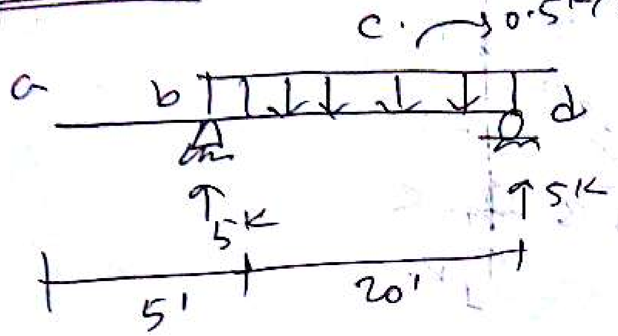
$$\Sigma Q \cdot \delta = \int \frac{M_P M_Q}{EI} \cdot dx + \Sigma F_P \cdot F_Q \frac{L}{AE} + \Sigma F_Q \alpha_t t L$$

↳ B.M. ↳ A.F ↳ Temp.

Virtual work (beam)

Assignment:



$E = 30\,000$ Ksi

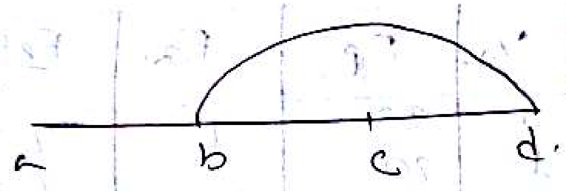
$I = 200$ in^4.

compute

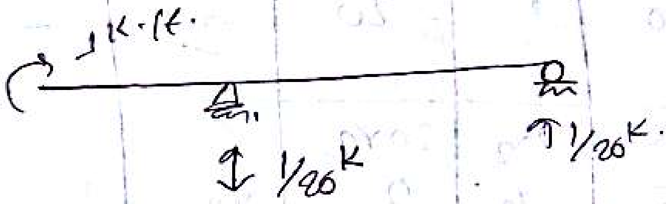
(i) ↳ change in slope at a.

(ii) deflection at c.

BMD.



M_P.

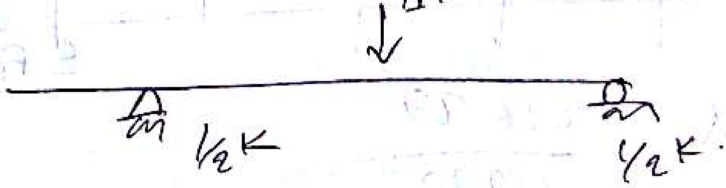


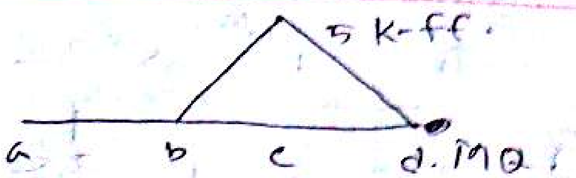
$M_{ab} = 5x - x^2/4$

for (i)

M_Q

for (ii)





For (i)

Segment.	Ma.	Mp.
ab.	1	0
db.	$\frac{x}{20}$.	$5x-\frac{x^2}{4}$.

For (ii)

Segment	Ma	Mp.
ab	0	0
cb.	$5-x/2$	$5x-x^2/4$
dc.	$x/2$	$5x-x^2/4$.

Applying principal of virtual work for (i).

$$\alpha_a = \int_a^b \frac{M_p M_a}{EI} ds + \int_d^b \frac{M_p M_a}{EI} ds.$$

$$= \left[\int_0^5 (1\times 0)\, dx + \int_0^{20} \frac{x}{20}\cdot(5x - x^2/4)\, dx\right] \frac{1}{EI}$$

$$= \frac{500}{3} \times \frac{12^2}{30\,000\times 200} = 0.004 \text{ radian}.$$

