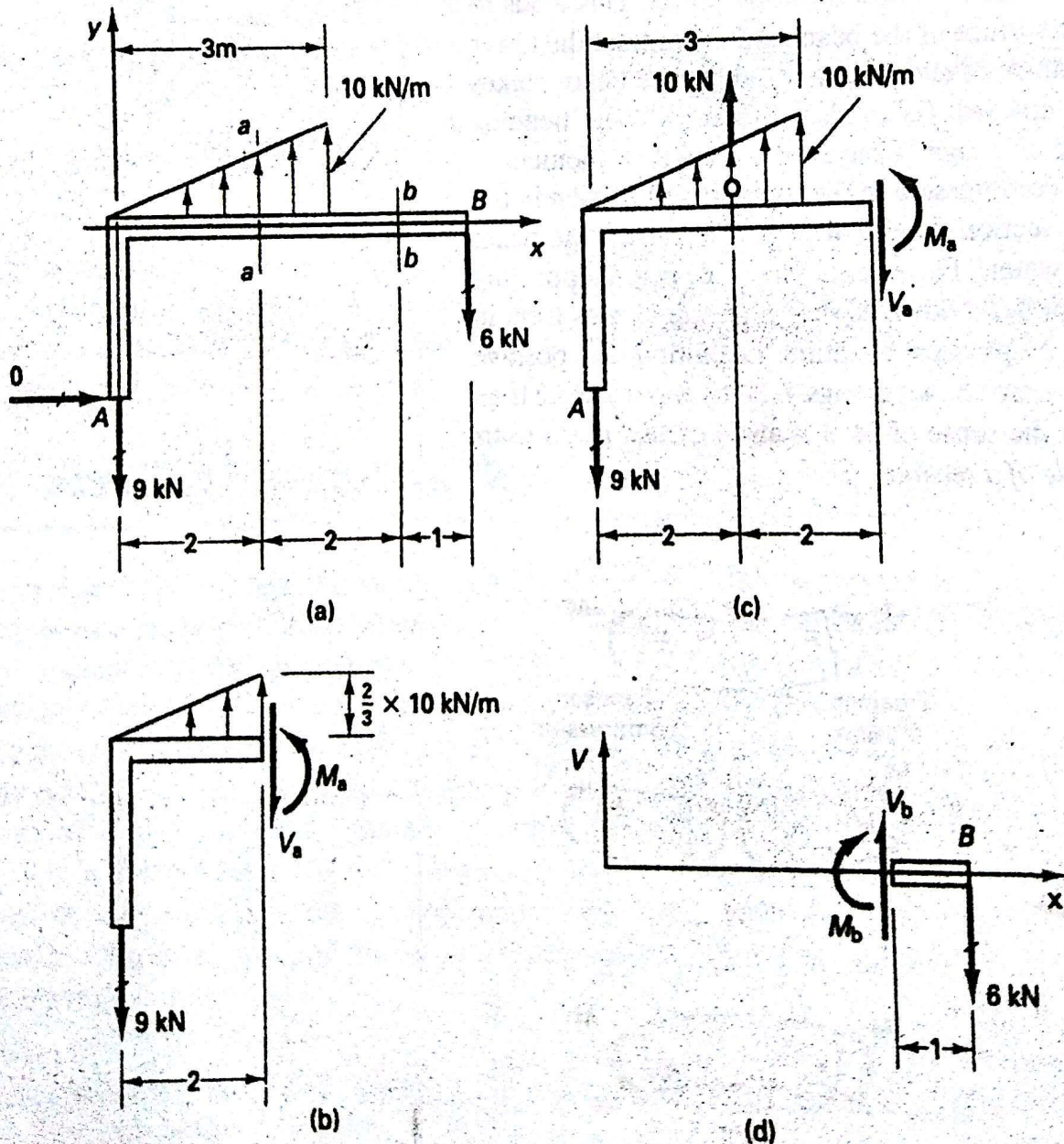


Fig. 7-15

and

$$M_a = -9 \times 2 + \frac{1}{2} \times 2 \times \frac{2}{3} \times 10 \times \frac{1}{3} \times 2 = -13.6 \text{ kN} \cdot \text{m}$$

These forces have opposite senses from those shown in the figure.

A free body to the left of section  $b-b$  is shown in Fig. 7-15(c), and to the right in Fig. 7-15(d). It is evident that the second free body is simpler for calculations, giving directly

$$V_b = +6 \text{ kN}$$

and

$$M_b = -6 \times 1 = -6 \text{ kN} \cdot \text{m}$$

The same procedure can be used for frames consisting of several members rigidly joined together as well as for curved bars. In all such cases, the sections must be perpendicular to the axis of a member.

---

be given. Algebraic expressions for these functions along a beam will

A systematic method for rapidly constructing shear and moment diagrams will be discussed in the next part of this chapter.

**Example 7-5**

Construct axial-force, shear, and bending-moment diagrams for the beam shown in Fig. 7-16(a) due to the inclined force  $P = 5$  k.

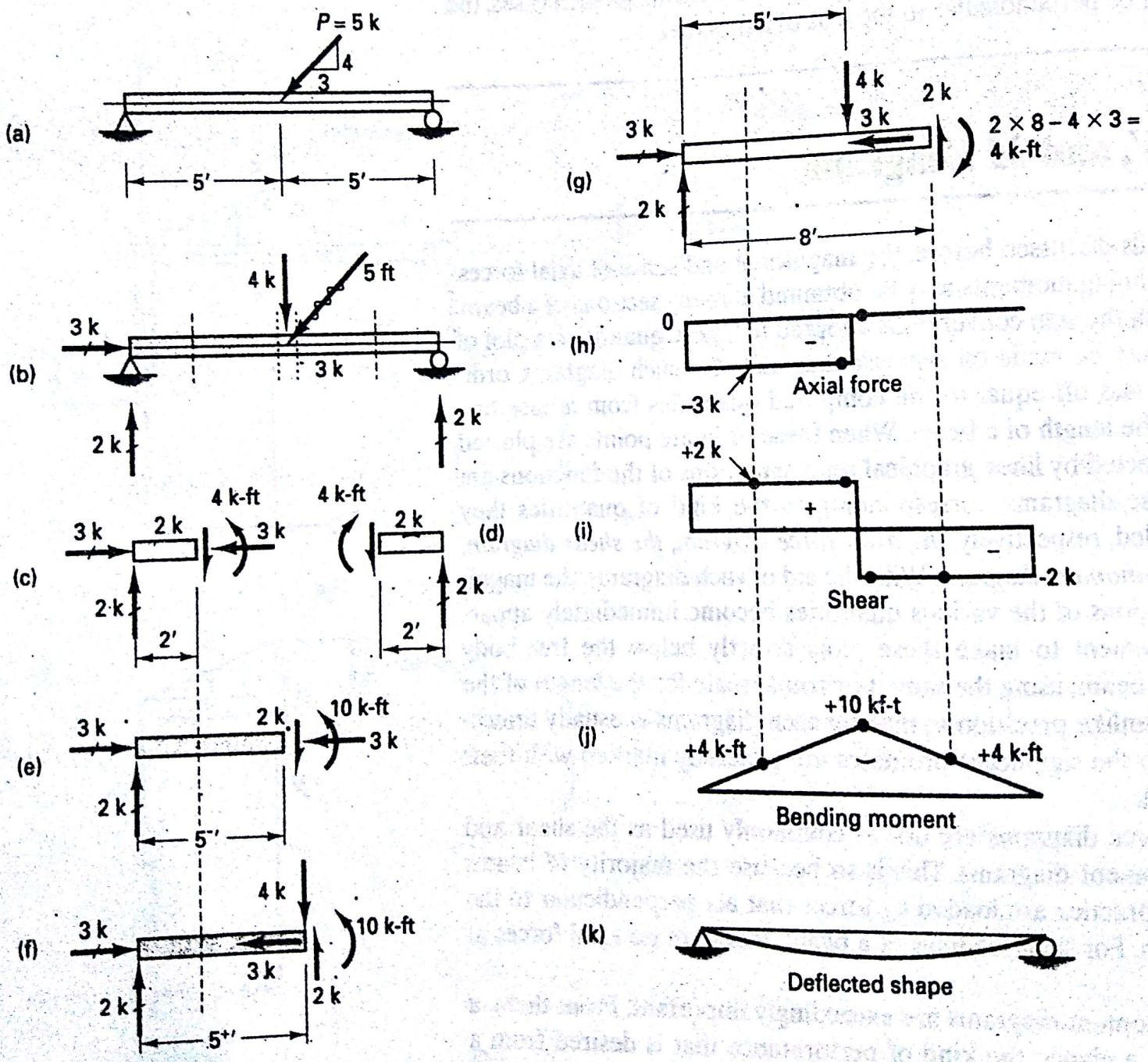
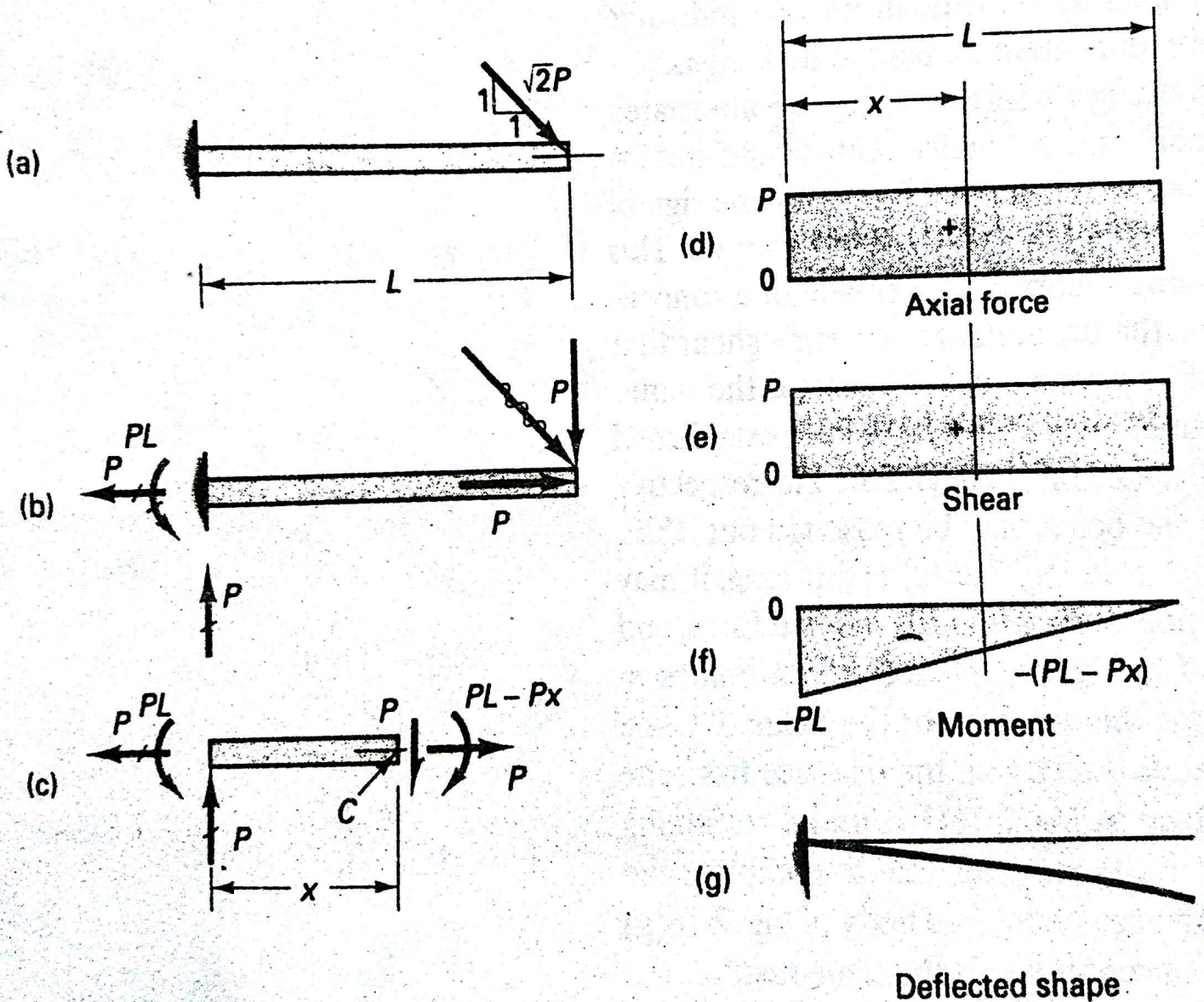


Fig. 7-16

**Example 7-6**

Determine axial-force, shear, and bending-moment diagrams for the cantilever loaded with an inclined force at the end; see Fig. 7-17(a).

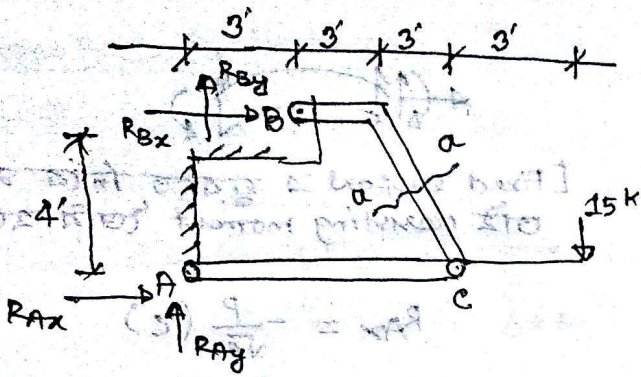


**Fig. 7-17**

**SOLUTION**

First, the inclined force is replaced by the two components shown in

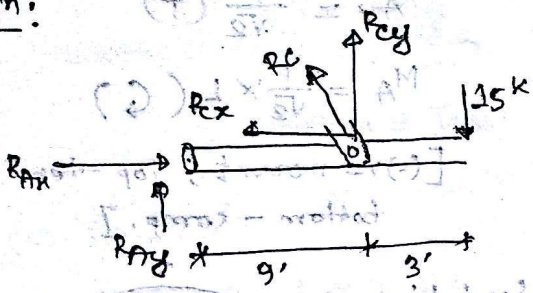
Problem (HW)



Find

- Internal forces
- Shear force
- Axial force
- Bending moment
- at sec a-a

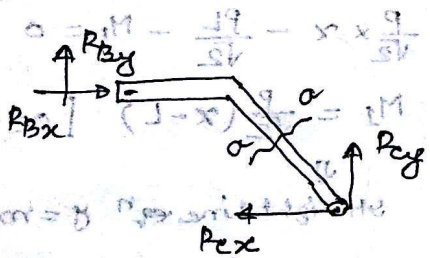
Soln:



$$\sum M_A = 0$$

$$\Rightarrow 15 \times 12 - R_{Cy} \times 9 = 0$$

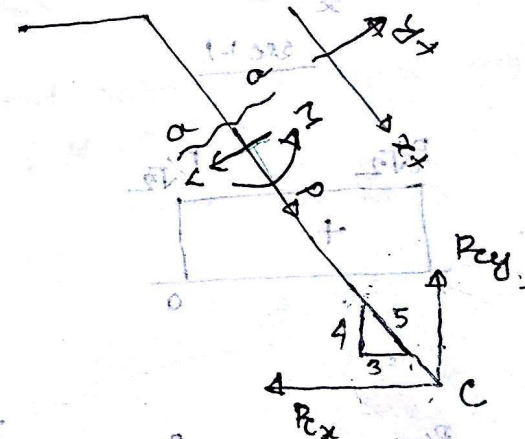
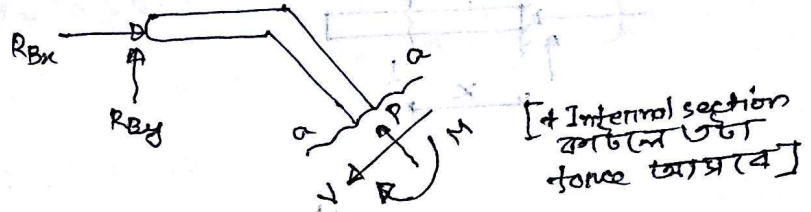
$$\therefore R_{Cy} = 20 \text{ k } (\uparrow)$$



$$\sum M_B = 0$$

$$\Rightarrow P_{cx} \times 4 = 20 \times 6 =$$

$$\therefore P_{cx} = 30 \text{ k } (\leftarrow)$$



$$\sum F_x = 0$$

$$\Rightarrow P - P_{cx} \times \frac{3}{5} - P_{cy} \times \frac{4}{5} = 0$$

$$\therefore P = 34 \text{ k } (\rightarrow)$$

$$\sum F_y = 0$$

$$\Rightarrow -V - P_{cx} \times \frac{4}{5} + P_{cy} \times \frac{3}{5} = 0$$

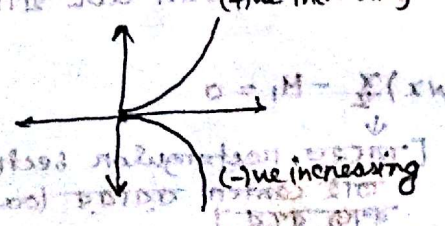
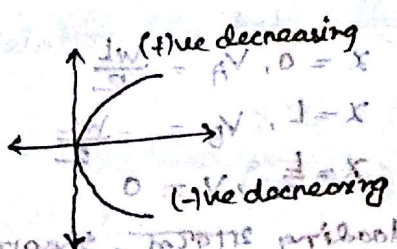
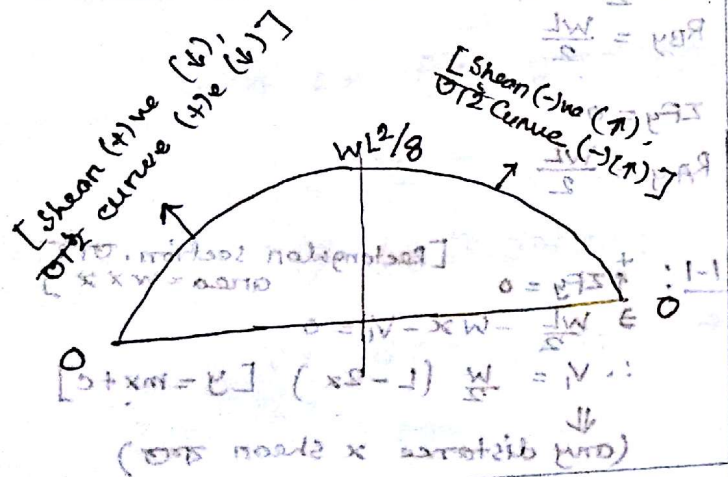
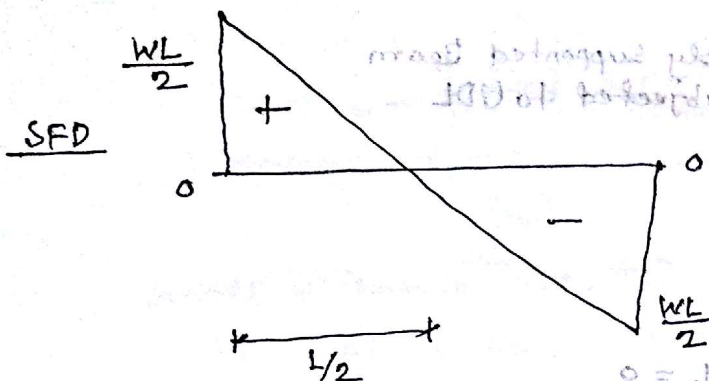
$$\therefore V = 12 \text{ k } (\uparrow)$$

$$\sum M_{a-a} = 0$$

$$\Rightarrow -P_{cx} \times 2 - P_{cy} \times 1.5 - M = 0$$

$$\therefore M = 30 \text{ k-ft } (\curvearrowright)$$

\* Summation approach



+ Upward force (+ve shear)

$$+ \frac{WL}{2}$$

$$- \frac{WL}{2}$$

$$0$$

$$- \frac{WL}{2}$$

$$+ \frac{WL}{2}$$

$$0$$

(downward force, or (-ve))

