Taufiq Sir	Zakaria Sir	Rouf Sir
1) 14 Feb	1) 23 Feb	1) 25 Feb
2) 28 Feb	2) 1 Mar	2) 4 Mar
3) 7 Mar	3) 2 Mar	3) 11 Mar
4) 21 Mar	4) 8 Mar	4) 25 Mar
5) 28 Mar	5) 15 Mar	5) 1 Apr
6) 25 Apr	6) 16 Mar	6) 22 Apr
7) 27 Apr	7) 18 Mar	7) 29 Apr
8) 2 May	8) 22 Mar	8) 6 May
9) 9 May	9) 29 Mar	9) 13 May
10) 11 May	10) 11 Apr	
	11) 12 Apr	
	12) 26 Apr	
	13) 4 May	
	14) 10 May	
	15) 16 May	

Taufiq Sir	Zakaria Sir	Rouf Sir
11) 23 May	16) 18 May	10) 20 May
12) 30 May		11) 27 May
		12) 30 May

23 February 2015

Zakaria Sir-1

Statically Determinate Suspended Bridge

[elastic theory of suspension bridge]

- Dead Loads are entirely carried by cable
- " " come from roadways
- " " are assumed to be uniform per horizontal foot of span

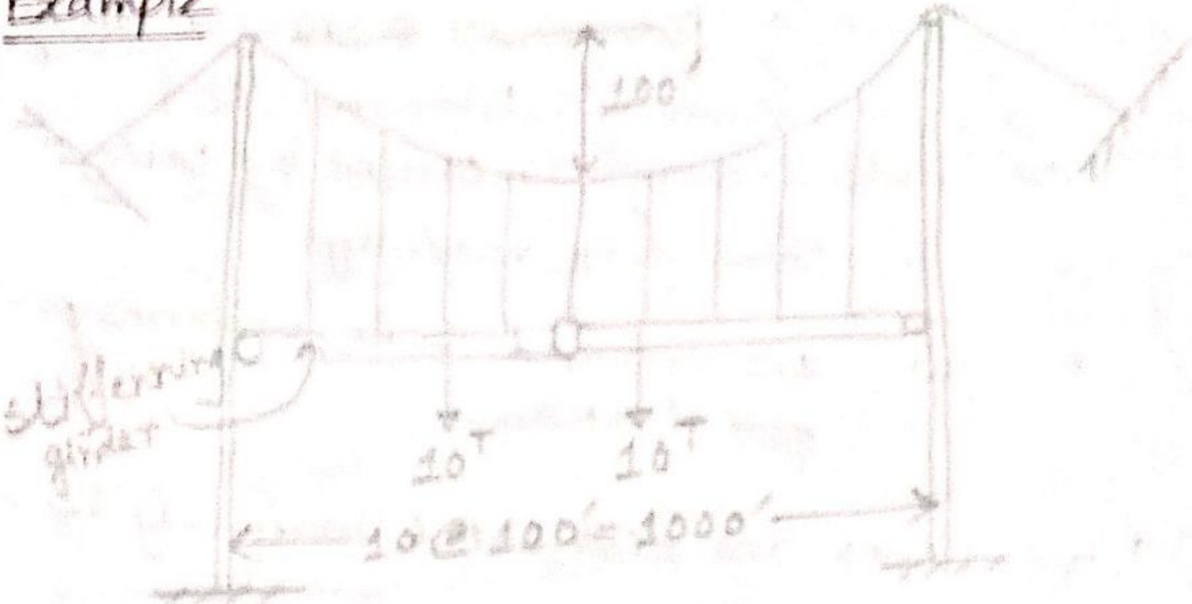
- Live Loads are distributed uniformly by stiffenning Truss or Girders

- Total load is uniform per horizontal foot of this span

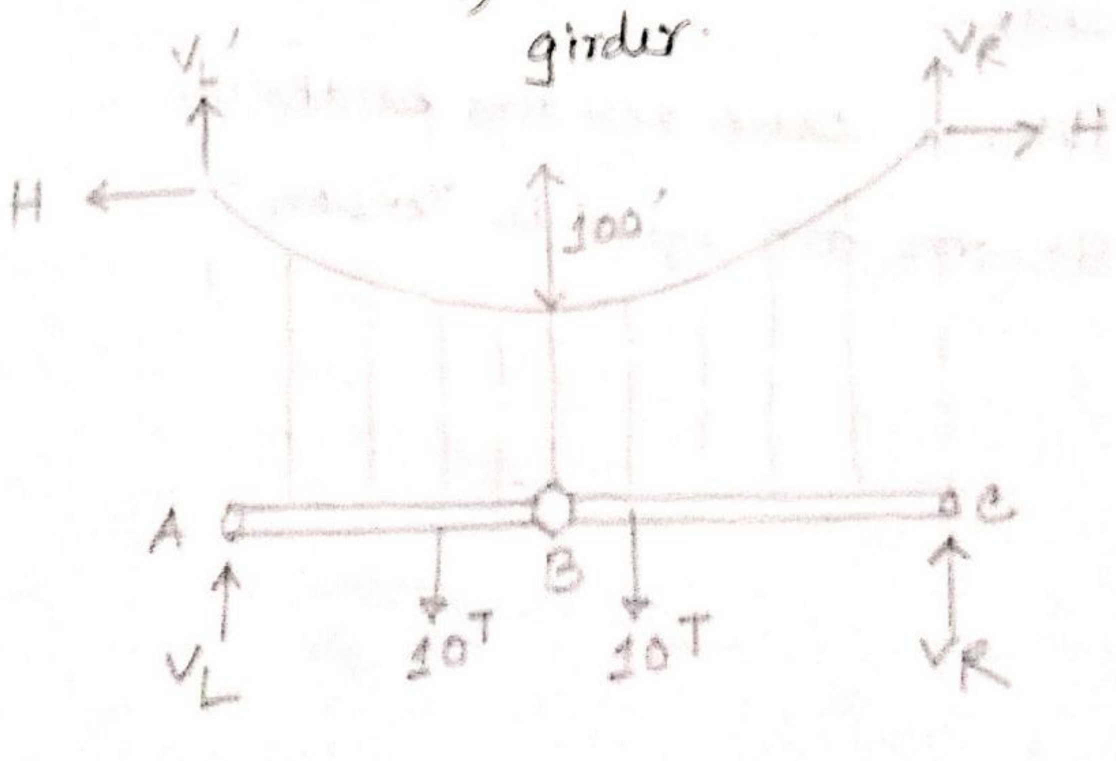
∴ Hence cable remains parabolic.

Hangers are equal in Tension

Example



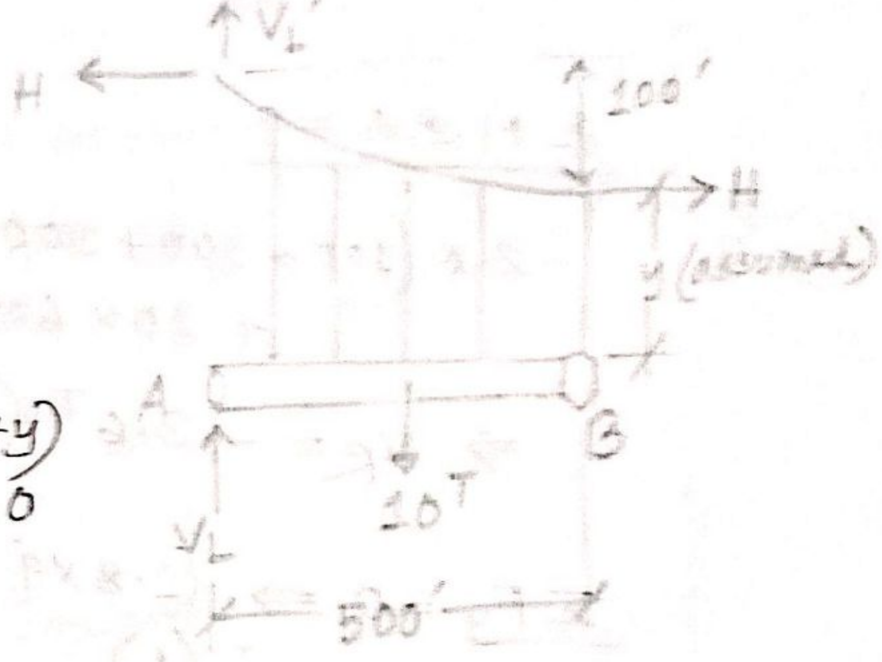
1) Find hanger force
2) Draw SFD & BMD of the girder.



$\Sigma M@C = 0$ +ve↻

$$(V_L + V_L') \times 1000 - 10 \times 700 - 10 \times 400 = 0$$

$$\therefore V_L + V_L' = \frac{11000}{1000} = 11^T$$



$\Sigma M@B = 0$ +ve↻

$$(V_L + V_L') \times 500 - H(100+y)$$
$$+ Hy - 10 \times 200 = 0$$

$$\Rightarrow 11 \times 500 - 100H$$
$$- \cancel{Hy} + \cancel{Hy} - 2000 = 0$$

$$\therefore H = \frac{5500 - 2000}{100} = 35^T$$

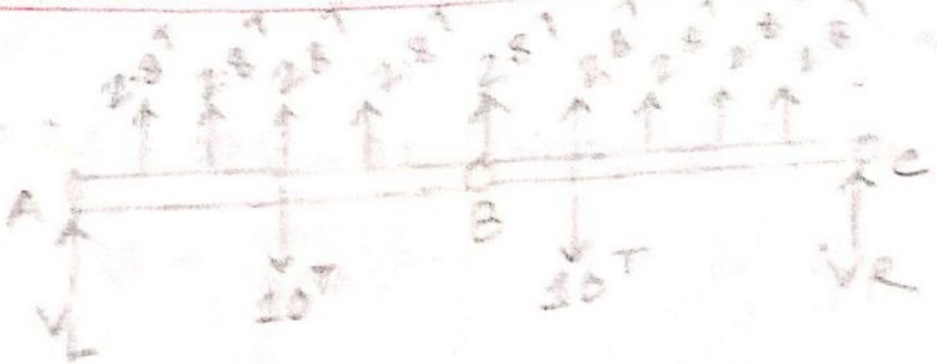
From G.C.T, $H = \frac{\omega L^2}{8h}$

$$\Rightarrow \omega = \frac{8Hh}{L^2} = \frac{8 \times 35 \times 100}{(1000)^2}$$

$$= 0.028 \text{ ton/foot}$$

Hanger Force = ω × spacing of hanger

$$= 0.028 \times 100$$
$$= 2.8^T$$



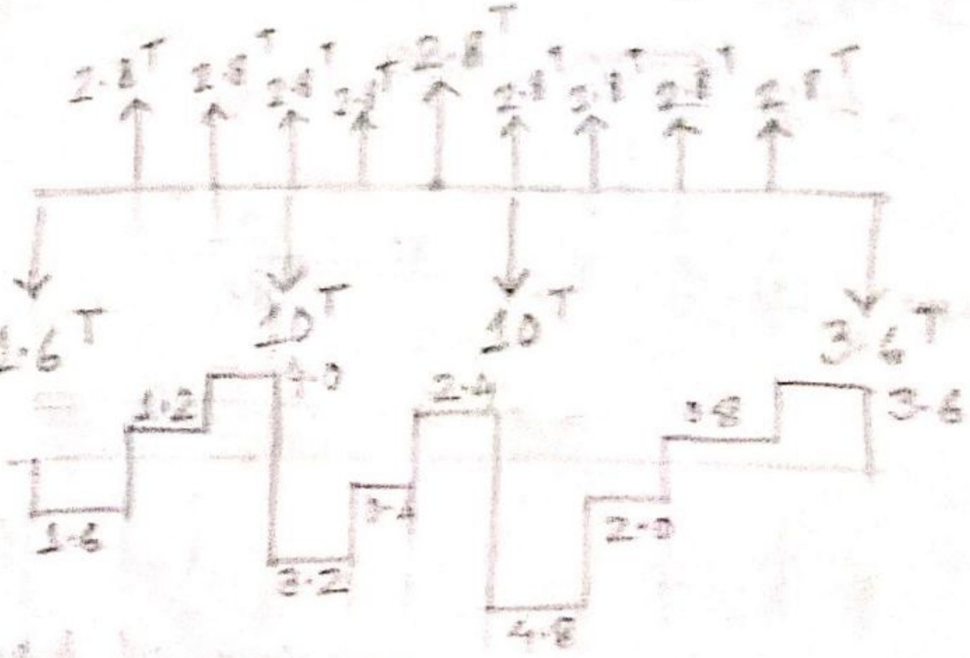
$\underline{\Sigma M\ @A = 0}$

$$-2.8(100+200+300+....+900)+10\times300$$
$$+10\times600-V_R\times1000=0$$

$$\Rightarrow V_R = -\,3.6^T\ (\downarrow)$$

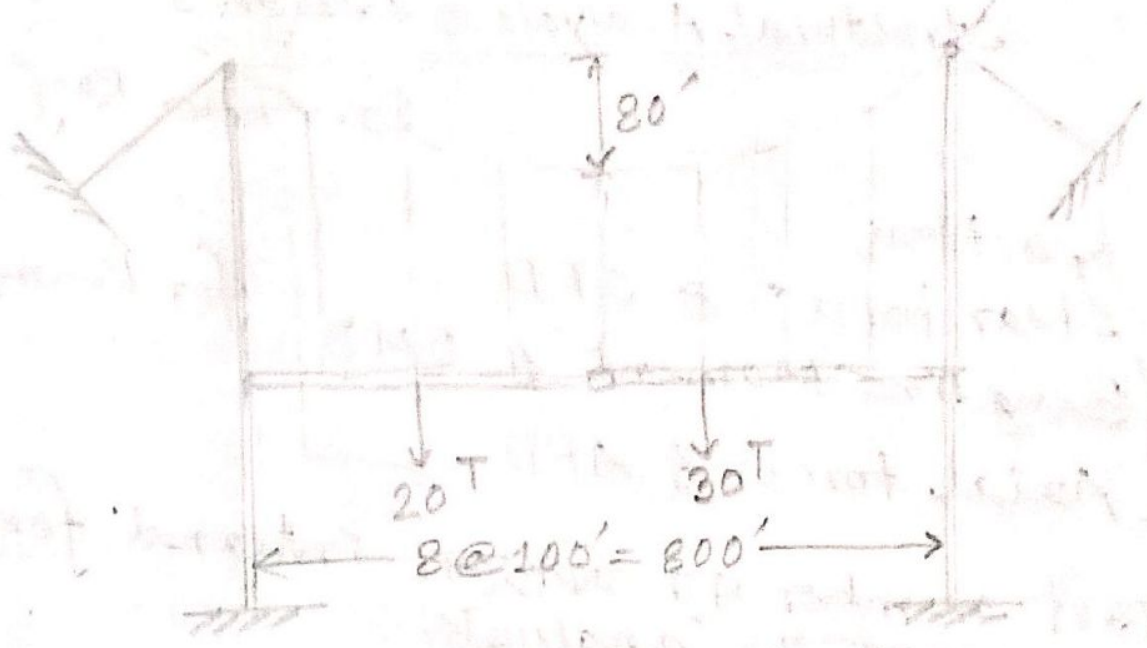
$$\underline{\Sigma F_y = 0} \Rightarrow (2.8\times9)-3.6-(10\times2)+V_L=0$$

$$\Rightarrow V_L = -1.6^T\ (\downarrow)$$



SFD

BMD



(a) Hanger Force

(b) Max^{m} cable Tension

(c) SFD for girder

(d) BMD for girder

1 March 2015

Zakaria Sir-2

Principle of Virtual Work

The external virtual work by a system of virtual $\mathcal{g}$ forces acting on deformation by real force is equal to the internal virtual work by the internal virtual $\mathcal{g}$ stresses on actual deformation.

$$W_s = W_d$$

Application - (1) to calculate deflection
(2) " " rotation etc.

P → real or Actual force system
$\mathcal{g}$ → virtual force system

deformation by actual force

$$W_s = \sum \mathcal{g}\,\delta$$

$$W_d = \underbrace{\int_0^L F_{\mathcal{g}}\, e_0\, ds}_{\text{axial force effect}} + \underbrace{\int_0^L \frac{M_{\mathcal{g}} M_P}{EI} ds}_{\text{bending effect}}$$

For truss member (member with axial force only)

coefficient of thermal expansi

member এর axial force

$$e_0 = \frac{F_P}{AE} + \alpha_t t$$

$$\left[\Delta L = \frac{PL}{AE} = \int \frac{F_P}{AE} ds\right]$$

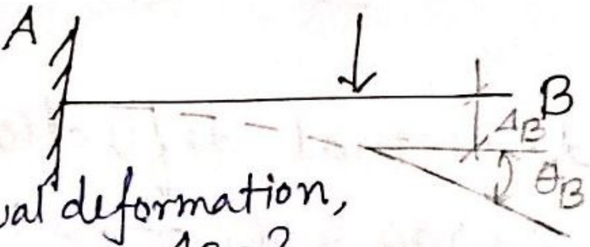
For Truss Structures

$$W_d = \sum \frac{F_Q F_P L}{AE} + \sum F_Q \alpha_t t L$$

যেই point-এ deflection চাচ্ছি সেই point-এ virtual force apply করতে হবে।

Q = unit force

এজন্য এই method কে unit force method ও বলা হয়।



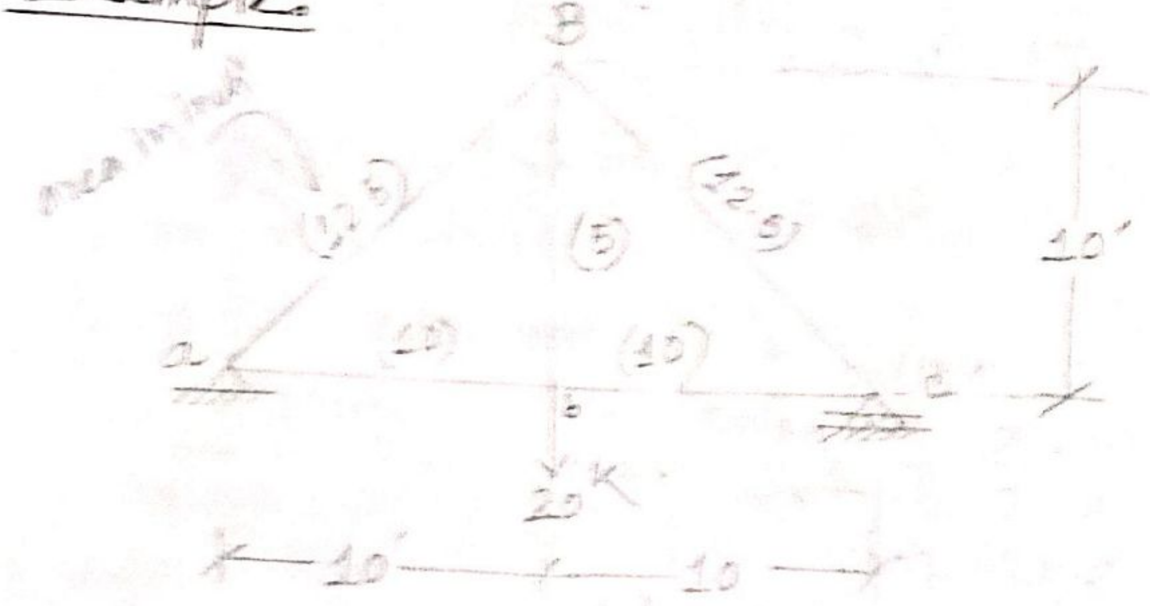
Virtual deformation, $\Delta_B = ?$

$\left\{ \begin{array}{l} \text{deflection} \rightarrow 1 \text{ unit force দিতে হবে।} \\ \text{rotation} \rightarrow 1 \text{ unit moment দিতে হবে।} \end{array} \right.$

Elementary Structural Analysis
– Norris, Wilbur, Utku

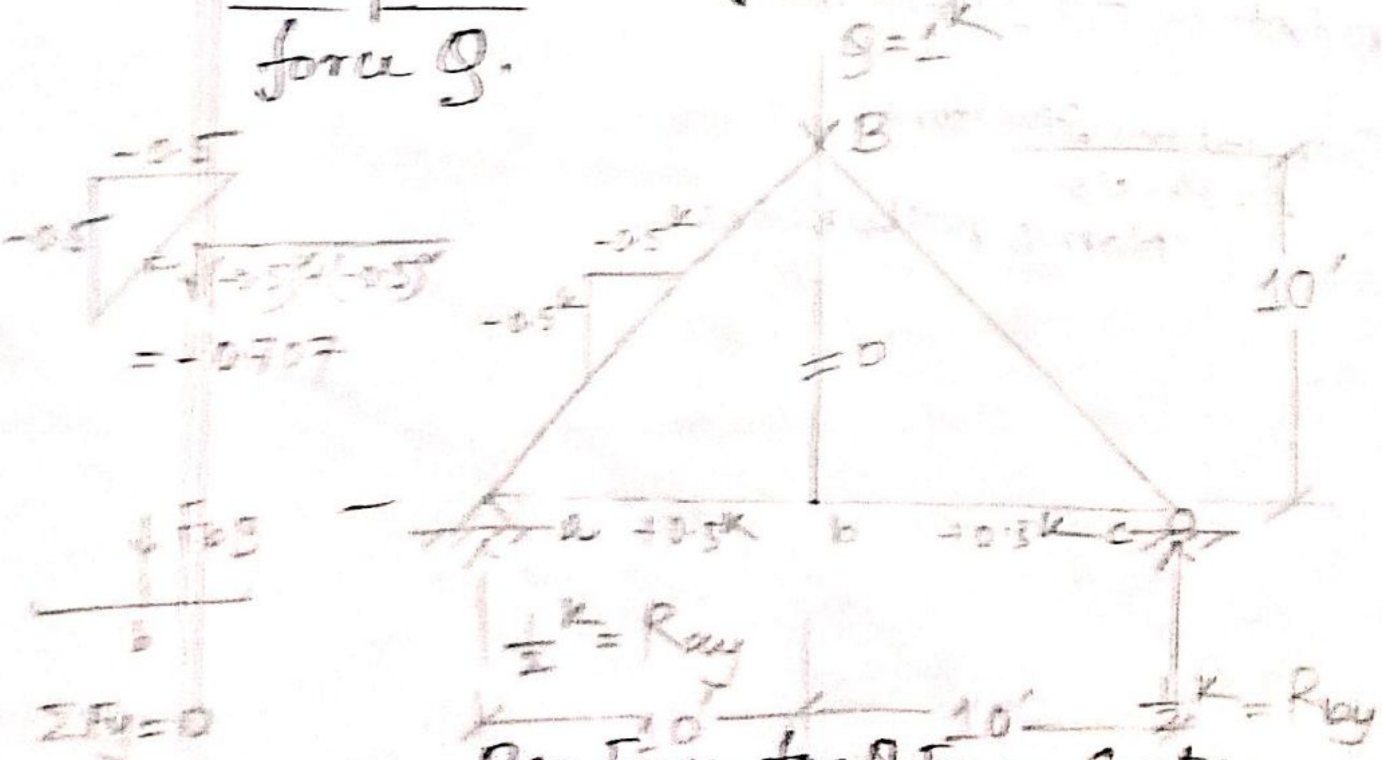
{ Solve: all problems in th
book (examples)

Example:



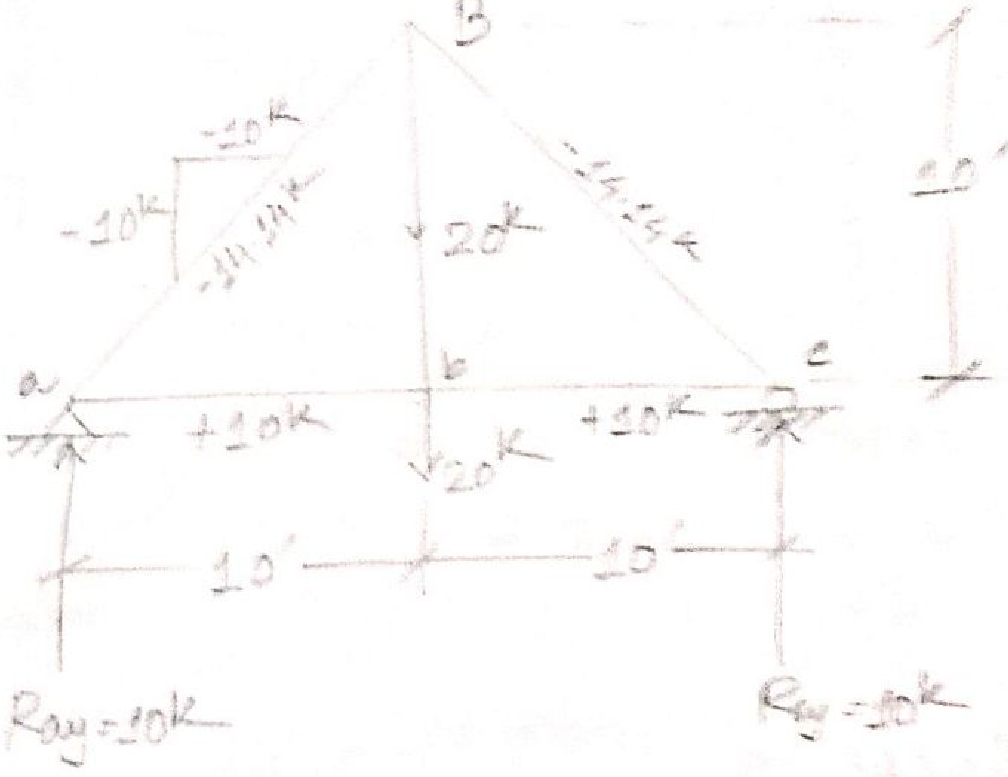
Find virtual deflection at B. Given E = 29000 ksi. Area in parentheses (inch²)

Step-1: Analyze Structure for Virtual force Q.

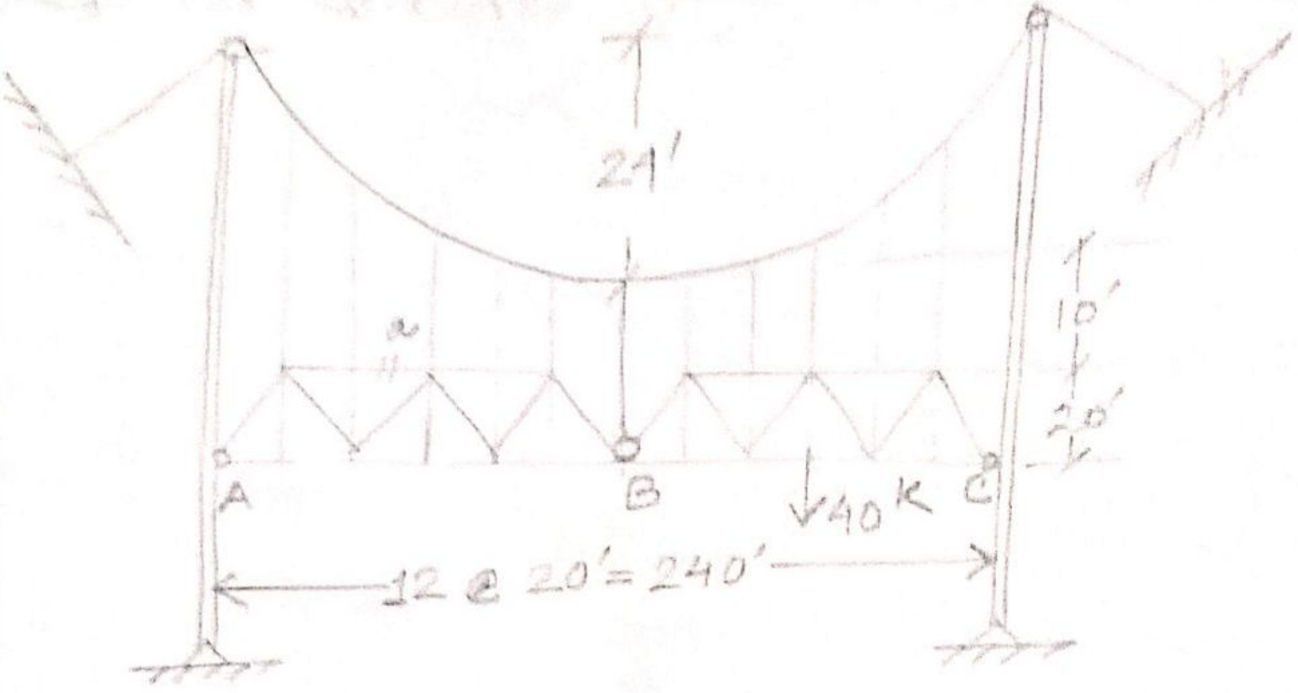


Bar Force for Q-Force System

Step-2: Analyze structure for Real force P.



