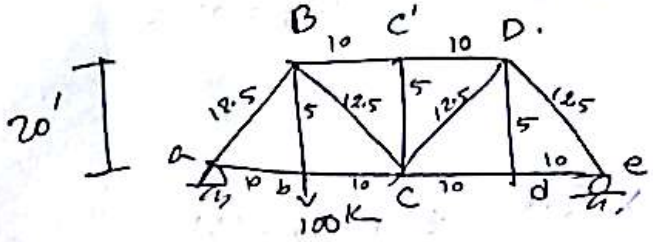
Virtual Work Trusses

Example-8.1

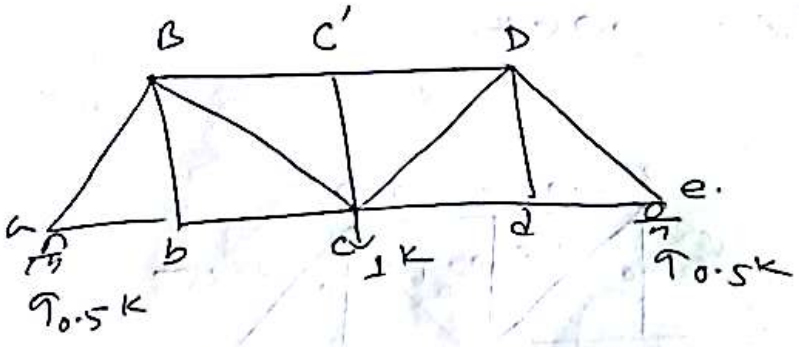


$\alpha = \frac{1}{150\,000}$.

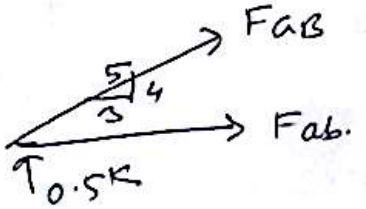
$4 @ 15' = 60'$

deflection at C.

② force analysis:



Joint a,



$\Sigma F_y = 0$

$\Rightarrow F_{aB} \times \frac{4}{5} + 0.5 = 0$.

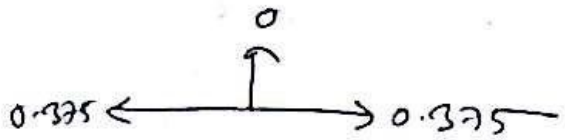
$\Rightarrow \boxed{F_{aB} = -0.625\,K}$.

$\Sigma F_x = 0$.

$\Rightarrow F_{ab} + F_{aB} \times \frac{3}{5} = 0$.

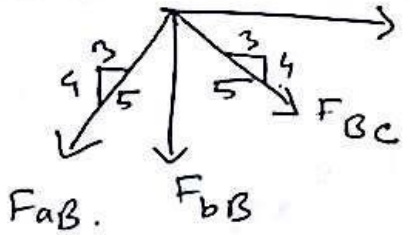
$\Rightarrow \boxed{F_{ab} = 0.375\,K}$.

joint b



$F_{bB} = 0$

Joint B



$\Sigma F_y = 0$. (↓)

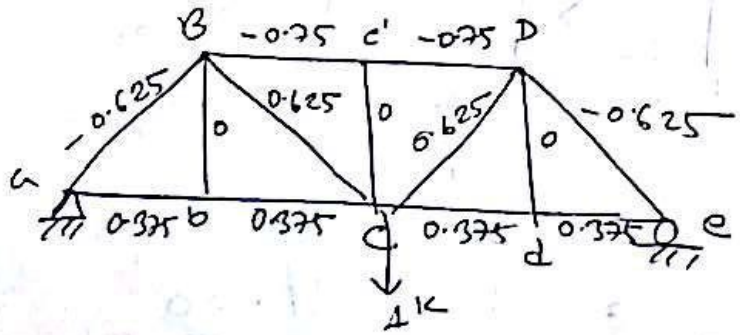
$\Rightarrow F_{aB} \times \frac{4}{5} + F_{BC} \times \frac{4}{5} = 0$.

$\Rightarrow F_{BC} = +0.625$

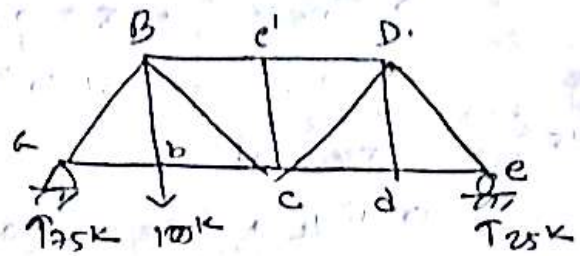
$\Sigma F_x = 0$.

$\Rightarrow -F_{aB} \times \frac{3}{5} + F_{BC} \times \frac{3}{5} + F_{BC'} = 0$.

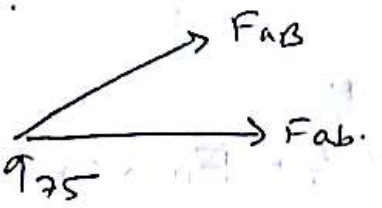
$\Rightarrow F_{BC'} = -0.75$ K.



P force analysis:



Joint a.



$\Sigma F_y = 0$.

$\Rightarrow 75 + F_{aB} \times \frac{4}{5} = 0$.

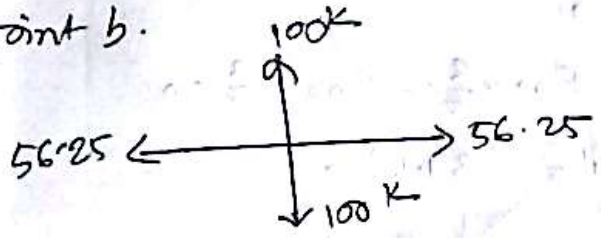
$\Rightarrow F_{aB} = -93.75K$

$\Sigma F_x = 0$.

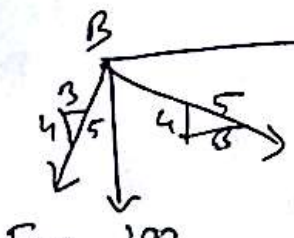
$\Rightarrow F_{ab} + F_{aB} \times \frac{3}{5} = 0$.

$\Rightarrow F_{ab} = 56.25K$

Joint b.



Joint B.



$\Sigma F_y = 0$. (2)

$\Rightarrow F_{aB} \times \frac{4}{5} + 100 + F_{Bc} \times \frac{4}{5} = 0$.

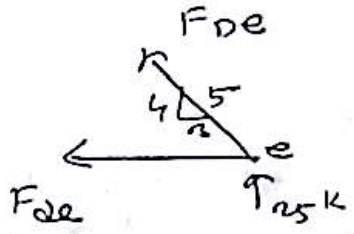
$\Rightarrow F_{Bc} = -31.25K$.

$\Sigma F_x = 0$.

$\Rightarrow -F_{aB} \times \frac{3}{5} + F_{Bc'} + F_{Bc} \times \frac{3}{5} = 0$.