Or

$$+ \frac{WL}{2}$$

$$- WL$$

$$- WL/2$$

$$+ WL/2$$

$$0$$

+ Total load W  
अधिक  $\frac{WL}{2}$

+ loading diagram में (मात्रा में) शून्य shear का value

+ UDL में shear diagram linear

+ SFD और BMD के बीच SFD का area का moment (+ve shear में (+ve moment

$$+ \frac{1}{2} \times \frac{WL}{2} \times \frac{L}{2}$$

$$\downarrow$$

$$\frac{WL^2}{8}$$

$$- \frac{WL^2}{8}$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

+ Area मात्र बराबर रहे, तब moment 0 रहे

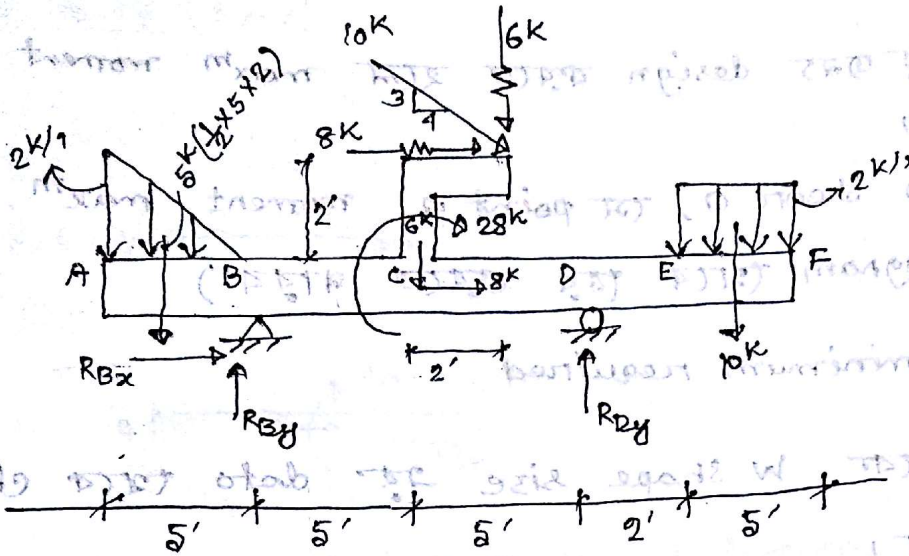
+ SFD linear में MD रहे 1st order parabola

$$y = mx \rightarrow y = mx^2$$

यदि  $y = mx^2$  रहे  $\rightarrow y = mx^3$  रहे

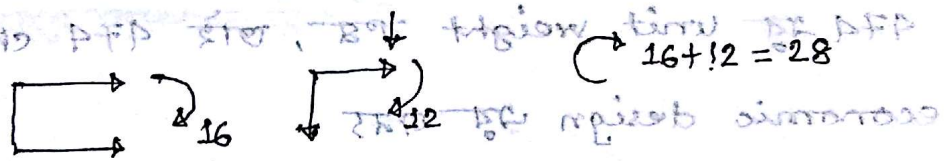
# Problem

(making beam rigid) Sub making diagram



# Sol<sup>n</sup>:

parallelly force c point o নামাই জানলে Moment দিবে।



$$\sum F_x = 0$$

$$\Rightarrow R_{Bx} = -8k (\leftarrow)$$

$$\sum M_B = 0$$

$$\Rightarrow -3.33 \times 5 + 6 \times 5 + 28 - R_{By} \times 10 + 10 \times 14.5 = 0$$

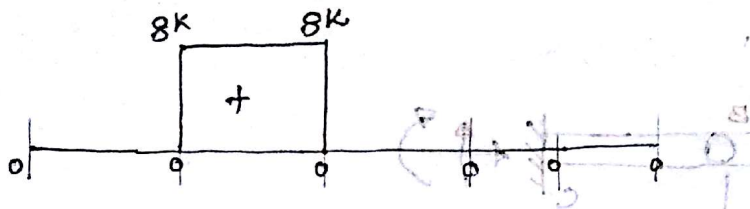
$$\therefore R_{By} = 18.635k$$

$$\sum F_y = 0$$

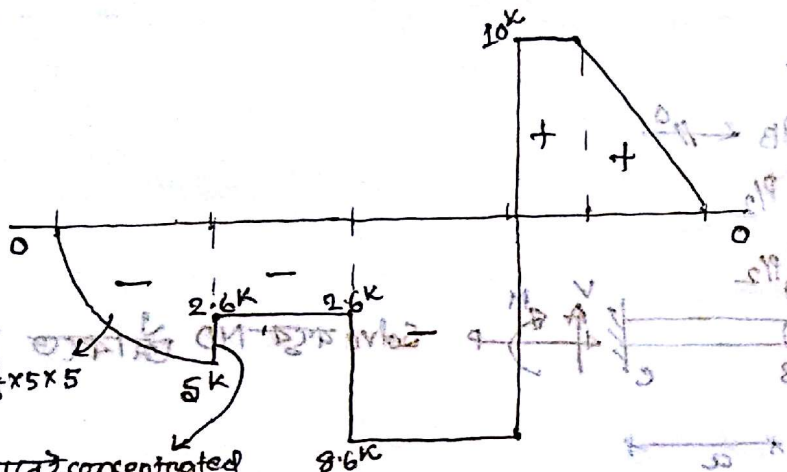
$$\Rightarrow -5 + R_{By} - 6 + 18.635 - 10 = 0$$

$$\therefore R_{By} = 2.4k (\uparrow)$$

AFD (K)



SFD (K)



$A_1 = \frac{2}{3} \times 5 \times 5$

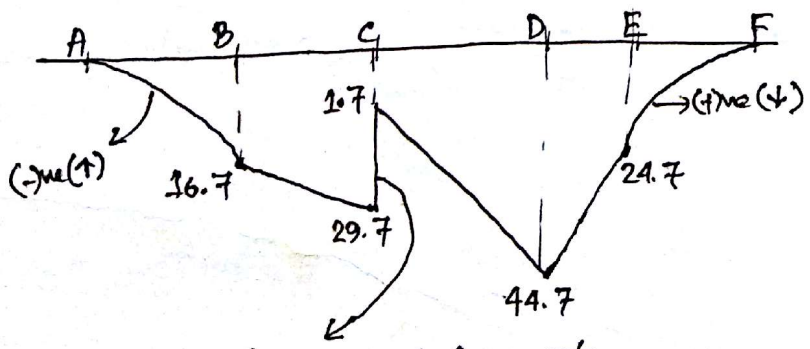
[যেখানে concentrated force সেখানে jump of shear.]

(↑)  $\frac{q}{L} = V$

0 = q

(↓)  $\frac{q \cdot x}{2} = M$

\* BMD (K-ft)



[যেখানে concentrated moment সেখানে jump]

AFD (K)

at B → +8K  
at C → -8K

SFD (K)

-5K (B point just to the left)  
+2.4K (B " " " " right)  
-2.6K (C just left)  
-6K (C " " right)  
-8.6K  
+18.6K [upward force (+)ve shear]  
+10K  
-10K [downward force (-)ve shear]  
0

(ultimately 0 আসতে হবে)

[যদি concentrated force ও moment থাকে তাহলে just to the left & right এ হিসাব করতে হবে]

+ B ও C য় মাঝে কোন vertical force নাই। তাই shear same থাকবে।

+ UDL হলে linear  
UVL হলে parabolic

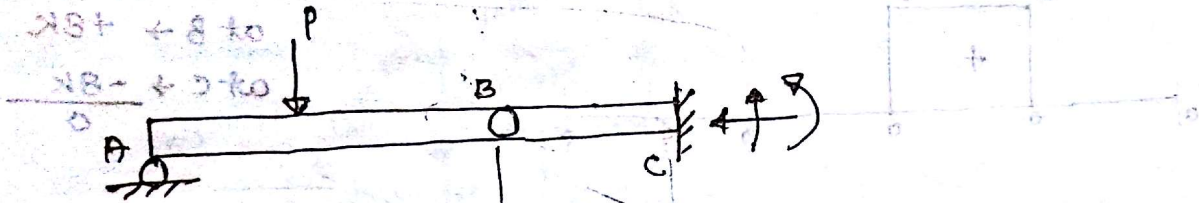
+\* BMD আঁকতে সতর্ক  
concentrated moment (8K)  
কিনা খেয়াল রাখতে  
হবে। C point এ concentrated  
moment আছে।

BMD

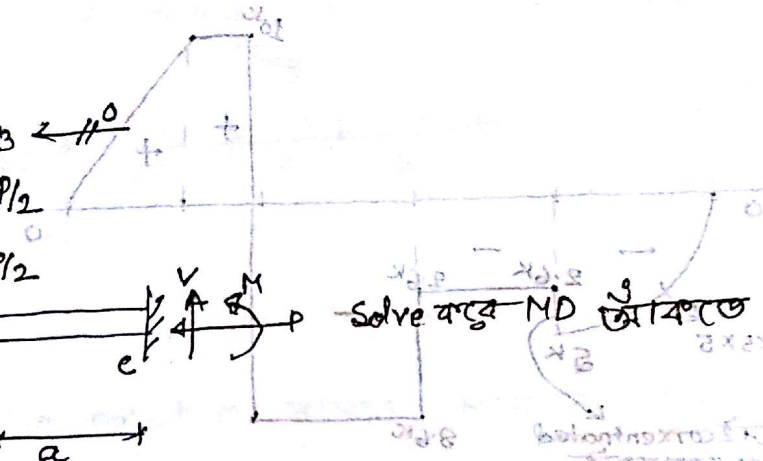
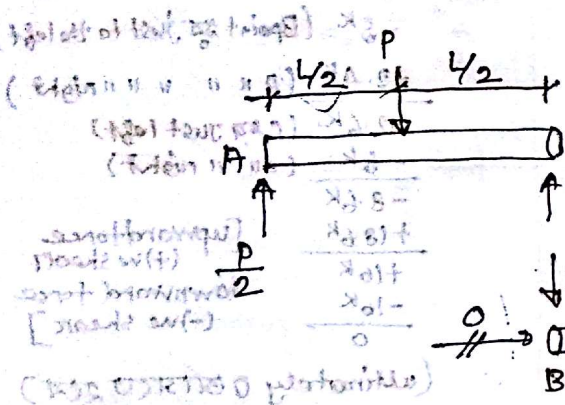
-16.7  
-13.0  
-29.7  
+28  
-1.7K  
-43  
-44.7  
+20  
-24.7  
+25  
+0.3 ≈ 0

[Rounding এর জন্য 0.3 আসবে, তাই 0 হবে]

(\*) Q7A



Internal hinge (fixed ball bearing (दया जाछे))  
 ↳ moment 0



Solve करुं MD जायते सब ।

$$V = \frac{P}{2} (\uparrow)$$

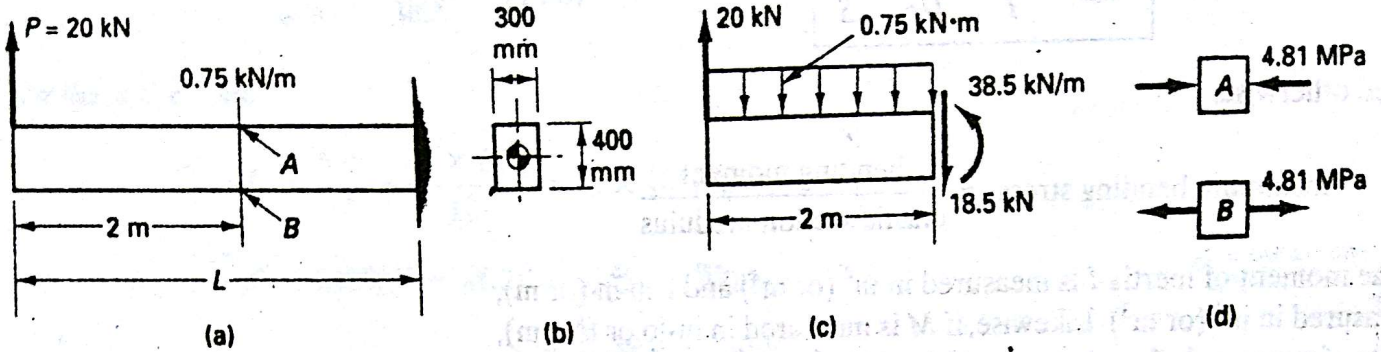
$$P = 0$$

$$M = -\frac{Pa}{2} (2)$$



**Example 8-4**

A 300-by-400-mm wooden cantilever beam weighing 0.75 kN/m carries an upward concentrated force of 20 kN at the end, as shown in Fig. 8-12(a). Determine the maximum bending stresses at a section 2 m from the free end.



**Figure 8-12**

**SOLUTION**

A free-body diagram for a 2-m segment of the beam is shown in Fig. 8-12(c). To keep this segment in equilibrium requires a shear of  $20 - (0.75 \times 2) = 18.5$  kN and a bending moment of  $(20 \times 2) - (0.75 \times 2 \times 1) = 38.5$  kN·m at the section. Both of these quantities are shown with their proper sense in Fig. 8-12(c). The distance from the neutral axis to the extreme fibers  $c = 200$  mm. This is applicable to both the tension and the compression fibers.

From Eq. 8-19,

$$I_z = \frac{bh^3}{12} = \frac{300 \times 400^3}{12} = 16 \times 10^8 \text{ mm}^4$$

From Eq. 8-13,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{38.5 \times 10^6 \times 200}{16 \times 10^8} = \pm 4.81 \text{ MPa}$$

From the sense of the bending moment shown in Fig. 8-12(c), the top fibers of the beam are seen to be in compression and the bottom ones in tension. In the answer given, the positive sign applies to the tensile stress and the negative sign applies to the compressive stress. Both of these stresses decrease at a linear rate toward the neutral axis, where the bending stress is zero. The normal stresses acting on infinitesimal elements at A and B are shown in Fig. 8-12(d). It is important to learn to make such a representation of an element as it will be frequently used in Chapters 11 and 13.

**ALTERNATIVE SOLUTION**

If only the maximum stress is desired, the equation involving the section modulus may be used. The section modulus for a rectangular section in algebraic form is

$$S = \frac{I}{c} = \frac{bh^3}{12} \frac{2}{h} = \frac{bh^2}{6} \quad (8-22)$$

In this problem,  $S = 300 \times 400^2/6 = 8 \times 10^6 \text{ mm}^3$ , and by Eq. 6-21,

$$\sigma_{\max} = \frac{M}{S} = \frac{38.5 \times 10^6}{8 \times 10^6} = 4.81 \text{ MPa}$$

Both solutions lead to identical results.

### ✓ Example 9-5

Find the stress distribution at section  $ABCD$  for the block shown in mm in Fig. 9-11(a) if  $P = 64$  kN. At the same section, locate the line of zero stress. Neglect the weight of the block.

### ✓ SOLUTION

In this problem, it is somewhat simpler to recast Eq. 9-5 with the aid of Eq. 8-22, defining the elastic section modulus  $S = I/c$  as  $bh^2/6$ . The normal stress at any  $i$ th corner of the block can be found directly from such a transformed equation. This equation reads

$$\sigma_i = \frac{P}{A} - \frac{M_z}{S_z} + \frac{M_y}{S_y} \quad (9-8)$$

where  $S_z = bh^2/6$  and  $S_y = hb^2/6$ .

The forces acting on section  $ABCD$ , Fig. 9-11(c), are  $P = -64 \times 10^3$  N,  $M_y = -64 \times 10^3 \times 150 = -9.6 \times 10^6$  N·mm, and  $M_z = -64 \times 10^3 \times (75 + 75) = -9.6 \times 10^6$  N·mm. The cross-sectional area has the following properties:  $A = 150 \times 300 = 45 \times 10^3$  mm<sup>2</sup>,  $S_z = 300 \times 150^2/6 = 1.125 \times 10^6$  mm<sup>3</sup>, and  $S_y = 150 \times 300^2/6 = 2.25 \times 10^6$  mm<sup>3</sup>.

The normal stresses at the corners are found using Eq. 9-8, assigning signs for the stresses caused by moments by inspection. For example, from Fig. 9-11(c), it can be seen that due to  $M_y$ , the stresses at corners  $A$  and  $D$  are compressive. Other cases are treated similarly. Using this approach,

$$\begin{aligned} \sigma_A &= -\frac{64 \times 10^3}{45 \times 10^3} - \frac{9.6 \times 10^6}{1.125 \times 10^6} - \frac{9.6 \times 10^6}{2.25 \times 10^6} \\ &= -1.42 - 8.53 - 4.27 = -14.2 \text{ MPa} \end{aligned}$$

$$\sigma_B = -1.42 - 8.53 + 4.27 = -5.7 \text{ MPa}$$

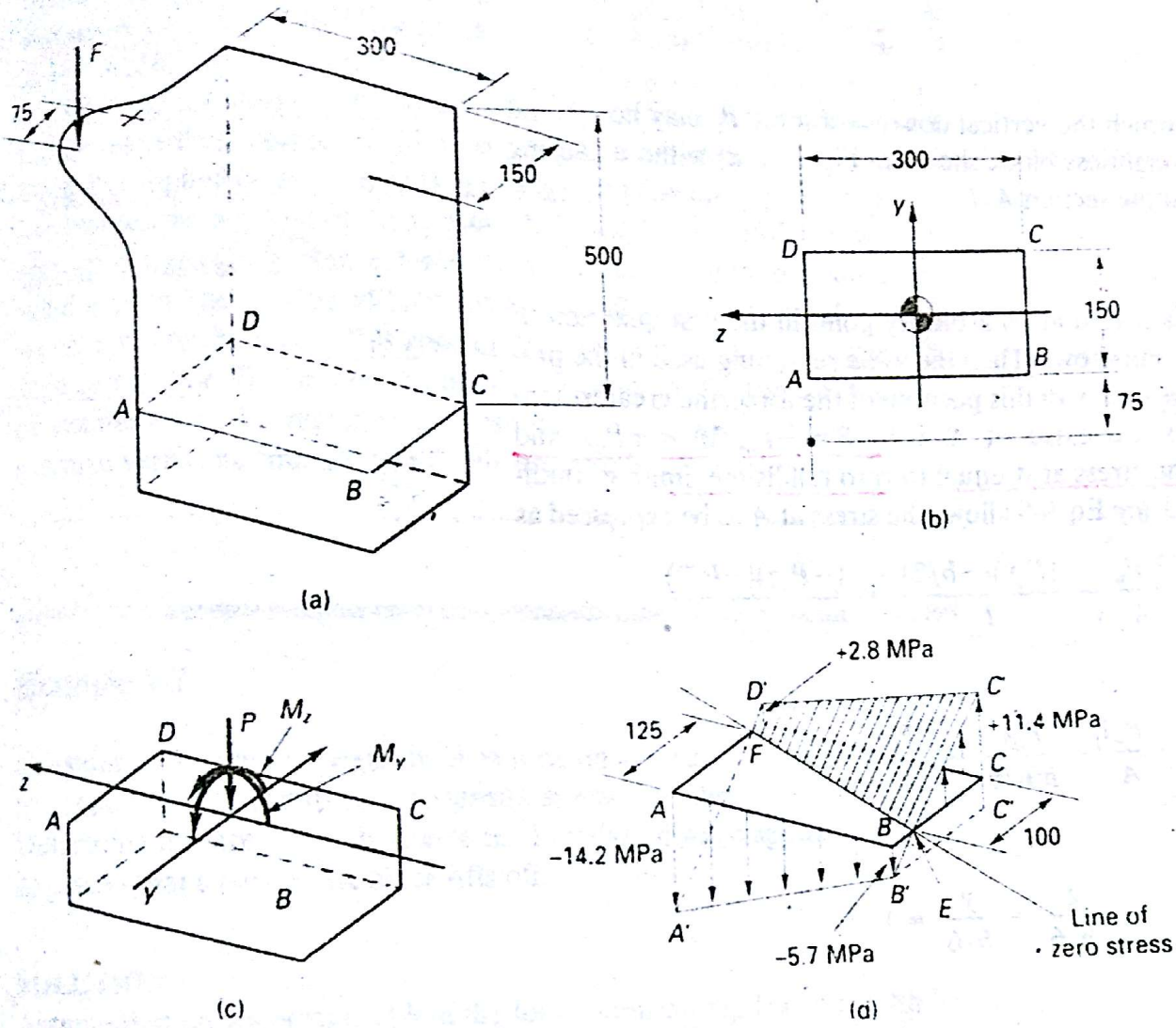


Fig. 9-11

$$\sigma_c = -1.42 + 8.53 + 4.27 = +11.4 \text{ MPa}$$

$$\sigma_D = -1.42 + 8.53 - 4.27 = +2.8 \text{ MPa}$$

These stresses are shown in Fig. 9-11(d). The ends of these four stress vectors at  $A'$ ,  $B'$ ,  $C'$ , and  $D'$  lie in the plane  $A'B'C'D'$ . The vertical distance between planes  $ABCD$  and  $A'B'C'D'$  defines the total stress on any point on the cross section. The intersection of plane  $A'B'C'D'$  with plane  $ABCD$  locates the line of zero stress  $FE$ .