Bar Force for P- Force System



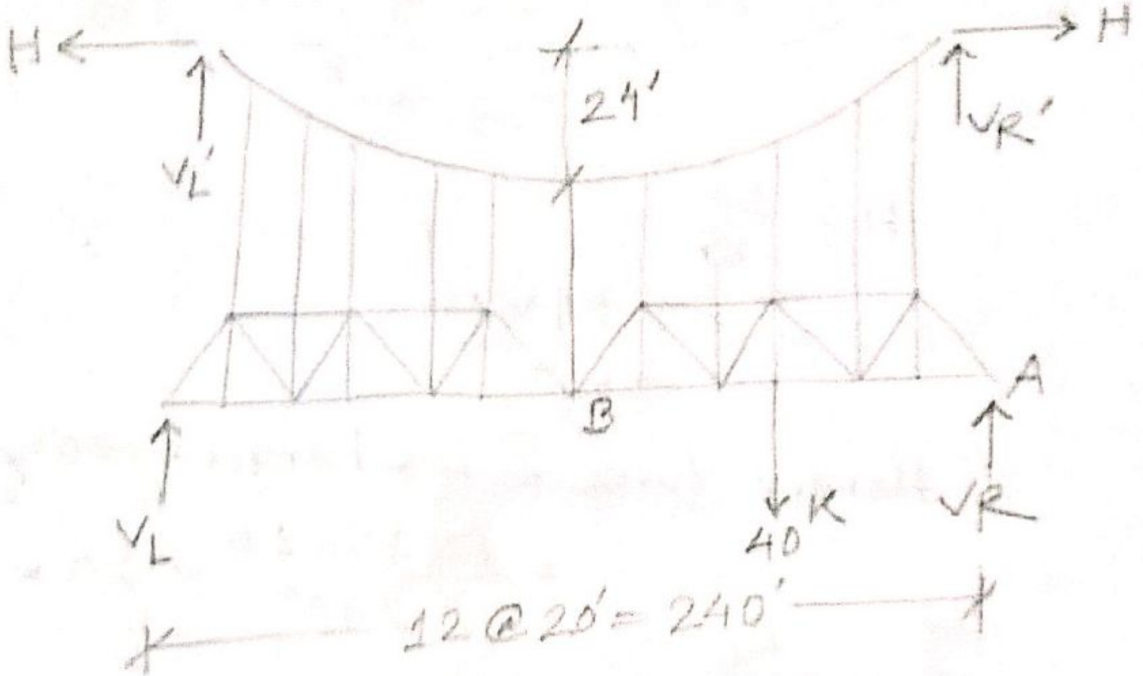
Question

(1) Hanger Force $= 3.33^K$

(2) $T_{max} = H\left[1+16\theta^2\right]^{1/2}$

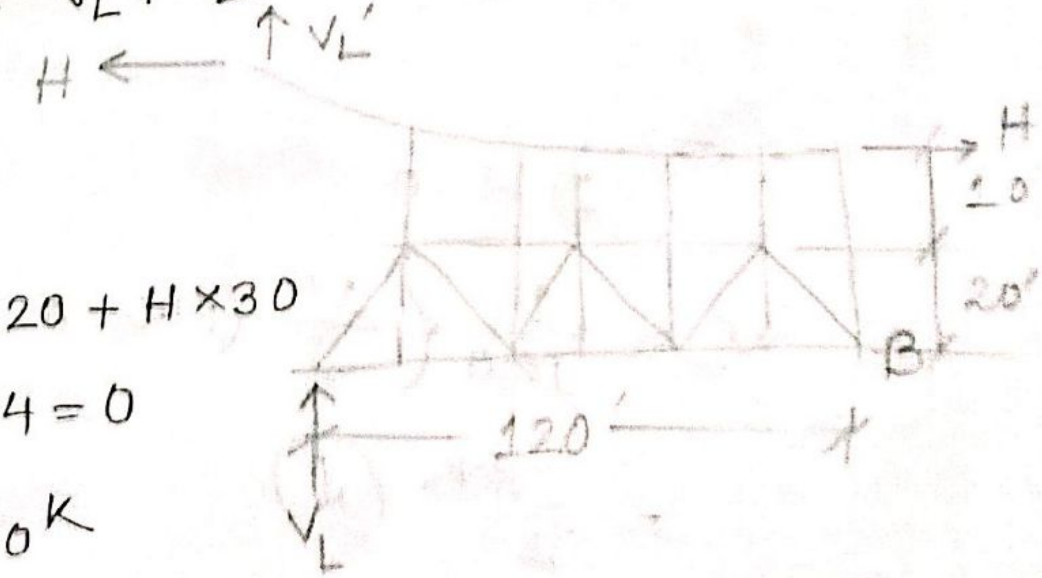
$= 53.85^K$

(c) Bass Force $Fa = 13.33^K (+)$



$$(V_L + V_L') \times 240 - 40 \times 60 = 0$$

$$\therefore\ V_L + V_L' = 10^K$$



$\Sigma M = 0$

$$(V_L + V_L') \times 120 + H \times 30 - H \times 54 = 0$$

$$\therefore\ H = 50^K$$

$$T_{max} = H\left(1+16\theta^2\right)^{\frac{1}{2}} = 50\left\{1+16\left(\frac{24}{240}\right)^2\right\}^{1/2}$$

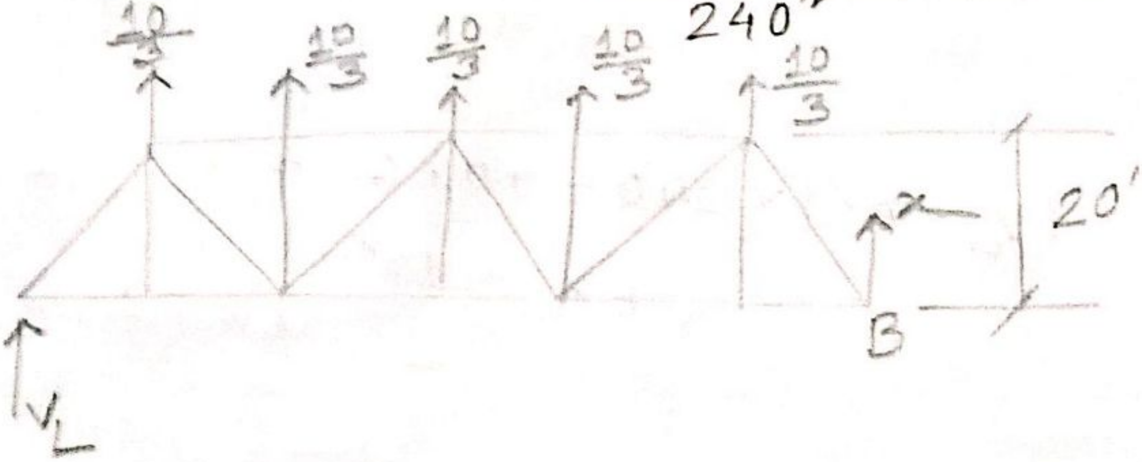
$$= 53.85^K$$

$$H = \frac{wL^2}{8h}$$

$$\Rightarrow w = \frac{8\times24\times50}{240^2}$$

$\therefore$ Hanger force = $w \times$ hanger spacing

$$= \frac{8\times24\times50}{240^2}\times20 = \frac{10}{3}^K$$

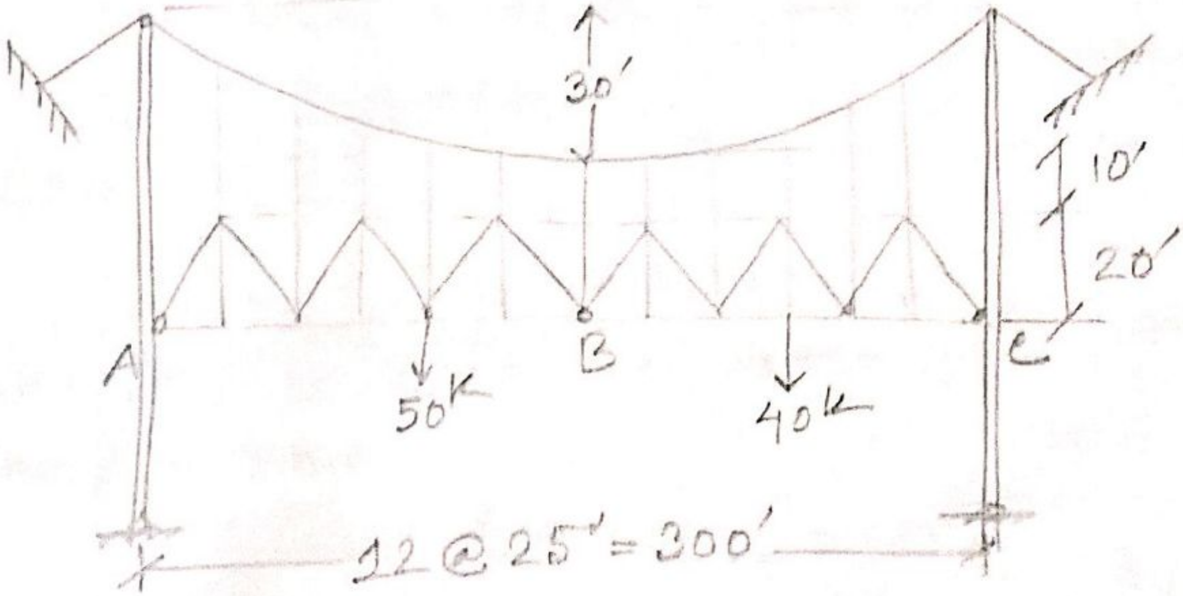


$$\Sigma M = 0$$

$$V_L = \frac{1}{120}\left\{\frac{10}{3}\cdot\left(100+80+60+40+20\right)\right\}$$

$$= \frac{25}{3}^K\ (\downarrow)$$

Assignment



Find: 1) Hanger Force
2) Tmax
3) Fa, Fb, Fc

Zakaria Sir-3

2 March 2015

	Bar	L (ft)	A (in²)	L/A	F_g (K)	F_p (K)	$F_g F_p \frac{L}{A}$	t	$F_g t L$
Horizontal Bar	ab	10	10	1	+0.5	+10	+5		
	bc	10	10	1	+0.5	+10	+5		
Diagonal Bar	aB	14.14	12.5	1.13	-0.707	-14.14	+11.3		
	cB	14.14	12.5	1.13	-0.707	-14.14	+11.3		
Vertical Bar	bB	10	5	2	0	+20	0		
							$\Sigma F_g F_p \frac{L}{A} = 32.6$		

AE = unit

$W \neq L/A$

$$W_d = \sum \frac{F_g F_p L}{AE} + \sum F_g \alpha_t t L$$

$$W_s = W_d$$

$$\Sigma g \delta_B = \Sigma \frac{F_g F_p L}{AE} + \Sigma \cancelto{0}{F_g} \alpha_t t L$$

$$\Rightarrow 1 \cdot \delta_B = \frac{32.6}{E} = \frac{32.6}{29000}$$

$$\therefore \delta_B = 0.00112 \text{ ft}$$

(downward)

Example 2:

Consider the problem in example 1 with 50°F temparature decrease in bottom chord member.

Given, $\alpha_t = \frac{1}{150,000}$ per°F

	Bar	L (ft)	A (in²)	$\frac{L}{A}$	F_Q (K)	F_P (K)	$F_Q F_P \frac{L}{A}$	t	$F_Q t L$
Horizontal Bar	ab	10	10	1	+0.5	+10	+5	-50	-25
	bc	10	10	1	+0.5	+10	+5	-50	-25
Diagonal Bar	aB								
	cB								
Vertical Bar	bB								

Using Principle of Virtual Work,

$$W_s = W_d$$

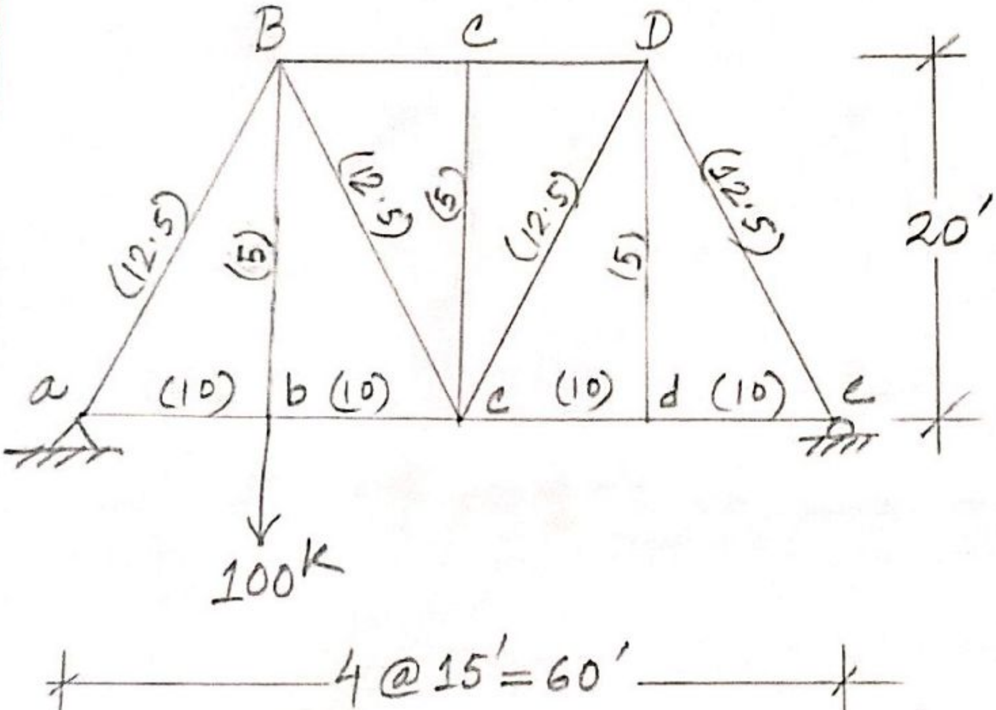
$$\Sigma Q \cdot \delta_B = \Sigma \frac{F_Q F_P L}{AE} + \Sigma \frac{F_Q \alpha_t t L}{}$$

$$= \frac{32.6}{E} + -500 \times \alpha_t$$

$$1 \cdot \delta_B = \frac{+32.6}{29000} - \frac{500 \times 1}{150000}$$

$$= -0.00221 \text{ ft (upward)}$$

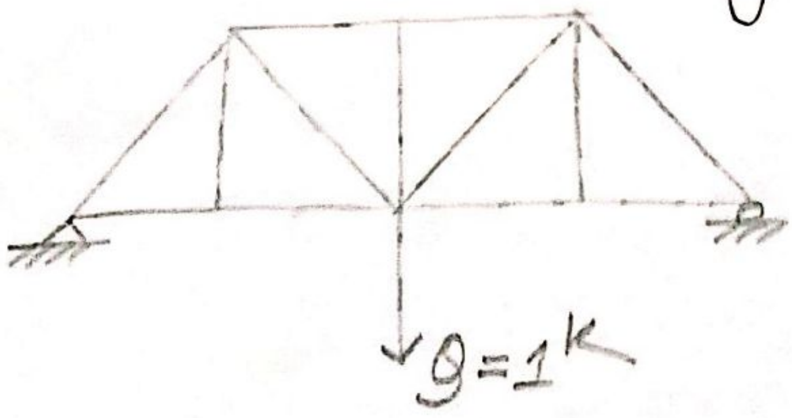
Example 8.1:



(a) Compute vertical deflection of joint c (bottom) chord. Given, E = 300,000 ksi

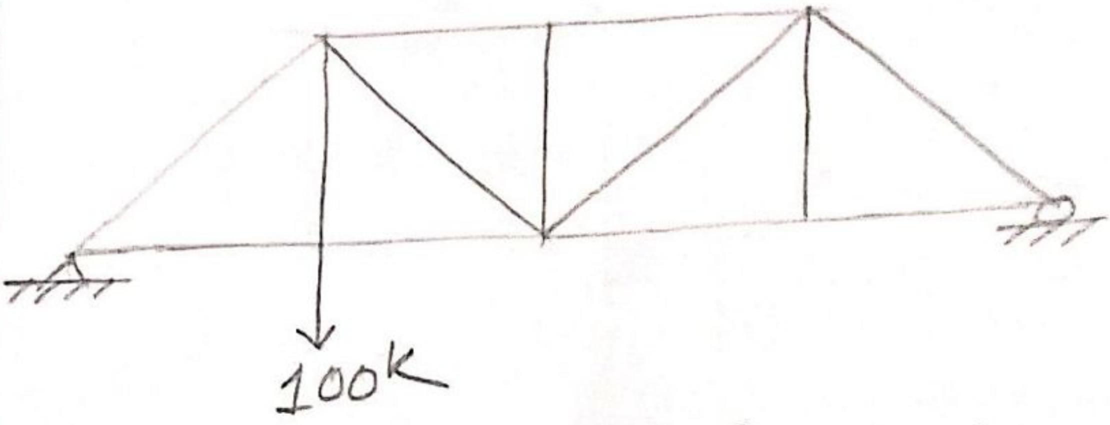
(b) Compute - - - - - . . . δ_c due to decrease in temparature of 50°F in the bottom chord only.

Step-1:



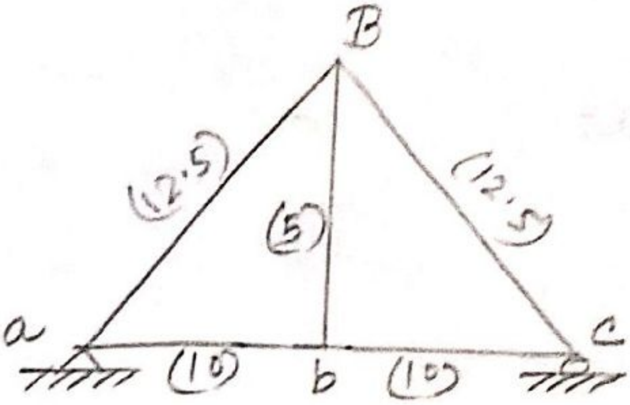
Bar Force for Q - Force system

Step - 2:



Bar Force for P-force system

Assignment 3

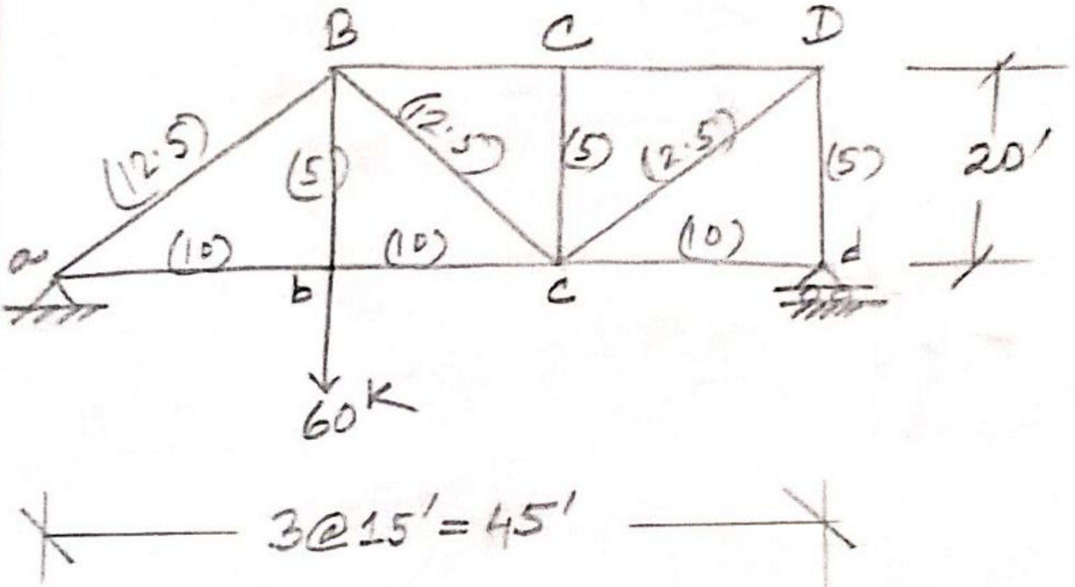


Temp. increase in Diagonal member = 100°F
Temp. decrease " horizontal " = 60°F

Compute vertical deflection at Joint 'b' of bottom chord.

$$\alpha_t = \frac{1}{150,000} \text{ per } ^\circ F$$

Assignment 4



(a) Compute vertical deflection of joint c (bottom chord) for the load shown and due to temp increase of 60°F in bottom chord & vertical members

$$E = 30000$$

$$\alpha_t = \frac{1}{150,000} \text{ per } {}^\circ F$$

Zakaria Sir - 4

8 March 2015

Truss with Support Movement

Principle of Work

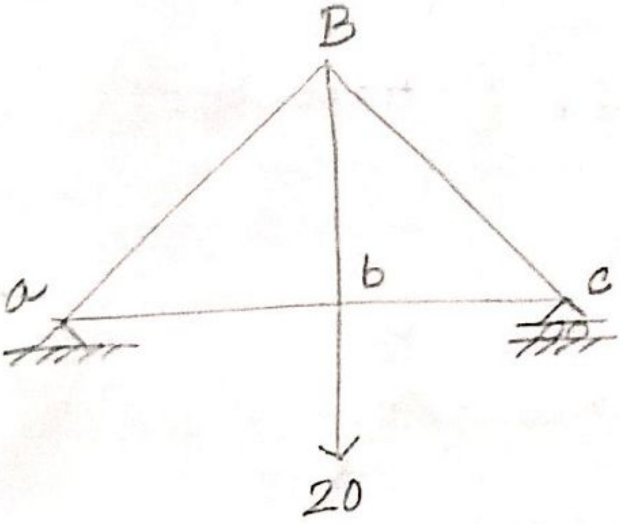
External Virtual Work by Virtual Reaction

$$\Sigma Q\delta + W_R = \Sigma \frac{F_Q F_P L}{AE} + \Sigma F_Q \cancel{F} \alpha_t t L$$

ΔL for F_P

ΔL for temp change

Problem



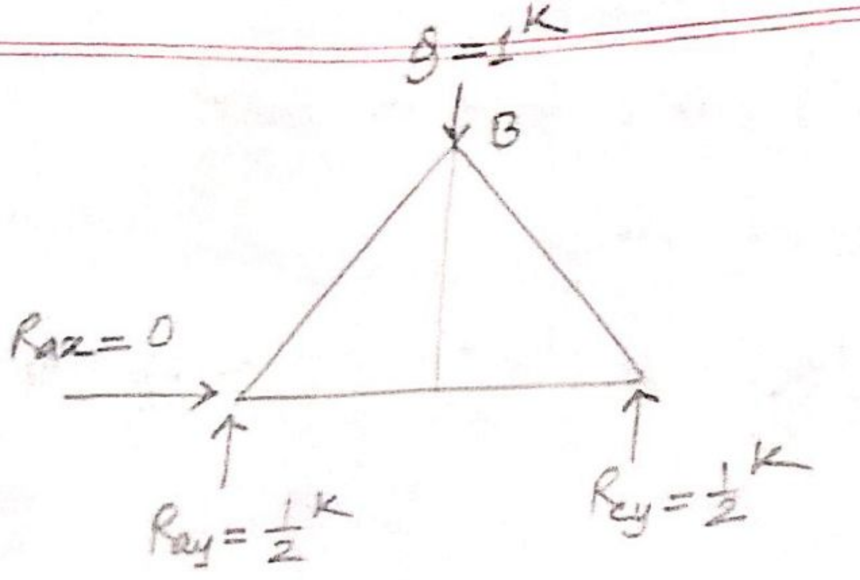
In addition to 20" load at 'b'.