$\Rightarrow F_{Bc'} = -37.5K$.

joint e,



$\Sigma F_y = 0$.

$\Rightarrow F_{De} \times \frac{4}{5} + 25 = 0$.

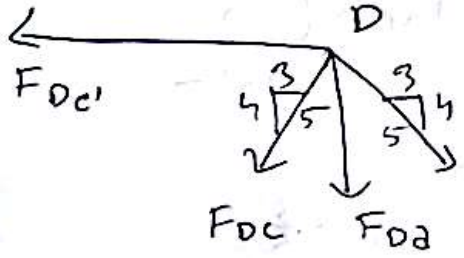
$\Rightarrow F_{De} = -31.25\text{K}$

$\Sigma F_x = 0$.

$\Rightarrow F_{de} + F_{De} \times \frac{3}{5} = 0$.

$\Rightarrow F_{de} = 18.75$

joint D.



$\Sigma F_y = 0$.

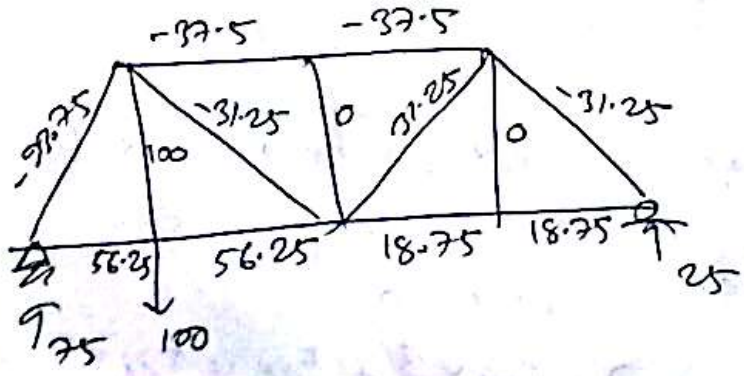
$\Rightarrow F_{De} \times \frac{4}{5} + F_{Dc} \times \frac{4}{5} +$ $F_{Da} = 0$ → 0.

$\Rightarrow F_{De} = 31.25\text{K}$

$\Sigma F_x = 0$.

$\Rightarrow F_{Dc'} + F_{Dc} \times \frac{3}{5} - F_{De} \times \frac{3}{5} = 0$.

$\Rightarrow F_{Dc'} = -37.5\text{K}$



	Bar	L	A	$\frac{L}{A}$	Fa	Fp	$\frac{FpFaL}{A}$	ε	FaLε
Hor.	Ab	15	10	1.5	0.375	56.25	31.64	−50	−281.25
	bc	15	10	1.5	0.375	56.25	31.64	−50	−281.25
	cd	15	10	1.5	0.375	18.75	70.55	−50	−281.25
	de	15	10	1.5	0.375	18.75	10.55	−50	−281.25
	Bc'	15	10	1.5	−0.25	−37.5	42.19	0	0
	c'D.	15	10	1.5	−0.25	−37.5	42.19	0	0
Dig	aB	25	12.5	2	−0.625	−93.75	117.19	0	0
	Bc	25	12.5	2	0.625	−31.25	39.06	0	0
	cD	25	12.5	2	0.625	31.25	39.06	0	0
	De	25	12.5	2	−0.625	−31.25	39.06	0	0
Vert	Bb	20	5	4	0	100	0	0	0
	c'c	20	5	4	0	0	0	0	0
	Dd	20	5	4	0	0	0	0	0

$\Sigma \frac{FpFaL}{A} = 325.01$ $\quad \Sigma FaL\varepsilon = -1125$

$$Q.\delta = \frac{\Sigma FpFaL}{AE} + \Sigma Fa.L\varepsilon.$$

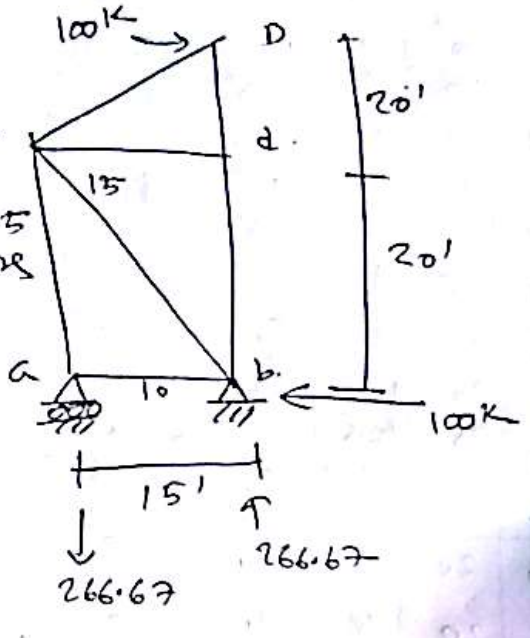
$$= \frac{325.01}{30\times10^3} - \frac{1125}{150\,000} = 0.1008 \text{ ft.}$$

Assignment

Temp change

Vertical member : 40°F decrease

Diagonal " : 50°F increase



Support movement;

'a' Vertical = 0.8" (↑)

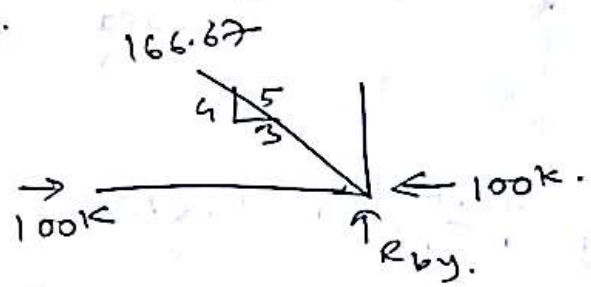
'b' " = 0.9" (↓)

'b' horizontal = 0.3" (→)

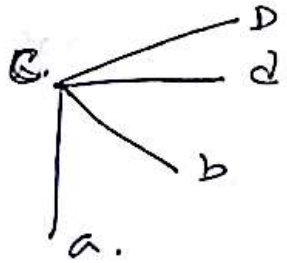
① Compute horizontal deflection at D.

② " Vertical " " D

P force analysis.



Joint c.



