F → Flexi T → theory N → not mention.
M → mom
S →

16-17

1 (a) → theory
(b) → Flexi
2 (a) → T
(b) → F/N
3 (a) → F/N
(b) → F
4 (a) → S
5 (a) → M
(b) → M
6 (a) → M
(b) → M
7 (a) → M
(b) → S
8 (a) → S
(b) → S

15-16

1 (a) → M
(b) → M
2 (a) S
(b) S
3 (a) S
(b) M
4 (a) M
(b) S
5 (a) F
(b) F
6 (a) F
(b) F
7 (a) F
(b) S
8 (a) N
(b) S

14-15

1 (a) F
(b) F
2 (a) F
(b) F
3 (a) S
(b) S
4 (a) S
(b) → T
5 (a) T
(b) M
6 (a) M
(b) M
7 (a) S
(b) F
8 (a) S
(b) S

13-14

1 (a) M
(b)
2 → F
3 → M
4 → F
5 → M
6 → F
7 → M
8 → S
9 → S
10 → S
11 → S
12 → S
13 → S
14 → F

12-13

1 (a) M
(b)
2 (a) M
(b) S
3 (a) S
(b) S
4 (a) S
5 (a) F
(b) F/T
6 (a) F
(b) F
7 (a) F
(b) F
8 F.

11-12

1 (a) F
(b) S
2 (a) F
(b) S
3 (a) N
(b) S
4 → S
5 (a) M
(b) F
6 (a) F Scope
(b) M
7 (a) M
8 (a) Scope
(b) —

S → 4
F → 4
M → 5

S → 5
F → 5
M → 4

13 → 14

S → 6
F → 5
M → 5

S → 6
F → 4
M → 4

S → 4
F → 6
M → 2

S → 4
F → 3
M → 3

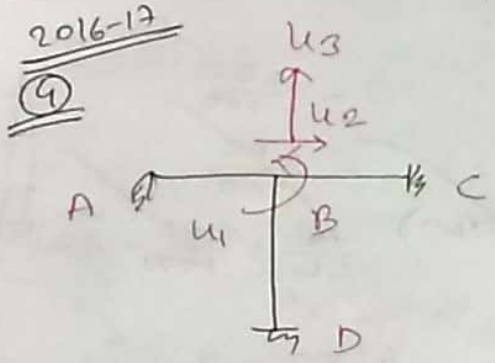
1112
1011
89

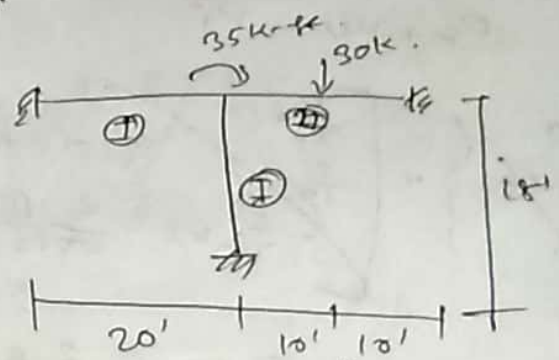
Nothing is certain.

Frame problem

2016-17

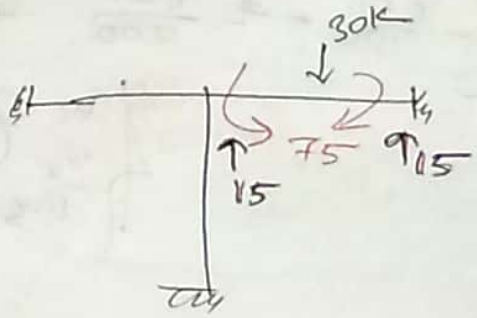
(4)





DOKI = 3.

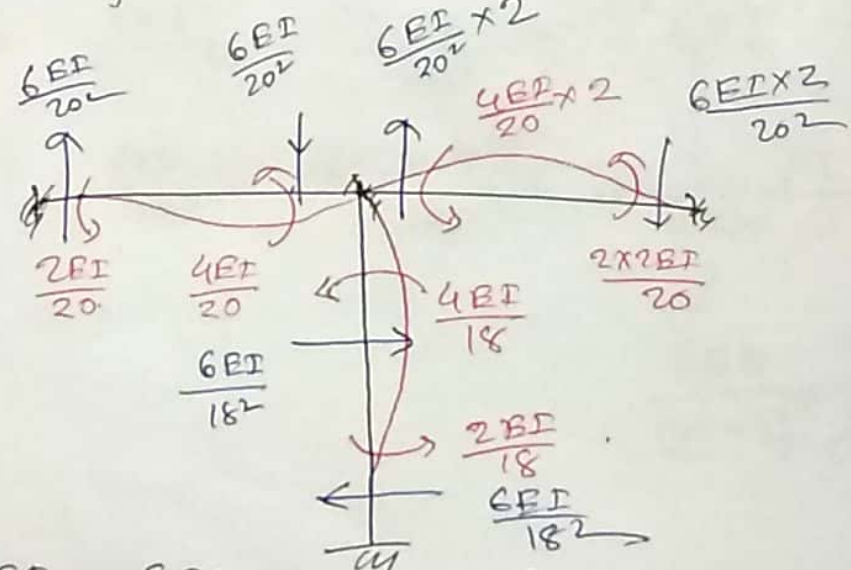
$u_1 = u_2 = u_3 = 0$



$\frac{PL}{8} = \frac{30 \times 20}{8} = 75$

$$P_m = \begin{Bmatrix} 75 \\ 0 \\ 15 \end{Bmatrix}$$

$u_1 = 1$

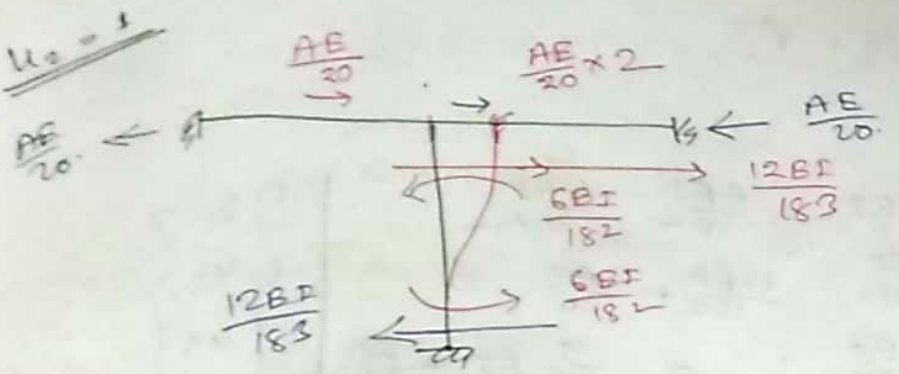


$$K_{11} = \frac{4EI}{18} + \frac{8EI}{20} + \frac{4EI}{20} = \frac{37}{45} EI$$

$$K_{21} = \frac{6EI}{18^2} =$$

$$K_{31} = \frac{12EI}{20^2} - \frac{6EI}{20^2} = \frac{6EI}{20^2}$$

$u_2 = 1$

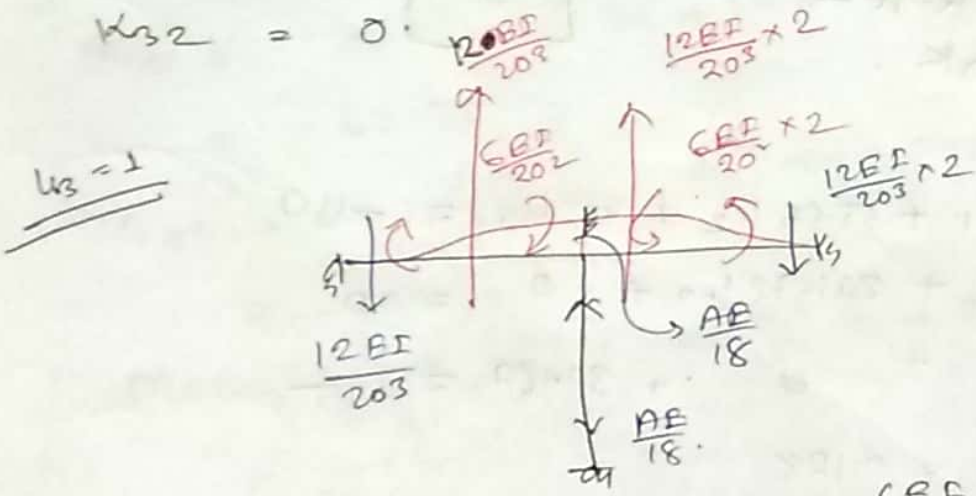


$$K_{12} = \frac{6EI}{182}$$

$$K_{22} = \frac{12EI}{183} + \frac{AE}{20} + \frac{2AE}{20} = \frac{12EI}{183} + \frac{3AE}{20}$$

$$K_{32} = 0$$

$u_3 = 1$



$$K_{13} = \frac{6EI}{202} \times 2 - \frac{6EI}{202} = \frac{6EI}{202}$$

$$K_{23} = 0$$

$$K_{33} = \frac{12EI}{203} + \frac{12EI}{203} \times 2 + \frac{AE}{18}$$

$$= \frac{9}{2000} EI + \frac{AE}{18}$$

$$P_m + Ku = P_s$$

$$\begin{Bmatrix} 75 \\ 0 \\ 15 \end{Bmatrix} + \begin{bmatrix} \frac{37}{45}EI & \frac{6EI}{18^2} & \frac{6EI}{20^2} \\ \frac{6EI}{18^2} & \frac{12EI}{18^3} + \frac{3AE}{20} & 0 \\ \frac{6EI}{20^2} & 0 & \frac{9}{2000}EI + \frac{AE}{18} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$= \begin{bmatrix} -35 \\ 0 \\ 0 \end{bmatrix}$$