Applying principal of virtual work (ii)

$$1.\delta_c = \int_a^b \cdot \frac{M_p M_a}{EI}.ds. + \int_c^b \frac{M_p M_a}{EI} ds + \int_d^c \frac{M_p M_a}{EI} ds$$

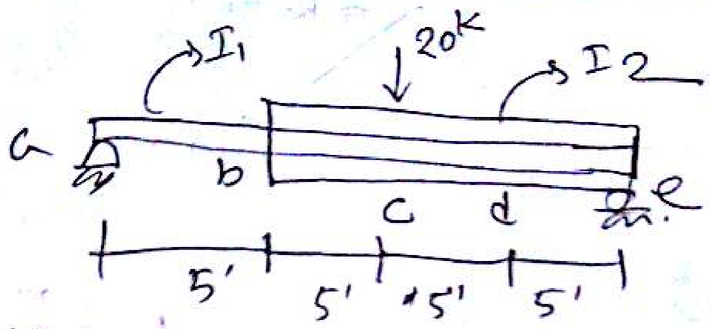
$$= \left[\int_0^5 0.\, dx + \int_0^{10} (-x/2+5).(5x - x^2/4)\, dx + \int_0^{10} \frac{x}{2}(5x - x^2/4)\, dx\right] \cdot \frac{1}{EI}$$

$$= \frac{2500}{3} \times \frac{12^2}{30\,000 \times 200} = 0.02 \text{ ft}.$$

Scanned by CamScanner

Teacher's choich.

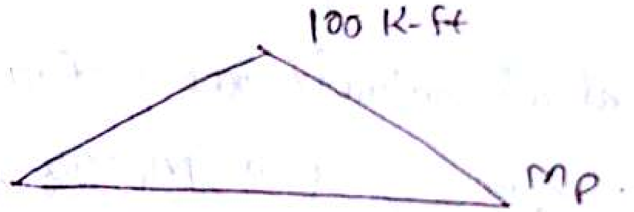
Assignment,



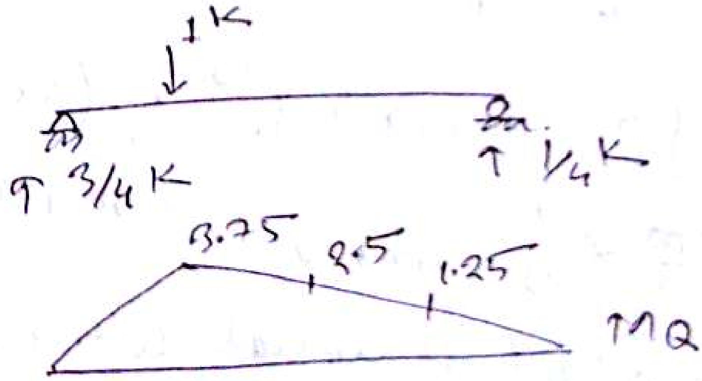
E = 30000ksi

$I_1 = 200 \text{ in}^4$ $\quad I_2 = 300 \text{ in}^4$.

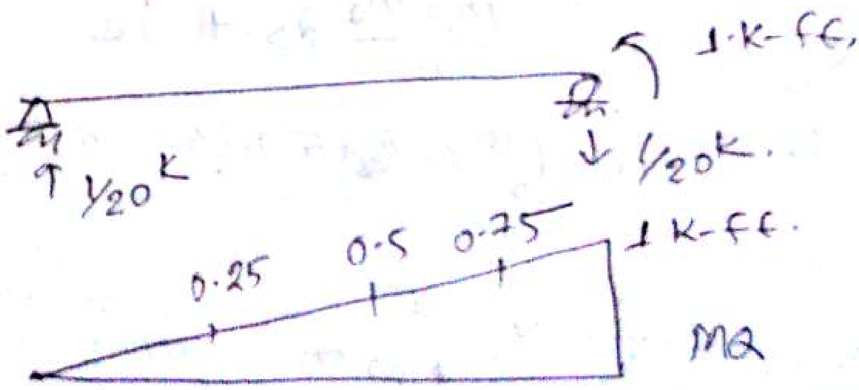
(i) deflection at b.
(ii) change in slop at e



for (i)



for (ii)