F_{ac} = 266.67 (↑)

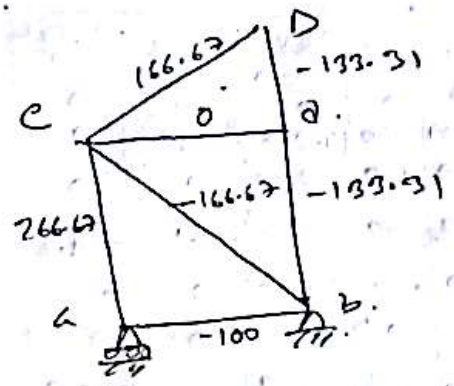
F_{ab} = 100K (→)

F_{bd} = 133.31 (↓)

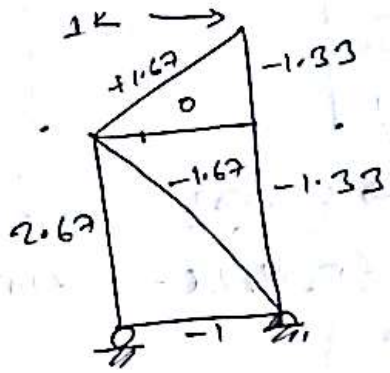
F_{be} = 166.67 (→)

F_{dD} = 133.31 (↑)

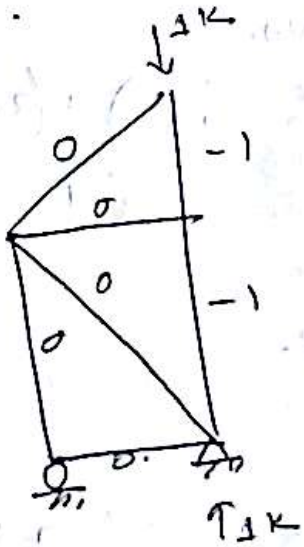
F_{cD} = 166.67 (→)



Q force analysis for 1K horizontal:



Q force for vertical 1K:



Bar	L	A	L/A	FP	FQ (hor.)	FPFQL/A	ε	FQαLε (→)	FQ (Vertical)	FPFQ²L/A	FQαLε.
ab	15	10	1.5	-100	-1	150	0	0	0	0	0
ac	20	5	4	266.67	2.67	2844.04	-40	14.24×10^{-3}	0	0	0
bc	25	15	5/3	-166.67	-1.67	4639	+50	13.92×10^{-3}	0	0	0
bd	20	5	4	-133.33	-1.33	709.2	-40	7.09×10^{-3}	-1	533.2	5.33×10^{-3}
cD	25	15	5/3	166.67	+1.67	4639	+50	13.92×10^{-3}	0	0	0
cd	15	10	1.5	0	0	0	0	0	0	0	0
Dd	20	5	4	-133.33	-1.33	709.2	-40	7.09×10^{-3}	-1	533.2	5.33×10^{-3}

$\Sigma F_Q \alpha_t L\varepsilon = 10.66\times10^{-3}$

$\Sigma \frac{F_P F_Q^2 L}{A} = 10664$

$\Sigma \frac{F_P F_Q L}{A} = 5344.24$

$\Sigma F_Q \alpha_t L\varepsilon = -6\times10^{-5}$

For horizontal.

$W_S + W_R = W_d.$

$\Rightarrow 1.8 + \left(2.67 \times \frac{-0.8}{12}\right) + \left(2.67 \times 10 \times \frac{-0.9}{12}\right) + \left(1 + \frac{-0.3}{12}\right).$

$= \frac{5344.24}{30000} + (-6\times10^{-5})$

$\Rightarrow \delta = 0.178$ ft.

For Vertical.

$W_S + W_R = W_d.$

$\Rightarrow 1.8 + \left(1 \times \frac{-0.9}{12}\right) = \frac{1066.4}{30000} + 10.66\times10^{-3}$

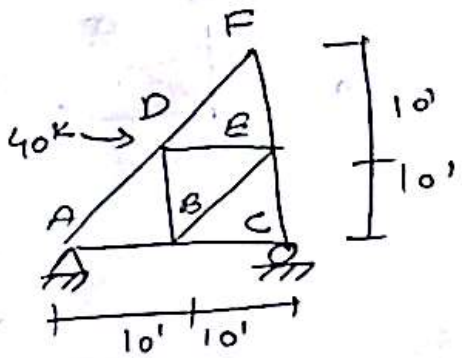
$\Rightarrow \delta = 0.12$ ft

2015-16 N.A

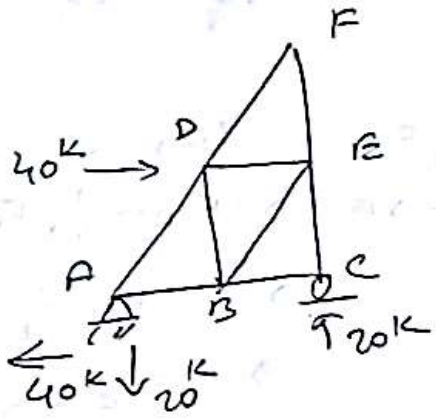
2014-15

(9)

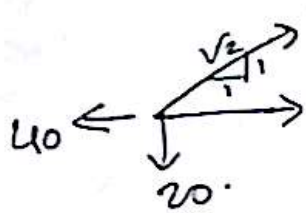
hor Area = 10 in²
other " = 5 in².
E = 30 000 ksi



p force analysis.



JOINT A.



$\Sigma F_y = 0$.

$\Rightarrow \frac{1}{\sqrt{2}} F_{AD} - 20 = 0$.

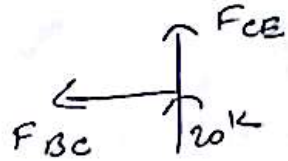
$\Rightarrow F_{AD} = 20\sqrt{2} = 28.28$ K

$\Sigma F_x = 0$.

$\Rightarrow -40 + \frac{1}{\sqrt{2}} F_{AD} + F_{AB} = 0$.

$\Rightarrow F_{AB} = 20$K.

Joint C



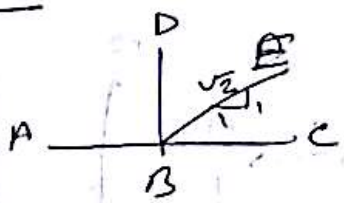
$\Sigma F_y = 0.$

$\Rightarrow F_{CE} = -20^K$

$\Sigma F_x = 0$

$\Rightarrow F_{BC} = 0.$

Joint B



$\Sigma F_x = 0.$

$\Rightarrow -F_{AB} + F_{BC} + F_{BE} \times \frac{1}{\sqrt{2}} = 0$

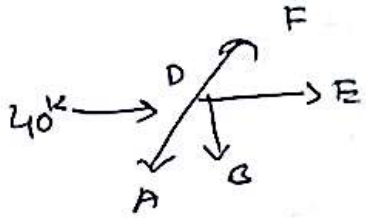
$\Rightarrow F_{BE} = 20\sqrt{2}.$

$\Sigma F_y = 0.$

$\Rightarrow F_{BE} \times \frac{1}{\sqrt{2}} + F_{BD} = 0.$

$\Rightarrow F_{BD} = -20^K.$

Joint D



$\Sigma F_y = 0.$

$\Rightarrow F_{BD} - \frac{1}{\sqrt{2}} F_{DF} + \frac{1}{\sqrt{2}} F_{AD} = 0.$

$\Rightarrow -20 - \frac{1}{\sqrt{2}} F_{DF} + \frac{20\sqrt{2}}{\sqrt{2}} = 0$

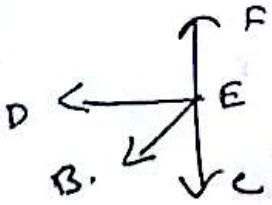
$\Rightarrow F_{DF} = 0.$

$\Sigma F_x = 0.$

$\Rightarrow 40 + \frac{F_{DF}}{\sqrt{2}} + F_{DE} - \frac{F_{AD}}{\sqrt{2}} = 0.$

