

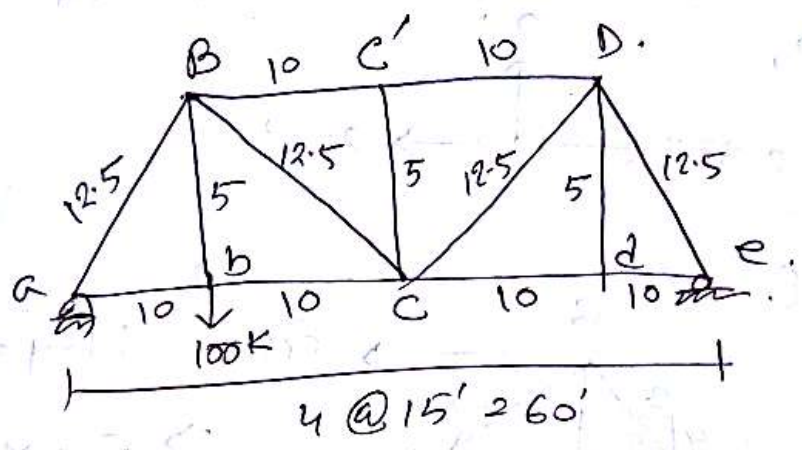
Virtual work:

2-3 to quest' answers

For truss structure:

$$\delta Q \cdot \delta = \sum \frac{F_p \cdot \delta a}{AE} + \sum \alpha_t \cdot t \cdot L \cdot F_a$$

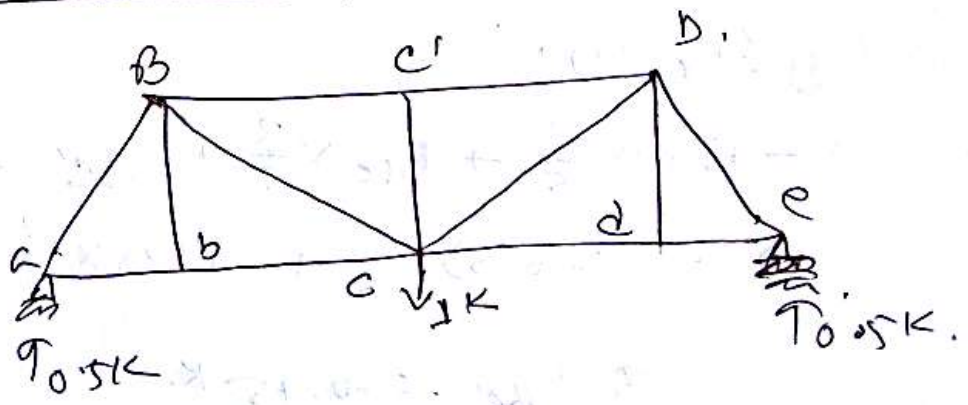
Example-8.1



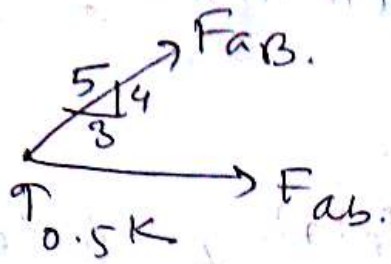
determine deflection at C.

$$\alpha = \frac{1}{150000}$$

Q Force analysis



Joint a,



$$\sum F_y = 0.$$

$$\Rightarrow F_{aB} \times \frac{4}{5} + 0.5 = 0.$$

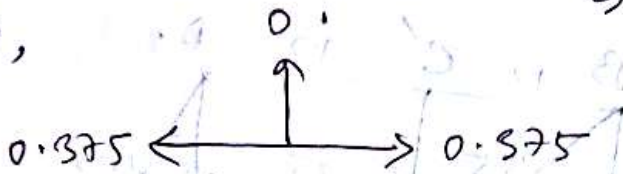
$$\Rightarrow F_{aB} = -0.625 \text{ k}.$$

$$\sum F_x = 0.$$

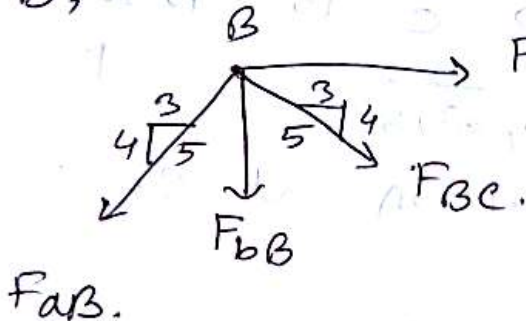
$$F_{ab} + F_{aB} \times \frac{3}{5} = 0.$$

$$\Rightarrow F_{ab} = 0.375 \text{ k}.$$

Joint b,



Joint B,



$$\sum F_y = 0 \quad (\downarrow +ve)$$

$$\Rightarrow F_{aB} \times \frac{4}{5} + F_{bB} + F_{Bc} \times \frac{4}{5} + 0 = 0.$$

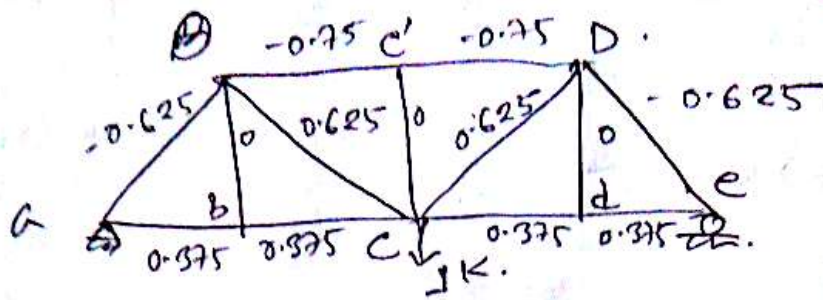
$$\Rightarrow F_{Bc} = -F_{aB} = 0.625 \text{ k}.$$