Scanned by CamScanner

For (I)

Segment	m_Q	m_P
ab	$\frac{3}{4}x$	$10x$
bc	$-\frac{x}{4}+3.75$	$10x+50$
cd	$-\frac{x}{4}+2.5$	$-10x+100$
de	$-\frac{x}{4}+1.25$	$-10x+50$

For (II)

Segment	m_Q	m_P	I
ab	$\frac{x}{20}$		I_1
bc	$\frac{x}{20}+0.25$		$1.5I_1$
cd	$\frac{x}{20}+0.5$		$1.5I_1$
de	$\frac{x}{20}+0.75$		$1.5I_1$

For (I) Applying principal of virtual work.

$$1.\delta_b = \Big[\int_0^5 \left(\tfrac{3}{4}x\right)\cdot 10x\cdot dx + \int_0^5 \left(-\tfrac{x}{4}+3.75\right)(10x+50)\frac{dx}{1.5} + \int_0^5 \left(-\tfrac{x}{4}+2.5\right)(-10x+100)\frac{dx}{1.5} + \int_0^5 \left(-\tfrac{x}{4}+1.25\right)(-10x+50)\frac{dx}{1.5}\Big]\frac{1}{EI_1}$$

$$= \left(\frac{625}{2}+\frac{6875}{9}+\frac{4375}{9}+\frac{625}{9}\right)\times\frac{12^2}{30000\times 200}$$

$$= 0.0392\text{ ft.}$$

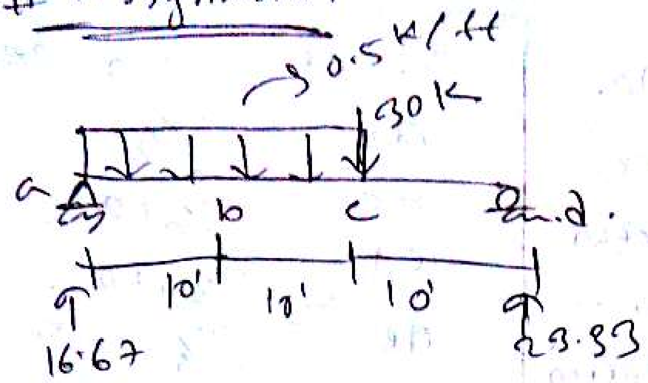
For (II) Applying principal of virtual work.

$$\alpha_e = \Big[\int_0^5 \left(\frac{x}{20}\times 10x\right)dx + \int_0^5 (10x+50)\left(\tfrac{x}{20}+0.25\right)\frac{dx}{1.5} + \int_0^5 (-10x+100)\times\left(\frac{x}{20}+0.5\right)\frac{dx}{1.5} + \int_0^5 (-10x+50)\times\left(\tfrac{x}{20}+0.75\right)\frac{dx}{1.5}\Big]\times\frac{1}{EI_1}$$

$$= \left[\frac{125}{6}+\frac{875}{9}+\frac{1375}{9}+\frac{625}{9}\right]\times\frac{12^2}{30000\times 200}$$

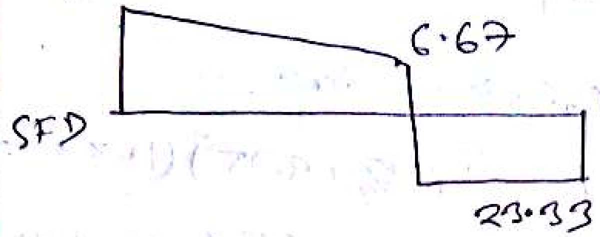
$$= 0.00862\text{ rad } (\circlearrowright)$$

Assignment:



(i) defletion at b
(ii) " " c
(iii) change in slop at a
(iv) " " " d.

EI → Cost^n.