$EI = 3000\ k\text{-}ft^2$

$AE = 300^k$.

$$\therefore \quad 2466.67 u_1 + 55.56 u_2 + 45 u_3 = -110$$

$$55.56 u_1 + 241.72 u_2 + 0 = 0$$

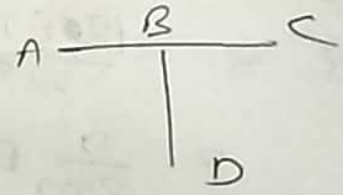
$$45 u_1 + 0 + 30.167 = -15$$

$$u_1 = 0.128$$

$$u_2 = -5.03$$

$$u_3 = -0.689$$

Moment



$$M_{AB} = 0 + \frac{2EI}{20}u_1 + 0u_2 - \frac{6EI}{20^2}u_3$$

$$= 69.405 \text{ K-ft.}$$

$$M_{BA} = 0 + \frac{4EI}{20}u_1 + 0\,u_2 + \frac{6EI}{20^2}u_3.$$

$$= 107.8 \text{ K-ft}$$

$$M_{BC} = 75 + \frac{8EI}{20}u_1 + 0u_2 + \frac{12EI}{20^2}u_3$$

$$= 166.59$$

$$M_{CB} = -75 + \frac{4EI}{20}u_1 + 0u_2 + \frac{12EI}{20^2}u_3$$

$$= -60.21$$

$$M_{BD} = 0 + \frac{4EI}{18}u_1 + \frac{6EI}{18^2}u_2 + 0u_3$$

$$= -194.11$$

$$M_{DB} = 0 + \frac{2EI}{18}u_1 + \frac{6EI}{18^2}u_2 + 0u_3$$

$$= -236.77$$

$$R_{AX} = 0 + 0 - \frac{AE}{20}u_2 + 0u_3 = -75.45 \ (\rightarrow)$$

$$R_{AD} = 0 + \frac{6EI}{20^2}u_1 + 0 + \frac{12EI}{20^3}u_3 = 8.86$$

$$R_{BX} = 0 + \frac{6EI}{18^2}u_1 + \frac{12EI}{18^3}u_2 + 0u_3 = -23.94$$

$$R_{BD} = 15\left(-\left(\frac{6EI}{20^2} + \frac{6EI}{20^2} \times 2\right)u_1 + \frac{12EI}{20^3}(2+1)u_2 + \frac{AE}{18}u_3\right.$$

$$= -58.63$$

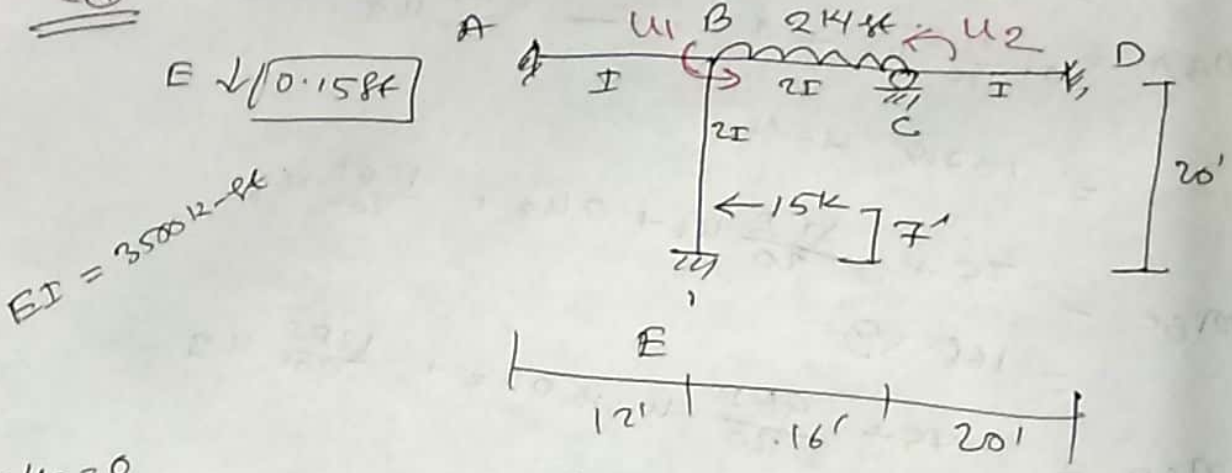
rest do it yourself.

~~2014-15~~

2015-16

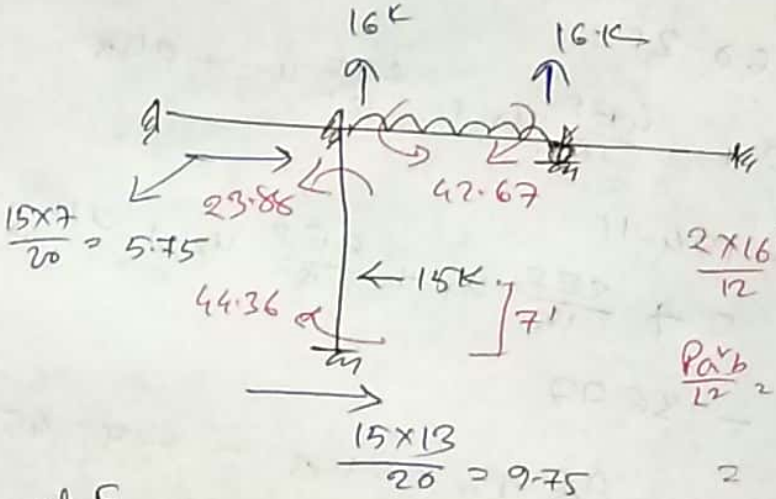
$F_j' = F_m + F_s + K u_j$ Support Settlement

2(a)



$EI = 3500 k\text{-}ft^2$

$u_1 = u_2 = 0$



$\frac{15 \times 7}{20} = 5.75$

$\frac{15 \times 13}{20} = 9.75$

$\frac{2 \times 16^2}{12} = 42.67$

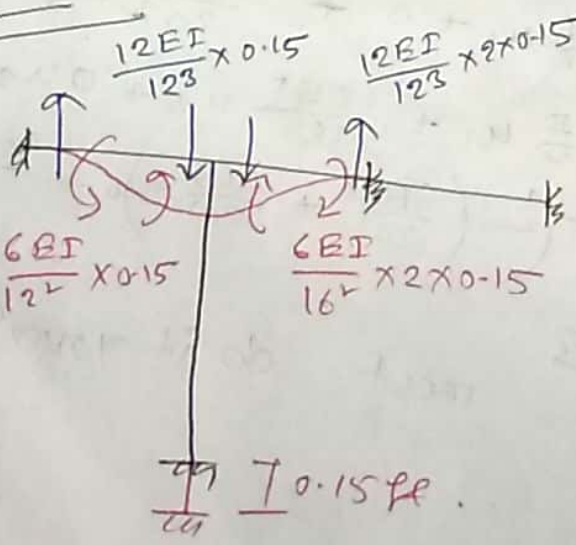
$\frac{Pa^2b}{L^2} = \frac{15 \times 13^2 \times 7}{20^2}$

