

16-12

F → Flexi
M → mem
S →

T → theory

N → not mention

1 (a) → theory
(b) → Flexi

2 (a) → T
(b) → F/N

3 (a) → F/N
(b) → F

4 (a) → S

5 (a) → M
(b) → M

6 (a) → M
(b) → M

7 (a) → M
(b) → S

8 (a) → S
(b) → S

15-16

1 (a) → M
(b) → M

2 (a) S
(b) S

3 (a) S
(b) M

4 (a) M
(b) S

5 (a) F
(b) F

6 (a) F
(b) F

7 (a) F
(b) S

8 (a) N
(b) S

14-15

1 (a) F
(b) F

2 (a) F
(b) F

3 (a) S
(b) S

4 (a) S
(b) T

5 (a) T
(b) M

6 (a) M
(b) M

7 (a) S
(b) F

8 (a) S
(b) S

13-14

1 (a) M
(b) F

2 → F
3 → M
4 → F

5 → M
6 → F
7 → M

8 → S
9 → S

10 → S
11 → S
12 → S

13 → S
14 → F

12-13

1 (a) M
(b) S

2 (a) M
(b) S

3 (a) S
(b) S

4 (a) S
(b) F/T

6 (a) F
(b) F

7 (a) F
(b) F

8 (a) F
(b) F

11-12

1 (a) F
(b) S

2 (a) F
(b) S

3 (a) N
(b) S

4 → S
5 (a) M
(b) F

6 (a) F scope
(b) M

7 (a) M
(b) scope

8 (a) scope
(b) —

S → 4
F → 4
M → 5

S → 5
F → 5
M → 4

S → 6
F → 5
M → 3

S → 6
F → 4
M → 4

S → 4
F → 6
M → 2

S → 4
F → 3
M → 3

13-14

11 12
10 11
8 9

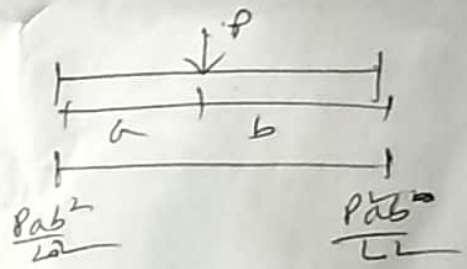
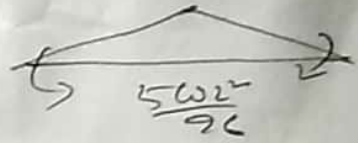
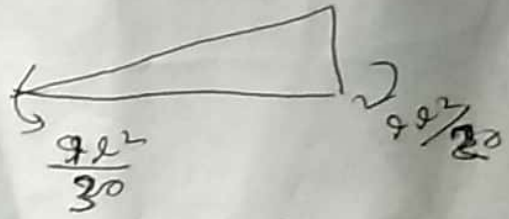
Nothing is certain.

10-11

- 1 @ IF
- ⑤ M
- 2 @ S
- ⑤ M
- 3 @ M
- ⑤ IF
- ⑤ M
- ⑤ T
- 5 @ F
- ⑤ S
- 6 @ S
- ⑤ S
- 7 @ S
- ⑤ S
- 8 @ S

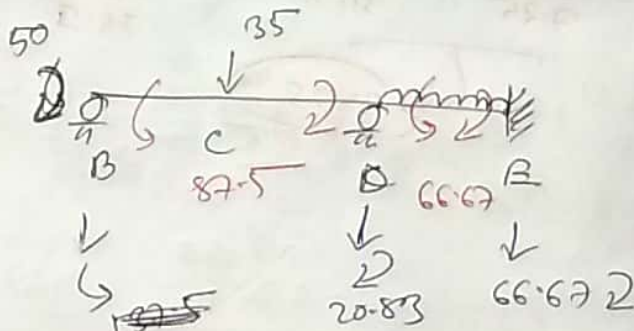
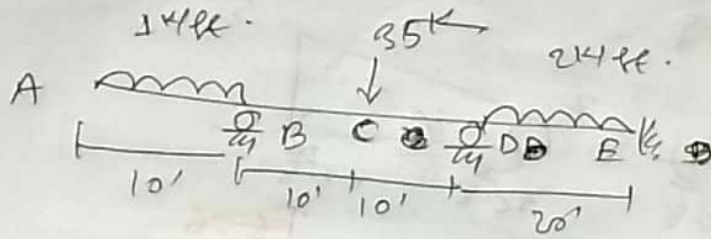
9-10

- 1 @ F/S
- ⑤ IF
- 2 @ M
- ⑤ M
- 3 @ S
- ⑤ M
- 4 @ IF
- ⑤ M
- 5 @ S
- ⑤ S
- 6 @ S
- ⑤ F
- 7 @ S
- ⑤ S
- 8 @ S
- ⑤ F

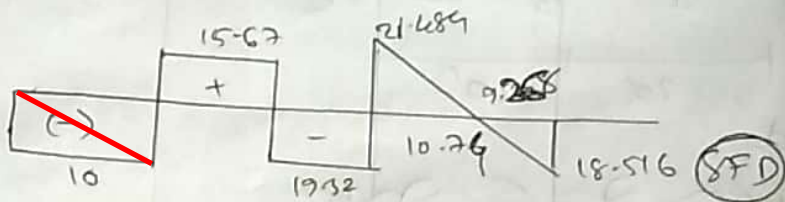
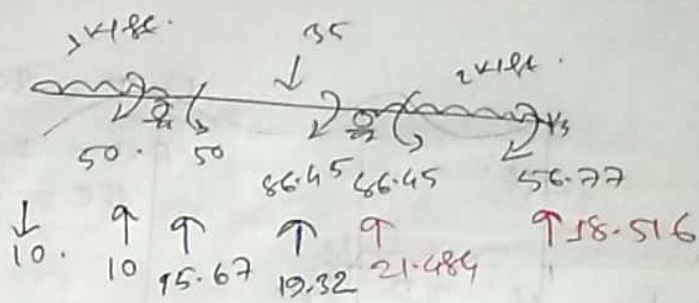