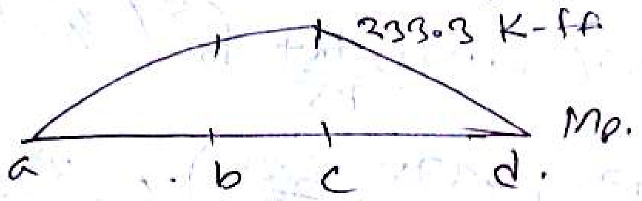
$dM = V\,dx$

$V = mx + C$

$= \dfrac{16.67 - 6.67}{0 - 20}x + c$

at $x = 0$, $c = 16.67$

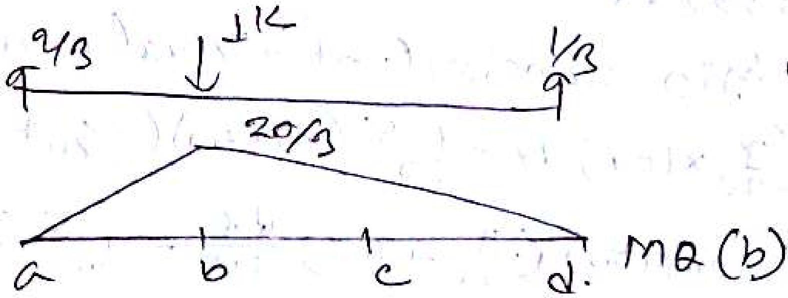
$V = -\dfrac{x}{2} + 16.67$



$M = \int V\,dx$

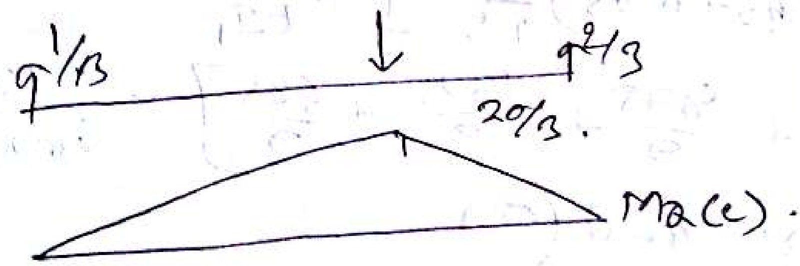
$= -\dfrac{x^2}{4} + 16.67x$

(i)



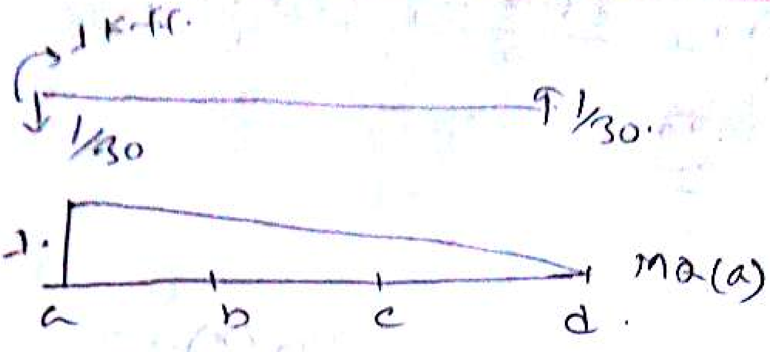
for BC section.

(ii)

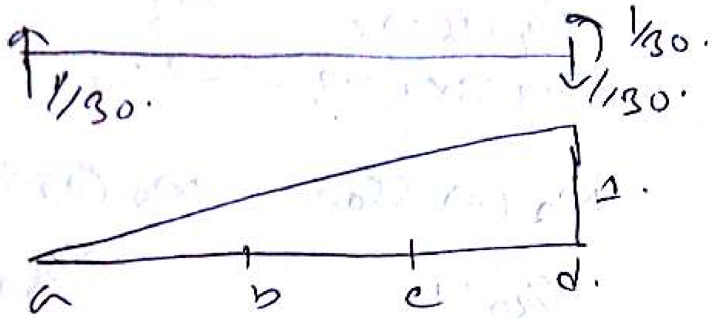


$M = -\dfrac{(x+10)^2}{4} + 16.67(x+10)$

(iii)



