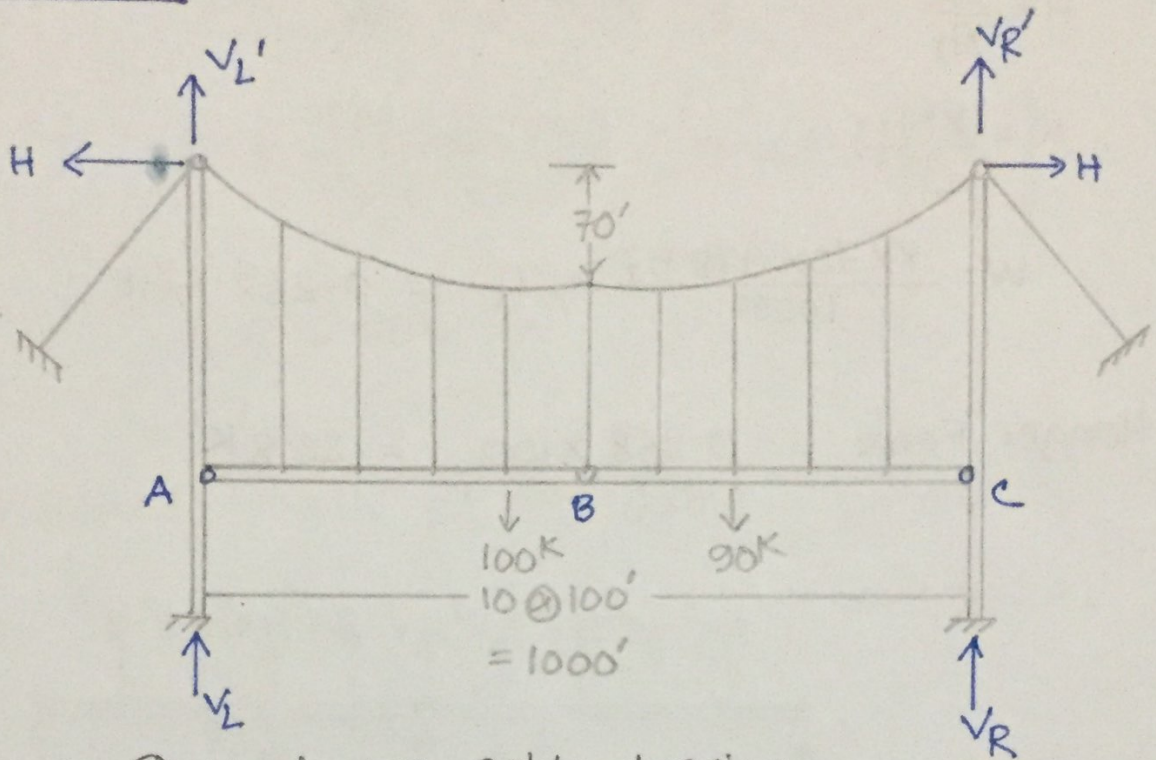


Assignment - 1



- Determine:
- ① maximum cable tension
 - ② Hanger force
 - ③ Shear Force Diagram
 - ④ Bending moment diagram
 - ⑤ Unstressed Length of Cable.

Solution: $\sum M_A = 0$

$$100 \times 400 + 90 \times 700 - (V_R + V_R') \times 1000 = 0$$

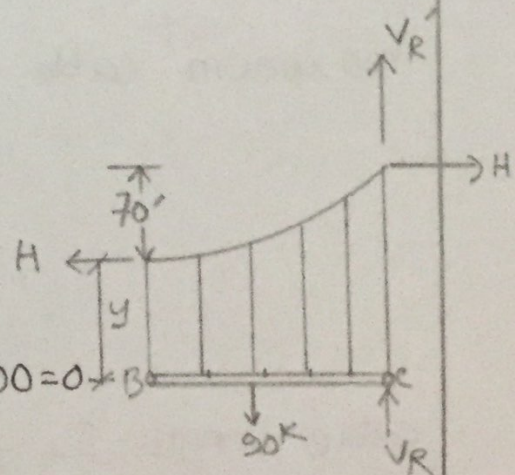
$$V_R + V_R' = \frac{40000 + 63000}{1000}$$

$$\therefore V_R + V_R' = 103 \text{ K}$$

$\sum M_B = 0$

$$-Hy + H(y + 70) - (V_R + V_R') \times 500 + 90 \times 200 = 0$$

$$\therefore H = 478.57 \text{ K}$$

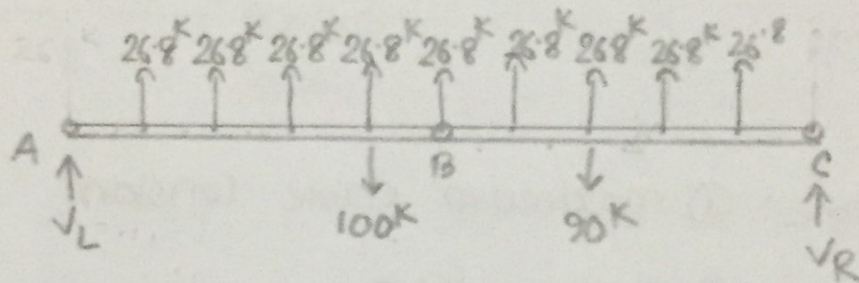


$$H = \frac{wL^2}{8h}$$

$$w = \frac{8hH}{L^2}$$

$$\therefore w = \frac{8 \times 70 \times 478.57}{1000^2} \text{ K/ft} = 0.268 \text{ K/ft}$$

$$\text{Hanger Force} = 0.268 \times 100 = 26.8 \text{ K}$$



$$\sum M_A = 0$$

$$-26.8(100 + 200 + 300 + 400 + 500 + 600 + 700 + 800 + 900) + 100 \times 400 + 90 \times 700 = V_R \times 1000$$

$$\therefore V_R = -17.6 \text{ K} \quad \therefore V_R = 17.6 \text{ K} (\downarrow)$$

$$\therefore V_L = 33.6 \text{ K} (\downarrow)$$

$$\text{maximum cable tension, } T = H(1 + 16\theta^2)^{\frac{1}{2}}$$

$$T = 478.57 \times \left(1 + 16 \cdot \frac{h^2}{L^2}\right)^{\frac{1}{2}} \quad [\theta = 0]$$

$$\therefore T = 476.207 \text{ K}$$

$$\text{cable length, } S = \frac{L}{2}(1 + 16\theta^2)^{\frac{1}{2}} + \frac{L}{8\theta} \left(4\theta + (1 + 16\theta^2)^{\frac{1}{2}}\right) = 1012.92 \text{ ft.}$$

cable stretch, $\Delta s = \frac{HL}{AE} [1 + 16Q^2/3]$

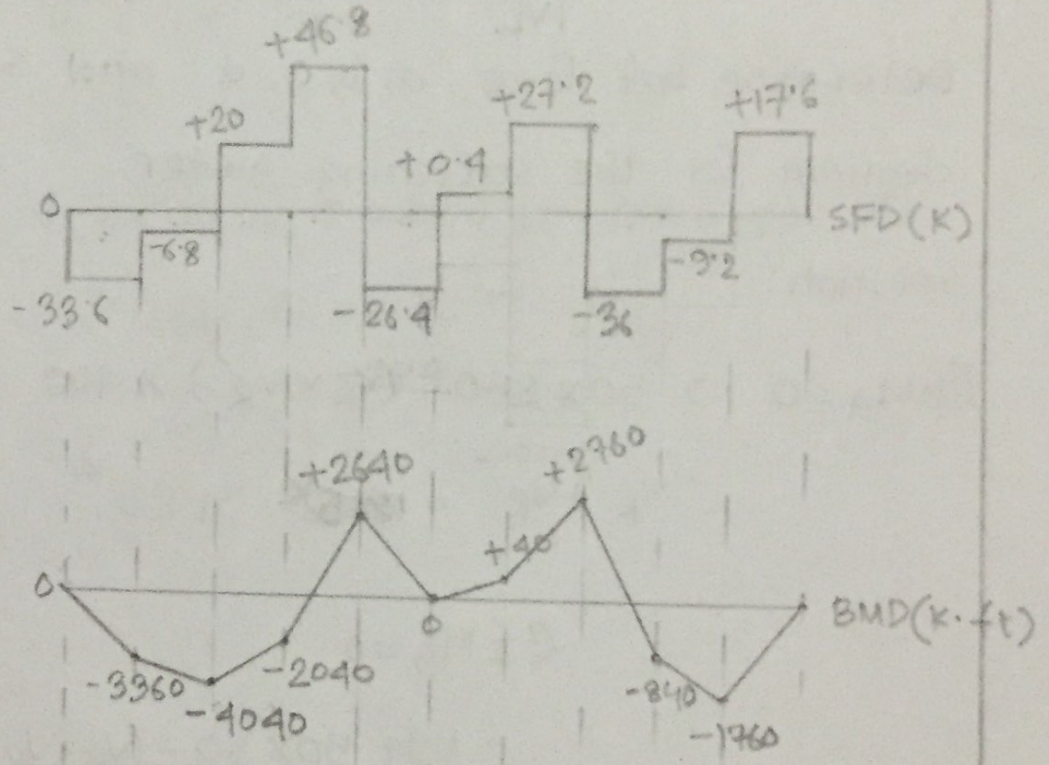
$= \frac{478.57 \times 1000}{50 \times 27 \times 10^6} [1 + 16/3 (\frac{70}{1000})^2]$

$= 0.004 \text{ ft.}$

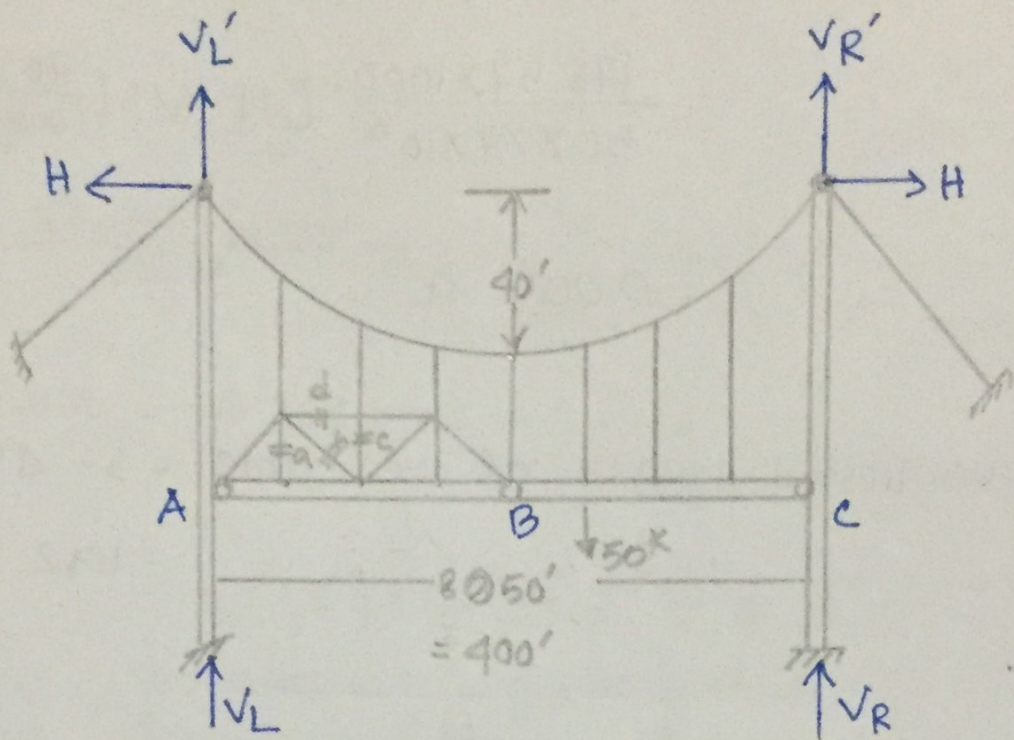
\therefore unstressed length of the cable = $s - \Delta s$

$= 102.916 \text{ ft.}$

Ans:



Assignment-2



Determine bar force a, b, c, d and SF and BM diagram for the stiffening girder.

Solution:

$$\sum M_A = 0 \Rightarrow 50 \times 250 - (V_R \times V_R') \times 400 = 0$$

$$\therefore V_R + V_R' = 31.25 \text{ K}$$

$$\sum M_B = 0$$

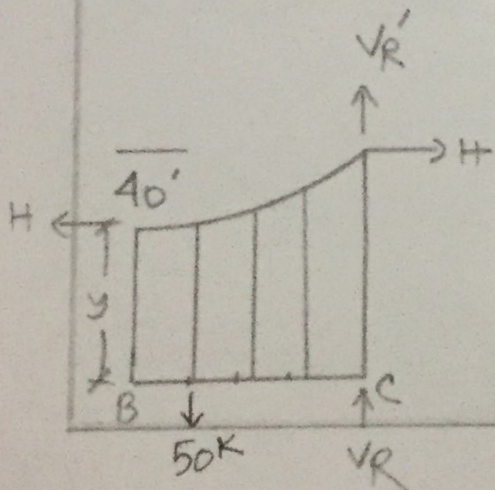
$$- H \times 40 + 50 \times 50 - (V_R + V_R') \times 200 +$$

$$H \times (40 + 40) = 0.$$

$$H = 0.9375 \text{ K}$$

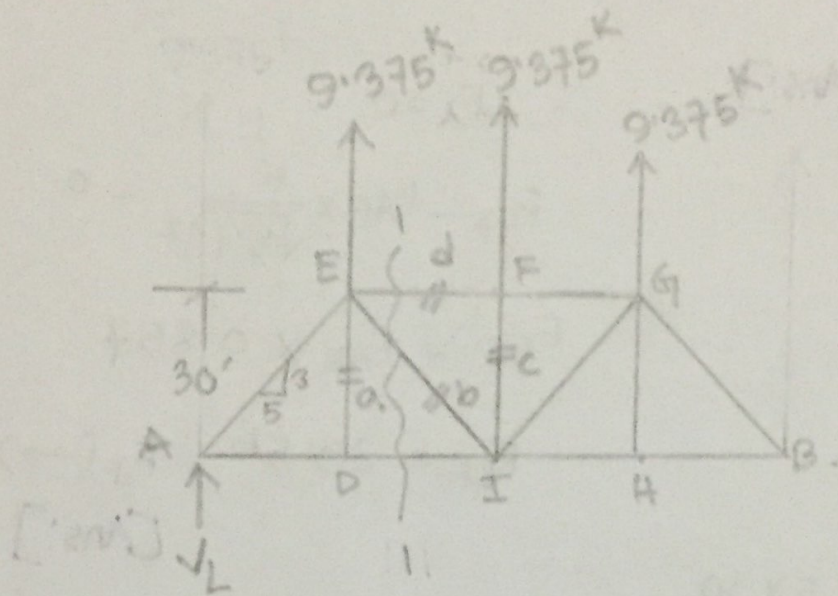
$$H = \frac{wL^2}{8h}$$

$$\therefore w = \frac{8 \times 40 \times 0.9375}{400^2} = 0.1875 \text{ K/ft.}$$



$$\therefore \text{Hanger force} = 0.1875 \times 400/8 = 9.375 \text{ K}$$

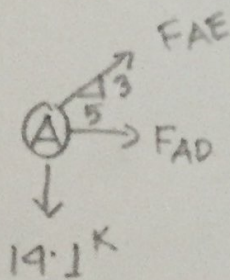
$$T_{\text{max}} = H(1+16d^2)^{1/2} = 93.75(1+16 \times 0.1^2)^{1/2} = 101 \text{ K}$$



$$\overset{+}{\curvearrowright} \sum M_B = 0 \Rightarrow V_L \times 200 + 9.375(50 + 100 + 150) = 0$$

$$\therefore V_L = -14.1 \text{ K}$$

$$\therefore V_L = 14.1 \text{ K} (\downarrow)$$



$$\rightarrow + \sum F_x = 0$$

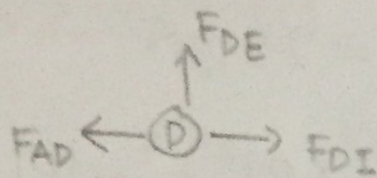
$$F_{AD} \times \frac{5}{\sqrt{5^2+3^2}} \times F_{AE} = 0$$

$$F_{AD} = -F_{AE} \times 0.857$$

$$\uparrow + \sum F_y = 0$$

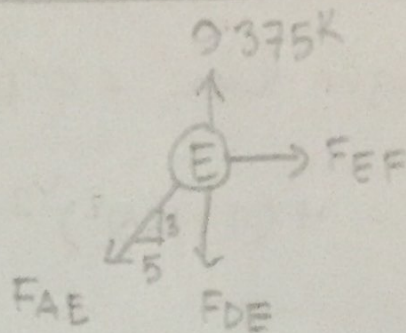
$$-14.1 + F_{AE} \times \frac{3}{\sqrt{5^2+3^2}} = 0$$

$$F_{AE} = \frac{14.1}{0.51} = 27.4 \text{ K}$$



$$\uparrow \sum F_y = 0$$

$$\therefore F_{DE} = F_a = 0 \text{ [Ans:]}$$



$$\rightarrow \sum F_x = 0$$

$$F_{EF} - F_{AE} \times \frac{5}{\sqrt{5^2+3^2}} = 0$$

$$F_{EF} = F_{AE} \times 0.857$$

$$F_{EF} = 23.5 \text{ K} = F_d (\rightarrow) \text{ [Ans:]}$$

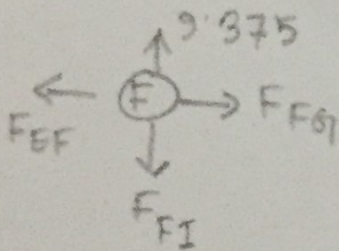
From section 1-1

$$\curvearrow \sum M_D = 0$$

$$F_{EF} \times 30 + F_{EI} \times \frac{5 \times 30}{\sqrt{5^2+3^2}} = 0$$

$$F_{EI} = -\frac{\sqrt{5^2+3^2}}{150} \times 30 F_{EF}$$

$$F_{EI} = -27.4 \text{ K} = F_b (\swarrow) \text{ [Ans:]}$$



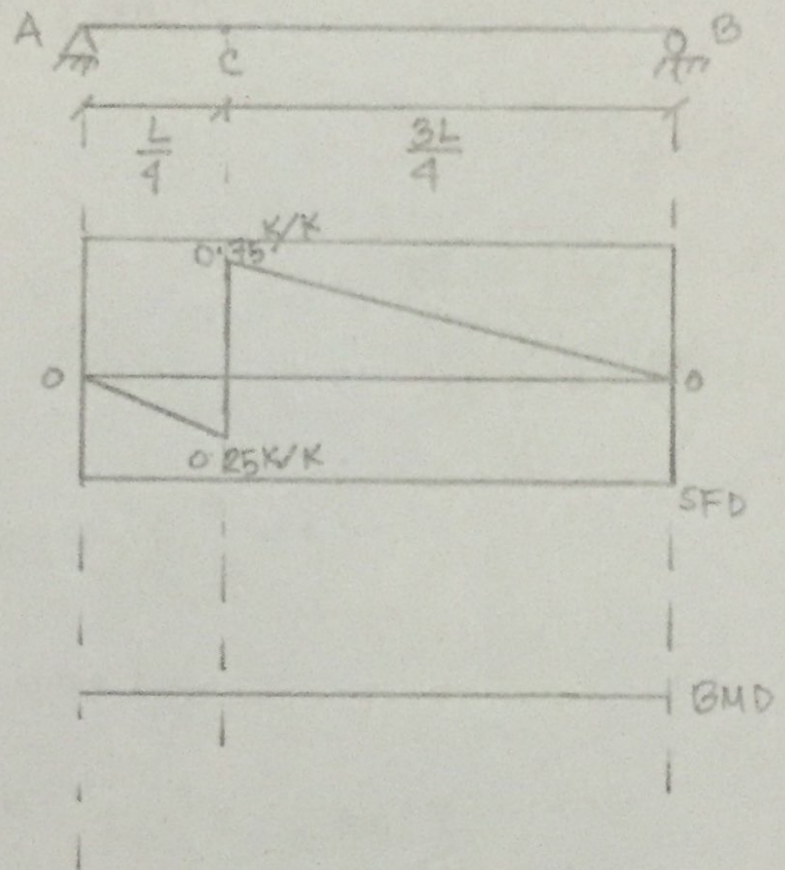
$$\uparrow \sum F_y = 0$$

$$9.375 - F_{FI} = 0$$

$$\therefore F_{FI} = 9.375 \text{ K} (\uparrow) \text{ [Ans:]}$$

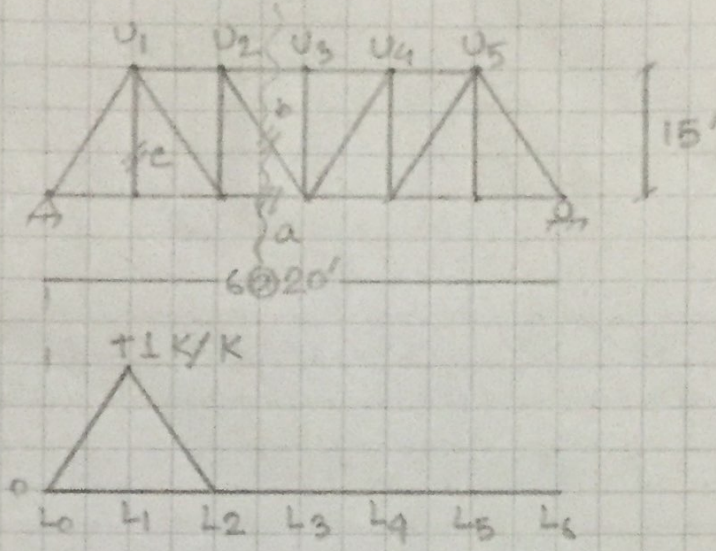
Assignment - 3

Draw I.L for shear and Moment at C.





Assignment - 4

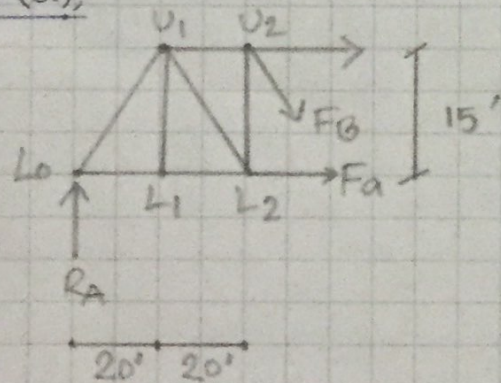


IL Diagram for bar c

For bar force (a),

case: 1

When \$1 \text{ k}\$ load moves from \$L_0\$ to \$L_2\$



$$\sum M_{U_2} = 0$$

$$R_A \times 40 - 1 \times x - F_a \times 15 = 0$$

$$\therefore F_a = \frac{40 R_A - x}{15}$$

case 2: when \$1 \text{ k}\$ load moves from \$L_2\$ to \$L_6\$

$$\sum M_{U_2} = 0 \quad R_A \times 40 - F_a \times 15 = 0 \quad \therefore F_a = \frac{40 R_A}{15}$$

For bar force (b),

case: 1

Load moves from \$L_0\$ to \$L_2\$

$$\sum F_y = 0$$

$$R_A - 1 - \frac{3}{5} F_b = 0$$

$$\therefore F_b = \frac{5}{3} (R_A - 1)$$

case: 2

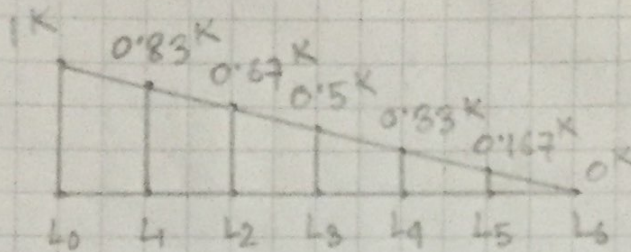
Load moves from \$L_2\$ to \$L_6\$

$$\sum F_y = 0 \Rightarrow R_A - \frac{3}{5} R_b = 0$$

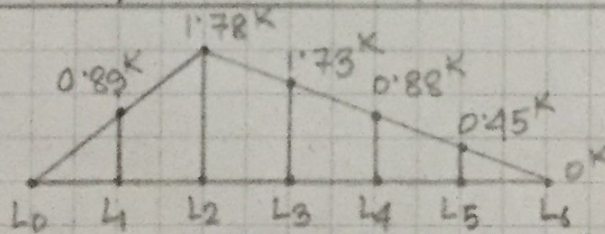
$$F_b = \frac{5}{3} R_A$$



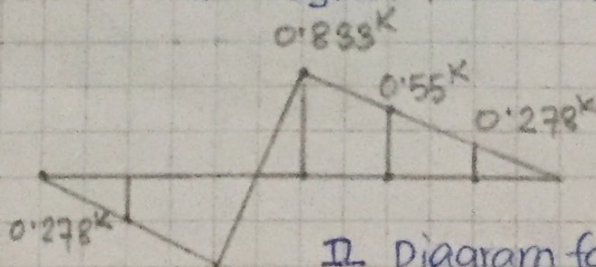
IL Diagram For R_A .



position of 1K Load	bar force 'a)	bar force 'b)
L ₀	$F_a = 0$	$F_b = 0$
L ₁	$F_a = 0.89K$	$F_b = -0.278K$
L ₂	$F_a = 1.78K$	$F_b = -0.56K$
L ₃	$F_a = 1.73K$	$F_b = 0.833K$
L ₄	$F_a = 0.88K$	$F_b = 0.55K$
L ₅	$F_a = 0.45K$	$F_b = 0.278K$
L ₆	$F_a = 0K$	$F_b = 0$



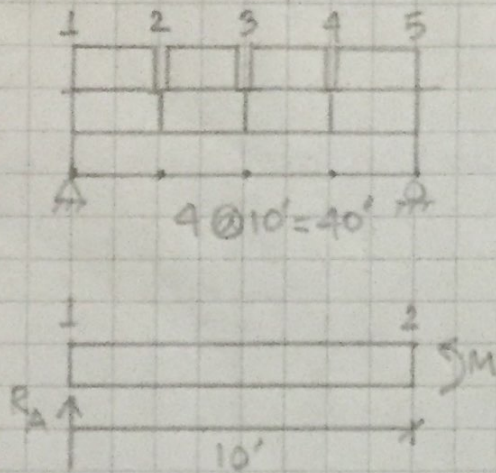
IL Diagram For bar force 'a)



IL Diagram for bar force 'b)



Assignment - 5



Draw IL Diagram for moment at panel point 2 of the girder.

Case 1: When the load moves from 1 to 2

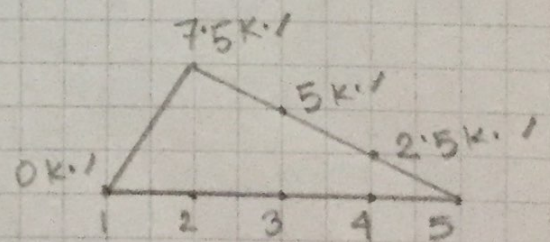
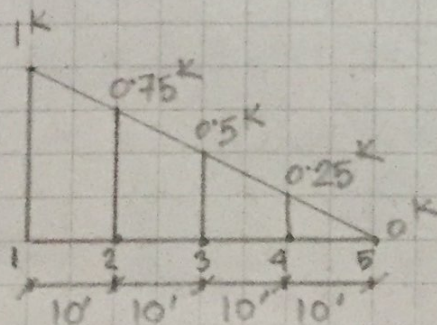
$$R_A \times 10 - 1 \times x = M$$

$$\therefore M = 10R_A - x$$

Case 2: When the load moves from 2 to 5,

$$R_A \times 10 - M = 0$$

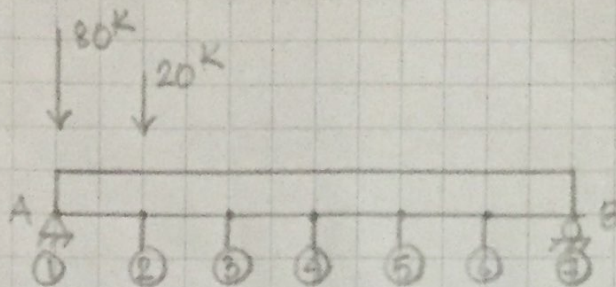
$$M = 10R_A$$



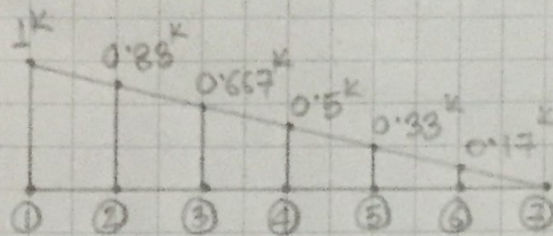
IL diagram for moment at panel point 2:



Assignment - 6



What is the maximum reaction for moving truck?



Value of $IL \times Load = Reaction$

1) When 80^k wheel at point 1 and 20^k wheel at point 2 [GOVERNS]

$$R = 80 \times 1 + 20 \times 0.83 = 96.67^k$$

2) When 80^k " " " 2 and 20^k " " " 3 $R = 79.98^k$

3) When 80^k " " " 3 and 20^k " " " 4 $R = 69.36^k$

4) When 80^k " " " 4 and 20^k " " " 5 $R = 46.6^k$

5) When 80^k " " " 5 and 20^k " " " 6 $R = 29.78^k$

6) When 80^k " " " 6 and 20^k " " " 7 $R = 13.34^k$

Zakaria Sir

CT-2

Moving Loads

1. Position of wheel to produce max^m reaction.

$$\Delta R = \frac{\sum P d}{L} + \frac{P' e}{L} - P_1 \quad \sum P = \text{that stay on span before and after mov.}^m$$

P' = wheel that comes on span e = distⁿ of P'

P_1 = wheel that left the span

2. Position of wheel to produce max^x shear:-

$$\Delta V = \frac{\sum P d}{L} + \frac{P' e}{L} + \frac{P_2 e_1}{L} - P_1$$

P' = wheel that comes on span. e = distance of P'

P_2 = wheel that left span. e_1 = distance of P_2

$P_1 = P_1$ = wheel that cross point C

3. Maximum moment at a section for a simple span:-

$$\frac{W_1}{a} = \frac{W}{L}$$

wheel placed at max^m moment (IL) point

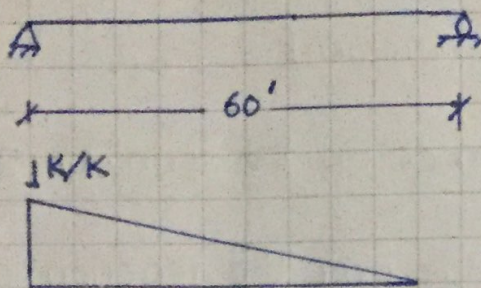
Just right $\frac{W}{L} > \frac{W_1}{a}$ is

Just left $\frac{W}{L} < \frac{W_1}{a}$

-this criteria is satisfied



□ position of wheel to produce maximum reaction for a simply supported span



Observation:

- cycle-1 1) R_a in bridges increases till wheel ① comes on A
- cycle-1 2) R_a suddenly decreases as wheel ② moves off the span
- cycle-2 3) R_a in bridges again increases till wheel ② comes on A
- cycle-2 4) R_a suddenly decreases as wheel ② moves from the span
- 5) (Repeated)
- 6) (Repeated)

change in reaction ' ΔR ' for distance ' d ' travelled :

$$\Delta R = \frac{\sum P d}{L} + \frac{P' e}{L} - P_1$$

where, $\sum P$ = sum of wheels that stay on the span before and after the movement

P' = wheel that comes on the span during movement.

