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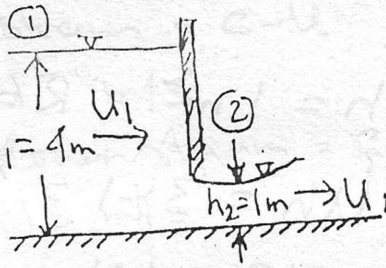
OPEN CHANNEL FLOW
Solutions to Problems of Chap. 1

1.1(a) Rectangle $b = 6\,m$

$h_1 = 4\,m$, $h_2 = 1\,m$,

$U_1 = 2\,m$, $U_2 = 8\,m/s$



Upstream

$A_1 = bh_1 = 24\,m^2$, $P_1 = b + 2h = 14\,m$, $R_1 = A_1/P_1 = 1.714\,m$

$$Re_1 = \frac{U_1 R_1}{\nu} = \frac{2 \times 1.714}{10^{-6}} = 3.43 \times 10^6 > 12{,}500$$

$$Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{U_1}{\sqrt{gh_1}} = \frac{2}{\sqrt{9.81 \times 4}} = 0.32 < 1$$

Hence, the flow upstream of the sluice gate is subcritical turbulent.

Downstream (at the vena contracta)

$A_2 = bh_2 = 6\,m^2$, $P_2 = b + 2h_2 = 8\,m$, $R_2 = \frac{A_2}{P_2} = 0.75\,m$

$$Re_2 = \frac{U_2 R_2}{\nu} = \frac{8 \times 0.75}{10^{-6}} = 6 \times 10^6 > 12{,}500$$

$$Fr_2 = \frac{U_2}{\sqrt{gD_2}} = \frac{U_2}{\sqrt{gh_2}} = \frac{8}{\sqrt{9.81 \times 1}} = 2.55 > 1$$

Hence, the flow downstream of the sluice gate is supercritical turbulent.

(b) $A = 33{,}500\,m^2$; $Q = 56{,}200\,m^3$, $B = 3820\,m$

The river is wide.

$U = Q/A = 56200/33500 = 1.68\,m/s$

$h = R = D = A/B = 33500/3820 = 8.77\,m$

$$Re = \frac{UR}{\nu} = \frac{Uh}{\nu} = \frac{1.68 \times 8.77}{10^{-6}} = 14.73 \times 10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{U}{\sqrt{gh}} = \frac{1.68}{\sqrt{9.81 \times 8.77}} = 0.18 < 1$$

Hence, the state of flow is subcritical turbulent.

1.2 $h = 1\,m \quad U = 3\,m/s$

i) <u>Wide</u>

$$A = h = 1\,m^2 \quad R = D = h = 1\,m$$

$$q = Uh = 3 \times 1 = 3\,m^3/s/m \text{ or } m^2/s$$

$$Re = \frac{UR}{\nu} = \frac{Uh}{\nu} = \frac{3 \times 1}{10^{-6}} = 3 \times 10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{U}{\sqrt{gh}} = \frac{3}{\sqrt{9.81 \times 1}} = 0.96 < 1$$

Hence, the flow is subcritical turbulent.

$$c = \sqrt{gD} = \sqrt{gh} = \sqrt{9.81 \times 1} = 3.13\,m/s$$

$\therefore$ Speed of wave front upstream $= c - U$
$= 3.13 - 3 = 0.13\,m/s$

Speed of want front downstream $= c + U$
$= 3.13 + 3 = 6.13\,m/s$

ii) <u>Rectangle</u> $b = 6\,m$

$$A = bh = 6 \times 1 = 6\,m^2 \quad P = b + 2h = 6 + 2 \times 1 = 8\,m$$

$$R = A/P = 6/8 = 0.75\,m \quad D = A/B = h = 1\,m$$

$$Q = AU = 6 \times 3 = 18\,m^3/s$$

$$Re = \frac{UR}{\nu} = \frac{3 \times 0.75}{10^{-6}} = 2.25 \times 10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{U}{\sqrt{gh}} = \frac{3}{\sqrt{9.81 \times 1}} = 0.96 < 1$$

Hence, the flow is subcritical turbulent.