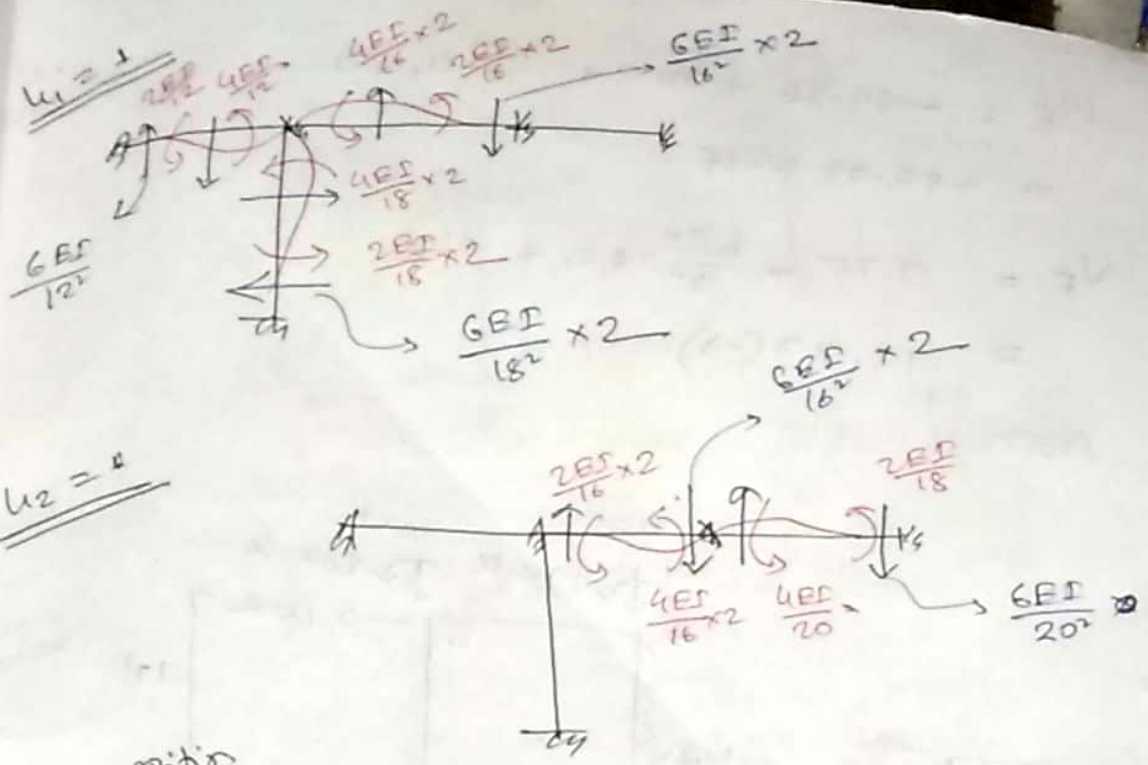
$= 44.36$

$\frac{15 \times 13 \times 7^2}{20^2}$

$= 23.86$

Applying 0.15ft downward at E





By Superposition

$$F_j = F_m + F_s + K u \quad \rightarrow$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 23.88 + 42.67 \\ -42.67 \end{bmatrix} + \begin{bmatrix} \frac{6EI}{12^2} \times 0.15 - \frac{6EI}{16^2} \times 2 \times 0.15 \\ -\frac{6EI}{16^2} \times 2 \times 0.15 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{4}{12} + \frac{4\times2}{16} + \frac{4\times2}{18} & \frac{2\times2}{16} \\ \frac{2\times2}{16} & \frac{4\times2}{16} + \frac{4}{20} \end{bmatrix} EI \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Putting $EI = 3500$ K-ft.

$$3833.33\, u_1 + 875\, u_2 = -63.82$$
$$875\, u_1 + 2450\, u_2 = 67.28$$

$$u_1 = -0.0249 \text{ K-ft}$$
$$u_2 = 0.0369 \text{ K-ft.}$$

$$M_E = -44.36 + 0 + \frac{2EI}{18} u_1 + 0$$

$$= -54.08 \text{ k-ft}$$

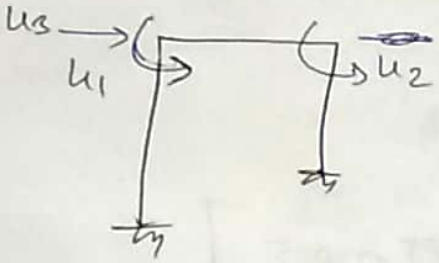
$$V_F = 9.75 - \frac{6EI}{18^2} \times 2 u_1 + 0 u_2$$

$$= 12.99 (\rightarrow)$$

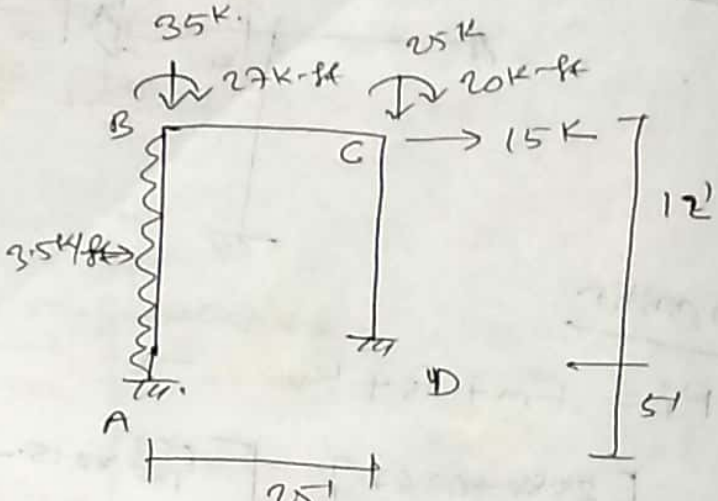
A

15-16

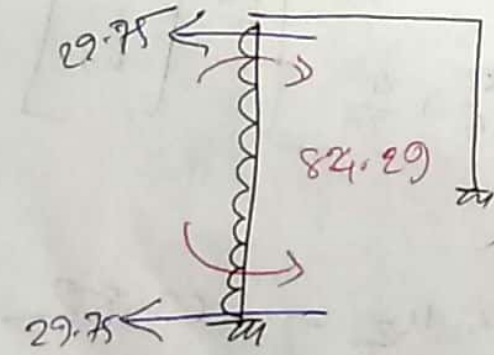
2(b)