At support "a" horizontal moment = 0.5" to left

" " " vertical " = 0.75" down

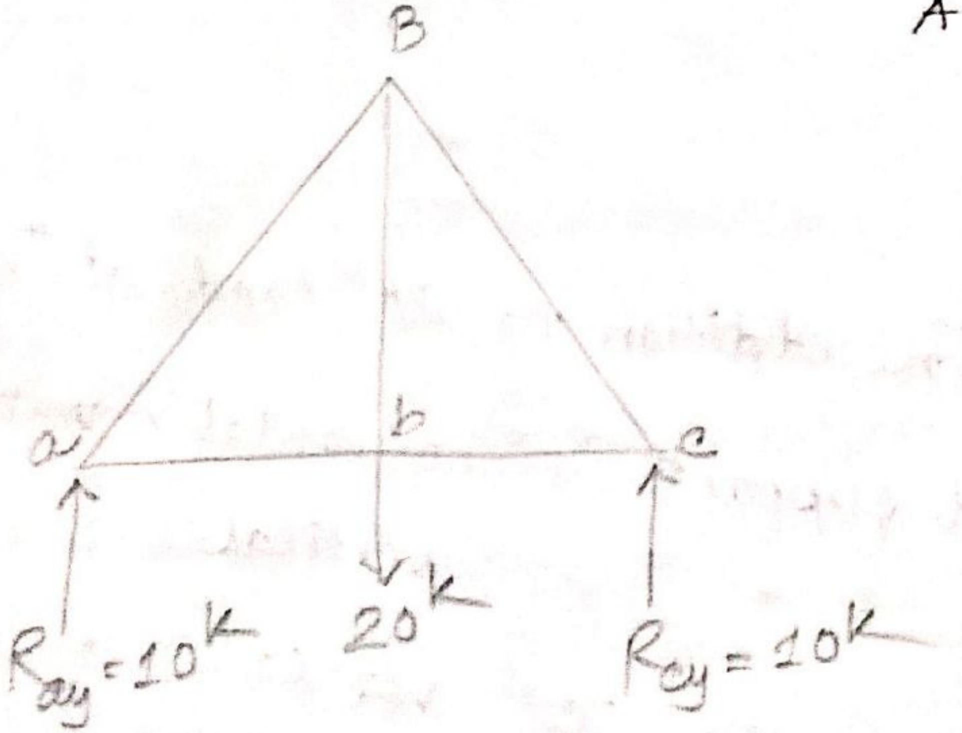
" " "c" vertical " = 0.3" down

Calculate vertical deflection at B (Top chord)



Bar Force for Q - Force system

$$\frac{\sum F_Q F_P L}{A} = 32.6$$



Bar Force for P - force system

Using principle of virtual work. (No temp change)

$$\sum 1\cdot\delta_B + W_R = \sum \frac{F_\alpha F_p L}{AE} + \sum F_\delta \alpha_t \, tL$$

$$\Rightarrow \delta_B + R_{ax}\Delta_{ax} + R_{ay}\Delta_{ay} + R_{cy}\Delta_{cy} = \frac{32.6}{E}$$

$$\Rightarrow \delta_B + 0 \times \frac{0.5}{12} + \frac{1}{2} \times \left(-\frac{0.75}{12}\right) + \frac{1}{2} \times \left(-\frac{0.3}{12}\right) = \frac{32.6}{29000}$$

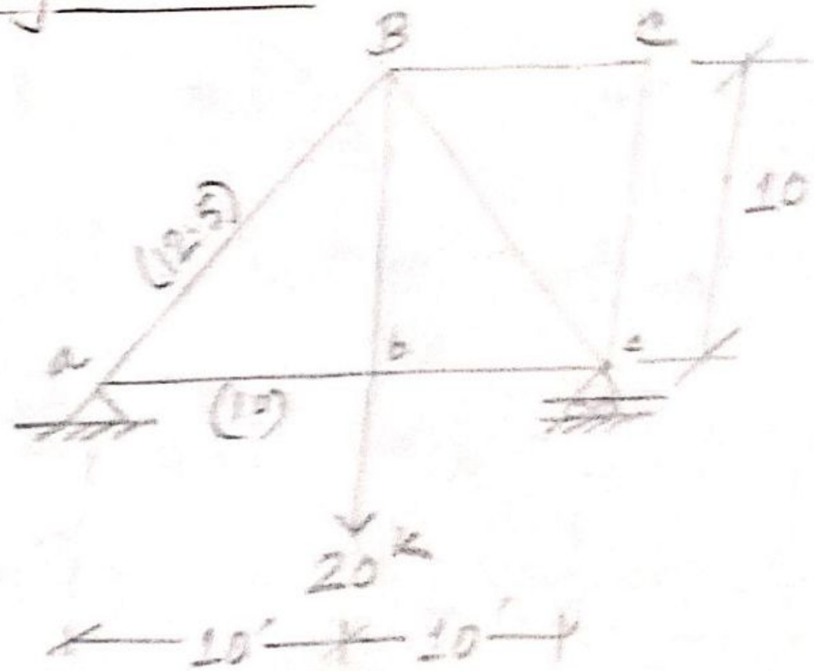
Reaction হয়েছিল movement এর opposite direction এ হলে তাই (-)ve হবে।

$$\delta_B = 0.00112' + 0.04375'$$

<u>Sign convention</u> { movement এর dir^n along reaction → (+)v
" " " opposite of " → (-)v

See Example Problems { 8.1
8.2
8.3
8.4

Assignment 5:



Load case #1 In addition to 20k load at B

Load case #2 For support moment

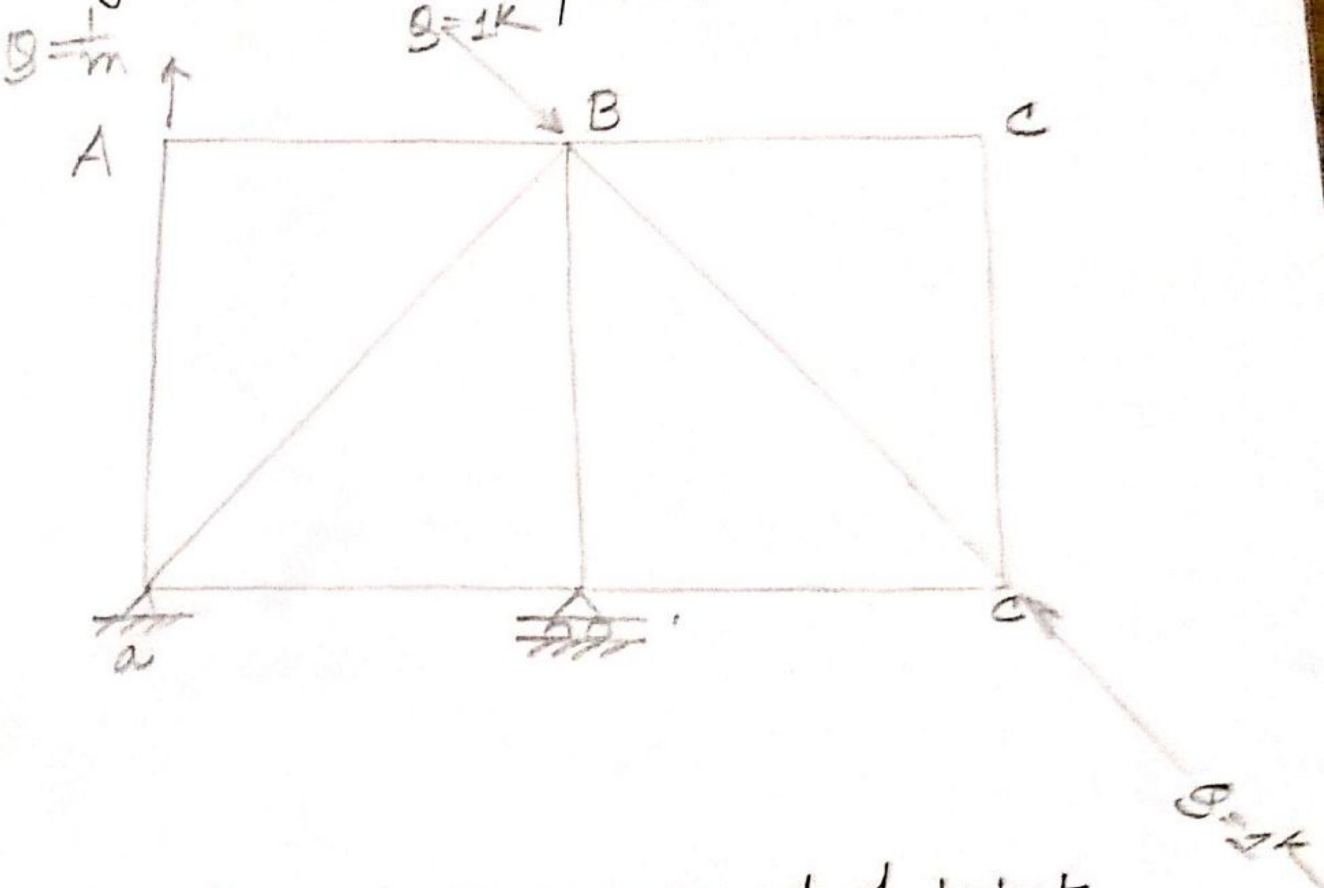
At support "a" horizontal moment = 0.5" to Left
" " " vertical " = 0.75" down
" " "c" " " = 0.3" down

Load case #3 50°F temp rise in bottom chord members.

Given, $g_t = \dfrac{1}{150,000}$ per °F

Find horizontal deflection at joint 'C' of top chord.

Fig 8.10 → Important



1) What is the relative movement of joint C & B?

2) What is the rotation of member AB?

$$\sum \overset{1K}{\cancel{\beta}}\ \delta_{C-B}$$

Zakaria Sir-5 15 March 2015

Deflection, Rotation for Beams & Frames

By virtual work

(If support movement)

$$\Sigma Q\delta + W_R = \Sigma \frac{F_Q F_P L}{AE} + \int \frac{M_Q M_P \, ds}{EI} + \Sigma F_Q \alpha_t t L$$

support movement না থাকলে এই term zero.

(for temparature change)

For beams with no axial force & temparature change and support movement,

$$\Sigma Q\delta = \int \frac{M_Q M_P}{EI} ds$$

real force এর জন্য moment.

virtual force এর জন্য moment

Book Example for Truss

Example 8.1

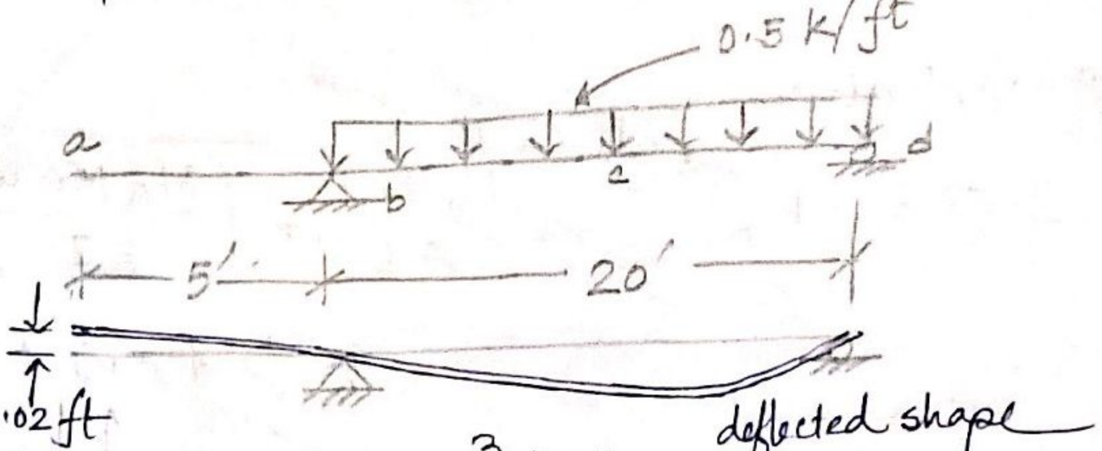
8.2

8.3

8.4

(very important for exam)

Example 8.5:

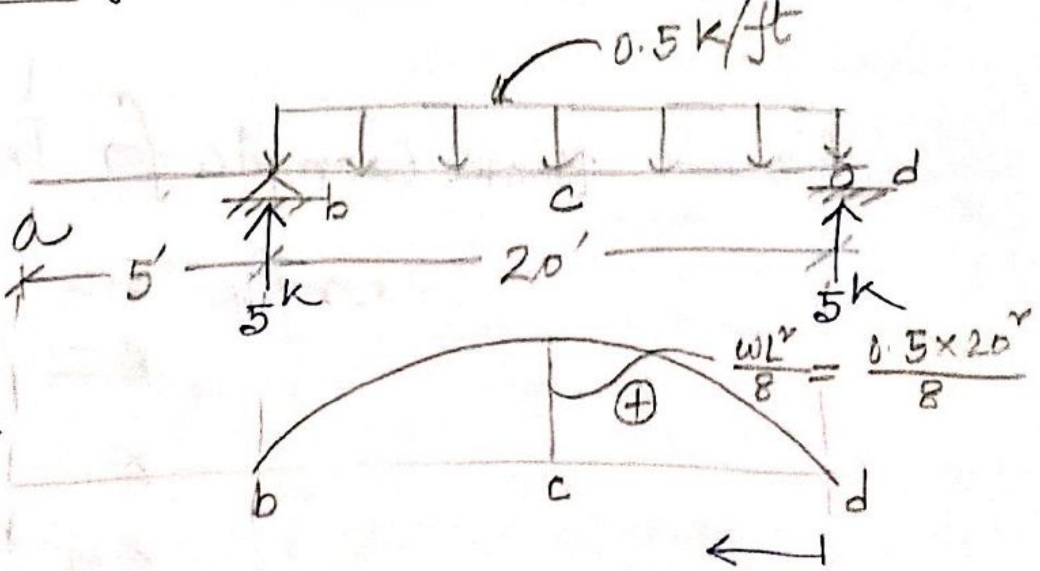


Given, $E = 30 \times 10^3$ ksi

$I = 200 \text{ in}^4$

Mp, MB একই সেকশনে এবং আলাদা করে হবে। Compute vertical deflection at point a

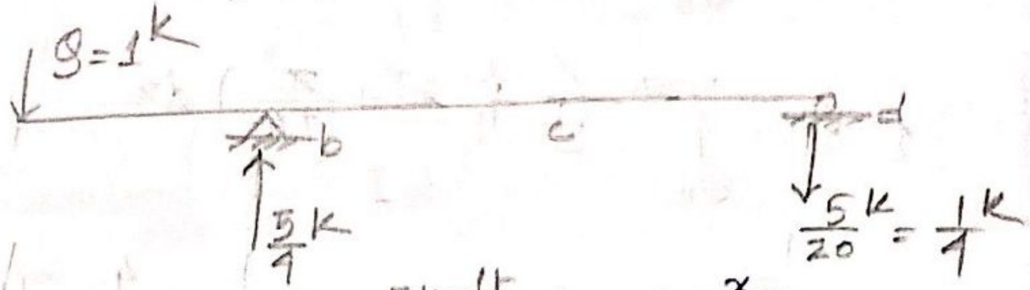
Solve for P-Force



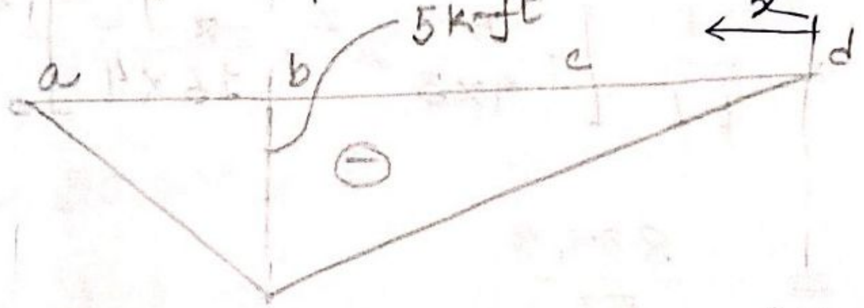
Mp diagram

{ Why from 'd' to 'b'?
{ For converience of formulating the equation

Solve for Q- force



Mq diagram



By Virtual Work,

$$\Sigma Q\delta = \cancelto{0}{\int_a^b \frac{M_Q M_P}{EI}dx} + \int_d^b \frac{M_Q M_P}{EI}dx \quad (\because M_P = 0)$$

For segment 'ab' $(a < x \le b)$ $\quad 0 < x \le 5'$

$$M_P = 0$$
$$M_Q = 1x$$

For segment 'db' $(d < x \le b)$ $\quad 0 < x \le 20'$

$$M_P = 5x - 0.5x \times \frac{x}{2} = 5x - \frac{x^2}{4}$$

$$M_Q = -\frac{1}{4}x$$

$$\Sigma q\cdot\delta = \int_0^{20} \frac{-\frac{1}{4}x\left(5x-\frac{x^2}{4}\right)dx}{EI}$$

$$1\cdot\delta = \int_0^{20} \frac{\left(-\frac{5}{4}x^2+\frac{x^3}{16}\right)dx}{EI}$$

$$= \frac{1}{EI}\left[-\frac{5}{4\times3}x^3+\frac{x^4}{16\times4}\right]_0^{20}$$

$$= -\frac{833.3}{EI}$$

$E = 30{,}000$ ksi

$= 30{,}000 \times \boxed{144}$ ksf $\rightarrow (12)^2$

$I = 200$ in^4

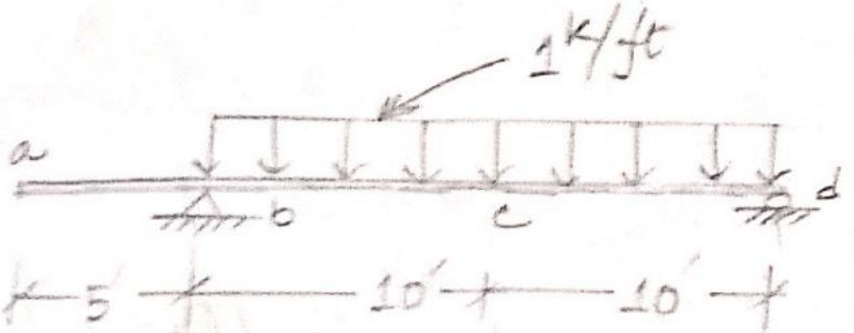
$= \dfrac{200}{\boxed{144\times144}}$ ft^4 $\rightarrow (12)^4$

$$= -\frac{833.3}{30{,}000\times144\times\frac{200}{144\times144}}$$

$= -0.02$ ft (upward)

↑ এই দুটো উল্লেখ করতেই হবে, ↑

Assignment #6



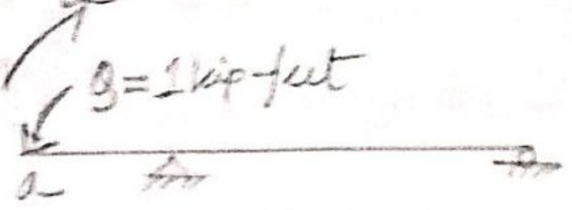
$E = 30,000$ ksi

$I = 300$ in^4.

Compute:

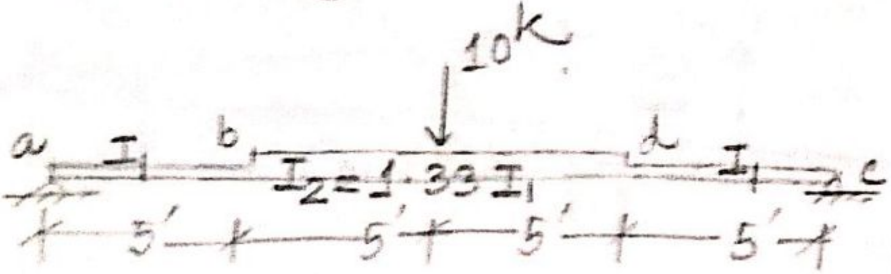
(a) vertical deflection at "c" (b to c & d to c)

(b) rotation or slope at point "a".



* See example 8.6

θ_a এর answer আসবে radians

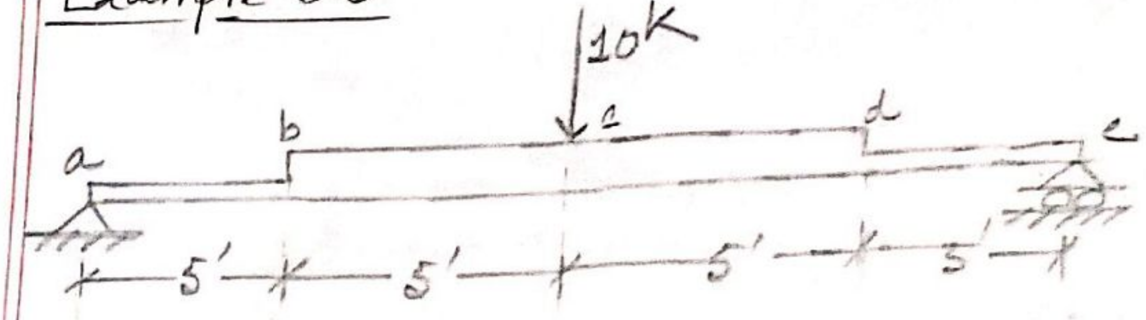


Given, $E = 30,000$ ksi

$I_1 = 150$ in^4

Compute change in slope at point 'a'

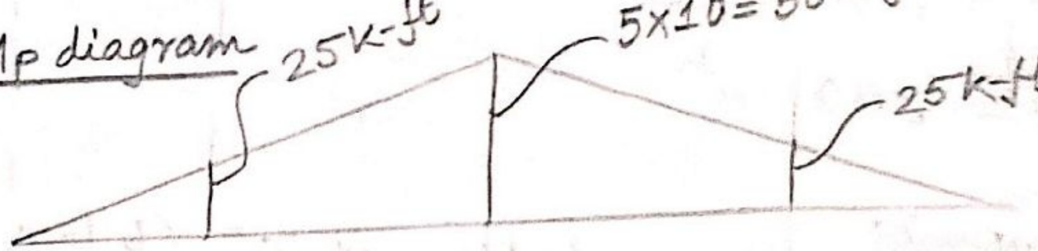
Example 8.6

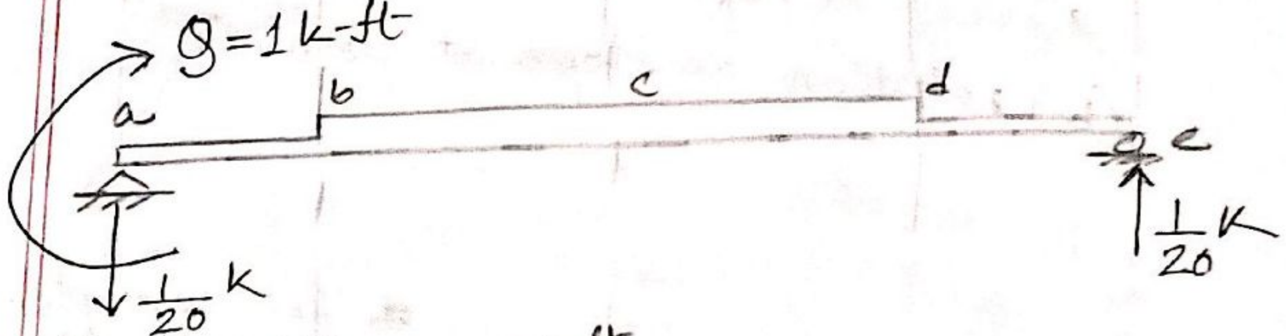


Find change in slope 'a' (θa=?)

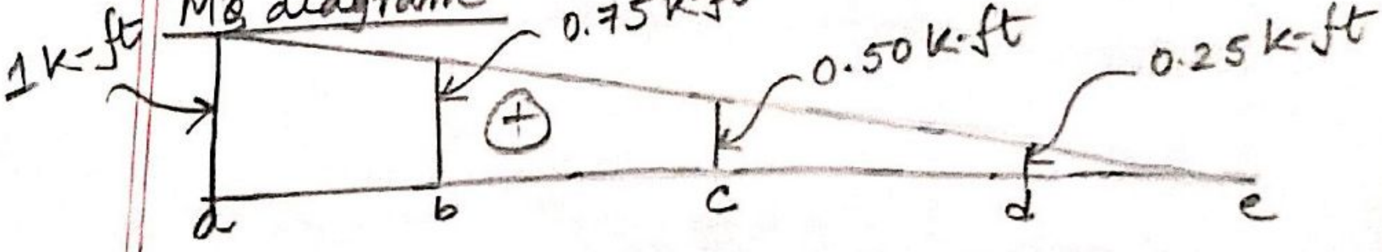
Solve for P-Force

Mp diagram





Mq diagram



By Virtual Work Method,

θa, change in slope

$$\sum Q\,\theta_a = \int_a^b \frac{M_Q M_P}{EI_1}\,dx + \int_b^c \frac{1}{EI_2}\,dx + \int_d^c \frac{1}{EI_2}\,dx + \int_e^d \frac{1}{EI_1}\,dx$$

Segment 'ab' $(0 < x \le 5)$

$M_P = +5x$

$M_Q = +\left(1 - \frac{1}{20}x\right)$

Segment 'bc' $(0 < x \le 5)$

$M_P = +(25 + 5x)$

$M_Q = +\left(0.75 - \frac{1}{20}x\right)$

Segment 'dc' $(0 < x \le 5)$

$M_P = +(25 + 5x)$

$M_Q = +\left(0.25 + \frac{1}{20}x\right)$

Segment 'ed' $(0 < x \le 5)$

$M_P = +5x$

$M_Q = +\frac{1}{20}x$

$$\Sigma g\,\theta_a = \int_a^b \frac{M_Q M_P}{EI_1}dx + \int_b^c \frac{1}{EI_2}dx$$

$$+ \int_d^c \frac{1}{EI_2}dx + \int_c^d \frac{1}{EI_1}dx$$

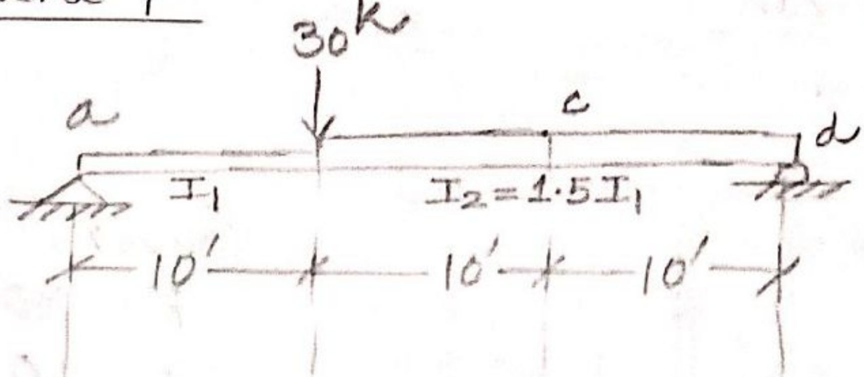
$$= \int_0^5 \frac{(5x)\left(1-\frac{x}{20}\right)dx}{EI_1} + \int_0^5 \frac{\qquad}{EI_2}dx$$

$$+ \int_0^5 \frac{\qquad}{EI_2}dx$$

$$+ \int_0^5 \frac{\qquad}{EI_1}dx$$

$\Rightarrow 1\cdot\theta_a = +0.0065$ radian (clockwise)

Assignment 7



$E = 30,000$ ksi

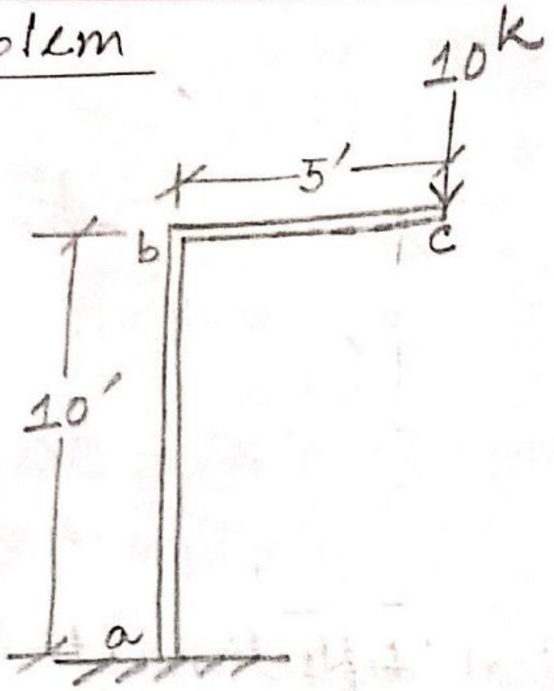
$I_1 = 400$ in^4

Compute (a) Vertical deflection at "c"

(b) Vertical deflection at "b"

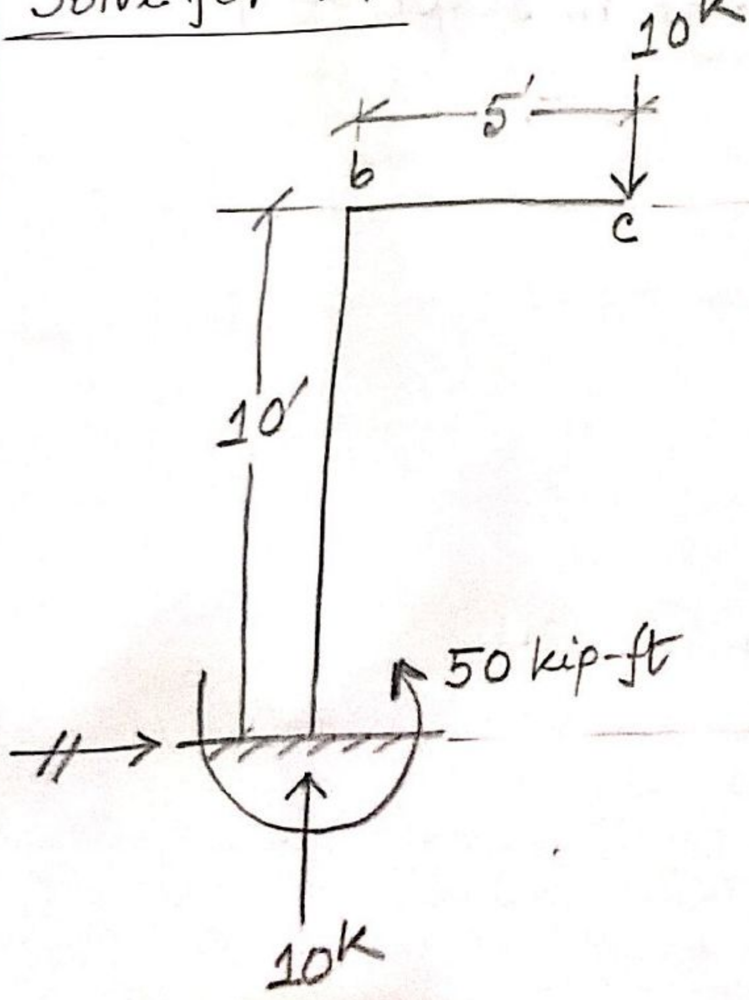
(c) change in slope at point "d"