$\Rightarrow F_{DE} = -20.$

Joint E



$\Sigma F_x = 0$

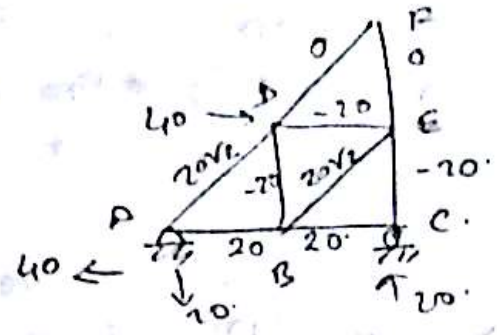
$\Rightarrow F_{DE} + \frac{1}{\sqrt{2}} F_{BE} = 0.$

$\Rightarrow -20 + \frac{1}{\sqrt{2}} \times 20\sqrt{2} = 0.$ [OK]

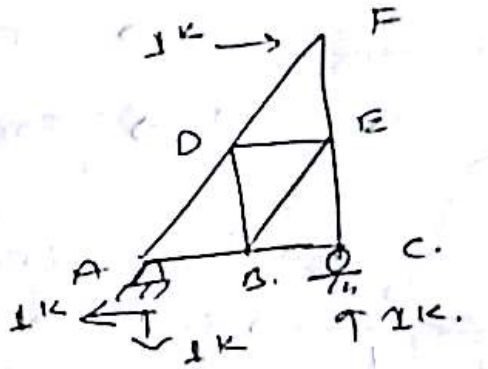
$\Sigma F_y = 0.$

$\Rightarrow F_{EF} - F_{EC} - \frac{1}{\sqrt{2}} F_{BE} = 0$

$\Rightarrow F_{EF} = 0.$



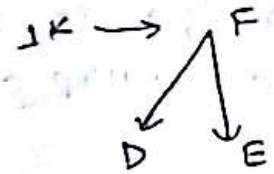
Q force analysis:



$1 \times 20 - R_c \times 20 = 0$

$\Rightarrow R_c = 1.$ (↑)

Joint F



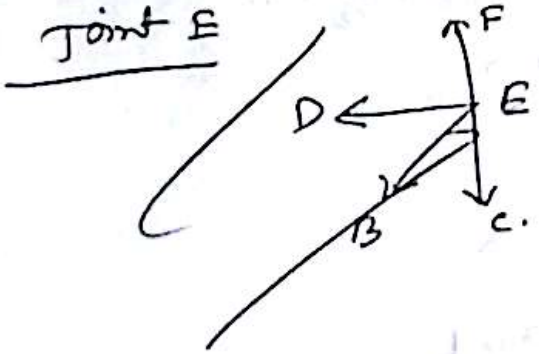
$\Sigma F_x = 0 \Rightarrow F_{DF} \times \frac{1}{\sqrt{2}} = 1.$

$\Rightarrow F_{DF} = \sqrt{2}.$

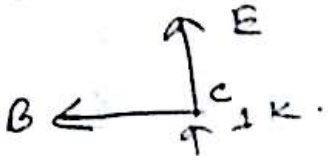
$\Sigma F_y = 0. \Rightarrow F_{DF} \times \frac{1}{\sqrt{2}} + F_{EF} = 0.$

$\Rightarrow F_{EF} = -1.$

Joint E



joint C



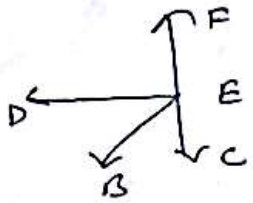
$\Sigma F_y = 0$

$\Rightarrow F_{CE} = -1K.$

$\Sigma F_x = 0.$

$F_{BC} = 0.$

Joint E



$\Sigma F_x = 0$.

$\Rightarrow F_{DE} + \frac{1}{\sqrt{2}} F_{BE} = 0$. —①

$\Sigma F_y = 0$.

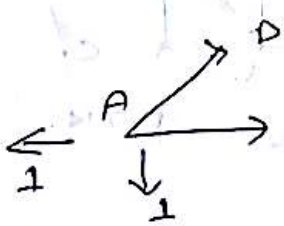
$F_{EF} - F_{CE} - \frac{1}{\sqrt{2}} F_{BE} = 0$.

$\Rightarrow -1 - (-1) - \frac{1}{\sqrt{2}} F_{BE} = 0$.

$\Rightarrow F_{BE} = 0$.

① $\Rightarrow F_{DE} = 0$.

Joint A



$\Sigma F_y = 0$.

$\Rightarrow F_{AD} \times \frac{1}{\sqrt{2}} = 1$.

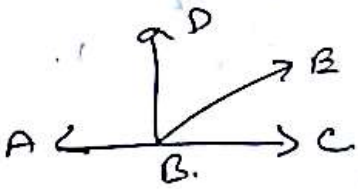
$\Rightarrow F_{AD} = \sqrt{2}$

$\Sigma F_x = 0$.

$\Rightarrow -1 + F_{AD} \times \frac{1}{\sqrt{2}} + F_{AB} = 0$.

$\Rightarrow F_{AB} = 0$ ~~0.41142~~

Joint B



$\Sigma F_y = 0$. → 0.

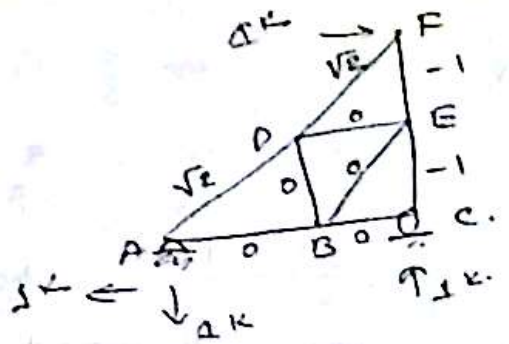
$\Rightarrow F_{BD} + F_{BE} \times \frac{1}{\sqrt{2}} = 0$.

$\Rightarrow F_{BD} = 0$.

$\Sigma F_x = 0$.

$\Rightarrow F_{BC} - F_{AB} + \frac{F_{BE}}{\sqrt{2}} = 0$.