$$\sum F_x = 0.$$

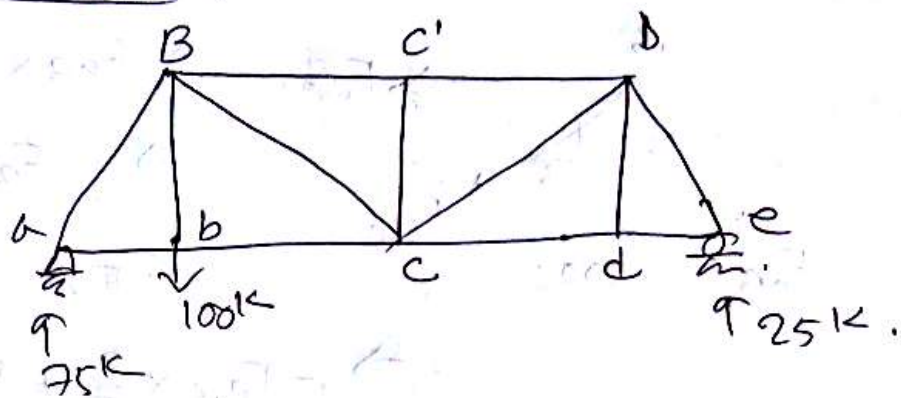
$$-F_{aB} \times \frac{3}{5} + F_{Bc} \times \frac{3}{5} + F_{Bc'} = 0$$

$$\Rightarrow -(-0.625) \times \frac{3}{5} + 0.625 \times \frac{3}{5} + F_{Bc'} = 0$$

$$\Rightarrow F_{Bc'} = -0.75 \text{ k}.$$



P. Force analysis



Joint A,

$$\sum F_y = 0$$

$$\Rightarrow 75 + F_{AB} \times \frac{4}{5} = 20$$

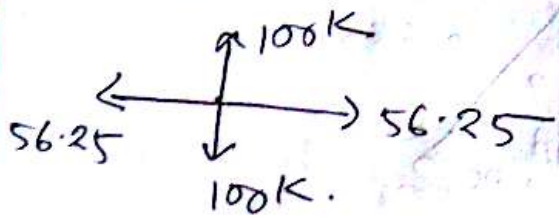
$$\Rightarrow F_{AB} = -93.75 \text{ k}$$

$$\sum F_x = 0$$

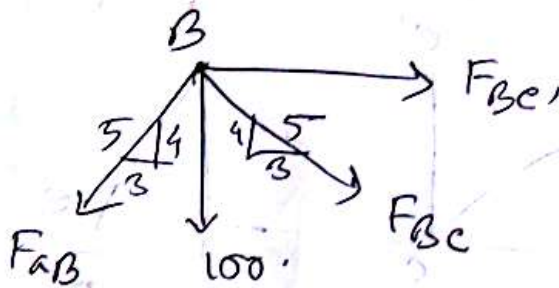
$$\Rightarrow F_{ab} + F_{AB} \times \frac{3}{5} = 20$$

$$\Rightarrow F_{ab} = 56.25 \text{ k}$$

Joint b,



Joint B,



$$\sum F_y = 0 \quad (\downarrow +ve)$$

$$\Rightarrow F_{AB} \times \frac{4}{5} + 100 + F_{BC} \times \frac{4}{5} = 0$$

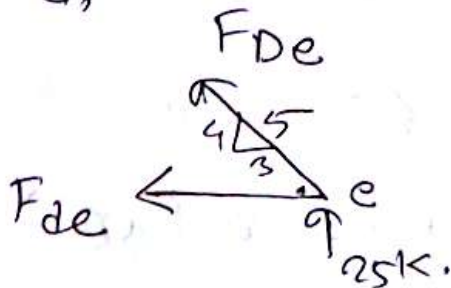
$$\Rightarrow F_{BC} = -31.25K$$

$$\sum F_x = 0$$

$$\Rightarrow -F_{AB} \times \frac{3}{5} + F_{BC'} + F_{BC} \times \frac{3}{5} = 0$$

$$\Rightarrow F_{BC'} = -37.5K$$

Joint e,



$$\sum F_y = 0$$

$$F_{De} \times \frac{4}{5} + 25 = 0$$

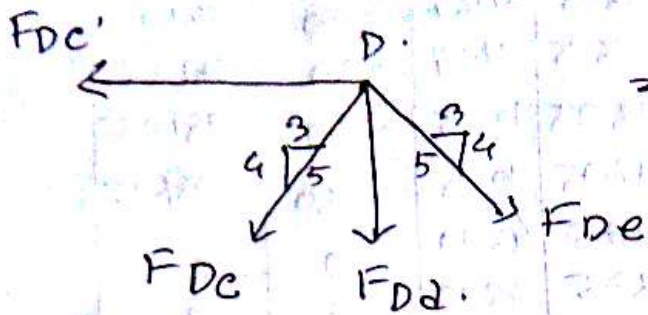
$$\Rightarrow F_{De} = -31.25K$$

$$\sum F_x = 0$$

$$\Rightarrow F_{de} + F_{De} \times \frac{3}{5} = 0$$

$$\Rightarrow F_{de} = 18.75K$$

Joint D,



$$\sum F_y = 0 \quad (\downarrow)$$

$$\Rightarrow F_{De} \times \frac{4}{5} + F_{De} \times \frac{4}{5} +$$

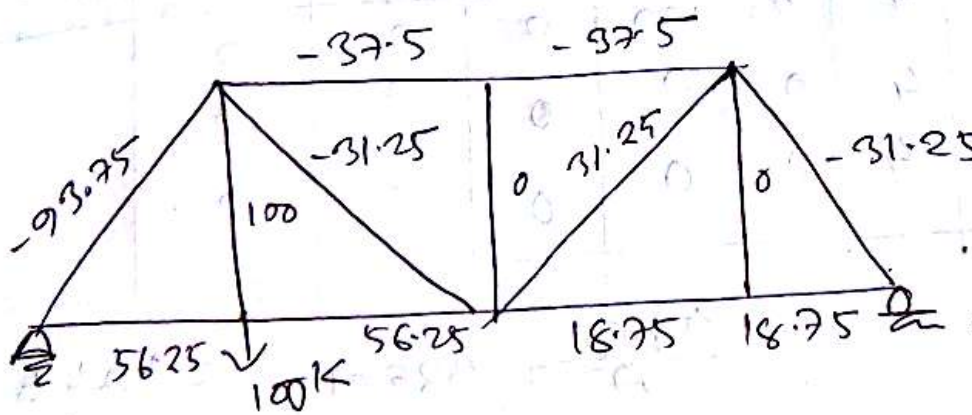
$$F_{Dd} = 0.$$

$$\Rightarrow F_{De} = 31.25 \text{ K.}$$

$$\sum F_x = 0.$$

$$\Rightarrow F_{Dc'} + F_{De} \times \frac{3}{5} - F_{De} \times \frac{3}{5} = 0.$$

$$\Rightarrow F_{Dc'} = -37.5 \text{ K.}$$



$$\left( \frac{2500}{10000} - \frac{10000}{20000} \right) =$$

$$0.10803 \cdot 10^3$$

$$7.5 \times 10^{-3}$$

Hor.