By drawing a line  $B'C''$  parallel to  $BC$ , similar triangles  $C'B'C''$  and  $C'EC$  are obtained: thus, the distance  $CE = [11.4 / (11.4 + 5.7)] 150 = 100$  mm. Similarly, distance  $AF$  is found to be 125 mm. Points  $E$  and  $F$  locate the line of zero stress. If the weight of the block is neglected, the stress distribution on any other section parallel to  $ABCD$  is the same.

where  $A_o$  is the cross-sectional area of the elastic part of the cross section. The shear stress distribution for the elastic-plastic case is shown in Fig. 10-15(c). This can be contrasted with that for the elastic case, shown in Fig. 10-15(d). Since equal and opposite normal stresses occur in the plastic zones, no unbalance in longitudinal forces occurs and no shear stresses are developed.

This elementary solution has been refined by using a more carefully formulated criterion of yielding caused by the simultaneous action of normal and shear stresses.<sup>8</sup>

### Example 10-5

An I beam is loaded as shown in Fig. 10-16(a). If it has the cross section shown in Fig. 10-16(c), determine the shear stresses at the levels indicated. Neglect the weight of the beam.

### SOLUTION

From the free-body diagram of the beam segment in Fig. 10-16(b), it is seen that the vertical shear at all sections is 50 kips. Bending moments do not enter directly into the present problem. The shear flow at the various levels of the beam is computed in the following table using Eq. 10-5. Since  $\tau = q/t$ , from Eq. 10-6, the shear stresses are obtained by dividing the shear flows by the respective widths of the beam:

$$I = 6 \times 12^3/12 - 5.5 \times 11^3/12 = 254 \text{ in}^4$$

For use in Eq. 10-5, the ratio  $V/I = 50,000/254 = 197 \text{ lb/in}^4$ . Some results at selected horizontal sections are given in the following table:

<sup>8</sup>D. C. Drucker, "The effect of shear on the plastic bending of beams," *J. Appl. Mech.*, 23 (1956), 509-514.

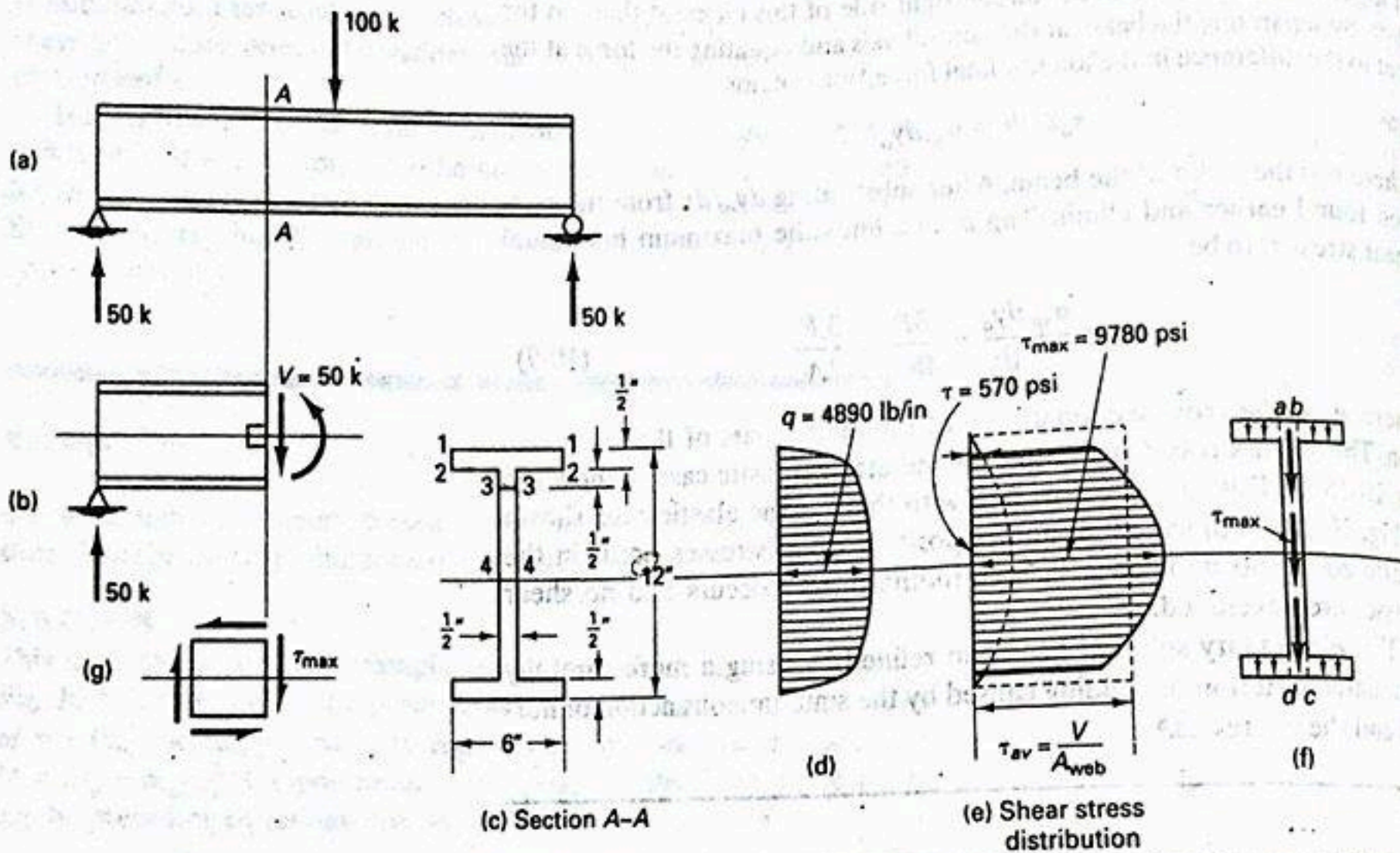


Fig. 10-16

Level	$A_{fchj}^a$	$y^b$	$Q = A_{fchj}y$	$q = VQ/I$	$t$	$\tau$ (psi)	
1-1	0	6	0	0	6.0	0	
2-2	$0.5 \times 6 = 3.00$	5.75	17.25	3400	0.5	6800	
3-3	$\begin{cases} 0.5 \times 6 = 3.00 \\ 0.5 \times 0.5 = 0.25 \end{cases}$	$\begin{cases} 5.75 \\ 5.25 \end{cases}$	$\begin{cases} 17.25 \\ 1.31 \end{cases}$	18.56	3650	0.5	7300
4-4	$\begin{cases} 0.5 \times 6 = 3.00 \\ 0.5 \times 5.5 = 2.75 \end{cases}$	$\begin{cases} 5.75 \\ 2.75 \end{cases}$	$\begin{cases} 17.25 \\ 7.56 \end{cases}$	24.81	4890	0.5	9780

<sup>a</sup> $A_{fchj}$  is the partial area of the cross section above a given level in  $\text{in}^2$ .

<sup>b</sup> $y$  is distance in mm from the neutral axis to the centroid of the partial area.

The positive signs of  $\tau$  show that, for the section considered, the stresses act downward on the right face of the elements. This sense of the shear stresses coincides with the sense of shear force  $V$ . For this reason, a strict adherence to the sign convention is often unnecessary. It is always true that  $\int_A \tau dA$  is equal to  $V$  and has the same sense.

Note that at level 2-2, two widths are used to determine the shear stress—one just above the line 2-2, and one just below. A width of 6 in cor-

responds to the first case, and 0.5 in to the second. This transition point will be discussed in the next section. The results obtained, which by virtue of symmetry are also applicable to the lower half of the section, are plotted in Figs. 10-16(d) and (e). By a method similar to the one used in the preceding example, it may be shown that the curves in Fig. 10-16(e) are parts of a second-degree parabola.

The variation of the shear stress indicated by Fig. 10-16(e) may be interpreted as shown in Fig. 10-16(f). The maximum shear stress occurs at the neutral axis, and the vertical shear stresses throughout the web of the beam are nearly of the same magnitude. The vertical shear stresses occurring in the flanges are very small. For this reason, the maximum shear stress in an I beam is often approximated by dividing the total shear  $V$  by the cross-sectional area of the web, with the web height assumed equal to the beam's overall height, area  $abcd$  in Fig. 10-16(f). Hence,

$$(\tau_{\max})_{\text{approx}} = \frac{V}{A_{\text{web}}} \quad (10-10)$$

In the example considered, this gives

$$(\tau_{\max})_{\text{approx}} = \frac{50,000}{0.5 \times 12} = 8330 \text{ psi}$$

This stress differs by about 15% from the one found by the accurate formula. For most cross sections, a much closer approximation to the true maximum shear stress may be obtained by dividing the shear by the web area between the flanges only. For this example, this procedure gives a stress of 9091 psi, which is an error of only about 8%. It should be clear from the foregoing that division of  $V$  by the whole cross-sectional area of the beam to obtain the shear stress is not permissible.

An element of the beam at the neutral axis is shown in Fig. 10-16(g). At levels 3-3 and 2-2, bending stresses, in addition to the shear stresses, act on the vertical faces of the elements. No shear stresses and only bending stresses act on the elements at level 1-1.

Structural Stability and Determinacy - Indeterminacy

(a) A structure is statically :

(i) unstable if (total unknown forces) < (total equation)

(ii) Stable & Determinant if ( " ) = ( " )

(iii) Stable & Indeterminant if ( " ) > ( " )

and  $D^{\circ}$  of Indeterminacy = ( " )  $\ominus$  ( " )

\* Geometrical  $\rightarrow$  determinacy नहीं,  $\rightarrow$   $\rightarrow$  instability.

(b) Total unknown forces = Member forces + R's

$\leftarrow$  (AF) =  $1b + r$  for a (2D) truss

$\leftarrow$  (AF, SF, BM) =  $3m + r$  for a (2D) frame

=  $r$  for a beam/col.

(c) Total eq<sup>n</sup>s = Eq<sup>n</sup>s for joints + Eq<sup>n</sup> of conditions

$\leftarrow$   $\Sigma F_x, \Sigma F_y$  =  $2j + 0$  for a (2D) truss

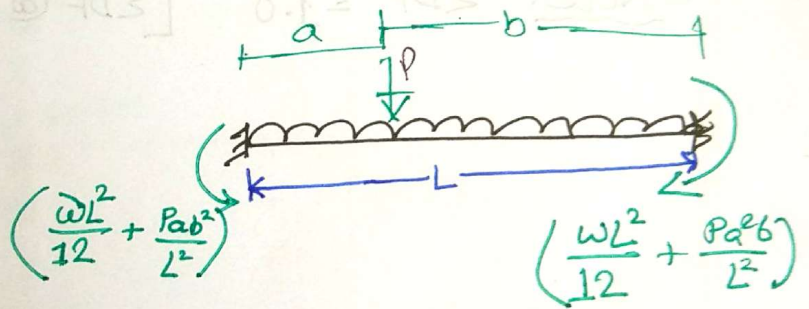
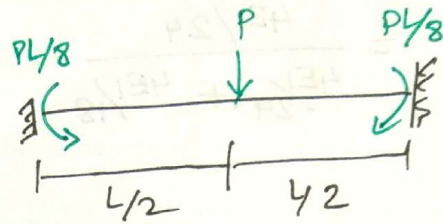
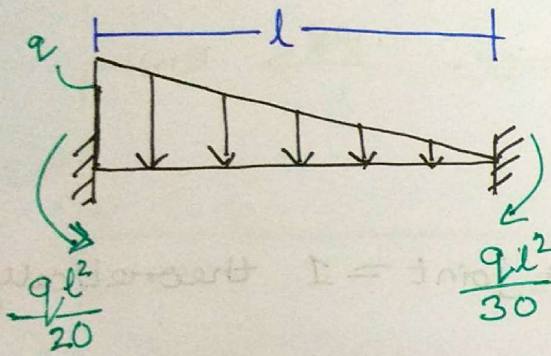
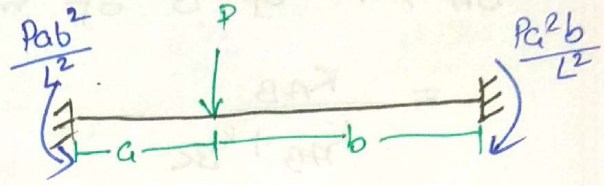
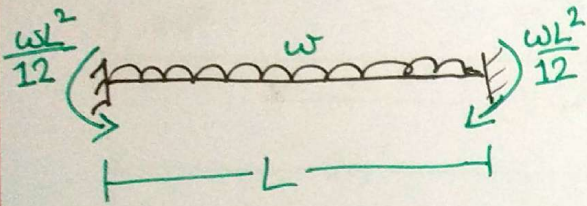
=  $3j + c$  for a (2D) frame

$\leftarrow$   $\Sigma F_x, \Sigma F_y, \Sigma M$  =  $3 + c$  for a beam/col.

wed

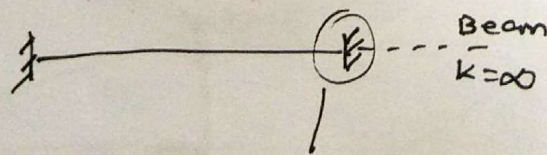
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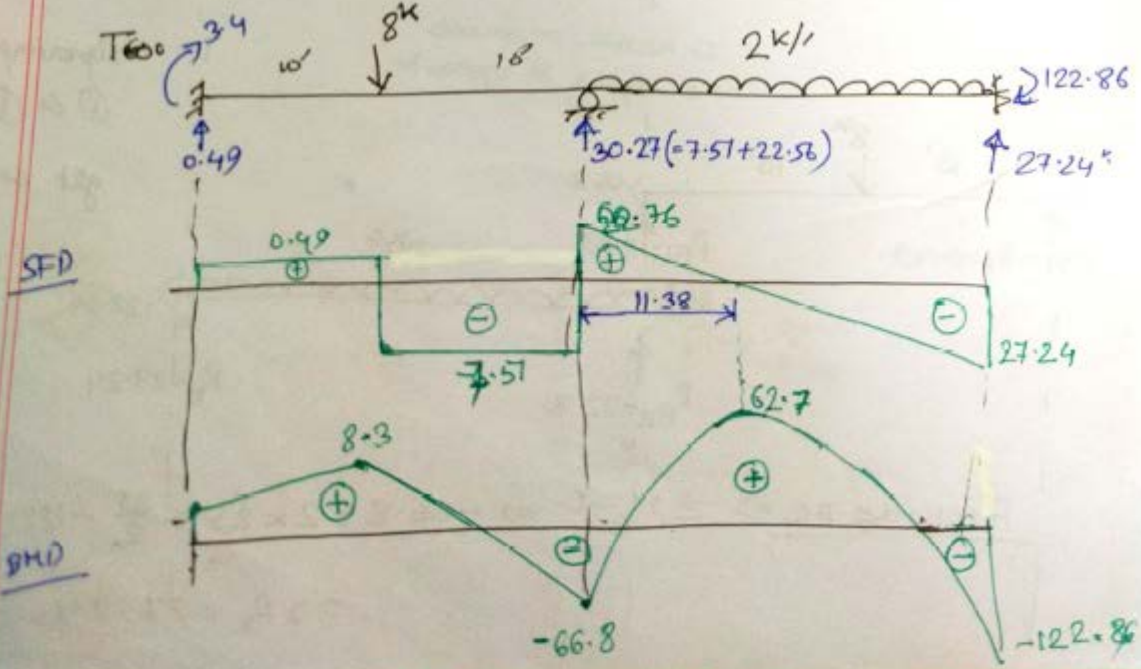


sign

(+) →

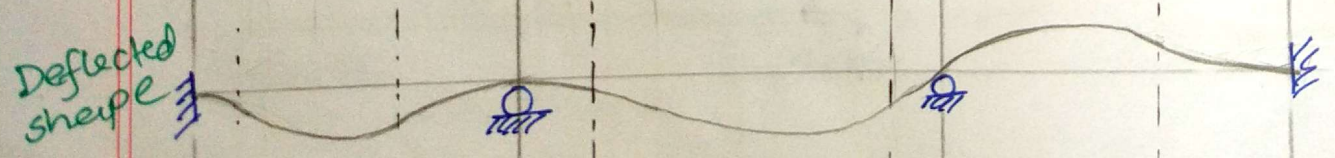
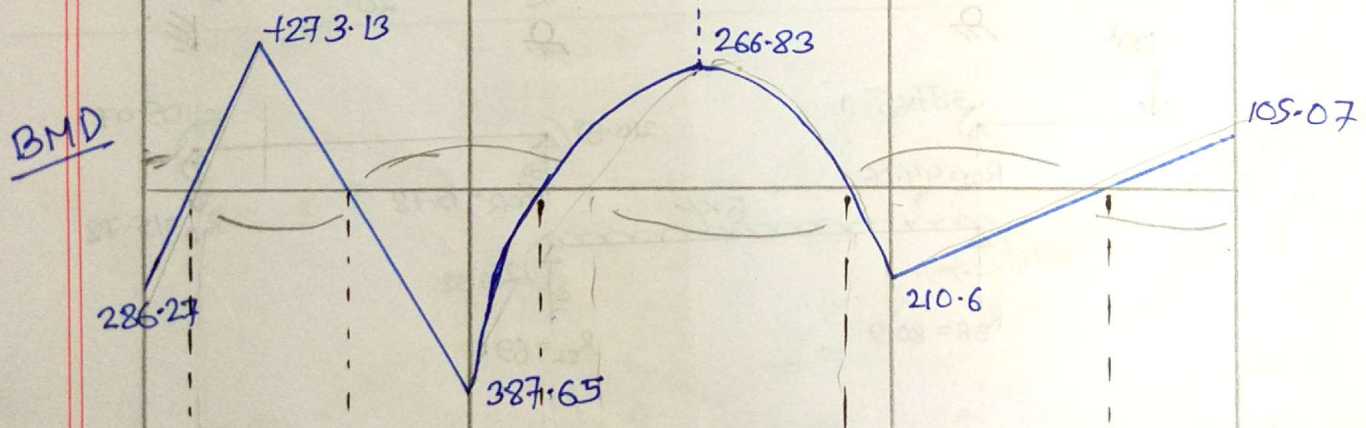
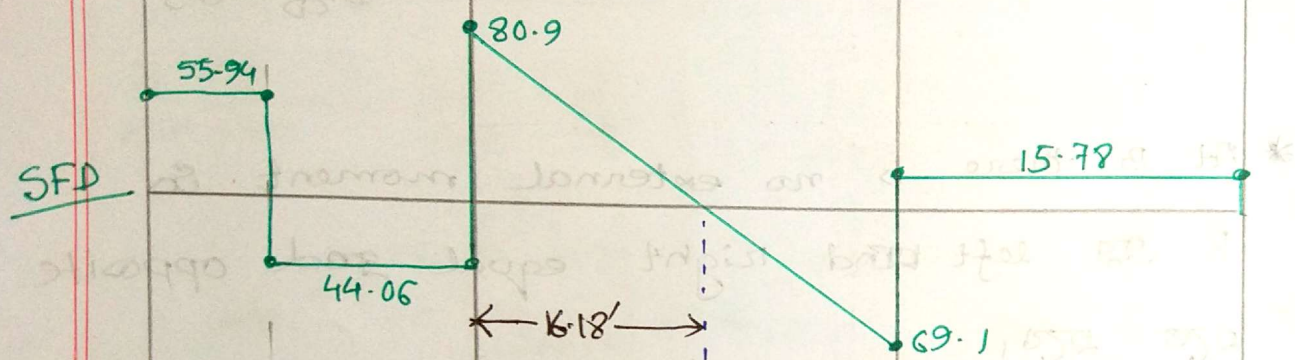
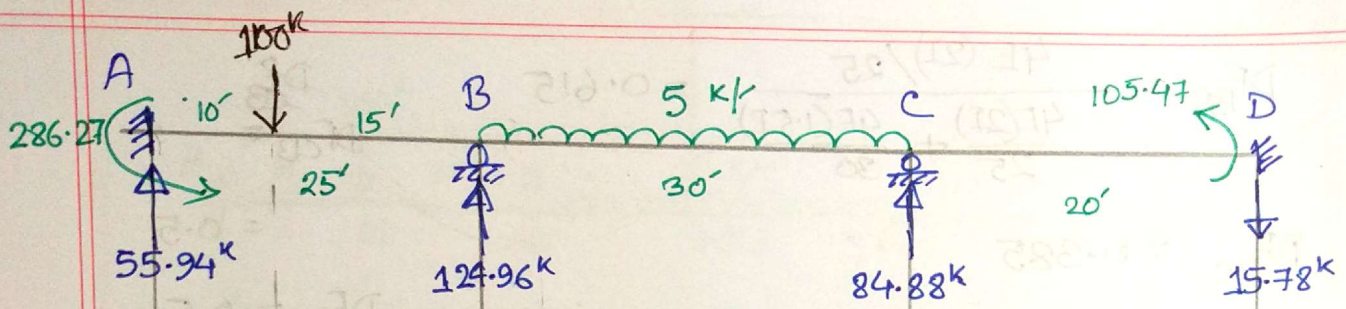