$\Rightarrow 0 - 0 + 0 = 0$. OK



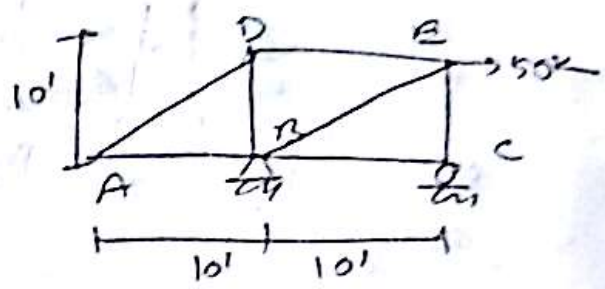
	Bar	L	A	$\frac{L}{A}$	Fp	Fq	$\frac{Fp\,Fq\,L}{A}$
Hor.	AB	10	10	1	20	0	0
	BC	10	10	1	20	0	0
	DE	10	10	1	-20	0	0
Diag.	AD	$10\sqrt{2}$	5	$2\sqrt{2}$	$20\sqrt{2}$	$\sqrt{2}$	$80\sqrt{2}$
	DF	$10\sqrt{2}$	5	$2\sqrt{2}$	0	$\sqrt{2}$	0
	BE	$10\sqrt{2}$	5	$2\sqrt{2}$	$20\sqrt{2}$	0	0
Vert.	BD	10	5	2	-20	0	0
	CE	10	5	2	-20	-1	40
	EF	10	5	2	0	-1	0

$\Sigma \frac{Fp\,Fq\,L}{A} = 153.1$

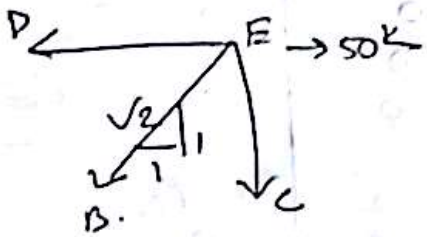
$$1.\delta = \frac{153.137}{30000} = 0.0051 \text{ ft.}$$

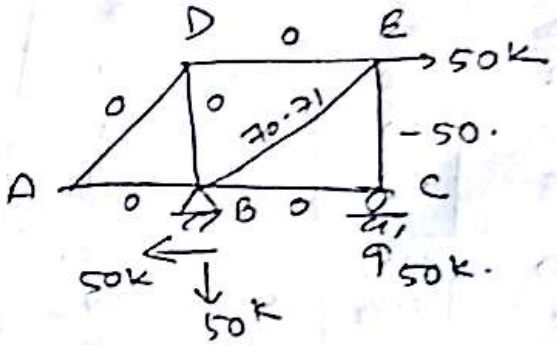
2013-14

A = 5in²
E = 30000 Kn/in²
at D (hor)



P force analysis:



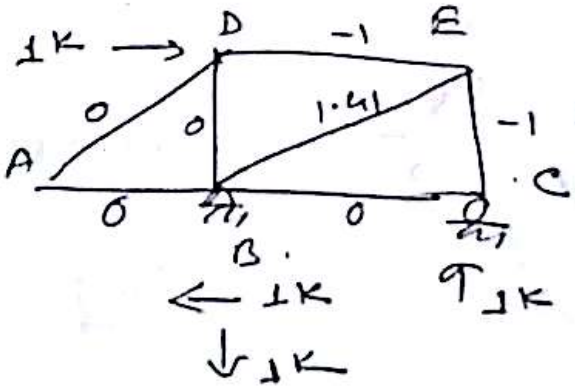


$\Sigma F_y = 0$.

$\Rightarrow -50 + BE \times \frac{1}{\sqrt{2}} = 0$

$\Rightarrow BE = 70.71$

Q force analysis;



Bar	L	A	L/A	FP	FQ	$\frac{FQFPL}{A}$
						0
DE	10	5	2	0	-1	281.333
BE	$10\sqrt{2}$	5	$2\sqrt{2}$	30.21	1.41	120
CE	10	5	2	-50	-1	
						$\sum \frac{FQFPL}{A} = 281.333$

$$\theta \cdot \delta = \frac{\sum FQFPL}{AE}$$

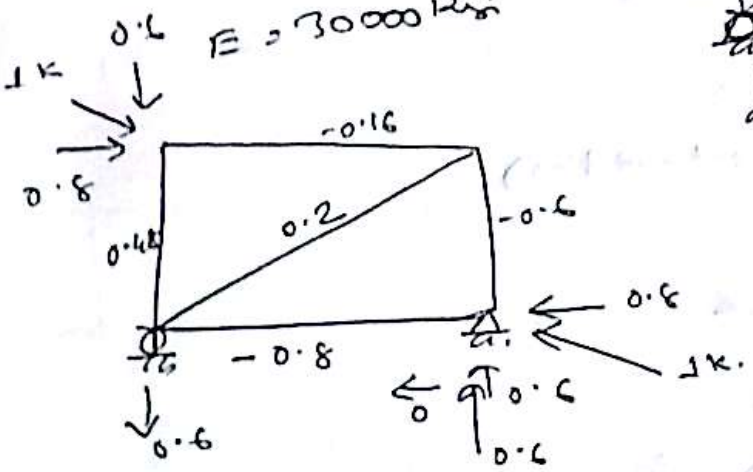
$$\Rightarrow 1 \cdot \delta = \frac{281.333}{30000} = 1.73 \times 10^{-3} \text{ ft}$$

12-13

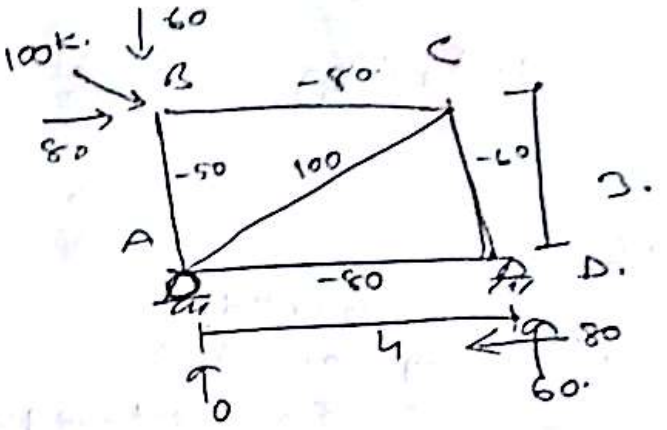
$A_C = 0.5 \text{ in}^2$

other $= 1 \text{ in}^2$

$E = 30000 \text{ k/in}^2$



Q force analysis.



P force.

Bar	L	A	L/A	F_Q	F_P	$\frac{F_P F_Q L}{A}$
AD	4	1	4	-0.8	-80	256
BC	4	1	4	-0.6	-80	51.2
AB	3	1	3	0.48	-60	-86.4
DC	3	1	3	-0.6	-60	108
AC	5	0.5	10	0.2	100	200

$$\frac{\Sigma F_P F_Q L}{A} = 528.6$$

$$\therefore \delta_{BD}(\rightarrow) = \frac{\Sigma F_P F_Q L}{AE} = \frac{528.6}{30000} = 0.0176 \text{ in}$$