Bar	L	A	$\frac{L}{A}$	$F_Q$	$F_P$	$\frac{F_P F_Q L}{A}$	$\epsilon$	$F_Q L \epsilon$
ab	15	10	1.5	0.375	56.25	31.64	-50	-281.25
bc	15	10	1.5	0.375	56.25	31.64	-50	-281.25
cd	15	10	1.5	0.375	18.75	10.55	-50	-281.25
de	15	10	1.5	0.375	18.75	10.55	-50	-281.25
bc'	15	10	1.5	-0.75	-37.5	42.19	0	0
c'd	15	10	1.5	-0.75	-37.5	42.19	0	0

Diag

ab	25	12.5	2	-0.625	-98.75	17.19	0	0
bc	25	12.5	2	0.625	-31.25	-39.06	0	0
cd	25	12.5	2	0.625	31.25	+39.06	0	0
de	25	12.5	2	-0.625	-31.25	39.06	0	0

vert.

Bb	20	5	4	0	100	0	0	0
Cc	20	5	4	0	0	0	0	0
Dd	20	5	4	0	0	0	0	0

$$\sum \frac{F_P F_Q L}{A} = 325.01$$

$$\sum F_Q L \epsilon = -1125$$

$$\therefore \Delta \delta = \frac{\sum F_P F_Q L}{AE} + \sum F_Q L \epsilon$$

$$= \left( \frac{325.01}{30 \times 10^3} - \frac{1125}{150000} \right)$$

$$= \frac{3.25 \times 10^{-3}}{10} = 0.1008 \text{ ff.}$$

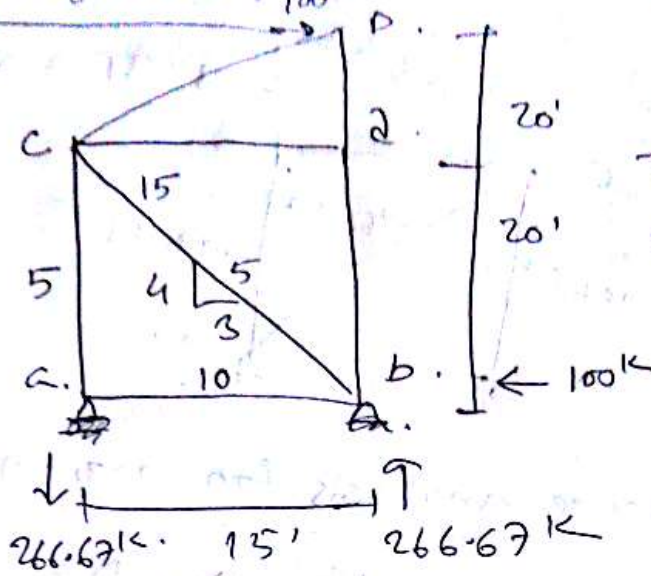
# From Assignment: 100k.

Support moment:

a vertical = 0.8" (↑)

b vertical = 0.9" (↓)

b horizontal = 0.3" (→)



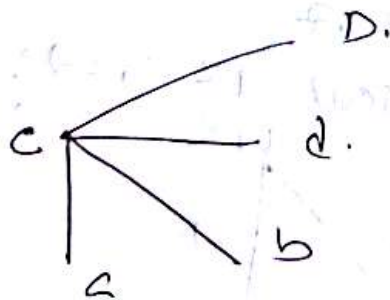
Temp. change.  
Vertical member  
40°F decrease  
Diagonal member.  
50°F increase

ⓐ compute horizontal deflection at D.

ⓑ " " vertical " " at D.

# Force analysis;

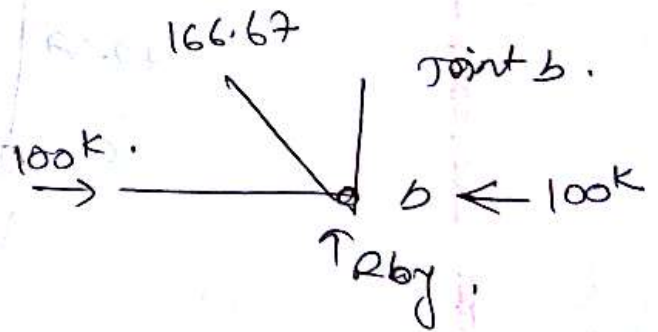
Joint c.



$F_{ac} = 266.67$  (↑)

$F_{ab} = 100k$  (→)

$F_{bd} = 133.31$  (↓)

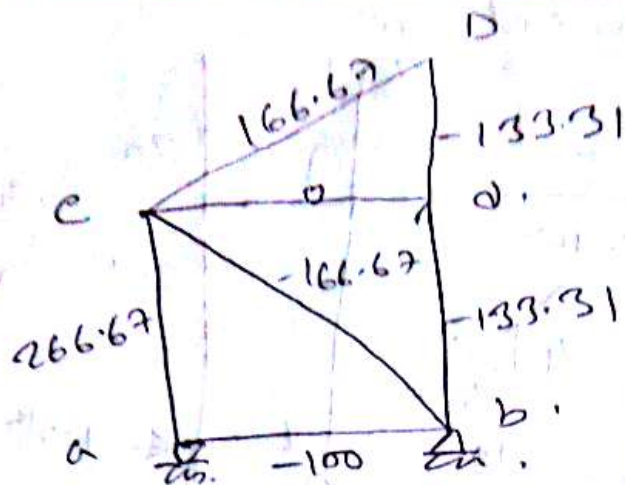


$F_{bc} = 166.67$  (→)