2

$c = \sqrt{gD} = \sqrt{gh} = \sqrt{9.81 \times 1} = 3.13$ m/s

$\therefore$ Speed of wave front upstream $= c - u$

$= 3.13 - 3 = 0.13$ m/s

Speed of wave front downstream $= c + u$

$= 3.13 + 3 = 6.13$ m/s

iii) <u>Trapezoid</u> $b = 6\text{m}$ $s = 2$

$A = (b + sh)h = (6 + 2 \times 1) \times 1 = 8\ \text{m}^2$

$P = b + 2\sqrt{1+s^2}\,h = 6 + 2\sqrt{1+2^2} \times 1 = 10.47\text{m}$

$B = b + 2sh = 6 + 2 \times 2 \times 1 = 10\text{ m}$

$R = A/P = 8/10.47 = 0.76\text{m}$

$D = A/B = 8/10 = 0.8\text{ m}$

$Q = AU = 8 \times 3 = 24\ \text{m}^3/\text{s}$

$Re = \frac{UR}{\nu} = \frac{3 \times 0.76}{10^{-6}} = 2.28 \times 10^6 > 12,500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 0.8}} = 1.07 > 1$

Hence, the flow is supercritical turbulent.

$c = \sqrt{gD} = \sqrt{9.81 \times 0.8} = 2.80$ m/s $< U$

Hence, both the wave fronts move downstream.

Speed of one wave front $= U - c = 3 - 2.80$

$= 0.20$ m/s

Speed of another wave front $= U + c$

$= 3 + 2.80 = 5.80$ m/s

) <u>Triangle</u> $s = 2$

$A = sh^2 = 2\times1^2 = 2\,m^2$ $\quad P = 2\sqrt{1+s^2}\,h = 2\sqrt{1+2^2}\times1$

$B = 2sh = 2\times2\times1 = 4\,m$ $\quad = 4.47\,m$

$R = A/P = 2/4.47 = 0.45\,m$ $\quad D = A/B = 2/4 = 0.50\,m$

$Q = AU = 2\times3 = 6\,m^3/s$

$$Re = \frac{UR}{\nu} = \frac{3\times0.45}{10^{-6}} = 1.35\times10^6 > 12,500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81\times0.50}} = 1.35 > 1$$

Hence, the flow is supercritical turbulent.

$C = \sqrt{gD} = \sqrt{9.81\times0.50} = 2.21\,m/s < U$

Hence, both the wave fronts move downstream.

Speed of one wave front $= U - c = 3 - 2.21 = 0.79\,m/s$

Speed of another wave front $= U + c = 3 + 2.21$
$= 5.21\,m/s$

) <u>Parabola</u> $\quad B = 4\,m,\ h = 1\,m,\ \frac{4h}{B} = \frac{4\times1}{4} = 1$

$A = \frac{2}{3}Bh = \frac{2}{3}\times4\times1 = 2.667\,m^2$

$$P = \frac{B}{2}\left[\sqrt{1+(4h/B)^2} + \frac{B}{4h}\ln\left(\frac{4h}{B} + \sqrt{1+(4h/B)^2}\right)\right]$$
$$= \frac{4}{2}\left[\sqrt{1+1^2} + 1\times\ln\left(1+\sqrt{1+1^2}\right)\right] = 4.591\,m$$

$R = A/P = 2.667/4.591 = 0.58\,m$

$D = A/B = 2.667/4 = 0.667\,m$

$Q = AU = 2.667\times3 = 8\,m^3/s$

$$Re = \frac{UR}{\nu} = \frac{3\times0.58}{10^{-6}} = 1.74\times10^6 > 12,500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81\times0.667}} = 1.17 > 1$$

Hence, the flow is supercritical turbulent.

$c = \sqrt{gD} = \sqrt{9.81 \times 0.667} = 2.56 \text{ m/s} < U$

Hence, both the wave fronts move downstream.

Speed of one wave front $= U - c = 3 - 2.56 = 0.44$ m/s

Speed of another wave front $= U + c = 3 + 2.56$
$= 5.56$ m/s

vi) <u>Circle</u> $d_o = 2.5$ m

$\omega = 2\cos^{-1}\left(1 - \frac{2h}{d_o}\right) = 2\cos^{-1}\left(1 - \frac{2 \times 1}{2.5}\right) = 2.739$ rad

$A = \frac{1}{8}(\omega - \sin\omega) d_o^2 = \frac{1}{8}(2.739 - \sin 2.739) 2.5^2 = 1.833 \text{ m}^2$

$P = \frac{1}{2}\omega d_o = \frac{1}{2} \times 2.739 \times 2.5 = 3.424$ m

$B = \left(\sin\frac{\omega}{2}\right) d_o = \left(\sin \frac{2.739}{2}\right) \times 2.5 = 2.449$ m

$R = A/P = 1.833/3.424 = 0.535$ m

$D = A/B = 1.833/2.449 = 0.748$ m

$Q = AU = 1.833 \times 3 = 5.50 \text{ m}^3/\text{s}$

$Re = \frac{UR}{\nu} = \frac{3 \times 0.535}{10^{-6}} = 1.61 \times 10^6 > 12{,}500$

$Fr = \frac{U}{\sqrt{gD}} = \frac{3}{\sqrt{9.81 \times 0.748}} = 1.11 > 1$

Hence, the flow is supercritical turbulent.