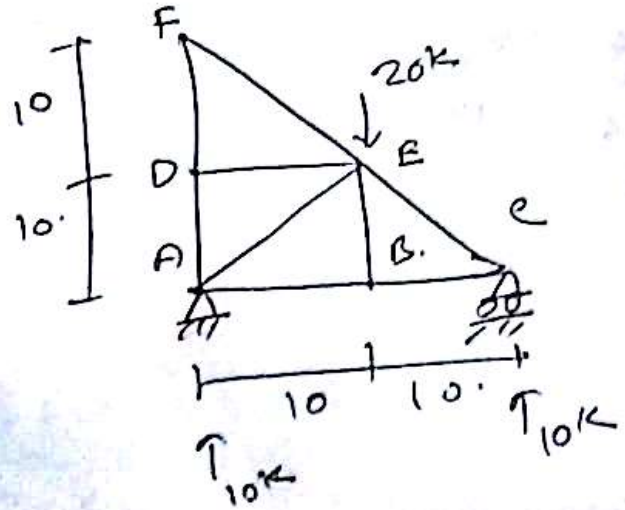
2011-12

(4)

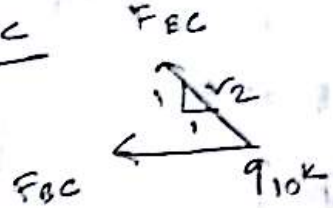
$E = 30000 \text{ ksi}$

$A_1 = 10 \text{ in}^2$ (hor)

$A_2 = 15 \text{ in}^2$ (vert and diag)



Joint C



$\Sigma F_y = 0$.

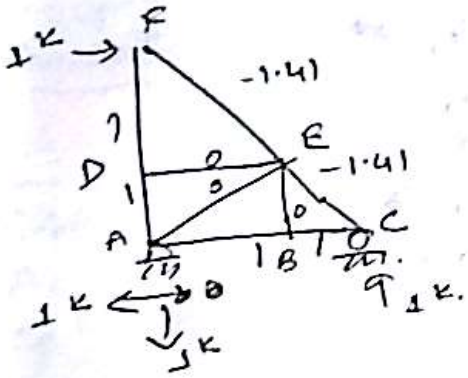
$\Rightarrow 10 + F_{BC} \times \frac{1}{\sqrt{2}} = 0$.

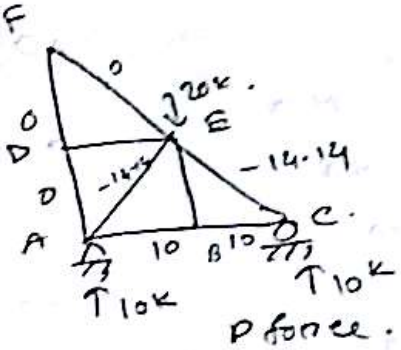
$\Rightarrow F_{BC} = -14.14$ K.

$\Sigma F_x = 0$.

$\Rightarrow F_{BC} + F_{EC} \times \frac{1}{\sqrt{2}} = 0$.

$\Rightarrow F_{BC} = 10$K.





Bar.	L	A	$\frac{A}{A}$	F_Q	F_P	$\frac{F_P L}{A}$	ϵ	$F_Q \cdot L \epsilon$
AB	10	10	1	1	10	10	0	0
BC	10	10	1	1	10	10	0	0
DE	10	10	1	0	0	0	0	0
AD	10	15	0.67	1	0	0	0	0
BE	10	15	0.67	0	0	0	0	0
DF	10	15	0.67	1	0	0	0	0
AE	14.14	15	0.94	0	-14.14	0	0	0
CE	14.14	15	0.94	-1.41	-14.14	18.74	0	0
BF	14.14	15	0.94	-1.41	0	0	0	0

$\frac{\Sigma F_0 F_{aL}}{A} = 38.74$

$\Sigma F_{aL} t = 0.$

$$\text{Q.8} = \frac{\Sigma F_0 F_{aL}}{AE} + \Sigma F_{aL} t \alpha.$$

$$\Rightarrow 1.8 = \frac{38.74}{30000} + 0 = 1.29 \times 10^{-3} \text{ ft}$$

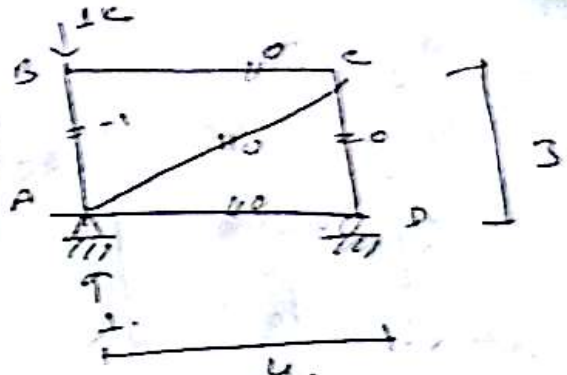
10-11	NA.
9-10	NA
8-9	NA

RAD-2

10-11 →	11, 12, 13
9-10 →	4, ~~7~~
8-9 →	3,
7-8 →	10.
6-7 →	2, 7
5-6 →	1, 6

2011-12

(12) Vertical 1K load.



$F_{AB} = -1K.$

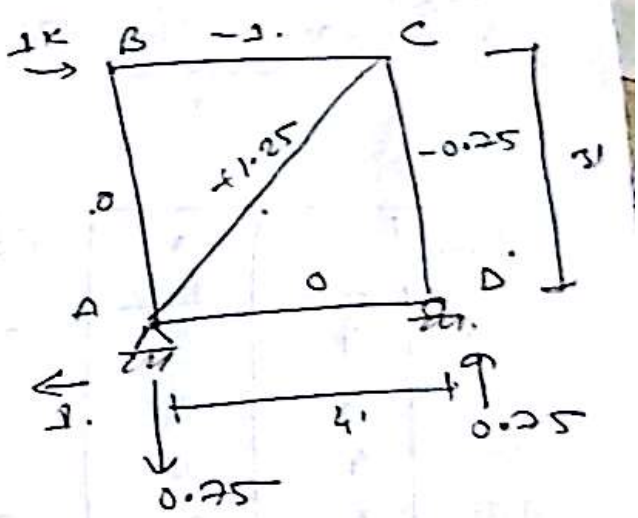
$Q \cdot \delta = \sum F_Q L \alpha$

$\Rightarrow 1 \cdot \delta_B = -1 \times (-50) \times 3 \times \frac{1}{150000}$

$= 0.001$ ft. (down ward)

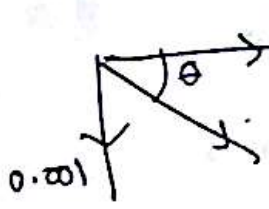
Horizontal 1K.