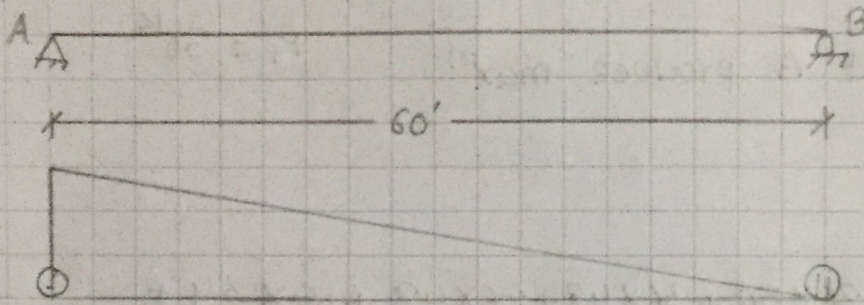
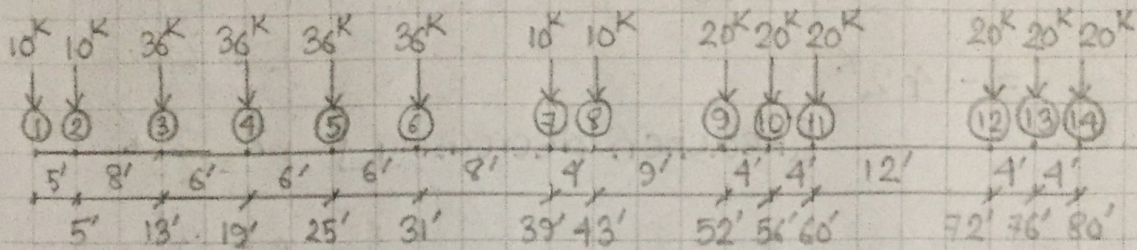
P_1 = the wheel which was over the reaction and is moved off the span.

e = distance travelled by P' on the span.



Problem:

Find maximum reaction for a simple span of 60' for given wheel load



Trial 1: Move wheel ② at A

Here,

$$\begin{aligned} \Delta R &= \frac{\sum Pd}{L} + \frac{P'e}{L} - P_i \\ &= \frac{234 \times 5'}{60'} + \frac{0 \times 0}{60'} - 10^K \\ &= +9.5^K \quad [\text{Reaction Increasing}] \end{aligned}$$

$$\sum P = \textcircled{2} - \textcircled{11} = 234^K$$

$$d = 5'$$

$$P' = 0 \quad P_i = \textcircled{1} =$$

Trial 2: move wheel ③ to A

Here,

$$\begin{aligned} \Delta R &= \frac{\sum Pd}{L} + \frac{P'e}{L} - P_i \\ &= \frac{224^K \times 8'}{60'} + \frac{20^K \times 1'}{60'} - 10^K \\ &= +20.2^K \quad [\text{Reaction Increasing}] \end{aligned}$$

$$\sum P = \textcircled{3} - \textcircled{11} = 224^K$$

because ⑫ before was not on span.

Trial 3: Move wheel ④ to A

$$\begin{aligned} \Delta R &= \frac{\sum Pd}{L} + \frac{P'e}{L} - P_1 \\ &= \frac{208' \times 6'}{60'} + \frac{20^k \times 3'}{60'} - 36^k \\ &= -14.2^k \end{aligned}$$

[Reaction decreasing]

Here, [∵ 13 নতুন আঁকছে]

$$\sum P = ④ - ⑫ = 208^k$$

$$d = 6'$$

$$L = 60'$$

$$P' = 20^k$$

$$e = 3'$$

$$P_1 = 36^k$$

Hence, wheel ③ at A produce max^m

Reaction.

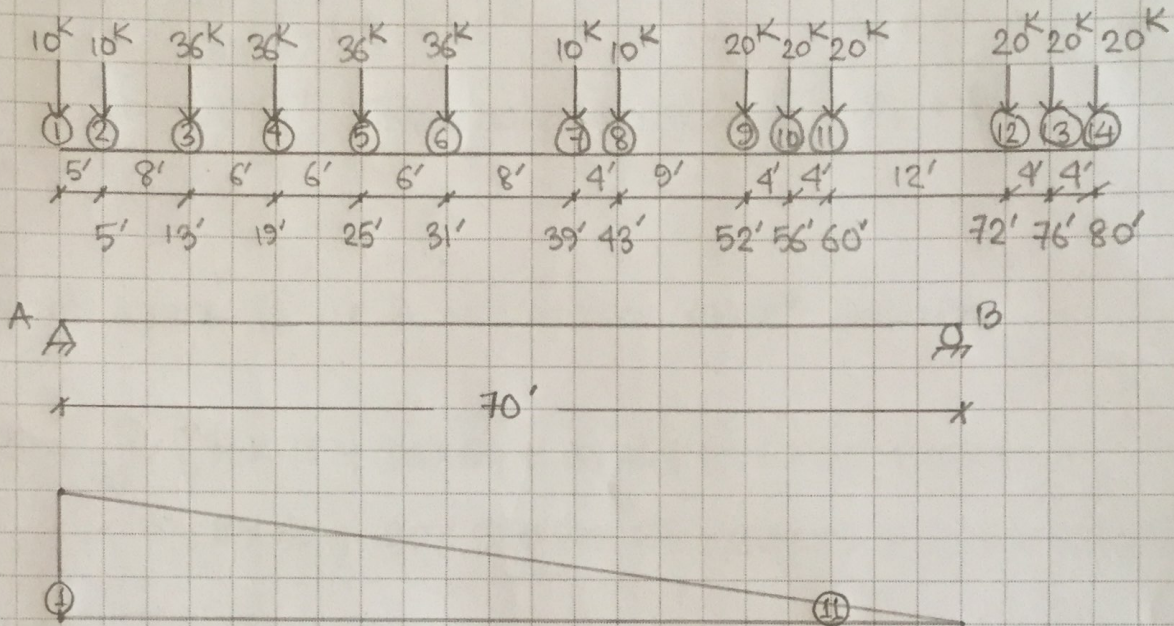
$$R_A = \frac{1}{60} [36 \times 60 + 36 \times 54 + 36 \times 48 + 36 \times 42 + 10 \times 34 + 10 \times 30 + 20 \times 21 + 20 \times 17 + 20 \times 13 + 20 \times 1]$$

$$R_A = 150.4^k \text{ (maximum Reaction)}$$



Assignment 7

Find maximum reaction for a simple span of 70' for given wheel Load



Trial 1: Move wheel ② to A

$$\Delta R = \frac{\sum P \cdot d}{L} + \frac{P' \cdot e}{L} - P_1$$

$$= \frac{234 \times 5}{70} + \frac{20 \times 03'}{70} - 10K$$

$$= 757K \text{ [Reaction Increasing]}$$

Here, $\sum P = ② - ⑪ = 234K$
 $d = 5'$ $P' = 20K$ $e = 03'$
 $L = 70'$ $P_1 = 10K$

Trial 2: Move wheel ③ to A

$$\Delta R = \frac{\sum P d}{L} + \frac{P' e}{L} - P_1$$

$$= \frac{244 \times 8}{70} + \frac{3 \times 20 + 7 \times 20}{70} - 10$$

$$= 2079K \text{ [Reaction Increasing]}$$

Here, $\sum P = ③ - ⑫ = 244K$
 $d = 8'$ $P' = ⑬ = 20K$
 $e = 3'$ $L = 70'$ $P_1 = 10K$
 $e' = 7'$ $P'' = ⑭ = 20K$



Trial 3: Move wheel ④ to A

$$\begin{aligned}\Delta R &= \frac{\sum P \cdot d}{L} + \frac{P' \cdot e}{L} - P_i \\ &= \frac{248^k \times 6'}{70'} + 0 - 36^k \\ &= -14.74^k \text{ [Reaction Decreasing]}\end{aligned}$$

$$\sum P = \textcircled{4} - \textcircled{14} = 248^k$$

$$d = 6' \quad P' = 0$$

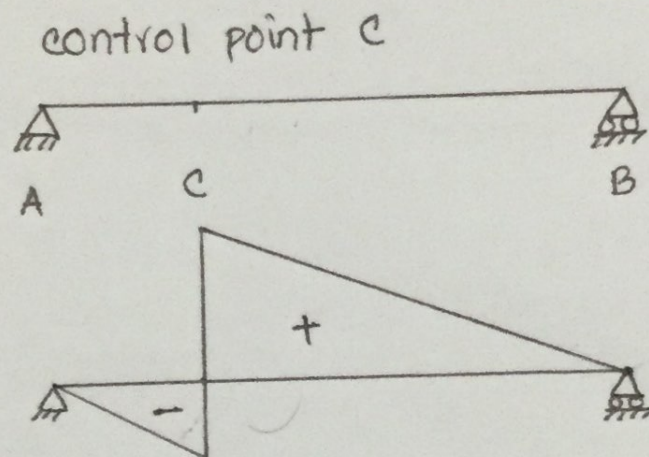
$$P_i = 36^k$$

Hence wheel ③ at A will produce max^m reaction.

$$\begin{aligned}R_A &= \frac{1}{70} \times [36 \times 70 + 36 \times 64 + 36 \times 58 + 36 \times 52 + 10 \times 44 + \\ &\quad 10 \times 40 + 20 \times 31 + 20 \times 27 + 20 \times 23 + 20 \times 11 + 20 \times 7 + 20 \times 3]\end{aligned}$$

$$\therefore R_A = 166.62^k \text{ (max}^m \text{ Reaction)}$$

□ Position of wheel to produce \max^m shear at a section of simple span (Article 62)



$$\Delta V = \frac{\sum P d}{L} + \frac{P' e}{L} + \frac{P_2 e_1}{L} - P_1$$

where, $\sum P$ = summation of wheel loads that stay on the span before and after the movement

d = distance travelled

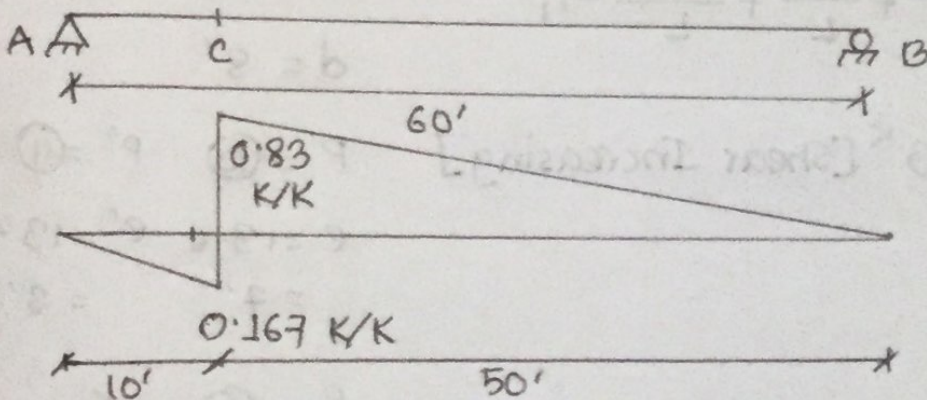
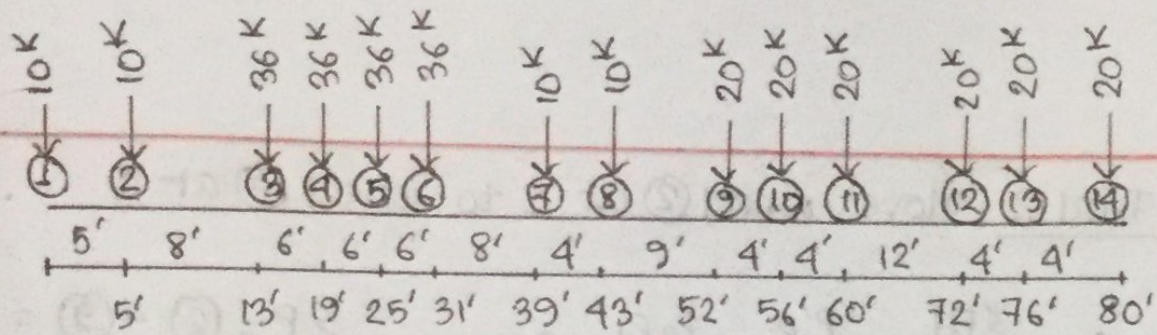
e = distance travelled by P' on the span.

P' = wheel that comes on during the movement

P_2 = wheel that move of the span

e_1 = distance travelled by P_2 before moving of the span

P_1 = wheel that cross point c



Find maximum shear at a distance 10' from the left support. Assume a span length of 60'

Trial 1: Move wheel ① at c to wheel ② at c

$$\Delta V = \frac{\sum P d}{L} + \frac{P' e}{L} + \frac{P_2 e_1}{L} - P_1$$

Here, $\sum P = \text{①} - \text{⑧} = 184^k$ $e =$

$d = 5'$

$P' = \text{⑨} = 20^k$

$e = 3'$

$P_2 = 0$

$P_1 = \text{①} = 10^k$

$$\therefore \Delta V = \frac{184^k \times 5'}{60'} + \frac{20 \times 3}{60} + 0 - 10 = +6.33^k \quad \left[\begin{array}{l} \text{Shear} \\ \text{Increasing} \end{array} \right]$$

Trial 2: Move wheel ② at c to wheel ③ at c

$$\Delta V = \frac{\sum P d}{L} + \frac{P' e}{L} + \frac{P_2 e_1}{L} - P_1$$

$$= 20.03^K \text{ [Shear Increasing]}$$

$$\sum P = \textcircled{2} - \textcircled{9} = 194^K$$

$$d = 8'$$

$$P' = \textcircled{10} \quad P'' = \textcircled{11}$$

$$e = 13 - 6 \quad e' = 13 - 10$$

$$= 7' \quad = 3'$$

$$P_2 = \textcircled{1} = 10^K$$

$$e_1 = \text{আছে আর 3 5'}$$

$$\text{আরও 8' 50 5'}$$

$$P_1 = \text{wheel } \textcircled{2} = 10^K$$

Trial 3: Move wheel ③ at c to wheel ④ at c

$$\Delta V = \frac{\sum P d}{L} + \frac{P' e}{L} + \frac{P_2 e_1}{L} - P_1$$

$$= -12.1^K \text{ [Shear Decreasing]}$$

$$\sum P = \textcircled{3} - \textcircled{11} = 224^K$$

$$d = 6' \quad P' = 0$$

$$P_2 = \textcircled{2} = 10^K$$

$$e_1 = 9'$$

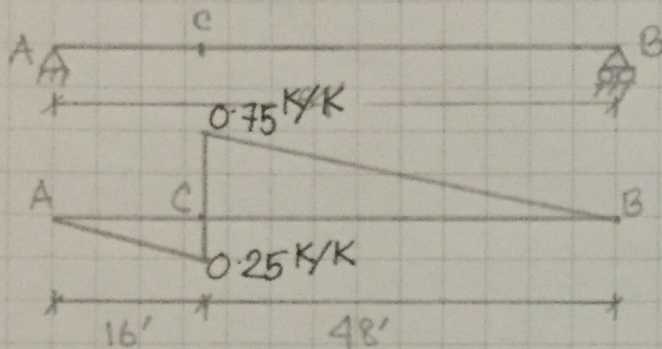
$$P_1 = 36^K$$



Assignment 8

Find maximum shear at quarter point of span, $L = 64'$

Loading is same as shown in previous assignment.



Trial 1: Move wheel ① at C to wheel ② at C

$$\begin{aligned} \Delta V &= \frac{\sum P d}{L} + \frac{P' e}{L} + \frac{P_2 e'}{L} - P_1 & \sum P &= \text{wheel ①} - \text{wheel ⑧} = 184^K \\ &= \frac{184 \times 5}{64} + \frac{20 \times 1}{64} - 10 & d &= 5' \quad L = 64' \quad P' = \text{wheel ⑨} = 20^K \\ &= 4.6875^K \quad [\text{Shear Increasing}] & e &= 4' \quad P_2 = 0 \quad P_1 = \text{wheel ①} = 10^K \end{aligned}$$

Trial 2: Move wheel ② at C to wheel ③ at C

$$\begin{aligned} \Delta V &= \frac{\sum P d}{L} + \frac{P' e}{L} + \frac{P_1 e'}{L} - P_1 & \sum P &= \text{wheel ①} - \text{wheel ⑨} = 204^K \\ &= \frac{204 \times 8}{64} + \frac{20 \times 5 + 20 \times 1}{64} - (10 + 10) & d &= 8' \quad L = 64' \quad P' = \text{⑩} = 20^K \quad e = 5 \\ &= 17.375^K \quad [\text{Shear Increasing}] & P_2 &= 0 & P'' &= \text{⑪} = 20^K \quad e'' = 1 \\ & & P_1 &= \text{wheel ②} - \text{⑫} = 20^K - 10^K \end{aligned}$$

Trial 3: Move wheel ③ at C to wheel ④ at C

$$\begin{aligned} \Delta V &= \frac{234 \times 6}{64} + \frac{0}{64} + \frac{10^K \times 3'}{64} - 36^K & \sum P &= \text{②} - \text{⑪} = 234^K \quad d = 6' \\ &= -13.59^K \quad [\text{Shear Decreasing}] & P' &= 0, \quad P_2 = \text{①} = 10^K \quad e' = 3' \\ & & P_1 &= 36^K \end{aligned}$$

Hence wheel ④ at C will produce max^m shear.

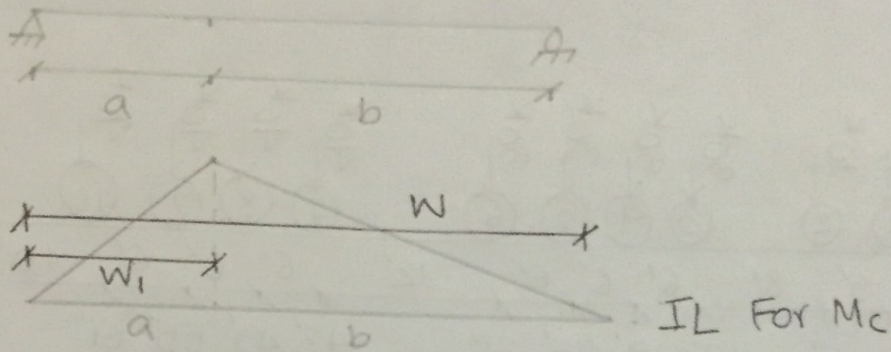
$$\therefore \text{Maximum shear} = \frac{3}{4} \times \frac{1}{48} (36 \times 48 + 36 \times 42 + 36 \times 36 + 36 \times 30 + 10 \times 22 + 10 \times 8 + 20 \times 9 + 20 \times 5 + 20 \times 1)$$

$$- \frac{0.25}{16} (10 \times 8 + 10 \times 3)$$

$$= 86.97 \text{ K}$$

[Ans.]

Maximum moment at a section for a simple span beam



Required Criteria for maximum moment is given by.

$$\frac{W_1}{a} = \frac{W}{L}$$

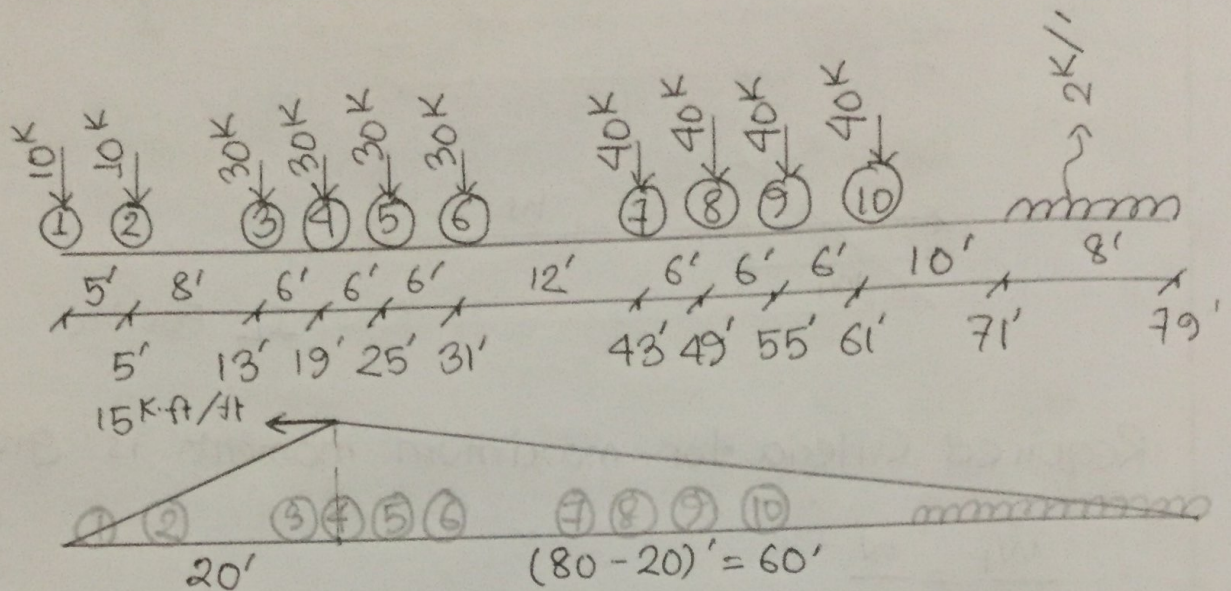
average load on Left of section = average load on span.

The wheel will be placed at c

- Just to the right if $\frac{W}{L} > \frac{W_1}{a}$
- Just to the Left if $\frac{W}{L} < \frac{W_1}{a}$

this criteria is obtained.

Find maximum moment at quarter point of span
 Length 80'. Loading as following:



Hence wheel ④ at C produces maximum moment at quarter point of the 80' span.

Maximum moment at C =

$$\frac{15}{20} [20 \times 14 + 10 \times 8 + 10 \times 1] +$$

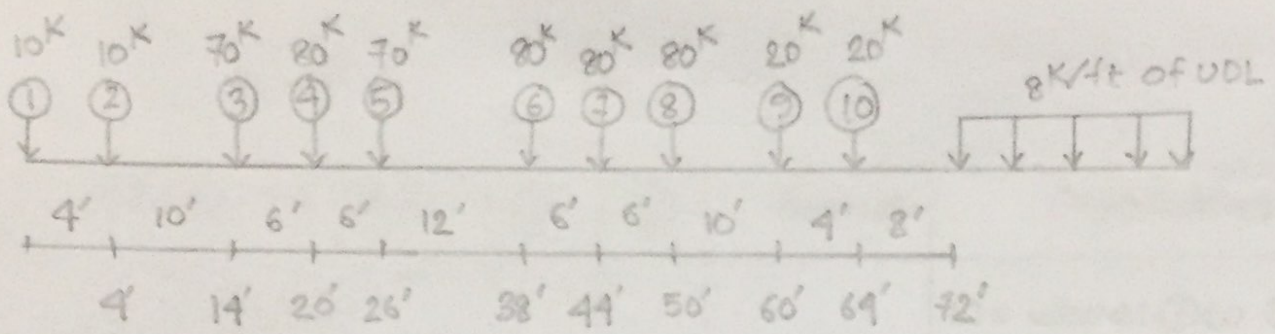
$$\frac{15}{60} [30 \times 60 + 30 \times 54 + 30 \times 48 + 40 \times 36 + 40 \times 30$$

$$+ 40 \times 24 + 40 \times 18] +$$

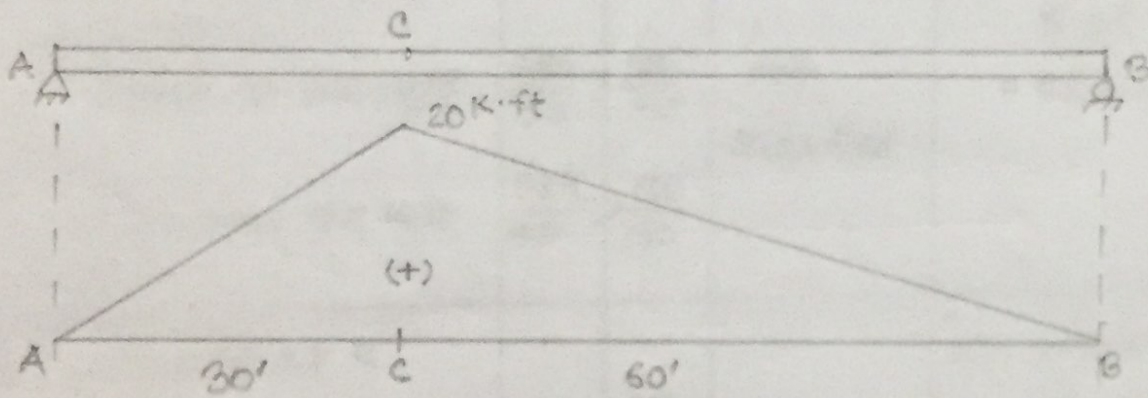
$$\frac{1}{2} \times 8 \times \left(\frac{15}{60} \times 8\right) \times 2 \text{ K}'$$

=

Trial No. #	Position of wheel at c	Remarks	Calculation
1	wheel ③ — just to the left $\frac{304}{80} > \frac{20}{20}$ — just to the right $\frac{304}{80} > \frac{50}{20}$	Criteria not satisfied	JTL $W = \text{wheel ①} - \text{wheel ⑩}$ $+ 2' \text{ from the udl}$ $= 300^k + 4^k$ $= 304^k$ $W_1 = 20^k$ JTR $W = 304^k$ $W_1 = 50^k$
2	wheel ④ — Just to the left $\frac{316}{80} > \frac{50}{20}$ — Just to the right $\frac{316}{80} < \frac{80}{20}$	Criteria satisfied	JTL $W = \text{wheel ①} + \text{⑩} + 8' \text{ of the}$ $\text{udl} = 300^k + 16^k$ $= 316^k$ $W_1 = 50^k$ JTR $W = 316^k \quad W_1 = 80^k$



For the axial load shown, calculate maximum moment at $\frac{1}{3}$ point of a simply supported beam of span $90'$



Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{W_1}{a}$	Remark	Calculation
1	wheel ③ at C a) Just to the right b) Just to the left	$\frac{536}{90}$ $\frac{536}{90}$	$> \frac{20}{30}$ $> \frac{90}{30}$	not satisfied	$W = \text{wheel } \textcircled{1} \text{ to } \textcircled{10} + 2' \text{ of UDL}$ $= 520 + 2 \times 8 = 536^k$ $W_1 = \text{wheel } \textcircled{1} \text{ and } \textcircled{2}$ $= 20^k$
2	wheel ④ at C a) Just to the right b) Just to the left	$\frac{584}{90}$ $\frac{584}{90}$	$> \frac{90}{30}$ $> \frac{170}{30}$	not satisfied	$W = \text{wheel } \textcircled{1} \text{ to } \textcircled{10} + 8' \text{ of UDL}$ $= 520 + 8 \times 8 = 584^k$
3	wheel ⑤ at C a) Just to the right b) Just to the left	$\frac{632}{90}$ $\frac{632}{90}$	$> \frac{170}{30}$ $< \frac{240}{30}$	Criteria satisfied	$W = \text{wheel } \textcircled{1} \text{ to } \textcircled{10} + 14' \text{ of UDL}$ $= 632^k$ $W_1 = \text{wheel } \textcircled{1} \text{ to } \textcircled{4}$ $= 170^k$

Hence, maximum moment occurs when wheel ⑤ is at C

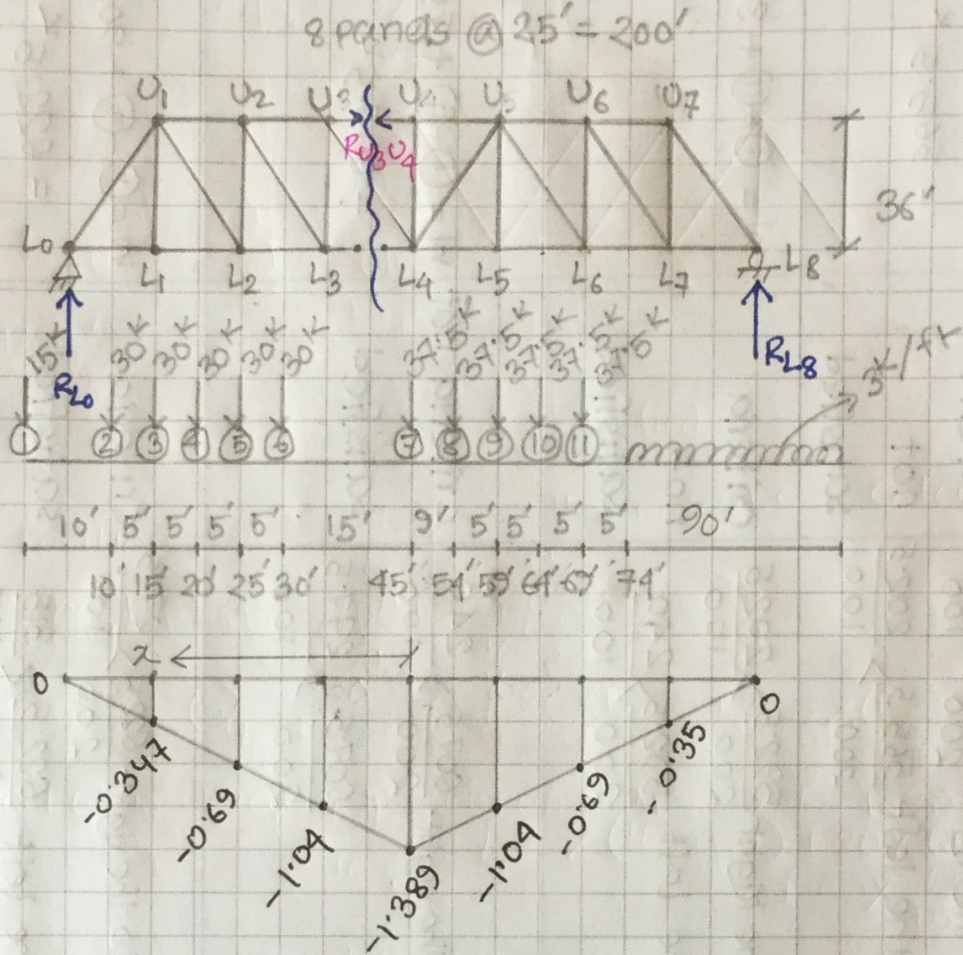
$$\begin{aligned} \therefore \text{Maximum moment} &= \frac{20}{60} [70 \times (60) + 80 \times (48 + 42 + 36) + 20 \times (26 + 22)] \\ &+ \frac{20}{30} [80 \times 24 + 70 \times 14 + 10 \times (8 + 4)] \\ &+ \frac{1}{2} \times 8 \times 14 \times \left(\frac{20}{60} \times 14\right) \end{aligned}$$

$$\therefore M_{\max} = 7541.33 \text{ kip-ft. [Ans!]}$$

Assignment - 9



Find Maximum stress in U_3U_4 for the Truss and Loads:-



$$\sum M_{L4} = 0 \Rightarrow R_{L0} \times 100 - 1 \times x + R_{U3U4} \times 36 = 0$$

Position of Load	R_{L0}	R_{U3U4}
L0	1	0
L1	0.875	-0.347
L2	0.75	-0.69
L3	0.625	-1.04
L4	0.5	-1.389
L5	0.375	-1.04
L6	0.25	-0.69
L7	0.125	-0.347
L8	0	0