$c = \sqrt{gD} = \sqrt{9.81 \times 0.748} = 2.71 \text{ m/s} < U$

Hence, both the wave fronts move downstream.

Speed of one wave front $= U - c = 3 - 2.71 = 0.29$ m/s

Speed of another wave front $= U + c = 3 + 2.71 = 5.71$ m/s

a) Wide $\quad h = 5\,m \quad$ Distance $= 30\,km$

Speed of tidal wave, $c = \sqrt{gD} = \sqrt{gh}$

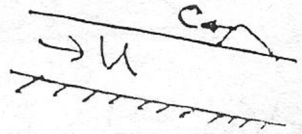
($\because$ for a wide channel, $D = h$)

$\therefore c = \sqrt{9.81 \times 5} = 7.00\ m/s$

i) $t = \dfrac{\text{Distance}}{c} = \dfrac{30 \times 1000}{7.00} = 4286\,s$

$= 1.19$ hrs

ii) $U = 1\,m/s$



$t = \dfrac{\text{Distance}}{c - U} = \dfrac{30 \times 1000}{7 - 1} = 5000\,s$

$= 1.39$ hrs

b) Let c be the celerity of the waves
hich travel a distance equal to 50 m in
) s.

$\therefore c \times 10 = 50 \qquad \therefore c = 50/10 = 5\ m/s$

But $c = \sqrt{gD} = \sqrt{gh}$ (assuming the pond is wide)

$\therefore h = \dfrac{c^2}{g} = \dfrac{5^2}{9.81} = 2.548\ m$

$\therefore$ Volume of water in the pond

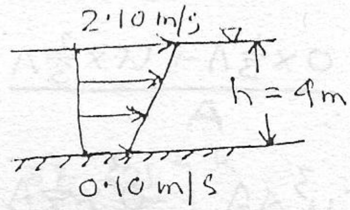
$=$ surface area of the pond $\times$ depth

$= \pi r^2 \times h = 3.14 \times 50^2 \times 2.548 = 20{,}015\,m^3$

The volume computed is approximate
ince it cannot be expected that the wave
travels at a uniform speed from the centre of
the lake to its edge.

1.4 (a) Since the river is wide,

$R = D = h = 4\,m$



i) $\bar{U} = \dfrac{0.1 + 2.10}{2} = 1.10\ m/s$

$\therefore\ q = \bar{U}h = 1.10 \times 4 = 4.40\ m^2/s$

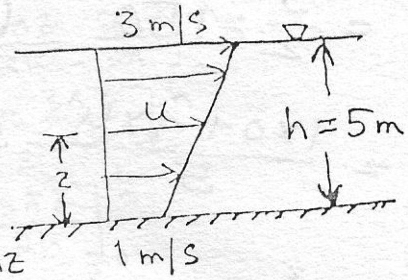
ii) $Re = \dfrac{\bar{U}R}{\nu} = \dfrac{\bar{U}h}{\nu} = \dfrac{1.10 \times 4}{10^{-6}} = 4.40 \times 10^{6} > 12{,}500$

$Fr = \dfrac{\bar{U}}{\sqrt{gD}} = \dfrac{\bar{U}}{\sqrt{gh}} = \dfrac{1.10}{\sqrt{9.81 \times 4}} = 0.18 < 1$

Hence, the flow is subcritical turbulent.

(b) Since the river is wide,

$R = D = h = 5\,m$



i) $\bar{U} = \dfrac{1}{h}\displaystyle\int_0^h u\,dz = \dfrac{1}{h}\int_0^h (1 + 2z/h)\,dz$

$= \dfrac{1}{h}\left[z + \dfrac{2z^2}{2h}\right]_0^h = \dfrac{1}{h}\left[h + h\right] = 2\,m/s$

or, using the trapezoidal rule

$\bar{U} = \dfrac{1+3}{2} = 2\ m/s$

$\therefore\ q = \bar{U}h = 2 \times 5 = 10\ m^2/s$

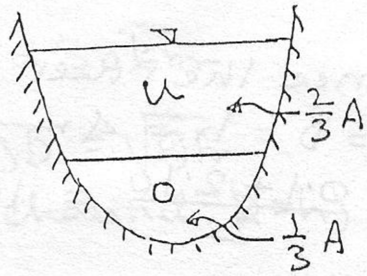
ii) $Re = \dfrac{\bar{U}R}{\nu} = \dfrac{\bar{U}h}{\nu} = \dfrac{2 \times 5}{10^{-6}} = 10 \times 10^{6} > 12{,}500$

$Fr = \dfrac{\bar{U}}{\sqrt{gD}} = \dfrac{\bar{U}}{\sqrt{gh}} = \dfrac{2}{\sqrt{9.81 \times 5}} = 0.29 < 1$

Hence, the flow is subcritical turbulent.