2016-17

5(a)

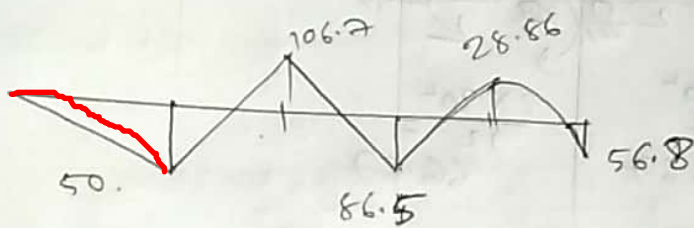


DF	(A)	(B)	(D)	(E)
FEM			0.5 0.5	
	-50	87.5	-87.5	66.67 -66.67
		-37.5	10.42	10.42
			-18.75	5.21
			+9.375	+9.375
				4.688
	-50	+50	-86.45	86.45 -56.77



$$\frac{x}{21.484} = \frac{20-x}{18.516}$$

$$x = 10.74$$



(BMD)



(S)

10	20	30	40	50
10	20	30	40	50
10	20	30	40	50

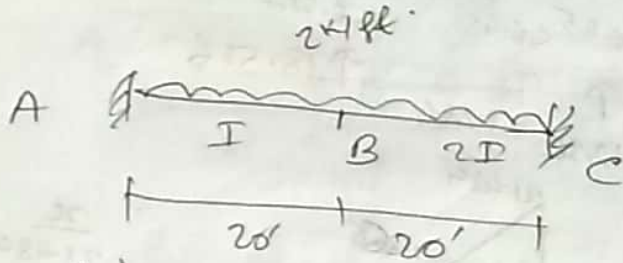
2016-17

5(b)

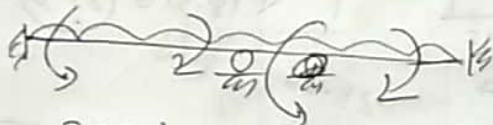
$E I = 3000$

$AB = I$

$BC = 2I$



Part 2
translation prevented



$\frac{2 \times 20^2}{12}$
 $= 66.67$

$\frac{2 \times 20^2}{12} =$
 66.67

DF

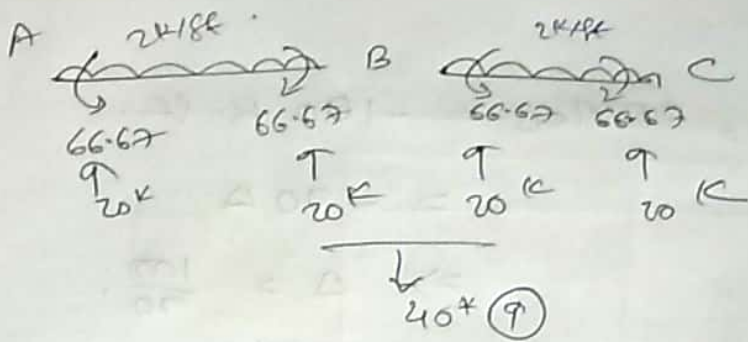
$DF_{AB} = 0$

$DF_{BA} = \frac{I/20}{I/20 + 2I/20} = 0.33$

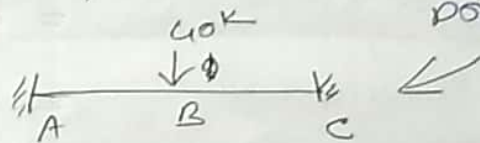
$DF_{BC} = 0.67$

$DF_{CB} = 0$

DF	0	0.67	0.33	0
FEM	66.67	-66.67	66.67	-66.67
Σ	66.67	-66.67	66.67	-66.67

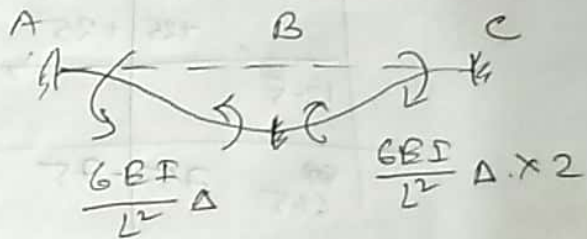


We need to apply $40k (\downarrow)$ to stabilize the point B.



Part-2

translation permitted.



We assume

$$\begin{aligned}
 FEM_{BC} = FEM_{CB} &= -\frac{6EI \times 2l}{L^2} \Delta \\
 &= -\frac{6 \times 3000 \times 2}{20^2} \times \Delta \\
 &= -90 \Delta
 \end{aligned}$$

$$\begin{aligned}
 FEM_{AB} = FEM_{BA} &= \frac{6EI}{L^2} \Delta \\
 &= \frac{6 \times 3000}{20^2} \times \Delta = 45 \Delta
 \end{aligned}$$

We assume

$$FEM_{BC} = FEM_{CB} = -100 \text{ K-ft}$$

$$= -90 \Delta$$

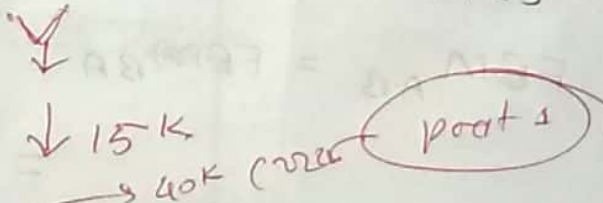
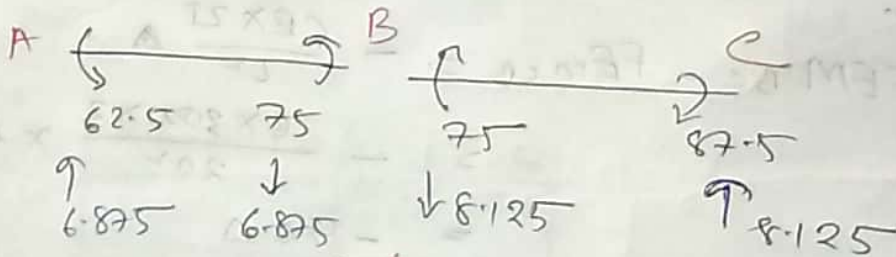
$$\Rightarrow \Delta = \frac{100}{90}$$