(iv)



For (1)

Segment	M_p.	M_q
ab	$-\frac{x^2}{4}+16.67x$	$\frac{2}{3}x$
bc	$-\frac{(x+10)^2}{4}+16.67(x+10)$	$-\frac{1}{3}x+\frac{20}{3}$
cd.	$-23.33x+233.3$	$-\frac{1}{3}x+\frac{10}{3}$.

applying princ

$$1\cdot\delta_b = \Big[\int_0^{10}\left(-\frac{x^2}{4}+16.67x\right)\times\frac{2}{3}x\,dx + \int_0^{10}\left\{-\frac{(x+10)^2}{4}+16.67(x+10)\right\}\times\frac{20-x}{3}\,dx + \int_0^{10}(-23.33x+233.3)\times\frac{10-x}{3}\,dx\Big]\frac{1}{EI}$$

$$= \left(\frac{29590}{9} + \frac{83980}{9} + \frac{23330}{9}\right)\frac{1}{EI}$$

$$= \frac{136900}{EI}$$

For (11)

Segment.	mp.	ma(c).
ac	$-x^2/4 + 16.67x$	$x/3$
cd.	$-23.33x + 233$	$-2x/3 + \frac{20}{3}$

	ma(a) slop.	ma(d) slop
ab.	$-x/30 + 1$	$x/30$.
cd	$-x/30 + \frac{1}{3}$	$x/30 + 2/3$.

(11) Applying principal of virtual work

$$1.\delta_c = \left[\int_0^{20} \left(-x^2/4 + 16.67x\right) \cdot x/3\, dx + \int_0^{10} \left(-23.33x + 233.3\right)\left(-\frac{2x}{3} + \frac{20}{3}\right) dx\right]\frac{1}{EI}$$

$$= \left[\frac{103360}{9} + \frac{46660}{9}\right]\frac{1}{EI}$$

$$\delta_c = \frac{150020}{EI}$$

(iii) Applying principal of virtual work

$$1.\alpha_a = \left[\int_0^{20}\left(-\frac{x^2}{4}+16.67x\right)\cdot x\left(1-\frac{x}{30}\right)dx\right.$$

$$\left.+\int_0^{10}\left(-23.33x+233\right)\cdot\left(-\frac{x}{30}+\frac{1}{3}\right)dx\right]\frac{1}{EI}$$

$$= \left[\frac{13670}{9}+\frac{2333}{9}\right]\frac{1}{EI}$$

$$= \frac{16003}{9EI}$$

(iv) Applying principal of virtual work

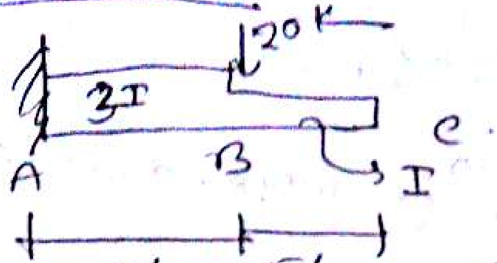
$$1.\alpha_d = \left[\int_0^{20}\left(-\frac{x^2}{4}+16.67x\right)\cdot\frac{x}{30}dx+\right.$$

$$\left.\int_0^{10}\left(-23.33x+233.3\right)\left(\frac{x}{30}+\frac{2}{3}\right)dx\right]\frac{1}{EI}$$

$$\alpha_d = \left[\frac{10336}{9}+\frac{16331}{18}\right]\frac{1}{EI}$$

$$= \frac{37003}{18EI}$$

Assignment (from 13)



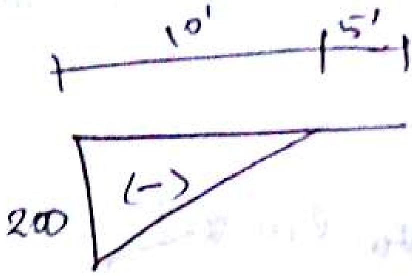
$E = 29000$ ksi

$I = 100$ in^4.