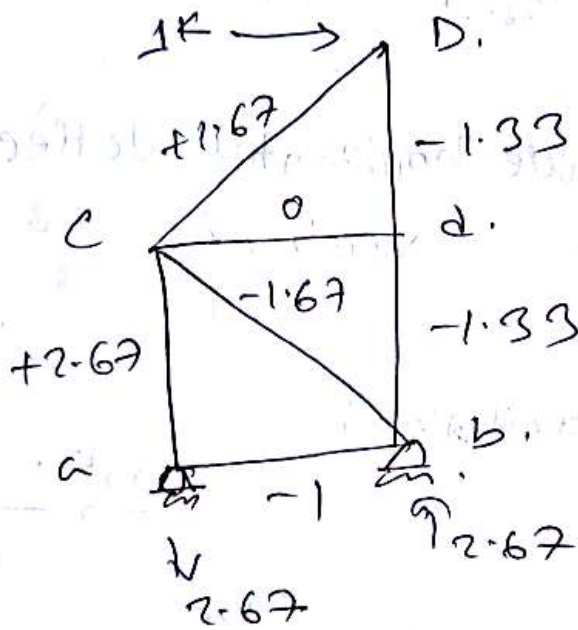
$F_{bd} = 133.31$  (↑)

$F_{cd} = 0$

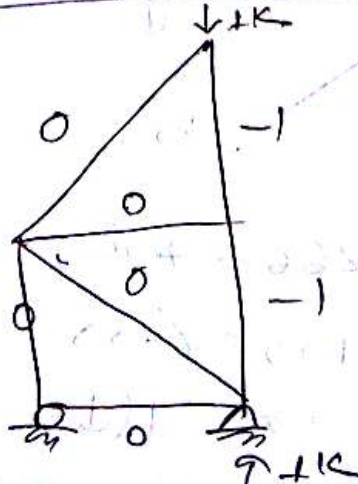
$F_{cd} = 166.67$  (→)



Q Force analysis for horizontal 1K load:



Q Force for vertical 1K load:



horizontal, temperature change, vertical

Bar	L	A	L/A	F <sub>P</sub>	F <sub>Q</sub> (→)	F <sub>P</sub> F <sub>Q</sub> L/A	ε	F <sub>Q</sub> Δt (→)	F <sub>Q</sub> (↓)	F <sub>Q</sub> F <sub>P</sub> L/A	F <sub>Q</sub> ΔtLε
ab	15	10	1.5	-100	-1	150	0	0	0	0	0
ac	20	5	4	266.67	2.67	2880.4	-40	14.24 × 10 <sup>-3</sup>	0	0	0
bc	25	15	5/3	-166.67	-1.67	463.9	+50	13.92 × 10 <sup>-3</sup>	0	0	0
bd	20	5	4	-133.33	-1.33	709.2	-40	7.09 × 10 <sup>-3</sup>	-1	533.2	5.33 × 10 <sup>-3</sup>
cd	25	15	5/3	166.67	+1.67	463.9	+50	13.92 × 10 <sup>-3</sup>	0	0	0
cd	15	10	1.5	0	0	0	0	0	0	0	0
Dd	20	5	4	-133.33	-1.33	709.2	-40	7.09 × 10 <sup>-3</sup>	-1	533.2	5.33 × 10 <sup>-3</sup>

$$\sum \frac{F_P F_Q L}{A} = 5344.24$$

$$\sum F_Q \Delta t L \epsilon = -6 \times 10^{-5}$$

$$\frac{\sum F_P F_Q L}{A} = 1066.4$$

$$\sum F_Q \Delta t L \epsilon = 10.66 \times 10^{-3}$$

For Q (→)

$$W_s + W_P = W_Q$$

$$\Rightarrow 1.8 + \left( 2.67 \times \frac{-0.8}{12} \right) + \left( 2.67 \times \frac{-0.9}{12} \right) + \left( 1 \times \frac{-0.3}{12} \right) = \frac{5344.24}{30000} + (-6 \times 10^{-5})$$

$$\Rightarrow \delta = 0.58 \text{ ft} = 6.98''$$

For Q (↓)

$$W_s + W_P = W_Q$$

$$1.8 + \left( 1 \times \frac{-0.9}{12} \right) = \frac{1066.4}{30000} + 10.66 \times 10^{-3}$$

$$\Rightarrow \delta = 0.12 \text{ ft} = 1.44''$$

Q: focus.

2015-16  $\rightarrow$  Na.  
2014-15  $\rightarrow$  9  
2013-14  $\rightarrow$  5  
2012-13  $\rightarrow$  10.  
2011-12  $\rightarrow$  4  
2010-11  $\rightarrow$  Na.  
2009-10  $\rightarrow$  Na.

2008-09  $\rightarrow$  Na.

16-17  $\rightarrow$  must

6mcs



28 marks.

2011-12

(9)

horizontal  
deflection at

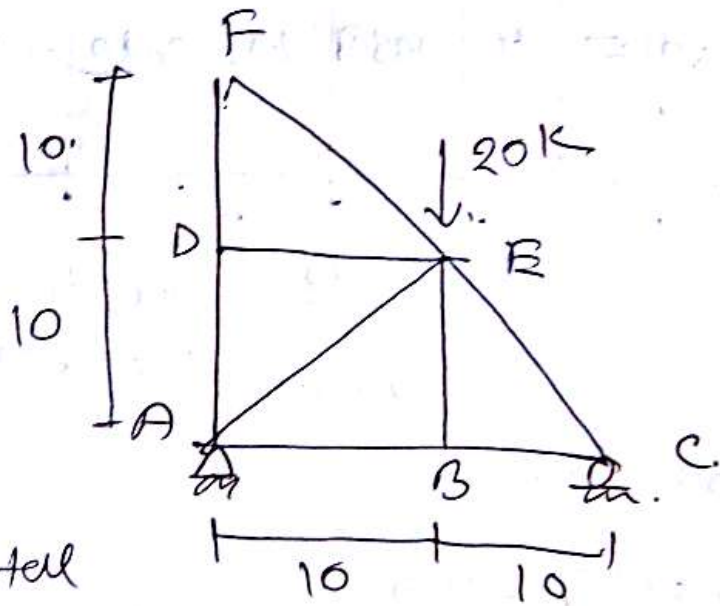
F

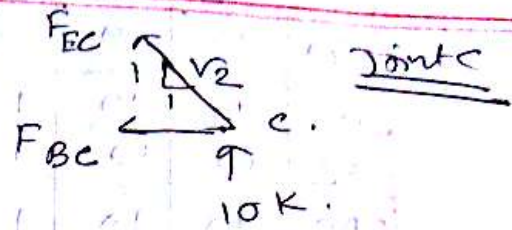
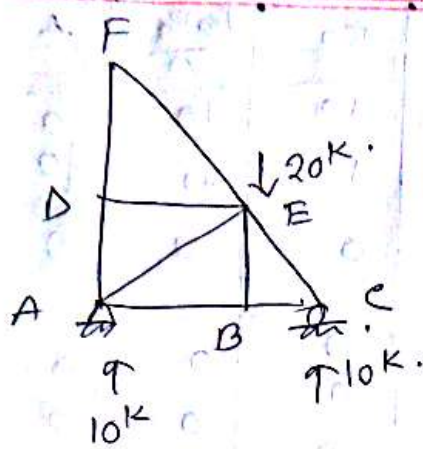
$B = 30000 \text{ Ksi}$