7

5(a) $$U = \frac{0\times\frac{1}{3}A + u\times\frac{2}{3}A}{A} = \frac{2}{3}u$$



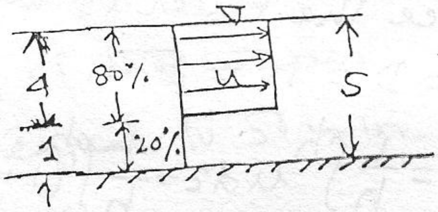
$$\alpha = \frac{\Sigma u^3 \Delta A}{U^3 A} = \frac{0^3\times\frac{1}{3}A + u^3\times\frac{2}{3}A}{\left(\frac{2}{3}u\right)^3 A}$$

$$= \frac{u^3\times\frac{2}{3}\times A}{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times u^3\times A} = \frac{9}{4} = 2{\cdot}25$$

$$\beta = \frac{\Sigma u^2 \Delta A}{U^3 A} = \frac{0\times\frac{1}{3}A + u^2\times\frac{2}{3}A}{\left(\frac{2}{3}u\right)^2 A} = \frac{u^2\times\frac{2}{3}\times A}{\frac{2}{3}\times\frac{2}{3}\times u^2\times A}$$

$$= \frac{3}{2} = 1{\cdot}50 \qquad \left(\frac{\alpha-1}{\beta-1} = \frac{2{\cdot}25-1}{1{\cdot}5-1} = \frac{1{\cdot}25}{{\cdot}5} = 2{\cdot}5\right)$$

(b) $$\bar{U} = \frac{1\times 0 + 4\times u}{5} = \frac{4}{5}u$$



$$\alpha = \frac{\Sigma u^3 \Delta A}{\bar{U}^3 A} = \frac{0^3\times 1 + u^3\times 4}{(4/5\,u)^3\times 5}$$

$$= \frac{4u^3\times 5^3}{4^3\times u^3\times 5} = \frac{5^2}{4^2} = \frac{25}{16} = 1{\cdot}5625$$

$$\beta = \frac{\Sigma u^2 \Delta A}{\bar{U}^2 A} = \frac{0^2\times 1 + u^2\times 4}{(4/5\,u)^2\times 5} = \frac{u^2\times 4\times 5^2}{4^2\times u^2\times 5}$$

$$= \frac{5}{4} = 1{\cdot}25 \qquad \left(\frac{\alpha-1}{\beta-1} = \frac{1{\cdot}5625-1}{1{\cdot}25-1} = \frac{0{\cdot}5625}{0{\cdot}25} = 2{\cdot}25\right)$$

6(a) $u = u_0 \sin \frac{\pi z}{2h}$

$$\bar{U} = \frac{1}{h}\int_0^h u_0 \sin \frac{\pi z}{2h}\, dz = \frac{u_0}{h}\left[-\cos \frac{\pi z}{2h}\cdot \frac{2h}{\pi}\right]_0^h$$

$$= -\frac{2u_0}{\pi}[0-1] = \frac{2u_0}{\pi}$$

$$\alpha = \frac{\int_A u^3 dA}{\bar{U}^3 A} = \frac{\pi^3}{8u_0^3 h}\int_0^h u_0^3 \sin^3 \frac{\pi z}{2h}\, dz$$

$$= \frac{\pi^3}{8h}\int_0^h \frac{1}{4}\left(3\sin \frac{\pi z}{2h} - \sin \frac{3\pi z}{2h}\right) dz$$

$$= \frac{\pi^3}{32h}\int_0^h \left[+3\sin \frac{\pi z}{2h} - \sin \frac{3\pi z}{2h}\right] dz$$

$$= \frac{\pi^3}{32h}\left[-3\cos \frac{\pi z}{2h}\cdot \frac{2h}{\pi} + \cos \frac{3\pi z}{2h}\cdot \frac{2h}{3\pi}\right]_0^h$$

$$= \frac{\pi^3}{32h}\left[-\frac{6h}{\pi}\cos \frac{\pi z}{2h} + \frac{2h}{3\pi}\cos \frac{3\pi z}{2h}\right]_0^h$$