Bar.	L	F_Q.	$F_Q L$.
BC	4	-1	+80 -320
AC	5	+1.25	+80 ·500
CD	·3	-0.25	·-50 ·112.5
			292.5



$Q \cdot \delta_B = \sum F_Q L \alpha$

$\Rightarrow 1 \cdot \delta_B = \frac{292.5}{150000} = 0.00195$ ft. (to right)

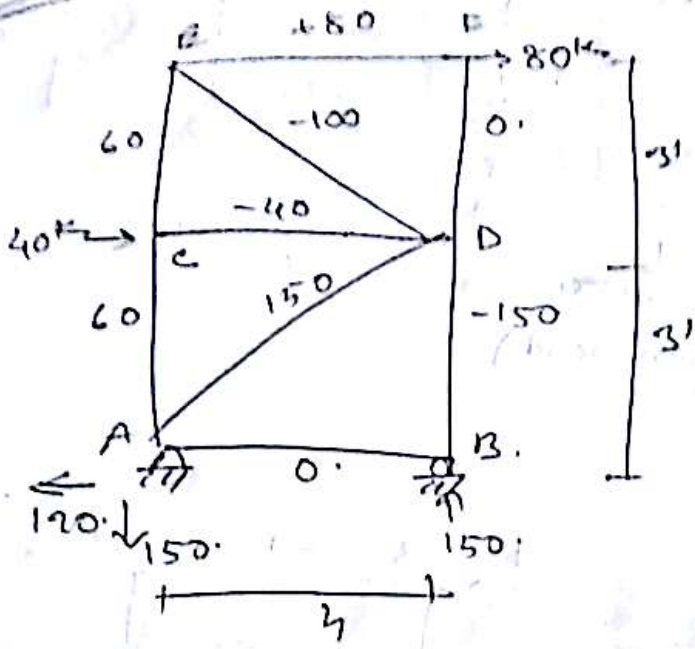


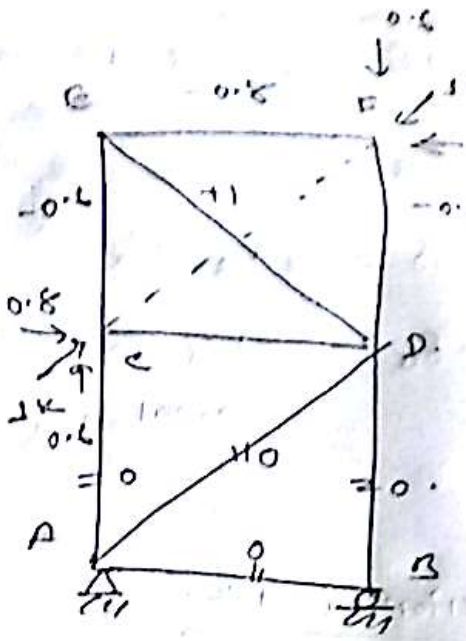
translation $= \sqrt{0.001^2 + 0.00195^2}$

$= 0.0022$ ft.

$\theta = \tan^{-1} \frac{0.001}{0.00195} = 27.15°$

2010-11 (13)





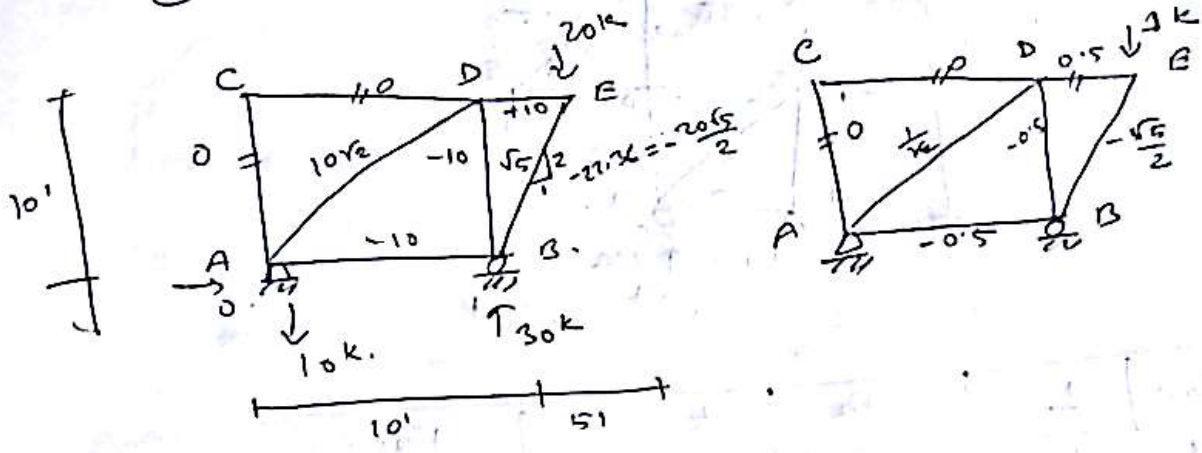
Bar.	L	A	Fa	Fo	$\frac{FoFaL}{A}$
AB	4	2	0	–	–
CD	4	2	-0.8	-40	64
EF	4	2	-0.8	+80	-128
AC	3	3	0	–	–
BD	3	3	0	–	–
CE	3	3	-0.6	+60	-36
DF	3	3	-0.6	0	–
AD	5	2	0	–	–
ED	5	2	+1	-100	-250

$\Sigma Q.\delta = \Sigma \frac{FoFaL}{AE}$ $\qquad \Sigma \frac{FoFaL}{A} = -350$

$\therefore 1.\ \delta_{CF}^{\downarrow} = \frac{-350}{30000} = -0.01167$ ft (apart)

9-10

(4)



Bar	L	A	FQ	Fp.	FQFpL/A.
AB	10	10	-0.5	-10	5
CD	10	10	0	-	0
DE	5	10	+0.5	+10	2.5
AC	10	10	0	-	0
BD.	10	10	-0.5	-10.	5
AD	14.142	10	$\frac{1}{\sqrt{2}}$	$+10\sqrt{2}$	14.142
BE	11.18	10'	$-\sqrt{5}/2$	$-10\sqrt{5}$	27.95

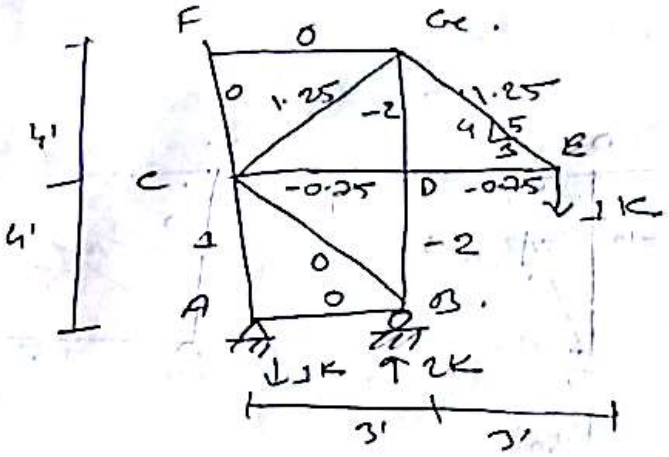
$\Sigma = 54.592$

$$Q.\delta = \frac{\Sigma F_p F_Q L}{AE}$$

$$\Rightarrow 1.\delta_E = \frac{54.592}{30000} = 0.00182 \text{ ft.} \quad (\downarrow)$$

6-9

(3)



Bar	L	F_Q	t	$F_Q t L$.
AB	3	0	-20	0
CD.	3	-0.25.	-20	45
GF	3	0	-20	0
DE	3	-0.25	-20.	45
AC	4	0	60	240
EF	4	0	60	0
BD	4	-2	60	-480
DG	4	-2	60	-480.
BC	5	0	-20	0.
CG	5	1.25	-20	-125
GB	5	1.25	-20.	-125