$$FEM_{AB} = FEM_{BA} = 45 \Delta$$

$$= 45 \times \frac{100}{90}$$

$$= 50 \text{ K-ft}$$

DF	A	B	C
FEM	0	0.5 0.5	0
	50	50 -100	-100
		+25 +25	
	12.5		12.5
	62.5	75 -75	-87.5

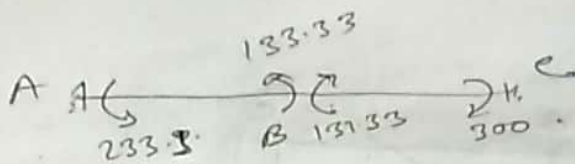


$$M_{AB} = 66.67 + \frac{40}{15} \times 62.5 = 233.33 \text{ K-ft}$$

$$M_{BA} = -66.67 + \frac{40}{15} \times 75 = 133.33 \text{ K-ft}$$

$$M_{BC} = 66.67 + \frac{40}{15} \times (-75) = -133.33$$

$$M_{CB} = -66.67 + \frac{40}{15} \times (-87.5) = -300$$



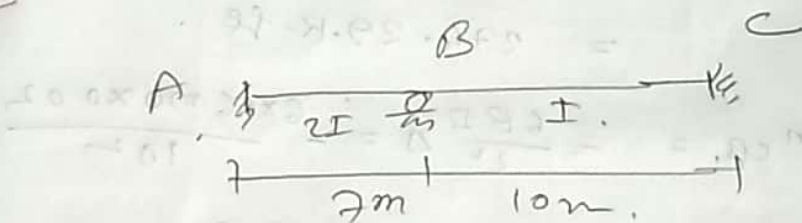
~~FEM_{AB}~~

$$\Delta = \frac{233.33 \times 20^2}{6 \times 3000} = 2.96 \text{ k}$$

$$FEM_{BA} = \frac{6EI}{L^2} \Delta = 133.33$$

$$\Rightarrow \Delta = 2.96 \text{ k}$$

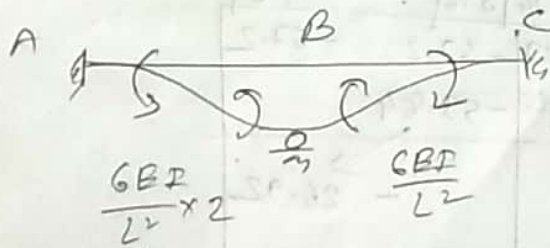
6(b)



$E = 70 \text{ kPa}$
 $I = 800 \times 10^6 \text{ mm}^4$

Settles at

$\delta = 20 \text{ mm}$



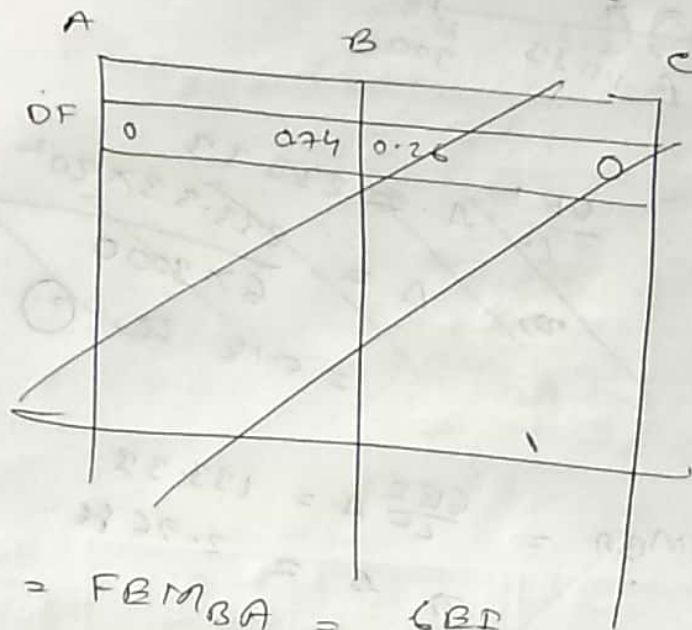
$EI = 70 \times 800$
 $= 56000 \text{ KN-m}$

$DF_{AB} = 0$

$DF_{BA} = \frac{2I/7}{2I/7 + I/10} = 0.74$

$DF_{BC} = 1 - 0.74$
 $= 0.26$

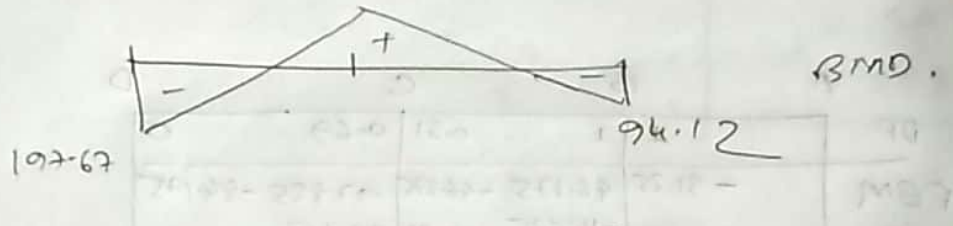
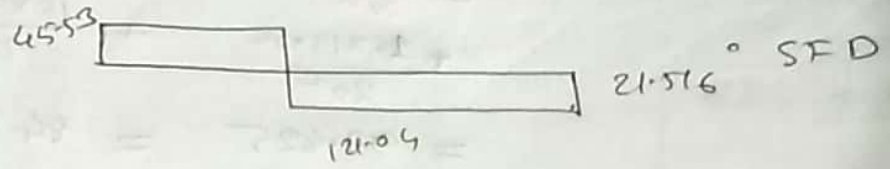
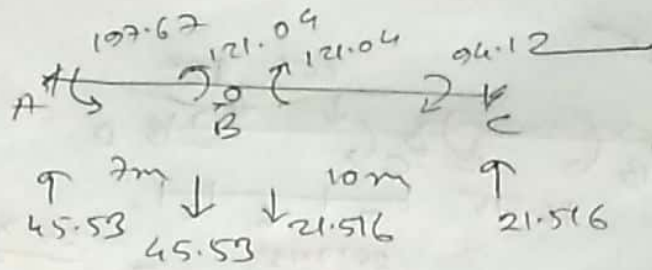
$DF_{CB} = 0$