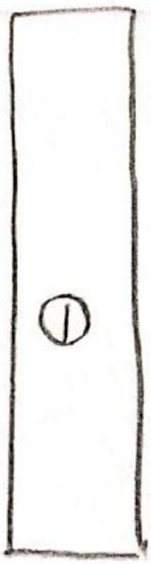
Problem



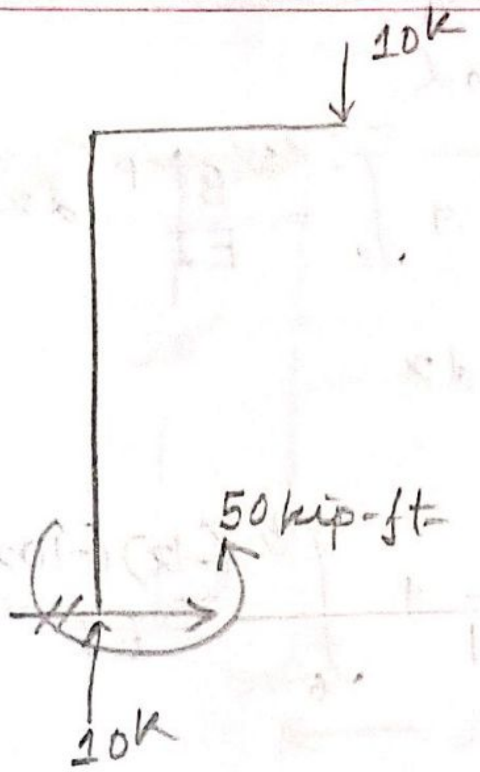
Given,
$E = 30,000$ ksi
$A = 10$ in^2
$I = 200$ in^4

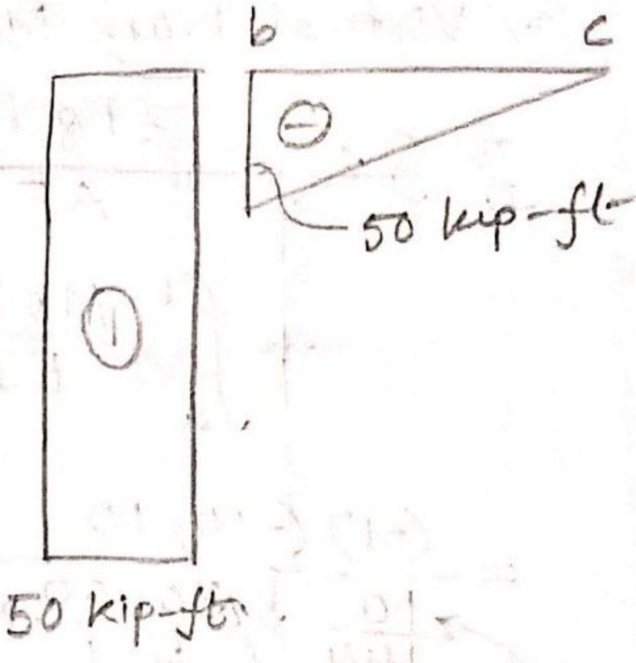
Solve for P:





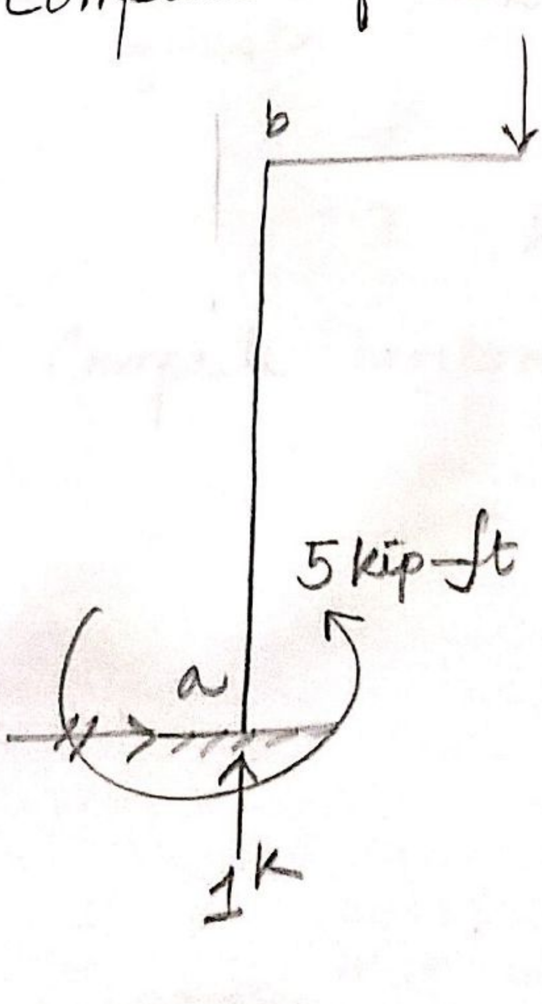
Axial Force Diagram
For P Force





Mp - diagram

Compute deflection at c.



over number of members

By Virtual Work Method,

$$\overset{\text{over number of members}}{\sum} g\,\delta_c = \frac{\sum F_g F_p L}{AE} + \int_c^b \frac{M_g M_p}{EI}\,dx$$

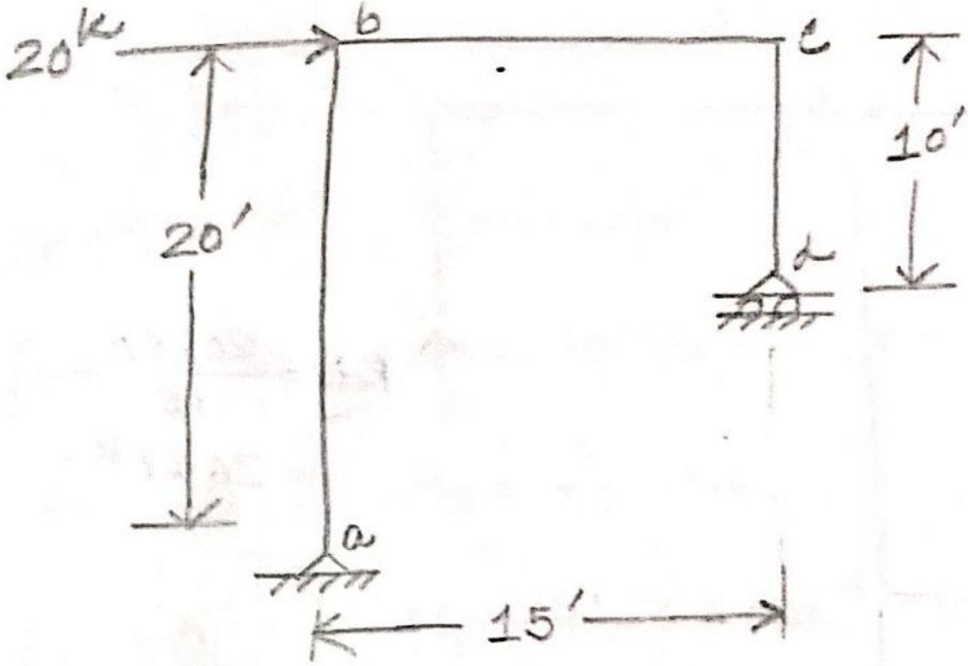
$$+ \int_a^b \frac{M_g M_P}{EI}\,dx$$

$$= \frac{(-1)(-10)10}{\underbrace{\frac{10}{144}}_{A} \times \underbrace{\frac{30000}{\cancel{144}}\times 144}_{E}} + \int_0^5 \frac{(-1x)(-10x)}{EI}\,dx$$

$$+ \int_0^{10} \frac{(-5)(-50)}{EI}\,dx \leftarrow \text{this expression may be wrong}$$

Zakaria Sir-7

18 March 2015



Problem

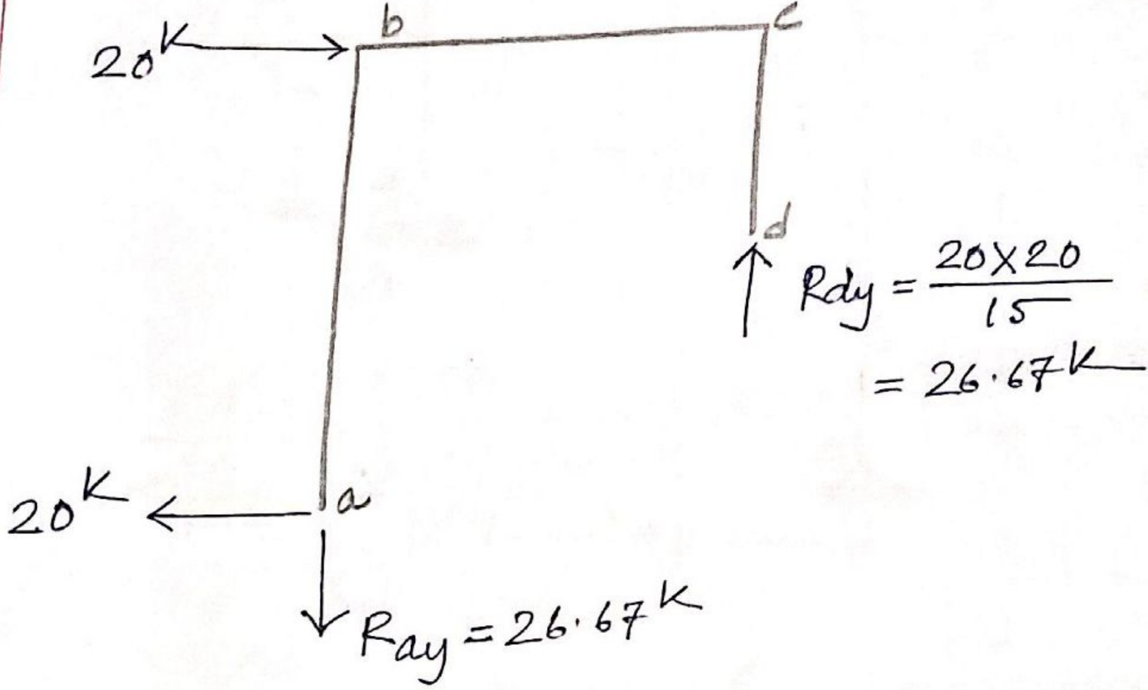
Given, $E = 30,000$ ksi

$A = 10 in^2$

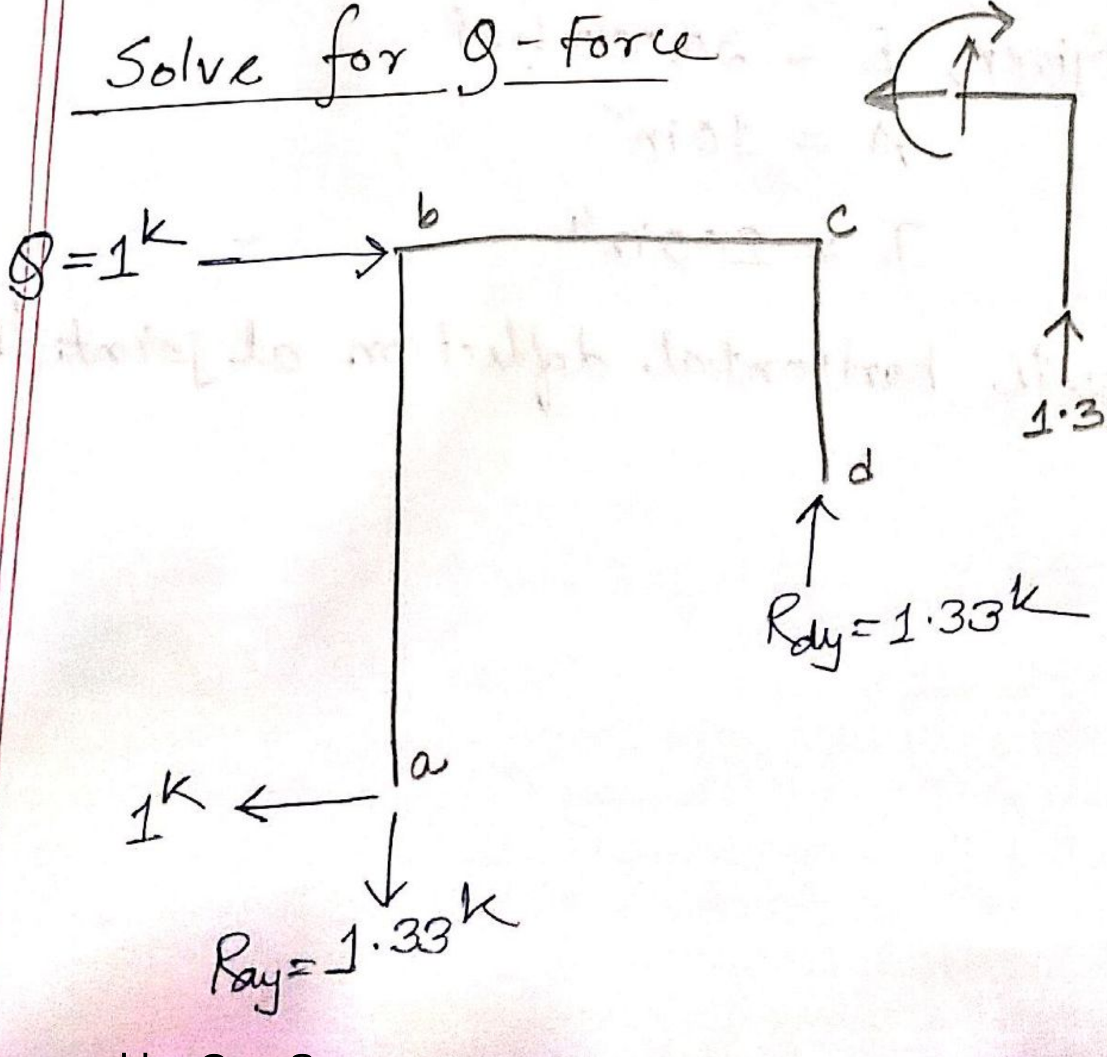
$I = 200 in^4$

Compute horizontal deflection at joint 'b'

Solve for P-Force



Solve for Q-force



From a to b (segment ab) $0 < y \leq 20'$

tension

$F_Q = +1.33^k$ | $M_Q = +1y$

$F_p = +26.67^k$ | $M_p = +20y$

From c to b (segment cb) $0 < x \leq 15'$

$F_Q = 0$ | $M_Q = +1.33x$

$F_p = 0$ | $M_p = +26.67x$

From d to c $(0 < y \leq 10')$

$F_Q = -1.33^k$ | $M_Q = 0$

$F_p = -26.67^k$ | $M_p = 0$

By Virtual Work Method,

$$\sum Q \cdot \delta_{hb} = \sum \frac{F_Q F_p L}{AE} + \int_a^b \frac{M_Q M_p}{EI}\, dy$$

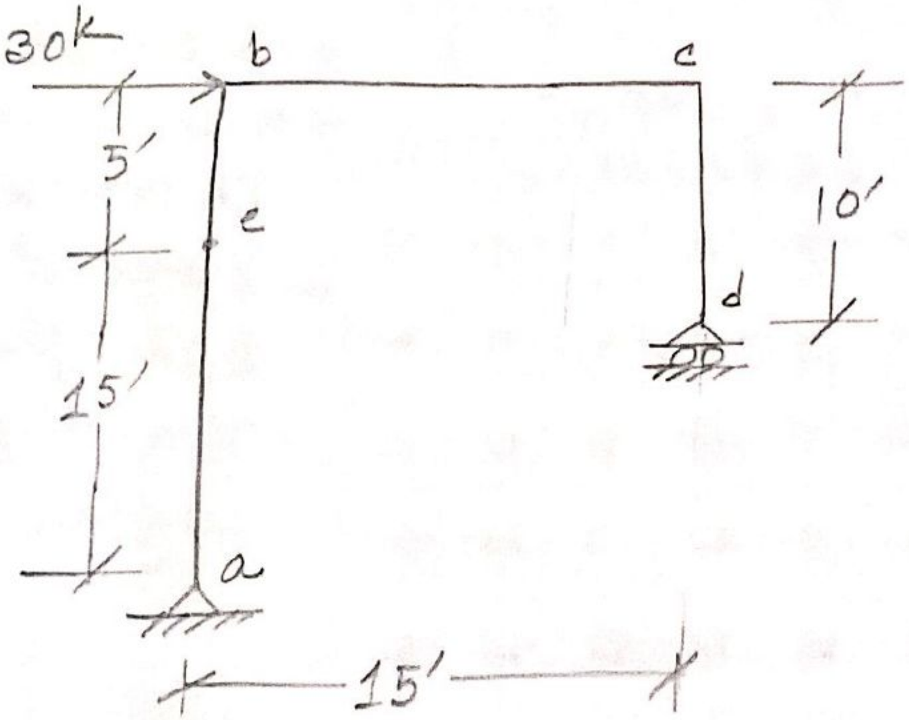
$$+ \int_c^b + \int_d^c = 0$$

$$\Rightarrow 1 \cdot \delta_{hb} = \frac{(+1.33)(26.67)\,\overset{L}{20}}{\underset{A}{\left(\frac{10}{144}\right)}(30{,}000 \times 144)} + \text{for bc moment} \rightarrow 0$$

$$+ \frac{(-1.33)(-26.67) \times 10}{\left(\frac{10}{144}\right)(30{,}000 \times 144)},$$

$$+ \int_0^{20} \frac{(y)(20y)}{EI}\,dy + \int_0^{15} \frac{(1.33x)(26.67x)}{EI}\,dx + 0$$

Assignment#8



$E = 30000\ ksi$

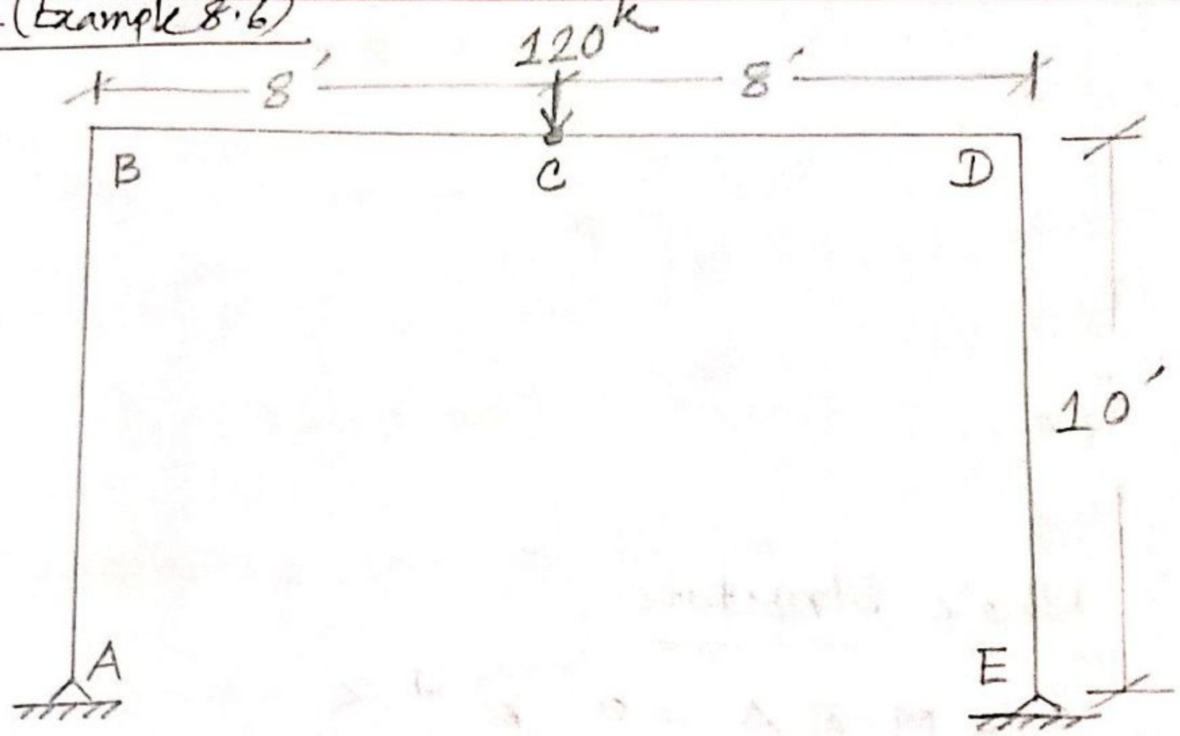
$A = 10\ in^2$

$I = 300\ in^4$

Compute horizontal deflection at point 'c'

Zakaria Sir - 8 22 March 2015

Book (Example 8.6)

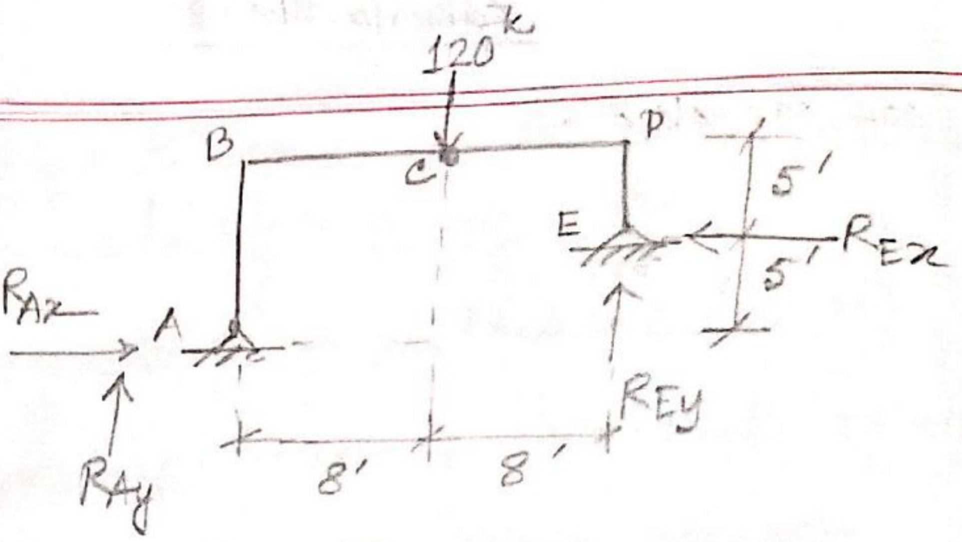


Given, $E = 30{,}000\,ksi$

$A = 20\,in^2$

$I = 2500\,in^4$

Compute the change in slope of the cross-section on the left side of the hinge 'C'.



Unknown

R_{Ax}
R_{Ay}
R_{Ex}
R_{Ey}

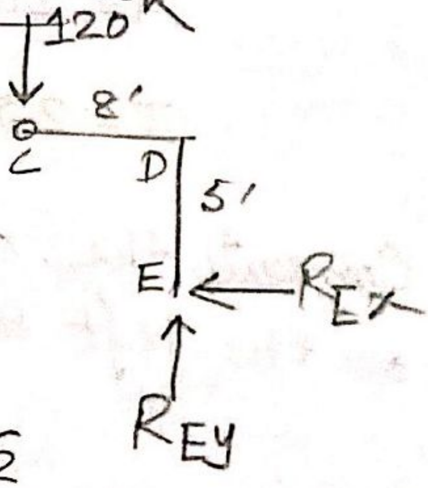
Whole Structure

$\Sigma M @ A = 0$ ↺ +ve

$\Rightarrow 120 \times 8 - R_{EX} \times 5 - R_{EY} \times 16 = 0$

$\Rightarrow 5R_{EX} + 16R_{EY} = 960$ ---(1

Right Part of Structure/Frame



$\Sigma M @ C = 0$ ↺ +ve

$R_{Ex} \times 5 - R_{Ey} \times 8 = 0$

$\Rightarrow 5_{EX} - 8 R_{EY} = 0$ ---(2

Solving equations 1) & 2),

$① - ② \Rightarrow 24R_{Ey} = 960$

$\therefore R_{Ey} = 40^{k}$

$R_{Ay} = 80^{k}$

Substituting value of R_{Ey} in 2),

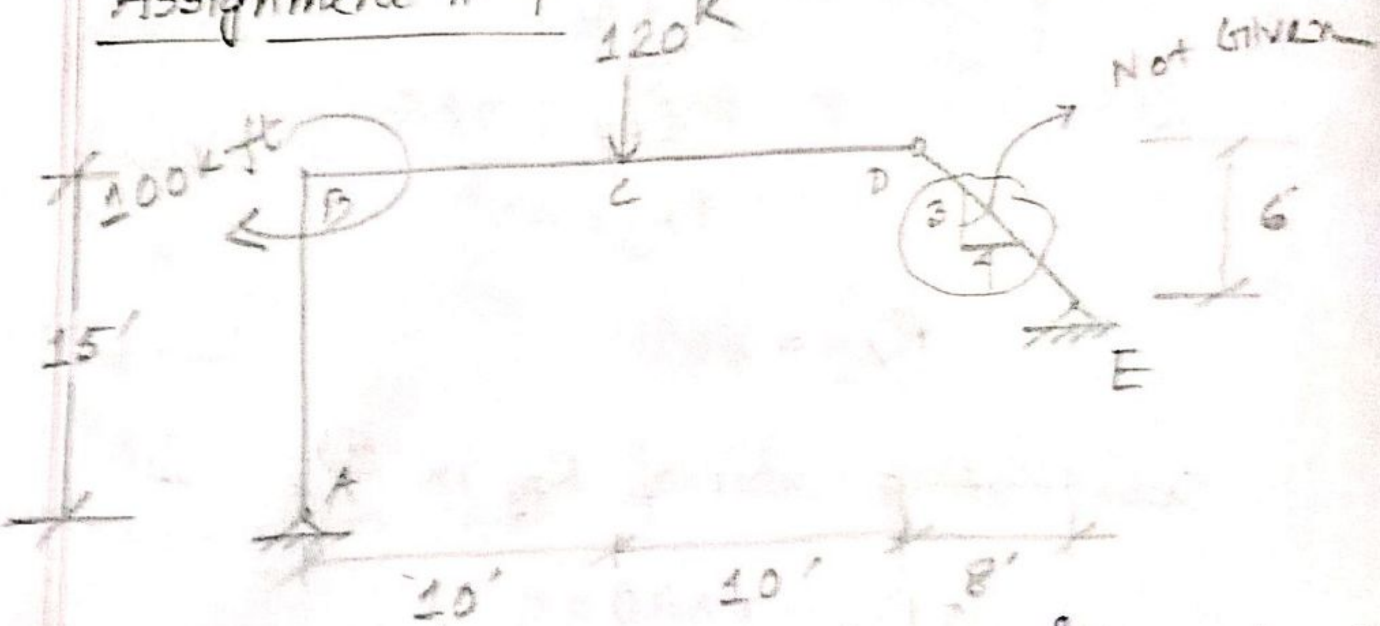
$5R_{Ex} - 8 \times 40 = 0$

$\therefore R_{Ex} = 64^{k} (\leftarrow)$

$\therefore R_{Ax} = 64^{k} (\rightarrow)$

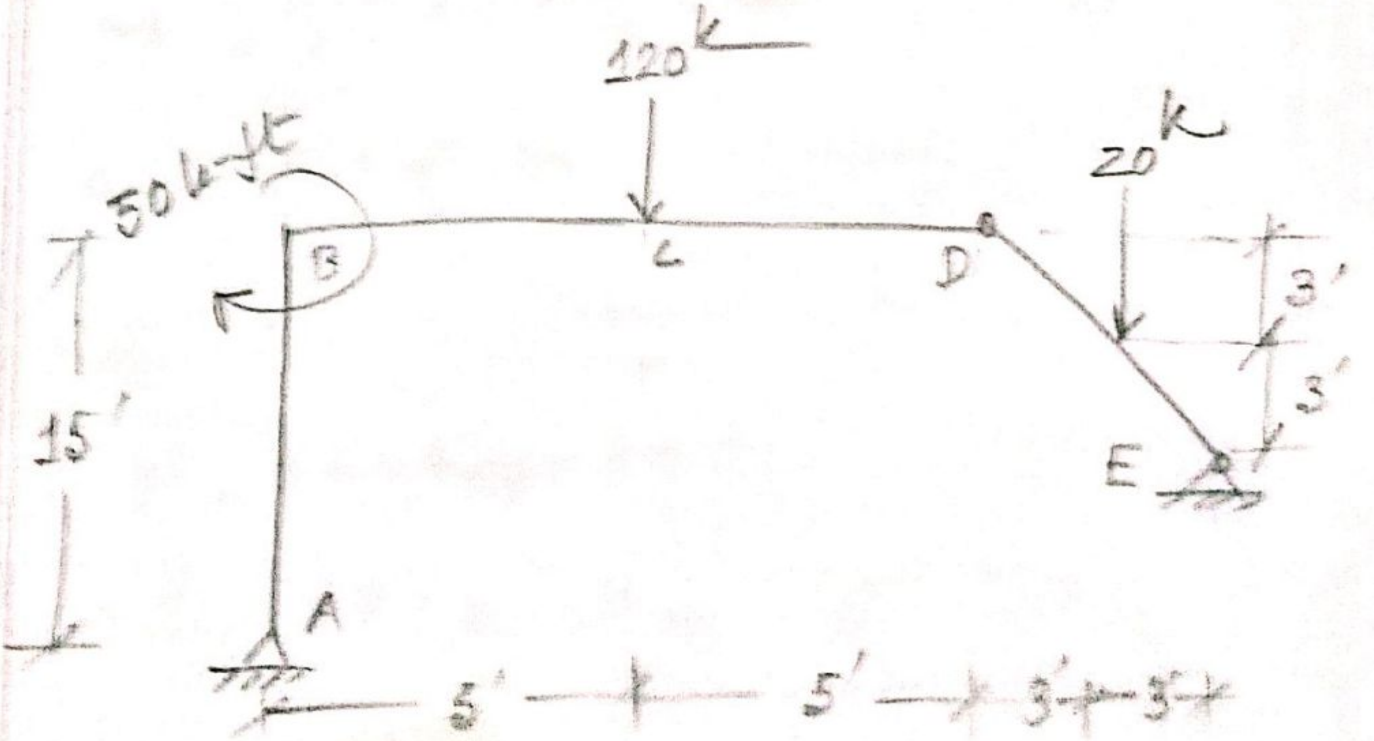
* Frame এর reaction, SFD, BMD
— একটি problem থাকবেই,