$$= \frac{\pi^3 \times 2h}{3\pi \times 32h}\left[-9\cos \frac{\pi z}{2h} + \cos \frac{3\pi z}{2h}\right]_0^h$$

$$= \frac{\pi^2}{48}[-0+9+0-1] = \frac{\pi^2}{6} = 1.645$$

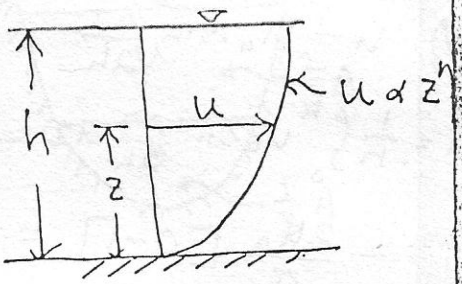
$$\beta = \frac{\int u^2 dA}{\bar{U}^2 A} = \frac{\pi^2}{4u_0^2 h}\int_0^h u_0^2 \sin^2 \frac{\pi z}{2h}\, dz$$

$$= \frac{\pi^2}{4h}\int_0^h \frac{1}{2}\left(1-\cos \frac{2\pi z}{2h}\right) dz = \frac{\pi^2}{8h}\left[z - \sin \frac{\pi z}{h}\cdot \frac{h}{\pi}\right]_0^h$$

$$= \frac{\pi^2}{8h}[h-0-0+0] = \frac{\pi^2 h}{8h} = \frac{\pi^2}{8} = 1.234$$

$$\frac{\alpha-1}{\beta-1} = \frac{1.645-1}{1.234-1} = \frac{0.645}{0.234} = 2.76$$

6(b) $u = kz^n$, k = constant



$$\overline{U} = \frac{1}{h}\int_0^h u\,dz = \frac{1}{h}\int_0^h kz^n\,dz$$

$$= \frac{k}{h}\left[\frac{z^{n+1}}{n+1}\right]_0^h = \frac{kh^n}{n+1}$$

$$\alpha = \frac{1}{\overline{U}^3 h}\int_0^h u^3\,dz = \frac{1}{\overline{U}^3 h}\int_0^h k^3 z^{3n}\,dz$$

$$= \frac{k^3}{\overline{U}^3 h}\left[\frac{z^{3n+1}}{3n+1}\right]_0^h = \frac{k^3}{\overline{U}^3 h}\,\frac{h^{3n+1}}{3n+1}$$

$$= \frac{(n+1)^3 k^3}{k^3 h^{3n} h}\cdot\frac{h^{3n+1}}{3n+1} = \frac{(n+1)^3}{3n+1}$$

$$\beta = \frac{1}{\overline{U}^2 h}\int_0^h u^2\,d \quad = \frac{1}{\overline{U}^2 h}\int_0^h k^2 z^{2n}\,dz$$

$$= \frac{k^2}{\overline{U}^2 h}\left[\frac{z^{2n+1}}{2n+1}\right]_0^h = \frac{k^2}{\overline{U}^2 h}\,\frac{h^{2n+1}}{2n+1}$$

$$= \frac{(n+1)^2 k^2}{k^2 h^{2n} h}\cdot\frac{h^{2n+1}}{2n+1} = \frac{(n+1)^2}{2n+1}$$

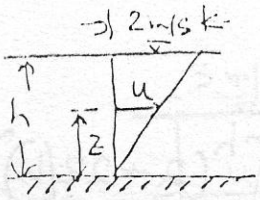
$$\therefore \frac{\alpha-1}{\beta-1} = \frac{\frac{(n+1)^3}{3n+1} - 1}{\frac{(n+1)^2}{2n+1} - 1} = \frac{\frac{(n+1)^3 - (3n+1)}{3n+1}}{\frac{(n+1)^2 - (2n+1)}{2n+1}}$$

$$= \frac{(n^3 + 3n^2 + 3n + 1 - 3n - 1)(2n+1)}{(n^2 + 2n + 1 - 2n - 1)(3n+1)} = \frac{(2n+1)(n+3)}{3n+1}$$

When $n = \frac{1}{7}$, $\alpha = 1{\cdot}045$, $\beta = 1{\cdot}016$ and $\frac{\alpha-1}{\beta-1} = 2{\cdot}83$