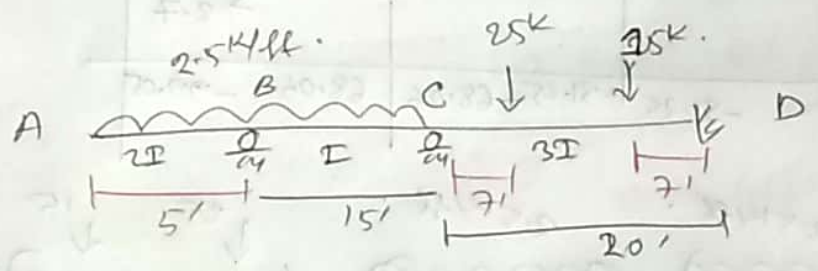
$$\begin{aligned}
 FEM_{AB} = FEM_{BA} &= \frac{6EI}{L^2} \Delta \times 2 & D = 20\text{mm} \\
 & & = 0.02\text{m} \\
 &= \frac{6 \times 56000}{7^2} \times 0.02 \times 2 \\
 &= 274.29 \text{ K-ft}
 \end{aligned}$$

$$\begin{aligned}
 FEM_{BC} = FEM_{CB} &= -\frac{6EI}{L^2} \Delta = -\frac{6 \times 56000 \times 0.02}{10^2} \\
 &= -67.2
 \end{aligned}$$

DF	0	0.74	0.26	0
FEM	274.29	274.29	-67.2	-67.2
		-153.25	-53.84	
	276.625			-26.92
	197.67	121.04	-121.04	-94.12



2015-16
10



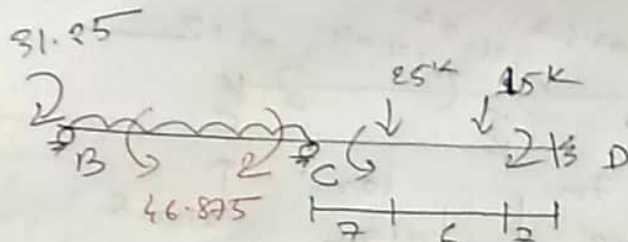
$$DF_{BC} = 1.$$

$$DF_{DC} = 0.$$

$$DF_{CD} = \frac{\frac{3I}{20}}{\frac{3I}{20} + \frac{I}{15}} = 0.69$$

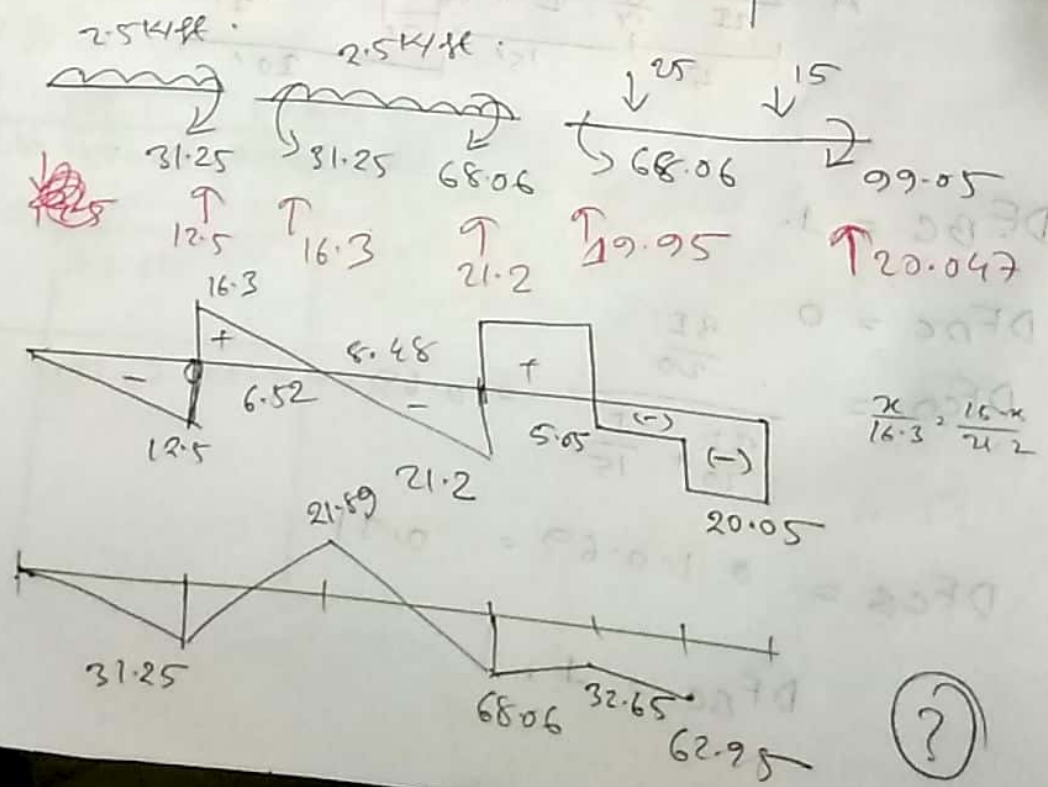
$$DF_{CB} = 1 - 0.69 = 0.31$$

⑤ $DF_{BC} = 1.$



$$\begin{aligned}
 &= \frac{25 \times 7 \times 15^2}{20^2} + \frac{15 \times 7 \times 15^2}{20^2} \\
 &= \frac{25 \times 13 \times 7^2}{20^2} + \frac{15 \times 7 \times 13^2}{20^2} \\
 &= 97.825 = 84.175
 \end{aligned}$$