Assignment # 9

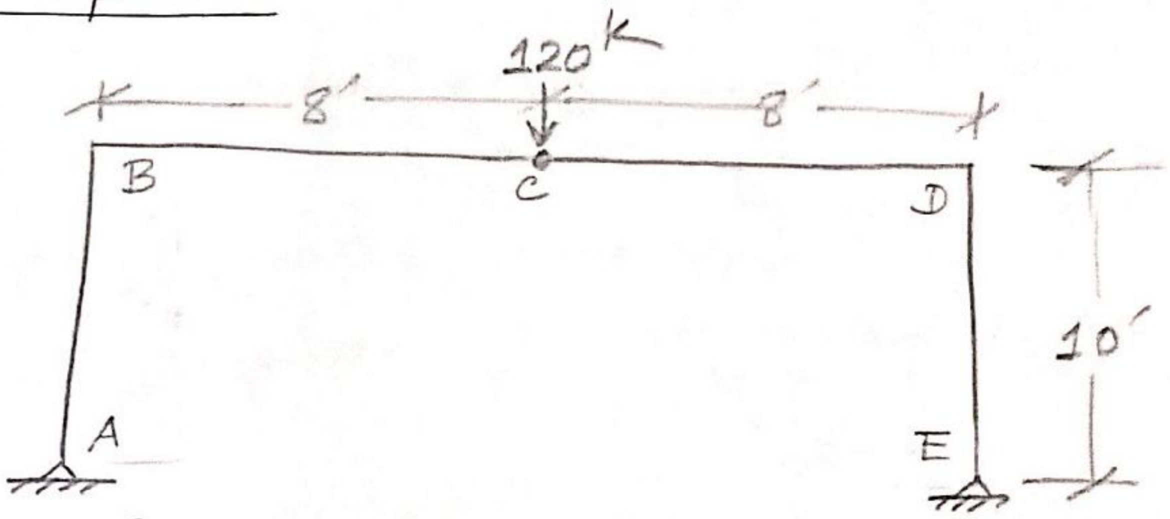


(দুইটি pin এর মধ্যের member এ force না থাকলে এবার truss / member or two force member)

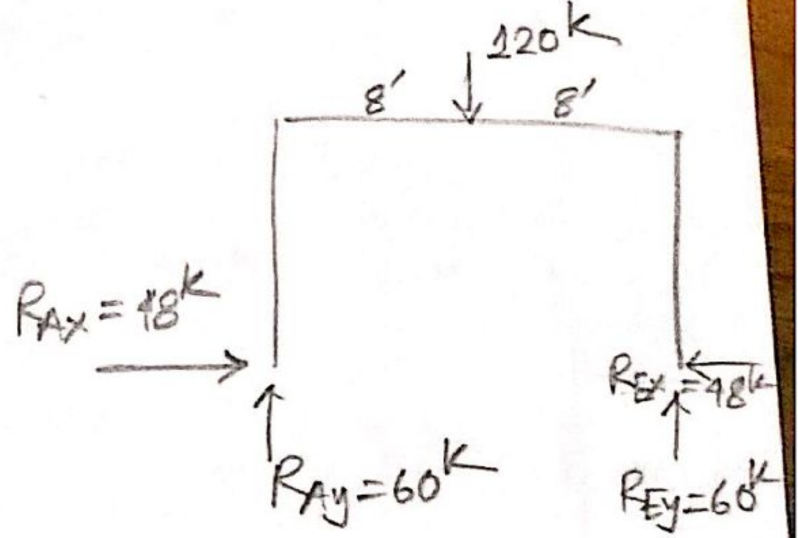
Assignment # 10

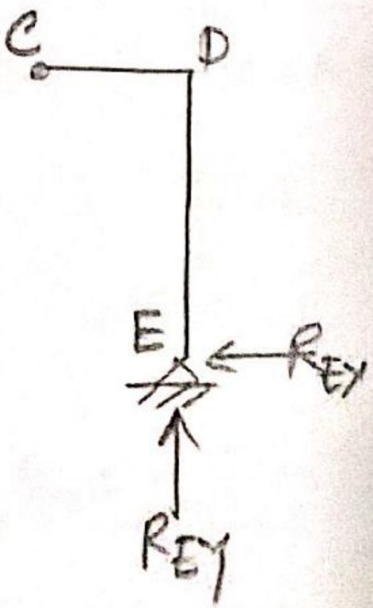


Example 8.6



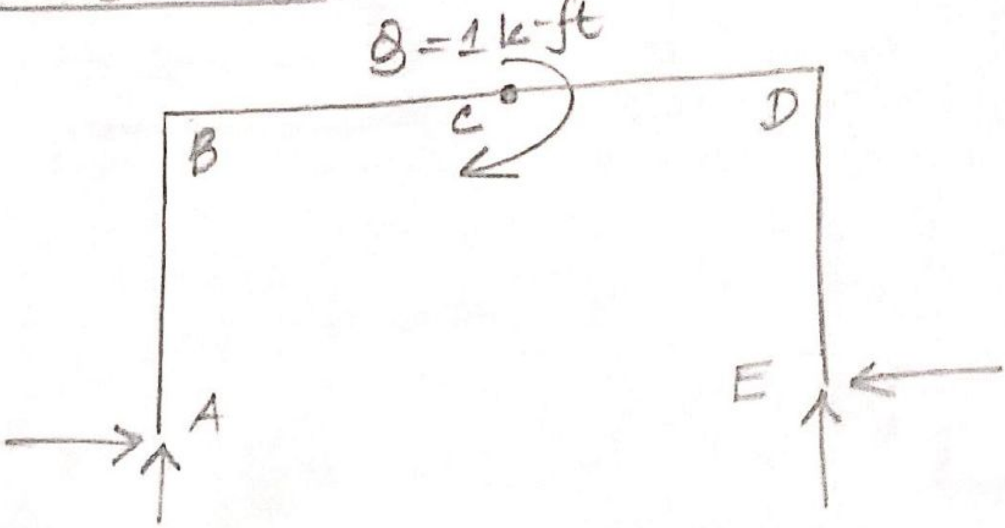
Solve for P-Force



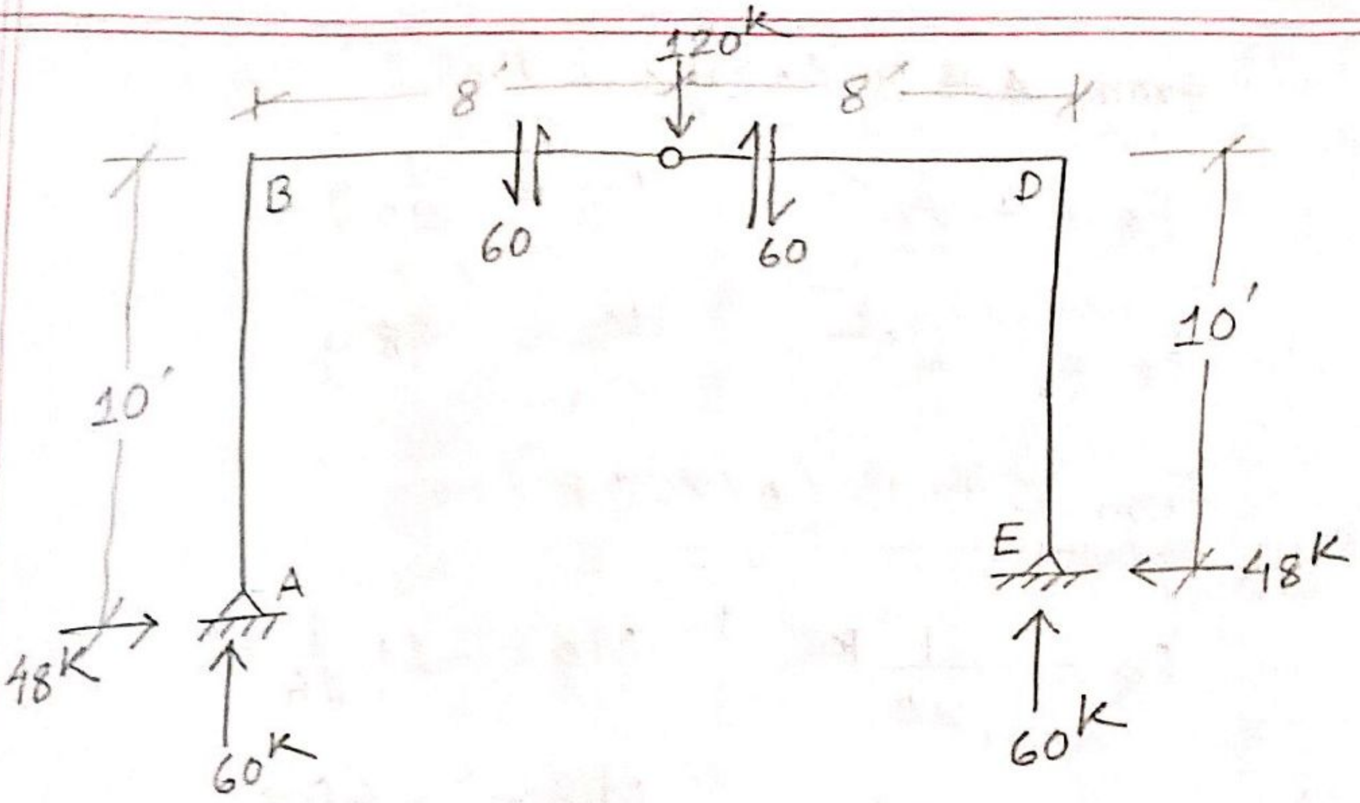


Scanned by CamScanner

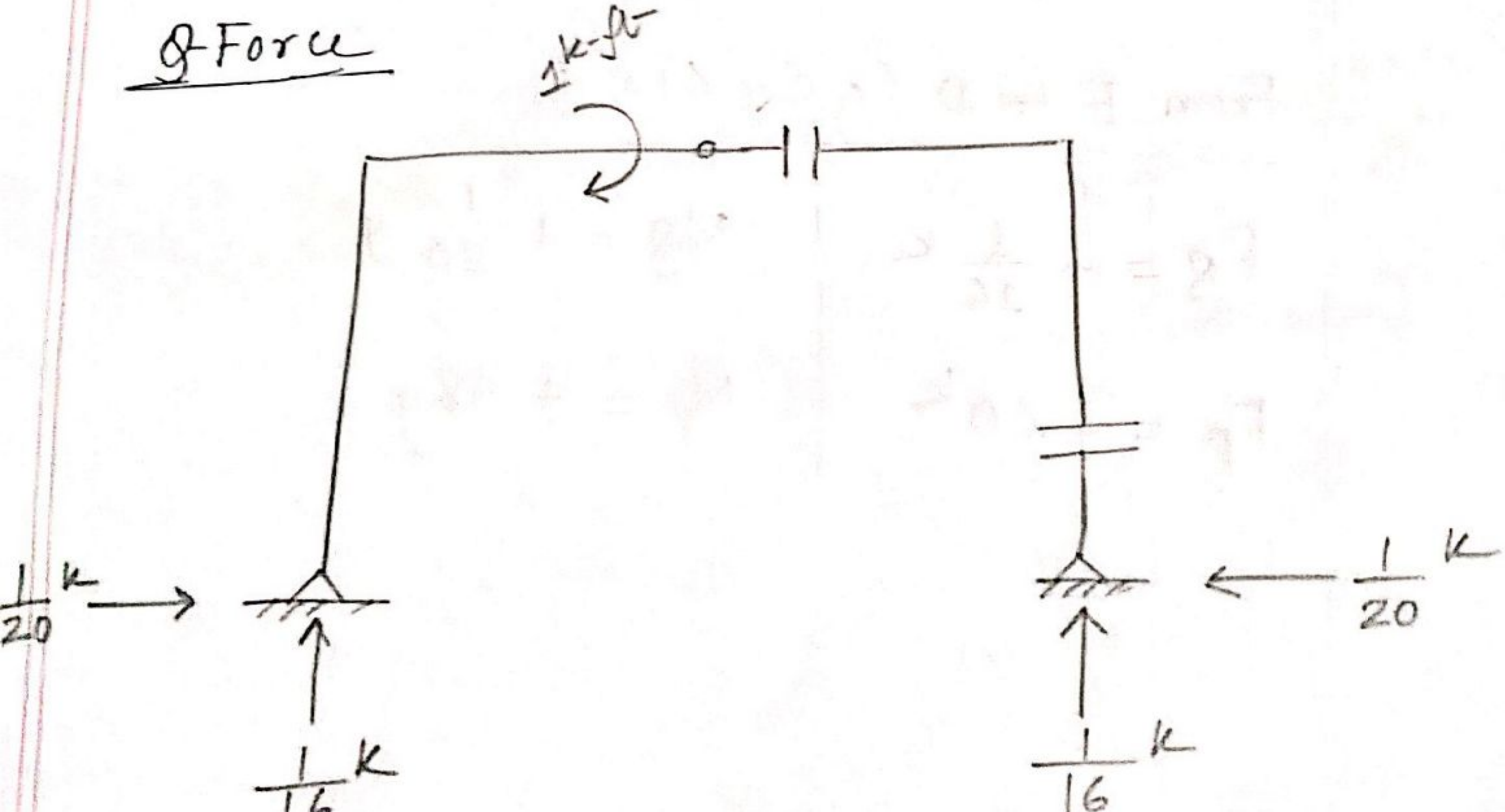
Solve for Q - Force



Zakaria Sir - 9 29 March 2015



Q Force



From A to B $(0 < y \le 10')$

$F_g = +\frac{1}{16}k$

$F_p = -60^k$

$M_g = -\frac{1}{20}y$

$M_p = -48y$

From C to B $(0 < x \le 8')$

$F_g = -\frac{1}{20}k$

$F_p = -48^k$

$M_g = -1 + \frac{1}{16}x$

$M_p = -60x$

From E to D $(0 < y \le 10')$

$F_g = -\frac{1}{16}k$

$F_p = -60^k$

$M_g = +\frac{1}{20}y$

$M_p = +48y$

$$\Sigma Q \cdot \theta_{CL} = \sum \frac{F_Q F_P L}{AE} + \int_A^B \frac{M_Q M_P}{EI} dy + \int_C^B + \int_C^D + \int_E^D$$

$$\therefore \theta_{CL} = \frac{\left(\frac{1}{16}\right)(-60) \times 10}{20 \times 30000} + \frac{\left(-\frac{1}{20}\right)(-48) \times 8}{20 \times 30000}$$

$$+ \frac{\left(-\frac{1}{20}\right)(-48) \times 8}{20 \times 30000} + \frac{\left(-\frac{1}{16}\right)(-60)}{20 \times 30000} .$$

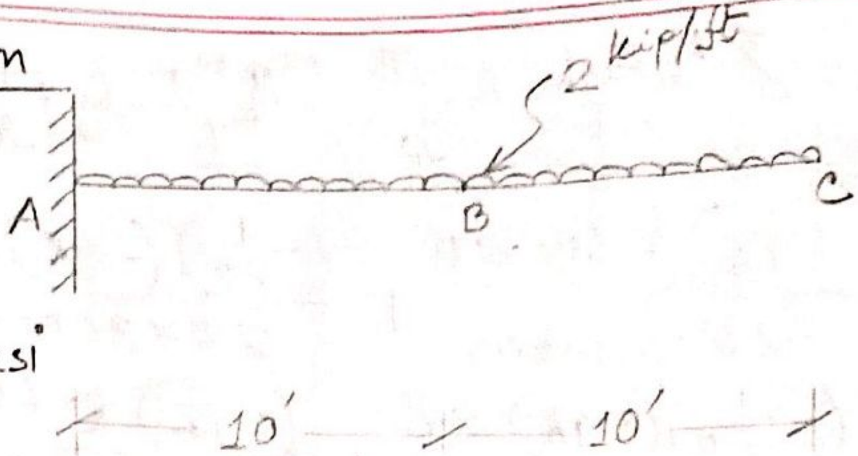
$$+ \int_0^{10} \frac{\frac{48}{20} y^2}{EI} dy + \int_0^8 \frac{\left(-1 + \frac{1}{12} x\right)(-60x)}{EI} dx$$

$$+ \int_0^8 \frac{\frac{60}{16} x^2}{EI} dx + \int_0^{10} \frac{\frac{48}{20} y^2}{EI} dy$$

$$\therefore \theta_{CL} = 7.75 \times 10^{-6} + \frac{1}{30000 \times \frac{2500}{144}} \times \frac{9920}{3}$$

$$\theta_{CL} = 6.3566 \times 10^{-3} \text{ radian (clockwise)}$$

Problem

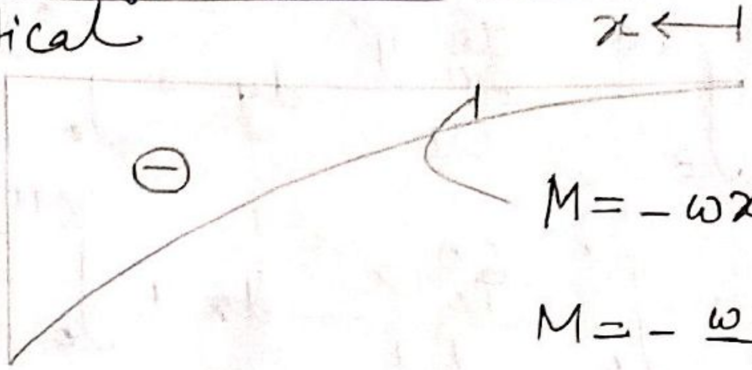


E= 30000ksi

I= 200in^4

\# Compute vertical Reaction at point B.

P-force: Moment Diagram



$$M = -\omega x \cdot \frac{x}{2}$$

$$M = -\frac{\omega x^2}{2}$$

$$= -\frac{2x^2}{2}$$

$$M = -x^2$$

is valid

$$\int_0^{10} + \int_{10}^{20}$$

Q-force Analysis

Moment Diagram

x axis start from c

M = 0 (from $0<x<10$)

M= $-x+10$ (from $10<x\leq 20'$

~~Fr~~

From B to A $(10 < x \leq 20)$

$$M_g = -x + 10$$
$$M_p = -x^2$$

By Virtual Work Method,

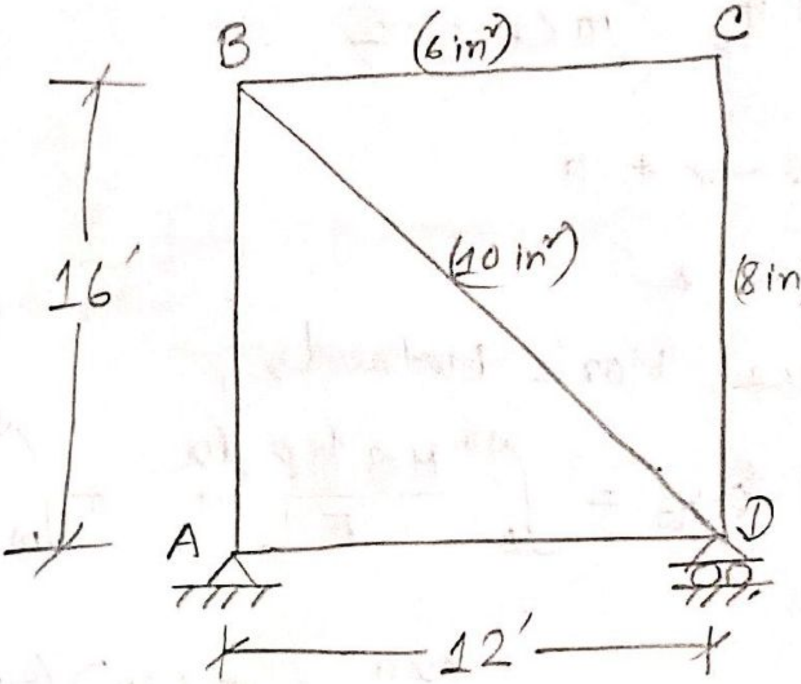
$$\Sigma g \, \delta_{VB} = \int_0^{10} \frac{M_g M_p \, dx}{EI} + \int_{10}^{20} \frac{M_g M_p \, d}{EI}$$

$$\therefore 1 \, \delta_{VB} = \int_{10}^{20} \frac{(-x+10)(-x^2)}{EI} dx$$

$$\delta_{VB} = \int_0^{20} \frac{(x^3 - 10x^2)}{EI} dx$$

Special Truss Problem

Truss with lack of Fit or Fabrication Defects

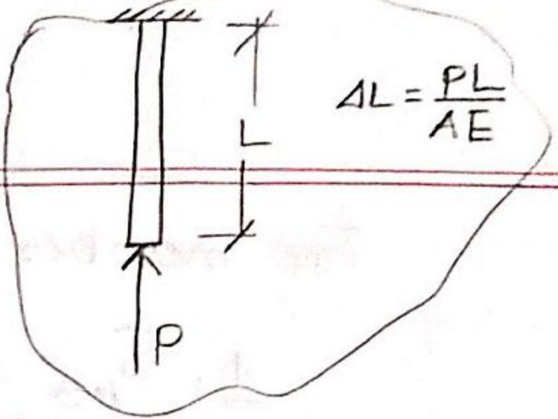


Problem: Calculate δ_{HB} & δ_{VB} if member BD is fabricated 1" larger than specified.