	α	β	$(\alpha-1)/(\beta-1)$
Laminar	1·64	1·234	2·76
Turbulent	1·045	1·016	2·83

1.7 (i) $u = 2z/h$



$$\bar{U} = \frac{1}{h}\int_0^h u\,dz = \frac{1}{h}\int_0^h \frac{2z}{h}\,dz \qquad \boxed{\text{or } \bar{U} = \frac{0+2}{2} = 1\ \text{m/s}}$$

$$= \frac{2}{h^2}\int_0^h z\,dz = \frac{2}{h^2}\left.\frac{z^2}{2}\right]_0^h = \frac{2}{h^2}\times\frac{h^2}{2} = 1\ \text{m/s}$$

$$\alpha = \frac{1}{\bar{U}^3 h}\int_0^h u^3\,dz = \frac{1}{\bar{U}^3 h}\int_0^h \frac{8z^3}{h^3}\,dz = \frac{8}{\bar{U}^3 h^4}\left.\frac{z^4}{4}\right]_0^h$$

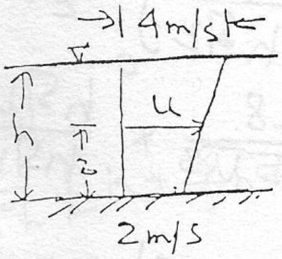
$$= \frac{8}{\bar{U}^3 h^4}\times\frac{h^4}{4} = \frac{2}{1^3} = 2$$

$$\beta = \frac{1}{\bar{U}^2 h}\int_0^h u^2\,dz = \frac{1}{\bar{U}^2 h}\int_0^h \frac{4z^2}{h^2}\,dz = \frac{4}{\bar{U}^2 h^3}\left.\frac{z^3}{3}\right]_0^h$$

$$= \frac{4}{1^2\times h^3}\times\frac{h^3}{3} = 1.333$$

$$\therefore (\alpha-1)/(\beta-1) = 1/.333 = 3.00$$

1.7 (ii) $u = 2 + 2z/h,\ \bar{U} = \frac{2+4}{2} = 3$ m/s



$$\alpha = \frac{1}{\bar{U}^3 h}\int_0^h (2+2z/h)^3\,dz$$

$$= \frac{1}{\bar{U}^3 h}\int_0^h (8 + 24z/h + 24z^2/h^2 + 8z^3/h^3)\,dz$$

$$= \frac{1}{\bar{U}^3 h}\left[8h + \frac{24}{h}\cdot\frac{h^2}{2} + \frac{24}{h^2}\cdot\frac{h^3}{3} + \frac{8}{h^3}\cdot\frac{h^4}{4}\right]$$

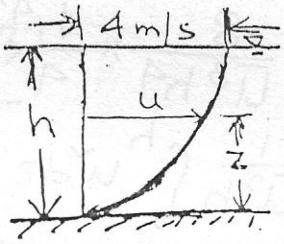
$$= \frac{1}{\bar{U}^3 h}\left[8h + 12h + 8h + 2h\right] = \frac{30h}{\bar{U}^3 h} = \frac{30}{27} = 1.111$$

$$\beta = \frac{1}{\bar{U}^2 h}\int_0^h (2+2z/h)^2\,dz = \frac{1}{\bar{U}^2 h}\int_0^h (4+8z/h+4z^2/h^2)\,dz$$

$$= \frac{1}{\bar{U}^2 h}\left(4h + \frac{8}{h}\cdot\frac{h^2}{2} + \frac{4}{h^2}\cdot\frac{h^3}{3}\right) = \frac{1}{\bar{U}^2 h}\left(4h+4h+\frac{4}{3}h\right)$$

$$= \frac{1}{\bar{U}^2 h}\times\frac{28h}{3} = \frac{28}{3\bar{U}^2} = \frac{28}{3\times 9} = \frac{28}{27} = 1.037$$

$$\therefore\ (\alpha-1)/(\beta-1) = (3/27)\div(1/27) = 3.000$$



(ii) $u = 4(z/h)^{1/2}$

$$\bar{U} = \frac{1}{h}\int_0^h u\,dz = \frac{1}{h}\int_0^h 4(z/h)^{1/2}\,dz$$

$$= \frac{4}{h^{3/2}}\int_0^h z^{1/2}\,dz = \frac{4}{h^{3/2}}\left.\frac{z^{3/2}}{3/2}\right]_0^h = \frac{8}{3}\frac{h^{3/2}}{h^{3/2}} = \frac{8}{3}\ \text{m/s}$$

$$\alpha = \frac{1}{\bar{U}^3 h}\int_0^h u^3\,dz = \frac{1}{\bar{U}^3 h}\int_0^h 64\,z^{3/2}/h^{3/2}\,dz$$

$$= \frac{64}{\bar{U}^3 h^{5/2}}\int_0^h z^{3/2}\,dz = \frac{64}{\bar{U}^3 h^{5/2}}\left.\frac{z^{5/2}}{5/2}\right]_0^h$$