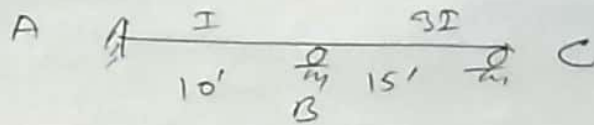
	A	B	C	D
DF		1	0.31	0.69
FEM	-31.25	46.875	-46.875	97.825
		-15.625	-15.79	-84.175
		-7.8125	-17.58	
		+2.42	+5.39	
				2.7
	-31.25	+31.25	-68.06	68.06
				-99.05



14-15

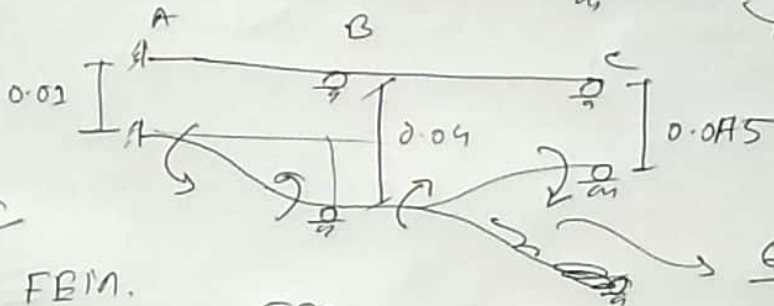
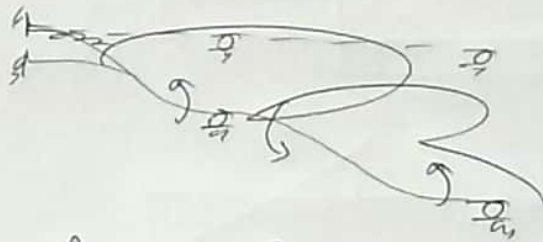
5(b) DIY → do it yourself.

5(c)



$$\begin{aligned} \theta_A &= 0.001 \text{ rad} \\ \Delta_A &= 0.01 \text{ ft} \downarrow \\ \Delta_B &= 0.04 \text{ ft} \downarrow \\ \Delta_C &= 0.0175 \text{ ft} \downarrow \end{aligned}$$

$$\begin{aligned} EI &= \frac{30 \times 10^3 \times 10^3}{254} \\ &= 30 \times 10^6 \text{ K-in}^2 \\ &= 208320 \text{ K-ft}^2 \end{aligned}$$



Deflection

FEM.

$$\begin{aligned} M_{BC} &= FEM_{CB} = -\frac{6EI}{L^2} \Delta = -\frac{6 \times 208320}{15^2} \times (0.0175 - 0.04) \\ &= -416.64 \text{ K-ft} \end{aligned}$$

FEM

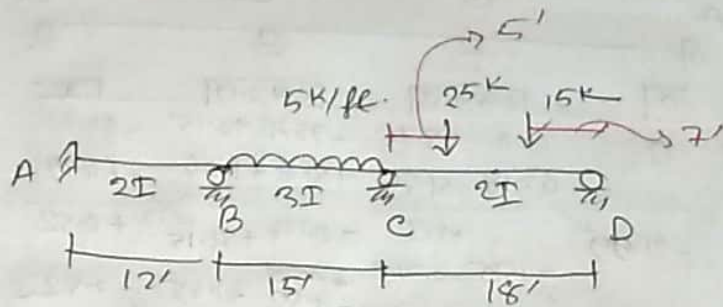
$$\begin{aligned} M_{AB} &= M_{BA} = \frac{6EI}{L^2} \Delta = \frac{6 \times 208320}{10^2} \times (0.04 - 0.01) \\ &= 374.976 \text{ K-ft} \end{aligned}$$

Rotation

Stiffness method & require

2013-14

①



$$DF_{BA} = \frac{2I/12}{\frac{2I}{12} + \frac{3I}{15}} = 0.45$$

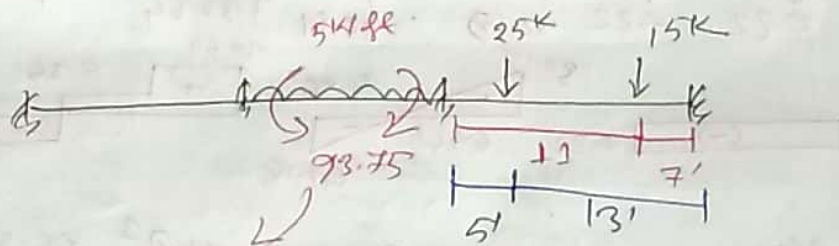
$$DF_{AB} = 0 \text{ (fixed)}$$

$$DF_{BC} = 1 - 0.45 = 0.55$$

$$DF_{CD} = \frac{\frac{2I}{18} \times \frac{3}{4}}{\frac{2I}{18} \times \frac{3}{4} + \frac{3I}{15}} = 0.29$$