$A_1 = 10 \text{ in}^2$

$\rightarrow$  horizontal

$A_2 = 15 \text{ in}^2$  vertical and diagonal.





$$\sum F_y = 0$$

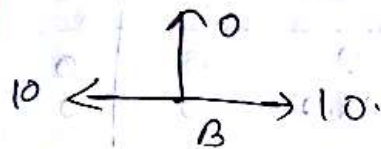
$$\Rightarrow 10 + \frac{F_{EC}}{\sqrt{2}} = 0$$

$$\Rightarrow F_{EC} = -14.14k$$

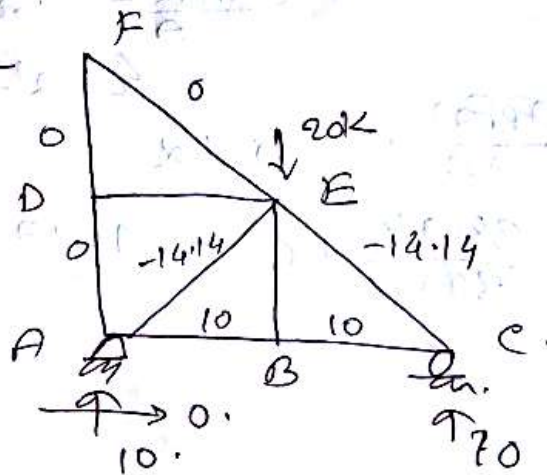
$$\sum F_x = 0$$

$$\Rightarrow F_{BC} + F_{EC} \times \frac{1}{\sqrt{2}} = 0$$

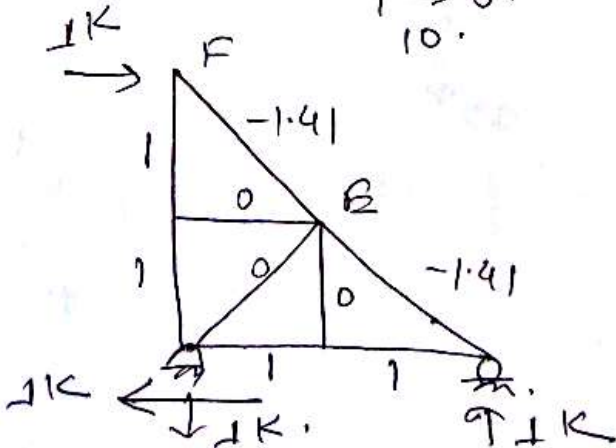
$$\Rightarrow F_{BC} = 10k$$



Joint B.



P Force



Q Force

Bar	L	A	$\frac{L}{A}$	$F_Q$	$F_P$	$\frac{F_P F_Q L}{A}$	$\epsilon$	$F_Q L \epsilon$
AB	10	10	1	1	10	10	0	0
BC	10	10	1	1	10	10	0	0
DE	10	10	1	0	0	0	0	0
AD	10	15	0.67	$\phi$	0	0	0	0
BE	10	15	0.67	0	0	0	0	0
DF	10	15	0.67	$\phi$	0	0	0	0
AE	14.14	15	0.94	$-14.14$	$-14.14$	0	0	0
CE	14.14	15	0.94	<del>14.14</del> $-1.41$	$-14.14$	18.74	0	0
EF	14.14	15	0.94	0	0	0	0	0

$$\sum \frac{F_P F_Q L}{A} = 38.74$$

$$\sum F_Q L \epsilon = 0$$

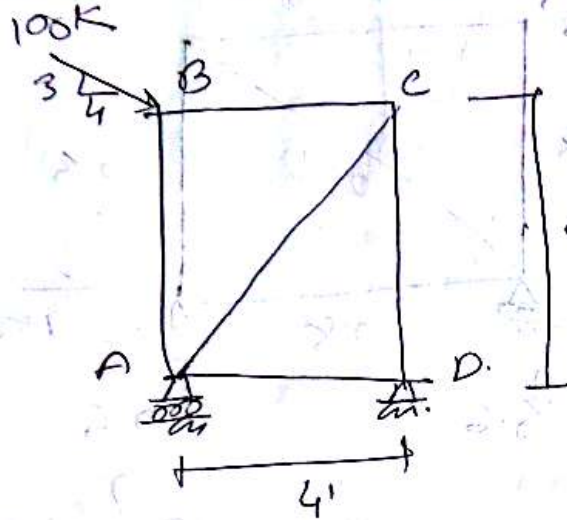
$$\Delta = \frac{\sum \frac{F_P F_Q L}{A}}{AE} + \sum F_Q L \epsilon$$

$$\Rightarrow \Delta = \frac{38.74}{30000} + 0 = 1.29 \times 10^{-3} \text{ ft.}$$



2012-13

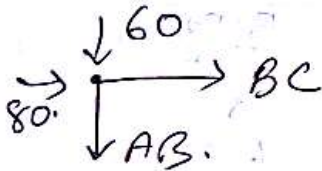
(10)