$$k = \frac{\cdot}{+\infty} = 0$$



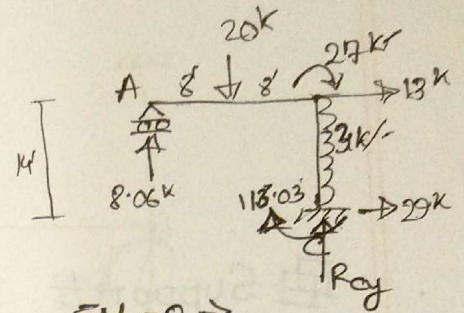
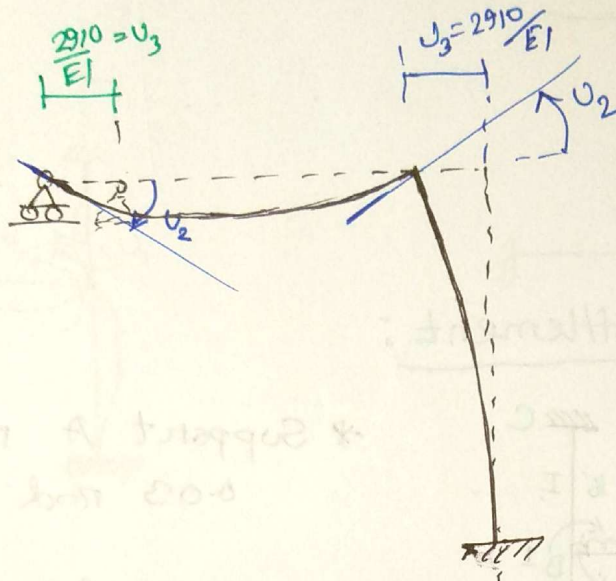
Deflected shape





Lec-15 Prob

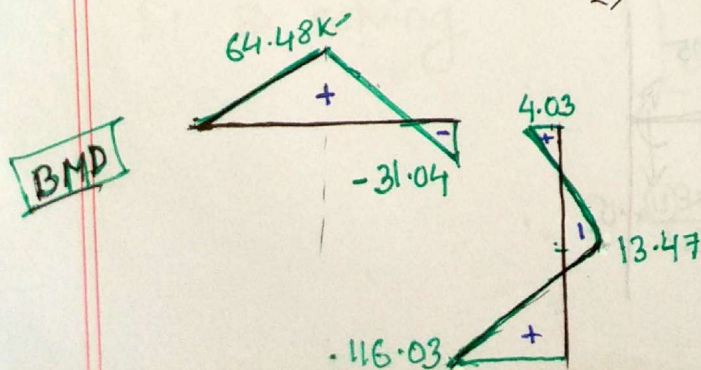
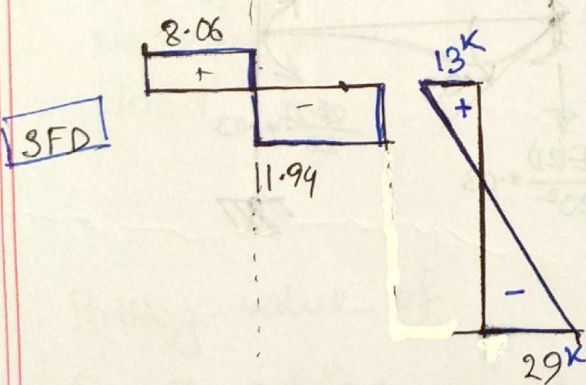
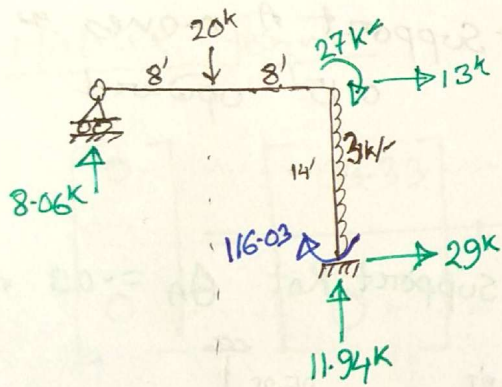
Deflected shape:



$$\sum M_A = 0 \Rightarrow$$

$$+27 + 118.03 + (20 \times 8) - (29 \times 14) + (3 \times 14 \times \frac{14}{2}) - 16R_{cy} = 0$$

$$\Rightarrow R_{cy} = 11.94 \text{ k (1)}$$



**EXAMPLE 3.5**

**Flexural strength of a given member.** A rectangular beam has width 12 in. and effective depth 17.5 in. It is reinforced with four No. 9 (No. 29) bars in one row. If  $f_y = 60,000$  psi and  $f'_c = 4000$  psi, what is the nominal flexural strength, and what is the maximum moment that can be utilized in design, according to the ACI Code?

**SOLUTION.** From Table A.2 of Appendix A, the area of four No. 9 (No. 29) bars is  $4.00 \text{ in}^2$ . Assuming that the beam is underreinforced and using Eq. (3.32),

$$a = \frac{4.00 \times 60}{0.85 \times 4 \times 12} = 5.88 \text{ in.}$$

The depth of the neutral axis is  $c = a/\beta_1 = 5.88/0.85 = 6.92$ , giving

$$\frac{c}{d_t} = \frac{6.92}{17.5} = 0.395$$

which is between 0.429 and 0.375, the values corresponding, respectively, to  $\epsilon_t = 0.004$  and  $\epsilon_t = 0.005$ , as shown in Fig. 3.10. Thus, the beam is, as assumed, underreinforced, and from Eq. (3.31)

$$M_n = 4.00 \times 60 \left( 17.5 - \frac{5.88}{2} \right) = 3490 \text{ in-kips}$$

The fact that the beam is unreinforced could also have been established by calculating  $\rho = 4.00/(12 \times 17.5) = 0.190$ , which just exceeds  $\rho_{0.005}$ , which is calculated using Eq. (3.30d).

$$\rho_{0.005} = 0.85 \times 0.85 \left( \frac{4}{60} \right) \left( \frac{0.003}{0.003 + 0.005} \right) = 0.0181$$

Because the net tensile strain  $\epsilon_t$  is between 0.004 and 0.005,  $\phi$  must be calculated:  $\epsilon_t = \epsilon_u(d - c)/c = 0.003 \times 17.5 - 6.92/6.92 = 0.00458$ . Using linear interpolation from Fig. 3.9,  $\phi = 0.87$ , and the design strength is taken as

$$\phi M_n = 0.87 \times 3490 = 3040 \text{ in-kips}$$

Review  
problem

The ACI Code limits on the reinforcement ratio

$$\rho_{\max} = 0.0206$$
$$\rho_{\min} = \frac{3\sqrt{4000}}{60,000} \geq \frac{200}{60,000} = 0.0033$$

are satisfied for this beam.

---

# Design Problem

**EXAMPLE 3.6 Concrete dimensions and steel area to resist a given moment.** Find the concrete cross section and the steel area required for a simply supported rectangular beam with a span of 15 ft that is to carry a computed dead load of 1.27 kips/ft and a service live load of 2.15 kips/ft, as shown in Fig. 3.12. Material strengths are  $f'_c = 4000$  psi and  $f_y = 60,000$  psi.

**SOLUTION.** Load factors are first applied to the given service loads to obtain the factored load for which the beam is to be designed, and the corresponding moment:

$$w_u = 1.2 \times 1.27 + 1.6 \times 2.15 = 4.96 \text{ kips/ft}$$

$$M_u = \frac{1}{8} \times 4.96 \times 15^2 \times 12 = 1670 \text{ in-kips}$$

The concrete dimensions will depend on the designer's choice of reinforcement ratio. To minimize the concrete section, it is desirable to select the maximum permissible reinforcement ratio. To maintain  $\phi = 0.9$ , the maximum reinforcement ratio corresponding to a net tensile strain of 0.005 will be selected (see Fig. 3.9). Then, from Eq. (3.30d)

$$\rho_{0.005} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.85 \times 0.85 \left( \frac{4}{60} \right) \left( \frac{0.003}{0.003 + 0.005} \right) = 0.0181$$

Using Eq. (3.30c) gives  $\rho_{\max} = 0.0206$ , but would require a lower strength reduction factor. Setting the required flexural strength equal to the design strength from Eq. (3.38), and substituting the selected values for  $\rho$  and material strengths,

# Determination of steel area

**EXAMPLE 3.7** **Determination of steel area.** Using the same concrete dimensions as were used for the second solution of Example 3.6 ( $b = 10$  in.,  $d = 17.5$  in., and  $h = 20$  in.) and the same material strengths, find the steel area required to resist a moment  $M_u$  of 1300 in-kips.

**SOLUTION.** Assume  $a = 4.0$  in. Then

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 2.0)} = 1.55 \text{ in}^2$$

Checking the assumed  $a$  gives

$$a = \frac{1.55 \times 60}{0.85 \times 4 \times 10} = 2.74 \text{ in.}$$

Next assume  $a = 2.6$  in. and recalculate  $A_s$ :

$$A_s = \frac{1300}{0.90 \times 60(17.5 - 1.3)} = 1.49 \text{ in}^2$$

No further iteration is required. Use  $A_s = 1.49 \text{ in}^2$ . Two No. 8 (No. 25) bars,  $A_s = 1.58 \text{ in}^2$ , will be used. A check of the reinforcement ratio shows  $\rho < \rho_{0.005}$  and  $\phi = 0.9$ .

# Doubly Reinforced beam

Nadim Hassoun

1. Calculate  $\rho$ ,  $\rho'$  and  $(\rho - \rho')$

$\rho_{max}$ ,  $\rho_{min}$

2. Calculate  $\bar{\rho}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'$

3. If  $\rho \geq \bar{\rho}_{cy}$  compression steel yields,  $f'_s = f_y$   
" " " does not yield  $f'_s < f_y$   
 $\rho < \bar{\rho}_{cy}$  " " "

4. If comp steel yields, then

a. Check that  $\rho_{max} \geq (\rho - \rho') \geq \rho_{min}$   $\phi = 0.9$   
or check  $\epsilon_t \geq 0.005$

b. Calculate  $a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b}$

c. Calculate  $\phi M_n = \phi \left[ (A_s - A'_s) f_y \left( d - \frac{a}{2} \right) + A'_s f_y (d - d') \right]$

5. If comp. steel does not yield, then

a.  $T = C$   
 $\Rightarrow A_s f_y = 0.85 f'_c b \beta_1 c + A'_s f'_s$   
 $A_s f_y = 0.85 \beta_1 b c f'_c + A'_s E_s \epsilon_u \frac{c-d'}{c}$

$\Rightarrow$  solve this quadratic equation to find  $c$

b. Find  $f'_s = E_s \epsilon_u \frac{c-d'}{c}$

c. Check  $\rho_{max} \geq \left( \rho - \rho' \cdot \frac{f'_s}{f_y} \right) \geq \rho_{min}$   $\phi = 0.9$   
 $\epsilon_t =$

d. Calculate  $a = \frac{A_s f_y - A'_s f'_s}{0.85 f'_c b}$  or  $a = \beta_1 c$  (check)

e. Calculate  $\phi M_n = \phi \left[ (A_s f_y - A'_s f'_s) \left( d - \frac{a}{2} \right) + A'_s f'_s (d - d') \right]$

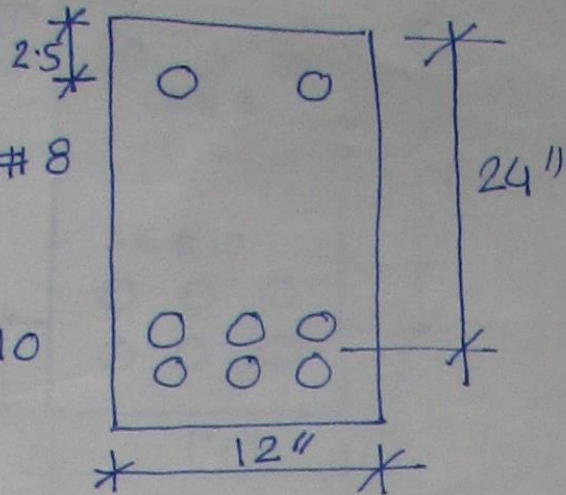
Find  $\varphi M_n$

Nilson

Ex 3.12

2 #8

6-#10



$$f'_c = 5000 \text{ psi}$$

$$f_y = 60,000 \text{ psi}$$

Design moment capacity = ?

$$\phi M_n = ?$$

Sol

$$A_s = 6 \times 1.27 = 7.62 \text{ in}^2$$

$$A_s' = 2 \times 0.79 = 1.58 \text{ in}^2$$

$$\rho = \frac{A_s}{bd} = 0.0265$$

$$\rho' = \frac{A_s'}{bd} = 0.0055$$

$$\rho_{\max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_y}{\epsilon_u + 0.004}$$

$$\rho_{\max} = 0.0243$$

$$\rho_{\min} = \frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}$$

$$\rho_{\min} = 0.00354$$

$$\bar{P}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'$$

$$= 0.85 * 8 \frac{5}{60} \frac{2.5}{24} \frac{.003}{.003 - .00207} + .0055 = 0.0245$$

$\rho > \bar{P}_{cy} \Rightarrow$  Comp bars yield when beam fails.

$$\rho_{max} = .0243 \quad \text{OK}$$

$$\rho - \rho' = 0.021$$

$$\rho_{min} = 0.00354$$

$$a = \frac{A_s - A_s'}{0.85f'_c b} f_y = 7.11$$

$$c = \frac{a}{\beta_1} = 8.88$$

$$\epsilon_t = 0.003 \frac{24 - 8.88}{8.88}$$

$$= 0.0051 \Rightarrow$$

$$\phi = 0.9$$

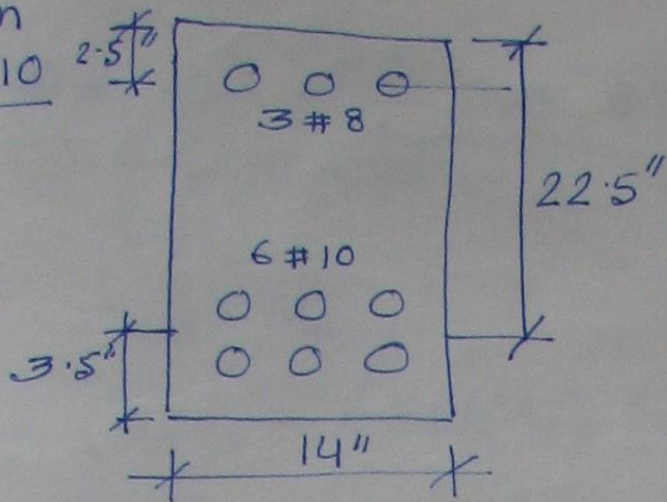
$$M_n = A_s' f_y (d - d') + (A_s - A_s') f_y (d - a/2)$$

$$= 1.58 * 60 (24 - 2.5) + 6.04 * 60 (24 - \frac{7.11}{2}) =$$

$$9447 \text{ k-in}$$

$$\phi M_n = 8503 \text{ k-in}$$

Nadim  
Ex 3.10



Find  $\phi M_n$

$f_c' = 5 \text{ ksi}$      $f_y = 60 \text{ ksi}$

$A_s' = 3 \times 0.79 = 2.37 \text{ in}^2$      $\rho' = 0.007524$

$A_s = 6 \times 1.27 = 7.62 \text{ in}^2$      $\rho = 0.0242$

$\rho - \rho' = 0.01667$

$\rho_{max} = 0.0243$

$\leftarrow (\rho - \rho')$  is ok.

$\rho_{min} = 0.00354$

2.  $\bar{\rho}_{cy} = 0.85 \times 0.8 \times \frac{5}{60} \frac{2.5}{22.5} \frac{0.003}{0.003 - 0.00207} + 0.007524 = 0.02783$

$\rho < \bar{\rho}_{cy} \Rightarrow$  Comp bar does not yield

to be  
more  
correct

$$P < P_{cy} \Rightarrow \text{Comp}$$

to be  
more  
correct

$$3. \quad T = C = C_s + C_c$$

$$\Rightarrow A_s f_y = 0.85 f_c' \beta_1 c \cdot b + A_s' E_s \epsilon_u \frac{c-d'}{c} - 0.85 f_c' A_s'$$

$$\Rightarrow 7.62 \times 60 = 0.85 \times 5 \times 14 \times c + 2.37 \times 29 \times 10^3 \times 0.003 \frac{c-2.5}{c} - 0.85 \times 5 \times 2.37$$

$$\Rightarrow 457.2 = 47.6c + 206.19 \frac{c-2.5}{c} - 10.07$$

$$\Rightarrow 47.6c^2 - 261.08c - 515.475 = 0$$

$$c = \frac{261.08 \pm \sqrt{261.08^2 + 4 \times 47.6 \times 515.475}}{2 \times 47.6}$$

$$= 7.026 \text{ in}$$

$$a = \beta_1 c = 5.62$$

$$f_s' = E_s \epsilon_u \frac{c-d'}{c} \\ = 56.04 \text{ ksi}$$

$$\rho - \rho' \frac{f_s'}{f_y} = 0.0243 - 0.007524 * \frac{56.04}{60}$$

$$= 0.01724 < \rho_{max} \quad \underline{OK.}$$

5. Find  $M_n$

$$M_n = (A_s' f_s' - 0.85 f_c' A_s) (d - d') + (A_s f_y - A_s' f_s') (d - \frac{a}{2})$$

$$= (2.37 * 56 - 0.85 * 5 * 2.37) (22.5 - 2.5)$$

$$+ (7.62 * 60 - 2.37 * 56.04 + 0.85 * 5 * 2.37) (22.5 - \frac{5.62}{2})$$

$$= 122.7 * 20.0 + 334.5 * 19.69$$

$$= 9040.3 \text{ k.in.}$$

Comp. in concrete =  $0.85 f_c' a b = C_c = 334.5 \text{ k}$

Comp. in steel =  $A_s' f_s' - \text{force in displaced concrete}$

$$= C_s = A_s' (f_s' - 0.85 f_c') = 122.7 \text{ k.}$$

Tension in steel =  $A_s f_y = 457.2 \text{ k}$

$$T = C \quad \underline{OK.}$$

6.

$$\frac{\epsilon_t}{d_t - c} = \frac{\epsilon_y}{c}$$

$$\epsilon_t = \frac{0.003}{67.026} (22.5 + 2.5)$$

$$= 0.00682 > 0.005 \quad \phi = 0.9$$

7.