$$DF_{DC} = 1$$

$$DF_{CB} = 1 - 0.29 = 0.71$$

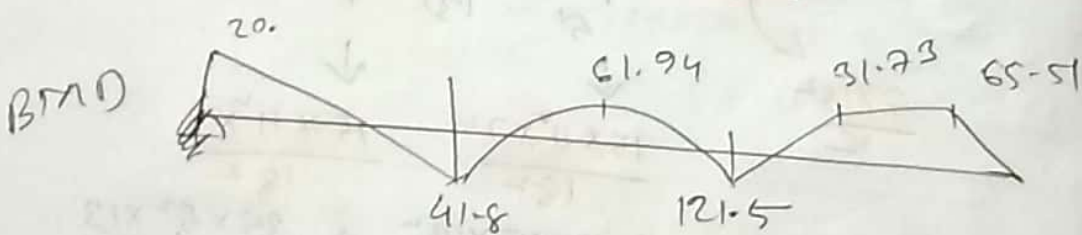
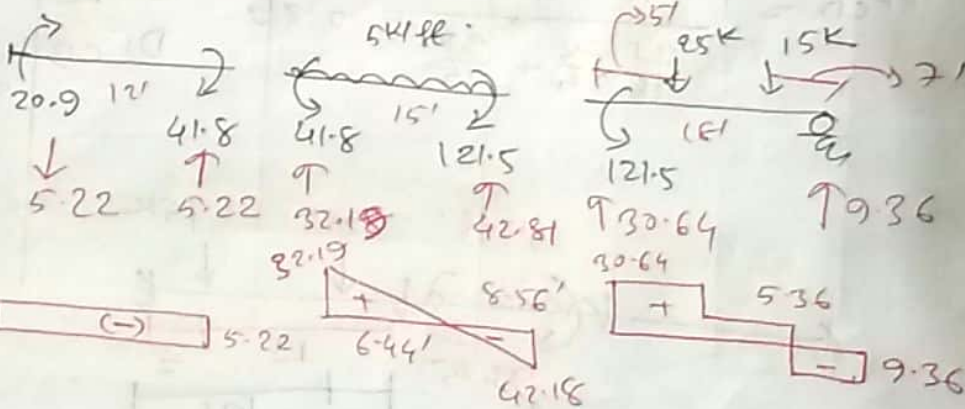


$$\frac{5 \times 15^2}{12}$$

$$\begin{aligned} & \frac{15 \times 11^2 \times 7^2}{18^2} + \frac{25 \times 5^2 \times 13^2}{18^2} + \frac{15 \times 11^2 \times 7^2}{18^2} \\ & = 90.15 \end{aligned}$$

$$= 64.29$$

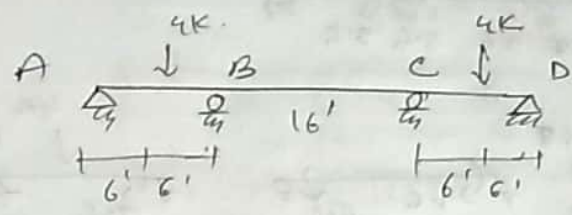
	A	B	C	D
DF	0	0	0	0
FEM	0	0	0	0
		0.45	0.55	
		93.75	-93.75	90.15
		-42.18	-51.56	+2.6
			+1.04	+64.29
	-21.09	+1.3	-25.78	+32.15
		-0.585	-0.715	+0.52
	-0.29		-4.52	-1.85
		-2.26	-0.37	-0.26
	1.01	1.24	+0.45	+0.18
	0.505	+0.23	0.62	0.465
		-0.1	-0.13	-0.09
	-20.9	-41.8	41.8	-121.5
				+121.5



2012-13

2(a)

Beam Problem (without settlement)



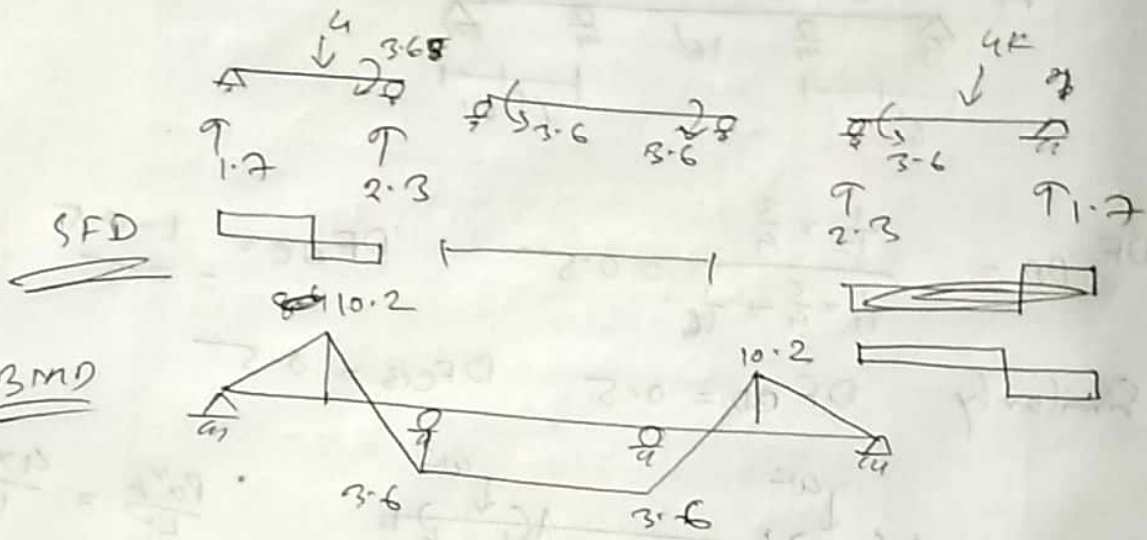
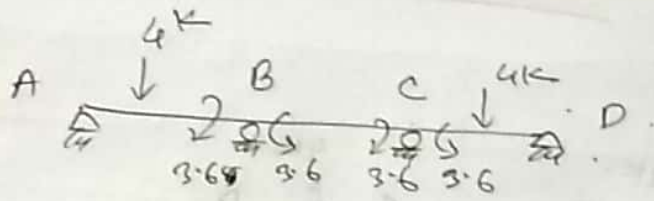
$$DF_{BA} = \frac{\frac{1}{12} \times \frac{3}{4}}{\frac{1}{12} \times \frac{3}{4} + \frac{1}{6}} = 0.5 \quad DF_{BC} = 1 - 0.5 = 0.5$$