(i) vertical deflection at C

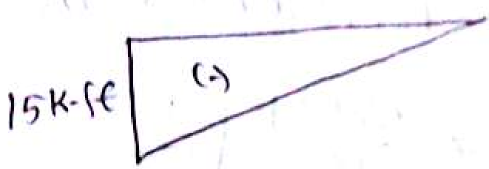
(ii) change in slope at C

(iii) change in slope at B



BMD.

(i)



AB

$M_P = -200 + 20x$

$M_Q = -15 + x$.

BC

$M_P = 0$.

$M_Q = -x$.

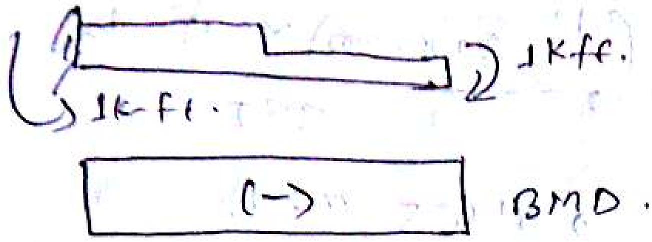
$$\therefore Q \cdot \delta_C = \int_A^B \frac{M_P M_Q}{EI}\,dx + \int \frac{M_P M_Q}{EI}\,dx.$$

~~Applying principle of virtual~~ work.

$$1 \cdot \delta_C = \int_0^{10} \frac{(-200+20x)(-15+x)}{3EI}\,dx + \int_0^5 \frac{0 \times (-x)}{EI}\,dx$$

$$= \frac{11666.67}{3 \times \frac{29000 \times 100}{144}} = 0.193'$$

(ii) Change in slope at C



AB segment

$M_P = -200+20x$

$M_Q = -1$.

BC segment

$M_P = 0$.

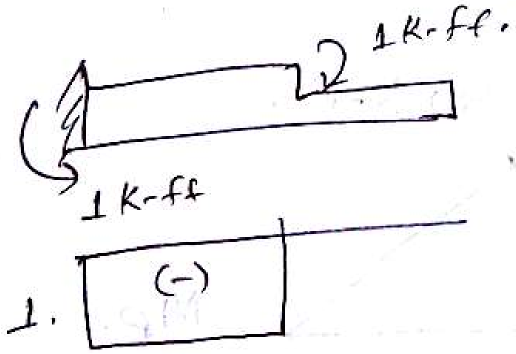
$M_Q = -1$.

$\therefore$ ~~Q.8~~ $Q.\alpha_C = \int_A^B \frac{M_P M_Q}{EI}\,dx + \int_B^C \frac{M_P M_Q}{EI}\,dx$.

$= \int_0^{10} \frac{(-200+20x)(-1)\,dx}{3EI} + \int_0^5 \frac{0\cdot(-1)\,dx}{EI}$

$\Rightarrow \alpha_C = 0.01655$ rad. (counter clockwise)

(iii) Change in slope at B.



AB

$M_P = -200+20x$

$M_Q = -1$

BC

$M_P = 0$

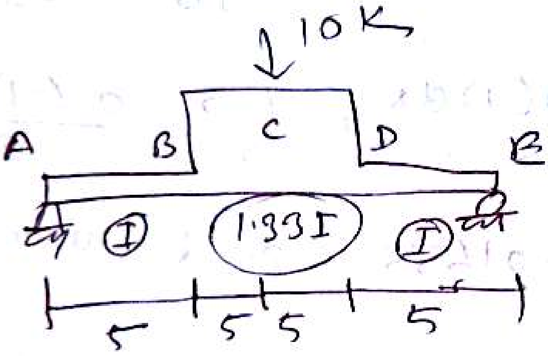
$M_Q = 0$

$$\Rightarrow Q\,\alpha_B = \int_A^B \frac{M_P\, M_Q}{EI}\, dx$$

$$\Rightarrow 1 \cdot \delta\alpha_B = \int_0^{10} \frac{(20x - 200) \times (-1)}{\delta EI}\, dx$$

$$= 0.01655 \text{ rad.}$$

Assignment (13 baten)