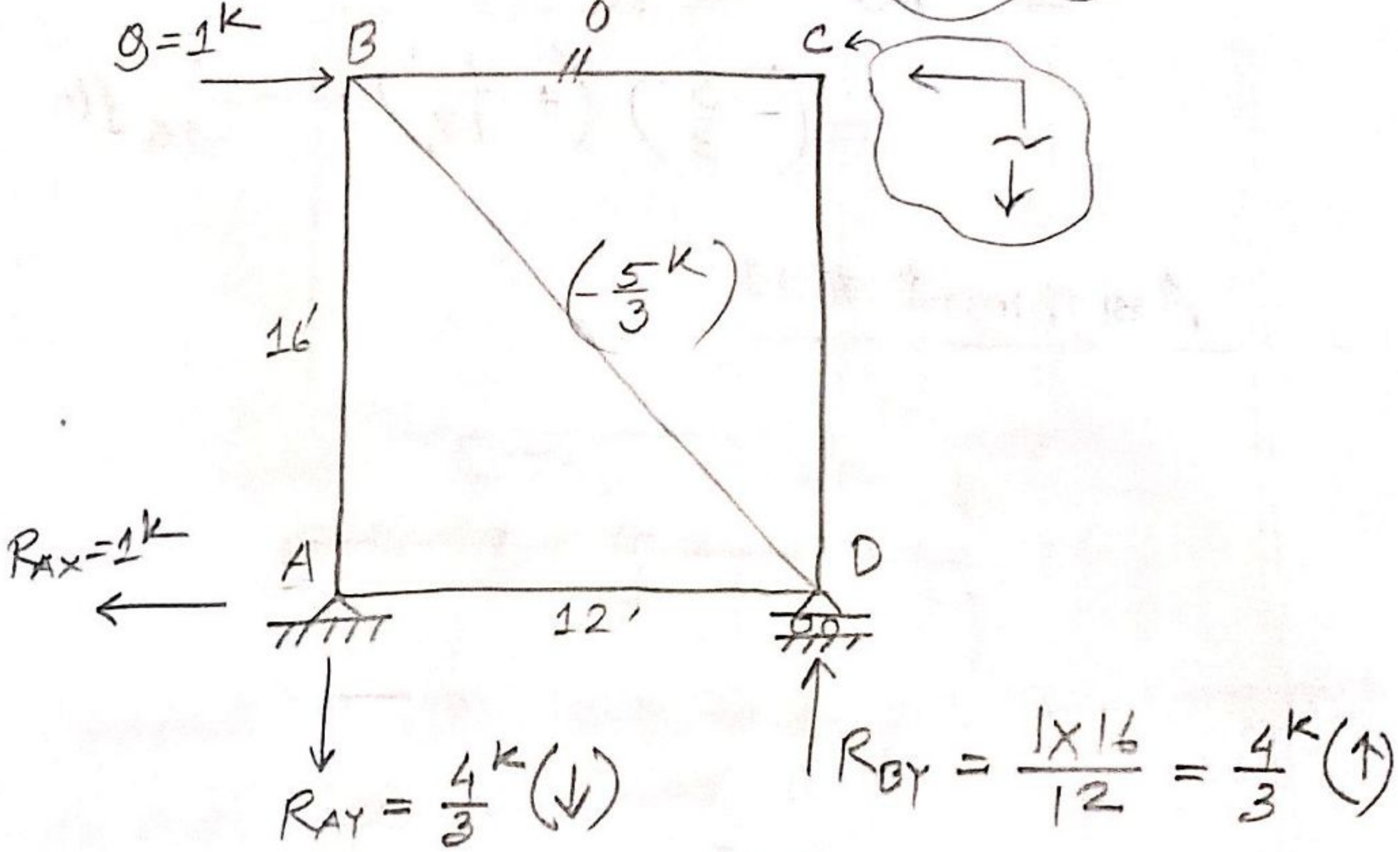
By Virtual Work Method,

$$\sum Q\,\delta_{HB} + \cancelto{0}{W_R} = \sum \frac{F_Q F_P L}{AE} + \cancelto{0}{F_Q \alpha_t t L}$$

$$= \sum F_Q \Delta L$$



Q-Force Analysis



$$(F_{BD})_v = \frac{4}{3}^K$$

$$\therefore F_{BD} = \frac{\frac{4}{3}}{16} \times \sqrt{16^2+12^2}$$

$$= \frac{\frac{4}{3}}{16} \times 20$$

$$= \frac{5}{3}^K (-)$$

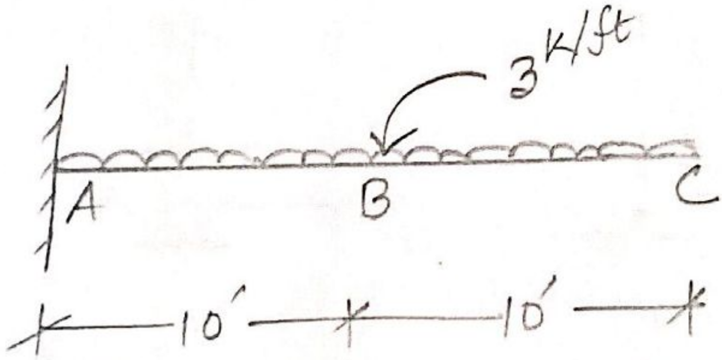
compression

For member AB, BC, CD and AD; $\Delta L = 0$

$$1.\ \delta_{HB} = \sum F_g \Delta L$$

$$= \left(-\frac{5}{3}\right)\left(+\frac{1}{12}\right) = -\frac{5}{36}\text{ ft}$$

Assignment # 11

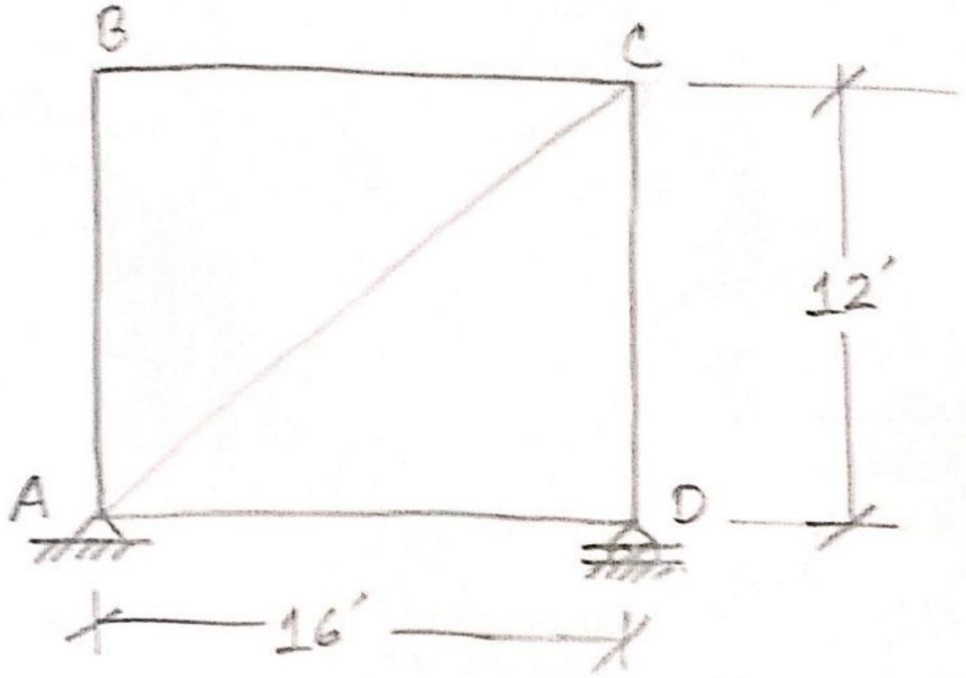


$E = 30{,}000$ ksi

$I = 300\ \text{in}^4$

Compute change in slope at point 'B'.

Assignment # 12



Compute horizontal and vertical component of deflection at Point 'B'.

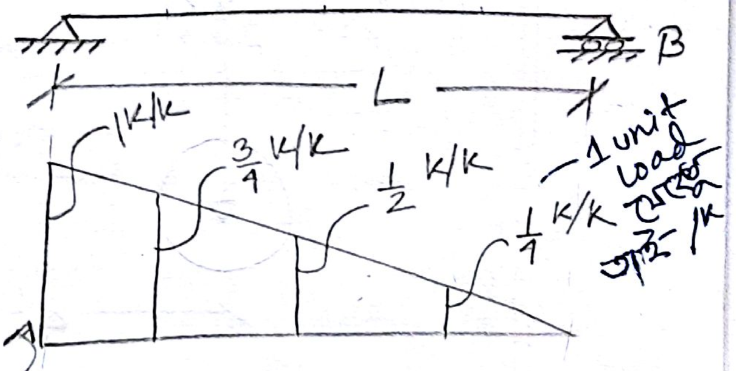
If member AC is fabricated 2" ~~lay~~ larger and member CD is fabricated 0.5" shorter than specified.

Zakaria Sir-10 11 April 2015

Maximum Reaction for a series of moving Loads

Influence Lines for Reaction, Shear, Moment etc.
↳ is drawn for a moving load of 1 unit.

$R_A = ?$ for different position of Unit load (1^k)



Influence Line for R_A

When unit load 1^k at,

$x = \frac{L}{4}$ (tanB), $R_A = \frac{1}{4}^K$

When unit load 1^k at,

$x = \frac{L}{2}$, $R_A = \frac{1}{2}^k$

When unit load 1^k at,

$x = \frac{3L}{4}$, $R_A = \frac{3}{4}k$.

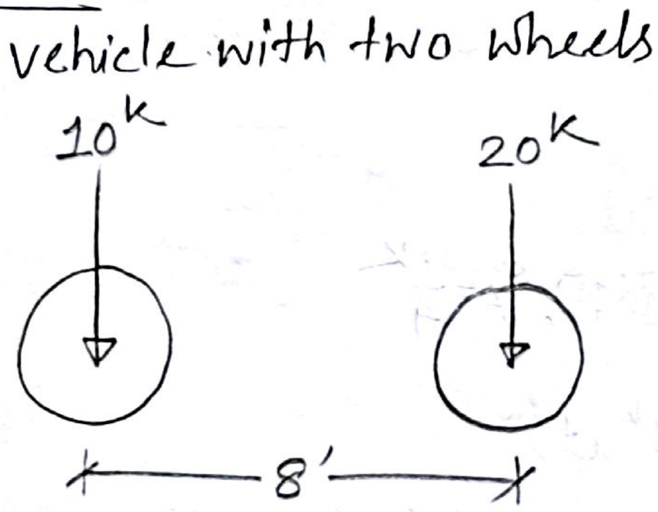
When unit load 1^K at,

$x = L \tan B$, $R_A = 1^k$

If. a wheel load of 10^k is placed at $x = \frac{L}{2}$, $R_A = ?$

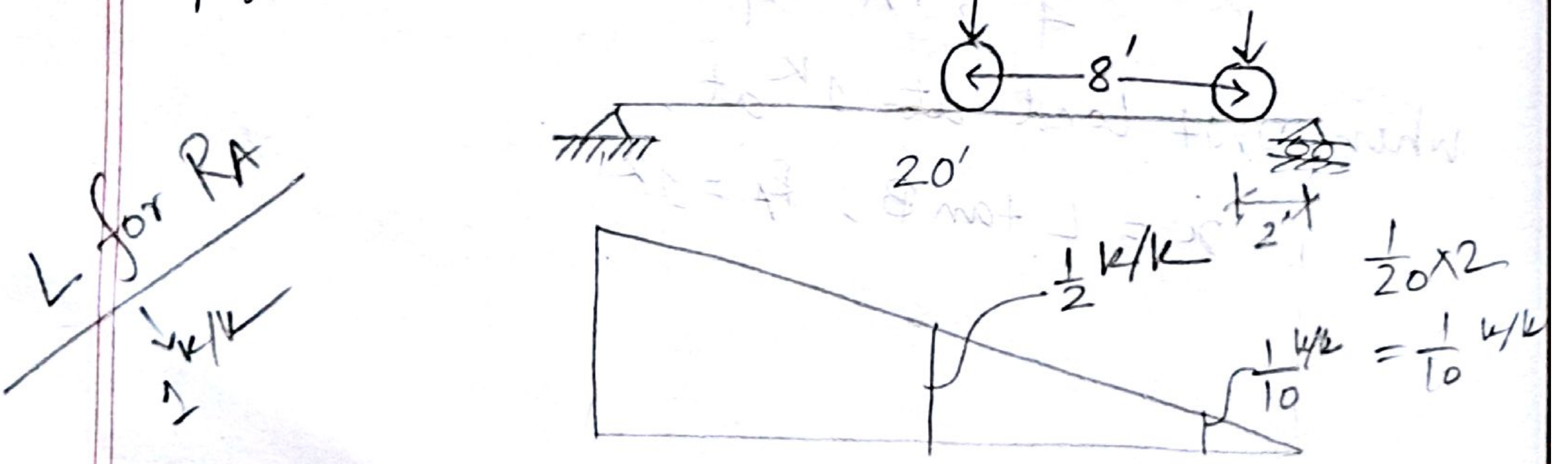
From IL diagram, $R_A = \frac{1}{2}^{k/k} \times 10^k = 5^k$.

Example



If, span, L= 20 ft, $R_A = ?$ if wheel ① is placed at $x = \frac{L}{2}$.

Maximum, $R_A = ?$



$$R_A = \frac{1}{2}^{k/k} \times 10^k + \frac{1}{10}^{k/k} \times 20^k$$
$$= 5 + 2 = 7^k$$

If, wheel ① is placed at A, $R_A = 1^{k/k} \times 10^k + (\frac{1}{20} \times 12)^{k/k} \times 20^k$
$= 22^k$

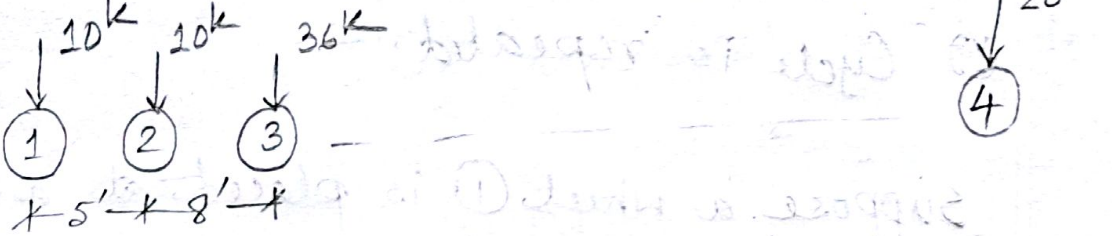
" " ② " " " A, $R_A = 1^{k/k} \times 20^k = 20^k$

→ Maximum $R_A = 22^k$.

Shedd & Vawter

Article 160: Position of wheel to Produce Maximum Reaction

Wheel Loads for a steam locomotive



Suppose train moves from right

Observations

(i) R_A increases till wheel ① comes in A.

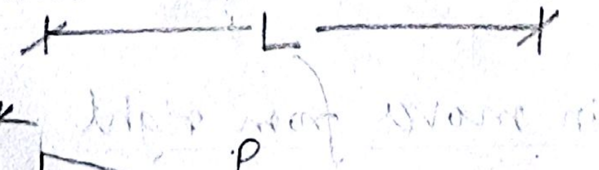
(ii) R_A suddenly decreases as wheel ① moves off the span

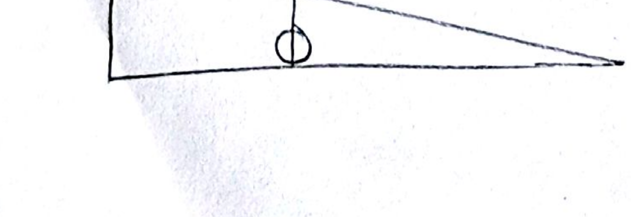
(iii) R_A increases as wheel ② approaches A.

(iv) R_A suddenly decreases as wheel ② moves off the span

(v) Cycle is repeated.

Suppose a wheel ① is placed at a distance 'x' from Right support B.





$$R_A = \frac{1}{L} x \cdot P = \frac{Px}{L}$$

if P moves a distance 'd' towards

$$R_A = \frac{P(x+d)}{L}$$

Change in reaction, $\Delta R = \frac{P(x+d)}{L} - \frac{Px}{L}$

$$= \frac{Pd}{L}$$

If 'N' no.s wheel have distance 'd' on the span,

$$\Delta R = \Sigma \frac{Pd}{L}$$

In general, if wheel P_1 moves off the span and wheel P' travels a distance "e" in the span during movement of 'd_1' distance, Change in Reaction, $\Delta R = \frac{\sum P y}{L} + \frac{P' e}{L} - P_1$

where,

$\sum P$ = sum of wheels that stay on the span during "y" movement.

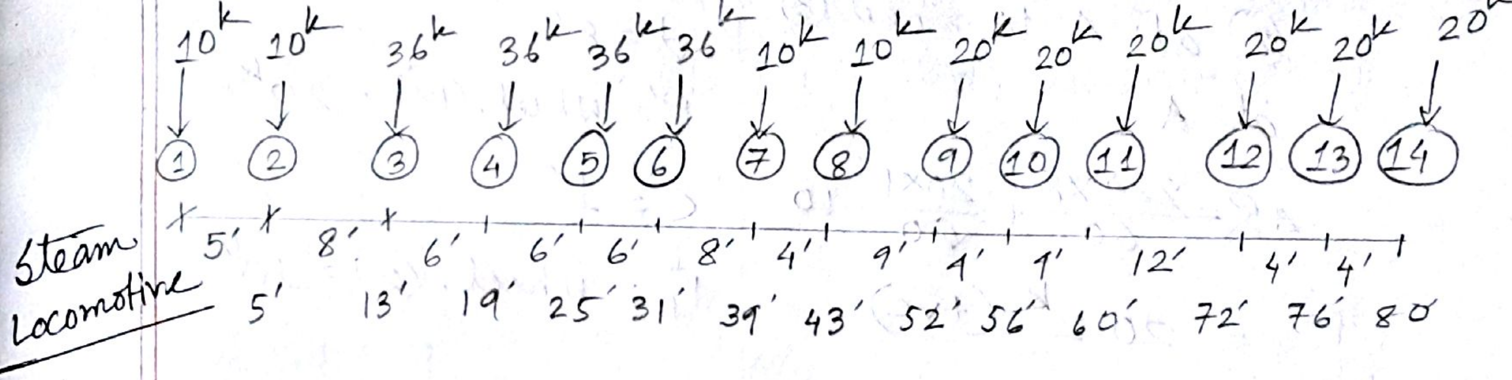
P' = wheel that comes in the span

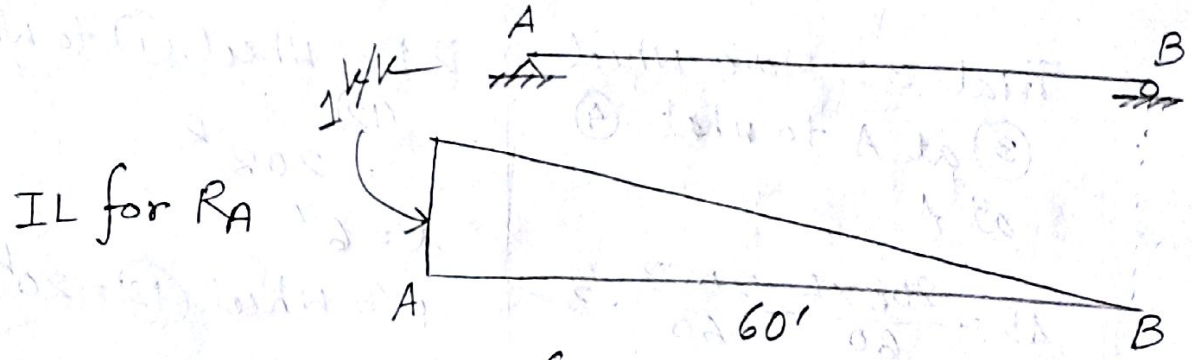
e = distance travelled by P'

P_1 = wheel that moves off the span

Zakaria Sir-11   12 April 2015

Problem:





Given: Simple span of 60ft.

Find maximum reaction

Trial-1: Move wheel ① at A to wheel ② at A

$$\Delta R = \frac{\Sigma Pd}{L} + \frac{P'e}{L} - P_1$$

or, $\Delta R = \frac{234\times 5'}{60} + 0 - 10^k$

$= 9.5$ (+ve)

ΣP = Wheels they stay on span before and after movement

ΣP = wheel ② to wheel ⑪

$\Sigma P = 234^k$

$d = 5'$

$P' = 0$, e = No need

P_1 = Wheel ① $= 10^k$

Trial-2: Move wheel ② at A to wheel ③ at A

$$\Delta R = \frac{224 \times 8}{60} + \frac{20 \times 1}{60} - 10$$

$$= 20.2^k \text{ (+ve)}$$

ΣP = wheel ③ to ⑪ = 224^k

$d = 8'$

P' = wheel ⑫ = 20^k

$e = 1'$

P_1 = wheel ② = 10^k

Trial-3: Move wheel ③ at A to wheel ④ at A

$$\Delta R = \frac{208 \times 6}{60} + \frac{20 \times 3}{60} - 36$$

$$= -14.2^k \text{ (+ve)}$$

ΣP = Wheel ④ to wheel ⑫

$= 208^k$

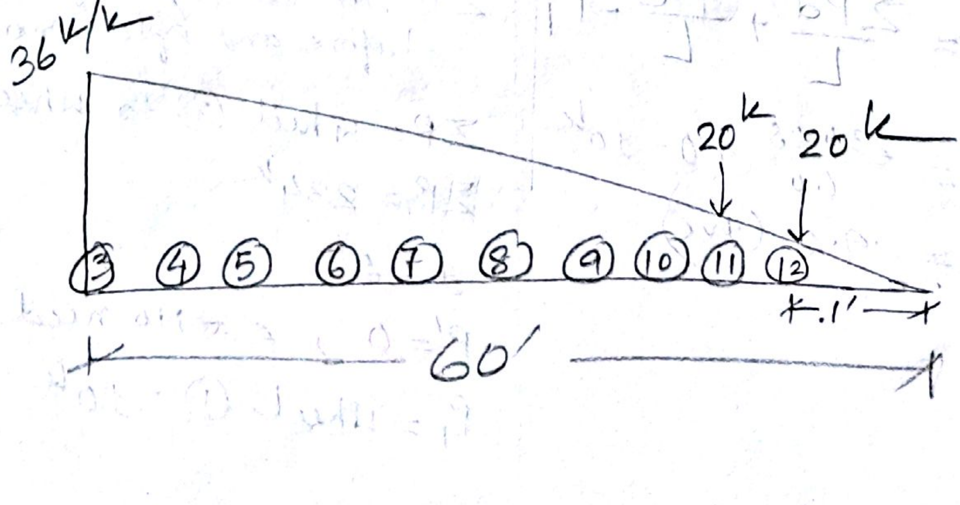
$d = 6'$

P' = Wheel ⑬ = 20^k

$e = 3'$

P_1 = Wheel ② = 36^k

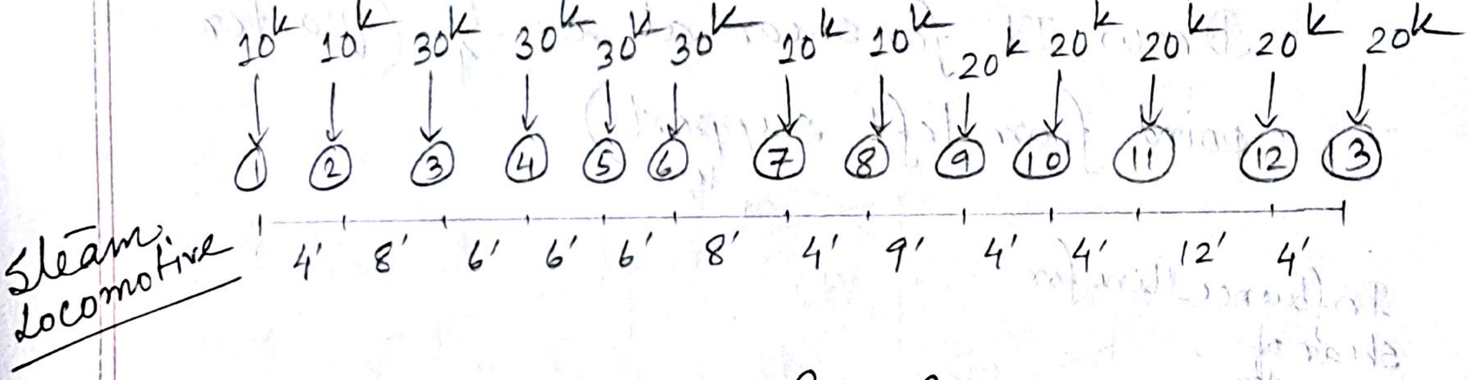
Wheel ③ at A Gives Maximum Reaction



$$\text{Maximum } R_A = \frac{1}{60}\left[60'\times 36^k + 54'\times 36^k + 48'\times 36^k + 42'\times 36^k + 34'\times 10^k + 30'\times 10^k + 21'\times 20^k + 17'\times 20^k + 13'\times 20^k + 1'\times 20^k\right]$$

$$R_A = 150.4^k$$

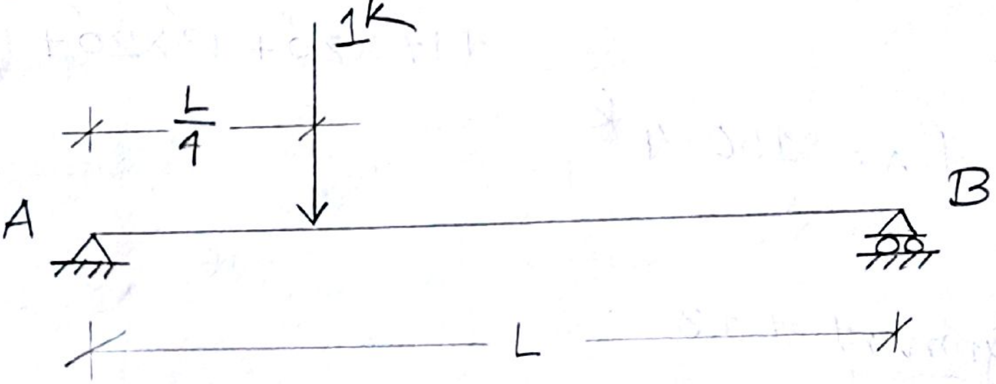
Assignment # 13



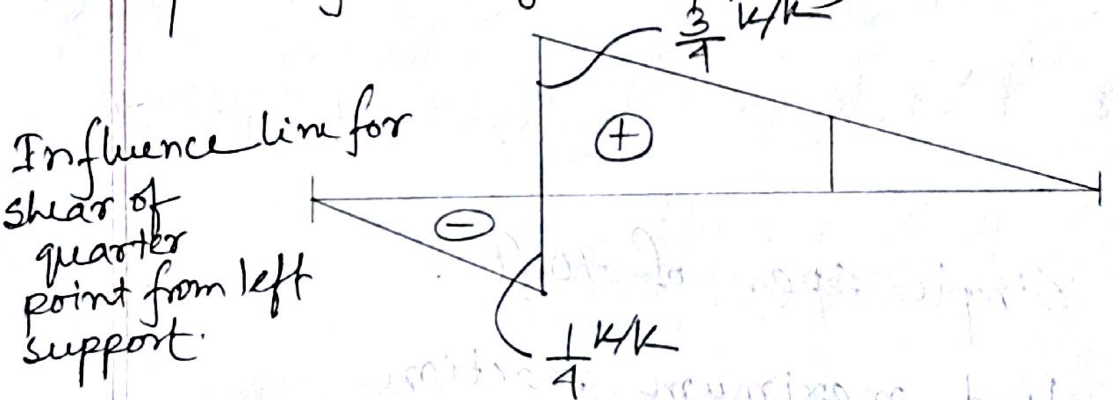
Given: Simple span of 70 ft.