DoKI = 3

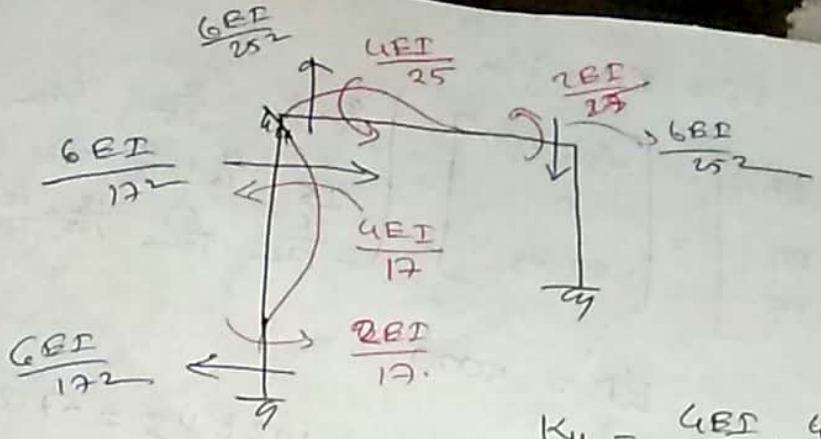


$u_1 = u_2 = 0 = u_3$



$$P_{fm} = \begin{Bmatrix} -84.29 \\ 0 \\ -29.75 \end{Bmatrix}$$

$u_1 = 1$

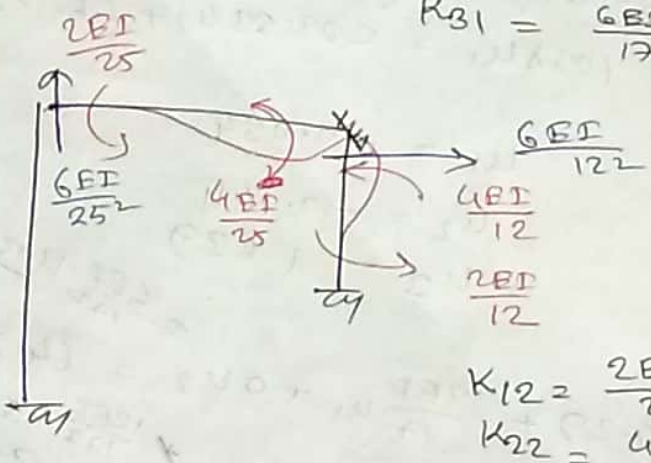


$$K_{11} = \frac{4EI}{25} + \frac{4EI}{17}$$

$$K_{21} = \frac{2EI}{25}$$

$$K_{31} = \frac{6EI}{17^2}$$

$u_2 = 1$

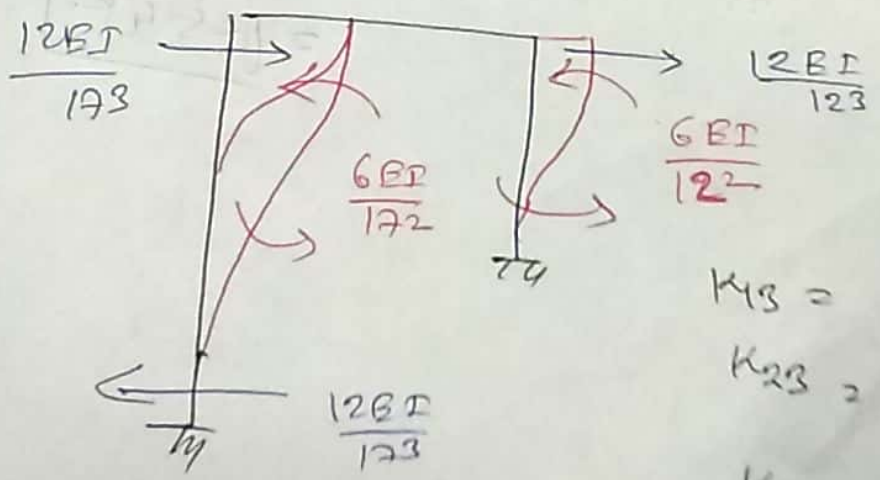


$$K_{12} = \frac{2EI}{25}$$

$$K_{22} = \frac{4EI}{25} + \frac{4EI}{12}$$

$$K_{32} = \frac{6EI}{12^2}$$

$u_3 = 1$



$$K_{13} = \frac{6EI}{17^2}$$

$$K_{23} = \frac{6EI}{12^2}$$

$$K_{33} = \frac{12EI}{17^3} + \frac{12EI}{12^3}$$

$$P_s^\circ = P_m + K_u$$

$$\Rightarrow \begin{bmatrix} -27 \\ -20 \\ 15 \end{bmatrix} = \begin{bmatrix} -84.29 \\ 0 \\ -29.75 \end{bmatrix} + \begin{bmatrix} \frac{4}{25}+\frac{4}{17} & \frac{2}{25} & 6/17^2 \\ \frac{2}{25} & \frac{4}{25}+\frac{4}{12} & \frac{6}{12^2} \\ \frac{6}{17^2} & \frac{6}{12^2} & \frac{12}{17^3}+\frac{12}{12^3} \end{bmatrix} EI \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$EI = 5000$$

$$1976.47u_1 + 400u_2 + 103.8u_3 = 57.29$$

$$400u_1 + 2466.67u_2 + 208.33u_3 = 0 \quad -26$$

$$103.8u_1 + 208.33u_2 + 46.93 = 44.75$$

$$u_1 = -0.034$$

$$u_2 = -0.1411$$

$$u_3 = 1.639$$

$$M_A = 84.29 + \frac{2EI}{17}u_1 + 0u_2 - \frac{6EI}{17^2}u_3 = 64.29 \text{ K-ft} + 170.13 = \boxed{234.42}$$

$$V_{AX} = -29.75 - \frac{6EI}{17^2}u_1 + 0u_2 + \frac{12EI}{17^3}u_3 = 33.28 \text{ K} - 20.66 = \boxed{12.6}$$

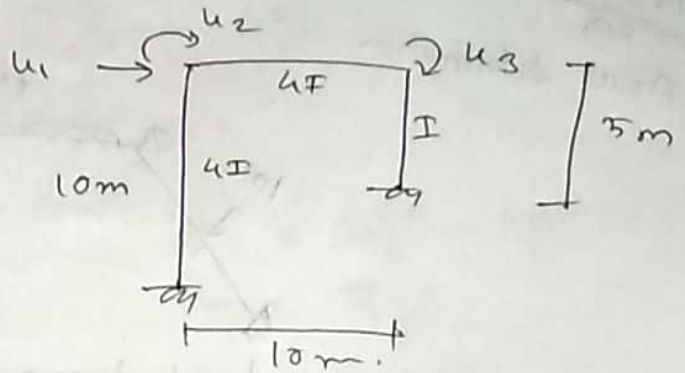
$$V_{AY} = 0 + \frac{6EI}{25^2}u_1 + \frac{6EI}{25^2}u_2 + 0u_3$$

$$= \boxed{-8.4 \text{ K}}$$

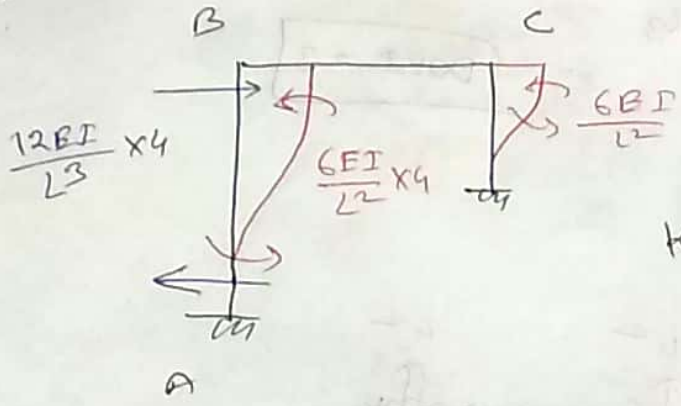
2014-15

3 (b)

$K_{11} = ?$



$u_1 = 1 \quad u_2 = u_3 = 0$



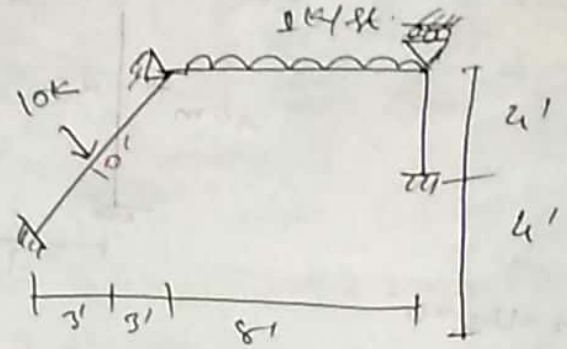
$$K_{11} = \frac{12EI}{10^3} \times 4$$

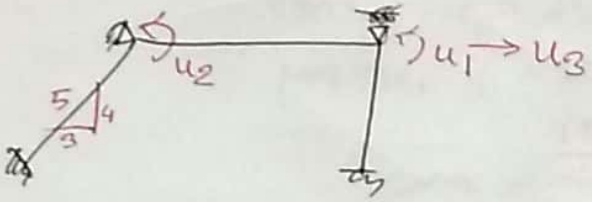
$$= \frac{48}{1000} EI$$

2013-14

8(b)

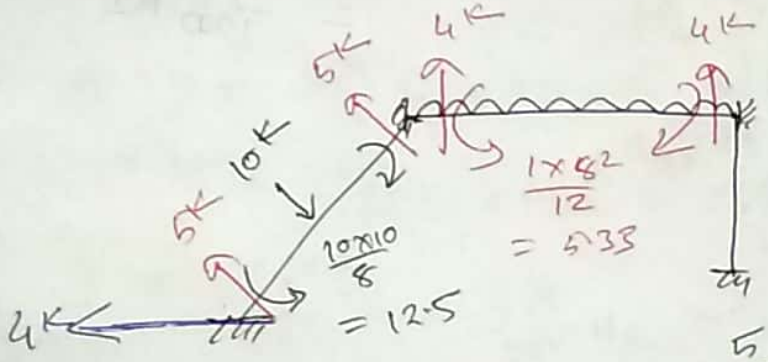
Axial deformation

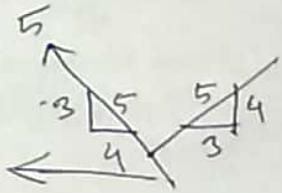




DOKI = 3

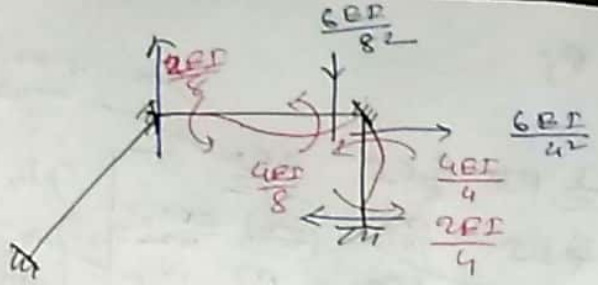
$u_1 = u_2 = u_3 = 0$





$$P_m = \begin{Bmatrix} -5.33 \\ 5.33 \\ -4 \end{Bmatrix}$$

$u_1 = 1$

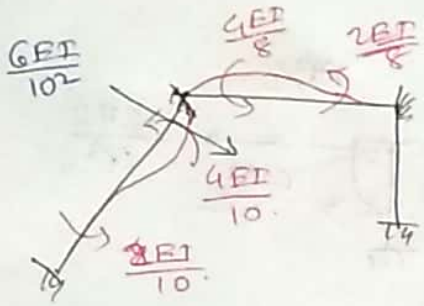


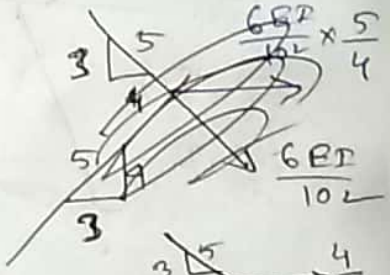
$$K_{11} = \frac{4EI}{8} + \frac{4EI}{4} = \frac{12EI}{8} = 3/2EI$$