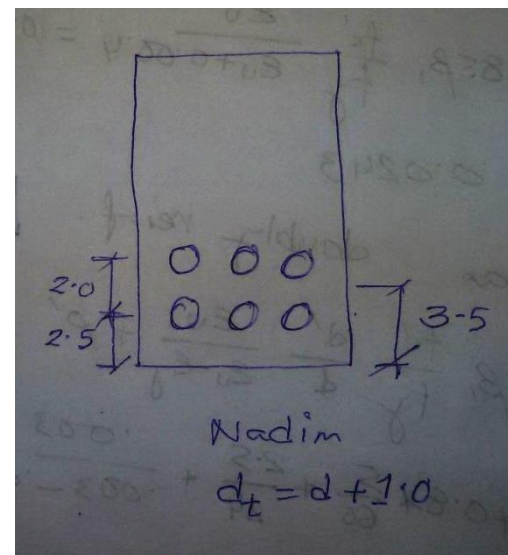
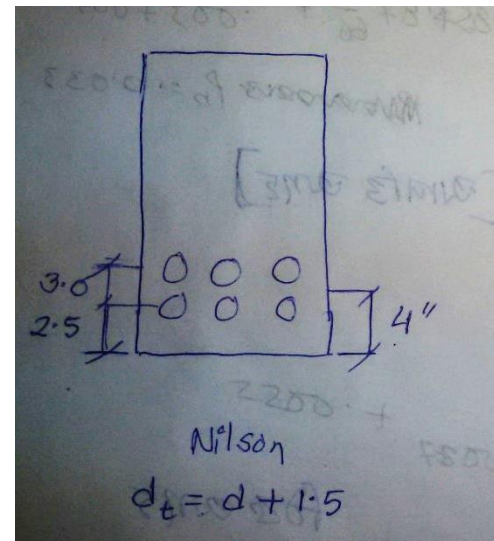
$$\phi M_n = 8136 \text{ k.in}$$

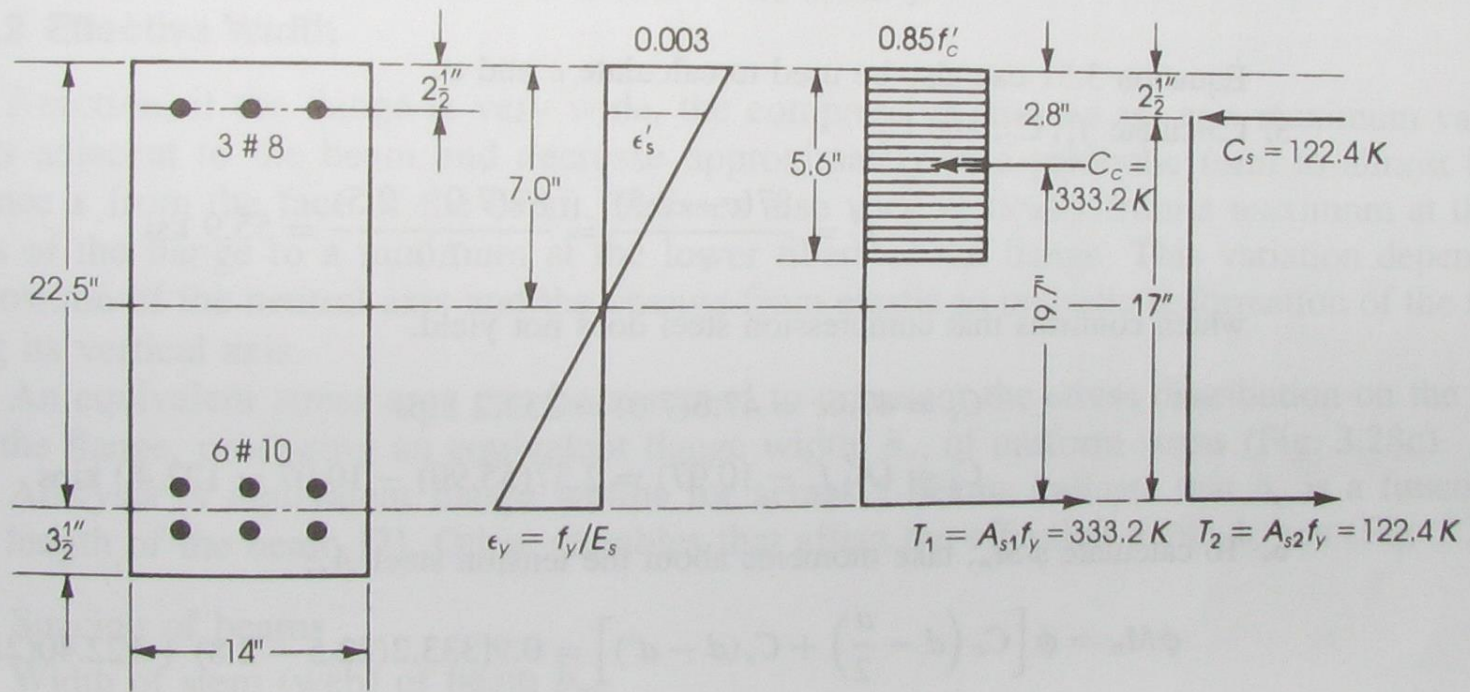
$$d_t = d + 1.0 \text{ if}$$

$$d = h - 3.5$$

$$d_t = d + 1.5 \text{ if}$$

$$d = h - 4.0$$





**Figure 3.27** Example 3.10 analysis solution.

# Design of Doubly Reinforced Beam

Design problem from Nadim

Ex 4.5 A beam section is limited to a width  $b=10$  in and total depth of  $h=22$  in and has to resist a factored moment of  $226.5$  k-ft. Calculate the required reinforcement. Given  $f_c' = 3$  ksi and  $f_y = 50$  ksi.

Sol  $\rho_{0.005} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} = 0.01625$

$$A_s = 0.01625 \times 10 \times 18.5 = 3 \text{ in}^2$$

$$d = 22 - 3.5 = 18.5 \text{ two layer}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = 5.88 \text{ in.}$$

$$M_u = 226.5 \times 12 = 2718 \text{ k-in}$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) = 2333.8 \text{ k-in.}$$

$$\phi M_n = 0.9 \times 2333.8 = 2100.4 \text{ k-in.}$$

Doubly Reinforced beam reqd.

$$\phi M_{n1} = 2100.4$$

$$\phi M_{n2} = 2718 - 2100.4 = 617.6 \text{ k''}$$

Assuming comp steel yield

$$A_{s2} = \frac{617.6}{\phi f_y (d - d')} = \frac{617.6}{0.9 \times 50 (18.5 - 2.5)} = 0.86 \text{ in}^2$$

$$\text{Total tension steel} = A_s = 3.0 + 0.86 = 3.86 \text{ in}^2$$

$$\text{Comp. steel } A_s' = 0.86 \text{ in}^2$$

$$\rho' = 0.00465$$

$$\rho^* = 0.02086$$

$$\rho - \rho' = 0.01621$$

Use actual area  
provided

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'$$

$$= 0.85 + 0.85 \frac{3}{50} \frac{2.5}{18.5} \frac{.003}{.003 - 0.001724} + 0.00465$$

$$= 0.01842$$

$\rho > \bar{\rho}_{cy}$  comp steel yields.

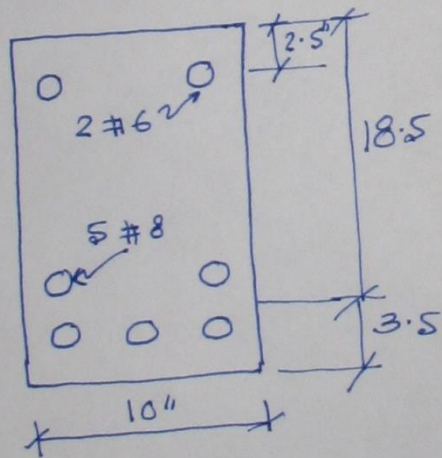
$$a = \frac{A_s - A_s'}{0.85f_c' b} f_y = \frac{3.86 - 0.86}{0.85 \times 3 \times 10} \times 50 = 5.88 \text{ in.}$$

$$c = \frac{a}{\beta_1} = 6.92 \text{ in.}$$

$$\epsilon_t = \epsilon_u \frac{d_t - c}{c} = 0.003 \frac{19.5 - 6.92}{6.92} = 0.00545$$

outer layer

$$\phi = 0.9 \quad \text{OK.}$$



Nilson  
Ex 3.13

Design

$$LL = 2.47 \text{ k/ft}$$

$$DL = 1.05 \text{ k/ft}$$

Simply supported span = 18'

Beam section  $\Rightarrow 10'' \times 20''$

$$f_c' = 4000 \text{ psi} \quad f_y = 60,000 \text{ psi}$$

find reinforcement.

Sol

$$W_u = 1.2 \times 1.05 + 1.6 \times 2.47 = 5.212 \text{ k/ft}$$

$$M_u = \frac{1}{8} W_u l^2 = \frac{1}{8} \times 5.212 \times 18^2 = 211.09 \text{ k'} = 2533 \text{ k''}$$

$$d = 20 - 4 = 16'' \text{ (two layer)}$$

$$d' = 2.5'' \text{ (if needed)}$$

First check if possible to design singly reinforced.

$$\epsilon_t = 0.005$$

$$\rho_{0.005} = 0.0181$$

$$A_s = \rho b d = 0.0181 \times 10 \times 16 = 2.89 \text{ in}^2$$

$$\frac{a}{d} = 6''$$

$$A_s = \rho b d = 0.0181 \times 10 \times 10 =$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = 5.1 \text{ in}$$

$$c = \frac{a}{\beta_1} = 6''$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) = 2.89 \times 60 \left( 16 - \frac{5.1}{2} \right) = 2332 \text{ k''}$$

$$\phi M_n = 0.9 \times 2332 = 2099 \text{ k''} < M_u$$

Doubly Reinf reqd.

$$\text{Remaining moment } \phi M_n = 2533 - 2099 = 434 \text{ k''}$$

$$M_n = 482.4 \text{ k''}$$

Assuming comp bars yield

$$A_{s2} = \frac{434}{0.9 \times 60 \times (16 - 2.5)} = 0.6 \text{ in}^2$$

$$\text{Total tension steel} = 2.89 + 0.6 = 3.49 \text{ in}^2$$

$$\text{Comp steel} = 0.6 \text{ in}^2$$

4 # 9  
4 in<sup>2</sup>

2 # 6  
0.88 in<sup>2</sup>

$$\rho = \frac{4}{10 \times 16} = 0.025$$

$$\rho' = \frac{0.88}{10 \times 16} = 0.0055$$

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'$$

$$= 0.85 * 0.85 \frac{4}{60} \frac{2.5}{16} \frac{.003}{.003 - .00207} + 0.0055$$

$$= 0.0298$$

$\rho < \bar{\rho}_{cy} \Rightarrow$  Comp steel does not yield

$$A_s f_y = 0.85 f_c' b \beta_1 c + A_s' f_s' - A_s' * 0.85 f_c'$$

$$\Rightarrow 4 * 60 = 0.85 * 4 * 10 * .85 c + A_s' * \left[ E_s \epsilon_u \frac{c-d'}{c} - 0.85 f_c' \right]$$

$$\Rightarrow 240c = 28.9c^2 - 2.992c + (76.56c - 191.4)$$

$$\Rightarrow 28.9c^2 - 166.43c - 191.4 = 0$$

$$c = 6.74 \text{ in.}$$

$$a = 5.73 \text{ in.}$$

$$f_s' = E_s \epsilon_u \frac{c-d'}{c} = 54.7 \text{ ksi.}$$

$$C_c = 0.85 f_c' \cdot b \cdot \beta_1 \cdot c = 194.8 \text{ k}$$

$$C_s = A_s' [f_s' - 0.85 f_c'] = 45.14 \text{ k}$$

checked.

$$T = A_s f_y = 4 \times 60 = 240 \text{ k}$$

$$M_n = C_c \left( d - \frac{a}{2} \right) + C_s (d - d')$$

$$= 194.8 \left( 16 - \frac{5.73}{2} \right) + 45.14 (16 - 2.5)$$

$$= 2558.5 + 609.44 = 3167.9 \text{ k''}$$

$$\epsilon_t = \epsilon_u \frac{d_t - c}{c} = 0.003 \frac{17.5 - 6.74}{6.74}$$
$$= 0.00479$$

$$\phi = 0.483 + 833 \epsilon_t = 0.882$$

$$\phi M_n = 2793.9 \text{ k} > M_u = 2533 \text{ k} \quad \underline{\text{OK}}$$

T-beam

- RC beam and slab are monolithically cast
- Beam stirrups and bent bars extend into the slab
- A part of slab act along with beam top to take longitudinal compression
- Slab forms the beam flange
- Part of beam below slab is called web/stem

# Effective flange width

The criteria for effective width given in ACI Code 8.12 are as follows:

1. For symmetric T beams, the effective width  $b$  shall not exceed one-fourth the span length of the beam. The overhanging slab width on either side of the beam web shall not exceed 8 times the thickness of the slab or go beyond one-half the clear distance to the next beam.
2. For beams having a slab on one side only, the effective overhanging slab width shall not exceed one-twelfth the span length of the beam, 6 times the slab thickness, or one-half the clear distance to the next beam.
3. For isolated beams in which the flange is used only for the purpose of providing additional compressive area, the flange thickness shall not be less than one-half the width of the web, and the total flange width shall not be more than 4 times the web width.

## 1. Symmetrical T beams

$$b < 16h_f + b_w$$

$$b < \text{Span}/4$$

$$b < c/c \text{ beam spacing}$$

## 2. Beam having slab on one side

$$b < \text{span}/12 + b_w$$

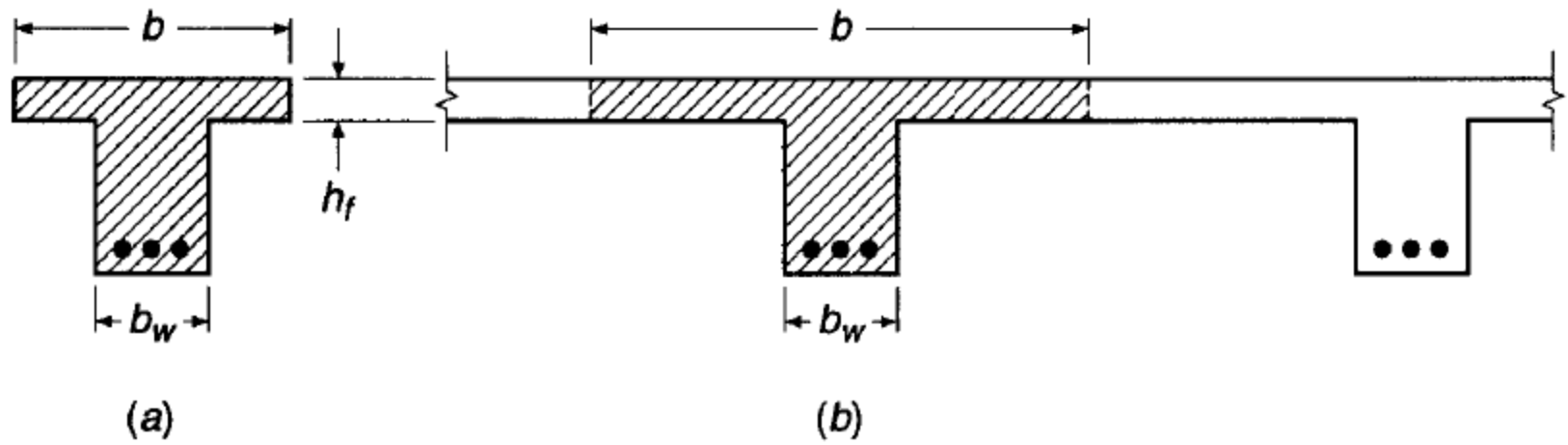
$$b < 6h_f + b_w$$

$$b < \text{Half the clear span} + b_w$$

## 3. Isolated T beam

$$h_f > b_w/2$$

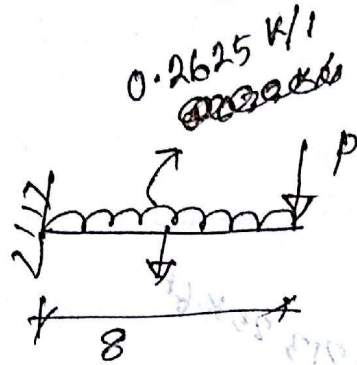
$$b < 4b_w$$



**FIGURE 3.17**  
Effective flange width of  
T beams.

# 13-14

7(b)



$$M = PL + \frac{WL^2}{2}$$

$$\Rightarrow 88.24 = 8P + \frac{0.2625 \times 8^2}{2}$$

$$\therefore \boxed{P = 9.98 \text{ kip}}$$

$0.45 f_c > f_r$   
 $\therefore$  Cracked

$$M = \left[ \frac{1}{2} f_c k j b d^2 \right]$$

$$\approx 88.24 \text{ k-ft}$$

$$M = \left[ A_s f_s j d \right]$$

$$= 96.12 \text{ k-ft}$$

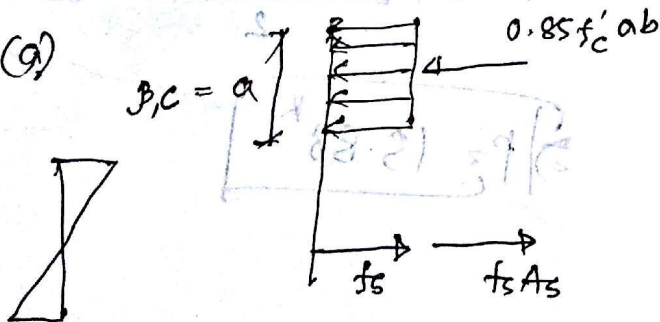
$$k = \sqrt{(pn)^2 + 2pn} - pn$$

$$\approx 0.34 \quad j = 0.89$$

$$p = \frac{A_s}{bd} = 0.011$$

$$SF FIA = \frac{8 \times 20000 \times 8^2}{2} + 8 \times 9.98 \times 8$$

8(c)



$$C = T$$

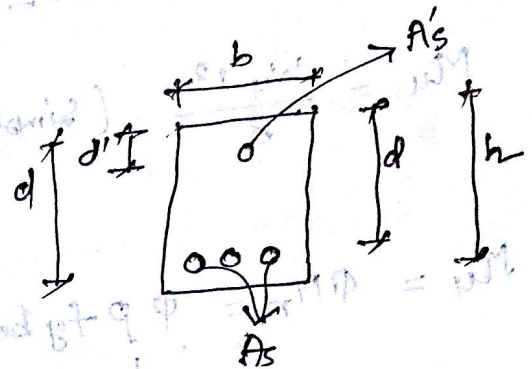
$$\Rightarrow 0.85 f'_c ab = f_s A_s = f_s p b d$$

$$\therefore p_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{f_y}{E_c + E_s}$$

$$C = \frac{E_c}{E_c + E_s} d$$

Overreinforced / DRB  
 (limited in construction for architectural reasons)