Find maximum reaction

Influence Line Diagram for Shear



Draw IL for shear at $x = \frac{L}{4}$ (quarter point from left support)



When unit load 1^k, just before $\frac{1}{4}$ (C^-)

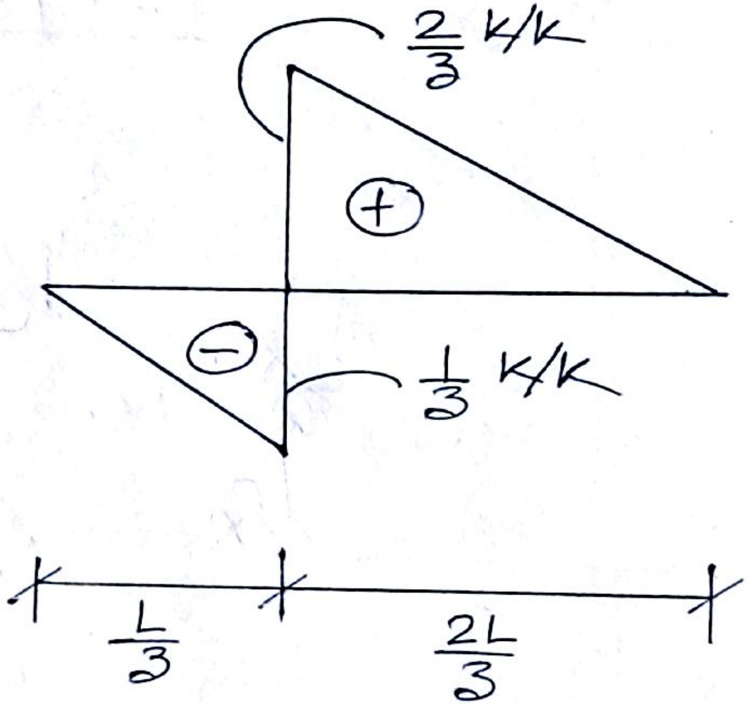
shear $= ? = -\frac{1}{4}^k$ (-ve)

when 1^k just after $\frac{1}{4}$ (C^+)

shear=

** If the load is at $\frac{L}{3}$ from left support

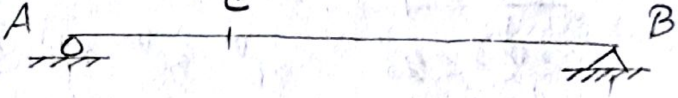
(IL for shear at third point from left support)



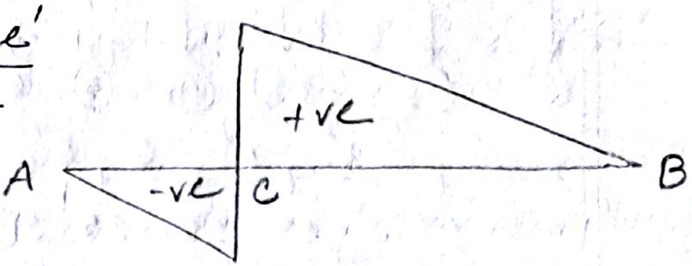
Zakaria Sir - 12

26 April 2015

Change in shear,



$$\Delta V = \frac{\Sigma P d_1}{L} + \frac{P' e}{L} - P_1 + \frac{P_2 e'}{L}$$



ΣP = Wheel loads, they stay on span before and after movement.

d_1 = distance travelled by ΣP

P' = Load that comes on the span

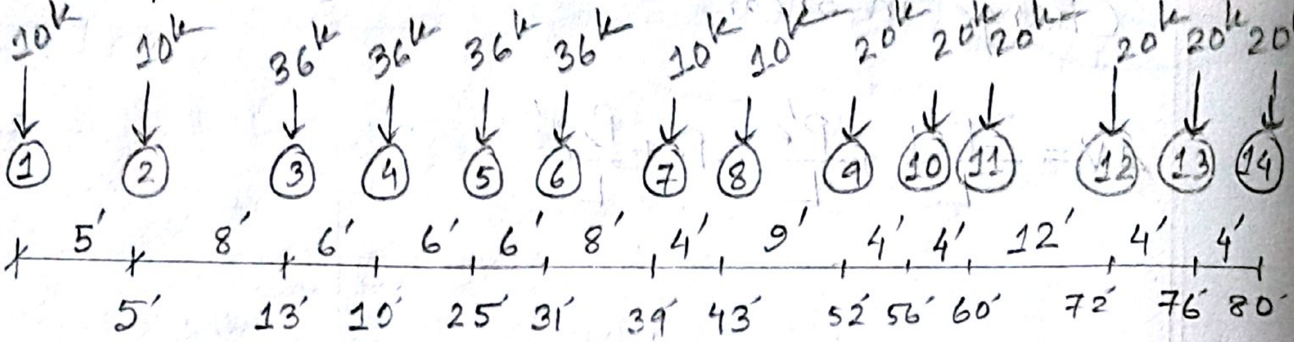
e = distance travelled by P'

P_1 = Load that crossed the control point "c"

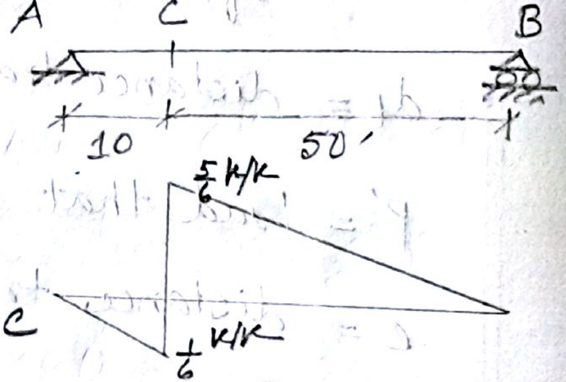
P_2 = load that moves off the span

e' = distance travelled by P_2 before moving off the span.

Example:



Find Maximum shear at 10 ft from left support of 60' span.



IL for shear at C

Trial 1: Wheel ① at C to wheel ② at C,

$$\Delta V = \frac{\Sigma P d_1}{L} + \frac{P' e}{L} - P_1 - \frac{P_2 e'}{L}$$

$$\Rightarrow \Delta V = \frac{184 \times 5}{60} + \frac{20 \times 3}{60} - 10 + 0$$

$$= +6.33^k \text{ (ve)}$$

ΣP = Wheel ① to wheel ⑧ $= 184^k$

$d_1 = 5'$

P' = wheel ④ $= 20^k$

$e = 3'$

P_1 = wheel ① $= 10^k$

$P_2 = 0$

Trial 2: Wheel ② at C to wheel ③ at C

$$\Delta V = \frac{194\times 8}{60} + \frac{20\times 7 + 20\times 3}{60} - 10 + \frac{10\times 5}{60}$$

$\Delta V = +20.03^k$ (+ve)

ΣP = wheel ② to wheel ⑨ $= 194^k$

$d_1 = 8'$

$P' =$ { wheel ⑩ $= 20^k$, $e_1 = 7'$; wheel ⑪ $= 20^k$, $e_2 = 3'$ }

P_1 = wheel ② $= 10^k$

P_2 = wheel ① $= 10^k$

$e' = 5'$

Trial 3: wheel ③ at C to wheel ④ at C

$$\Delta V = \frac{224\times 6}{60} + 0 - 36 + \frac{10\times 2}{60}$$

$= -13.26^k$ (-ve)

ΣP = wheel ③ to wheel ⑪ $= 224^k$

$d_1 = 6'$

$P' = 0$

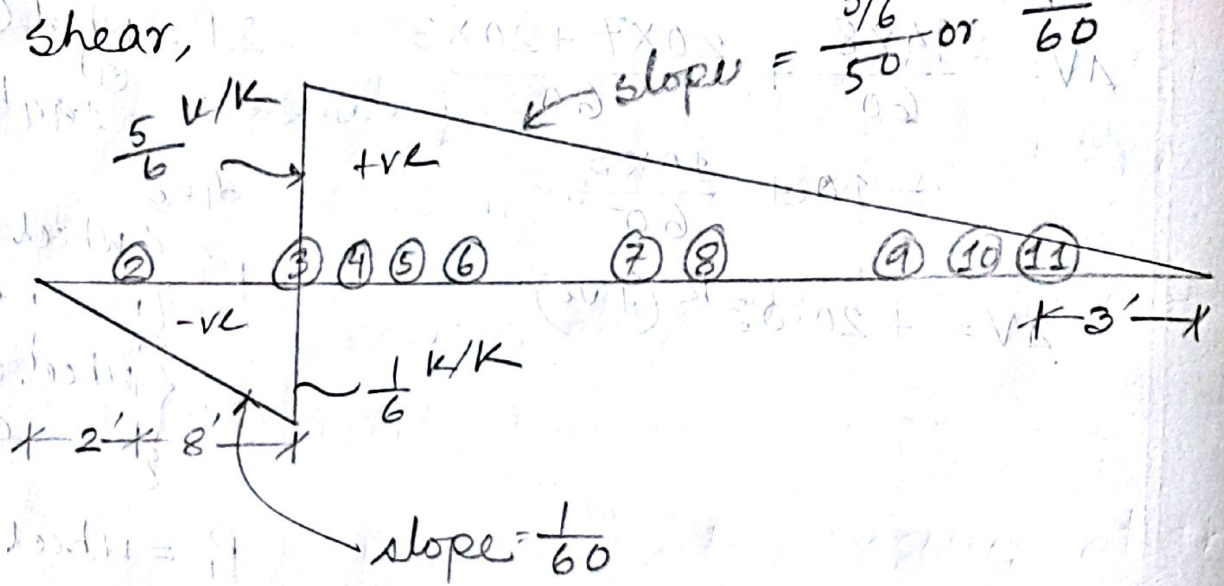
$e = 0$

P_1 = wheel ③ $= 36^k$

P_2 = wheel ② $= 10^k$

$e' = 2'$

Hence, wheel ③ at C gives maximum shear,



$$V_{max} = \frac{1}{60}\Big[50' \times 36^k + 44' \times 36^k + 38 \times 36^k + 32 \times 36^k + 24 \times 10^k + 20 \times 10^k + 11 \times 20^k + 7 \times 20^k + 3 \times 20^k\Big] - \frac{1}{60}\left[2' \times 10^k\right]$$

$$= 112.4^k$$

Assignment 14

* একই load একই span but ~~max~~^{m} shear বের করতে হবে quarter point-এ.

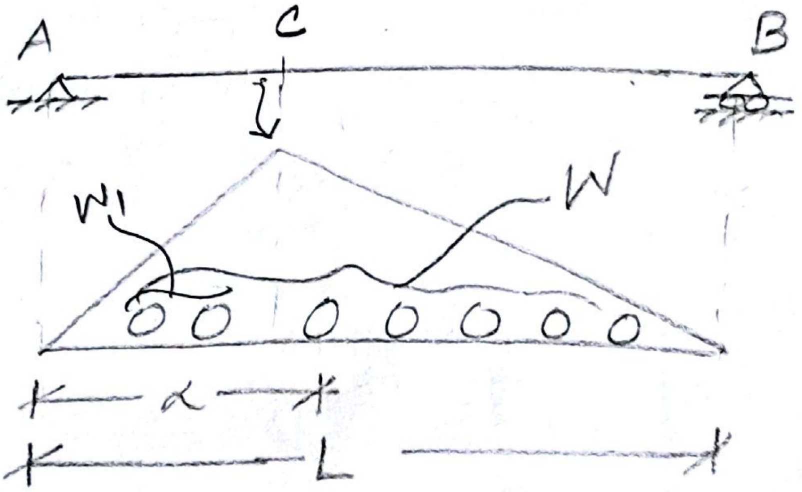
Zakaria-13 4 May 2015

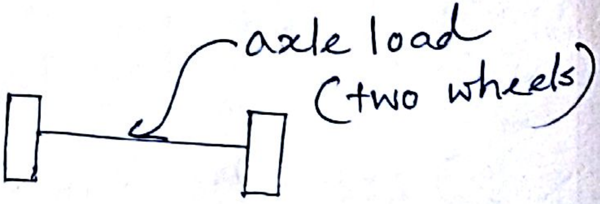
Article 65
(Shedd and Vawter)

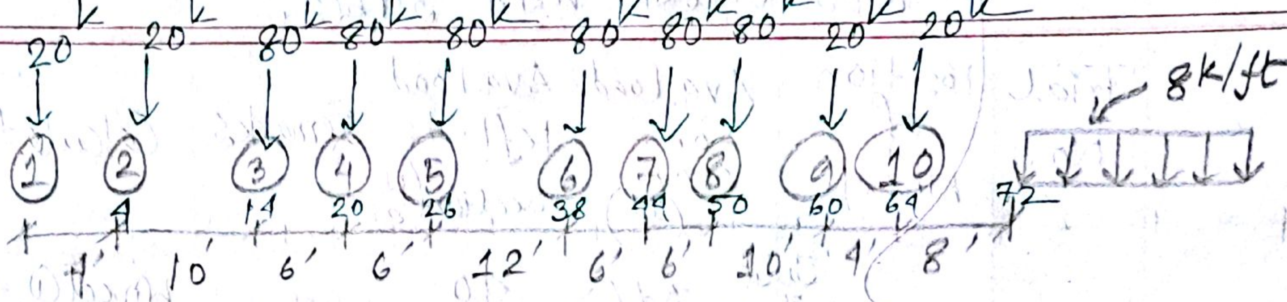
Position of Wheel to Produce Maximum Moment at a section (bgm) / Maximum Chord Stress of Truss

Criterion:

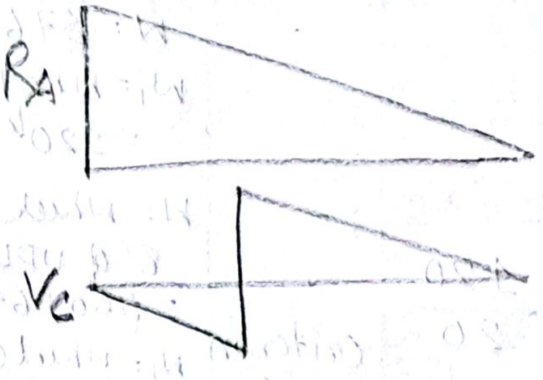
Criterion: $\boxed{\frac{W}{L} = \frac{W_1}{a}}$ = Average load on span = Average load left of section

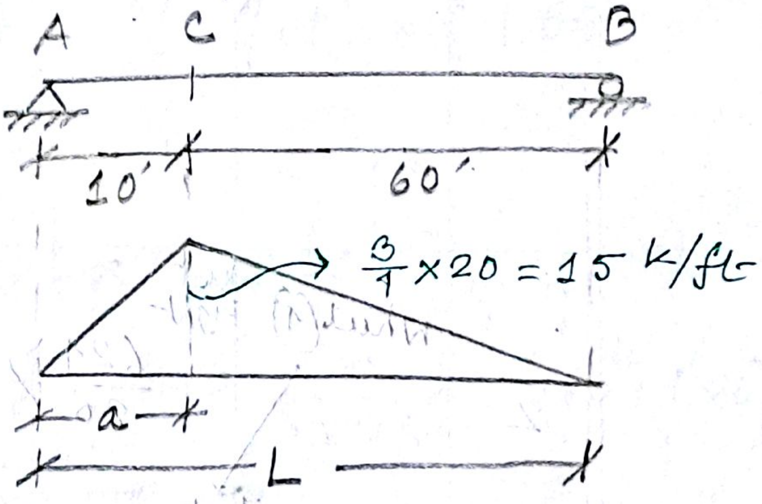






Find Maximum moment at 20' from left support of 80' span for given axle load.





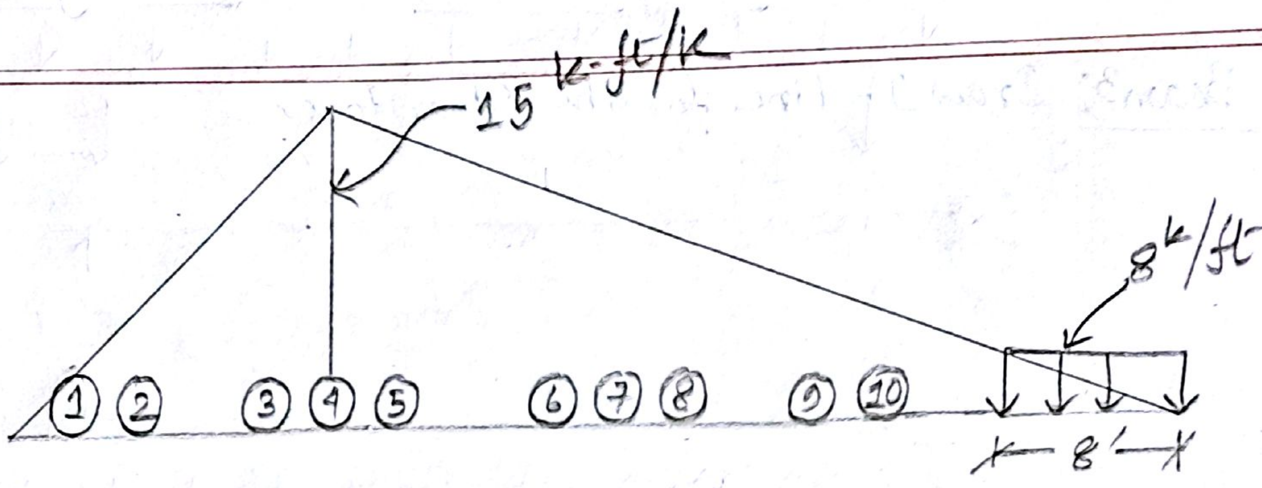
$a = 20'$

$h = 80'$

প্রথমবারে start করব
যাতে maximum হয়, 2 দিয়ে করা
উচিত তবুও করব 3 দিয়ে।

Trial No.	Position of Wheel	Avg. Load on span $\left(\frac{W}{L}\right)$	Avg. Load left to Section $\left(\frac{W}{a}\right)$	Remarks	Calculation
1	Wheel ③ Just Right	$\frac{576}{80} > \frac{40}{20}$		criterion not satisfied	Wheel ① + ⑩ + 2' of UDL $= 560 + 2\times8'$ $= 576^k$ W_1 = Wheel ① & ② $= 40^k$
	Just Left	$\frac{576}{80} > \frac{120}{20}$			$W = 576^k$ W_1 = Wheel ① to ③ $= 120^k$
2	Wheel ④ Just Right	$\frac{624}{80} > \frac{120}{20}$		criterion satisfied	W = Wheel ① to ⑩ + 8' of UDL $= 560 + 64 = 624^k$ W_1 = Wheel ① to ③ $= 120^k$
	Just Left	$\frac{604}{80} < \frac{180}{20}$			W = Wheel ② to ⑩ + 8' of UDL $= 604^k$ W_1 = Wheel ② to ④ $= 20 + 160$ $= 180^k$

Hence, wheel ④ at C gives maximum moment.



$$\text{Maximum Moment} = \frac{15}{60}\Big[60' \times 80^k + 54' \times 80^k + 42' \times 80^k + 36' \times 80^k + 30' \times 80^k + 20' \times 20^k \times 16' \times 20^k\Big]$$

$$+ \underbrace{\frac{1}{2} \times \frac{15}{60} \times 8 \times 8 \times \overset{\text{Intensity of UDL}}{8}}_{\text{area of diagram with UDL}} + \frac{15}{20}\Big[14' \times 80^k + 4' \times 20^k\Big]$$

$= 5584^{k\text{-}ft}$ for Axle Load

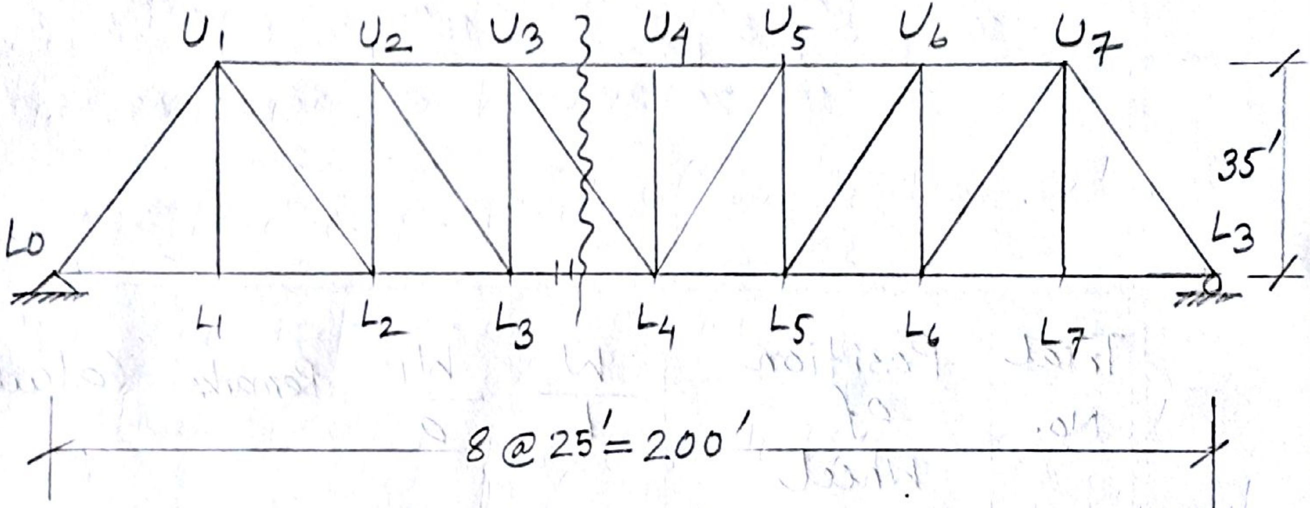
or $\frac{5584}{2} = 2792^k$ for wheel load

Assignment 15: (Load Same)

Calculate maximum moment at $\frac{1}{3}$rd point of sample span of 90 ft

Zakaria Sir-14 10 May 2015

Example: Calculate Maximum stress in L_3L_4 of the truss shown.

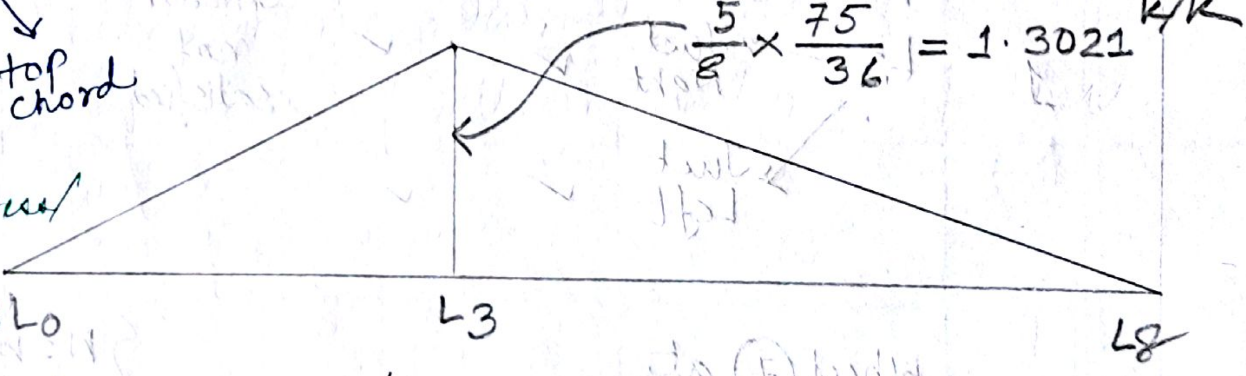


chord member

bottom chord

top chord

IL for stress/bar forces of members L_3L_4

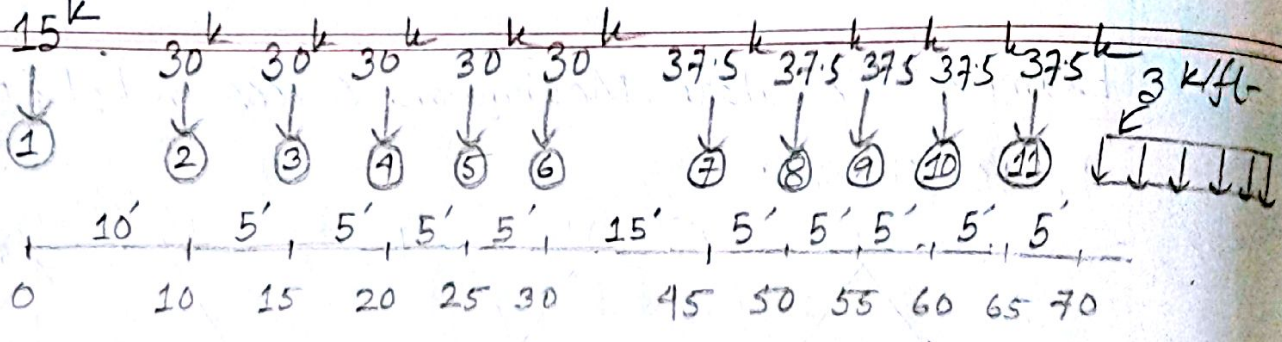


$\frac{5}{8} \times \frac{75}{36} = 1.3021^{k/k}$

$$F_{L_3L_4} = \frac{R_{L_3} \times 75'}{36}$$

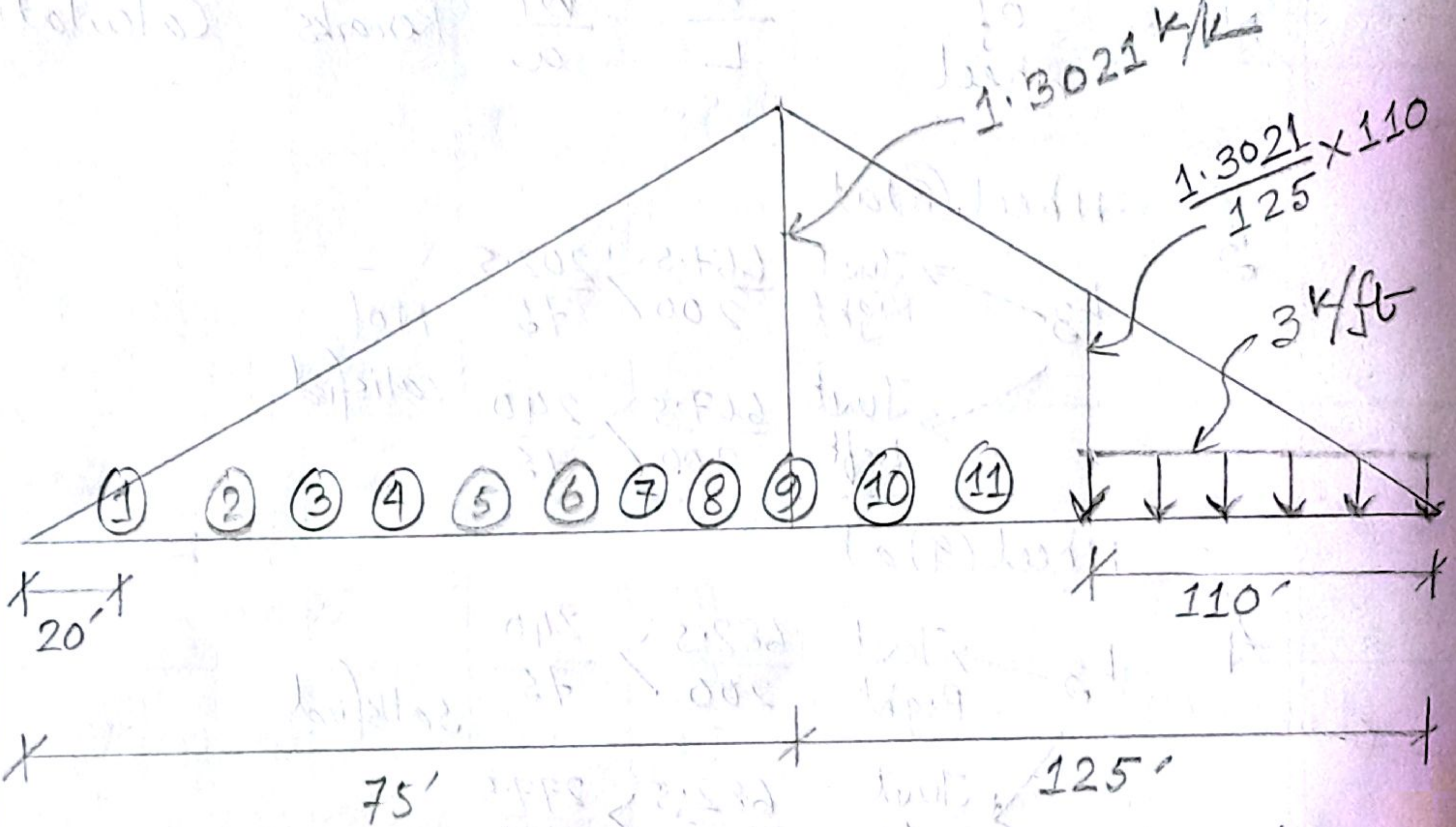
$$= \frac{\frac{5}{8} \times 75}{36}$$

$$= 1.3021^{k/k}$$



Trial No.	Position of Wheel	$\frac{W}{L}$	$\frac{W_1}{a}$	Remarks	Calculation
1	Wheel ⑥ at L3 → Just Right L3 → Just Left	✓ ✓	✓ ✓	Criterion not satisfied	
2	Wheel ⑦ at L3 → Just Right L3 → Just Left	$\frac{652.5}{200}$ > $\frac{652.5}{200}$ >	$\frac{165}{75}$ $\frac{202.5}{75}$		W = Wheel ① to ⑪ + 100' of UDL = 652.5^k W_1 = Wheel ① to ⑥ = 165^k W = 652.5^k W_1 = Wheel ① to ⑦ = 202.5^k

Trial No.	Position of wheel	$\frac{W}{L}$	$\frac{W_1}{a}$	Remarks	Calculation
3	Wheel ⑧ at $L_3 \rightarrow$ Just Right	$\frac{667.5}{200} >$	$\frac{202.5}{75}$	Not satisfied	
	$L_3 \rightarrow$ Just Left	$\frac{667.5}{200} >$	$\frac{240}{75}$		
4	Wheel ⑨ at $L_3 \rightarrow$ Just Right	$\frac{682.5}{200} >$	$\frac{240}{75}$	satisfied	
	$L_3 \rightarrow$ Just Left	$\frac{682.5}{200} >$	$\frac{277.5}{75}$		



Maximum stress/bar force $= \frac{1.3021}{125}\left[125' * 37.5^k\right.$

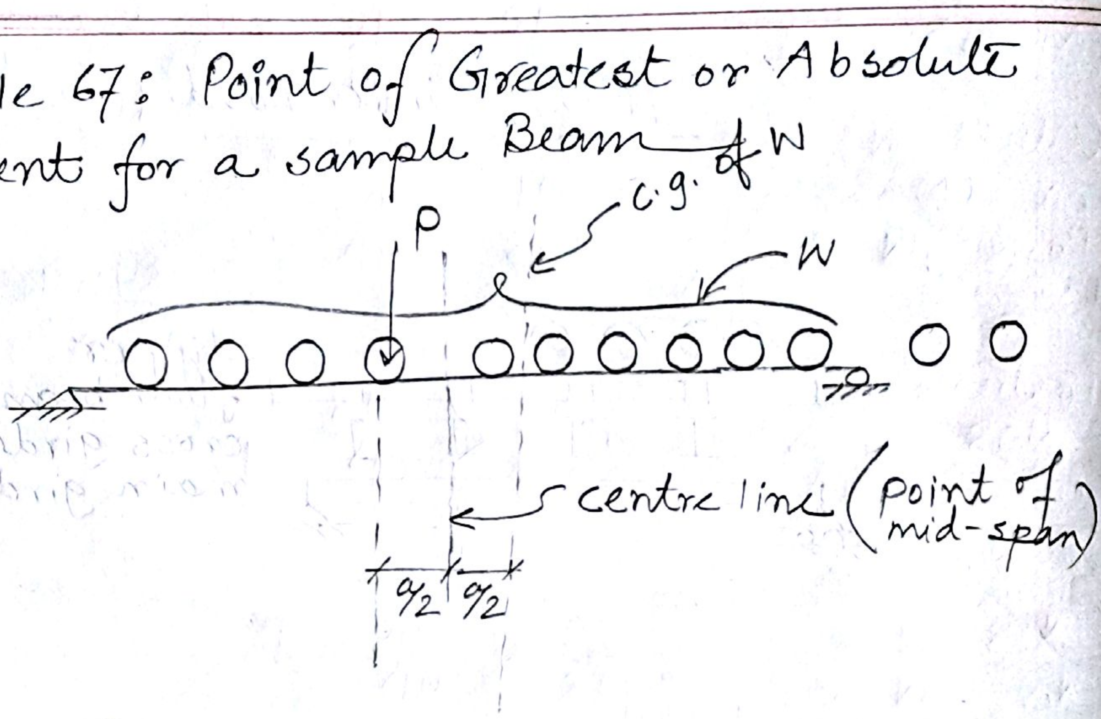
$\left.+ 120' * 37.5^k + 115' * 37.5^k\right] + \frac{1}{2}\left(\frac{1.3021}{125} \times 110\right) \times 110 \times 3^{k/ft}$

$+ \frac{1.3021}{75}\left[70 \times 37.5 + 65 \times 37.5 + 50 \times 30 + 45 + 30\right.$

$\left.+ 40 \times 30 + 35 \times 30 + 30 \times 30 + 20 \times 15\right]$

$= 526.96^k$ (Tension)

Article 67: Point of Greatest or Absolute Moment for a sample Beam



W = weight of wheel on the span

P = wheel that gives maximum moment at midspan

a = distance between P & C.G of W

Procedure:

(1) Find the wheel (P) that gives maximum moment at mid~~span~~-section

(2) Place this wheel "P" & CG of W (loads on the span) equidistant ($\frac{a}{2}$) from mid-section.