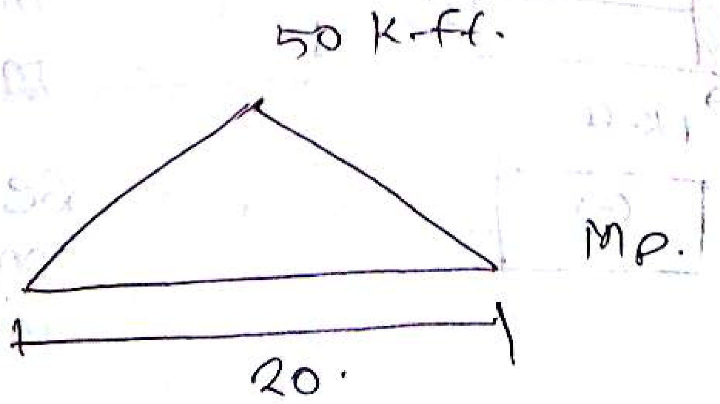


$E = 30 \times 10^3$

$I = 150\,in^4$

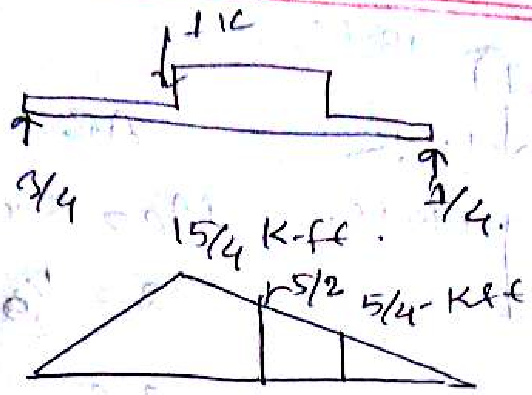
(i) Deflection at B.

(ii) change in slope at D.



$m = \underline{\text{vary}}$

①



AB Segment.

$m_P = 5x.$

$m_Q = \frac{3}{4}x.$

BC Segment.

$m_P = 5x + 25$

$m_Q = \frac{15}{4} - \frac{x}{4}.$

CD Segment.

$m_P = 50 - 5x.$

$m_Q = \frac{5}{2} - \frac{x}{4}.$

DE Segment. (←)

$m_P = 5x.$

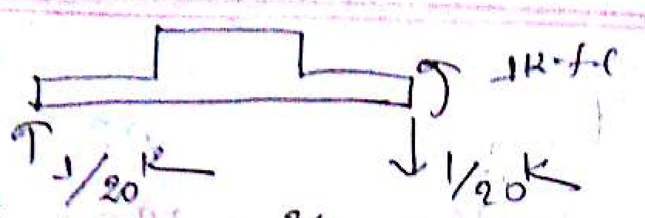
$m_Q = \frac{x}{4}.$

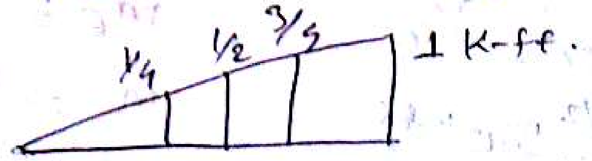
$$\Rightarrow \delta_B = \int_A^B \frac{m_P m_Q}{EI} dx + \int_B^C \frac{m_P m_Q}{EI} dx + \int_C^D \frac{m_P m_Q}{EI} dx + \int_E^D \frac{m_P m_Q}{EI} dx.$$

$$\Rightarrow \frac{1}{EI}\left[\int_0^5 5x \cdot \frac{3}{4}x\, dx + \int_0^5 \frac{25+5x}{1.33}\left(\frac{15}{4} - \frac{x}{4}\right) dx + \int_0^5 \frac{(50-5x)}{1.33}\cdot\left(\frac{5}{2} - \frac{x}{4}\right) dx + \int_0^5 5x \cdot \left(\frac{x}{4}\right) dx\right]$$

$$\Rightarrow \delta_B = \frac{919.22}{EI} = 0.029\text{ ft}.$$

(11)





AB.