Analysis  
 (Flexural strength)



\*  $P > P_{max} \Rightarrow$  DRB

$$\frac{A_s}{bd} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{3}{7}$$

[\*\*  $P < P_{man} \Rightarrow$  SRB  $\Rightarrow$   $f_y < f'_c$ ]  
 Compression steel  
 yield [ ]

$$* \frac{1}{\phi} Q = \frac{(A_s - A_s') f_y}{0.85 f'_c b}$$

$$* M_n = M_{n1} + M_{n2}$$

$$* M_{n1} = A_s' f_y (d - d')$$

$$\hookrightarrow f'_s (f'_s < f_y)$$

$$\hookrightarrow f_y (f'_s > f_y) \text{ [compression steel yields]}$$

$$\hookrightarrow \epsilon_s \times E_s \times \frac{c - d'}{c} \rightarrow \frac{(A_s - A_s') f_y}{0.85 f'_c b}$$

$$* M_{n2} = (A_s - A_s') f_y (d - \frac{a}{2}) \text{ [} f_s > f_y \text{]}$$

$$\hookrightarrow f_s = \epsilon_s \times E_s$$

$$\hookrightarrow \epsilon_s \times \frac{d - c}{c}$$

$$* S_n = \phi M_n$$

$$\hookrightarrow 0.9 \text{ or } 0.65 + (\epsilon_t - 0.002) \frac{250}{3}$$

$$\epsilon_s \geq 0.005$$

$$\epsilon_s < 0.005$$

# Shear & Diagonal Tension in Beams

→ (Shear force at factored loads)

\*  $V_u \leq \phi V_n$  (0.75)

\*  $V_u = \phi (V_c + V_s) = \phi V_c + \phi V_s$  → (web steel cont<sup>n</sup> to shear strength)

\*  $V_c = 2 \lambda \sqrt{f'_c} b_w d$

↓ (Concrete cont<sup>n</sup> to shear strength)      ↓ (3000-5000 psi)      ↓ (beam/web width)

$\lambda = 1$  (normal weight concrete)

\*  $A_{v, min} = \frac{50 b_w s}{f_y}$

↳ (yield strength of steel)

\*  $s = \frac{\phi A_v f_y d}{V_u - \phi V_c}$

\*  $s_{min} = 4"$



\*  $s_{max} = 24" = \frac{d}{2} = \frac{A_v f_y}{50 b_w} [V_s \leq 4 \sqrt{f'_c} b_w d]$

$= 12" = \frac{d}{4} = \frac{A_v f_y}{100 b_w} [V_s > 4 \sqrt{f'_c} b_w d]$

Vertical stirrups area  
#3 or #4 rod area x 2 legs

OR  
check  $\phi V_c$  to  $\frac{2}{3}$  part

\*  $V_u < \frac{\phi V_c}{2} \Rightarrow$  No web reinf.

\*  $V_u > \phi V_c \Rightarrow$  web reinf.

\*  $\frac{\phi V_c}{2} < V_u < \phi V_c \Rightarrow$  min<sup>m</sup> web reinf.

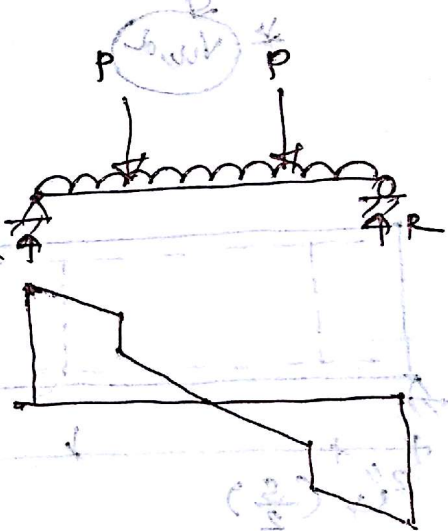
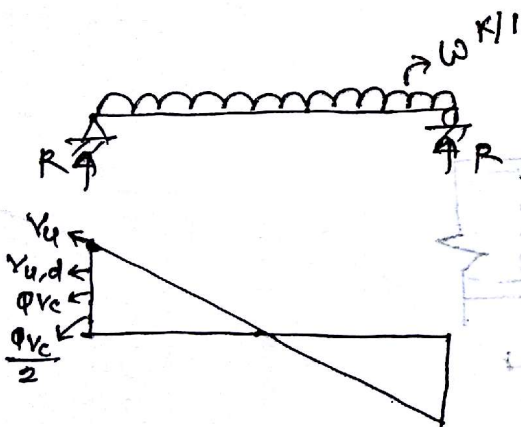
## # Beam without web reinforcement

$$* V_u = \frac{\phi V_c}{2} = \frac{\phi}{2} (2\lambda\sqrt{f'_c} b_w d) \Rightarrow b_w d$$

$$[b_w d \geq \frac{V_u}{\phi}] \quad \frac{b_w d}{b_w d} = \frac{b}{s} = d = 2b \quad (b = 18")$$

$$* V_u = \phi V_c = \phi (2\lambda\sqrt{f'_c} b_w d) \Rightarrow \text{if minimum steel is given}$$

## # Design of web reinforcement



\* SFD

\*  $V_u$  (from reaction)

\*  $V_{u,d}$  effective depth  $\frac{s}{2}$

$$* \phi V_c = \phi 2\lambda\sqrt{f'_c} b_w d$$

$$* \frac{\phi V_c}{2}$$

$$** V = (R-P) - (w \times x)$$

$$\frac{(R-P) - (w \times x)}{s}$$

\*  $S_{min} = 4''$

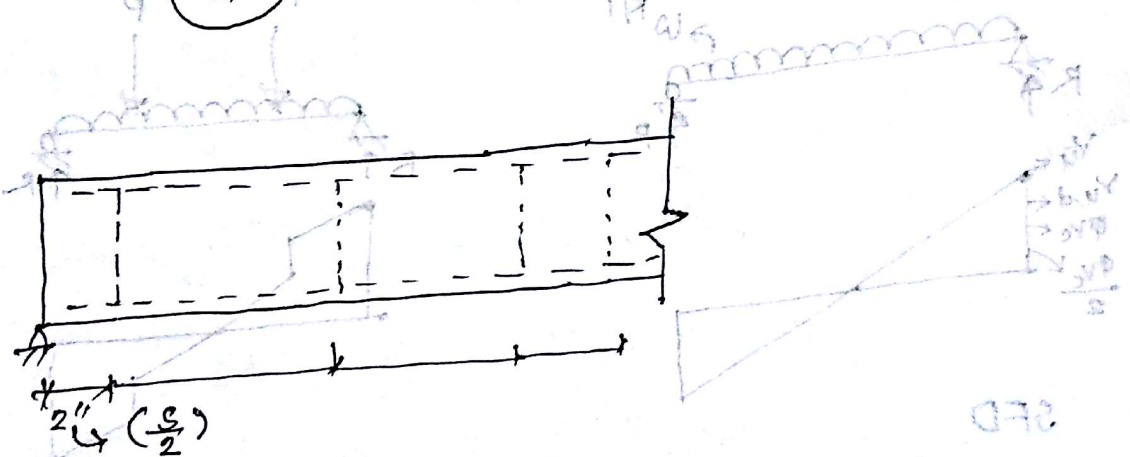
\*  $\phi V_s = V_u - \phi V_c$

\*  $S_{max} = 24'' = \frac{d}{2} = \frac{A_v f_y}{50 b_w}$

$[\phi V_s \leq \phi 4 \sqrt{f_c} b_w d]$

$12'' = \frac{d}{4} = \frac{A_v f_y}{100 b_w}$   $[\phi V_s > \phi 4 \sqrt{f_c} b_w d]$

\*  $S = \frac{\phi A_v f_y d}{V_u - \phi V_c} \Rightarrow$  [lower rounding]

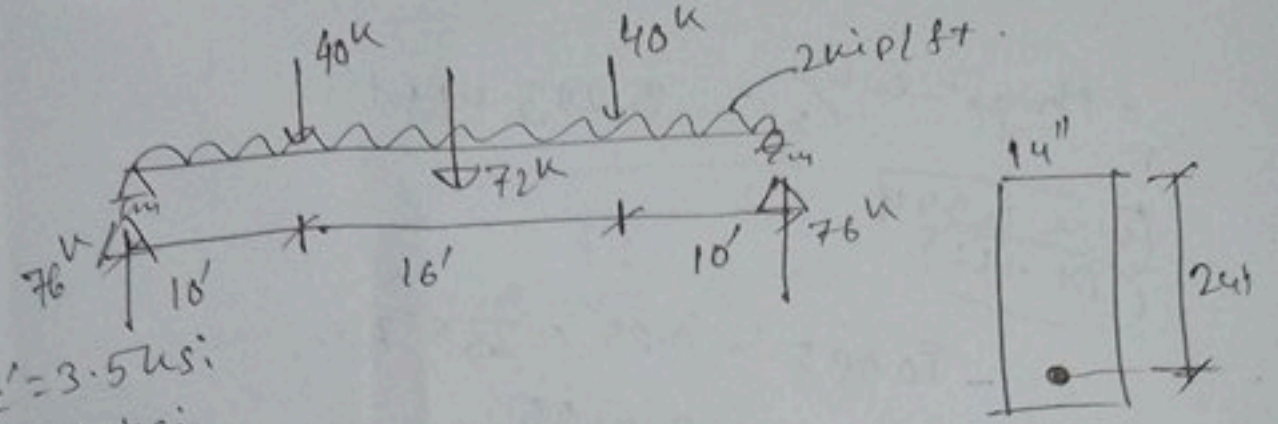


\*  $\frac{\text{distance for } \frac{\phi V_c}{2} - 2''}{3} = \frac{L_1}{S} \Rightarrow \frac{L_1}{S}$

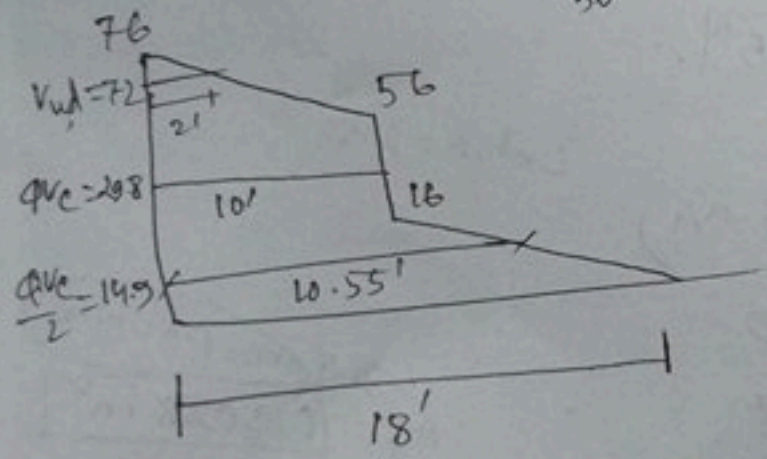
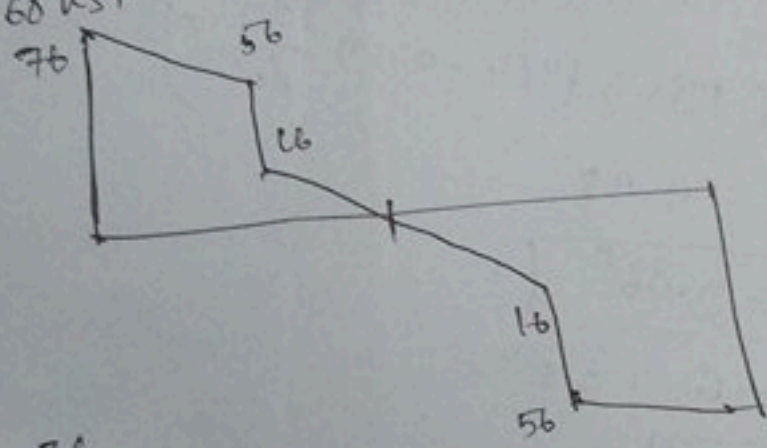
\*  $\frac{L - (x_1 + 2)}{2} = L_2 \Rightarrow \frac{L_2}{S} \Rightarrow \frac{\phi A_v f_y d}{V_u - \phi V_c} \Rightarrow (R-P) - (w \times x) \Rightarrow (x_1 + 2)$

\*  $\frac{L - (x_1 + x_2 + 2)}{S} \Rightarrow \frac{\phi A_v f_y d}{V_u - \phi V_c} \Rightarrow (R-P) - (w \times x) \Rightarrow (x_1 + x_2 + 2)$

② (a)



$f_c' = 3.5 \text{ ksi}$   
 $f_y = 60 \text{ ksi}$



$V_u = 76 \text{ k}$   
 $d = 24'' = 2'$

$V_{ud} = 76 - (2 \times 2) = 72 \text{ k}$

$\phi V_c = 0.75 \times 2 \sqrt{f_c'} b_w d = \underline{39.8 \text{ kips}}$   $\times 0.75$   
 must be psi  $\approx 29.8 \text{ kips}$  from figure distance = 10'

$\frac{\phi V_c}{2} = 14.9$ , distance  $\rightarrow 14.9 = 76 - 40 - (2 \times x)$   
 $\Rightarrow x = 10.55'$

$S_{min} = 4''$

$\phi V_s = V_{ud} - \phi V_c = 72 - 29.8 = 42.2 \text{ kips}$

$$4\phi\sqrt{f_c'}bw d = 59.6 \text{ kip} > \phi V_s$$

in Psi

$$\therefore S_{max} = \frac{29}{2} = 12''$$

$$= 24''$$

$$= \frac{A_v f_y t}{0.75\sqrt{f_c'}bw} \leq \boxed{\frac{A_v f_y t}{50bw}} = \frac{0.22 \times 60000}{50 \times 14}$$

$$= 18.86''$$

$$S_{max} = 12''$$

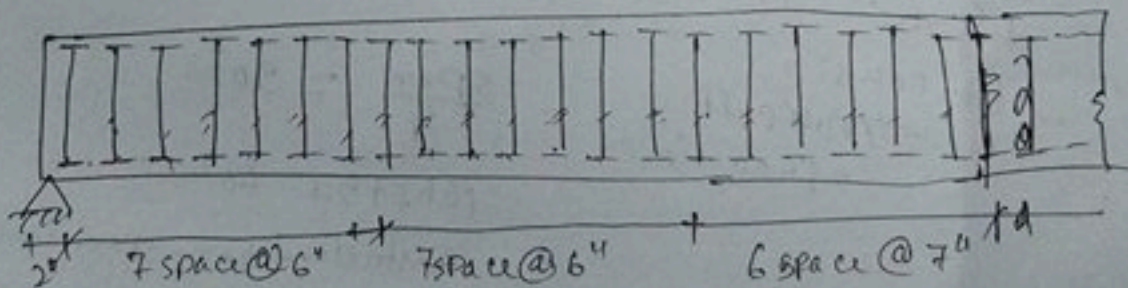
$$S = \frac{\phi A_v f_y t d}{V_u - \phi V_c} = 5.63'' \approx 6''$$

$$\therefore \boxed{S = 6''}$$

The following spacing pattern is satisfactory:

- 1 space at 2 in = 2''
- 7 space at 6 in = 42''
- 7 space at 6 in = 42''
- 6 space at 7 in = 42''

$$= 128'' \approx \boxed{10.66'}$$



\* ~~TABLE~~ 13.1  
Minimum thickness  $h$  of  
nonprestressed one-way slabs

---

Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

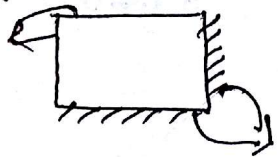
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# Two-way slab

(Design)

\*  $\frac{\text{Long direction}}{\text{Short direction}} < 2 \Rightarrow$  two way slab

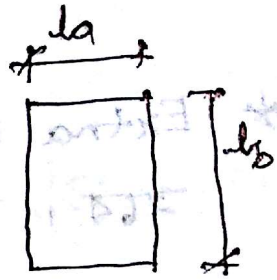
discontinuous



Hatched means continuous

\*  $M_a = C_a W l_a^2$  → short direction clear span  
 tabulated moment coefficients

\*  $M_b = C_b W l_b^2$  → long direction clear span  
 uniform load, psf



\*\* Continuous edge has negative moment

\*\* (-)ve M at discontinuous edge =  $\frac{1}{3} \times$  positive moment

\*\* short direction M > long direction M

\*\*  $h = \frac{\text{perimeter}}{180}$

min<sup>m</sup> slab thickness

\*\*  $s < 2h$

## ✓ Reinforcements –usual sizes

- ✓ Slab- No 3, 4, 5 (10mm, 12mm, 16mm)
- ✓ Beam- No 5,6, 7, 8 (16 20 ~~22~~ 25mm)
- ✓ Stirrup/tie- No, 3 4 (10 12mm)
- ✓ Column –No 5, 6 7 8 9 10 11 14 18 (16 20 ~~22~~ 25 28 32 ....)
- ✓ Mat- No 4,5,6,8 (12 16 20 25 mm)
- ✓ Smaller sizes preferred as long as there is no congestion

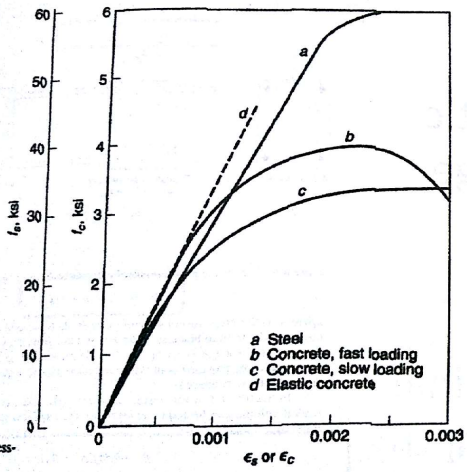


FIGURE 1.6  
Concrete and steel stress-strain curves.

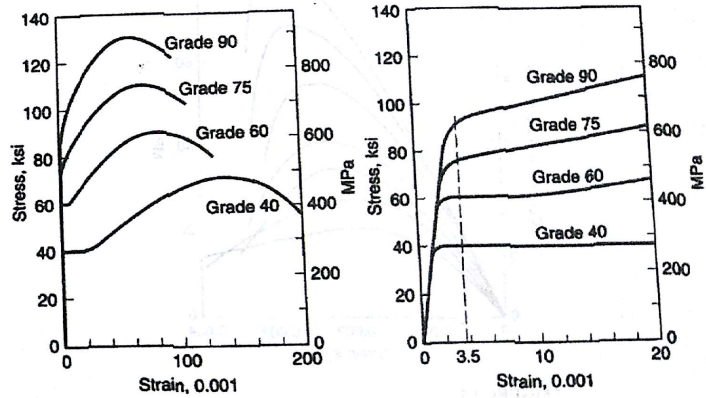
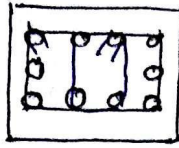


FIGURE 2.15  
Typical stress-strain curves for reinforcing bars.