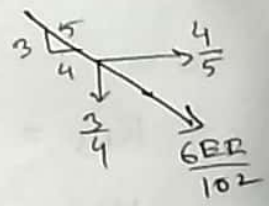
$$K_{21} = \frac{2EI}{8}$$

$$K_{31} = \frac{6EI}{4^2}$$

$u_2 = 1$





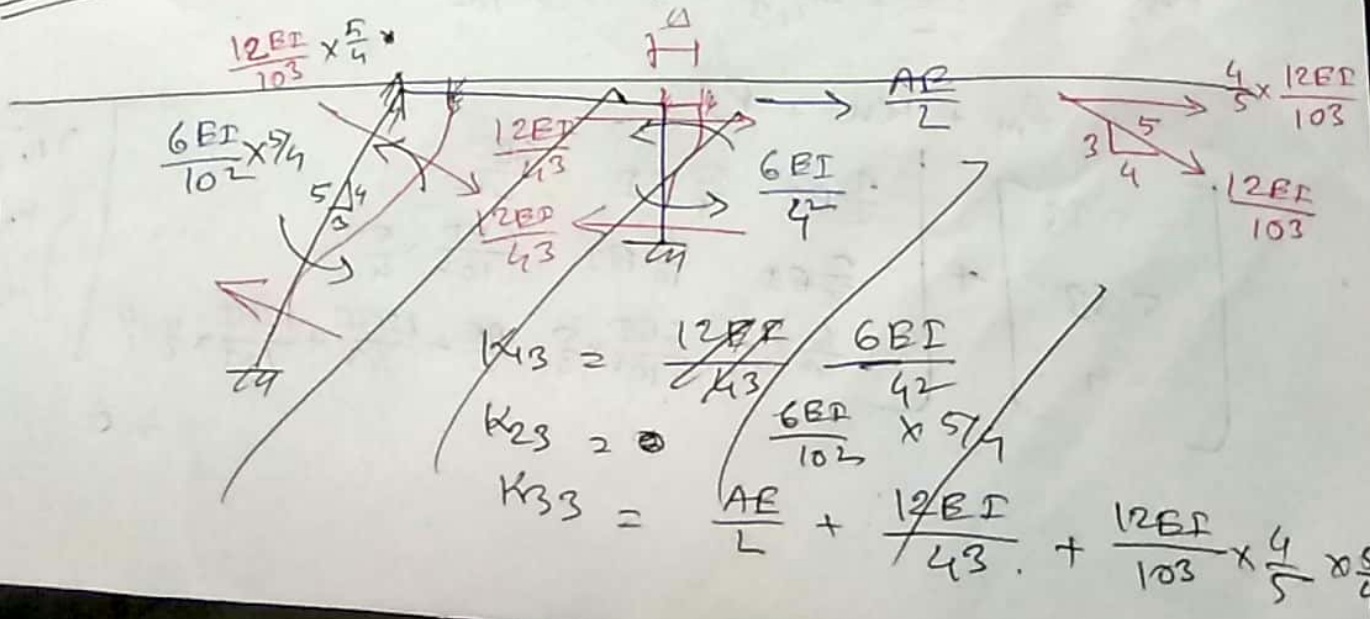


$$K_{12} = \frac{2EI}{8}$$

$$K_{22} = \frac{4EI}{8} + \frac{4EI}{10} = \frac{9}{10}EI$$

$$K_{32} = \frac{6EI}{10^2} \times \frac{4}{5}$$

$u_3 = 1$



(7)

$$K_{13} = \frac{12EI}{4^3} \quad \frac{6EI}{4^2}$$

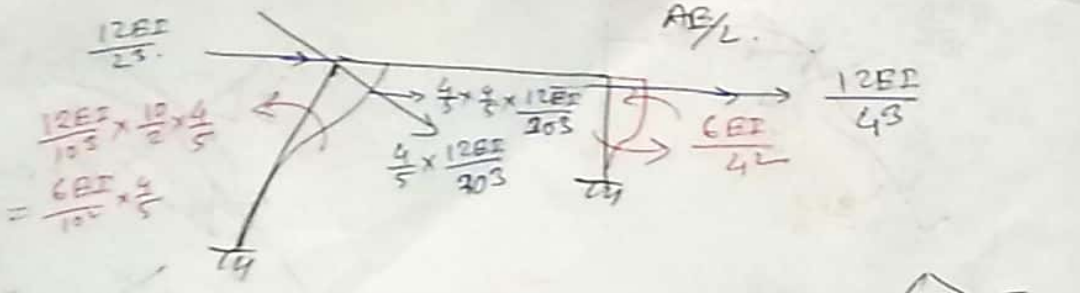
$$K_{23} = \frac{6EI}{10^2} \times 5/4$$

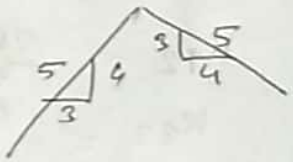
$$K_{33} = \frac{AE}{L} + \frac{12EI}{4^3} + \frac{12EI}{10^3} \times \frac{4}{5} \times \frac{5}{4}$$

$P_m + Ku = P_j$

$$\begin{bmatrix} -5.33 \\ 5.33 \\ -4 \end{bmatrix} + \begin{bmatrix} \frac{3}{2}EI & \frac{2}{8}EI & \frac{6EI}{4^2} \\ \frac{2}{8}EI & \frac{2}{10}EI & \frac{6EI}{10^2}\times\frac{5}{4} \\ \frac{6}{4^2}EI & \frac{6EI}{10^2}\times\frac{5}{4} & \frac{AE}{L}+\frac{12EI}{4^3}+\frac{12EI}{10^3}\times\frac{4}{5}\times\frac{5}{4} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$u_3 = 1$





$$K_{13} = \frac{6EI}{4^2}$$

$$K_{23} = \frac{6EI}{10^2}\times\frac{4}{5}$$

$$K_{33} = \frac{AE}{L} + \frac{12EI}{4^3} + \frac{4}{5}\times\frac{4}{5}\times\frac{12EI}{10^3}$$

$P_m + Ku = P_j$

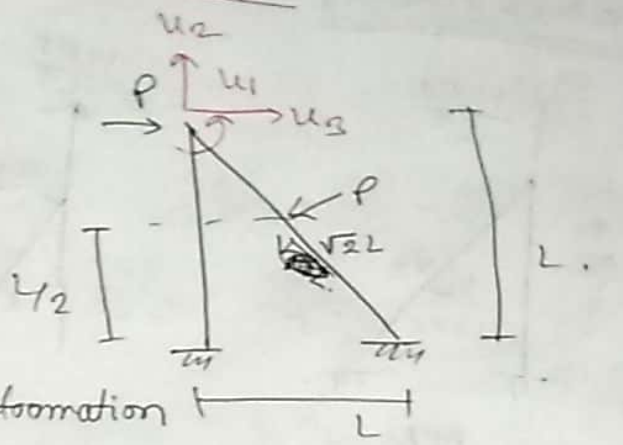
$$\begin{bmatrix} -5.33 \\ 5.33 \\ -4 \end{bmatrix} + \begin{bmatrix} \frac{3}{2}EI & \frac{2}{8}EI & \frac{6EI}{4^2} \\ \frac{2}{8}EI & \frac{2}{10}EI & \frac{6EI}{10^2}\times\frac{5}{4} \\ \frac{6}{4^2}EI & \frac{6EI}{10^2}\times\frac{5}{4} & \frac{AE}{L}+\frac{12EI}{4^3}+\frac{12EI}{10^3}\times\frac{4}{5}\times\frac{4}{5} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$= 0$

Frame Problem

2013-14

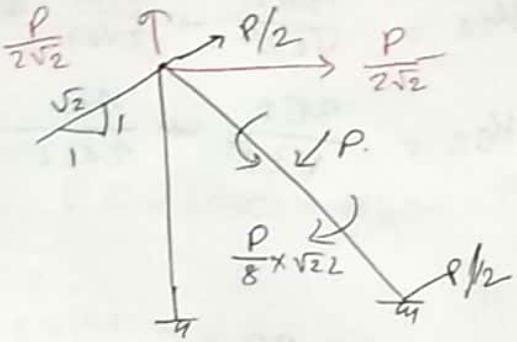
(7) Consider axial deformation



DOKI = 3.