Similarly $DF_{CD} = 0.5 \quad DF_{CB} = 0.5$



$$\frac{P \cdot a \cdot b^3}{L^3} = \frac{4 \times 6^3}{12^3}$$

DF	J	0.5	0.5	0.5	0.5	J.
FEM	6	-6	0	0	6	-6
	-6	+3	+3	-3	-3	+6
	+1.5	-3	-1.5	+1.5	+3	-1.5
	-1.5	+2.25	+2.25	-2.25	-2.25	+1.5
	+1.125	-0.8	-1.125	+1.125	+0.8	-1.125
	-1.125	+0.96	+0.96	-0.96	+0.96	+1.125
	+0.48	-0.57	-0.48	+0.48	+0.57	-0.48
	-0.48	+0.5	+0.5	-0.5	-0.5	+0.48
	+0.25	-0.24	-0.25	+0.25	+0.24	-0.25
	-0.25	+0.25	+0.25	-0.25	-0.25	+0.25
	+0.125	-0.125	-0.125	+0.125	+0.125	-0.125
	-0.125	+0.125	+0.125	-0.125	-0.125	+0.125
	0	-3.65	+3.6	-3.6	+3.6	0



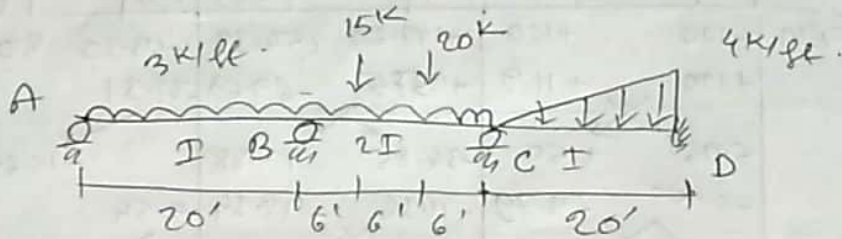
Segment	Start Point	End Point	Length (m)	Shear Force (kN)	Bending Moment (kNm)
1	A	B	3.6	1.7	0 to 10.2
2	B	C	3.6	2.3	10.2 to 0
3	C	D	3.6	1.7	0 to 10.2
4	D	A	3.6	1.7	10.2 to 0

2011-12

No beam

10-11

1 (B)



$$DF_{AB} = 1 \quad DF_{BA} = \frac{\frac{I}{20} \times \frac{3}{4}}{\frac{I}{20} \times \frac{3}{4} + \frac{2I}{18}} = 0.252 \approx 0.25$$

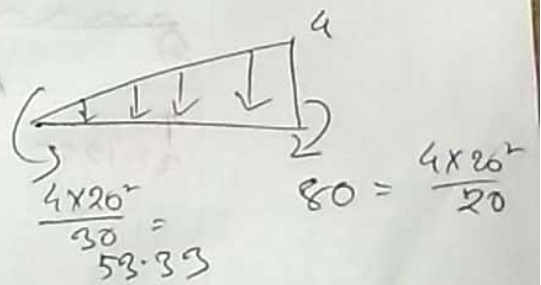
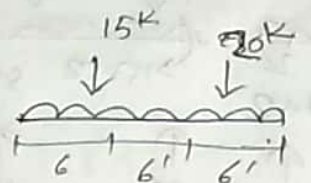
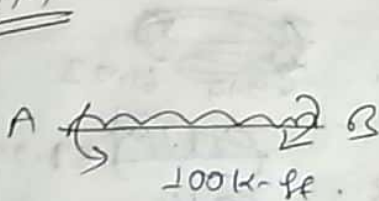
$$DF_{BC} = 1 - 0.252 = 0.748 \approx 0.75$$