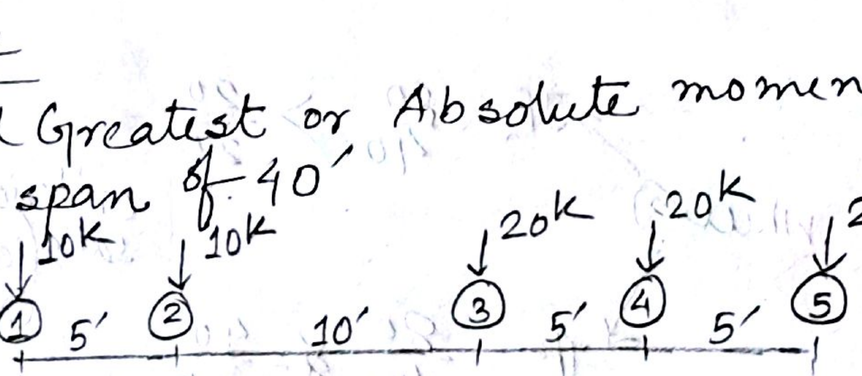
(3) Check criterion for maximum moment $\left(\frac{W}{L} - \frac{W}{a'}\right)$ at the section under load "P".

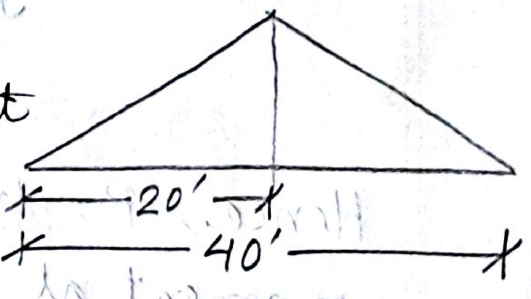
If OK, calculate greatest or Absolute moment.

If Not, try with next wheel.

Example

Find Greatest or Absolute moment for a sample span of 40′

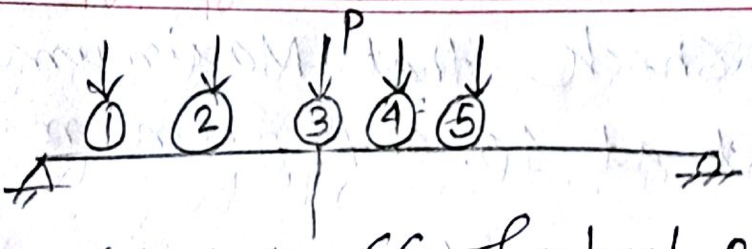




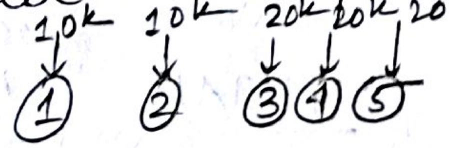
Step-1: Find the wheel that gives maximum moment at Mid-section

TRIAL	Position of Wheel	$\frac{W}{L}$		$\frac{W_1}{a'}$	Remarks	CALCULATIONS
1	Wheel ② at midspan → JR	$\frac{60}{40}$	$>$	$\frac{10}{20}$	Criterion not ok	W = wheel ① to ④ = 60k W_1 = wheel ① = 10k
	JL	$\frac{80}{40}$	$>$	$\frac{20}{20}$		W = wheel ① to ⑤ = 80k W_1 = wheel ① to ② = 20k
2	Wheel ③ → JR	$\frac{80}{40}$	$>$	$\frac{20}{20}$	criterion satisfied	W = 80k W_1 = 20k
	JL	$\frac{80}{40}$	$\cancel{>} =$	$\frac{40}{20}$		W = 80k W_1 = 40k

Hence, P = Wheel ~~②~~ ③ that gives maximum moment at Mid-Section



Step-2: Calculate CG of wheel on the span



Distance of CG from wheel ⑤

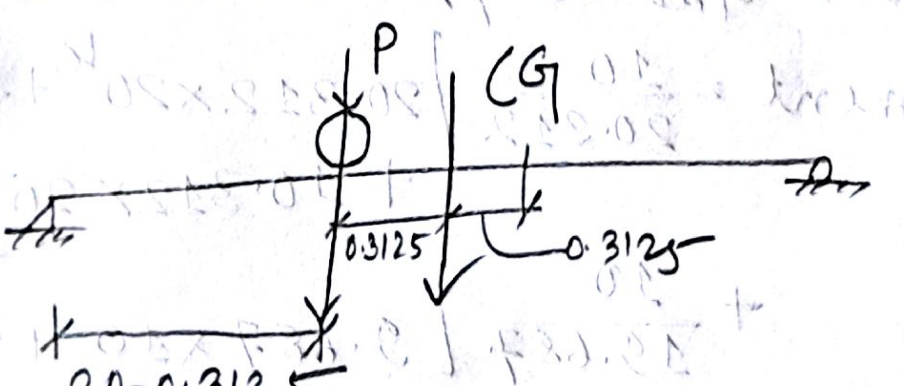
$$X = \frac{20\cancel{0}\times0 + 20\cancel{0}\times5 + 20\times10 + 10\times20 + 10\times25}{20 + 20 + 20 + 10 + 10}$$

$= 9.375'$ (from wheel ⑤)

a = distance between P & CG

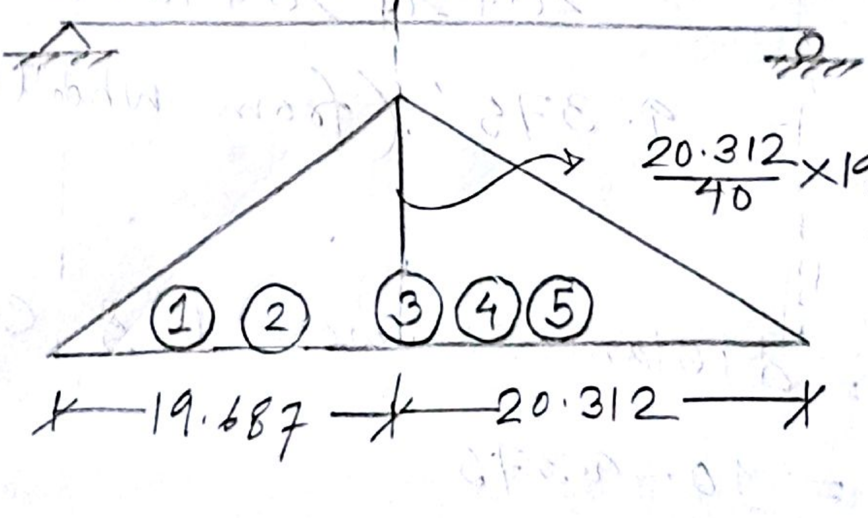
$= 10 - 9.375$

$= 0.625'$



Step-3: Check that Maximum Moment occur at at 19.687′ from mid-span.

TRIAL	Position of wheel		$\frac{W}{L}$	$\frac{W_1}{a'}$	Remarks
1	Wheel ③	→ JR	$\frac{80}{40}$	$> \frac{20}{20}$	criterion satisfied
		→ JL	$\frac{80}{40}$	$= \frac{40}{20}$	



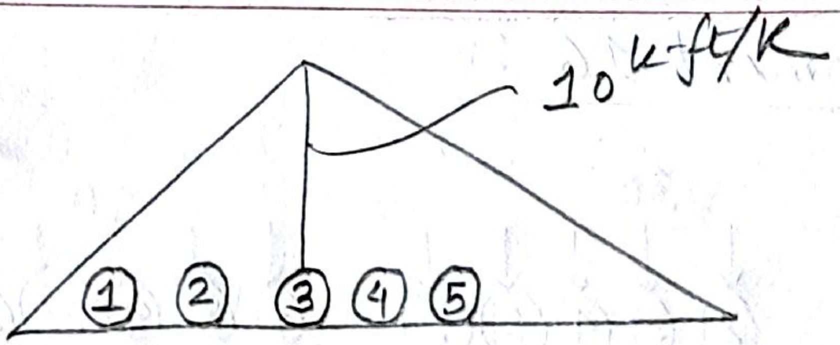
$\frac{20.312}{40} \times 19687 = 10^{k\text{-}ft/k}$

Hence, Greatest / Absolute

$$\text{moment} = \frac{10}{20.312}\left[20.312 \times 20^{k} + 15.312 \times 20 + 10.312 \times 20\right]$$

$$+ \frac{10}{19.687}\left[9.687 \times 10 + 4.687 \times 10\right]$$

$$= 525.317^{\,k\text{-}ft} > 525$$



Maximum Moment at Mid-Section

$$= \frac{10}{20}\left[20\times20+15\times20+10\times20\right]$$

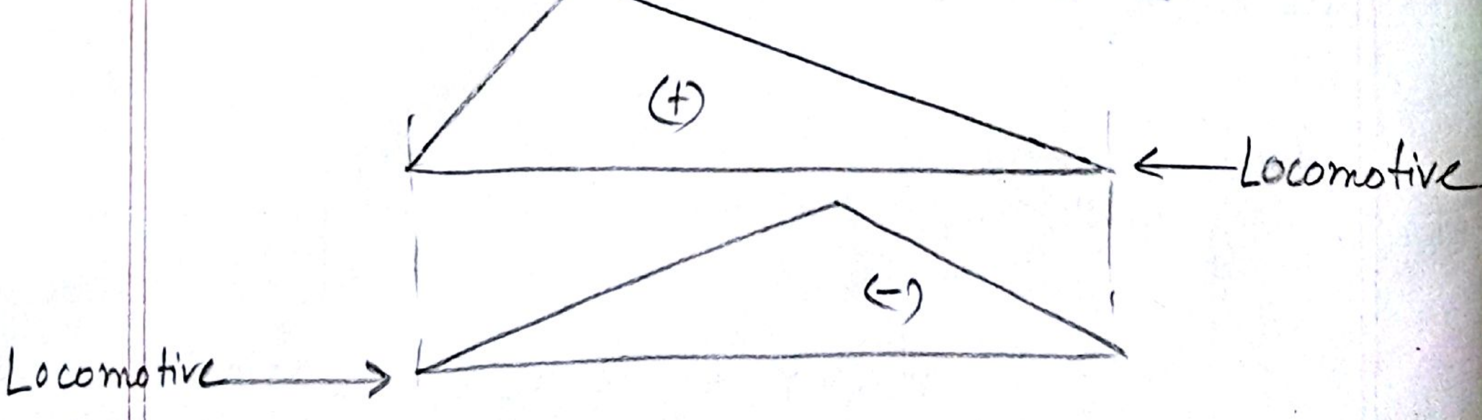
$$+ \frac{10}{20}\left[10\times10 \times 5\times10\right]$$

$$= 525^{k\text{-}ft}$$

Assignment 16 :

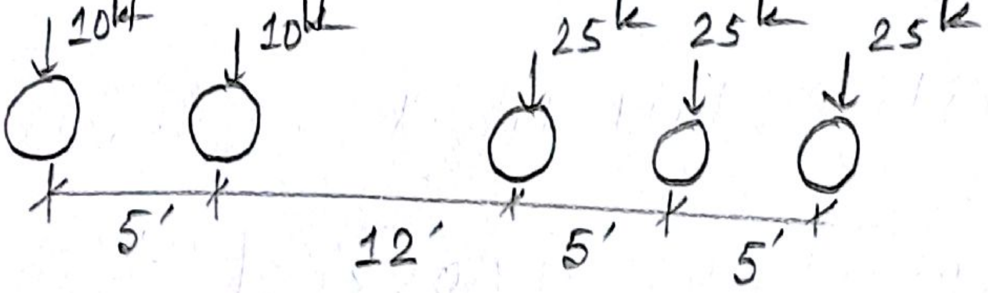
- Find U4U5 , distance same

Result: compression হবে। Locomotive may need to be moved from left to right.



Assignment 17:

Find absolute moment for a ~~samp~~ simple span of 50´



Article 69

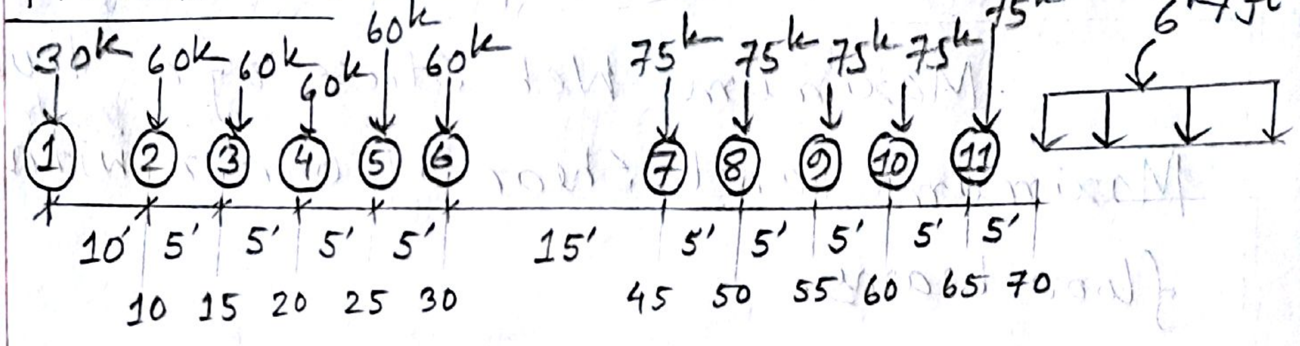
Maximum Web Stress of a Truss/ Maximum Panel Shear of Girder with floor beams.

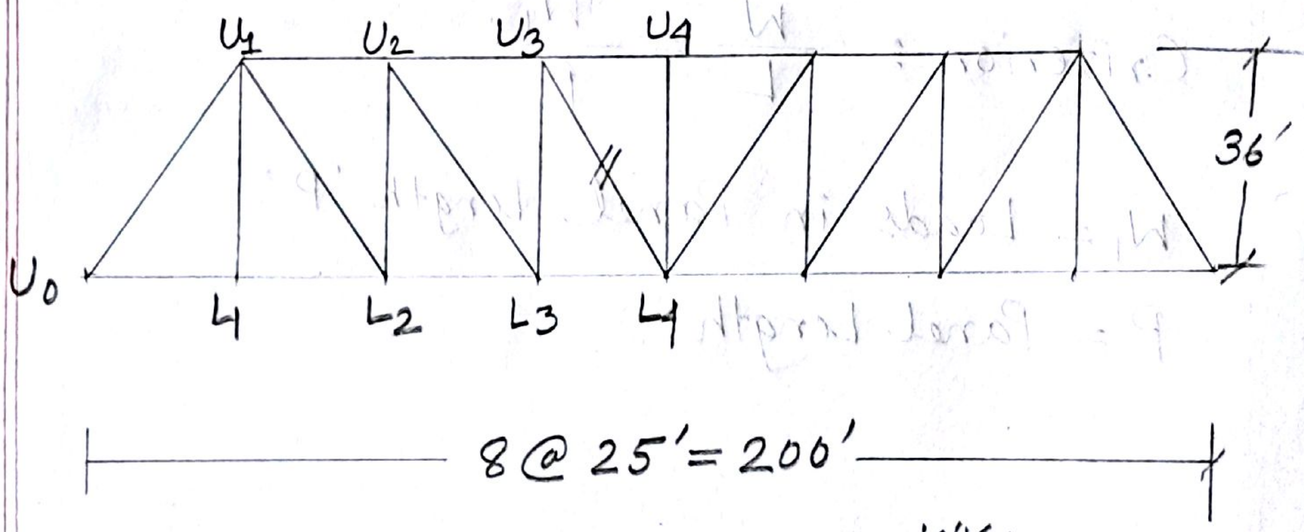
Criterion: $\frac{W}{L} = \frac{W_1}{P}$

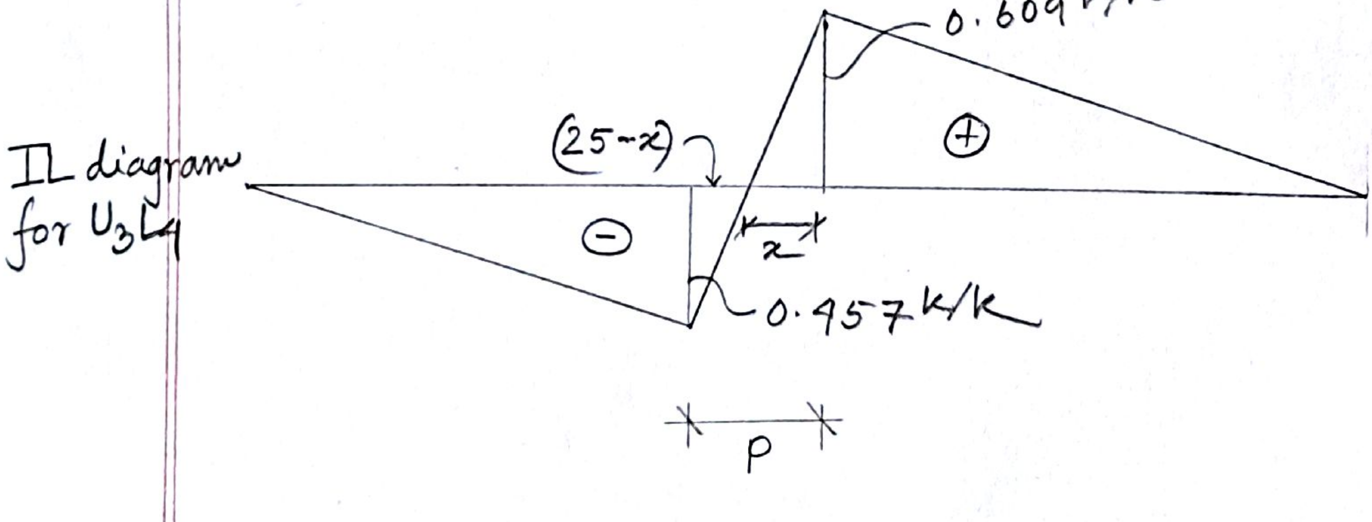
W_1 = Loads in Panel length 'P'

P = Panel Length

Problem 221: Calculate max^{m} tension in U_3L_4 of the beam:



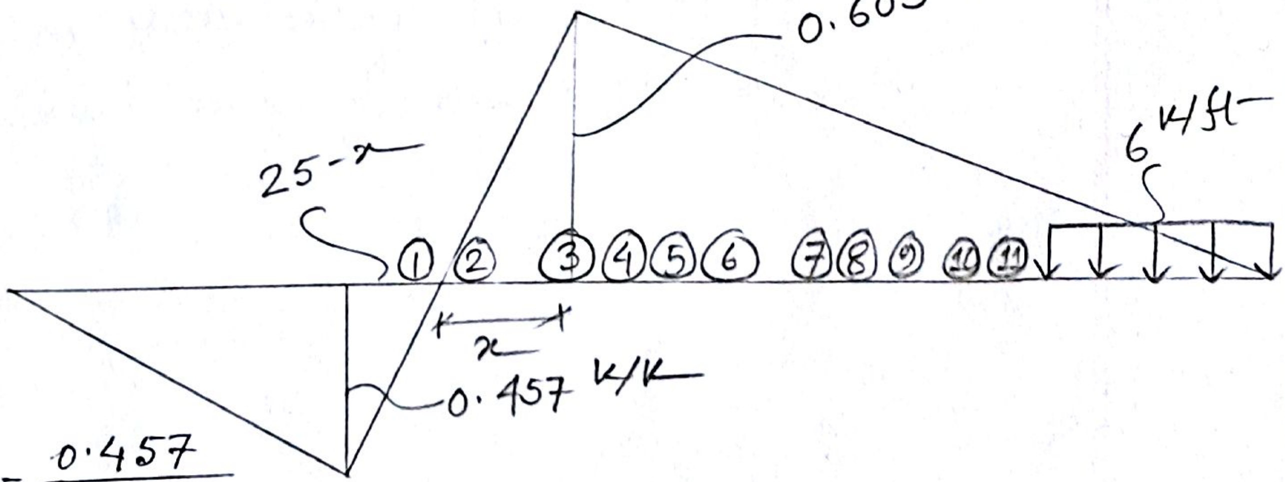




(1) For maximum Tension Load Moves from Right to Left

(2) For maximum Compression Load Moves from L→R

Trial No.	Position of Wheel	$\frac{W}{L}$	$\frac{W_1}{P}$	Remarks	Calculation
1	wheel② at L_4	JR $\frac{945}{200} >$ JL $\frac{945}{200} >$	$\frac{30}{25}$ $\frac{90}{25}$	criterion not ok.	W = wheel① to ⑪ +40' UDL = 945^k W_1 = wheel① = 30^k W = 945^k W_1 = wheel① to② = 90^k
2	wheel③ at L_4	JR $\frac{975}{200} >$ JL $\frac{975}{200} <$	$\frac{90}{25}$ $\frac{150}{25}$	criteria satisfied	W= W_1= W= W_1=



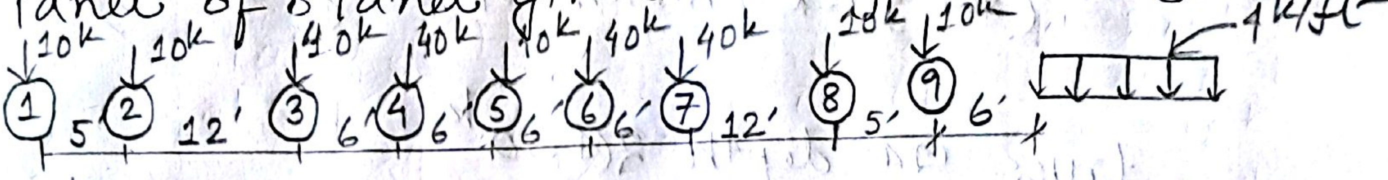
$$\frac{0.609}{x} = \frac{0.457}{25-x}$$

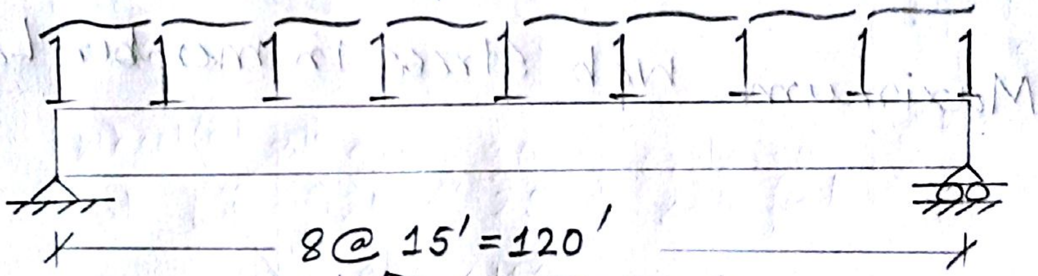
$\therefore x = 9.28$ ft

Maximum Tension =

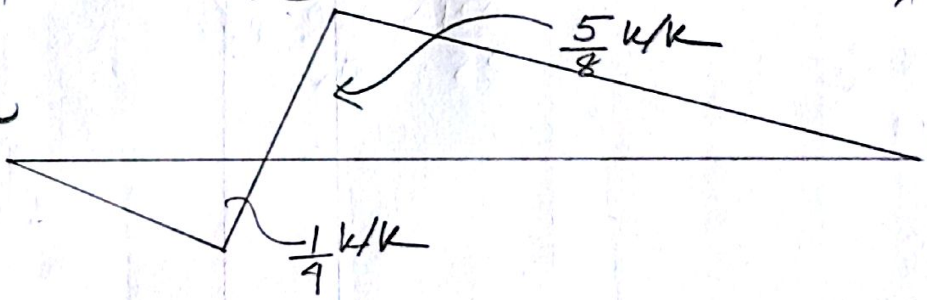
$= 166.04^{k}$

Problem 134: Calculate Maximum shear in 3rd Panel of 8 Panel girder with floor beams.





IL-diagram for 3rd panel



Maximum shear = +ve shear

Load move from Right to Left

Ans. 119.26^k

Assignment 18:

(** first problem টায় change হবে —)

truss এর depth হবে 30^k

60^k load হবে 65^k

Maximum web stress in member $L_4 U_5$

Assignment 19: Girder with floor beam

(** second problem-টির change হবে)

panel length হবে 18' (8 @ 18')

40k load এর পরিবর্তে 45k load দিতে হবে,