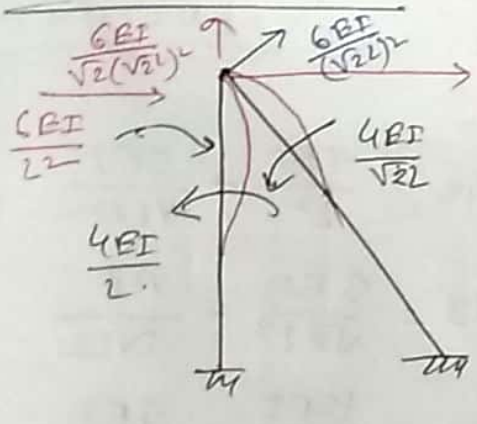
Considering axial deformation

$u_1 = u_2 = u_3 = 0$.



$$P_m = \begin{Bmatrix} \frac{\sqrt{2}PL}{8} \\ \frac{P}{2\sqrt{2}} \\ \frac{P}{2\sqrt{2}} \end{Bmatrix}$$

$u_1 = 1$, $u_2 = u_3 = 0$.



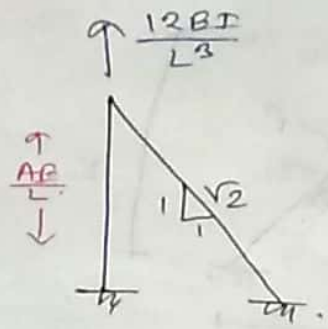
$$K_{11} = \frac{4EI}{L} + \frac{2\sqrt{2}EI}{L}$$

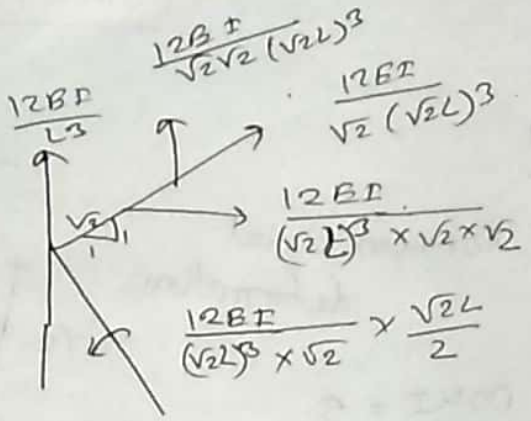
$$K_{21} = \frac{6EI}{2\sqrt{2}L^2} = \frac{3EI}{\sqrt{2}L^2}$$

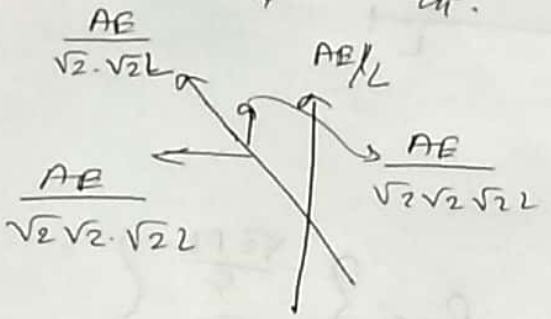
$$K_{31} = \frac{6EI}{L^2} + \frac{6EI}{\sqrt{2}.2L^2}$$

$$= \frac{6EI}{L^2} + \frac{3EI}{\sqrt{2}L^2}$$

$u_1 = 0 \quad u_2 = 1 \quad u_3 = 0$





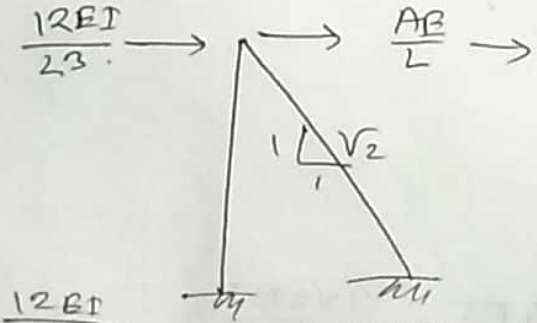


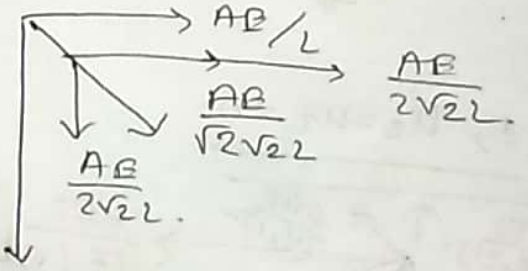
$$K_{12} = \frac{3EI}{\sqrt{2}L^2}$$

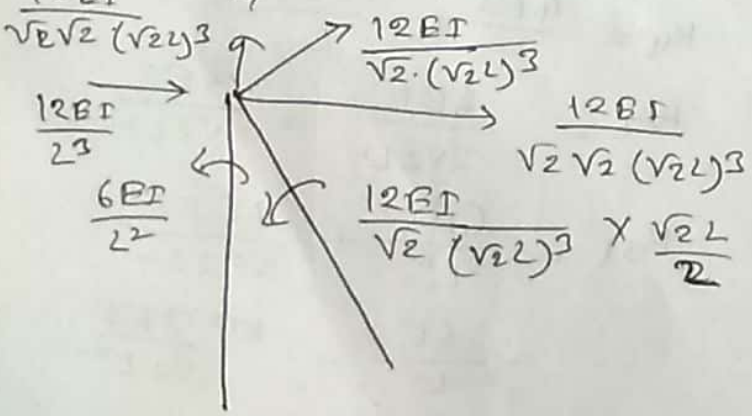
$$K_{22} = \frac{3EI}{\sqrt{2}L^3} + \frac{AE}{2\sqrt{2}L} + \frac{AE}{L}$$

$$K_{32} = \frac{3EI}{\sqrt{2}L^3} - \frac{AE}{2\sqrt{2}L}$$

$u_1 = 0 = u_2$
$u_3 = 1$







$$K_{13} = \frac{6EI}{L^2} + \frac{3EI}{\sqrt{2}L^2}$$

$$K_{23} = \frac{3EI}{\sqrt{2}L^3} - \frac{AE}{2\sqrt{2}L}$$

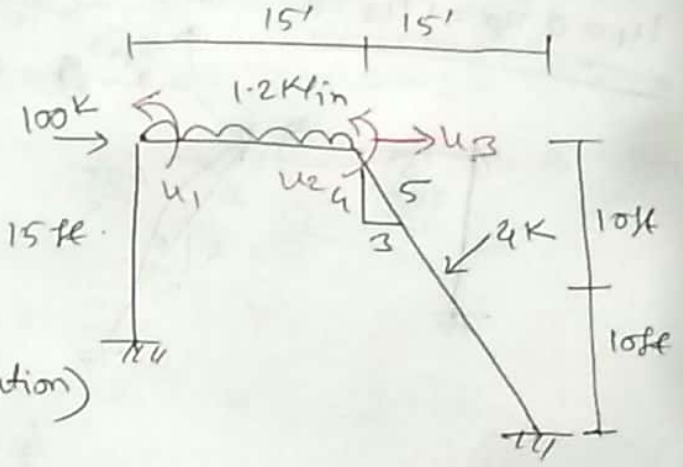
$$K_{33} = \frac{12EI}{L^3} + \frac{3EI}{\sqrt{2}L^3} + \frac{AE}{2\sqrt{2}L}$$

$P_m + KU = P_f$

$$\Rightarrow \begin{Bmatrix} \frac{\sqrt{2}P}{8} \\ \frac{P}{2\sqrt{2}} \\ \frac{P}{2\sqrt{2}} \end{Bmatrix} + \begin{bmatrix} \frac{4EI}{L} + \frac{2\sqrt{2}EI}{L} & \frac{3EI}{\sqrt{2}L^2} & \frac{6EI}{L^2} + \frac{3EI}{\sqrt{2}L^2} \\ \frac{3EI}{\sqrt{2}L^2} & \frac{3EI}{\sqrt{2}L^3} + \frac{AE}{2\sqrt{2}L} + \frac{AE}{L} & \frac{3EI}{\sqrt{2}L^3} - \frac{AE}{2\sqrt{2}L} \\ \frac{6EI}{L^2} + \frac{3EI}{\sqrt{2}L^2} & \frac{3EI}{\sqrt{2}L^3} - \frac{AE}{2\sqrt{2}L} & \frac{12EI}{L^3} + \frac{3EI}{\sqrt{2}L^3} + \frac{AE}{2\sqrt{2}L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ P \end{Bmatrix}.$$

2012-13

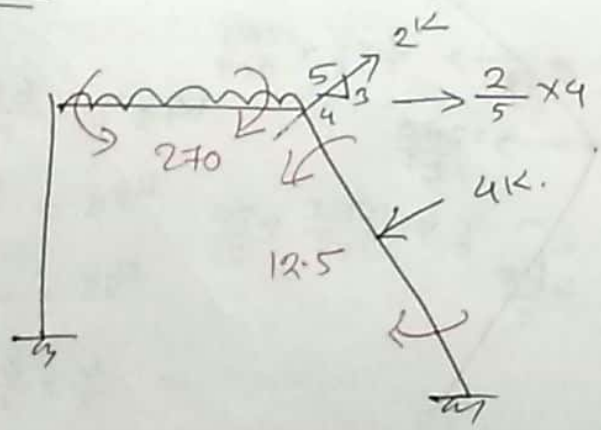
2(b) $EI = 10000\ K\text{-}in^2$



DoKI = 3.