# 3 types of column:

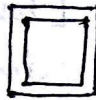
① Tied column



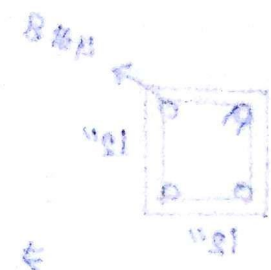
② Spiral column



③ Composite "

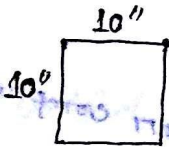


Elastic Range:



# Main reinforcement:

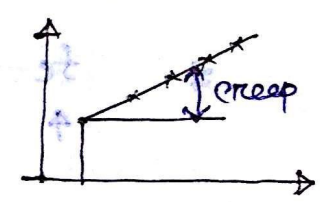
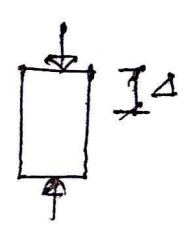
1 to 8%



$\Rightarrow A = 100 \text{ in}^2$

Min<sup>m</sup> 1 in<sup>2</sup>

Max 8 in<sup>2</sup>

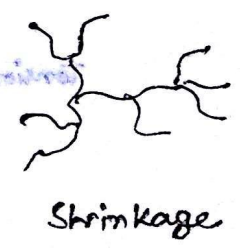


\* To reduce bending, creep & shrinkage

We give min<sup>m</sup> reinf.

Normally 2-3%

Prefarably not exceed 4-5%

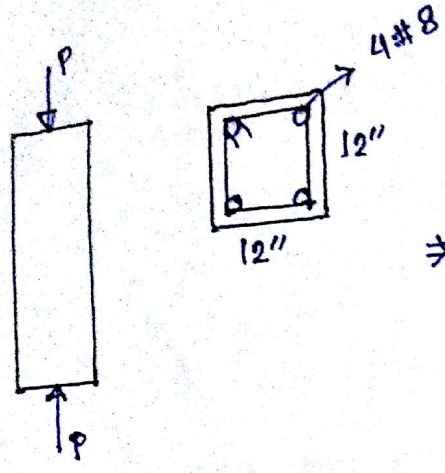


\* #5 and nine rod are used.

Min<sup>m</sup> 4 longitudinal

(2)

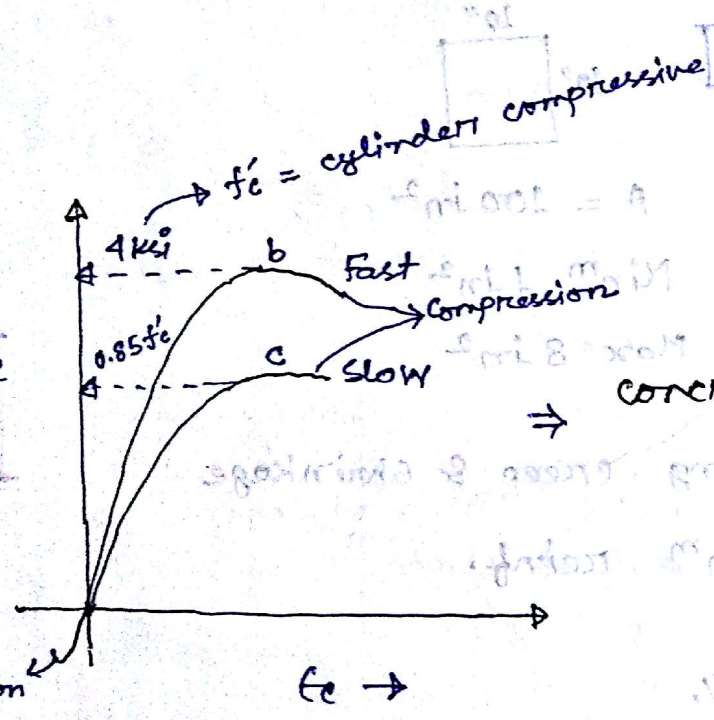
Elastic Range:



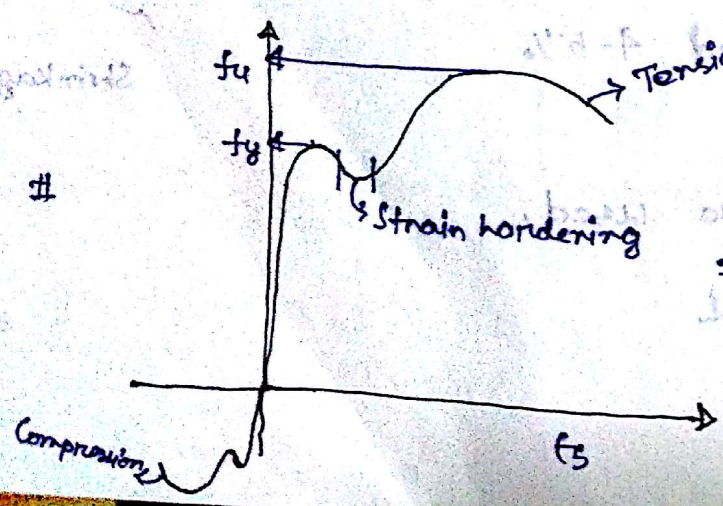
$$\Rightarrow \epsilon = \frac{\Delta}{L} = \epsilon_c = \epsilon_s$$

[because perfect bonding in concrete & steel]

$$P = P_c + P_s$$



Concrete tension = 1/10 of compression value



Tension & compression of steel are same shape graph

$$P_n = 0.85 f'_c A_c + A_{st} f_y \Rightarrow \text{slow loading}$$

$$\boxed{** P_n = 0.85 f'_c (A_g - A_{st}) + A_{st} f_y} \quad [\epsilon = 0.003]$$

Nominal Strength

slow loading & convert  
ବରମାତ୍ର

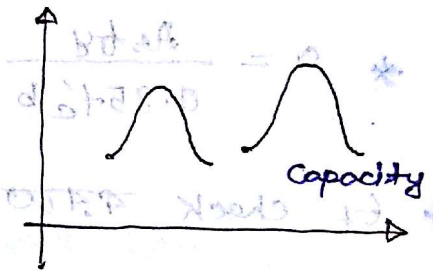
Net concrete section



\* Nominal  $\rightarrow$  Calculated strength

\*\* Capacity  $>$  Requirement (in design)  
because of uncertainty

\*  $\phi P_n \rightarrow$  Design Strength



$\Rightarrow$   $\phi$  ବରମାତ୍ର column  $\rightarrow$  ବରମ ହେବ because column  
fail ବରମେ building ଢେଙ୍କେ ଯାଏ ।

$$\boxed{* \phi \text{ Design strength} = \phi P_n}$$

\*  $\phi = 0.65$  for tied column

= 0.75 for spirally reinf. column

\*  $\alpha =$  for accidental eccentricity

= 0.8 for tied

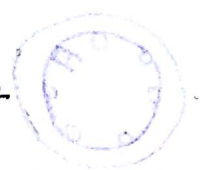
= 0.85 for spiral

(3)

# Purpose of ties :

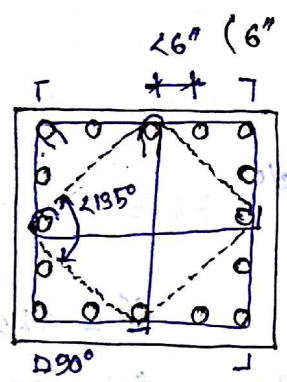
$$* P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$I = 4I + Ad^2$$



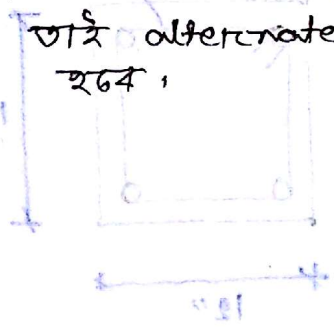
\* Tie prevent Buckling वक्र रूपा,

# ACI provisions for ties :



<math>6''</math> (6" clean spacing रूपा छयकि रूमेन जेजव rod व tie फिरोत रूवे) otherwise buckling रूवे

⇒ rod direct bending व जाओत वरि alternate tie फिरोत रूवे,



(1) All bars

# 3 tie for <math>\leq</math> # 10

# 4 tie for > # 10 & bundled bars

(2) spacing <math><</math> 16 diam bars

48 dia bars

least dimension of column (10x15)

least

\* corners always 90°. but max<sup>m</sup> limit 135°

\* circular tie arrangement व alternate tie

फिरोत रूवे ना।



$$\phi P_n = 0.8 * 0.65 * 716$$

$$= \boxed{372.3 \text{ K}}$$

\* Tie

#3 tie is OK because #9 bars

$$\text{spacing} \rightarrow = 16 + \frac{9}{8} = 18''$$

$$48 * \frac{3}{8} = 18''$$

$$12''$$

∴ #3 tie @ 12" c/c is OK.

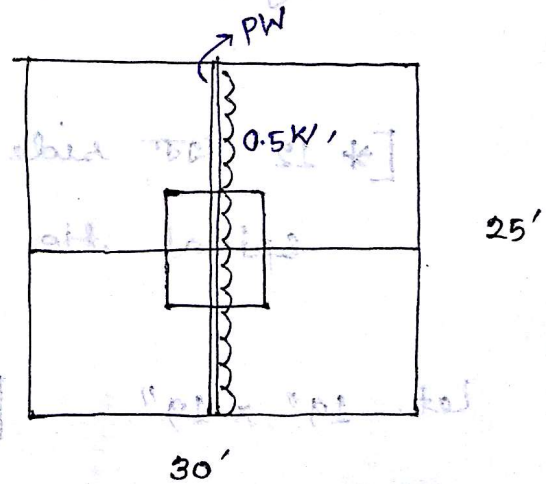
□ Problem 2: (Design Problem)

$$P_{DL} = 300 \text{ K}$$

$$P_{LL} = 200 \text{ K}$$

$$f_c = 3 \text{ ksi}$$

$$f_y = 60 \text{ ksi}$$



$$P_u = 1.2 P_{DL} + 1.6 P_{LL}$$

$$= 1.2 * 300 + 1.6 * 200$$

$$= 680 \text{ K}$$

slab thickness = 6"

$$\text{Floor finish} = \frac{2}{12} * 150$$

$$PW = \frac{5}{12} * 10' * 1' * 0.12$$

PF \* Assignment # DL & LL calculation

✓  
Design capacity  $\geq$  Demand

$$\Rightarrow \alpha \phi P_n \geq P_u$$

$$\Rightarrow P_u = \alpha \phi P_n = \alpha \phi^* [0.85 f_c (A_g - A_{st}) + A_{st} f_y]$$

$$\Rightarrow 680 = \alpha \phi A_g [0.85 f_c (1 - \rho_g) + \rho_g f_y]$$

↓

$$\rho_g = \frac{A_{st}}{A_g} = 1-8\%$$

$$\text{Let } \rho_g = 0.02$$

$$680 = 0.8^* 0.65 A_g [0.85^* 3 (1 - 0.02) + 0.02^* 60]$$

$$A_g = 353.5 \text{ in}^2 \Rightarrow 18.8'' \times 18.8''$$

[\* 12" side side design assignment

spiral tie

"]

$$\text{Let, } 19'' \times 19'' \quad [* (19 \times 19) \times 2\% \text{ कबल सब ना } ]$$

$$680 = 0.8^* 0.65^* [0.85^* 3^* (19 \times 19 - A_{st}) + A_{st}^* 60]$$

$$A_{st} = 6.73 \text{ in}^2$$

$$4\#9 \quad 4\#1$$

$$4\#8 \quad 4\#0.79$$

$$\underline{7.16 \text{ in}^2}$$

\* Tie

# 3 size as # 9 bar is used

⇒ [size এর ক্ষেত্রে বড় bar নিব]

spacing :

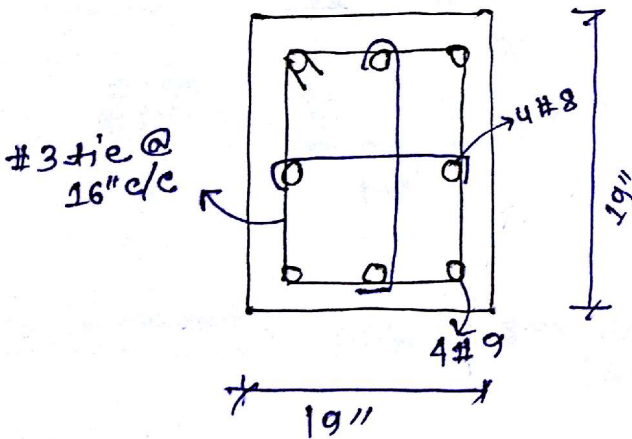
$$16 * \frac{8}{8} = 16''$$

$$48 * \frac{3}{8} = 18''$$

$$19 = 19''$$

\* [Spacing এর ক্ষেত্রে ছোট bar নিব কারণ ছোট bar 2 spacing বসে আসবে]

∴ # 3 tie @ 16" c/c



⇒ [বড় bar corner এ দিব]

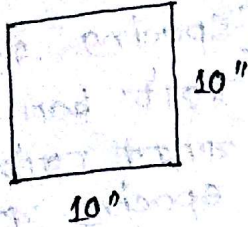
$$** 19'' - 2 * 1.5'' - 2 * \frac{3}{8} - 2 * \frac{9}{8} - 1 * \frac{8}{8} = 12'' \div 2 = 6''$$

↓ clean cover
↓ tie
↓ #9
↓ #8
↓ spacing

[\* 6.1" রকম must tie দিতে হবে।]

[\*\* spacing criteria must check করতে হবে।]

### Problem 3 (Design)



$$P_{DL} = 60 \text{ k}$$

$$P_{LL} = 30 \text{ k}$$

$$f'_c = 3 \text{ ksi}$$

$$f_y = 60 \text{ ksi}$$

$$\begin{aligned} P_u &= 1.2 P_{DL} + 1.6 P_{LL} \\ &= 1.2 * 60 + 1.6 * 30 \\ &= 120 \text{ k} \end{aligned}$$

$$\alpha \phi P_n \geq P_u$$

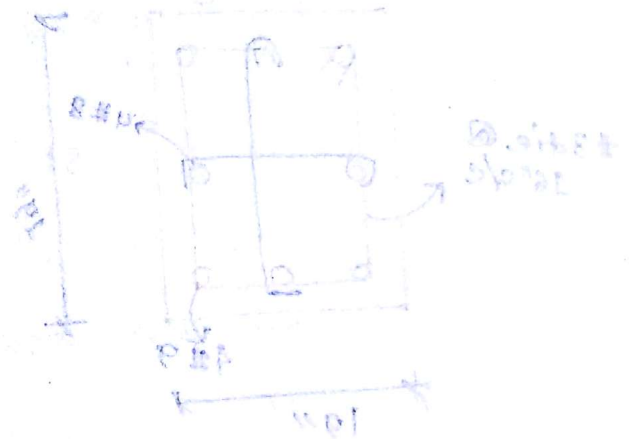
$$P_u = \alpha \phi P_n = \alpha \phi [0.85 f'_c (A_g - A_{st}) + A_{st} f_y]$$

$$\Rightarrow 120 = 0.8 * 0.65 * [0.85 * 3 (10 * 10 - A_{st}) + A_{st} * 60]$$

$$\Rightarrow \underline{A_{st} = -0.42 \text{ in}^2}$$

\* (-)ve आजा घाटन steel जागव ना ।

\* Min<sup>m</sup> 1% mod दिवत रहव ।



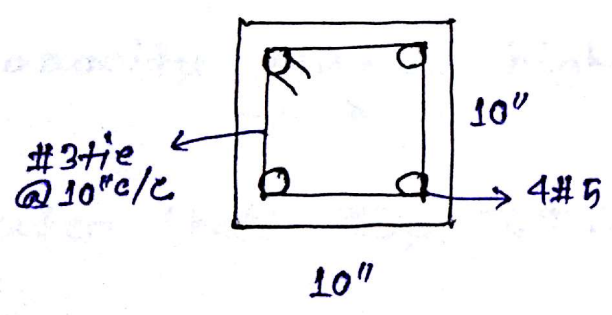
✓

Min  $A_s \Rightarrow 1\%$   
 $= 0.01 * 100$   
 $= \boxed{1 \text{ in}^2}$

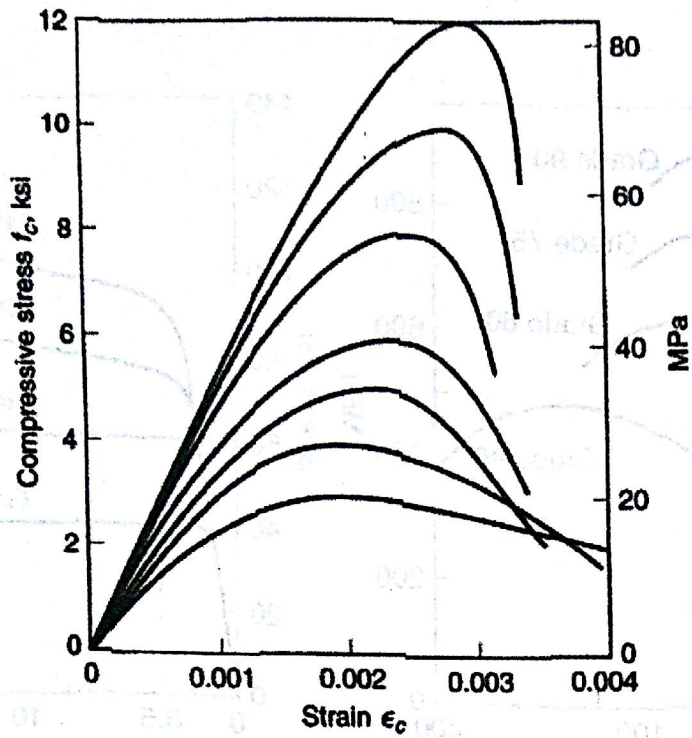
$\boxed{\text{min } 4\#5}$

Tie #3 spacing  $16 * \frac{5}{8} = 10''$   
 $48 * \frac{3}{8} = 18''$   
 $10'' = 10''$

$\boxed{\therefore \#3 \text{ tie @ } 10'' \text{ c/c}}$

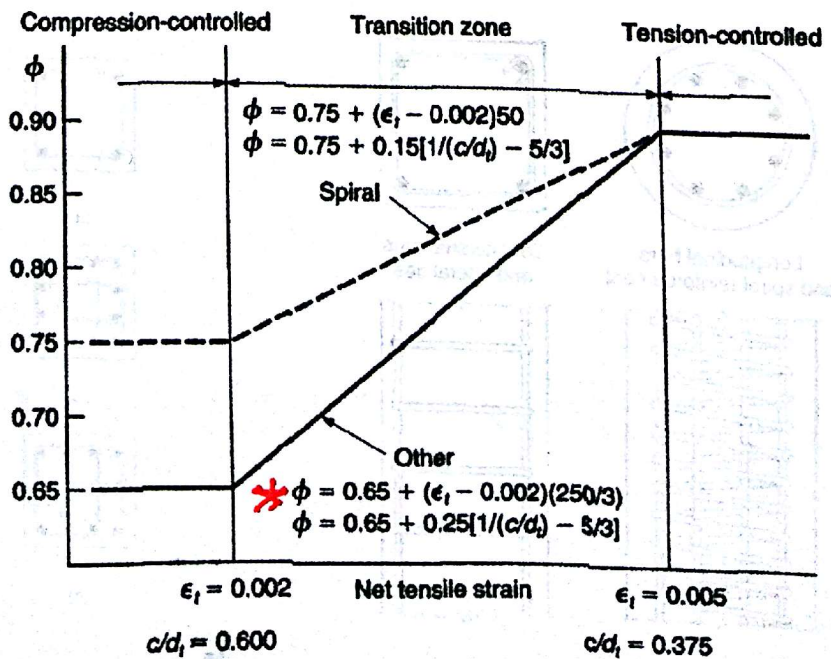


# Assignment: Min<sup>m</sup> load  $\frac{P}{A} = ?$   
 (section)  $\alpha P_n = ?$



**FIGURE 2.3**  
 Typical compressive stress-strain curves for normal-density concrete with  $w_c = 145$  pcf. (Adapted from Refs. 2.23 and 2.24.)