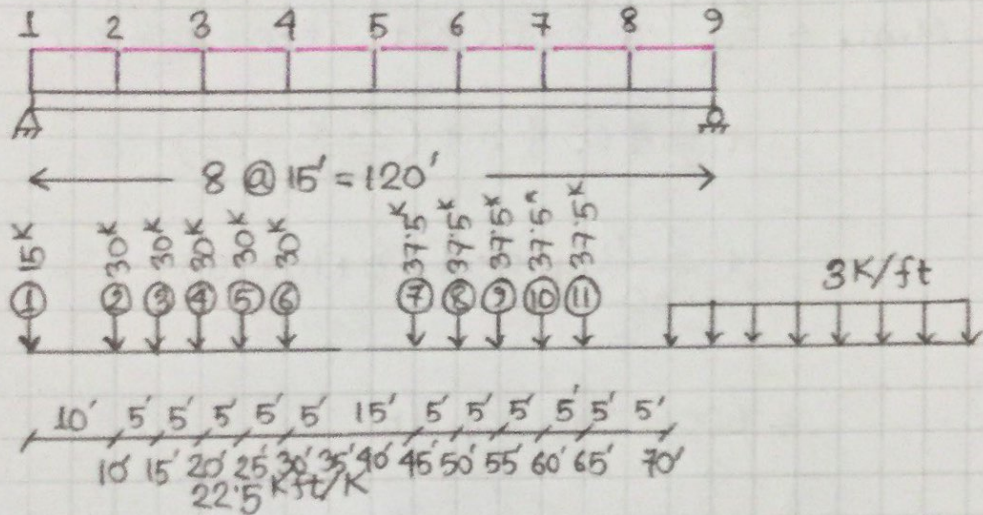
Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{Wl}{a}$	Remarks	Calculation
1	wheel ⑦ just to the right just to the left	$\frac{565.5}{200}$	$\frac{165}{100}$ $\frac{202.5}{100}$	criteria not satisfied	$W = \textcircled{1} - \textcircled{11} + 71'$ of UDL $= 565.5^k$
2	wheel ⑧ just to the right just to the left	$\frac{592.5}{200}$	$\frac{202.5}{100}$ $\frac{240}{100}$	criteria not satisfied	$W = \textcircled{1} - \textcircled{11} + 80'$ of UDL
3	wheel ⑨ just to the right	$\frac{607.5}{200}$	$\frac{240}{100}$ $\frac{277.5}{100}$	criteria not satisfied	$W = \textcircled{1} - \textcircled{11} + 85'$ of UDL $= 607.5^k$
4	wheel ⑩ just to the right just to the left	$\frac{622.5}{200}$	$\frac{277.5}{100}$ $\frac{315}{100}$	criteria satisfied ^_^	$W = \textcircled{1} - \textcircled{11} + 90'$ of UDL $= 622.5^k$

Hence, the maxm stress is = $\frac{1.38}{100} [37.5 \times 100 + 37.5 \times 95] + \frac{1}{2} \times 90 \times (\frac{1.38}{100} \times 90)$
 $= 515.94^k$
 [Ans.] $+ \frac{1.38}{100} [37.5 \times 95 + 37.5 \times 90 + 37.5 \times 85 + 30 \times 70 + 30 \times 65 + 30 \times 60 + 30 \times 55 + 30 \times 50 + 15 \times 40]$

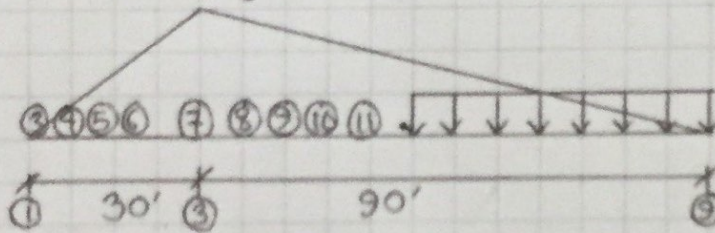
Assignment - 10



Find maximum moment at panel point 3-4 (3rd panel) of 8 panel span with floor beams. Panel Length 15'. Loading is shown below (axle)



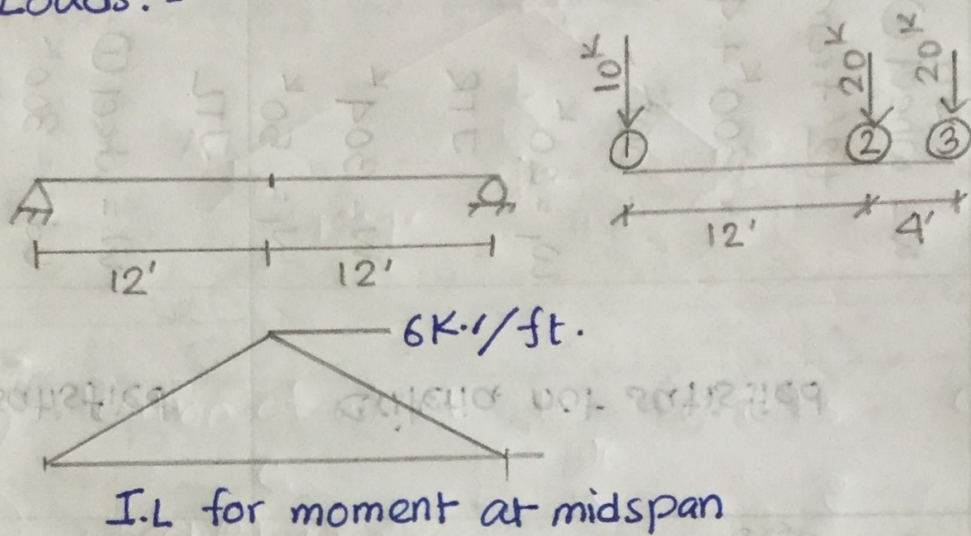
Solution:



Trial No	Position of axle load	$\frac{W}{L}$	$\frac{W_1}{a}$	Remarks	Calculation
1	wheel ⑥ at 3 a) Just to right	$\frac{502.5}{120}$	$\frac{135}{30}$	Criteria not satisfied	$W = \text{wheel } \textcircled{1} \text{ to wheel } \textcircled{11} + 50' \text{ of UDL}$ $= 352.5 + 50 \times 3 = 502.5^k$
	b) just to left	$\frac{487.5}{120}$	$\frac{150}{30}$		$W_1 = \text{wheel } \textcircled{1} - \text{wheel } \textcircled{5} = 135^k$ $W = \text{wheel } \textcircled{2} \text{ to wheel } \textcircled{11} + 50'$ $= 352.5 - 15 + 50 \times 3 = 487.5^k$
2	wheel ⑦ at 3 a) just to right	$\frac{502.5}{120}$	$\frac{120}{30}$	Criteria satisfied	$W = \text{wheel } \textcircled{3} - \textcircled{11} + 65' \text{ of UDL}$ $= 502.5^k$ $W_1 = \textcircled{3} \text{ to } \textcircled{6}$
	b) just to left	$\frac{472.5}{120}$	$\frac{127.5}{30}$		$W = \textcircled{4} \text{ to } \textcircled{11} + 65' \text{ of UDL}$ $= 475.5^k$
		394	< 425		$W_1 = \textcircled{4} \text{ to } \textcircled{7} = 127.5^k$

Lec - 7

Find Max^m moment for span 24'. Assume following axial Loads:-



Procedure

1. Find the wheel that gives max^m moment at the center.

When wheel ① is placed at midspan:-

$$M = 10 \times 6 = 60 \text{ k}\cdot\text{ft}$$

When wheel ② is placed at midspan:-

$$M = 20 \times 6 + 20 \times \frac{6}{12} \times 8 = 20 \times 6 + 20 \times 4 = 200 \text{ k}\cdot\text{ft}$$

Hence, wheel ② at midspan gives maximum moment.

2. Place the wheels and the C.G. of the wheels on the span such that they are equidistant from the center or midspan.

3. If the wheel does not satisfy the IL criterion for maximum moment at that point, descend the wheel and try with the next wheel.

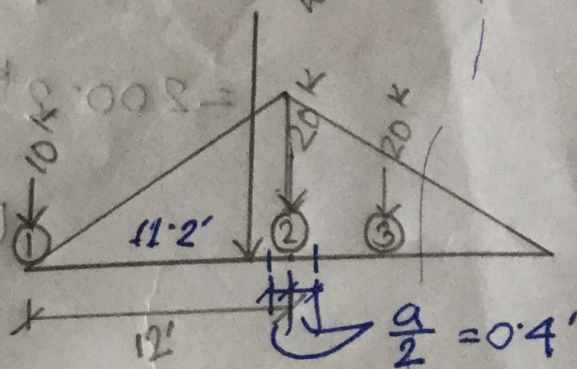
Let, C.G. of the wheels on span is 'x' ft from wheel ①

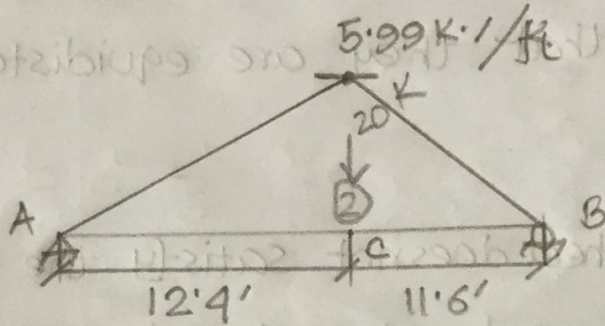
$$x = \frac{10^k \times 0 + 20^k \times 12 + 20^k \times 16}{10 + 20 + 20} = 11.2'$$

Let, a = distance between that wheel and C.G. of the wheels on span [Here wheel ②]

$$a = 12' - 11.2' = 0.8'$$

$$\frac{a}{2} = 0.4'$$





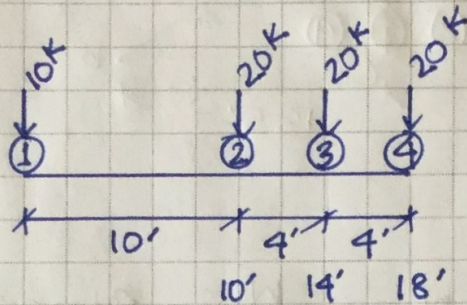
Trial No:	Position of wheel at	Remarks	Calculation
1	C		
	wheel ② J.T.T.L	$\frac{W}{L} > \frac{W_1}{a}$	Criteria satisfied
	J.T.T.R	$\frac{50}{24} < \frac{10+20}{12.4}$	

$$\begin{aligned} \text{max}^m \text{ moment, } M_{\text{max}} &= \frac{5.99}{11.6} [20 \times 11.6 + 20 \times (11.6 - 4)] \\ &+ \frac{5.99}{12.4} [10 \times (12.4 - 12)] \\ &= 200.2 \text{ k}\cdot\text{ft} \end{aligned}$$

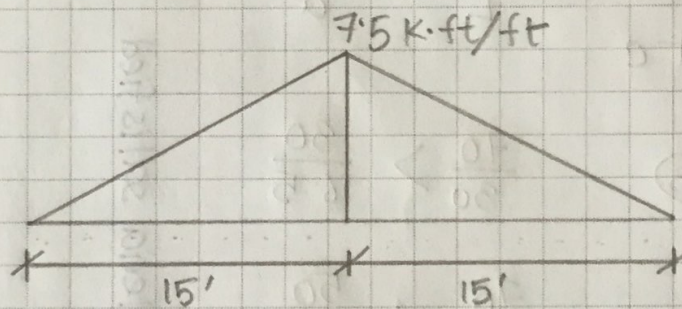
[Ans!]



Find Max^m moment for span 30'. Consider the Following axle Loads:-



Solⁿ:



IL diagram for bending moment at midspan

When wheel ① is placed at midspan:- $M = 10 \times 7.5 + 20 \times \frac{7.5}{15} + 20 \times \frac{7.5}{15} + 20 \times \frac{7.5}{15}$
 $= 135 \text{ K-ft/ft}$

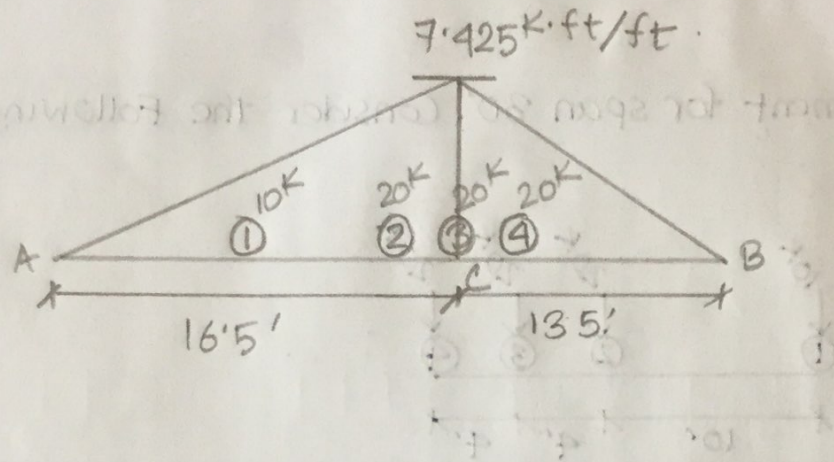
When wheel ② is placed at midspan, $M = 355 \text{ K-ft/ft}$

When wheel ③ is placed at midspan, $M = 375 \text{ K-ft}$

When wheel ④ is placed at midspan, $M = 330 \text{ K-ft}$

Hence, wheel ③ at C produces max^m moment.

$$x = 12' \quad a = 15' - 12' = 3' \quad a/2 = 1.5'$$



Trial	Position of Wheel at c	$\frac{W}{L}$	$\frac{Wl}{a}$	Remarks	Calculation
1	wheel ③	$\frac{70}{30} >$	$\frac{30}{16.5}$	Criteria satisfied	
		$\frac{70}{30} <$	$\frac{50}{16.5}$		

$$\text{Max}^m \text{ moment, } M_{\text{max}} = \frac{7.425}{13.5} [20 \times 13.5 + 20 \times (13.5 - 4)]$$

$$+ \frac{7.425}{16.5} [20 \times (16.5 - 4) + 20 \times (16.5 - 14)]$$

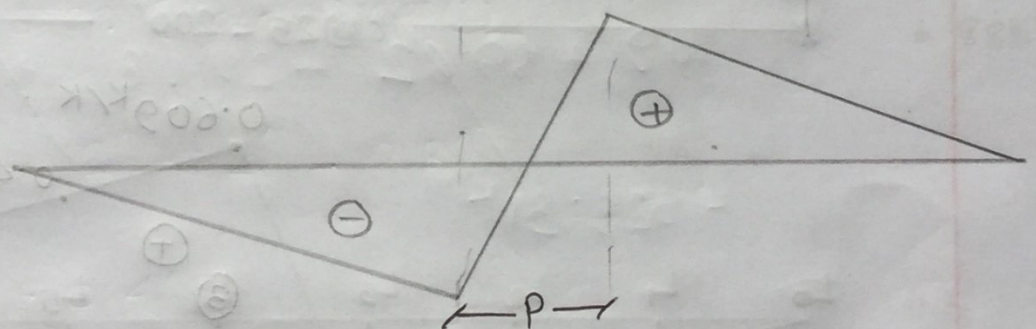
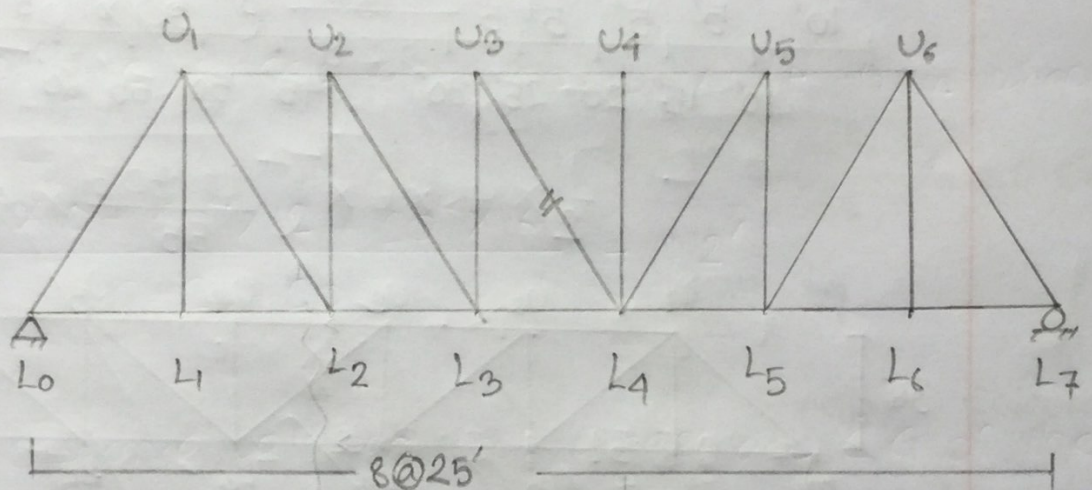
$$= 377.003 \text{ k}\cdot\text{ft}$$

[Ans.]

Art. 69 \square Position of wheel to produce maximum

web stress in
truss

panel shear
in span
with floor
beams.



IDEALIZED IL for stress in U_3L_4

Required criteria for max^m, $\frac{W}{L} = \frac{W_1}{P}$

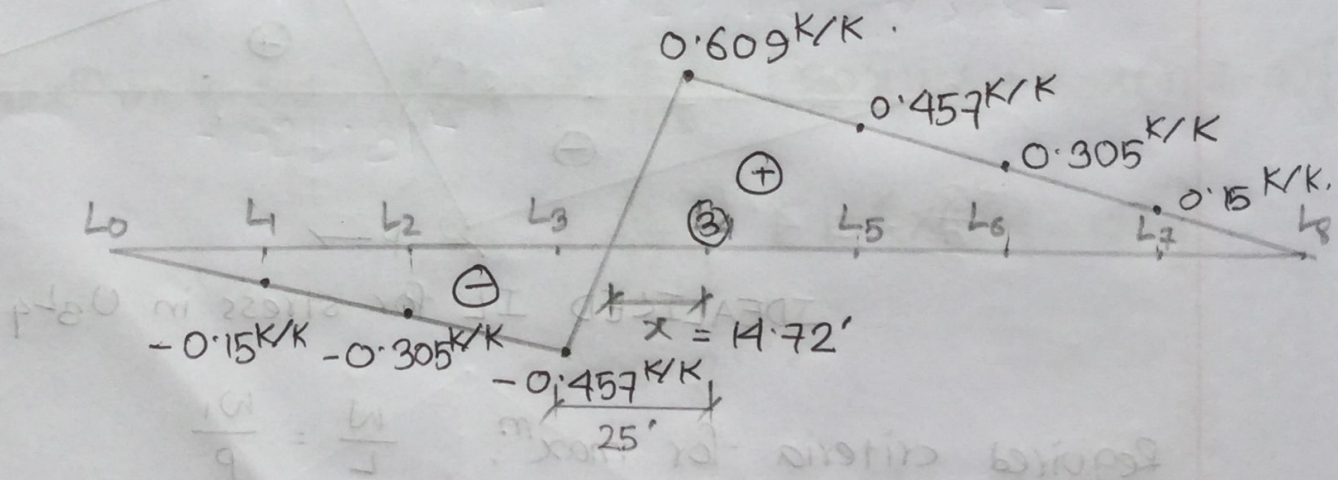
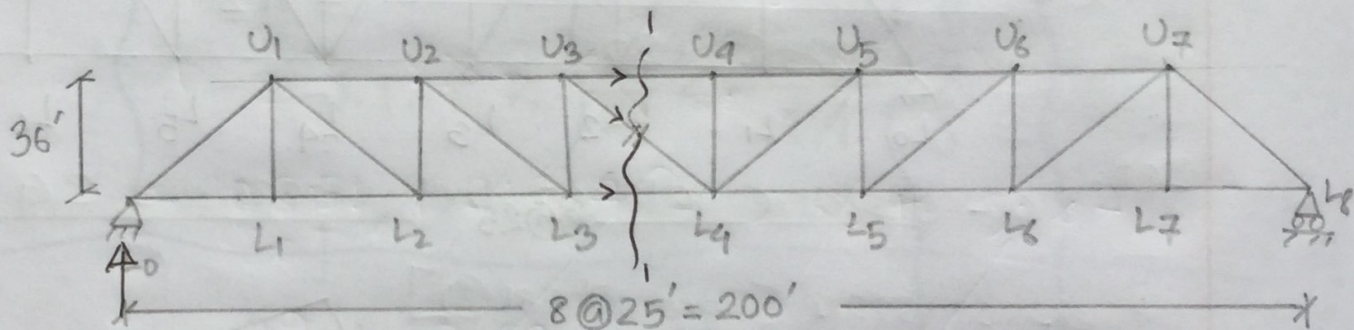
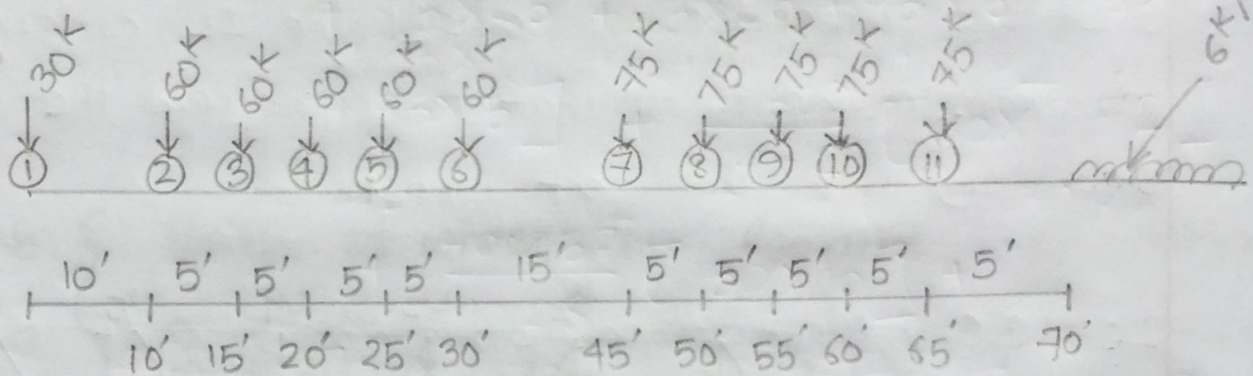
W = Load on span

W_1 = Loads on panel

L = span length

P = panel length

Prob 221 Find max^m Tension and Compression in U_3L_4 of the truss shown. for the following axle loads.



At Section 1-1 [When 1^k load moves from L₀ to L₃]

$$\uparrow \sum F_y = 0 \text{ [Load at } L_3]$$

$$R_{L0} - 1^k - U_{3L4} \times \frac{36}{\sqrt{36^2 + 25^2}} = 0$$

$$-\frac{5}{8} - 1 = \frac{U_{3L4}}{\sqrt{36^2 + 25^2}} \times 36$$

$$U_{3L4} = -0.457 \text{ k/k}$$

[1^k load at L₄] $U_{3L4} = (1 - 0.5) \times \frac{\sqrt{36^2 + 25^2}}{36} = 0.609 \text{ k/k}$

For max^m tension wheel will move from L₄

Trial No.	Position of wheel	$\frac{w}{L}$	$\frac{w_1}{P}$	Remarks	Calculation
1.	wheel ① at L ₄ J.T.T.B	$\frac{885}{200}$	$> \frac{0}{25}$		$W = 1 - 11 + 30'$ $= 885^k$
	J.T.T.L	$\frac{885}{200}$	$> \frac{30}{25}$		
2.	wheel ② at L ₄ J.T.T.R	$\frac{945}{200}$	$> \frac{30}{25}$		$W = 1 - 11 + 40'$
	J.T.T.L	$\frac{945}{200}$	$> \frac{90}{25}$		
3.	wheel ③ at L ₄ JTTR	$\frac{975}{200}$	$> \frac{90}{25}$	criteria satisfied.	$W = 1 - 11 + 45'$
	JTTL	$\frac{975}{200}$	$< \frac{150}{25}$		

Hence wheel ③ at L_4 will produce max^m tension.

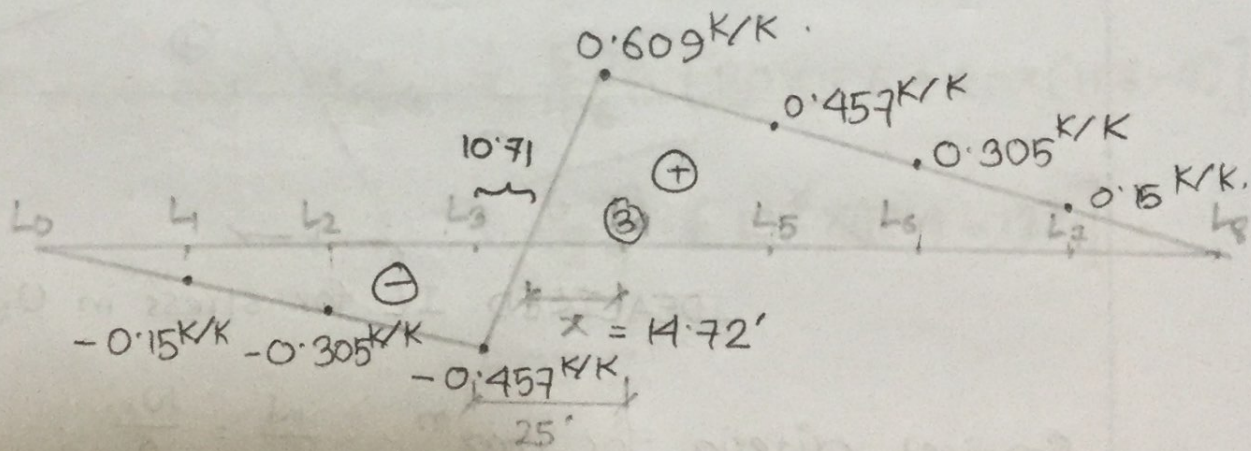
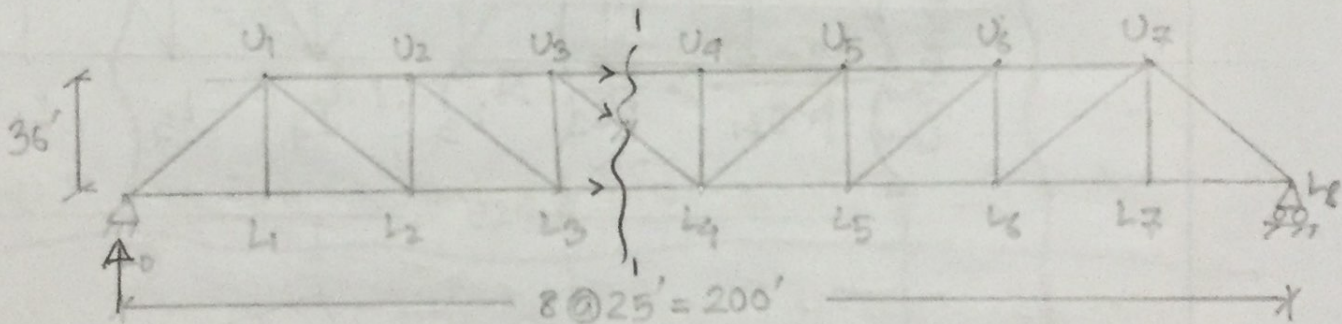
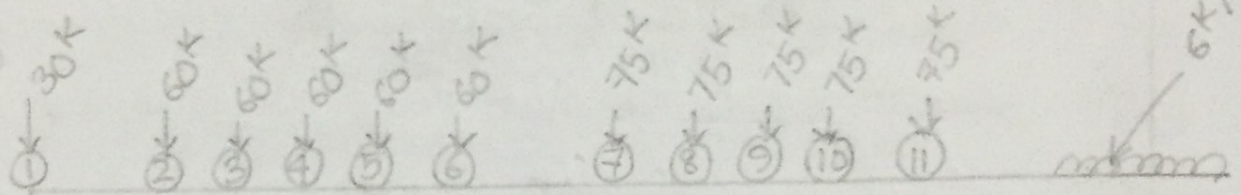
$$T_{\max} = \frac{0.609}{100} [60 \times (100 + 95 + 90 + 85) + 75 \times (70 + 65 + \dots)]$$

$$+ \frac{0.609}{14.72} [60 \times 9.72] - \frac{0.457}{10.28} [30 \times 10]$$

$$+ \frac{1}{2} \times 45 \times 6 \times \left(\frac{0.609}{100} \times 45 \right)$$

$$= 319.703^{\text{K}} = 320^{\text{K}} \quad [\text{Ans!}]$$

Prob 221 Find max^m Tension and Compression in U_3L_4 of the truss shown. for the following axle loads.



Compute maximum Compression in U_3L_4 of the truss

Load moving from L_6 to L_8 [Point to be noted]

Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{W_1}{P}$	Remarks	Calculation
1	wheel ① at L_3 a) just to the right b) just to the left	$\frac{735}{200}$	$> \frac{30}{25}$ > 0	criteria not satisfied	$W = \textcircled{1} - \textcircled{11} + 5'$ $= 735'$ $W_1 = \textcircled{1} = 30^k$ $W_1 = 735'$ $W = 0^k$
2	wheel ② at L_3 a) just to the right b) just to the left	$\frac{795}{200}$	$> \frac{90}{25}$ $> \frac{30}{25}$	criteria not satisfied	$W = \textcircled{1} \text{ to } \textcircled{11} +$ $15' \text{ of UDL}$ $= 705 + 90^k$ $= 795^k$ $W_1 = \textcircled{1} - \textcircled{2} = 90^k$
3	wheel ③ at L_3 a) just to the right b) just left	$\frac{825}{200}$	$< \frac{150}{25}$ $> \frac{90}{25}$	criteria satisfied	$W = \textcircled{1} - \textcircled{11} +$ $20' \text{ of UDL}$ $= 705 + 120$ $= 825^k$ $W_1 = \textcircled{1} - \textcircled{3}$ $= 150^k$ $W_1 = \textcircled{1} - \textcircled{2}$ $= 90^k$

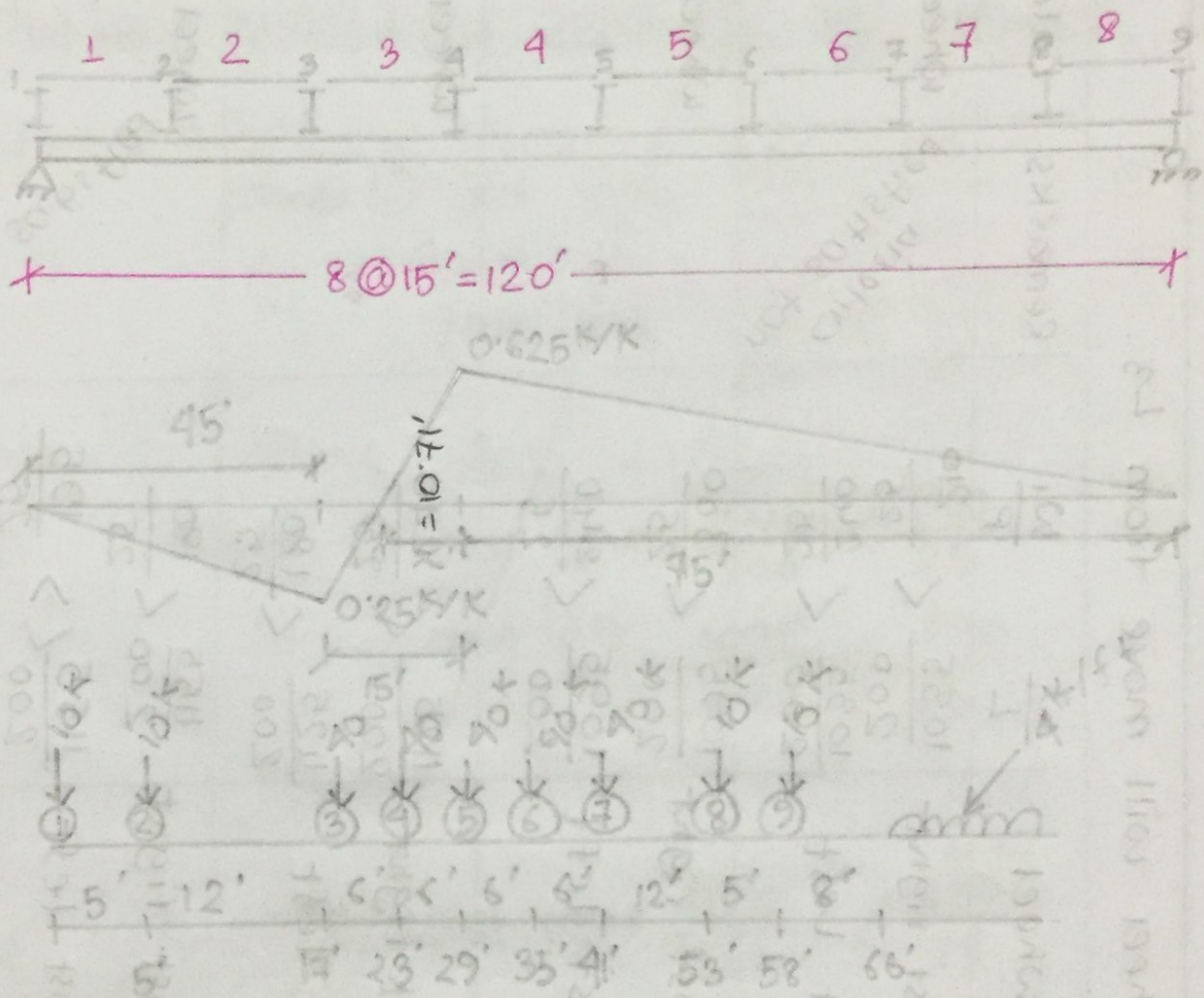
so, wheel ③ at L_3 gives maximum compression.