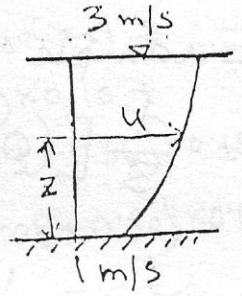
$$= \frac{128}{5\bar{U}^3}\times\frac{h^{5/2}}{h^{5/2}} = \frac{128\times3\times3\times3}{5\times8\times8\times8} = \frac{27}{20} = 1.350$$

$$\beta = \frac{1}{\bar{U}^2 h}\int_0^h u^2\,dz = \frac{1}{\bar{U}^2 h}\int_0^h \frac{16z}{h}\,dz = \frac{16}{\bar{U}^2 h^2}\int_0^h z\,dz$$

$$= \frac{16}{\bar{U}^2 h^2}\left.\frac{z^2}{2}\right]_0^h = \frac{8h^2}{\bar{U}^2 h^2} = \frac{8\times3\times3}{8\times8} = \frac{9}{8} = 1.125$$

$$\therefore\ \frac{\alpha-1}{\beta-1} = \frac{0.350}{0.125} = 2.80$$

7(iv) $u = 1 + 2(z/h)^{1/2}$



$$\bar{U} = \frac{1}{h}\int_0^h u\,dz = \frac{1}{h}\int_0^h \left[1 + 2(z/h)^{1/2}\right] dz$$

$$= \frac{1}{h}\left[z + \frac{2z^{3/2}}{(3/2)h}\right]_0^h = \frac{1}{h}\left[h + \frac{4}{3}h\right]$$

$$= 1 + \frac{4}{3} = \frac{7}{3} \text{ m/s}$$

$$\alpha = \frac{1}{\bar{U}^3 h}\int_0^h u^3\,dz = \frac{1}{\bar{U}^3 h}\int_0^h \left[1 + 2(z/h)^{1/2}\right]^3 dz$$

$$= \frac{1}{\bar{U}^3 h}\int_0^h \left[1 + \frac{6z^{1/2}}{h^{1/2}} + \frac{12z}{h} + \frac{8z^{3/2}}{h^{3/2}}\right] dz$$

$$= \frac{1}{\bar{U}^3 h}\left[z + \frac{6z^{3/2}}{(3/2)h^{1/2}} + \frac{12z^2}{2h} + \frac{8z^{5/2}}{(5/2)h^{3/2}}\right]_0^h$$

$$= \frac{1}{\bar{U}^3 h}\left[h + 4h + 6h + \frac{16}{5}h\right] = \frac{1}{\bar{U}^3}\left[1 + 4 + 6 + \frac{16}{5}\right]$$

$$= \frac{71}{5\bar{U}^3} = \frac{71 \times 3 \times 3 \times 3}{5 \times 7 \times 7 \times 7} = 1.118$$

$$\beta = \frac{1}{\bar{U}^2 h}\int_0^h u^2\,dz = \frac{1}{\bar{U}^2 h}\int_0^h \left[1 + 2(z/h)^{1/2}\right]^2 dz$$

$$= \frac{1}{\bar{U}^2 h}\int_0^h \left[1 + \frac{4z^{1/2}}{h^{1/2}} + \frac{4z}{h}\right] dz$$

$$= \frac{1}{\bar{U}^2 h}\left[z + \frac{4z^{3/2}}{(3/2)h^{1/2}} + \frac{4z^2}{2h}\right]_0^h = \frac{1}{\bar{U}^2 h}\left[h + \frac{8}{3}h + 2h\right]$$

$$= \frac{1}{\bar{U}^2 h} \times \frac{17h}{3} = \frac{17}{3\bar{U}^2} = \frac{17 \times 3 \times 3}{3 \times 7 \times 7} = \frac{51}{49} = 1.041$$

$$\therefore (\alpha - 1)/(\beta - 1) = 0.118/0.041 = 2.89$$

The numerical values of α and β are same for $h = 5$m and $h = 10$m.

8(a) Example 1.5 by Simpson's rule

$$A = \frac{d}{3}\Big[(O_0 + O_n) + 2(O_2 + O_4 + \cdots) + 4(O_1 + O_3 + \cdots)\Big]$$

To apply the Simpson's rule, no. of ordinates must be odd and no. of strips must be even.