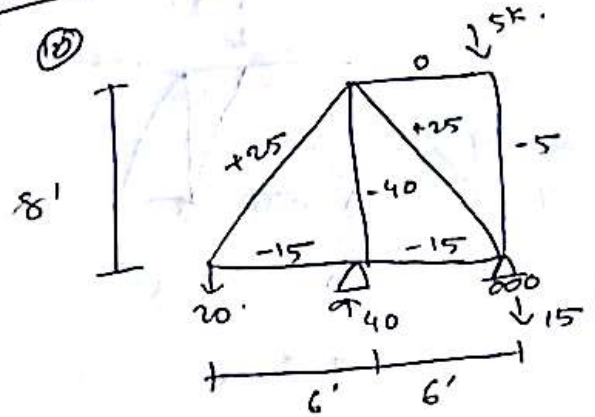
$\Sigma = -880.$

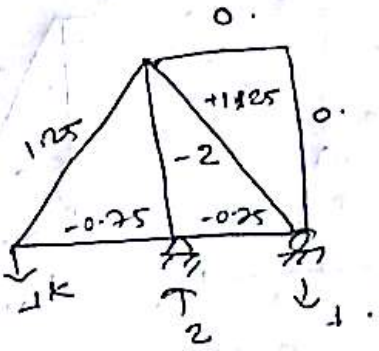
$$Q \cdot \delta_E = \Sigma F_Q t L \alpha. = -880 \times \frac{1}{150\,000}$$

$$\Rightarrow \delta_E = -0.005867 \text{ ft.} \quad (9).$$

7-8

(b)





Bar.	L	A	Fp	Fa	FpFaL/A
AB	6	7.5	-15	-0.75	9
BC	6	7.5	-15	-0.75	9
DE	6	7.5	0	0	0
BD	8	10	-40	-2	64
CE	8	10	-5	0	0
AD	10	12.5	+25	+1.25	25
ED	10	12.5	+25	+1.25	25

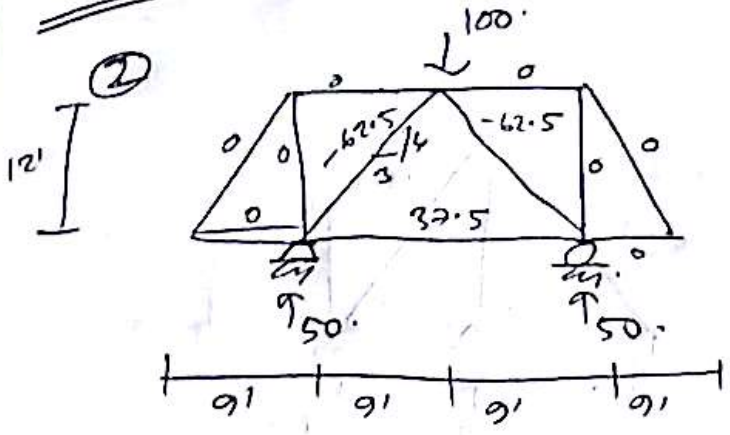
$\Sigma = 132$

$$\delta \times 1 - \frac{2 \times 0.25}{12} = \frac{132}{30000}$$

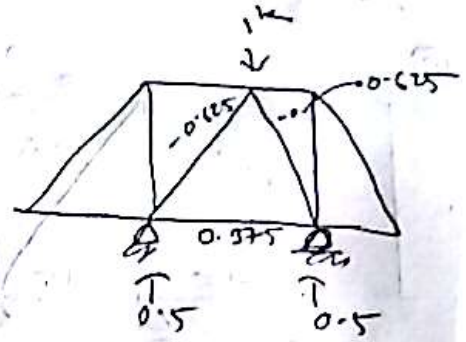
$$\Rightarrow \delta = 0.0461 \ (\downarrow)$$

6-7

(2)



P force.

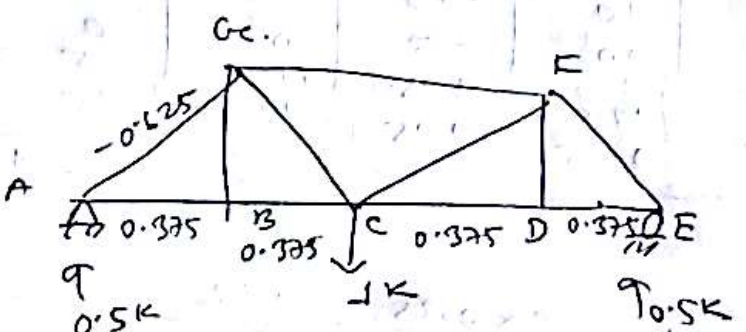


Q force

$$\delta_{1x} = \frac{62.5 \times 0.625 \times 2 \times 15}{10 \times 30 \times 10^3} + \frac{37.5 \times 0.375 \times 18}{10 \times 30000}$$

$$= 0.00435' \ (\rightarrow)$$

6-7 (3)

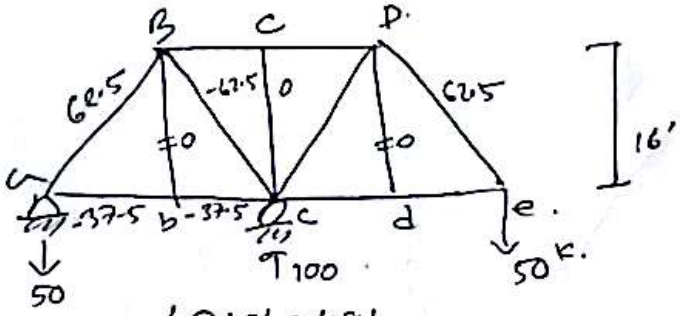


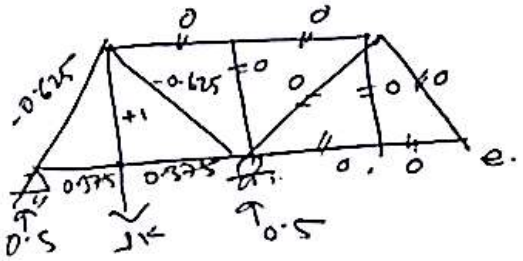
$$Q \cdot \delta_c \downarrow = -\Sigma F_Q L \epsilon \alpha.$$

$$\Rightarrow 1 \cdot \delta_c = 0.375 \times (-60) \times 15 \times \frac{1}{150000} \times 4$$

$$= 0.009 \text{ ft } (\downarrow)$$

5-6 ①





$$\delta_b \times 1 = -\frac{37.5 \times 0.375 \times 12}{2 \times 30 \times 10^3} \times 2 + \frac{62.5 \times 0.625 \times 20}{3 \times 30 \times 10^3} + \frac{62.5 \times 0.625 \times 20}{3 \times 30 \times 10^3}$$

$$= -5.625 \times 10^{-3} \cdot ft. \quad (\rightarrow)$$