∴ Max^m Compression =

$$\begin{aligned} & \frac{0.457}{75} [60(70+75+65+60) + 75(45+40+35+25)] \\ & + \frac{1}{2} \times 20 \times 6 \times \frac{0.457}{75} \times 20 + \frac{0.457}{10.71} \times 60 \times (10.71-8) \\ & - \frac{0.609}{14.28} (30 \times 4.28) \end{aligned}$$

$$= 195^k \text{ for axle load [Ans!].}$$

Find max^m shear in 3rd panel of span of 8 panels
 [panel length 15' each] Loading as following



Area of positive shear > area of negative shear.

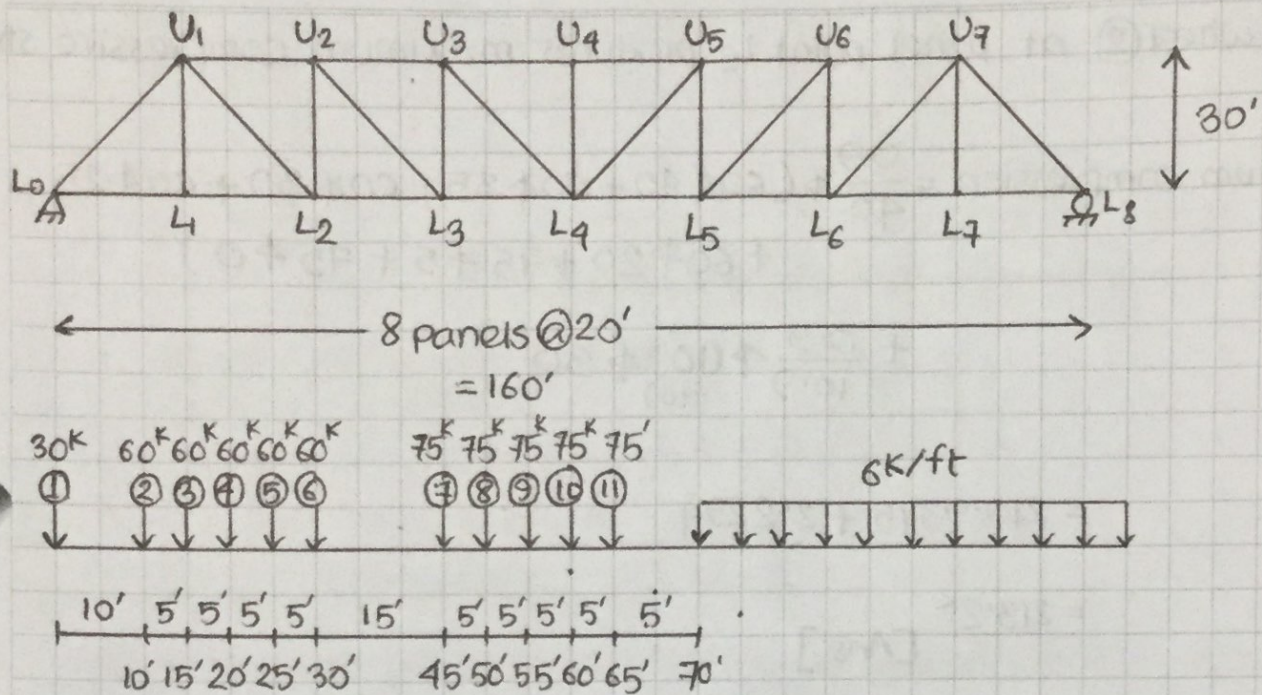
Hence wheel will move from right to left

Trial	Position of wheel	$\frac{W}{L}$	$\frac{W}{P}$	Remarks	Calculation
1	wheel ① Just right at point 4 Just Left	$\frac{276}{120} > \frac{20}{15}$ $\frac{276}{120} > \frac{10}{15}$	$\frac{20}{15}$ $\frac{10}{15}$	not satisfied	$240 + 9' \text{ of udl} = 276^k$
2	wheel ② Just right at point 4 Just left	$\frac{296}{120} > \frac{16}{15}$ $\frac{296}{120} > \frac{20}{15}$	$\frac{16}{15}$ $\frac{20}{15}$	not satisfied	$240 + 14' \text{ of udl} = 296^k$
3	wheel ③ Just right at point 4 Just left	$\frac{344}{120} > \frac{10}{15}$ $\frac{344}{120} < \frac{50}{15}$	$\frac{10}{15}$ $\frac{50}{15}$	Criteria is satisfied	$240 + (12+14)' \text{ of udl} = 344^k$

Hence, wheel 4 at panel point 4 will produce maximum shear

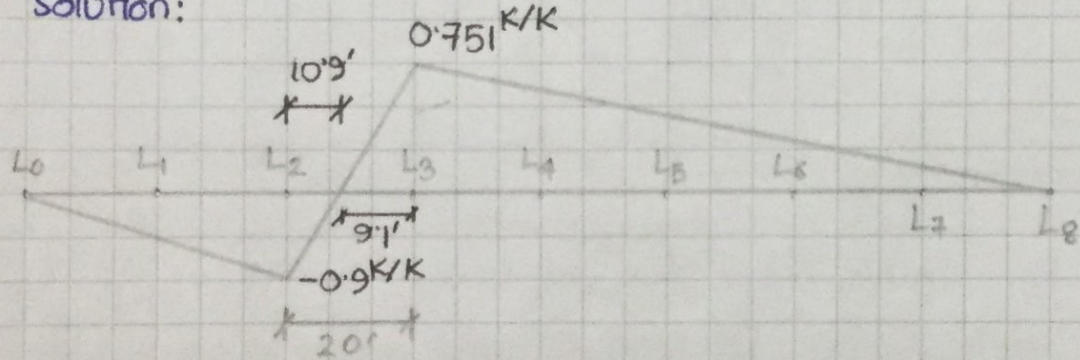
$$\begin{aligned}
 \max^m \text{ shear} &= \frac{0.625}{75} [40 \times (75 + 69 + 63 + 57 + 51) + 10 \times (39 + 34)] \\
 &+ \frac{0.625}{10.71} [0] - \frac{0.25}{4.29} [1.29 \times 10] - \frac{0.25}{45} [43 \times 10] \\
 &+ \frac{1}{2} \times 26 \times 4 \times \left(\frac{0.625}{75} \times 26 \right) \\
 &= 111.08 + 0 - 0.75 - 2.389 + 11.267 = 119.2 = 120^k
 \end{aligned}$$

Assignment -12



Find maximum stress in U_2L_3 of the truss shown. Axle Load given.

Solution:



Max^m stress occurs in Compression.

Load is moving from Left support to L_2

Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{W_1}{a}$	Remarks	Calculation
1	wheel ① at L_2 a) Just to the right b) Just to the left	$\frac{330}{160}$	$\frac{30}{20}$ $\frac{0}{20}$	not satisfied	$W = \text{wheel ①} - \text{wheel ⑥} = 330\text{K}$
2	② at L_2 a) right b) just to the left	$\frac{480}{160}$ $\frac{405}{160}$	$\frac{30}{20}$ $\frac{30}{20}$	satisfied	$W = \text{wheel ①} - \text{wheel ⑧} = 480\text{K}$ $W = \text{wheel ①} - \text{wheel ⑦} = 405\text{K}$

Hence, wheel ② at panel point L_2 produces maximum compressive stress

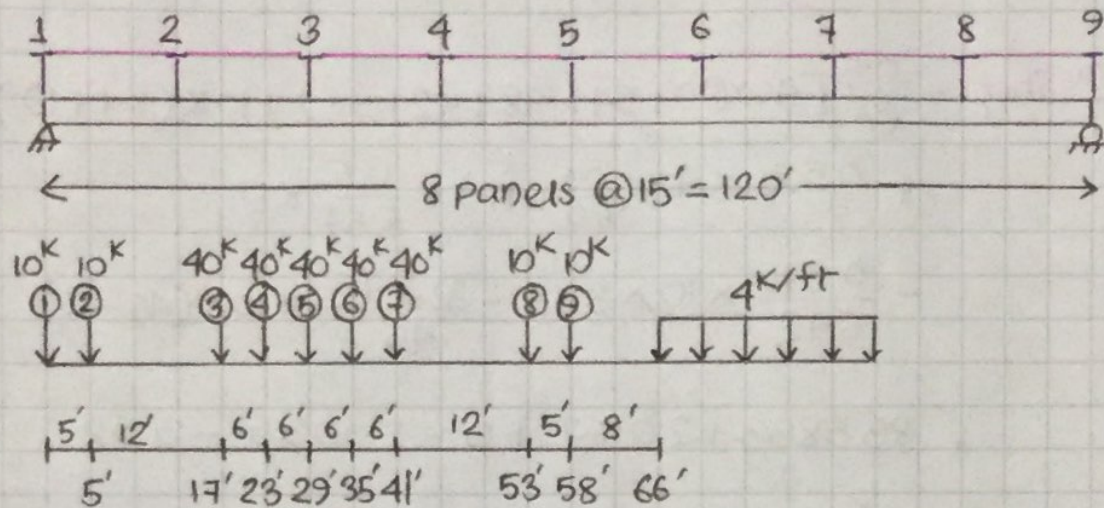
$$\text{Maximum compression} = \frac{0.9}{40} \times (60 \times 40 + 60 \times 35 + 60 \times 30 + 60 \times 25 + 60 \times 20 + 75 \times 5 + 75 \times 0)$$

$$+ \frac{0.9}{10.9} \times (10.9 \times 30 - 10)$$

$$= 210.9375 + 2.2294$$

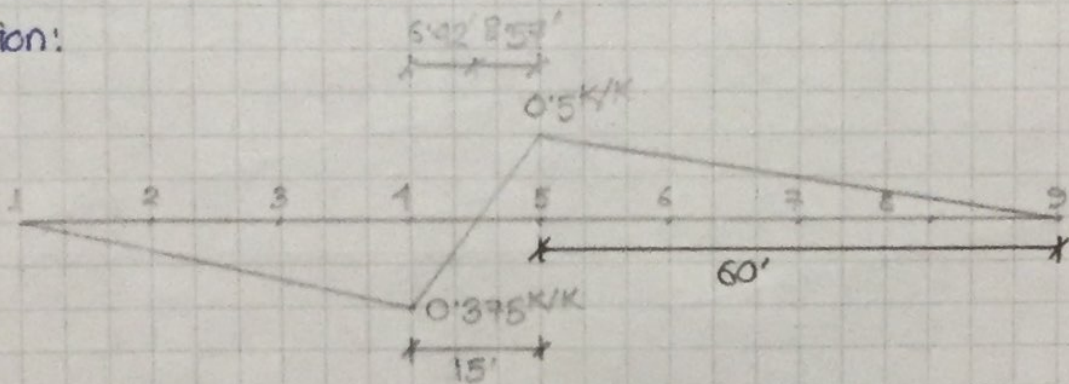
$$= 213.2^k \quad [\text{Ans.}]$$

Assignment-13



Calculate maximum shear in 4th Panel of a span of 120'. Assume the given axle loadings.

Solution:



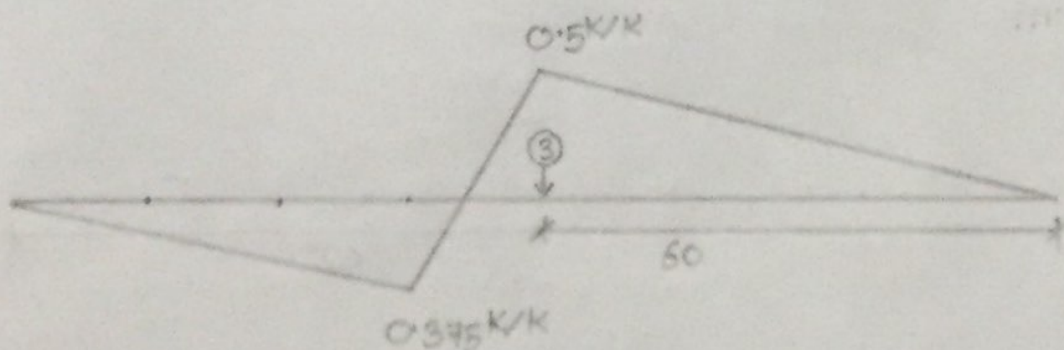
Here, area of positive shear > area of negative shear. Load will be on 5.

Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{W_1}{a}$	Remarks	Calculation
1	wheel ② at panel P.5 a) just to the right b) just to the left	$\frac{240}{120}$	$\frac{10}{15}$	not satisfied	$\left\{ \begin{aligned} N &= \text{wheel ①} - \text{wheel ③} = 240^K \quad W_1 = 10^K \\ W &= 240^K \quad W_1 = \text{wheel ①} + \text{wheel ②} \end{aligned} \right.$
2	wheel ③ at panel P.5 a) just to the right b) just to the left	$\frac{284}{120}$	$\frac{10}{15}$	criteria satisfied	$\left\{ \begin{aligned} W &= \text{wheel ①} - \text{wheel ②} + 11' \text{ of UDL} \\ &= 340 + 4 \times 11^K = 284^K \\ W_1 &= \text{wheel ②} + \text{wheel ③} = 50^K \end{aligned} \right.$

Hence, maximum shear occurs at 4th panel when wheel ③ is on panel point 5.

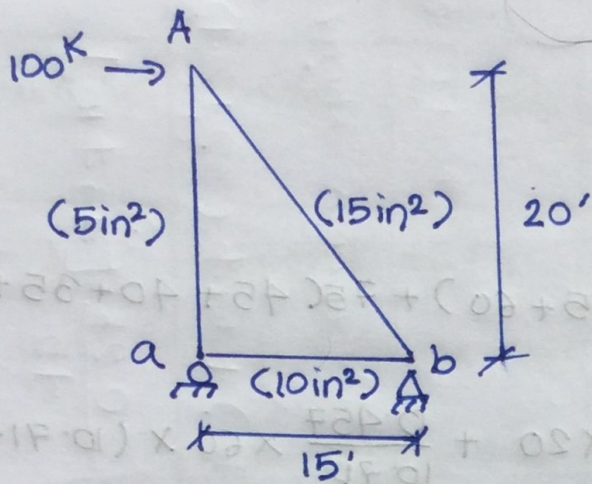
$$\begin{aligned} \therefore \text{Max}^m \text{ Shear} &= \frac{0.5}{60} [40 \times (60 + 54 + 48 + 42 + 36) + 10 \times (24 + 19)] \\ &+ 0.5 \times 11 \times 4 \times \left(\frac{0.5}{60} \times 11\right) + \frac{0.5}{8.57} \times 0 \\ &- \frac{0.375}{642} \times 10 \times 3.43 - \frac{0.375}{45} \times 10 \times (45 - 2) \\ &= 83.5833 + 2.0167 + 0 - 2.0035 - 3.58 \\ &= 80 \text{ K} \end{aligned}$$

[Ans.]



VIRTUAL WORK

Problem: 1



Compute Horizontal Deflection at A. The following data are given:-

$$E = 30,000 \text{ ksi}$$

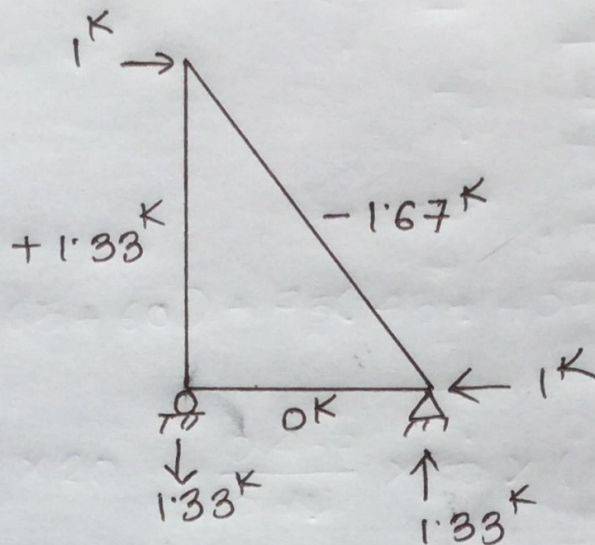
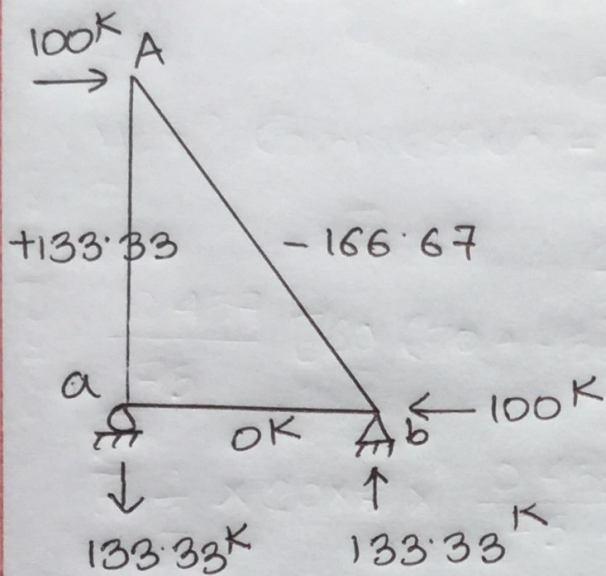
$$\alpha_t = \frac{1}{15,000} / ^\circ\text{F}$$

Temperature change for bottom chord member = 40°F Incr.

diagonal chord member = 30°F Decreasing

Vertical chord member = 20°F Decreasing

Soln: ① P-force Analysis:-



② Q-force Analysis:-

position	Bar	Length (ft)	Area (in ²)	$\frac{L}{A}$	F_Q (k)	F_P (k)	$F_Q F_P \frac{L}{A}$	t	$F_Q t L$
horizontal	ab	15	10	1.5	0	0	0	+40	0
Diagonal	Ab	25	15	1.67	-1.67	-166.67	463.16	-30	1250 25
Vertical	Aa	20	5	4	1.33	133.33	710.92	-20	-5332

$$\sum F_P F_Q \frac{L}{A} = 1174$$

$$\sum F_Q t L = 717$$

③ Applying principal of Virtual work:-

$$1 \cdot \delta = \frac{1174}{30,000} + 717 \times \frac{1}{15,000}$$

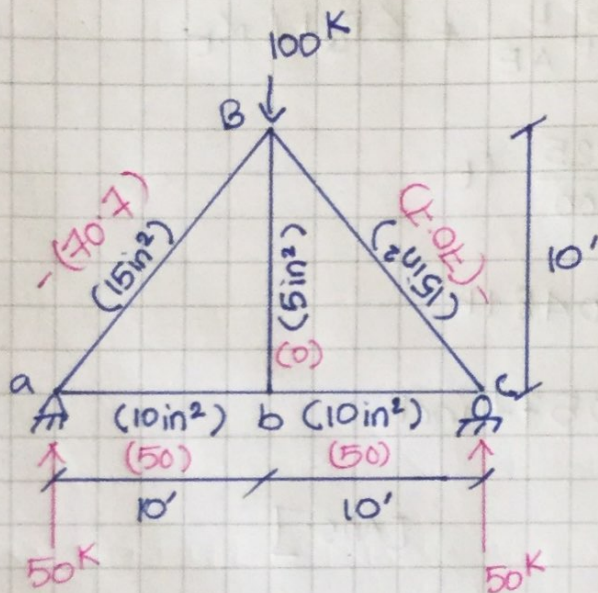
$$\therefore \delta = 0.087 \text{ ft} = 0.0439 \text{ ft} = 0.523 \text{ inch}$$

[Ans:]

Assignment - 16



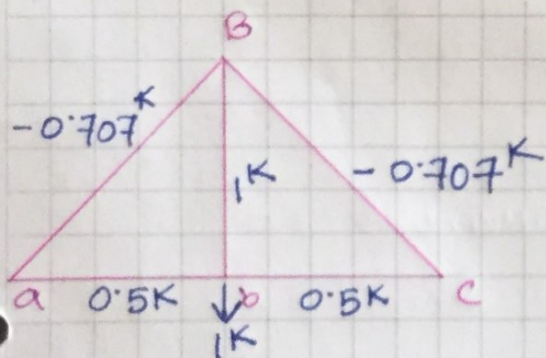
Solⁿ: step 1. Real force analysis



Compute Vertical Deflection at B

Given = $E = 30,000 \text{ Ksi}$

step 2: Virtual force analysis



position	bar	L	A	$\frac{L}{A}$	F_Q	F_P	$F_Q F_P \frac{L}{A}$	t	$F_Q t L$
horizontal bar	ab	10	10	1	+05	+50	+25	0	0
	bc	10	10	1	+05	+50	+25	0	0
diagonal bar	AB	14.14	15	0.94	-0.707	-70.7	47.12	0	0
	BC	14.14	15	0.94	-0.707	-70.7	47.12	0	0
vertical	Bb	10	5	2	1	0	0	0	0

Step 3: Applying Principle of Virtual Work:

$$W_S = W_D$$

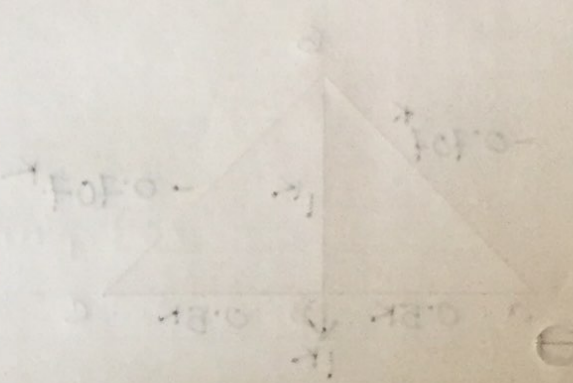
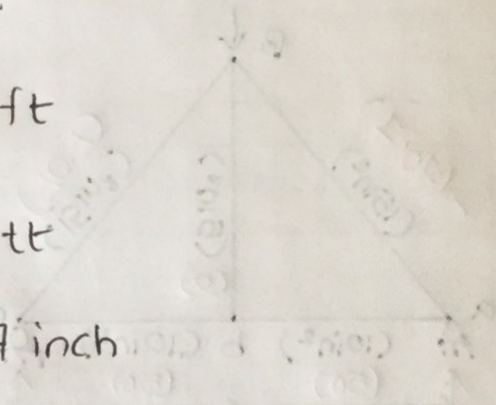
$$\Rightarrow Q \cdot \delta = \sum F_Q P_P \frac{L}{AE} + \sum P_Q t L \alpha_t$$

$$\Rightarrow 1 \cdot \delta = \frac{144.25}{30,000} \text{ ft}$$

$$\Rightarrow \delta = 0.0048 \text{ ft}$$

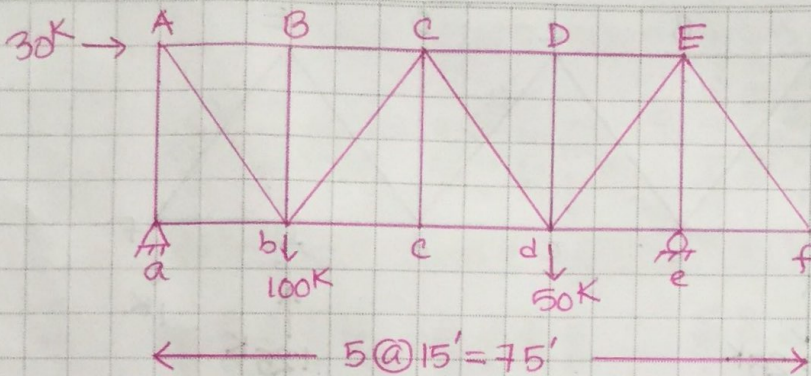
$$\therefore \delta = 0.0577 \text{ inch}$$

[Ans.]



Member	Length (ft)	Force (k)	Virtual Force (k)	Product (k-ft)
1	30	144.25	1	144.25
2	45	100	0	0
3	45	100	0	0
4	60	0	0	0
5	30	0	0	0
6	30	0	0	0
7	30	0	0	0
8	30	0	0	0
9	30	0	0	0
10	30	0	0	0
11	30	0	0	0
12	30	0	0	0
13	30	0	0	0
14	30	0	0	0
15	30	0	0	0
16	30	0	0	0
17	30	0	0	0
18	30	0	0	0
19	30	0	0	0
20	30	0	0	0
21	30	0	0	0
22	30	0	0	0
23	30	0	0	0
24	30	0	0	0
25	30	0	0	0
26	30	0	0	0
27	30	0	0	0
28	30	0	0	0
29	30	0	0	0
30	30	0	0	0
31	30	0	0	0
32	30	0	0	0
33	30	0	0	0
34	30	0	0	0
35	30	0	0	0
36	30	0	0	0
37	30	0	0	0
38	30	0	0	0
39	30	0	0	0
40	30	0	0	0
41	30	0	0	0
42	30	0	0	0
43	30	0	0	0
44	30	0	0	0
45	30	0	0	0
46	30	0	0	0
47	30	0	0	0
48	30	0	0	0
49	30	0	0	0
50	30	0	0	0

Assignment 17



Given,
 $E = 30,000 \text{ Ksi}$
 $\alpha_t = 1.150,000 / ^\circ\text{F}$

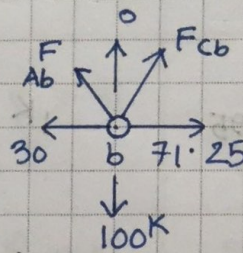
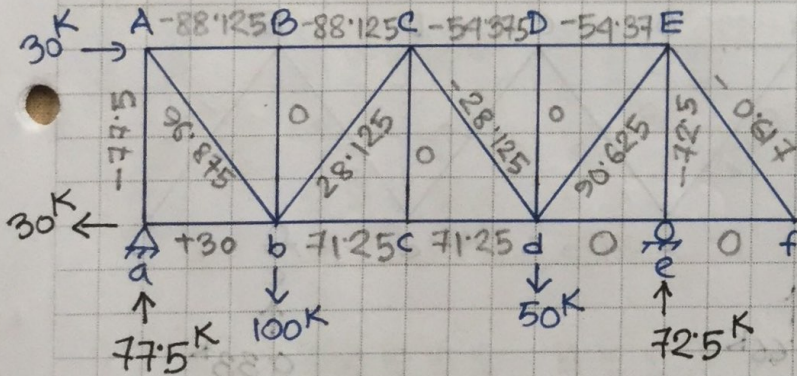
Given, Temperature change,

- ① bottom and top chord member 40°F increase
- ② Diagonal chord member 30°F decrease
- ③ Vertical chord member 20°F decrease

Compute (a) vertical deflection at f (b) Horizontal Deflection at A

Solution:

Step 1: P force analysis



$$\sum F_x = 0$$

$$-30 + 71.25 + F_{cb} \times \frac{15}{25} - F_{Ab} \times \frac{15}{25} = 0$$

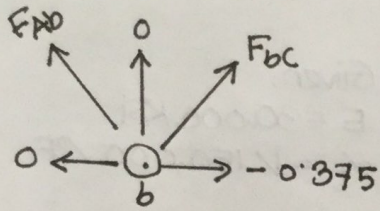
$$F_{cb} \times \frac{15}{25} - F_{Ab} \times \frac{15}{25} = -41.25 \quad \text{--- ①}$$

$$\sum F_y = 0$$

$$-100 + F_{cb} \times \frac{20}{25} + F_{Ab} \times \frac{20}{25} + 0 = 0$$

$$F_{cb} \times \frac{20}{25} + F_{Ab} \times \frac{20}{25} = 100 \quad \text{--- ②}$$

$$F_{cb} = 28.125 \text{ K} \quad F_{Ab} = 96.875 \text{ K}$$



$$\sum F_x = 0$$

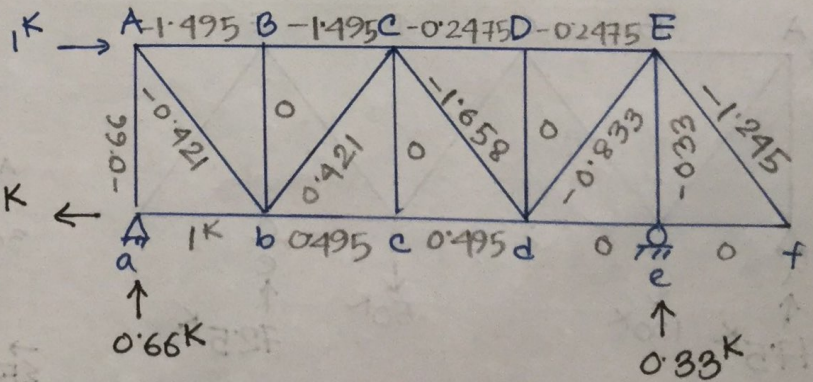
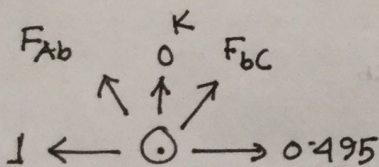
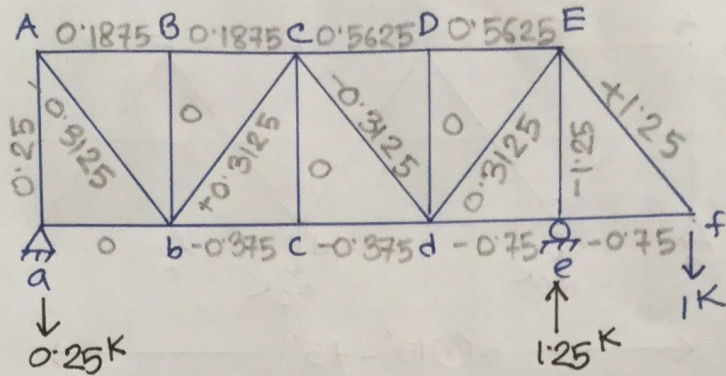
$$-0.375 + F_{bc} \times \frac{15}{25} - F_{ab} \times \frac{15}{25} = 0$$

$$15F_{bc} - 15F_{ab} = 9.375 \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$F_{bc} \times \frac{20}{25} + F_{ab} \times \frac{20}{25} = 0$$

$$20F_{bc} + 20F_{ab} = 0 \quad \text{--- (2)}$$



$$\sum F_x = 0$$

$$-1 + 0.495 + F_{bc} \times \frac{15}{205} - F_{ab} \times \frac{15}{205} = 0$$

$$F_{bc} \times \frac{15}{205} - F_{ab} \times \frac{15}{205} = 0.505 \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$F_{bc} \times \frac{20}{25} + F_{ab} \times \frac{20}{25} = 0 \quad \text{--- (2)}$$

$$F_{bc} = 0.421 \quad F_{ab} = -0.421$$

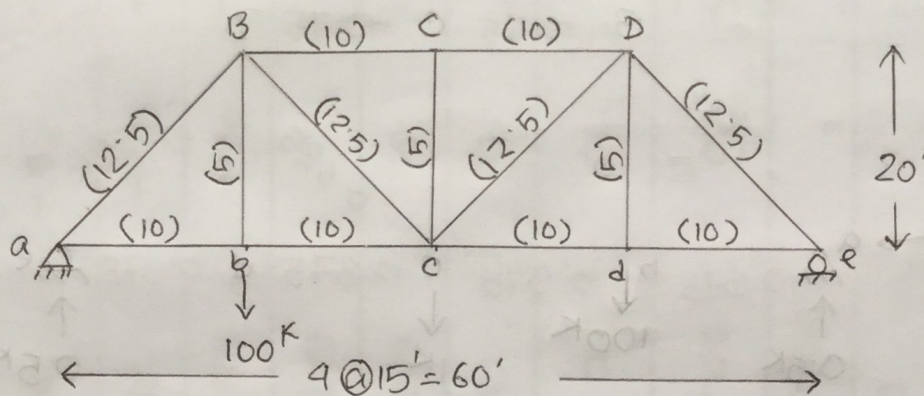
39 SQUARE HOUSE 33 SQUARE HOUSE WIDE 7.88" TALL 9.32" WIDE 200.01mm X TALL 234. mm

position	bar	L	A	L/A	Fp	FQ	FpFQ L/A	t	FatL
horizontal	ab	15	10	1.5	30	1	45	40	600
	bc	15	10	1.5	71.25	0.495	52.9	40	297
	cd	15	10	1.5	71.25	0.495	52.9	40	297
	de	15	10	1.5	0	0	0	40	0
	ef	15	10	1.5	0	0	0	40	0
	AB	15	10	1.5	-88.125	-1.495	197.62	40	-897
	BC	15	10	1.5	-88.125	-1.495	197.62	40	-897
	CD	15	10	1.5	-54.375	-0.2475	20.17	40	-148.5
	DE	15	10	1.5	-54.375	-0.2475	20.187	40	-148.5
	Diagonal	Ab	25	15	1.67	96.875	-0.421	-67.974	-30
bC		25	15	1.67	28.125	0.421	19.734	-30	-315.75
Cd		25	15	1.67	-28.125	-1.658	77.72	-30	1248.5
dE		25	15	1.67	90.625	-0.833	-125.82	-30	625.75
ef		25	15	1.67	-0.167	-1.245	0.3465	-30	933.75
Vertical	Aa	20	5	4	-77.5	-0.66	24.6	-20	264
	Bb	20	5	4	0	0	0	-20	0
	Cc	20	5	4	0	0	0	-20	0
	Dd	20	5	4	0	0	0	-20	0
	Ee	20	5	4	-72.5	-0.33	95.7	-20	132
							790.728		2301