(ignoring axial deformation)

$u_1 = u_2 = u_3 = 0$



$1.2\ K/in = 1.2 \times 12$

$= 14.4\ K/ft.$

$\frac{wL^2}{12} = \frac{14.4 \times 15^2}{12}$

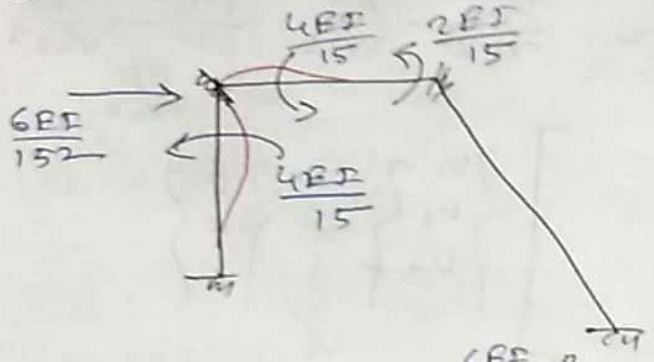
$= 270$

$\frac{PL}{8} = \frac{4 \times 25}{8}$

$= 12.5$

$$P_m = \begin{Bmatrix} 270 \\ -270+12.5 \\ 2\times\frac{4}{5} \end{Bmatrix} = \begin{Bmatrix} 270 \\ -257.5 \\ 1.6 \end{Bmatrix}$$

$u_1 = 1 \quad u_2 = u_3 = 0$:

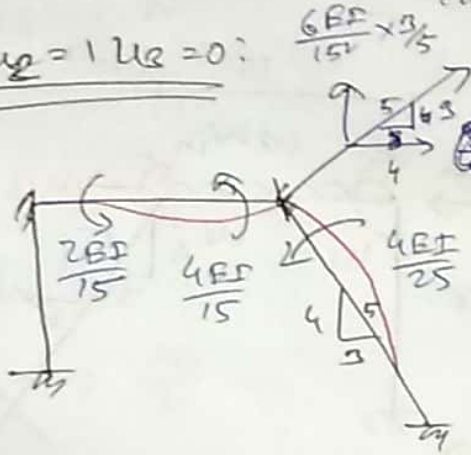


$$K_{11} = \frac{4EI}{15} + \frac{4EI}{15}$$

$$K_{21} = \frac{2EI}{15}$$

$$K_{31} = \frac{6EI}{15^2}$$

$u_1 = 0 \quad u_2 = 1 \quad u_3 = 0$:

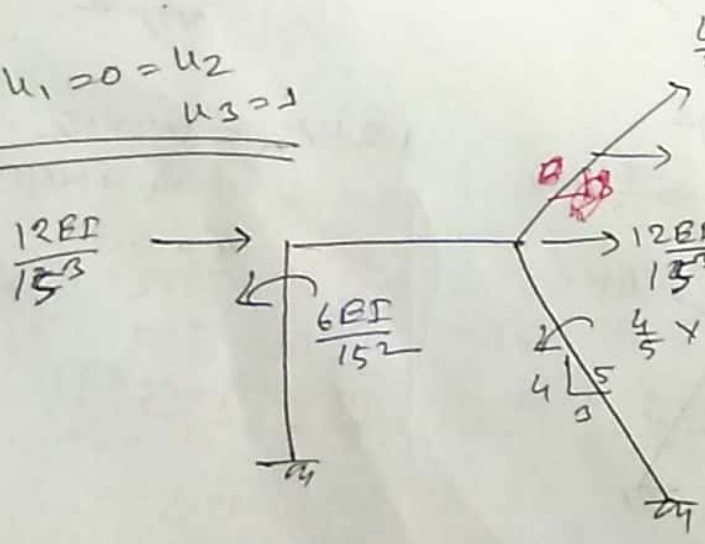


$$K_{12} = \frac{2EI}{15}$$

$$K_{22} = \frac{4EI}{15} + \frac{4EI}{25}$$

$$K_{32} = \frac{4}{5} \times \frac{6EI}{15^2}$$

$u_1 = 0 = u_2$
$u_3 = 1$



$$K_{13} = \frac{6EI}{15^2}$$

$$K_{23} = \frac{4}{5} \times \frac{6EI}{15^2}$$

$$K_{33} = \frac{12EI}{15^3} + \frac{4}{5} \times \frac{4}{5} \times \frac{12EI}{25^3}$$

$$P_m + Ku = P_j$$

$$\Rightarrow \begin{Bmatrix} 270 \\ -257.5 \\ 1.6 \end{Bmatrix} + \begin{bmatrix} \frac{4EI}{15} + \frac{4EI}{15} & \frac{2EI}{15} & \frac{6EI}{15^2} \\ \frac{2EI}{15} & \frac{4EI}{15} + \frac{4EI}{25} & \frac{4}{5} \times \frac{6EI}{25^2} \\ \frac{6EI}{15^2} & \frac{4}{5} \times \frac{6EI}{25^2} & \frac{12EI}{15^3} + \frac{4}{5} \times \frac{4}{5} \times \frac{12EI}{25^2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}$$

$EI = 10000 \text{ K-ft}^2 = \frac{10000}{144} \times 144$

$= 1440000 \text{ K-in}^2$

$$\therefore 768000 u_1 + 192000 u_2 + 384000 u_3 = -270$$

$$192000 u_1 + 614400 u_2 + 110592 u_3 = 257.5$$

$$38400 u_1 + 110592 u_2 + 58279 u_3 = 98.4$$

$$u_1 = -5.04 \times 10^{-4} \text{ K-ft}$$

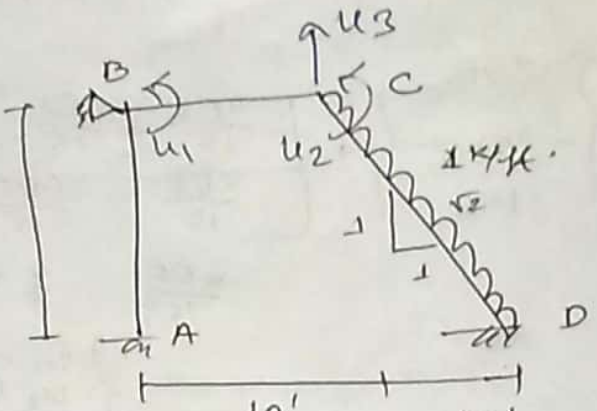
$$u_2 = 5.73 \times 10^{-4} \text{ K-ft}$$

$$u_3 = 1.912 \times 10^{-4} \text{ K}$$

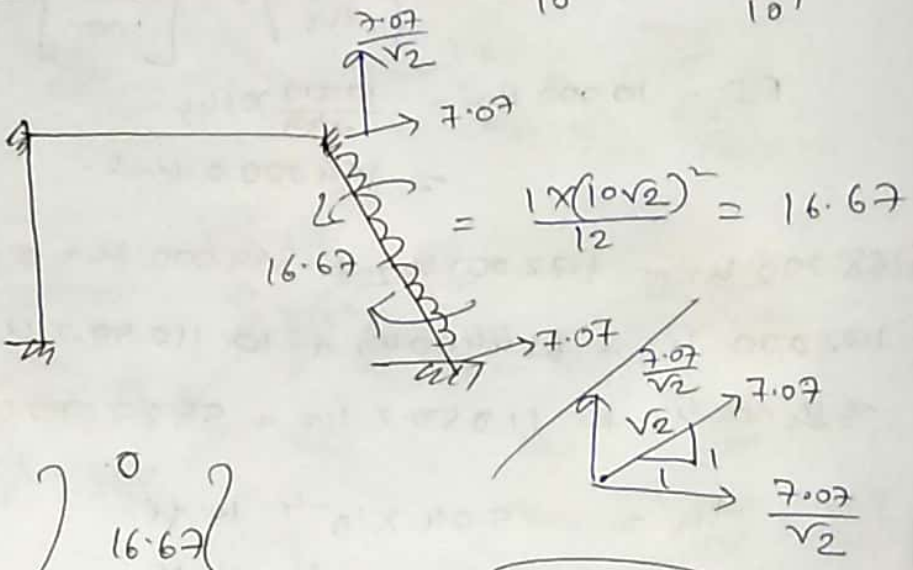
2011-12

3(b)

axial deformation in BC is ignored but not in CD. 2 coln.