$M_P = 5x.$

$M_Q = \frac{1}{20}x.$

$\cancel{1\cdot\alpha_D = \frac{1}{EI}\Big[\int}$

$$\Rightarrow Q\cdot\alpha_D = \int_A^B \frac{M_P M_Q}{EI}\,dx + \int_B^C \frac{M_P M_Q}{EI}\,dx + \int_C^D \frac{M_P M_Q}{EI}\,dx + \int_D^E \frac{M_P M_Q}{EI}\,dx.$$

BC

$M_P = 25+5x.$

$M_Q = \frac{1}{4} + x/20.$

CD.

$M_P = 50-5x.$

$M_Q = \frac{1}{2} + x/20.$

DE

$M_P = 5x.$

$M_Q = 1 - x/20$

$$= \frac{1}{EI}\left[\int_0^5 5x\cdot\left(\frac{1}{20}x\right)dx + \int_0^5 \frac{(25+5x)\left(\frac{1}{4}+\frac{x}{20}\right)}{1\cdot 33}dx + \int_0^5 \frac{(50-5x)\left(\frac{1}{2}+\frac{x}{20}\right)}{1\cdot 33}dx + \int_0^5 5x\cdot(1-x/20)\,dx\right]$$

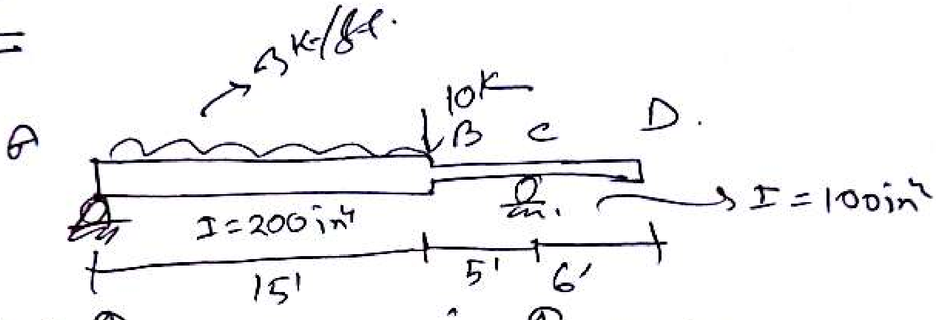
$= 0\cdot 0065$ rad.

Q. 2015-16 → (R)

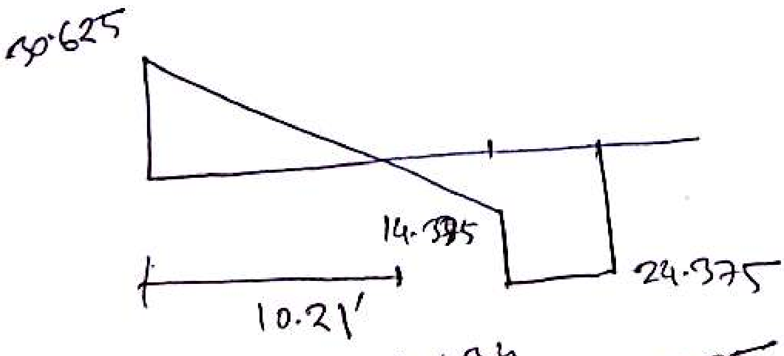
2014-15 → 10.
2013-14 → 6
2012-13 → Na.
2011-12 → 5, 6
2010-11 → Na.

09-10 → Na.
08-9 → Na.

2014-15



Slope at D



$$\frac{30.625}{14.375} = \frac{x}{15-x}$$