$\therefore \delta_A = 790/E + 2301 \times L = 0.0417 \text{ foot (right)}$

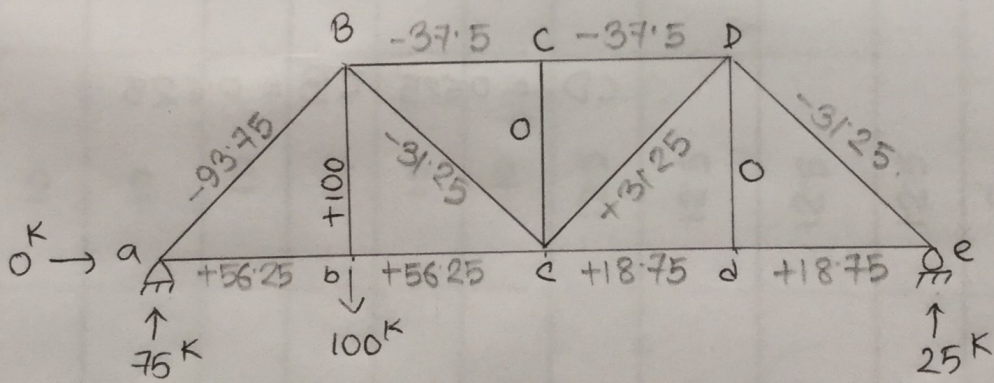
Example 8.1 [Page 256 Norris]



- a) Compute the vertical component of deflection due to applied load 100^k shown. $E = 30 \times 10^3 \text{ kip/in}^2$
- b) Compute the vertical component of deflection of joint c, due to a decrease of temperature of 50°F in the bottom chord only.
 $\alpha_t = \sqrt{150,000} \text{ per } ^\circ\text{F}$

Solution:

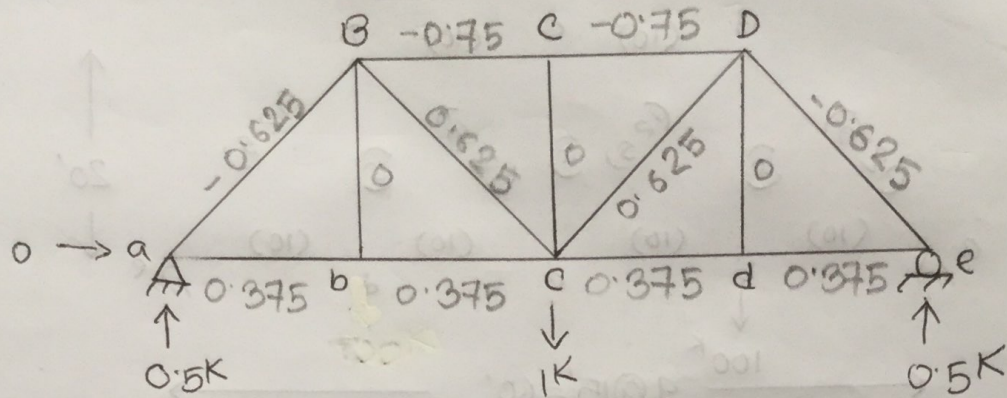
Step 1: Real Force analysis:



joint a, $\sum F_y = 0$
 $75 + F_{ab} \times \frac{20}{25} = 0$
 $F_{ab} = -93.75$ $F_{ab} = 56.25$
 $\sum F_x = 0$, $0 - 93.75 \times \frac{15}{25} + F_{ab} = 0$

joint B, $\sum F_x = 0$ $F_{De} = -31.25$
 $-F_{De} \times \frac{15}{25} - 18.75 = 0$
 $\sum F_y = 0$
 $-F_{Bc} \times \frac{20}{25} + 100 + (-93.75) \times \frac{20}{25} = 0$
 $F_{Bc} = -31.25$

Step: 2



100% shown $E = 30 \times 575 \times 10^{-6} = 17.25 \text{ mm}$

(d) Compute the vertical component of deflection of joint C due to a

decrease in length of top chord only

$$\sum F_x = 0 \Rightarrow 375 + 375 - CD \times \frac{15}{25} - CB \times \frac{15}{25} = 0$$

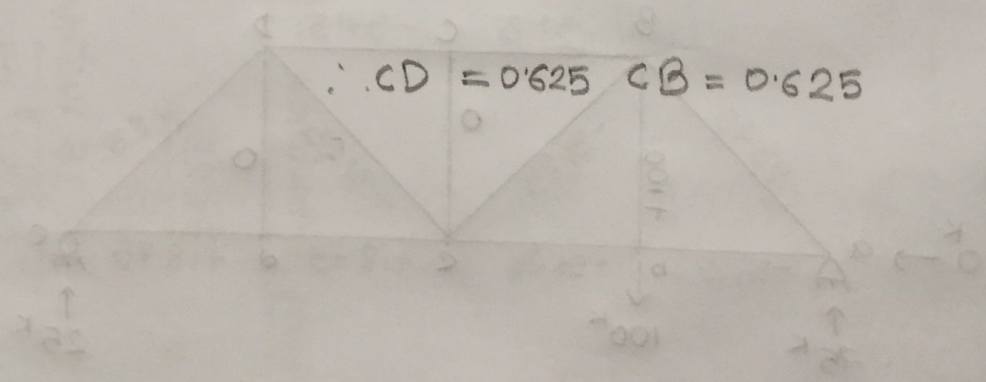
$$CD \times 15 - CB \times 15 = 0$$

Step 2: Real force analysis

$$\sum F_y = 0 \Rightarrow CB \times \frac{20}{25} + CD \times \frac{20}{25} - 1 = 0$$

$$CD \times 20 + CB \times 20 = 25$$

$\therefore CD = 0.625 \quad CB = 0.625$



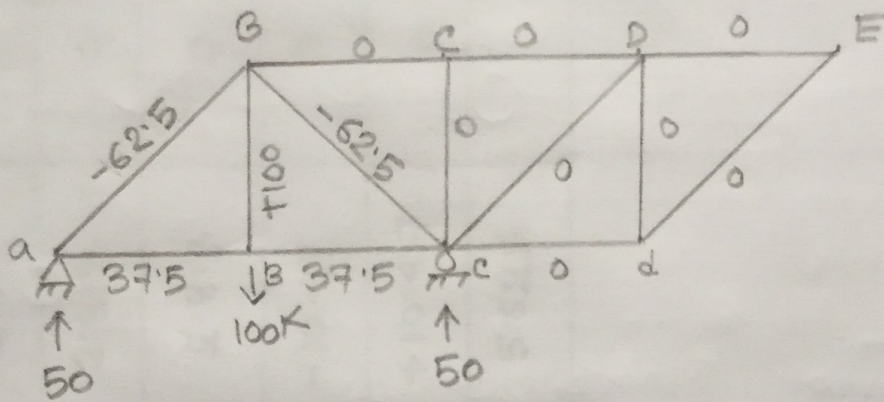
Bar	L	A	$\frac{L}{A}$	F_Q	F_P	$F_Q F_P \frac{L}{A}$	t	Force
ab	15	10	1.5	0.375	56.25	31.64	-50	-281.25
bc	15	10	1.5	0.375	56.25	31.64	-50	-281.25
cd	15	10	1.5	0.375	18.75	10.55	-50	-281.25
de	15	10	1.5	0.375	18.75	10.55	-50	-281.25
BC	15	10	1.5	-0.75	-37.5	+42.19	0	0
CD	15	10	1.5	-0.75	-37.5	+42.19	0	0
AB	25	12.5	2	-0.625	-93.75	+117.19	0	0
BC	25	12.5	2	+0.625	-31.25	-39.06	0	0
CD	25	12.5	2	+0.625	+31.25	+39.06	0	0
De	25	12.5	2	-0.625	-31.25	+39.06	0	0
Bb	20	5	4	0	+100	0	0	0
Cc	20	5	4	0	0	0	0	0
Dd	20	5	4	0	0	0	0	0
Σ						325.01		-1125

Horizontal

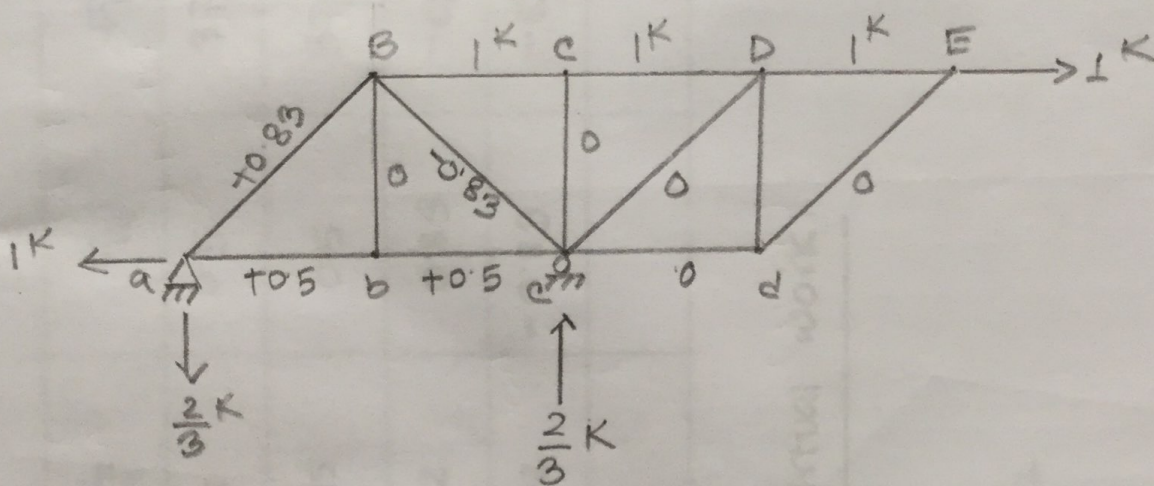
Diagonal

Vertical

Step: 1 Real Force Analysis



Step: 2 Virtual Force Analysis



$$0.06 = \frac{5005}{E}$$

$$= \frac{5005 \times 10^3}{2091.8 \times 10^3}$$

$$= 2.39$$

Bar	L	A	$\frac{L}{A}$	F_Q	F_P	$F_Q F_P \frac{L}{A}$	F	$F_Q L$
ab	1.5	10	0.15	0.5	37.5	2.813	0	0
bc	1.5	10	0.15	0.5	37.5	2.813	0	0
aB	2.5	12.5	0.2	0.83	-62.5	-10.417	0	0
Bc	2.5	12.5	0.2	-0.83	-62.5	+10.417	0	0
Σ						5.625		

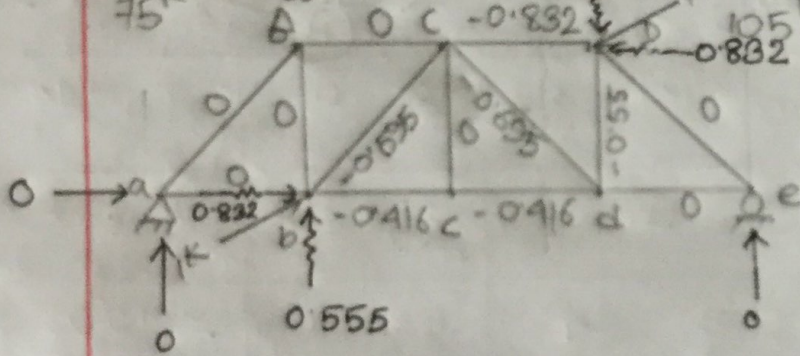
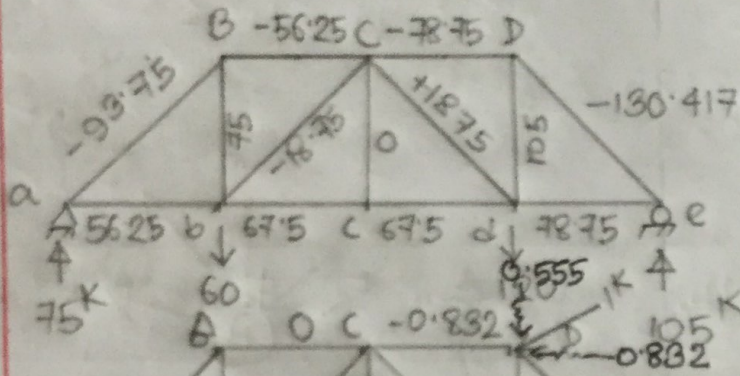
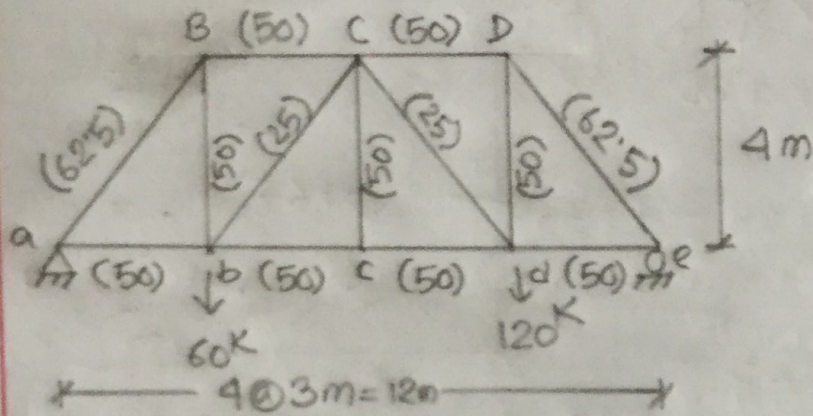
Step 3: Applying principles of virtual work:

$$1 \cdot \Delta_E = \frac{5.625}{E}$$

$$= \frac{5.625 \text{ kN}^2 \text{ m/cm}^2}{20.7 \times 10^3 \text{ kN/cm}^2}$$

$$\therefore \Delta_E = 0.00027174 \text{ m (right)}$$

Example 8.4



$-0.832 \times 4 - 2 \times 4 = 0$

Position	Bar	L	A	L/A	Fq	Fp	Fq Fp L/A	t	Fq t L
Horizontal	ab	3	50	0.06	0	56.25	0	-10	0
Horizontal	bc	3	50	0.06	-0.416	67.5	-1.6848	-10	12.48
Horizontal	cd	3	50	0.06	-0.416	67.5	-1.6848	-10	12.48
Horizontal	de	3	50	0.06	0	78.75	0	-10	0
Horizontal	BC	3	50	0.06	0	-56.25	0	40	0
Horizontal	CD	3	50	0.06	-0.832	-78.75	3.9312	40	-99.84
Diagonal	aB	5	62.5	0.08	0	-93.75	0	0	0
Diagonal	bC	5	25	0.2	-0.693	-18.75	2.59875	0	0
Diagonal	Cd	5	25	0.2	0.693	18.75	2.59875	0	0
Diagonal	De	5	62.5	0.08	0	-130.417	0	0	0
Vertical	Bb	4	50	0.08	0	75	0	0	0
Vertical	Cc	4	50	0.08	0	0	0	0	0
Vertical	Dd	4	50	0.08	-0.55	105	-4.62	0	0
							1.1391		-74.88

$$a) \sum Qd = \sum F_Q F_P \frac{L}{AE}$$

$$\Rightarrow 1^{\text{KN}} (\delta_b^{\rightarrow}) + 1^{\text{KN}} (\delta_D^{\leftarrow}) = \frac{1}{E} \times 1.1391$$

$$\Rightarrow 1^{\text{KN}} (\delta_{b-D}^{\rightarrow\leftarrow}) = \frac{1.1391}{20.7 \times 10^3}$$

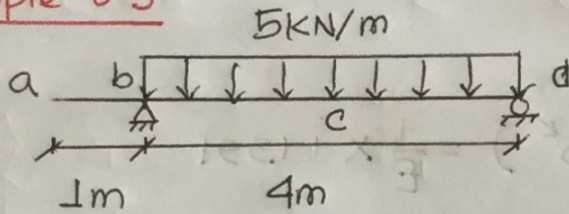
$$\therefore \delta_{b-D}^{\rightarrow\leftarrow} = 5.5 \times 10^{-5} \text{ m (together)}$$

$$b) \sum Qd = \sum F_Q t L \alpha_t$$

$$1^{\text{KN}} (\delta_{b-D}^{\rightarrow\leftarrow}) = \frac{1}{75,000} (-74.88)$$

$$(\delta_{b-D}^{\rightarrow\leftarrow}) = -9.984 \times 10^{-4} \text{ m (apart)}$$

Example 8.5

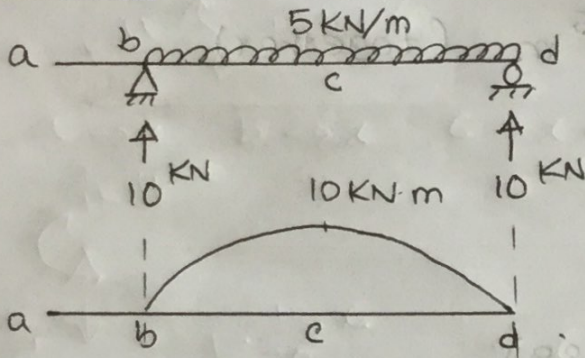


Compute the vertical deflection of 'a' due to the load shown.

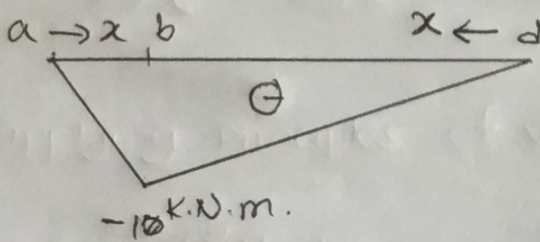
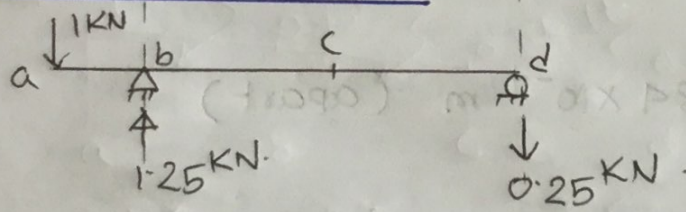
$E = 207 \times 10^3 \text{ MPa}$

$I = 10^{-4} \text{ m}^4$

Solution:



Virtual Force analysis:-



Segment	M_Q	M_P
ab	$-x$	0
$0 < x \leq 5/5$		
db	$-\frac{x}{4}$	$10x - 5x \cdot \frac{x}{2}$
$0 < x \leq 20/5$		

applying principles of virtual work:-

$$Q \cdot \delta = \int_a^b \frac{M_P M_Q}{EI} ds + \int_d^b \frac{M_P M_Q}{EI} ds$$

$$= 0 + \int_1^4 \frac{(10x - \frac{5x^2}{2}) (-\frac{x}{4})}{EI} dx$$

$$= \frac{1}{EI} \int_0^4 \left(-\frac{10x^2}{4} + \frac{5x^3}{8} \right) dx$$

$$= \frac{1}{EI} \left[-\frac{10x^3}{12} \right]_0^4 + \left[\frac{5x^4}{32} \right]_0^4$$

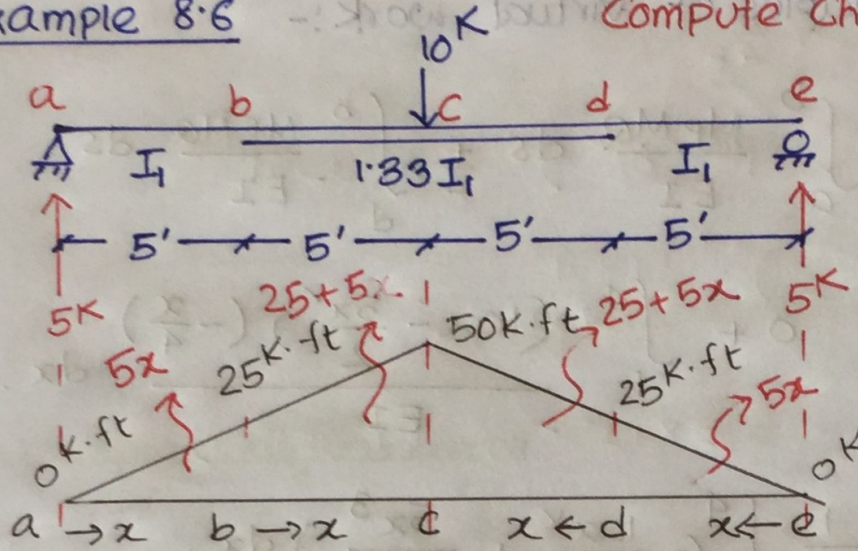
$$= \frac{1}{EI} \left[-\frac{160}{3} + 40 \right]$$

$$= \frac{1}{EI} \left(-\frac{40}{3} \right)$$

$$= -6.44 \times 10^{-4} \text{ m} \quad \text{upward}$$

Example 8.6

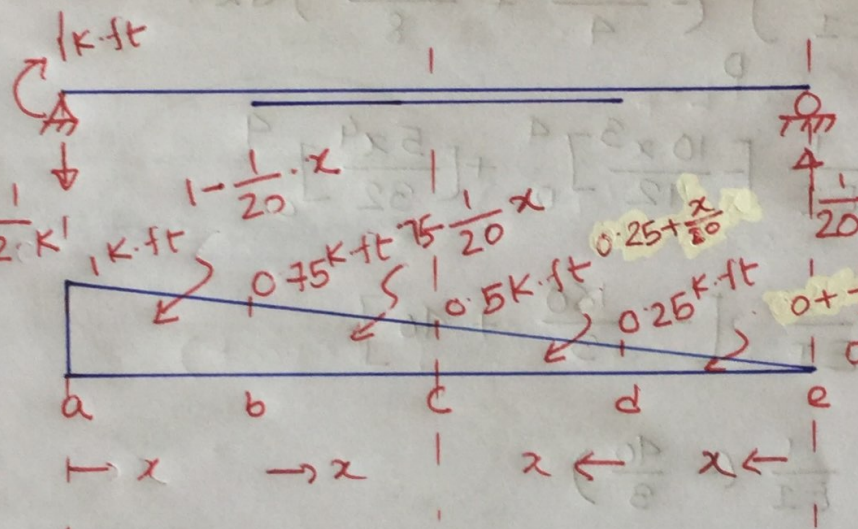
Compute Change in slope at 'a'



Given, $E = 30000 \text{ ksi}$

$I_1 = 150 \text{ in}^4$

$I_2 = 200 \text{ in}^4$



Bending moment for real Force

Virtual force analysis

Bending moment for virtual Force

Applying principles of virtual work :-

$$\Delta \alpha_a = \int_a^b \frac{M_p M_Q}{EI_1} dx + \int_b^c \frac{M_p M_Q}{EI_2} dx + \int_c^d \frac{M_p M_Q}{EI_2} dx + \int_d^e \frac{M_p M_Q}{EI_1} dx$$

$$= \int_a^b \frac{5x(1 - \frac{x}{20})}{EI_1} dx + \int_b^c \frac{(25+5x)(0.75 - \frac{x}{20})}{EI_2} dx$$

$$+ \int_c^d \frac{(25+5x)(0.25 + \frac{x}{20})}{EI_2} dx + \int_d^e \frac{5x \cdot \frac{x}{20}}{EI_1} dx.$$

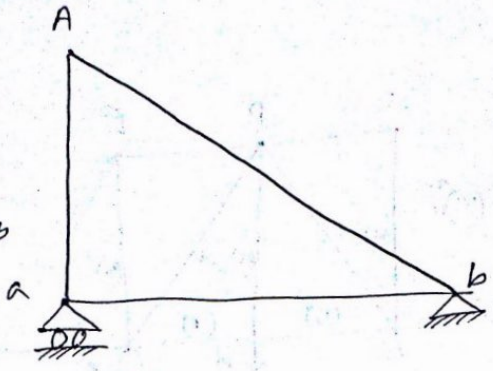
$$= \frac{625}{12EI_1} + \frac{1375}{12 \times 1.33EI_1} + \frac{875}{12 \times 1.33EI_1} + \frac{125}{12EI_1}$$

$$= 0.0065 \text{ radian } (\therefore \text{clockwise})$$

Support settlement effect :-

Example - 8.3

Given; following support movements



a reaction 18r so

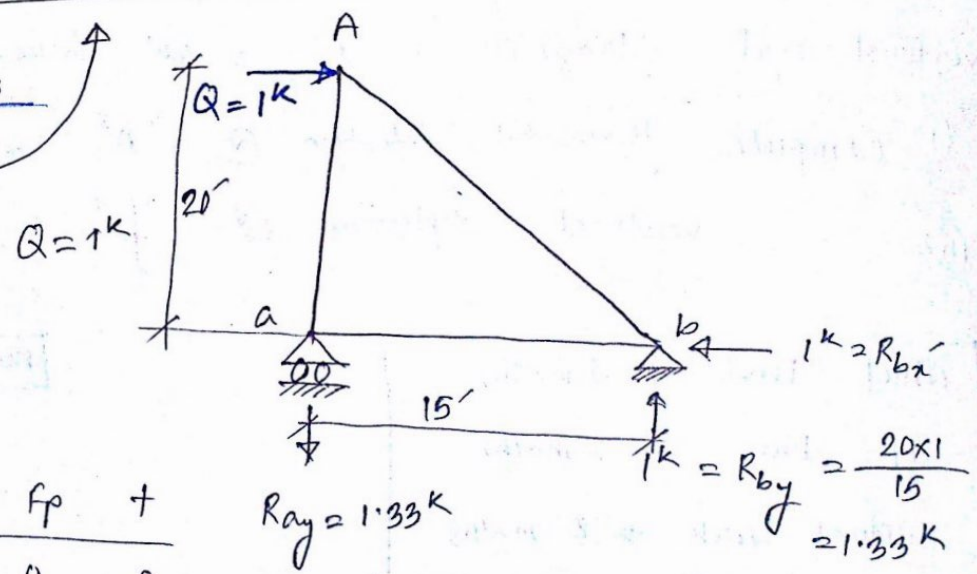
b reaction 28r so

- @ 'a' vertical = 0.5" to ~~left~~ down
- @ 'b' horizontal = 0.75" to ~~down~~ 0.15" to left
- @ 'b' vertical = 0.75" to down

here; for this prob. p-force analysis → X
p-force analysis

Compute horizontal deflection @ A

Q force analysis
Q force from horizontal direction → moment



Table

bars	Fp	+
	0	0
	0	0
	0	0

principle of virtual work (including support movements) :-

$$W_s = \sum Q \cdot \delta + W_R$$

external virtual work done by the reaction forces

For this problem;

$$W_s = W_e$$

$$\Rightarrow Q \cdot \delta + W_R = \frac{\sum F_Q \cdot P \cdot L}{EA} + \sum F_Q \cdot \delta_e \pm L = 0$$

$$Q_e \rightarrow \delta_e$$

Applied movement ↓
so ⊕

← why

force converting into feet

→ why

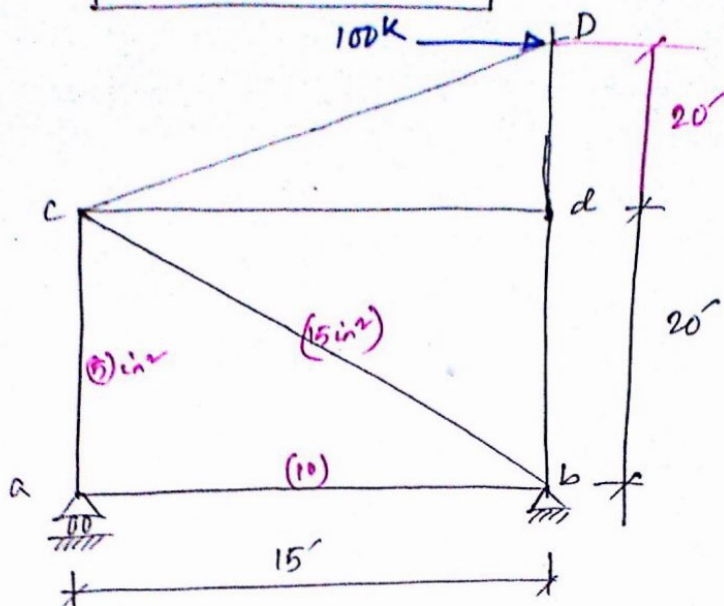
b/c R → ↑
Applied → ↓ 50 ⊖

$$\Rightarrow (1 \times 5)_{ACh} + (1.33k) \left(+ \frac{0.25}{12} \right) + (1k) \left(+ \frac{0.5}{12} \right) + (1.33k) \left(- \frac{0.75}{12} \right) = 0$$

$$\therefore \boxed{\delta_{Ah} = +0.01375 \text{ ft}}$$

Assign #19

VVI math



$$E = 30,000 \text{ ksi}$$

$$\alpha_L = \frac{1}{1,50,000}$$

Temp. change:

vertical member = 40°F decrease

Diagonal & = 50°F increase

support movement

@ 'a' vertical = 0.9" upward

@ "b" horizontal = 0.3" to right

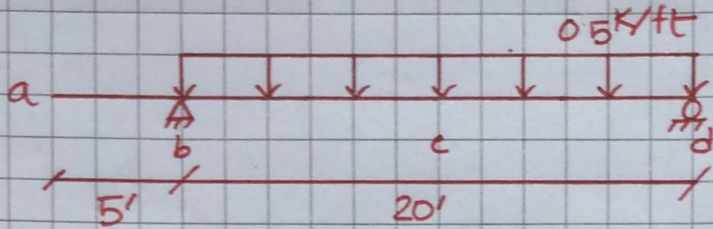
@ "b" vertical = 0.9" down

Q. → Compute horizontal deflection @ D
and vertical & @ D

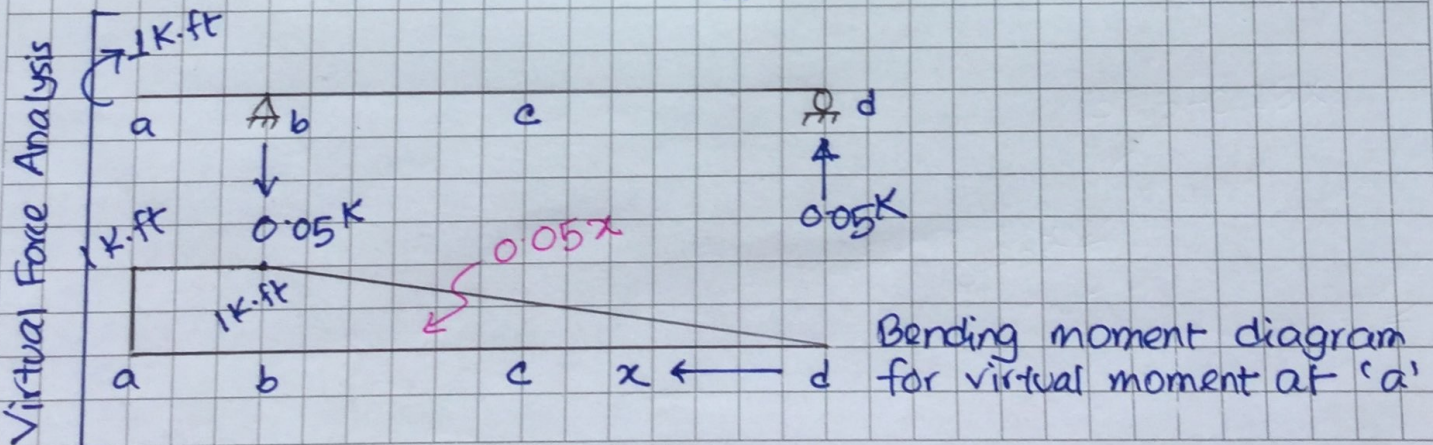
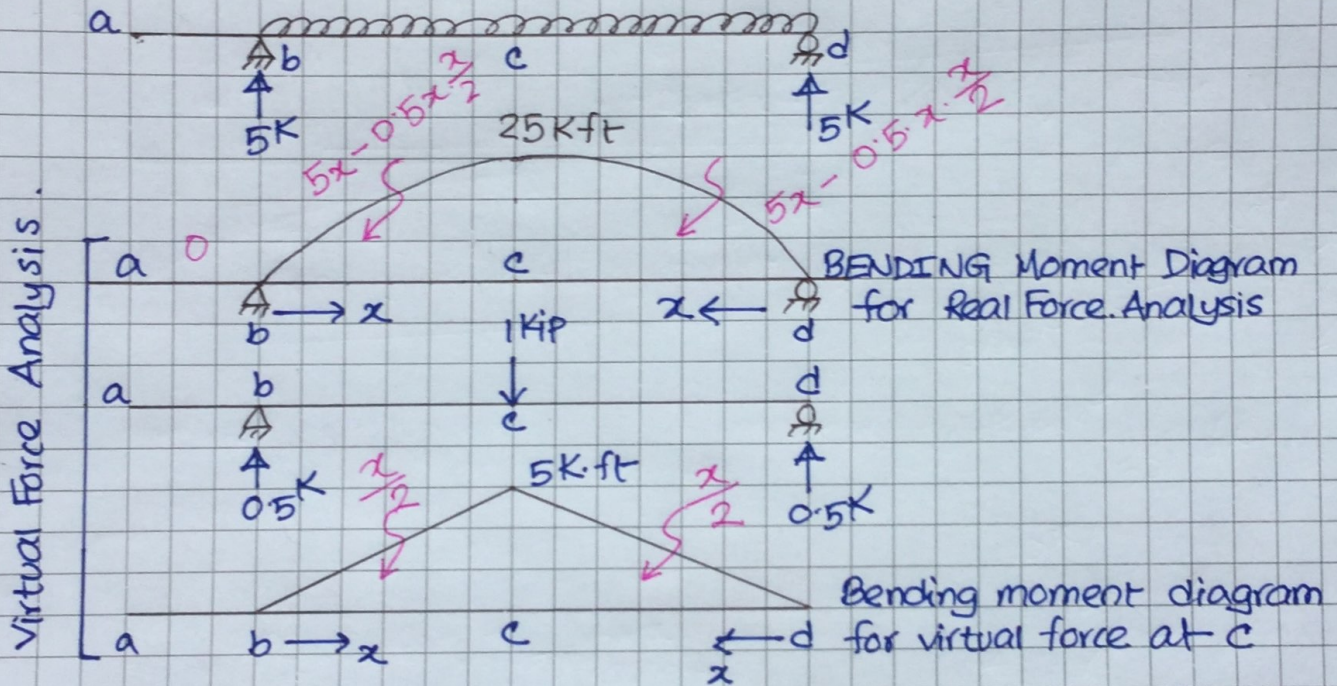
Assignment - 19



Given, $E = 30,000 \text{ Ksi}$, $I = 200 \text{ in}^4$ Compute change in slope at 'a' and deflection at C.



Solution: Real force analysis:



Change in slope at 'a'

$$\begin{aligned} Q \cdot \alpha_A &= \int_a^b \frac{M_p M_Q}{EI} dx + \int_d^b \frac{M_p M_Q}{EI} dx \\ &= \int_0^b \frac{20 \left(5x - 0.5 \frac{x^2}{2}\right) (0.05x)}{EI} dx \\ &= \frac{1}{EI} \times \frac{500}{3} \\ &= \frac{1}{(30000 \times 144) \times (200/144^2)} \times \frac{500}{3} \end{aligned}$$

$$\alpha_A = 4 \times 10^{-3} \text{ radian } (\therefore \text{clockwise})$$

deflection at 'c'

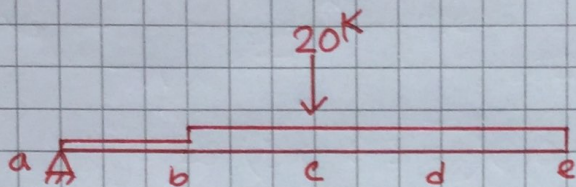
$$\begin{aligned} Q \cdot \delta_c &= \int_a^b \frac{M_p M_Q}{EI} dx + \int_b^c \frac{M_p M_Q}{EI} dx + \int_d^c \frac{M_p M_Q}{EI} dx \\ &= \int_0^{10} \frac{(5x - 0.5 \frac{x^2}{2}) (\frac{x}{2})}{EI} dx + \int_0^{10} \frac{(5x - 0.5 \frac{x^2}{2}) (\frac{x}{2})}{EI} dx \\ &= 2 \int_0^{10} \frac{(5x - 0.5 \frac{x^2}{2}) (\frac{x}{2})}{EI} dx \\ &= \frac{2}{EI} \times \frac{3125}{6} \end{aligned}$$

$$\delta_c = 0.025 \text{ ft } \therefore \text{(down)}$$

Assignment-20

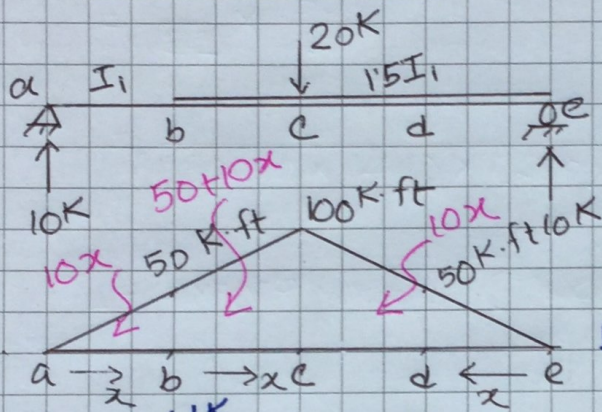


Compute deflection at 'b' and change in slope at e for the following beam shown. Given that, $E = 30000 \text{ Ksi}$, $I_1 = 200 \text{ in}^4$, $I_2 = 300 \text{ in}^4$

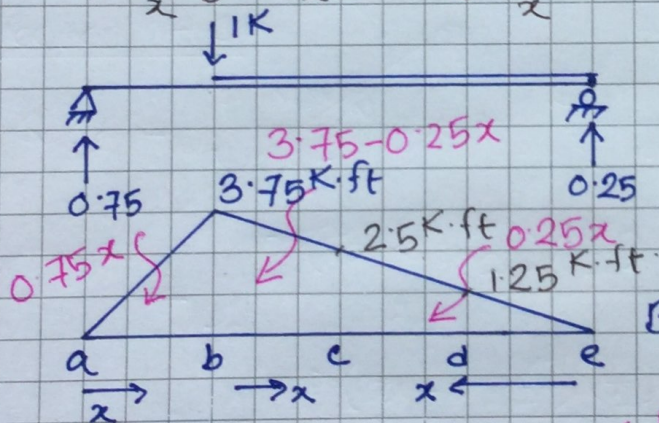


Solution:

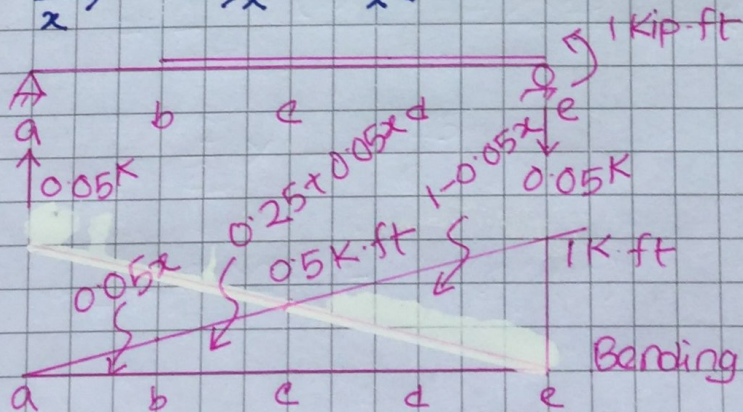
Real Force analysis:



Bending moment for Real Force



Bending moment for virtual force 'b'



Bending moment for virtual moment @ e



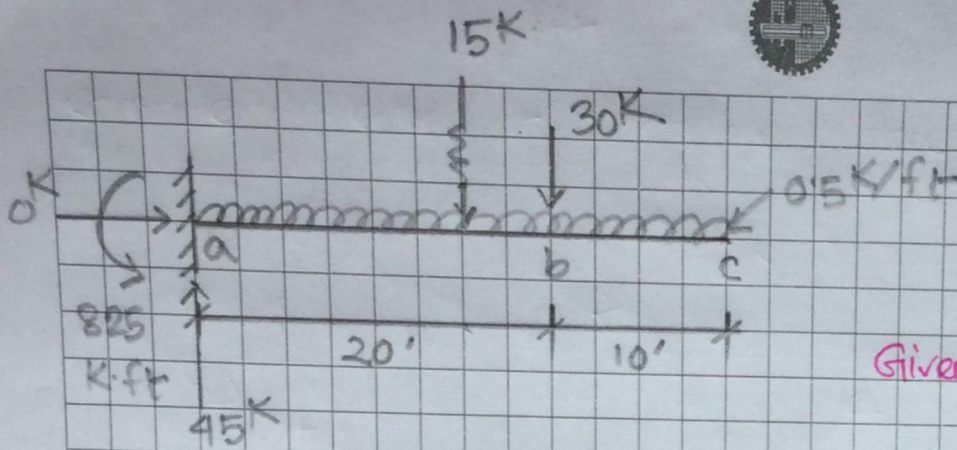
Deflection at 'b'

$$\begin{aligned} Q \cdot \delta_B &= \int_a^b \frac{M_{PMQ}}{EI_1} dx + \int_b^c \frac{M_{PMQ}}{EI_2} dx + \int_e^c \frac{M_{PMQ}}{EI_2} dx \\ &= \int_0^5 \frac{(10x)(0.75x)}{EI_1} dx + \int_0^5 \frac{(50+10x)(3.75-0.25x)}{EI_2} dx \\ &\quad + \int_0^{10} \frac{(10x)(0.25x)}{EI_2} dx \\ &= \frac{1}{EI_1} \times \frac{625}{2} + \frac{1}{EI_2} \times \frac{6875}{6} + \frac{2500}{3} \times \frac{1}{EI_2} \\ &= 0.039167 \text{ ft } (\therefore \text{down}) \end{aligned}$$

change in slope at 'e'

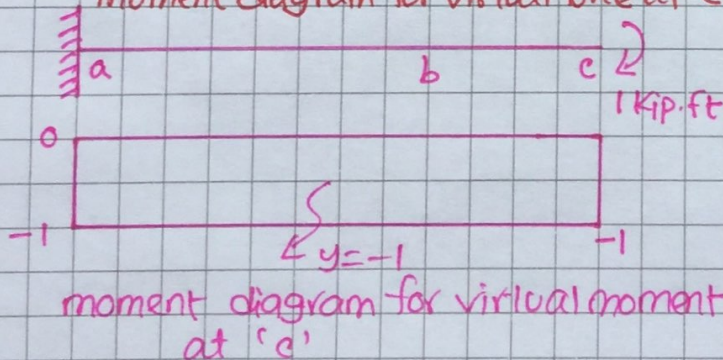
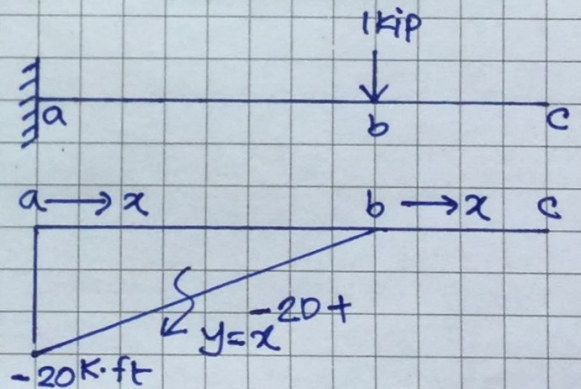
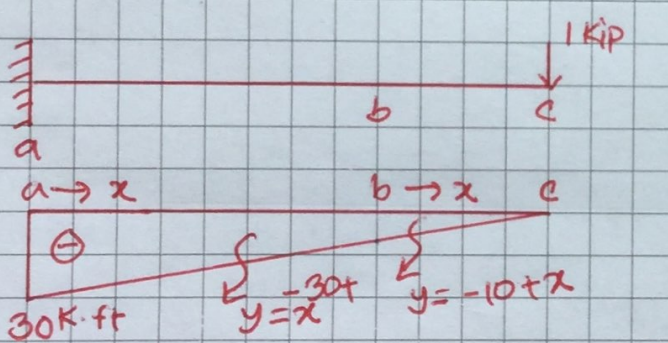
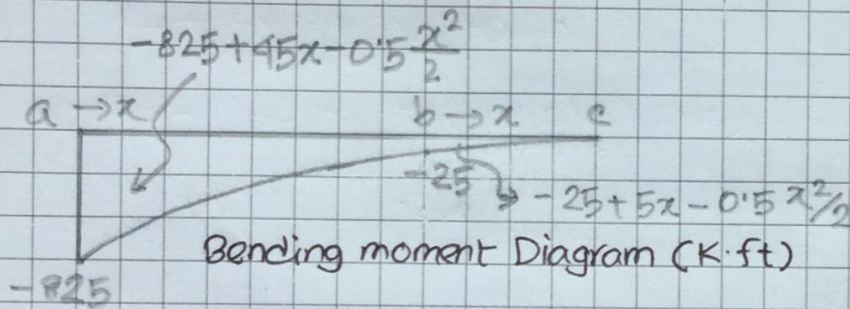
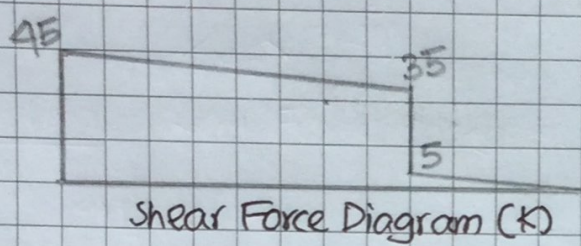
$$\begin{aligned} Q \cdot \alpha_e &= \int_a^b \frac{M_{PMQ}}{EI_1} dx + \int_b^c \frac{M_{PMQ}}{EI_2} dx + \int_e^c \frac{M_{PMQ}}{EI_2} dx \\ &= \int_0^5 \frac{(10x)(0.05x)}{EI_1} dx + \int_0^5 \frac{(50+10x)(0.25+0.05x)}{EI_2} dx \\ &\quad + \int_0^{10} \frac{(10x)(1-0.05x)}{EI_2} dx \\ &= \frac{1}{EI_1} \times \frac{125}{6} + \frac{1}{EI_2} \times \frac{875}{6} + \frac{1}{EI_2} \times \frac{1000}{3} \\ &= 0.012 \text{ radian } (\therefore \text{clockwise}) \end{aligned}$$

Assignment-21



Compute
 a) deflection at 'c'
 b) change in slope at 'c'
 c) deflection at 'b'

Given $E = 30000 \text{ Ksi}$ $I = 200 \text{ in}^4$



Deflection at C:

$$\begin{aligned} Q \cdot \delta_c &= \int_a^b \frac{M \cdot P \cdot M_Q}{EI} dx + \int_b^c \frac{M \cdot P \cdot M_Q}{EI} dx \\ &= \int_0^{20} \frac{(-825 + 45x - 0.5x^2/2)(-30+x)}{EI} dx + \int_0^{10} \frac{(-25 + 5x - 0.5x^2/2)(-10+x)}{EI} dx \\ &= \frac{190,000}{EI} + \frac{625}{EI} \\ &= \frac{190,625}{EI} = 4.575 \text{ ft (down)} \end{aligned}$$

change in slope at C:

$$\begin{aligned} Q \cdot \alpha_c &= \int_a^b \frac{M \cdot P \cdot M_Q}{EI} dx + \int_b^c \frac{M \cdot P \cdot M_Q}{EI} dx \\ &= \int_0^{20} \frac{(-825 + 45x - 0.5x^2/2)(-1)}{EI} dx + \int_0^{10} \frac{(-25 + 5x - 0.5x^2/2)(-1)}{EI} dx \\ &= \frac{1}{EI} \left(\frac{24500}{3} + \frac{250}{3} \right) \\ &= \frac{24750}{3EI} \\ &= 0.198 \text{ radian (clockwise)} \end{aligned}$$

Deflection at 'b':

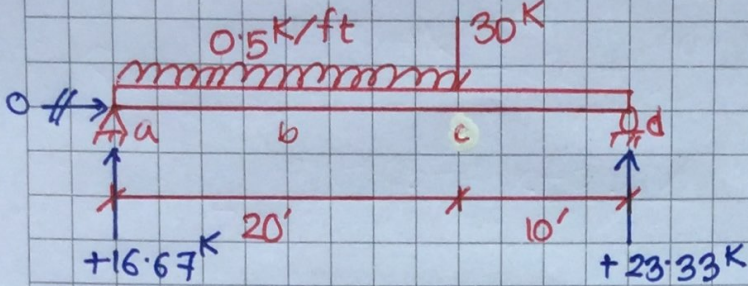
$$\begin{aligned} Q \cdot \delta_b &= \int_a^b \frac{M \cdot P \cdot M_Q}{EI} dx + \int_b^c \frac{M \cdot P \cdot M_Q}{EI} dx \\ &= \int_0^{20} \frac{325000}{3EI} dx \\ &= 2.6 \text{ ft } (\therefore \text{down}) \end{aligned}$$

Assignment -22

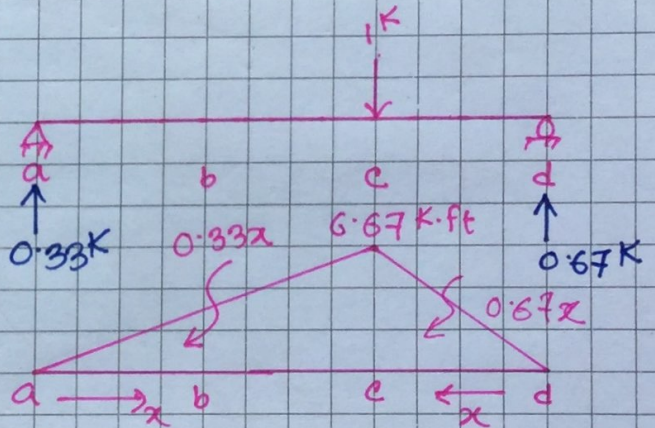
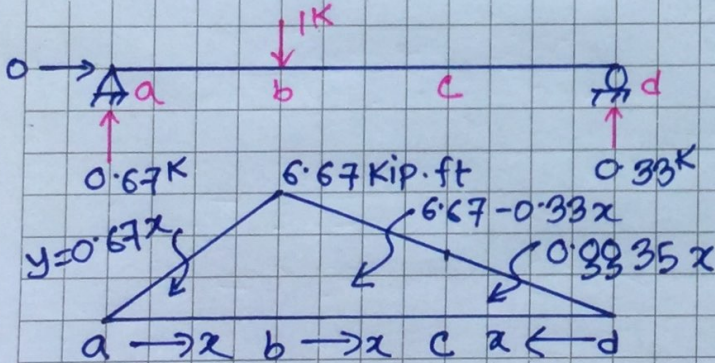
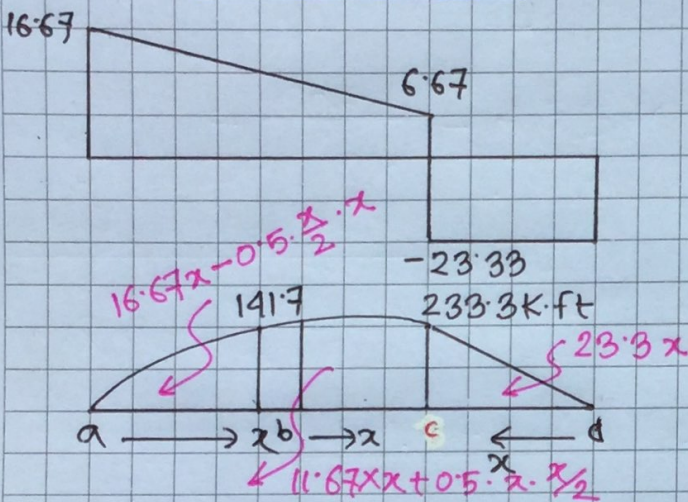


Given $E = 30000 \text{ ksi}$, $I = 200 \text{ in}^4$. Compute

- deflection at 'b'
- deflection at 'c'
- change in slope at 'a'
- change in slope at 'd'



Solution: Real Force Analysis:



deflection at 'b'

$$Q \cdot \delta_b = \int_{a=0}^{b=10} \frac{(16.67x - 0.5 \frac{x^2}{2})(0.67x)}{EI} dx + \int_{c=0}^{d=10} \frac{(141.7 + 11.67x - 0.5 \frac{x^2}{2})(6.67 - 0.33x)}{EI} dx$$

$$I \cdot \delta_b = \frac{3304.2167}{EI} + \frac{9682.5}{EI} + \frac{2563}{EI} + \int_0^{10} \frac{(23.33x)(0.33x)}{EI} dx$$

$$\delta_b = 0.373 \text{ ft (Ans.)}$$



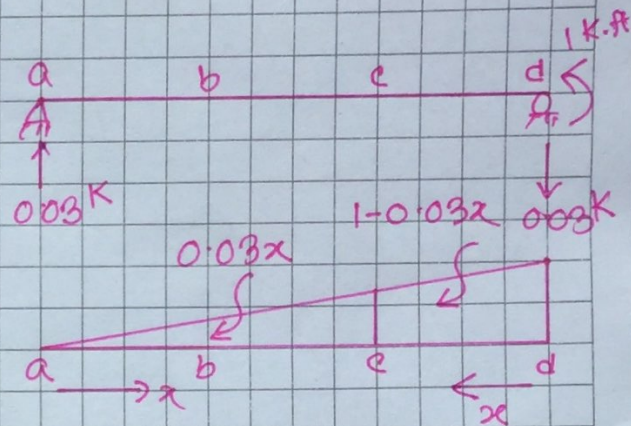
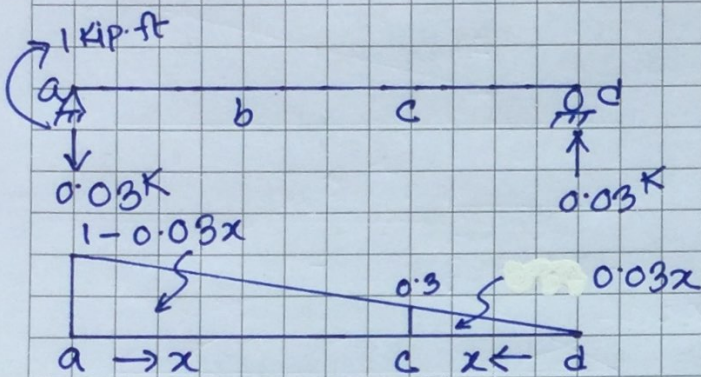
$$Q \cdot \Delta_c = \int_0^{20} \frac{(16.67x - 0.5x^2 \times 2 \times \frac{1}{2})(0.33x)}{EI} dx + \int_0^{10} \frac{(23.3x)(0.67x)}{EI} dx$$

$$= \frac{11369.6}{EI} + \frac{5203.67}{EI}$$

$$= 0.39 \text{ ft (down)}$$

change in slope at 'a'

change in slope 'd'



$$Q \cdot \alpha_A = \int_0^{20} \frac{(16.67x - 0.5x^2/2)(1 - 0.03x)}{EI} dx + \int_0^{10} \frac{(23.3x)(0.03x)}{EI} dx$$

$$= 0.0392 + 0.005592$$

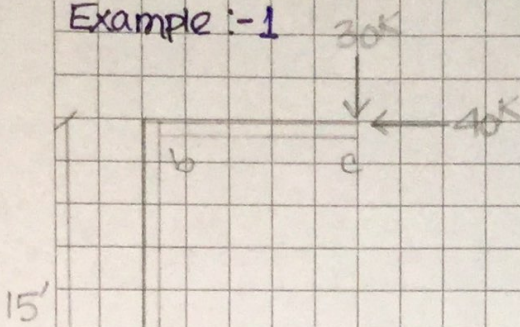
$$= 0.044792 \text{ ft (clockwise)}$$

$$Q \cdot \alpha_d = 0.0248044 + 0.022368$$

$$= 0.04717 \text{ radian (anticlockwise)}$$

VIRTUAL WORK METHOD FOR FRAMES

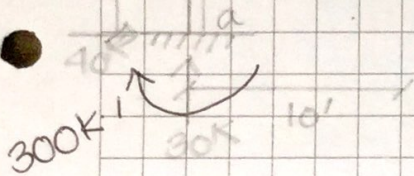
Example :- 1



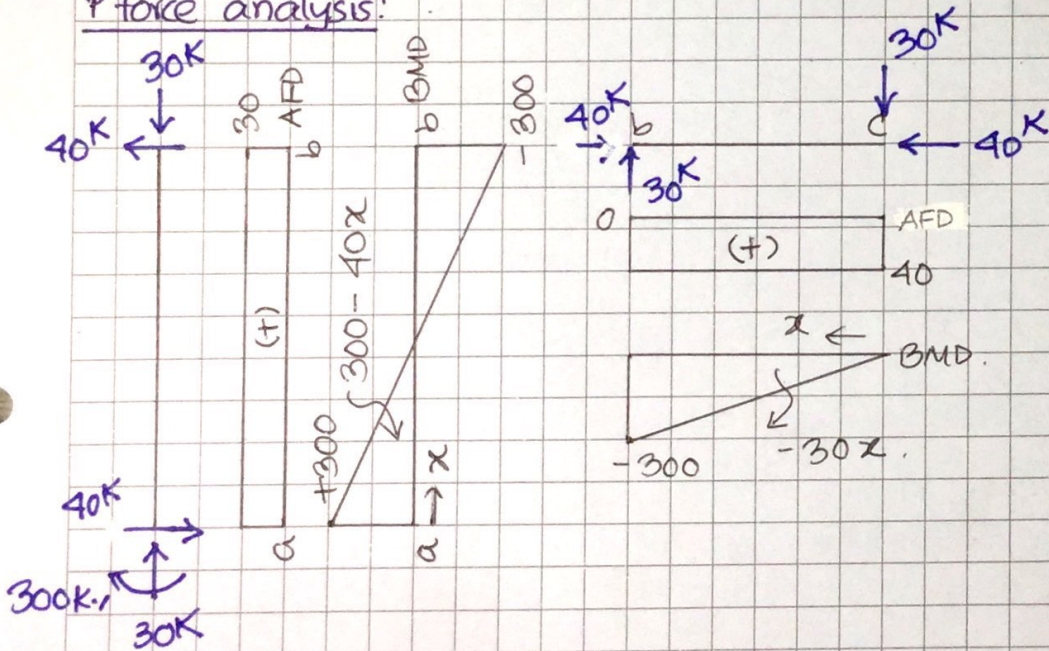
Compute Change in slope at 'c'.
vertical deflection at 'c'

Given, $E = 30,000 \text{ ksi}$, $A_{col} = 12 \text{ in}^2$, $I_{col} = 300 \text{ in}^4$
 $A_{beam} = 10 \text{ in}^2$, $I_{beam} = 200 \text{ in}^4$

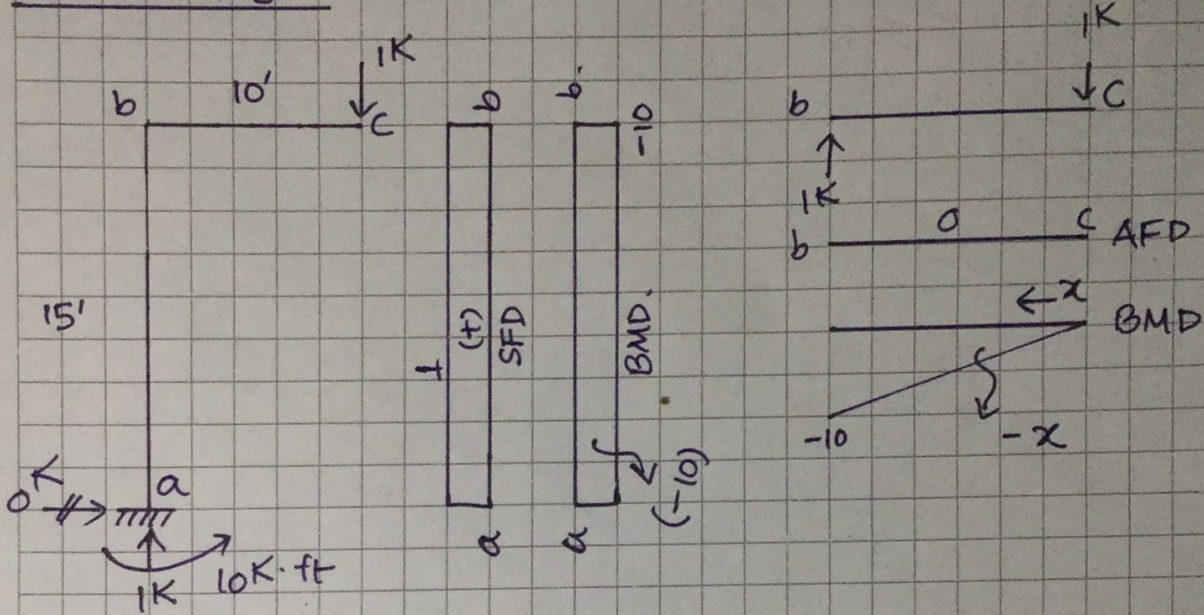
Solution:-



P force analysis:



Q force Analysis: For Vertical Deflection at 'a'



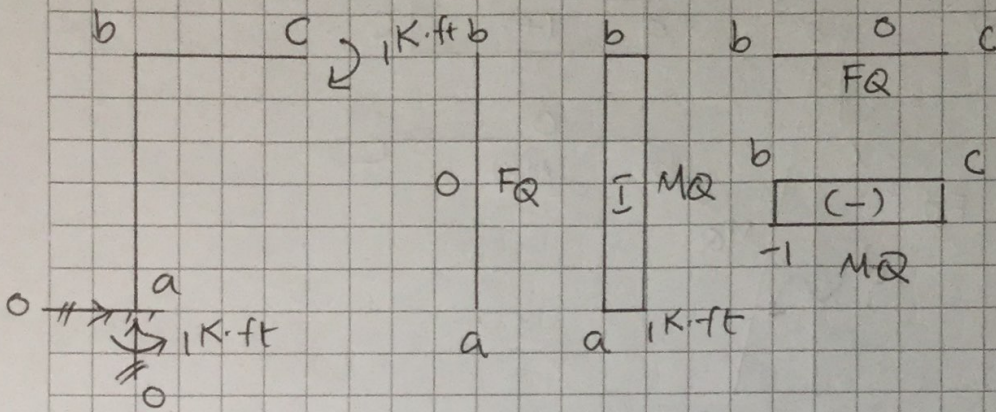
Applying Principles of Virtual Work:-

$$\begin{aligned} \sum Q \cdot \delta &= \left(\frac{F_P F_Q L}{EA} \right)_{ab} + \left(\frac{F_P F_Q L}{EA} \right)_{bc} + \int_a^b \frac{M_P M_Q}{EI} dx + \int_b^c \frac{M_P M_Q}{EI} dx \\ &= \frac{(+30)(+10)(15')}{30000 \times 12} + 0 + \int_0^{15} \frac{(300 - 40x)(-10)}{EI} dx + \int_0^{10} \frac{(-30x)(x)}{EI} dx \\ &= \frac{1}{800} + 0 + \frac{6}{25} \\ &= 0.2412 \text{ ft} \end{aligned}$$

[Ans.]

change in slope at 'c'

Q - force analysis:



$$\therefore \theta_c = \sum F_P F_Q L / AE + \sum F_Q t L + \int M P M_Q / EI dx$$

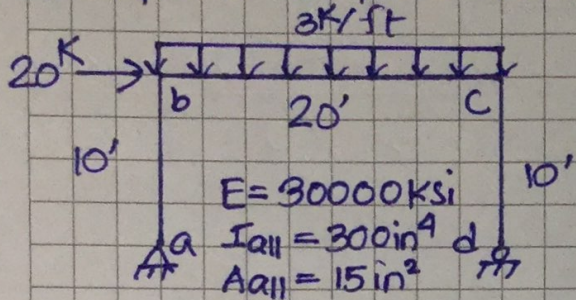
$$= \int_a^b \frac{M P M_Q}{EI} dx + \int_b^c \frac{M P M_Q}{EI} dx$$

$$= \int_0^{20} \frac{(-800)(-1)}{30000 \times \frac{300}{144}} dx + \int_0^{10} \frac{(-300-50x)(-1)}{30000 \times \frac{200}{144}} dx$$

$$= \frac{32}{125} + \frac{33}{250}$$

$$= 0.388 \text{ radian (clockwise)}$$

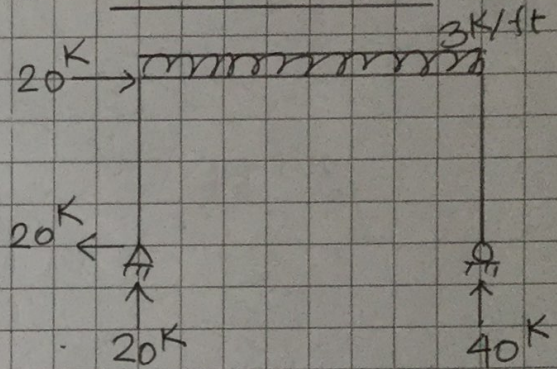
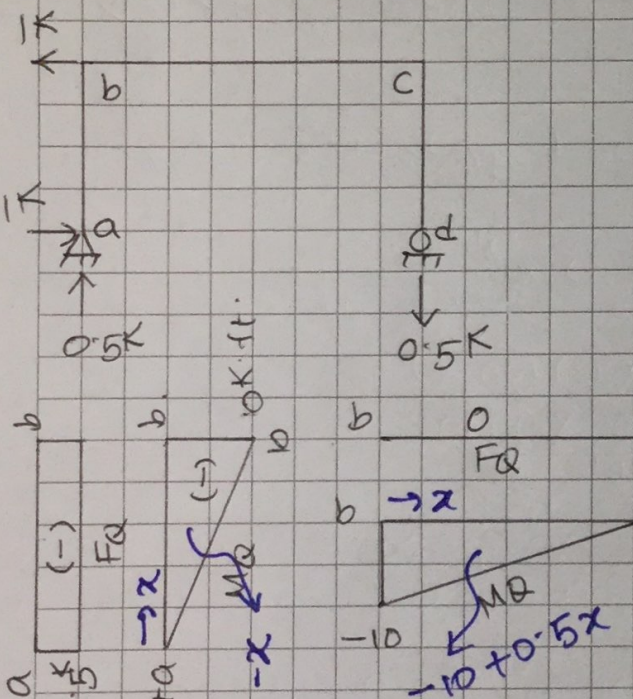
Example 2: Compute horizontal deflection at joint 'b'



Compute horizontal deflection at joint 'b'

Q force Analysis

P force analysis



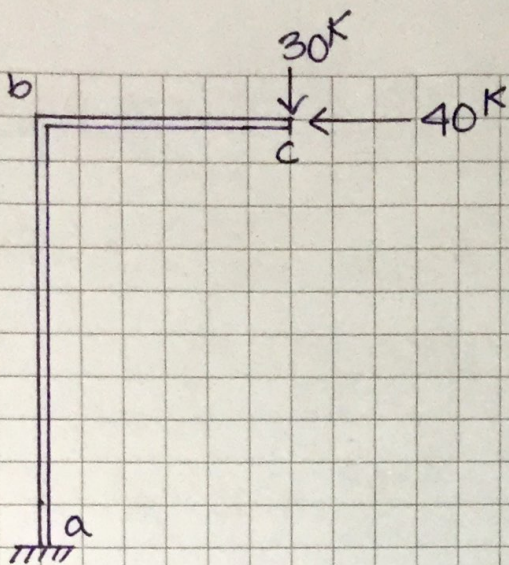
$$Q \times \delta_b = \frac{(0.5)(-20) \times 10}{15 \times 30 \times 10^3} + \frac{(0.5)(-40) \times 10}{15 \times 30 \times 10^3}$$

$$\int_0^{10} \frac{(20x)(-x)}{30 \times 10^3 \times 300} dx + \int_0^{20} \frac{(10+0.5x)(200 - 20x - 3 \frac{x^2}{2})}{30 \times 10^3 \times \frac{300}{144}} dx$$

$$= \frac{1}{75} + \frac{-1}{75}$$

$$= -0.48 \text{ ft (right)}$$

Assignment -23



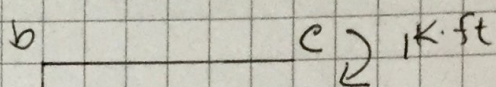
Compute change in slope at c

Given, $E = 30000 \text{ Ksi}$

$A_{ab} = 12 \text{ in}^2 \quad A_{bc} = 10 \text{ in}^2$

$I_{ab} = 300 \text{ in}^4 \quad I_{bc} = 200 \text{ in}^4$

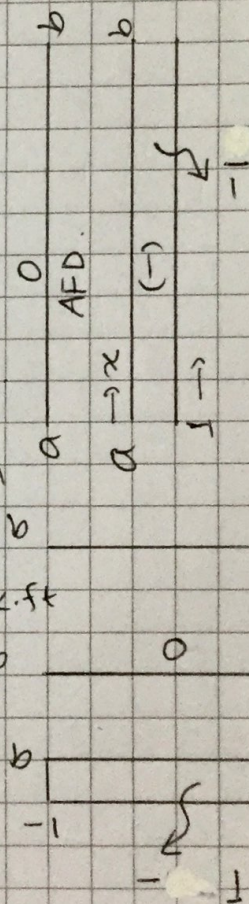
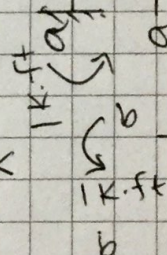
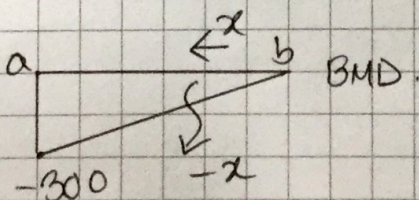
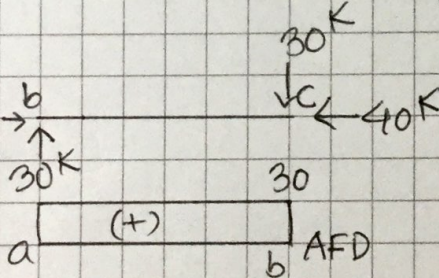
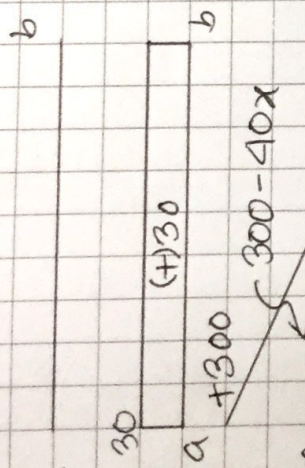
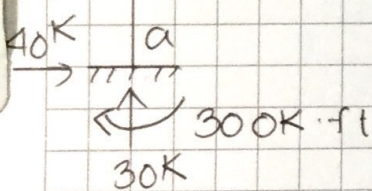
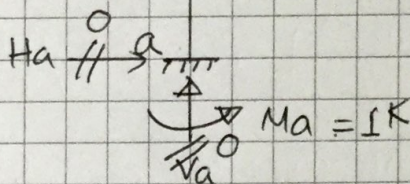
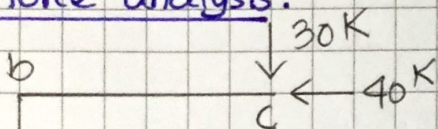
Q - force analysis:



$$\begin{aligned} \sum M_a &= 0 \\ -Ma + 1K &= 0 \\ \therefore Ma &= 1K \end{aligned}$$

Solution:-

P force analysis:

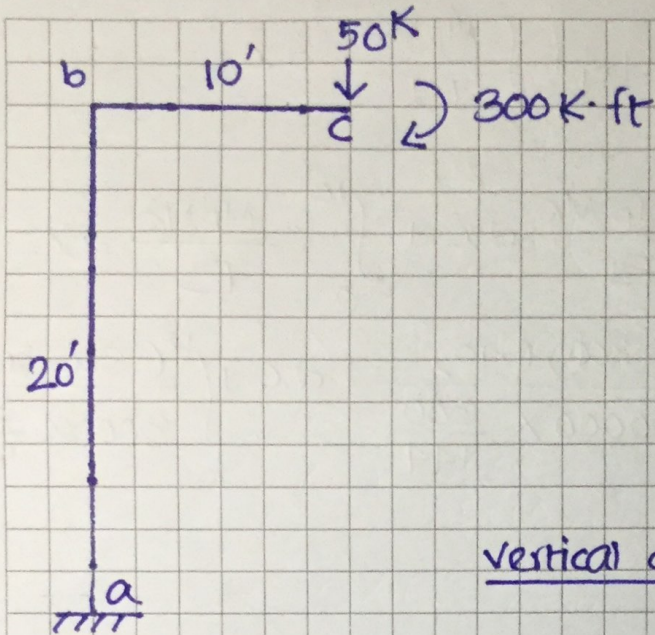


Applying principles of virtual work:

$$\begin{aligned}\sum Q \delta &= \left(\frac{F_P F_Q L}{EA} \right)_{ab} + \left(\frac{F_P F_Q L}{EA} \right)_{bc} + \int_0^{15} \frac{M_P M_Q}{EI} dx + \int_0^{10} \frac{M_P M_Q}{EI} dx \\ &= \int_0^{15} \frac{(300 - 40x)(-1)}{EI} dx + \int_0^{10} \frac{(-x)(-1)}{EI} dx \\ &= 0 + \frac{50 \times 3}{125000} = \frac{3}{2500} \\ &= 1.2 \times 10^{-3} \text{ radian (}\therefore \text{ clockwise)}\end{aligned}$$

[Ans:]

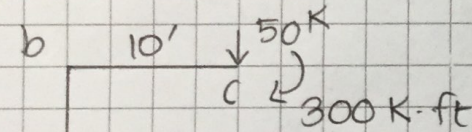
Assignment 24



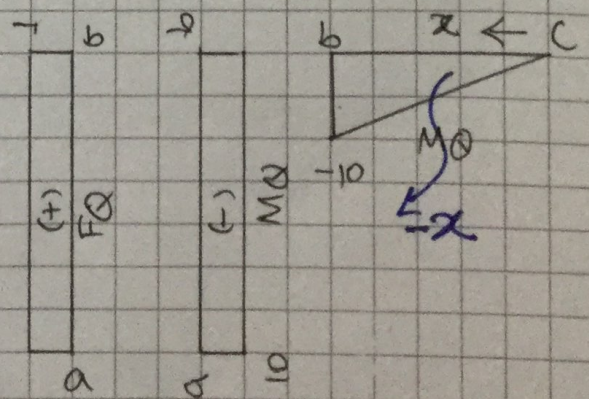
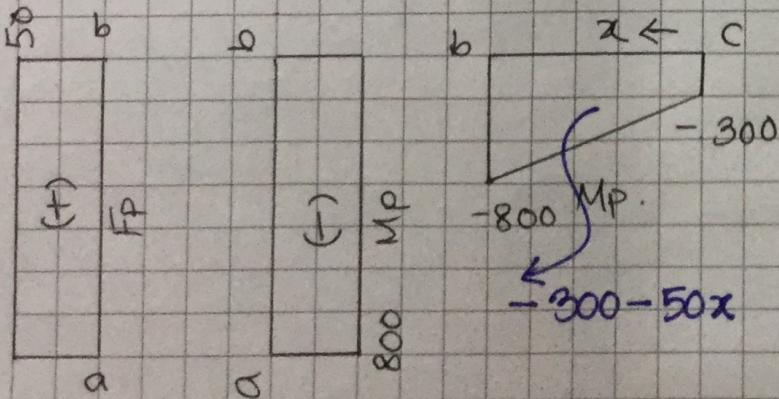
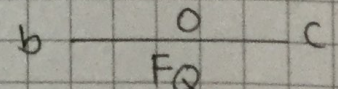
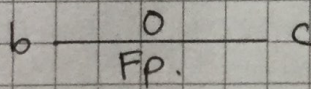
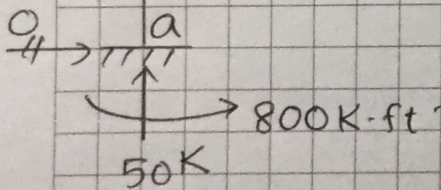
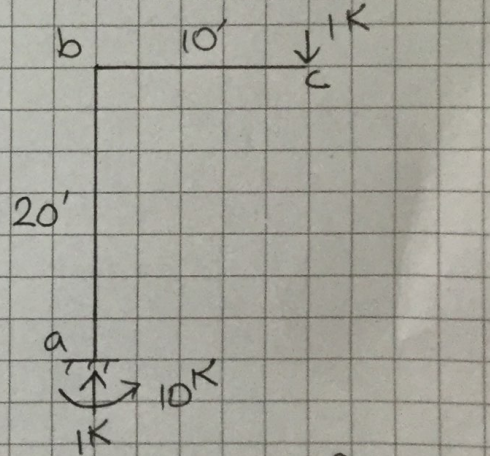
Compute vertical deflection at c
 b) change in slope at c
 c) horizontal deflection at 'b'

vertical deflection at 'c'

P force analysis:



Q force analysis:



As 100 mg 202A

$$Q \delta_c = \sum \frac{F_P F_Q L}{AE} + \sum F_Q t L_x + \int \frac{M_P M_Q}{EI} dx$$

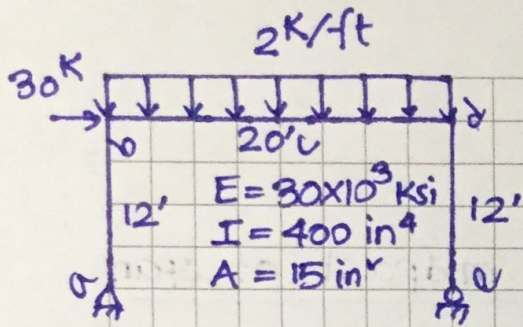
$$= \left(\frac{F_P F_Q L}{AE} \right)_{ab} + \int_0^{20} \frac{M_P M_Q}{EI} dx + \int_0^{10} \frac{M_P M_Q}{EI} dx.$$

$$= \frac{50 \times 1 \times 20}{12 \times 30000} + \int_0^{20} \frac{(-800)(-10)}{30000 \times \frac{300}{144}} dx + \int_0^{10} \frac{(-300-50x)(-x)}{30000 \times \frac{200}{144}} dx$$

$$= \frac{1}{360} + \frac{64}{25} + \frac{19}{25}$$

$$= 3.32 \text{ ft.}$$

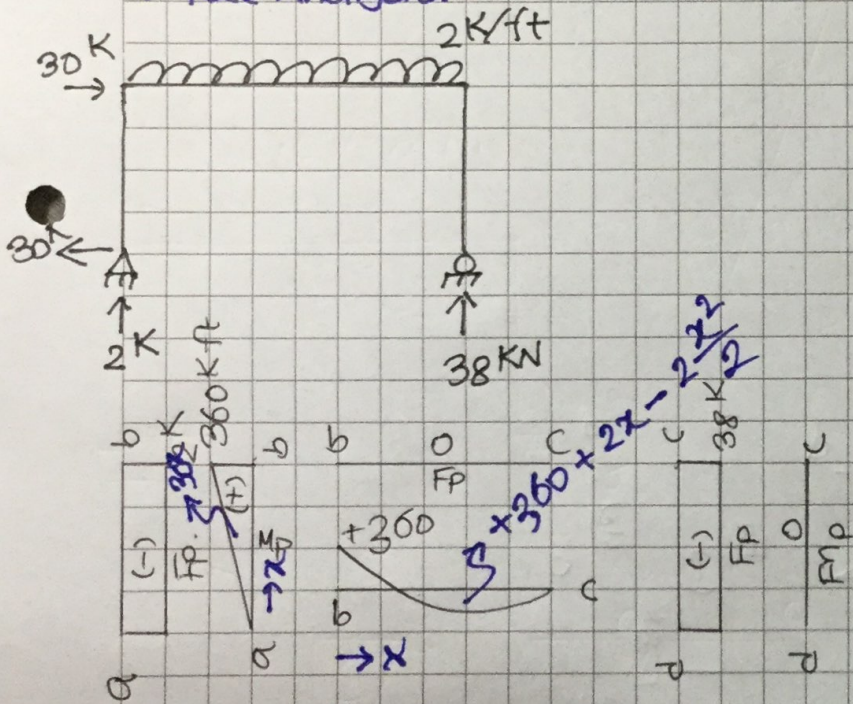
Assignment 25



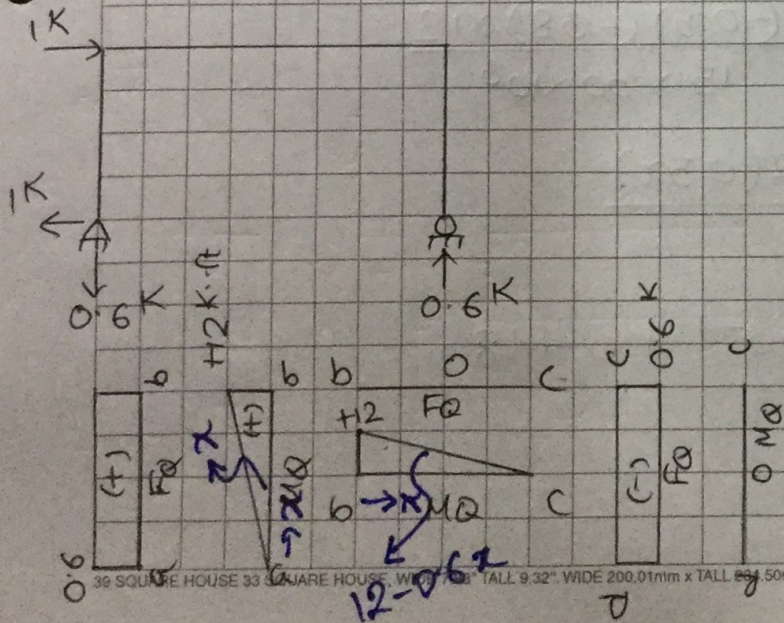
Compute Horizontal Deflection at 'b' and vertical deflection at the midpoint of beam

horizontal deflection at 'b'

P force Analysis:



Q force Analysis:



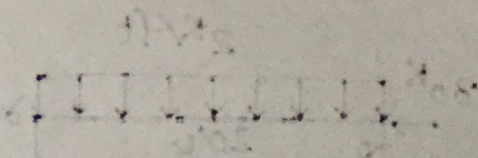
$$Q \Delta_b = \frac{(-2)(+0.6)12}{30 \times 10^3 \times 15} + \frac{(-38)(-0.6) \times 12}{15 \times 30 \times 10^3}$$

$$+ \int_0^{12} \frac{(30x)(x) dx}{30 \times 10^3 \times \frac{400}{144}} + \int_0^{20} \frac{(360 + 2x - 2x^2/2)(12 - 0.6x)}{144} dx$$

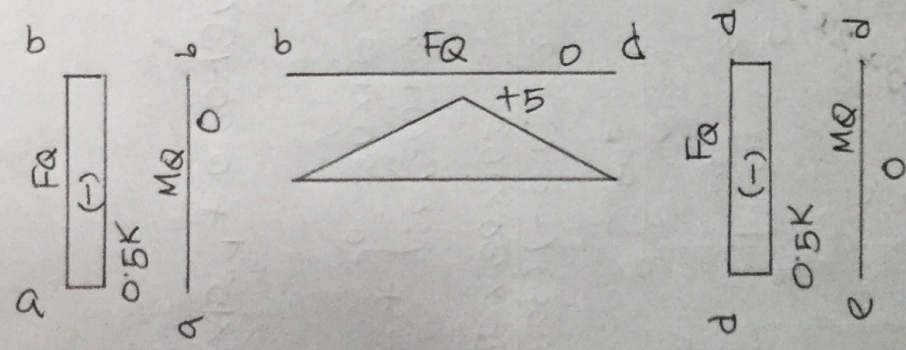
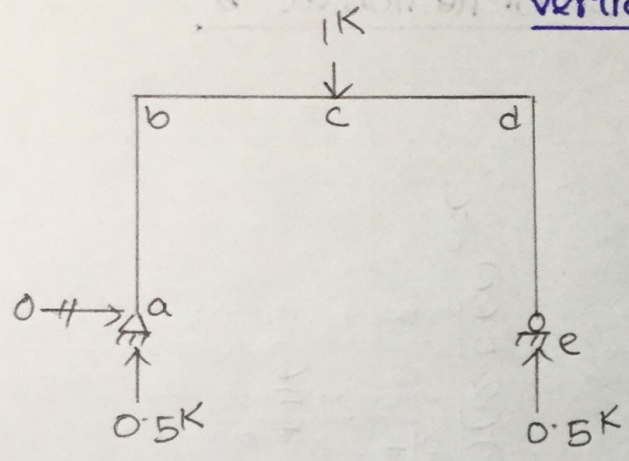
$$= -32 \times 10^{-6} + 6.08 \times 10^{-4} + 0.20736 + 0.4416$$

$$= 0.649536 \text{ ft}$$

Compute Horizontal Deflection of B and Vertical deflection at the support of beam



Vertical deflection at midpoint of span



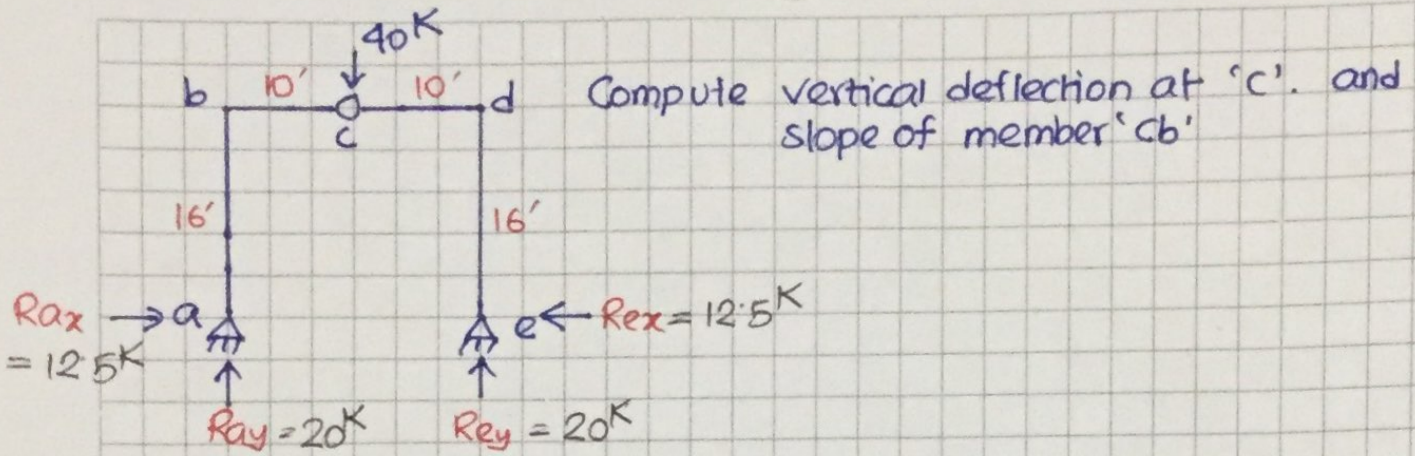
$$Q \delta_c = \frac{(-2)(-0.5) \times 12}{15 \times 30 \times 10^3} + \frac{(-0.5)(-38) \times 12}{15 \times 30 \times 10^3}$$

$$+ \int_0^{20} \frac{(360 + 2x - 2 \frac{x^2}{2}) \cdot 2(0.5x)}{30 \times 10^3 \times \frac{400}{144}}$$

$$= 2.67 \times 10^{-5} + 5.067 \times 10^{-4} + 0.448$$

$$= 0.4485334 \text{ ft.}$$

Assignment -26



P-force analysis:

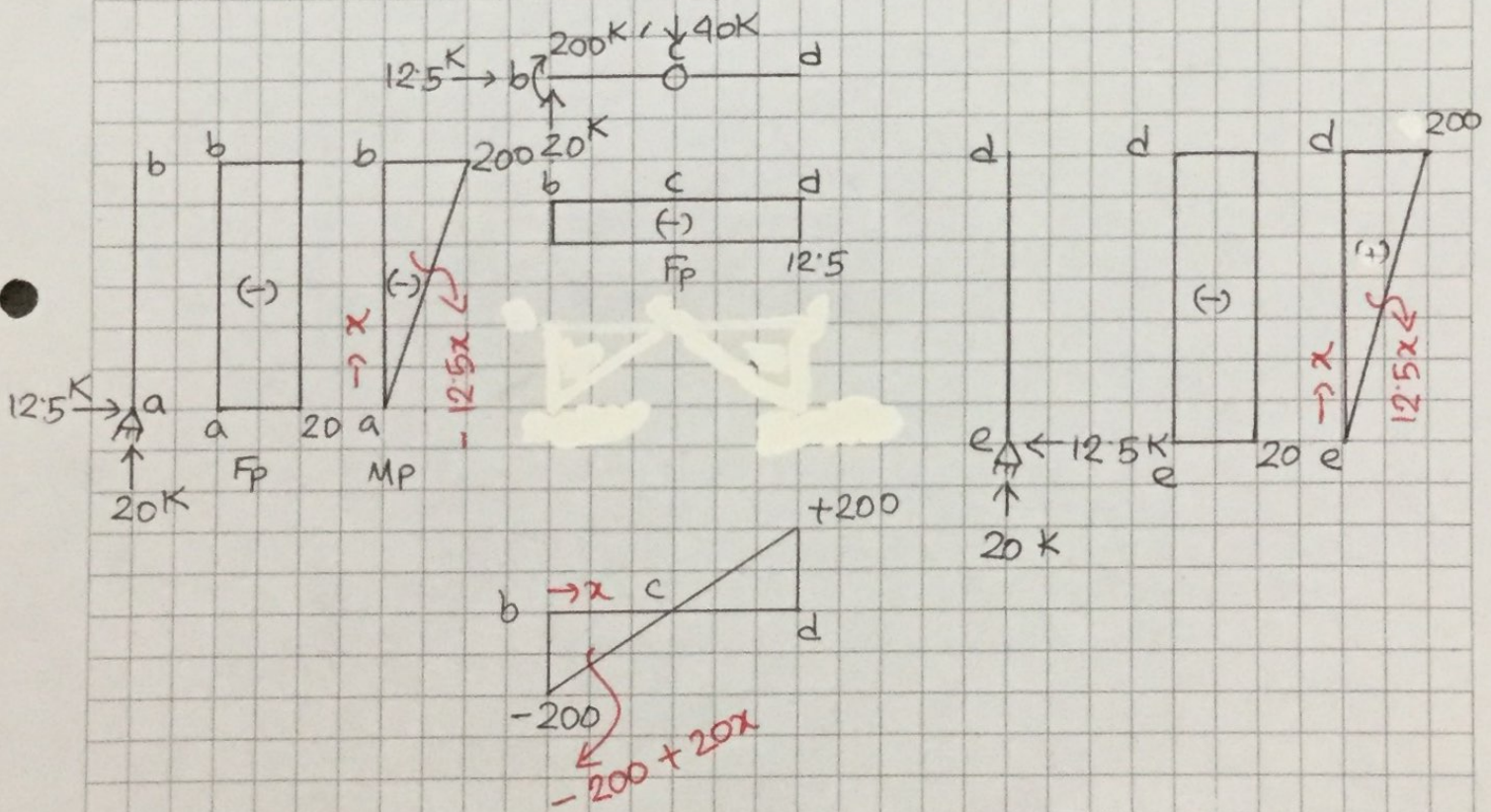
$$\sum M_A = 0 \quad 40 \times 10 - R_{ey} \times 20 = 0$$

$$\therefore R_{ey} = 20K \quad \therefore R_{ay} = 20K$$

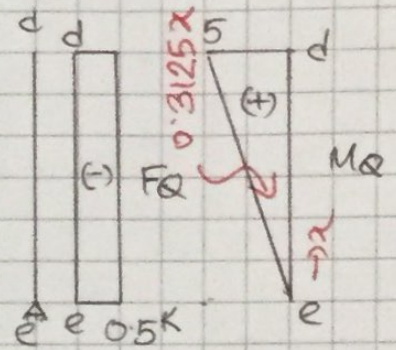
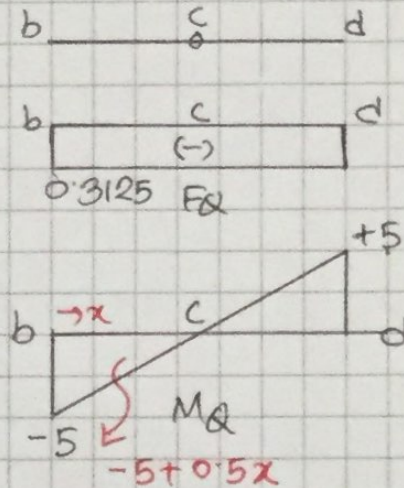
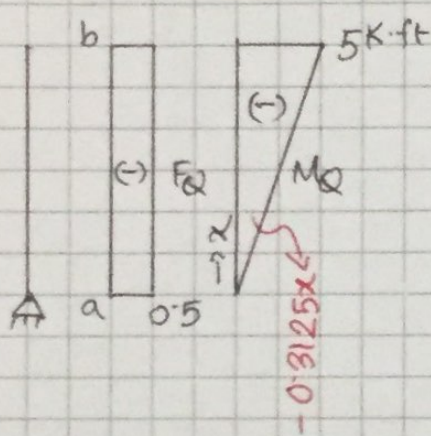
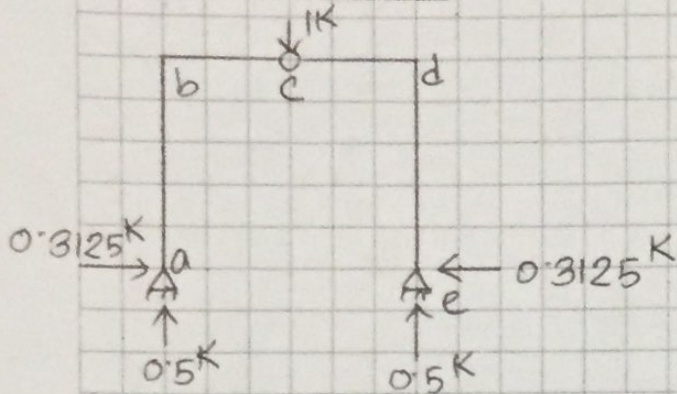
$$\sum M_c = 0 \text{ (Left portion)} \quad -R_{ey} \times 10 + R_{ex} \times 16 = 0$$

$$\therefore R_{ex} = 12.5K$$

$$\therefore R_{ax} = 12.5K$$



Q force analysis:



applying principles of virtual work:-

$$Q \cdot \delta c = \sum \frac{F_P F_Q L}{AE} + \sum \frac{F_Q t L \delta t}{I} + \int \frac{M_P M_Q}{EI} dx$$

$$1 \cdot \delta c = \frac{(-20)(-0.5)(16)}{15 \times 30 \times 10^3} + \frac{(-12.5)(-0.3125)(20)}{15 \times 30 \times 10^3} + \frac{(-0.5)(-20)(16)}{15 \times 30 \times 10^3}$$

$$+ \int_0^{16} \frac{(-12.5x)(0.3125x)}{30 \times 10^3 \times 400/144} dx + \int_0^{16} \frac{(12.5x)(0.3125x)}{30 \times 10^3 \times 400/144} dx$$

$$+ \int_0^{20} \frac{(-5+0.5x)(-200+20x)}{30 \times 10^3 \times 400/144} dx$$

$$= \frac{4}{5625} + \frac{1}{5760} + \frac{6}{125} + \frac{2}{25} = 0.1288847 \text{ ft}$$

EARTHQUAKE LOADING PROVISIONS IN BNBC

- Main Considerations
 - Strength Economy Probability
- Earthquake forces are selected such that
 - the building would not collapse in the event of rarely occurring major EQ, although it may be subjected to damage (safety consideration)
 - the building should be able to respond without structural damage, to shocks of moderate intensities (serviceability consideration).