**FIGURE 3.9**  
 Variation of strength reduction factor with net tensile strain in the steel.



- when load eccentricity, reinforced column show greater toughness i.e. greater ductility than tied column although this difference fades out as the eccentricity is increased.

#	Tied Column □	Spiral Column ○
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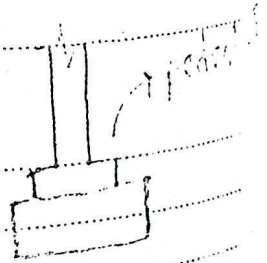
① It takes small amount of load compared to the Spiral Column	① It takes generally more loads compared to the tied column
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② Failure of tied column is abrupt & complete.	② Failure of spiral column is gradually & ductile manner.
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③ $\phi = 0.70 - 0.65$ (ACI CODE)	③ $\phi = 0.75 - 0.70$ (ACI CODE)
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# Function of a pedestal →

- A pedestal distributes the column load over a large area of footing & thus contributing to a more economical footing design.



2012

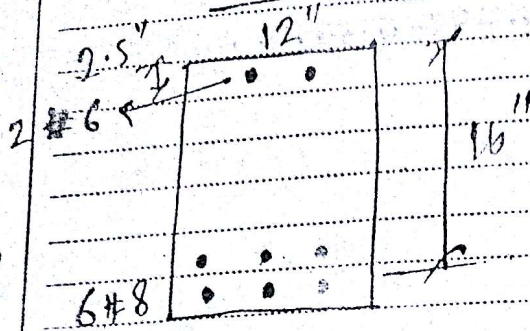
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সামকাল  
The Daily Samakal

## # Moment Calculation:



$$A_s = 6 \times 0.79 = 4.74 \text{ in}^2$$

$$A_s' = 2 \times 0.44 = 0.88 \text{ in}^2$$

$$f_c' = 4 \text{ ksi}, f_y = 60 \text{ ksi}$$

$$p_{max} = 0.85 \beta_1 f_c' / f_y \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$= 0.85 \times 0.85 \times 4 / 60 \times \frac{0.003}{0.003 + 0.004}$$

$$= 0.0206$$

$$p = A_s / bd = \frac{4.74}{12 \times 16} = 0.0247$$

$$p' = \frac{A_s'}{bd} = \frac{0.88}{12 \times 16} = 0.00458 = 0.0046$$

$$\bar{p}_{max} = p_{max} + p' = 0.0206 + 0.0046$$

$$= 0.0252 > p$$

So compression steel yield.

$$M_{n1} = A_s' f_y (d - d') = 0.88 \times 60 \times (16 - 2.5)$$

$$= 713 \text{ k-in}$$

$$M_{n2} = (A_s - A_s') f_y (d - a/n)$$

$$a = \frac{(A_s - A_s') f_y}{0.85 f_c' b} = \frac{(4.74 - 0.88) \times 60}{0.85 \times 4 \times 12} = 5.68''$$

$$M_{n2} = (A_s - A_s') f_y \left( d - \frac{a}{2} \right)$$

$$= (4.74 - 0.88) \times 60 \times \left( 16 - \frac{5.68}{2} \right)$$

$$= 3054 \text{ k-in}$$

$$\therefore M_n = M_{n1} + M_{n2} = 713 + 3054 = 3767 \text{ k-in}$$

$$\therefore M_n = \phi M_n$$

$$\epsilon_t = 0.003 \times \frac{d-c}{c} \quad c = \frac{a}{\beta_1}$$

$$= 0.003 \times \frac{16 - 6.68}{6.68} = \frac{5.68}{0.85}$$

$$= 0.00419 < 0.005 = 6.68$$

$$\therefore \phi = 0.483 + 83.3 \times 0.00419$$

$$= 0.83$$

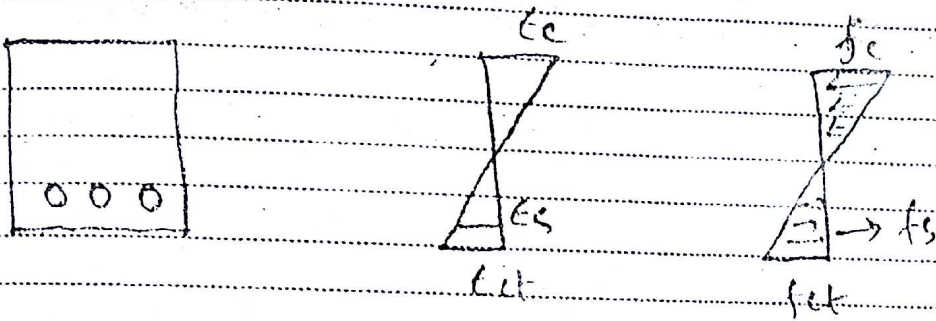
$$\therefore M_n = \phi M_n = 0.83 \times 3767$$

$$= 3128 \text{ k-in}$$

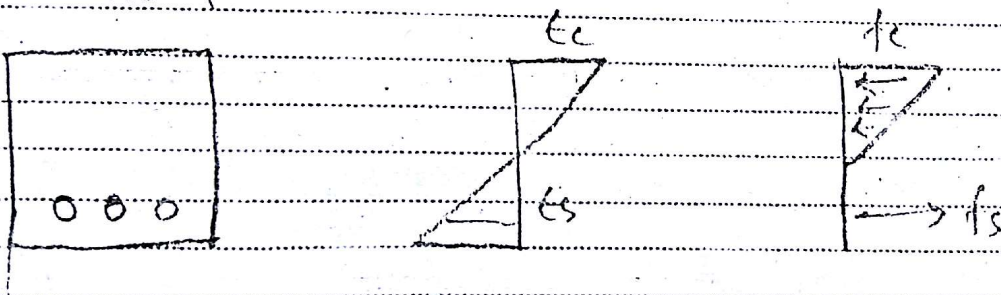
সংস্করণ প্রস্তুত করা হয়েছে

Draw sketches for strain & stress distribution diagrams of a reinforced concrete beam section when subjected to bending for uncracked, cracked & ultimate section,

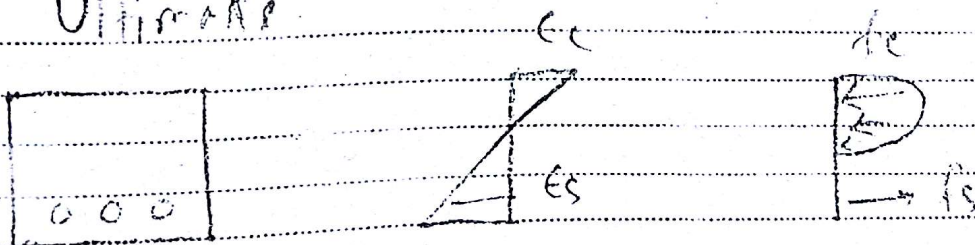
(a) Uncracked



(b) Cracked

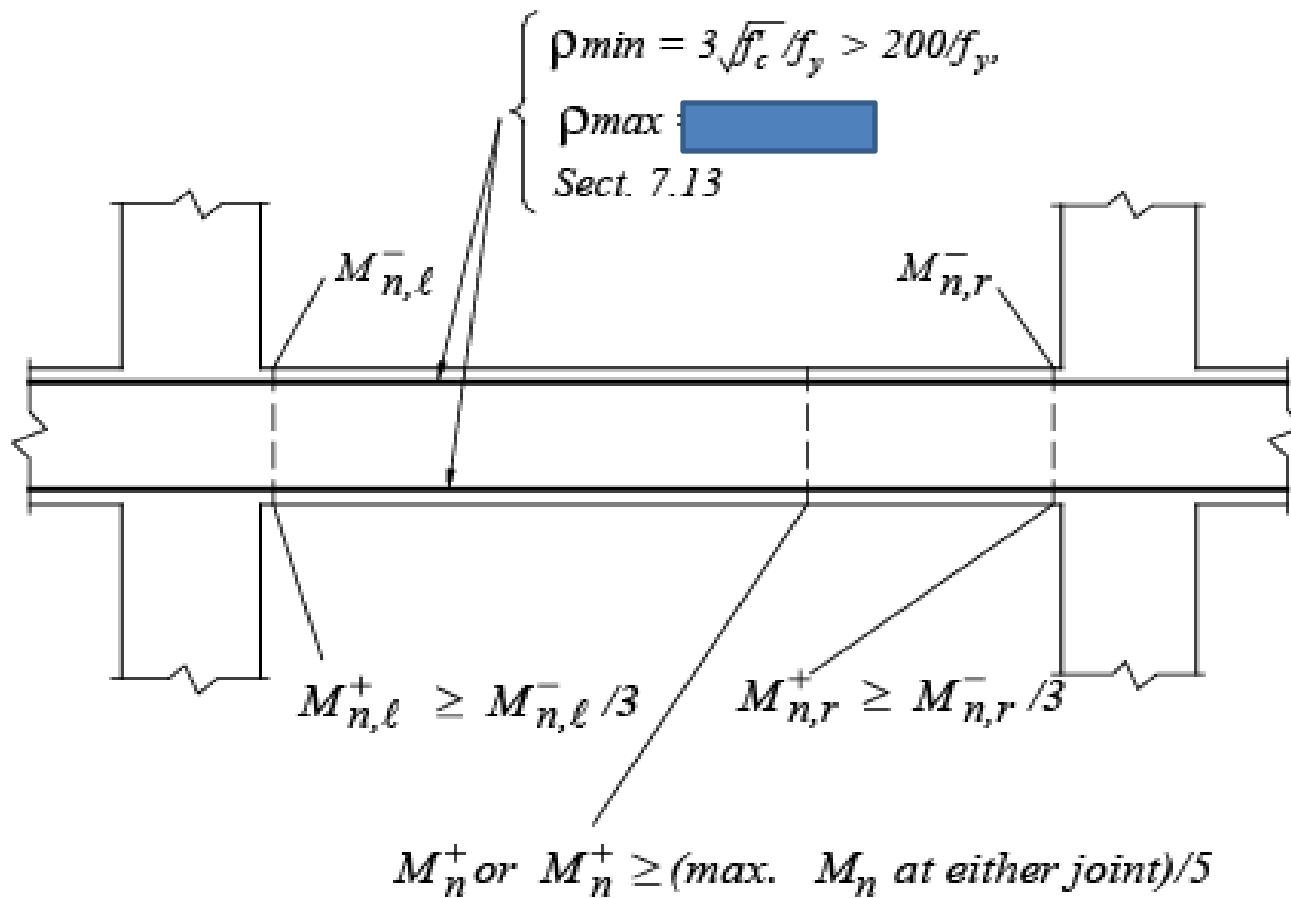


(c) Ultimate



## Requirements for Intermediate Moment Frames: Beams

- a) The positive moment strength at the face of the joint shall not be less than one-third the negative moment strength provided at that face (Fig.8.3.12). Neither the negative nor positive moment strength at any section along the length of the member shall be less than one-fifth of the maximum moment strength provided at the face of either joint.
- b) At both ends of the member, stirrups shall be provided over lengths equal to twice the member depth measured from the face of the supporting member toward midspan (Fig.8.3.13). The first stirrup shall be located not more than 50 mm from the face of the supporting member. Maximum stirrup spacing shall not exceed (a)  $d/4$ , (b) 8 times the diameter of the smallest longitudinal bar enclosed, (c) 24 times the diameter of the stirrup bar, and (d) 300 mm.
- c) Stirrups shall be placed at not more than  $d/2$  throughout the length of the member.



*Note: transverse reinforcement not shown for clarity*

Fig.8.3.12 Flexural Requirements for Beams

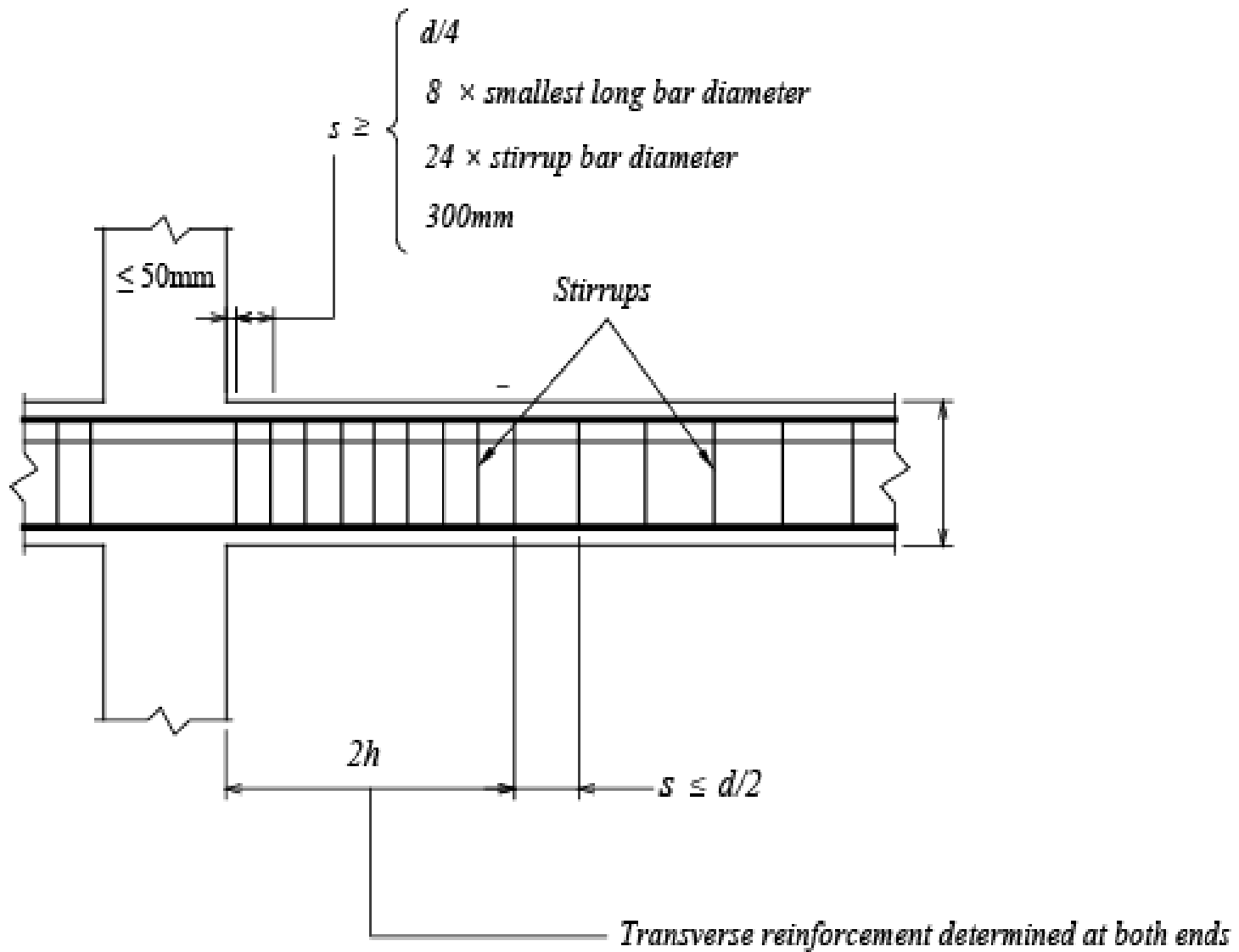


Fig.8.3.13 Transverse reinforcement Requirements for Beams

## Requirements for Intermediate Moment Frames: Columns

- a) Maximum tie spacing shall not exceed  $s_o$  over a length  $l_o$  measured from the joint face. The spacing  $s_o$  shall not exceed (i) 8 times the diameter of the smallest longitudinal bar enclosed, (ii) 24 times the diameter of the tie bar, (iii) one-half of the smallest cross-sectional dimension of the frame member, and (iv) 300 mm. The length  $l_o$  shall not be less than (i) one-sixth of the clear span of the member, (ii) maximum cross-sectional dimension of the member, and (iii) 450 mm.
- b) The first tie shall be located not more than  $s_o/2$  from the joint face.
- c) Joint reinforcement shall conform to 6.4.9.
- d) Tie spacing shall not exceed  $2s_o$  throughout the length of the member.

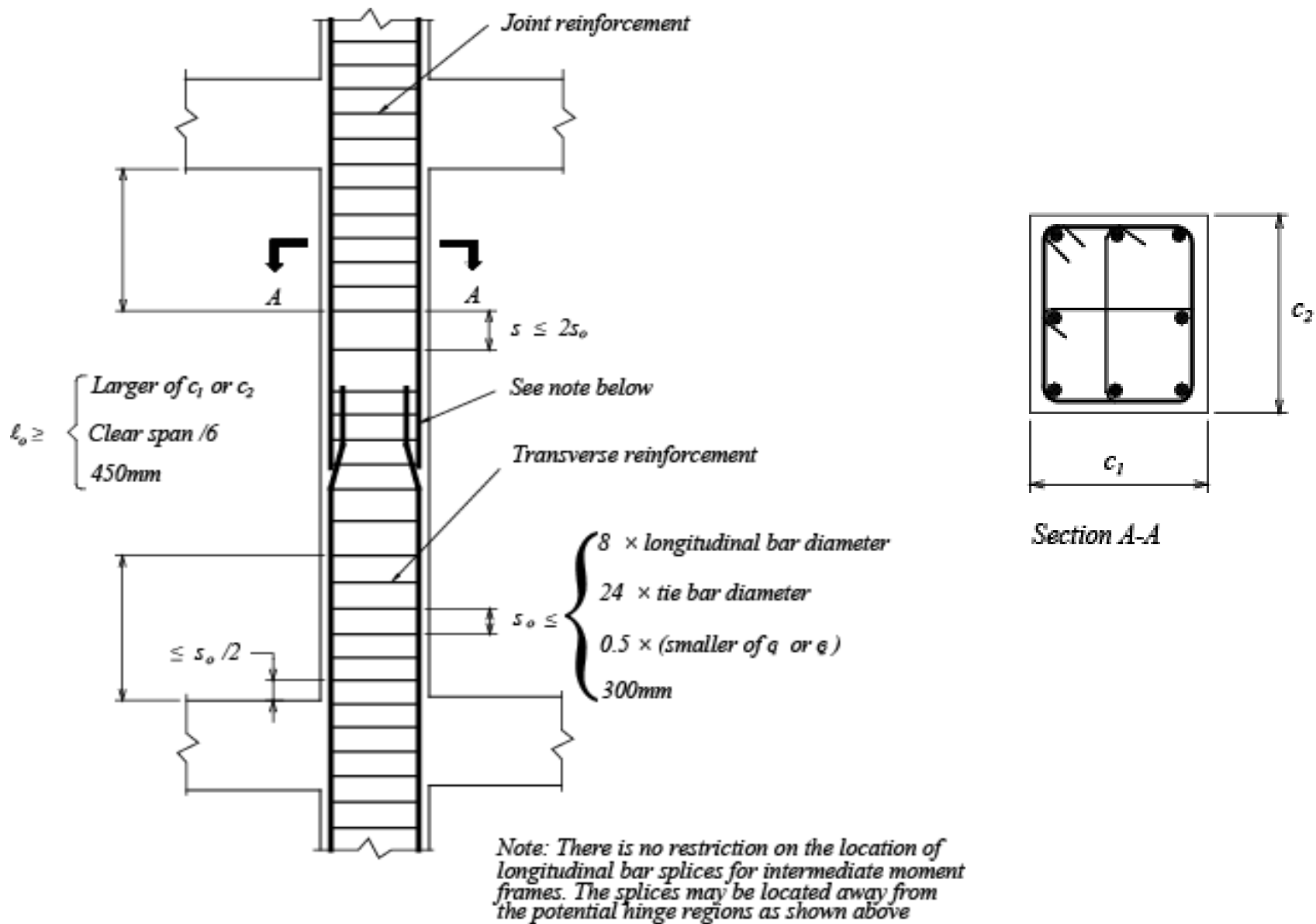


Fig.8.3.14 Transverse Reinforcement Requirements for Columns