$u_1 = u_2 = u_3 = 0$



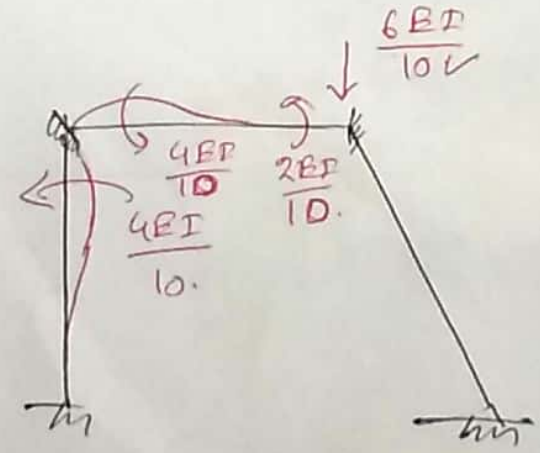
$= \frac{1 \times (10\sqrt{2})^2}{12} = 16.67$

$$P_m = \begin{Bmatrix} 0 \\ 16.67 \\ 5 \end{Bmatrix}$$

$\frac{7.07}{\sqrt{2}} = 5$

$u_1 = 1$

$u_2 = u_3 = 0$

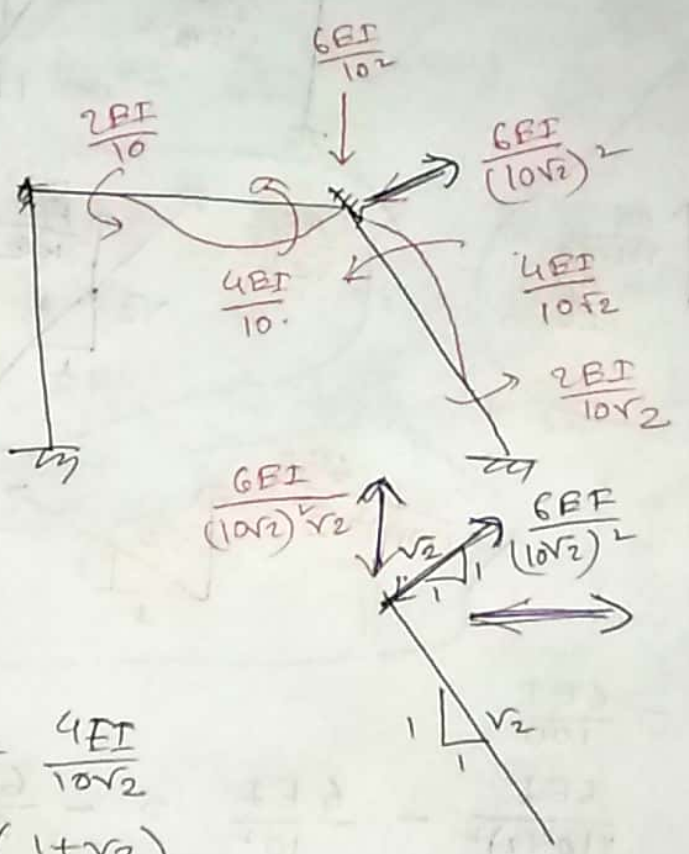


$$K_{11} = \frac{4EI}{10} + \frac{4EI}{10} = \frac{8}{10}EI$$

$$K_{21} = \frac{2EI}{10}$$

$$K_{31} = -\frac{6EI}{100}$$

$u_2 = 1$
$u_1 = u_3 = 0$



$$K_{12} = \frac{2EI}{10}$$

$$K_{22} = \frac{4EI}{10} + \frac{4EI}{10\sqrt{2}}$$

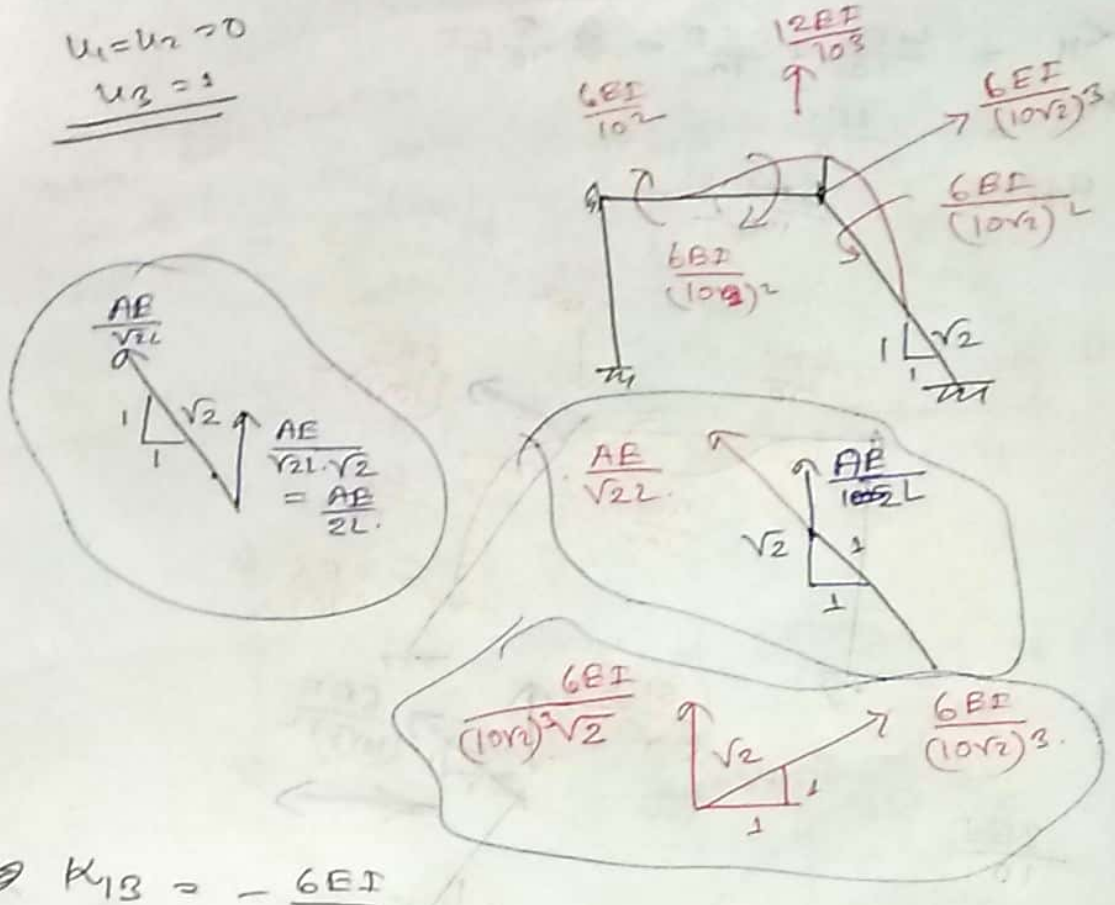
$$= \frac{4EI}{10}(1+\sqrt{2})$$

$$K_{32} = -6\frac{EI}{100} + \frac{6EI}{100\times 2\times\sqrt{2}} = -\frac{6EI}{100}\left(1\right.$$

$$= +\frac{6EI}{200\sqrt{2}}\left(+2\sqrt{2}\pm 1\right)$$

$u_1 = u_2 = 0$

$u_3 = 1$



$K_{13} = -\frac{6EI}{100}$

$K_{23} = \frac{6EI}{(10\sqrt{2})^2} - \frac{6EI}{10^2} = -\frac{6EI}{200}$

$K_{33} = \frac{AE}{2L} + \frac{12EI}{1000} + \frac{3EI}{2000}$

$\frac{6EI}{(10\sqrt{2})^3\sqrt{2}}$

Stiffness eqn

$$P_m + KU = P_j$$

$$\Rightarrow \begin{Bmatrix} 0 \\ 16.67 \\ 5 \end{Bmatrix} = \begin{bmatrix} \frac{8}{10}EI & \frac{2}{10}EI & -\frac{6BI}{100} \\ \frac{2}{10}EI & \frac{4}{10}EI(1+r_2) & -\frac{6BI}{200} \\ -\frac{6}{100}EI & \frac{6}{200r_2}EI(2r_2-1) & \frac{AB}{2L} + \frac{12BI}{1000} + \frac{3}{2000}EI \end{bmatrix}$$

$$\Rightarrow \begin{Bmatrix} 0 \\ 16.67 \\ 5 \end{Bmatrix} = \begin{bmatrix} \frac{8}{10}EI & \frac{2}{10}EI & -\frac{6}{100}EI \\ \frac{2}{10}EI & \frac{4BI}{10}(1+r_2) & \frac{6EI}{200r_2}(2r_2-1) \\ -\frac{6BI}{100} & -\frac{6BI}{200} & +\frac{AB}{2L} + \frac{12BI}{1000} + \frac{3}{2000}EI \end{bmatrix}$$

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$