In moderate zone economy can be provided with acceptable level of damage but save human lives.

- **Selection of EQ Force**

- Forces are applied as static forces for the following structures

- All structures in Seismic Zone 1; and in Zone 2, struc. having importance category IV (low risk)
- Regular structure under 75 m height
- Irregular structure not more than 20_^ in height.

- Alternatively, a Dynamic EQ analysis is required if,

- Structure is of 75 m or more height and is not located in Zone 1.
- Structure 20 m or more high in Zone 3 and not having the same structural system throughout their height
- Structures, located on Soil Profile Type S₄ (Table 6.2.25) , having a time period greater than 0.7 second.

Equivalent Static Force Method

- Base Shear, $V = (ZIC)W / R$

Z = seismic zone coefficient given in BNBC Table 6.2.22

I = structure importance coefficient given in BNBC Table 6.2.23

R = response modification coefficient for struc.. system given in Table 6.2.24

W = total seismic dead load

$$C = (1.25 S) / (T^{2/3})$$

$C \not> 2.75$ and $\frac{C}{R} \not< 0.075$

S = site coefficient for soil characteristics (Table 6.2.25)

T = fundamental period of vibration in second

Seismic Dead Load, W is the total dead load of a structure, including permanent partitions and applicable portions of other loads as listed below:

- 25% of the floor LL in storage or warehouse occupancies
- All loads of partitions, but not less than 0.6 kN/m²
- Total weight of permanent equipment

- 18
- **Structure Period, T** shall be calculated from one of the following methods:

Method A: $T = C_t (h_n)^{3/4}$

Where $C_t = 0.083$ for steel moment resisting frames

$= 0.073$ for RC moment resisting frames and eccentric braced steel frames

$= 0.049$ for all other structural systems

h_n = Height in metres above the base to level n .

Alternatively, the value of C_t for buildings with concrete or masonry shear walls may be taken as $0.031/\sqrt{A_c}$

$$A_c = \sum A_e [0.2 + (D_e/h_n)^2]$$

A_c = combined effective area, in sq. metres, of shear walls in first story of the structure.

- A_e = effective horizontal cross-sectional area, in sq. metre, of a shear wall in the first story of a structure
- D_e = Length, in metre, of a shear wall element in the first story in the direction of the applied forces
- D_e/h_n shall not exceed 0.9

Method B :

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^n w_i \delta_i^2}{g \sum_{i=1}^n f_i \delta_i}}$$

The values of f_i represent any static lateral force distributed approximately as EQ forces.

The elastic deflection δ_i shall be calculated using the applied lateral forces, f_i . The value of T determined by Method B shall not exceed that calculated by Method A by more than 40%.

Vertical Distribution of Lateral Forces

The base shear V shall be distributed along the height of the structure in accordance with the following formula:

$$V = F_t + \sum_{i=1}^n F_i$$

where F_i = lateral force applied at storey level i .

F_t = Concentrated lateral force considered at the top of the building in addition to the force F_n .

The concentrated lateral force F_t acting at the top of the building shall be determined as follows:

$$F_t = 0.07TV \leq 0.25 V \quad \text{when } T > 0.7 \text{ second}$$

$$F_t = 0.0 \quad \text{when } T \leq 0.7 \text{ second}$$

The remaining portion of the base shear ($V - F_t$) shall be distributed over the height of the building, including level n , according to the following relationship :

$$F_x = \frac{(V - F_t) w_x h_x}{\sum_{i=1}^n w_i h_i}$$

Structural Systems for Buildings

Bearing Wall System: System without a complete vertical load carrying space frame.

Dual System: A combination of Moment Resisting Frame and Shear Walls or Braced Frames to resist lateral loads.

Moment Resisting Frame: A frame in which members and joints are capable of resisting forces primarily by flexure.

Ordinary Moment Resisting Frame: A moment resisting frame not meeting special detailing requirements for ductile behaviour.

Shear Wall : A wall designed to resist lateral forces parallel to the plane of the wall (also known as vertical diaphragm or a structural wall).

• Desirable Behaviour of EQ Resistant Structures

- Ability to withstand high lateral force is not the only requirement.
- Building designed with small seismic force but having high flexibility and energy absorbing mechanism will perform better than a building designed with a very high lateral force but with little flexibility.
- Aim is to ensure maximum possible elastic behaviour by proper detailing of the joints etc.

SEISMIC ZONING MAP
Fig. 6.2.10

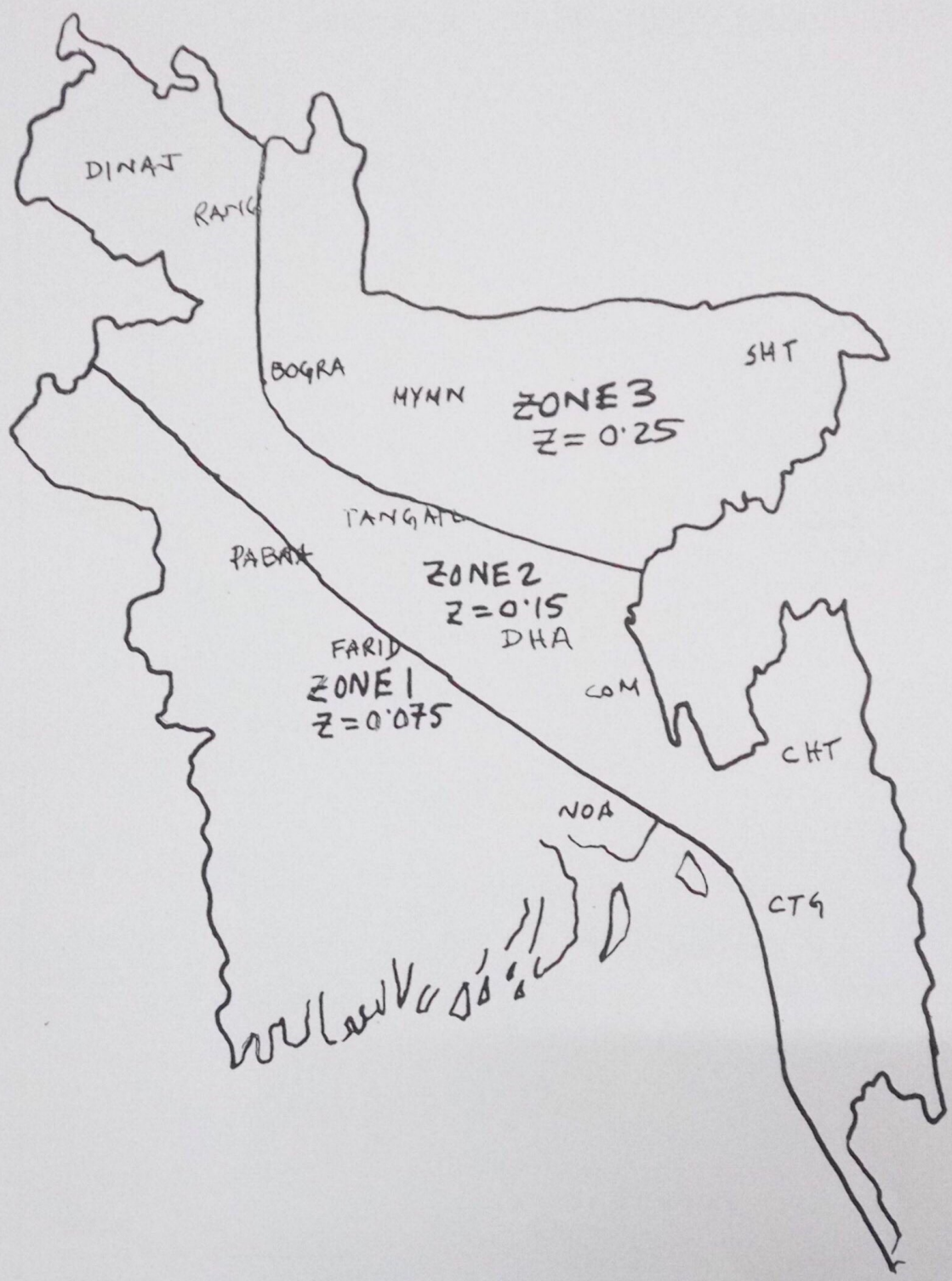


FIG. 6.2.10 : SEISMIC ZONING MAP OF BANGLADESH

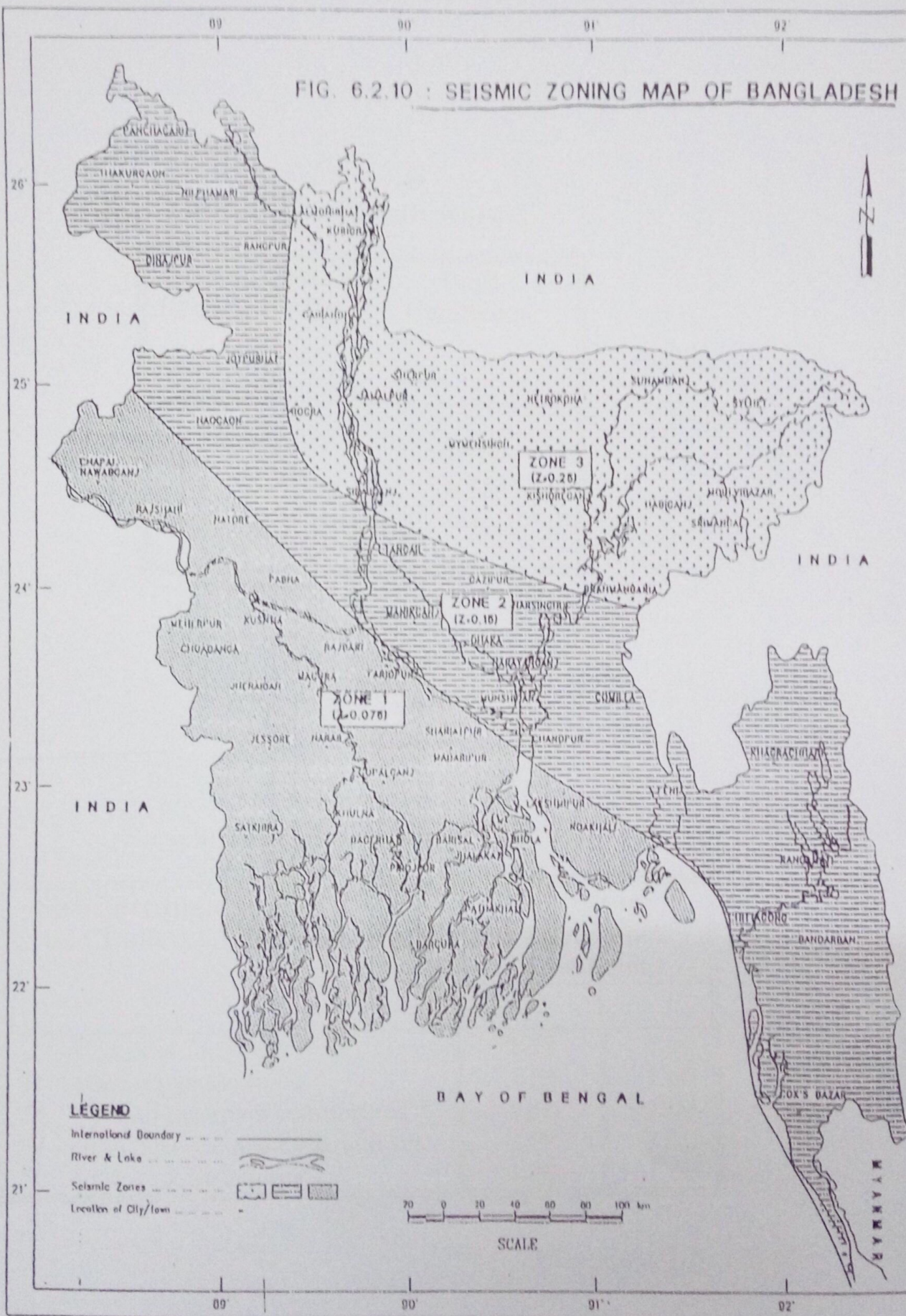


Table 6.2.22
Seismic Zone Coefficients, Z

Seismic Zone (see Fig 6.2.10)	Zone Coefficient
1	0.075
2	0.15
3	0.25

Table 6.2.23
Structure Importance Coefficients I, I'

Structure Importance Category (see Table 6.1.1 for occupancy)	Structure Importance Coefficient	
	I	I'
I Essential facilities	1.25	1.50
II Hazardous facilities	1.25	1.50
III Special occupancy structures	1.00	1.00
IV Standard occupancy structures	1.00	1.00
V Low-risk Structures	1.00	1.00

Table 6.2.24
Response Modification Coefficient for Structural Systems, $R \approx 4 \sim 12$

Basic Structural System ⁽¹⁾	Description of Lateral Force Resisting System	R ⁽²⁾	
a. Bearing Wall System	1. Light framed walls with shear panels i) Plywood walls for structures, 3 storeys or less ii) All other light framed walls	8 6	
	2. Shear walls i) Concrete ii) Masonry	6 6 4	
	3. Light steel framed bearing walls with tension only bracing	6	
	4. Braced frames where bracing carries gravity loads i) Steel ii) Concrete ⁽³⁾ iii) Heavy timber	4 4 4	
	b. Building Frame System	1. Steel eccentric braced frame (EBF)	10
		2. Light framed walls with shear panels i) Plywood walls for structures 3-storeys or less ii) All other light framed walls	9 7
		3. Shear walls i) Concrete ii) Masonry	8 8
		4. Concentric braced frames (CBF) i) Steel ii) Concrete ⁽³⁾ iii) Heavy timber	8 8 8
c. Moment Resisting Frame System		1. Special moment resisting frames (SMRF) i) Steel ii) Concrete	12 12 8
		2. Intermediate moment resisting frames (IMRF), concrete ⁽⁴⁾	6
		3. Ordinary moment resisting frames (OMRF) i) Steel ii) Concrete ⁽⁵⁾	5
d. Dual System		1. Shear walls i) Concrete with steel or concrete SMRF ⁽⁴⁾ ii) Concrete with steel OMRF ⁽⁴⁾ iii) Concrete with concrete IMRF ⁽⁴⁾ iv) Masonry with steel or concrete SMRF ⁽⁴⁾ v) Masonry with steel OMRF ⁽⁴⁾ vi) Masonry with concrete IMRF ⁽³⁾	12 6 9 8 6 7
	2. Steel EBF i) With steel SMRF ⁽⁴⁾ ii) With steel OMRF ⁽⁴⁾	12 6	
	3. Concentric braced frame (CBF) i) Steel with steel SMRF ⁽⁴⁾ ii) Steel with steel OMRF ⁽⁴⁾ iii) Concrete with concrete SMRF ⁽³⁾ iv) Concrete with concrete IMRF ⁽³⁾	10 6 9 6	
	e. Special Structural Systems	See Sec 1.3.2, 1.3.3, 1.3.5	

Notes : (1) Basic Structural Systems are defined in Sec 1.3.2, Chapter 1.
 (2) See Sec 2.5.6.6 for combination of structural systems, and Sec 1.3.5 for system limitations.
 (3) Prohibited in Seismic Zone 3.
 (4) Prohibited in Seismic Zone 3 except as permitted in Sec 2.5.9.3.
 (5) Prohibited in Seismic Zones 2 and 3. Sec 1.7.2.6.

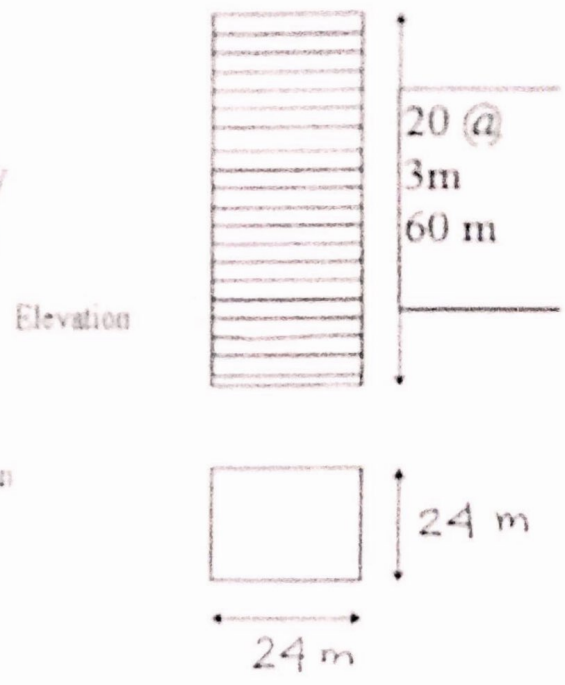
Table 6.2.25
Site Coefficient, S for Seismic Lateral Forces (1)

Site Soil Characteristics		Coefficient, S
Type	Description	
S_1	A soil profile with either : a) A rock-like material characterized by a shear-wave velocity greater than 762 m/s or by other suitable means of classification, or b) Stiff or dense soil condition where the soil depth is less than 61 metres	1.0
S_2	A soil profile with dense or stiff soil conditions, where the soil depth exceeds 61 metres	1.2
S_3	A soil profile 21 metres or more in depth and containing more than 6 metres of soft to medium stiff clay but not more than 12 metres of soft clay	1.5
S_4	A soil profile containing more than 12 metres of soft clay characterized by a shear wave velocity less than 152 m/s	2.0
<p>Note : (1) The site coefficient shall be established from properly substantiated geotechnical data. In locations where the soil properties are not known in sufficient detail to determine the soil profile type, soil profile S_3 shall be used. Soil profile S_4 need not be assumed unless the building official determines that soil profile S_4 may be present at the site, or in the event that soil profile S_1 is established by geotechnical data.</p>		

Example : Eq. Static EQ Force

Calculate the distribution of EQ forces on the 20 storied office building shown in figure.

- Location : Dhaka
- Soil Type : Soft to medium stiff clay
- DL including partition = 12 kN/sq.m
- Structure type : SMRF in concrete



Solution:
 Zone 2: $Z=0.15$ $I=1.0$

$$C = \frac{1.25S}{T^{2/3}}$$

Soil Type $S_3 = 1.5$

$$T = C_t (h_n)^{3/4}$$

$$h_n = 60 \text{ m}$$

$$C_t = 0.073$$

$$T = 0.073 \times (60)^{3/4}$$

$$= 1.57 \text{ Sec}$$

$$C = \frac{1.25 \times 1.5}{(1.57)^{2/3}}$$

$$C = 1.386 < 2.75$$

$$C \not\leq 2.75 \text{ and } (C/R) \not\leq 0.075$$

Here $R=12$

hence: $C/R=0.115 > 0.075$

Therefore, $C=1.386$ O.K

Seismic Dead Load

$$W = 12 \text{ kN/sq. m} \times (24 \times 24 \text{ sq. m}) \times 20 \text{ floors} \\ = 138240 \text{ kN}$$

$$\text{Base Shear, } V = (ZIC) \times W/R \\ = (0.15 \times 1 \times 1.386 / 12) \times W \\ = 0.0173 W \\ = 2392 \text{ kN}$$

Vertical Distribution of Forces:

The concentrated lateral force F_t at the top of the building:

$$F_t = 0.07TV \leq 0.25 V \quad \text{when } T > 0.7 \text{ second}$$

$$F_t = 0.0 \quad \text{when } T \leq 0.7 \text{ second}$$

$$F_t = 0.07 \times 1.57 \times 2392 = 264 \text{ kN}$$

here : $F_t \leq 0.25 V$ i.e. $F_t = 264 \text{ kN}$ O.K.

$$F_x = \frac{(V - F_t) w_x h_x}{\sum_{i=1}^n w_i h_i}$$

$w_x = 12 \times 576 \text{ kN}$ for each floor.

In this example w is same for all the floors.

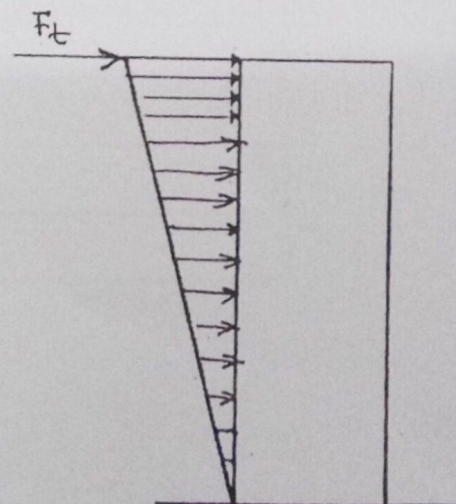
For any building for which w is same for each floor:

$$F_x = \frac{w_x (V - F_t) h_x}{w_i \sum_{i=1}^n h_i}$$

$$F_x = \frac{(2392 - 264) h_x}{(3 + 6 + 9 + \dots + 60)}$$

$$F_x = \frac{2128 h_x}{3 \times \frac{(20 \times 21)}{2}}$$

$$F_x = 3.38 h_x$$



EQ Forces at various levels

Floor Level	h_x , m	F_x , kN
20	60	202.7
19	57	192.5
18	54	182.4
17		
16		
15		
14		
13		
12		
11		
10		
9		
8		
7		
6		
5		
4		
3		
2		
1		

Wind Pressure and Coefficients

Basic Wind Pressure

The basic wind pressure on a surface is given by $q_b = \rho_{\text{air}} V_b^2 / 2$ (1)

where $\rho_{\text{air}} = \text{Density of air} = 0.0765/32.2 = 23.76 \times 10^{-4} \text{ slug/ft}^3$

$V_b = \text{Basic wind speed, ft/sec} = 1.467 \times \text{Basic wind speed, mph}$

$\therefore \text{Eq. (1)} \Rightarrow q_b = 23.76 \times 10^{-4} \times (1.467 V_b)^2 / 2 = 0.00256 V_b^2$ (2)

where q_b is in psf (lb/ft²) and V_b is in mph (mile/hr).

The basic wind speeds at different important locations of Bangladesh are given below. A more detailed map for the entire country is available in BNBC 1993.

Location	V_b (mph)
Dhaka	130
Chittagong	160
Rajshahi	95
Khulna	150

Sustained Wind Pressure

The wind velocity (and pressure) increases from zero at the base of the structure and is also a function of the exposure (i.e., open terrain or congested area). Moreover one has to account for the importance of the structure; i.e., design the sensitive structures more conservatively.

The sustained wind pressure on a building surface at any height z above ground is given by $q_z = 0.00256 C_1 C_z V_b^2$ (3)

where $C_1 = \text{Structural importance coefficient}$, $C_z = \text{Height and exposure coefficient}$.

Category	C_1
Essential facilities	1.25
Hazardous facilities	1.25
Special occupancy	1.00
Standard occupancy	1.00
Low-risk structure	0.80

Height z (ft)	C_z		
	Exp A	Exp B	Exp C
0~15	0.368	0.801	1.196
50	0.624	1.125	1.517
100	0.849	1.371	1.743
150	1.017	1.539	1.890
200	1.155	1.671	2.002
300	1.383	1.876	2.171
400	1.572	2.037	2.299
500	1.736	2.171	2.404
650	1.973	2.357	2.547
1000	2.362	2.595	2.724

Design Wind Pressure

The design wind pressure can be calculated by multiplying the sustained wind pressure by appropriate pressure coefficients due to wind gust and turbulence as well as local topography.

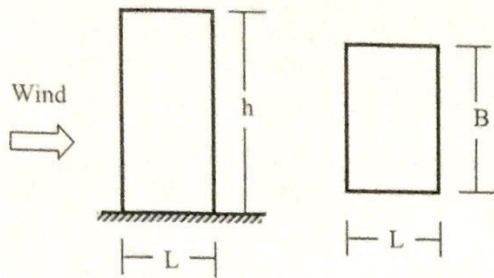
The design wind pressure on a surface at any height z above ground is given by $p_z = C_G C_t C_p q_z$ (4)

where $C_G = \text{Wind gust coefficient}$, $C_t = \text{Local topography coefficient}$, $C_p = \text{Pressure coefficient}$.

Height z (ft)	C_G (for non-slender structures)		
	Exp A	Exp B	Exp C
0~15	1.654	1.321	1.154
50	1.418	1.215	1.097
100	1.309	1.162	1.067
150	1.252	1.133	1.051
200	1.215	1.114	1.039
300	1.166	1.087	1.024
400	1.134	1.070	1.013
500	1.111	1.057	1.005
650	1.082	1.040	1.000
1000	1.045	1.018	1.000

The value of C_G for slender structures (height > 5 times the minimum width) would be determined by dynamic analysis. Although code-based formulae are available, it is unlikely to exceed 2.0.

The pressure coefficient C_p for rectangular buildings with flat roofs may be obtained as follows

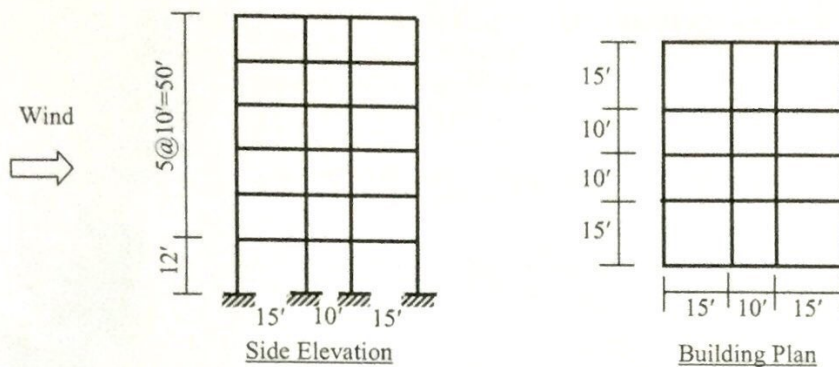


h/B	L/B					
	0.1	0.5	0.65	1.0	2.0	≥ 3.0
≤ 0.5	1.40	1.45	1.55	1.40	1.15	1.10
1.0	1.55	1.85	2.00	1.70	1.30	1.15
2.0	1.80	2.25	2.55	2.00	1.40	1.20
≥ 4.0	1.95	2.50	2.80	2.20	1.60	1.25

Calculation of Wind Load

Wind Load on a Building

Calculate the wind load at each story of a six-storied hospital building (shown below) located at a flat terrain in Dhaka. Assume the structure to be subjected to Exposure B.



Solution

The design wind pressure at a height z is given by $p_z = 0.00256 C_1 C_2 C_G C_t C_p V_b^2$

Since the building is located in Dhaka, the basic wind speed $V_b = 130$ mph

For the hospital building (essential facility), Structural importance coefficient $C_1 = 1.25$

In plane terrain, Local topography coefficient $C_t = 1.0$

Building height $h = 62'$, dimensions $L = 40'$ and $B = 50'$; i.e., $h/B = 1.24$ and $L/B = 0.80 \Rightarrow C_p \cong 1.98$

$\therefore p_z = 0.00256 \times 1.25 \times C_2 \times C_G \times 1.00 \times 1.98 \times (130)^2 = 107.08 C_2 C_G$

\therefore The corresponding force $F_z = B h_{eff} p_z = 50 h_{eff} p_z$; where h_{eff} = Effective height of the tributary area
 $h_{eff} = 6' + 5' = 11'$ at 1st floor, $(5' + 5' =) 10'$ between 2nd and 5th floor and $5'$ at 6th floor

The coefficients C_2 , C_G and the design wind pressure p_z and force F_z at different heights are shown below.

Story	z (ft)	C_2	C_G	p_z (psf)	F_z (kips)	F_{frames} (kips)				
						9.35	15.58	12.46	15.58	9.35
1	12	0.801	1.321	113.30	62.32	9.35	15.58	12.46	15.58	9.35
2	22	0.866	1.300	120.55	60.27	9.04	15.07	12.05	15.07	9.04
3	32	0.958	1.270	130.28	65.14	9.77	16.29	13.03	16.29	9.77
4	42	1.051	1.239	139.44	69.72	10.46	17.43	13.94	17.43	10.46
5	52	1.135	1.213	147.42	73.71	11.06	18.43	14.74	18.43	11.06
6	62	1.184	1.202	152.39	38.10	5.72	9.53	7.62	9.53	5.72