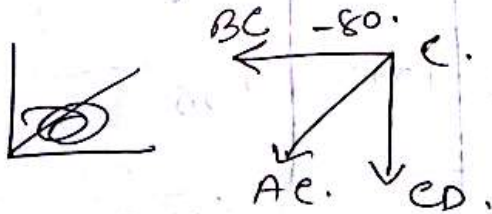
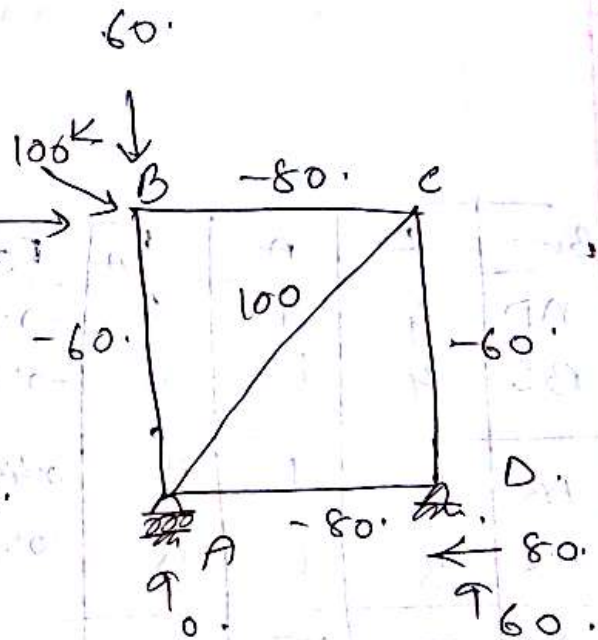
$A_{AC} = 0.5 \text{ in}^2$   
 other  $A = 1 \text{ in}^2$   
 $E = 30000 \text{ ksi}$

deflection along  
 BD (↓)

P-force analysis:



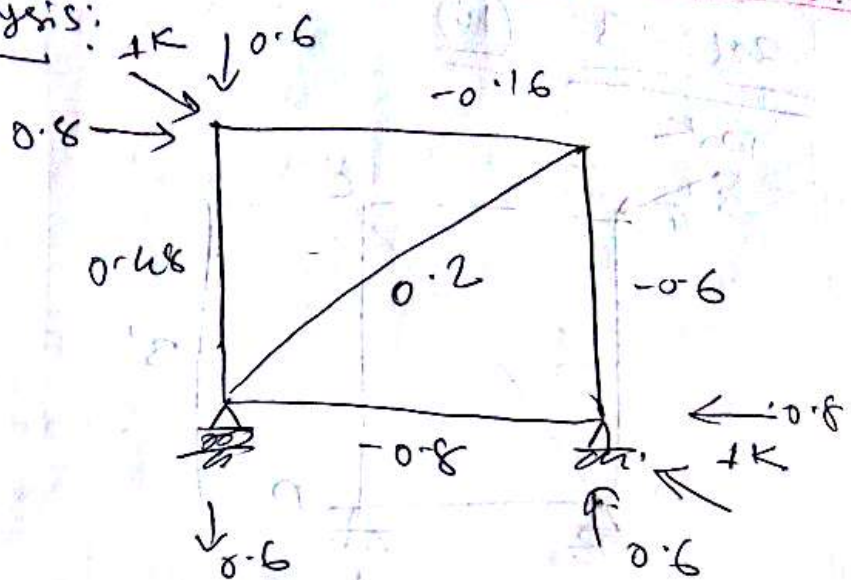
$\sum F_x = 0 \Rightarrow BC = -80$   
 $\sum F_y = 0 \Rightarrow AB = -60$



$\sum F_y = -80 + AC \times \frac{4}{5} = 0 \Rightarrow AC = 100$

(...)

Force analysis:



ସମସ୍ତ କାର୍ଯ୍ୟ (ସମସ୍ତ 23700 26200)   
 (5)

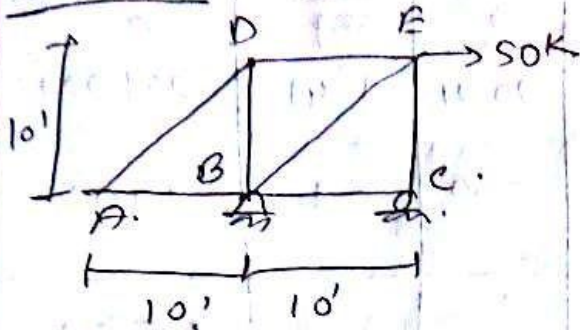
Mem.	L	A	L/A	F <sub>Q</sub>	F <sub>P</sub>	$\frac{F_P F_Q L}{A}$
AD	4	1	4	-0.8	-80	256
BC	4	1	4	-0.16	-80	51.2
AB	3	1	3	0.48	-60	-86.4
DE	3	1	3	-0.6	-60	108
AC	5	0.5	10	0.2	100	200

$$\frac{\sum F_P F_Q L}{A} = 328.8$$

$$\Delta_{BD} = \frac{\sum \frac{F_P F_Q L}{AE}}{30000} = \frac{328.8}{30000} \Rightarrow 0.01096 \text{ ft. } (\rightarrow \leftarrow)$$

2013-14

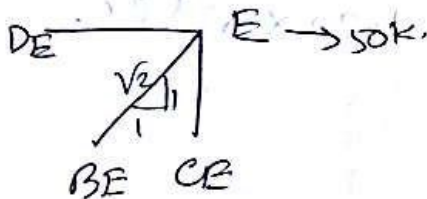
(5)



horizontal deflection at D.

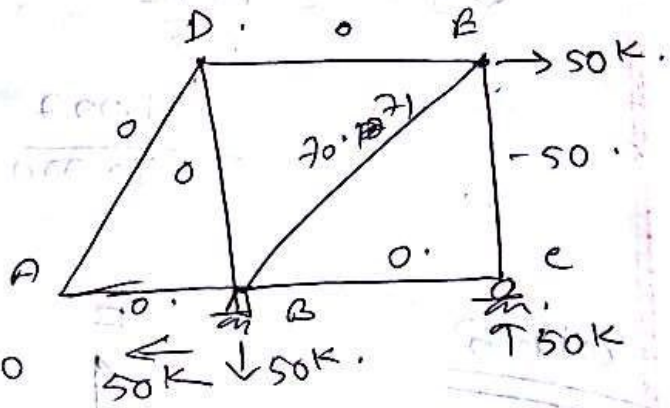
$A = 5 \text{ in}^2$   
 $E = 30000 \text{ ksi}$

P force analysis:

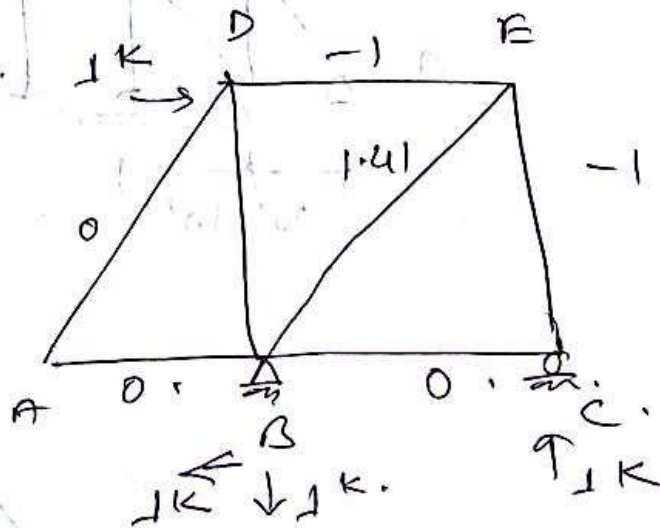


$$\sum F_y = -50 + BE \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow BE = 70.71$$



Q force analysis:



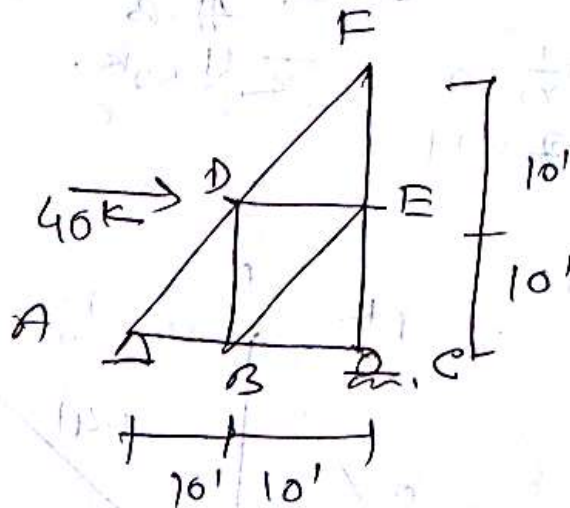
*Similar to Q*

Bar.	L	A	L/A	F <sub>P</sub>	F <sub>Q</sub>	$\frac{F_P F_Q L}{A}$
DE	10	5	2	0	-1	0
BE	$10\sqrt{2}$	5	$2\sqrt{2}$	70.71	1.41	281.997
CE	10	5	2	-50	-1	100

$$Q.8 = \frac{\sum F_P F_Q L}{AE}$$

$$\Rightarrow 1.8 = \frac{381.997}{30000} = 1.27 \times 10^{-3} \text{ ft.}$$

2014-15



horizontal deflection  
at F.

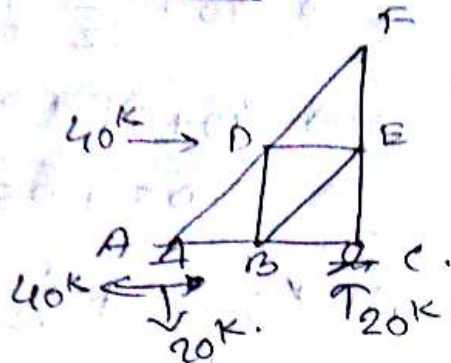
$$A = 10 \text{ in}^2 \text{ (horiz)}$$

$$A = 5 \text{ in}^2 \text{ (other)}$$

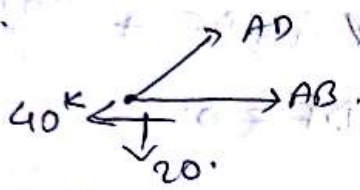
$$E = 30000 \text{ ksi}$$

Not Sure  
(?)

P force analysis:



Joint A.



$$\sum F_y = \frac{1}{\sqrt{2}} AD - 20 = 0$$

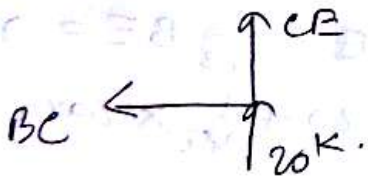
$$\Rightarrow AD = 20\sqrt{2} = 28.28k$$

$$\sum F_x = 0$$

$$\Rightarrow -40 + \frac{1}{\sqrt{2}} AD + AB = 0$$

$$\Rightarrow AB = 20k$$

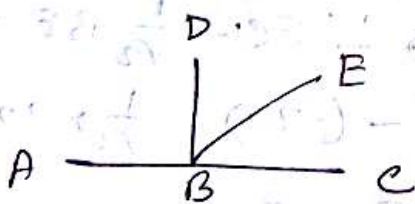
Joint C.



$$BC = 0$$

$$CE = -20k$$

Joint B.



$$\sum F_x = 0$$

$$\Rightarrow -AB + BC + BE \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow -20 + 0 + BE \times \frac{1}{\sqrt{2}} = 0$$

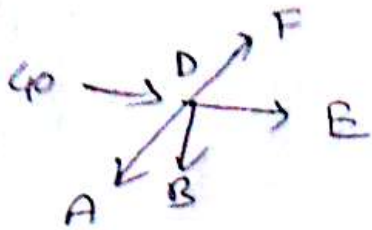
$$\Rightarrow BE = 20\sqrt{2}k$$

$$\sum F_y = 0$$

$$BE \times \frac{1}{\sqrt{2}} + BD = 0$$

$$\Rightarrow BD = -20k$$

Joint D.



$$\sum F_x = 0$$

$$\Rightarrow 40 + \frac{DE}{\sqrt{2}} + DE - \frac{AD}{\sqrt{2}} = 0$$

$$\Rightarrow 40 + DF + DE - \frac{20\sqrt{2}}{\sqrt{2}} = 0$$

$$\Rightarrow DF + DE = -20$$

$$\sum F_y = 0 \Rightarrow BD + \frac{1}{\sqrt{2}} DF + \frac{1}{\sqrt{2}} AD = 0$$