$$DF_{CD} = \frac{\frac{I}{20}}{\frac{I}{20} + \frac{2I}{18}} = 0.31$$

$$DF_{CB} = 1 - 0.31 = 0.69$$

$$DF_{DE} = 0$$

FEM

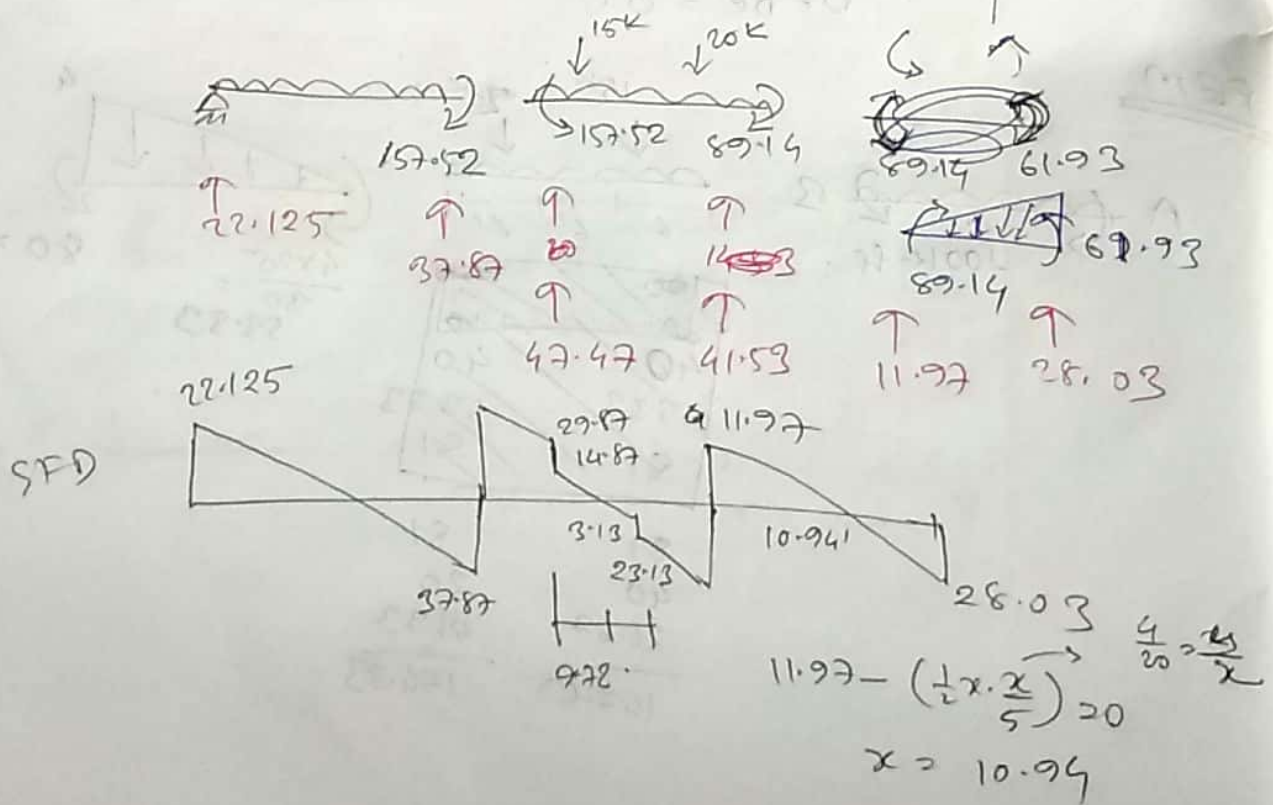


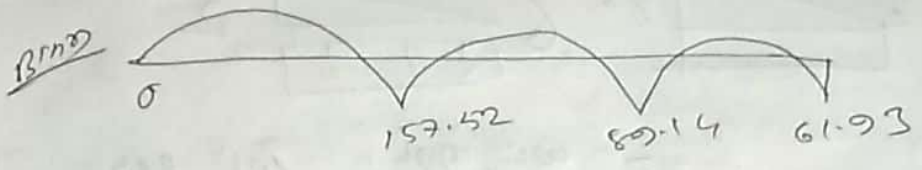
100	100
40	40
53.33	53.33
81	81

81	81
40	20
26.67	53.33
<hr/>	<hr/>
147.67	154.33

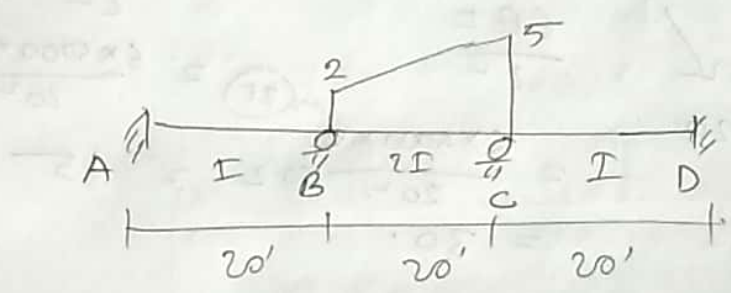
DF	1	0.25	0.75	0.69	0.31	0
FEM	-100	+100	-147.67	154.33	-53.33	80
	+100	+11.9	+33.75	-69.69	31.31	
	5.95	+50	-34.85	17.88		-15.65
	-5.95	-3.79	-11.36	-12.34	5.54	
	-1.875	-2.98	-6.17	-5.68		-2.77
	+1.875	2.29	6.86	3.92	1.26	
	1.15	0.95	1.96	3.43		+0.88
	-1.15	-0.73	-2.18	-2.37	-1.06	
	+0.37	-0.58	-1.185	-1.09		-0.53
	-0.37	-0.45	0.45	+0.75	+0.34	
	0	0.44	1.32			
	0	+157.52	-157.52	+89.14	-89.14	61.93

Need to
2-3 more
iteration.
please DJY





2010-11
4(b)



$EI = 5000 \text{ K-ft}^2$

A rotate 0.04 rad

C settles 0.2 ft

DF

$$DF_{AB} = 0$$

$$DF_{BA} = \frac{I/20}{I/20 + 2I/20} = 0.33$$

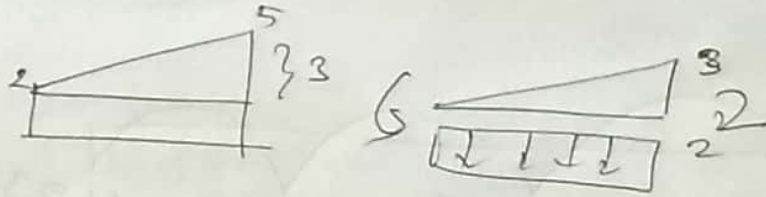
$$DF_{BC} = 1 - 0.33 = 0.67$$

$$DF_{CD} = \frac{I/20}{I/20 + 2I/20} = 0.33$$

$$DF_{CB} = 1 - 0.33 = 0.67$$

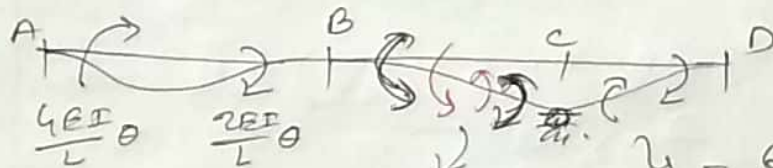
$$DF_{DC} = 0$$

FEM



$$= \frac{wL^2}{12} + \frac{qL}{30} \qquad \frac{wL^2}{12} + \frac{qL}{20}$$

$$= 126.67 \qquad = 126.67$$



$$\frac{4 \times 5000}{20} \times 0.04 = 20$$

$$= 40$$

$$\frac{6EI}{L^2} \Delta = \frac{6 \times 5000 \times 0.2}{20^2} = 15$$

$$= 30$$

DF	0	0.33	0.67	0.67	0.33	0
FEM	-40	-20	+136.67	-30	-15	-15
		-38.5	-78.17	-96.67	+36.85	
	-19.25		37.41	-39.1		+19.48
		-12.35	-25.06	26.2	12.9	
	-6.18		13.1	-12.51		6.45
		-4.92	-8.77	8.38	4.13	
	-2.16		4.19	-2.39		2.06
		-1.38	-2.81	1.45	2.94	
	-0.7		0.73	-1.4		1.07
		-0.24	-0.45	0.46	0.94	
	-68.29	-26.78	+76.8	-42.8	42.8	+14.41