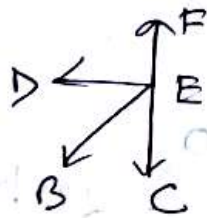
$$\Rightarrow -20 + \frac{1}{\sqrt{2}} DF + \frac{20\sqrt{2}}{\sqrt{2}} = 0$$

$$\Rightarrow DF = 0$$

$\therefore$

$$DE = -20$$

Joint E



$$\sum F_x = 0$$

$$\Rightarrow DE + \frac{1}{\sqrt{2}} BE = 0$$

$$\Rightarrow -20 + \frac{1}{\sqrt{2}} \times 20\sqrt{2} = 0$$

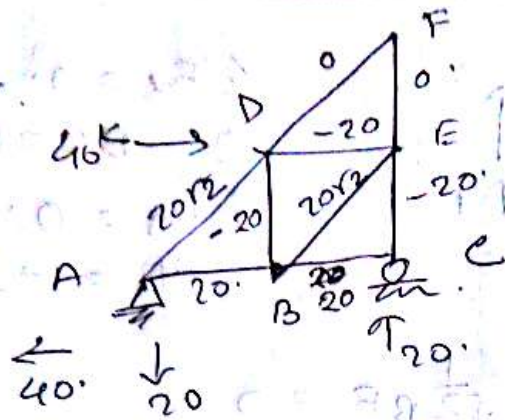
$$\sum F_y = 0$$

$$EF + FE - \frac{1}{\sqrt{2}} BE = 0$$

$$\Rightarrow EF - (-20) - \frac{1}{\sqrt{2}} \times 20\sqrt{2} = 0$$

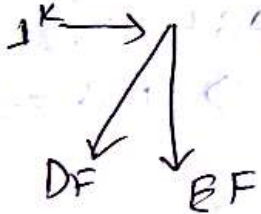
$$\Rightarrow EF = 0$$

OK



Q Force analysis:

Joint F.



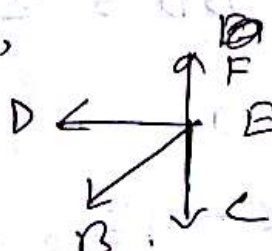
$$\sum F_x = 0$$

$$\Rightarrow DF \times \frac{1}{\sqrt{2}} = 1$$

$$\Rightarrow DF = \sqrt{2}$$

$$\sum F_y = 0 \Rightarrow DF \times \frac{1}{\sqrt{2}} + EF = 0 \Rightarrow EF = -1$$

Joint E.



$$\sum F_x = 0$$

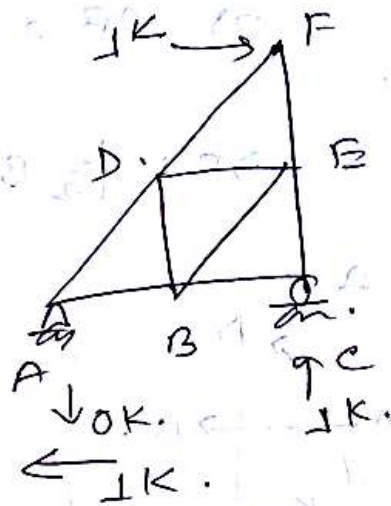
$$\Rightarrow DE + \frac{1}{\sqrt{2}} BE = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0$$

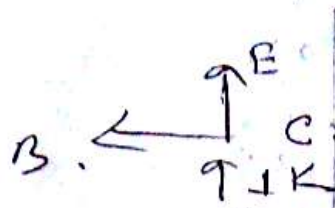
$$\Rightarrow EF - CE - \frac{1}{\sqrt{2}} BE = 0$$

$$\Rightarrow -1 - CE - \frac{1}{\sqrt{2}} BE = 0 \quad \text{--- (2)}$$

(u)



Joint C



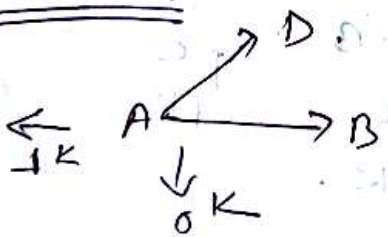
$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ BC &= -1 \end{aligned}$$

$$\textcircled{II} \Rightarrow -1 - 1 - \frac{1}{\sqrt{2}} BE = 0$$

$$\Rightarrow BE = -2\sqrt{2}$$

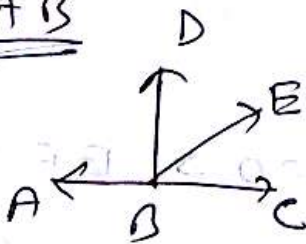
$$\textcircled{I} \Rightarrow DE + \frac{1}{\sqrt{2}} BE = 0 \Rightarrow DE = 2$$

Joint A



$$\begin{aligned} \sum F_x &= 0 \quad AB + AD/\sqrt{2} = 1 \\ \sum F_y &= 0 \quad AD = 0 \\ \Rightarrow AB &= 1 \end{aligned}$$

Joint B



$$\begin{aligned} \sum F_y &= 0 \\ \Rightarrow BD + \frac{BE}{\sqrt{2}} &= 0 \\ \Rightarrow BD &= 2 \end{aligned}$$

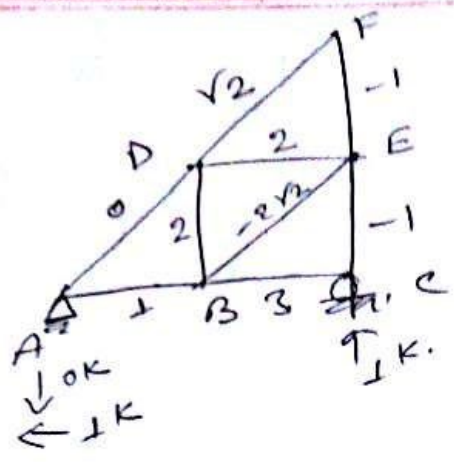
$$\sum F_x = 0$$

$$AB - BC - \frac{BE}{\sqrt{2}} = 0$$

$$\Rightarrow 1 - BC - \frac{(-2\sqrt{2})}{\sqrt{2}} = 0$$

$$\Rightarrow BC = 3 \quad (?)$$

in 2 days practice  
24/12/21  
22/12/21



Not Sune ☹️

Hor

Diag

Ver

Bar	L	A	C/A	F <sub>P</sub>	F <sub>Q</sub>	$\frac{F_P F_Q L}{A}$
AB	10	10	1	20	1	20
BC	10	10	1	20	3	60
DE	10	10	1	-20	2	-40
AD	10√2	5	2√2	20√2	0	0
DF	10√2	5	2√2	0	√2	0
BE	10√2	5	2√2	20√2	-2√2	-226.27
BD	10	5	2	-20	2	-80
CE	10	5	2	-20	-1	+40
EF	10	5	2	0	-1	0

$$\sum \frac{F_P F_Q L}{A} = -226.27$$

$$\Delta \delta = \frac{-226.27}{30000}$$

$$= -7.54 \times 10^{-3} \text{ ft}$$

Need more practice.