In this problem, no. of strips, $n = 6$ and no. of ordinates, $n+1 = 7$. Hence, the Simpson's rule can be used directly.

$$q = \frac{1}{3}\Big[(0 + 4.51) + 2(3.31 + 3.95) + 4(2.95 + 3.62 + 4.12)\Big]$$

$$= \frac{1}{3} \times 61.79 = 20.597 \ m^2/s$$

$$\bar{U} = q/h = 20.597/6 = 3.433 \ m/s$$

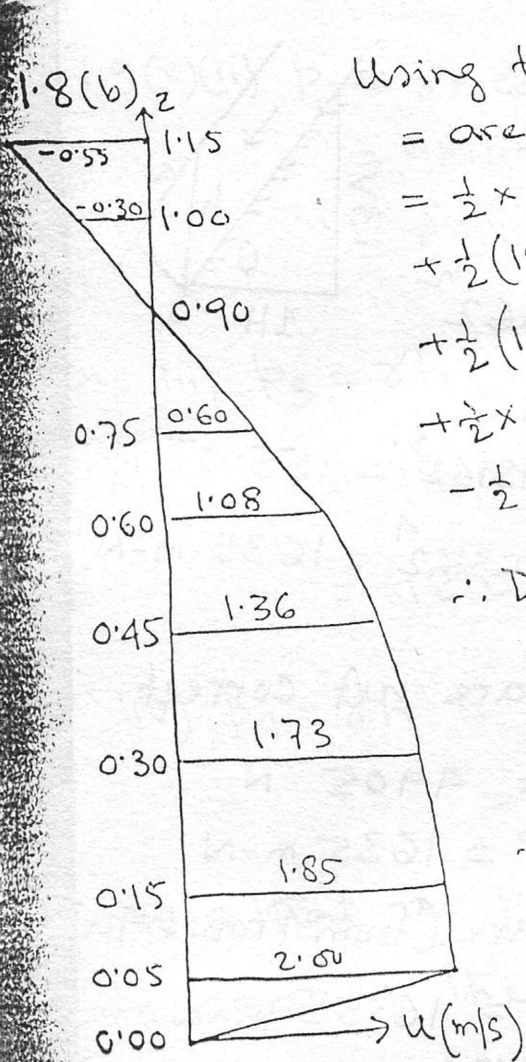
$$\alpha = \frac{1}{3.433^3 \times 6} \times \frac{1}{3}\Big[(0^3 + 4.51^3) + 2(3.31^3 + 3.95^3) + 4(2.95^3 + 3.62^3 + 4.12^3)\Big]$$

$$= \frac{859.702}{3.433^3 \times 6 \times 3} = 1.181$$

$$\beta = \frac{1}{3.433^2 \times 6} \times \frac{1}{3}\Big[(0^2 + 4.51^2) + 2(3.31^2 + 3.95^2) + 4(2.95^2 + 3.62^2 + 4.12^2)\Big]$$

$$= \frac{228.5825}{3.433^2 \times 6 \times 3} = 1.078$$

1.8(b)



Using trapezoidal rule, $\Sigma u \Delta z$

$= \text{area of the plot of } u \text{ against } z$

$= \frac{1}{2}\times 2 \times 0.05 + \frac{1}{2}(2.00+1.85)\times 0.10 + \frac{1}{2}(1.85+1.73)\times 0.15 + \frac{1}{2}(1.73+1.36)\times 0.15 + \frac{1}{2}(1.36+1.08)\times 0.15 + \frac{1}{2}(1.08+0.60)\times 0.15 + \frac{1}{2}\times 0.60 \times 0.15 - \frac{1}{2}\times 0.30\times 0.10 - \frac{1}{2}(0.30+0.55)\times 0.15 = 1.018$

$\therefore$ Discharge per unit width

$q = \Sigma u \Delta z = 1.018 \ m^2/s$

Total depth, $h = 1.15$ m

$\therefore$ Mean velocity, $\bar{U} = \frac{q}{h} = \frac{1.018}{1.15}$

$= 0.885$ m/s

$$\Sigma u^3 \Delta z = \frac{1}{2}\times 2^3 \times 0.05 + \frac{1}{2}(2.00^3+1.85^3)\times 0.10 + \frac{1}{2}(1.85^3+1.73^3)\times 0.15 + \frac{1}{2}(1.73^3+1.36^3)\times 0.15 + \frac{1}{2}(1.36^3+1.08^3)\times 0.15 + \frac{1}{2}(1.08^3+0.60^3)\times 0.15 + \frac{1}{2}\times 0.60^3\times 0.15 - \frac{1}{2}\times 0.30^3 \times 0.10 - \frac{1}{2}(0.30^3+0.55^3)\times 0.15 = 2.7509$$

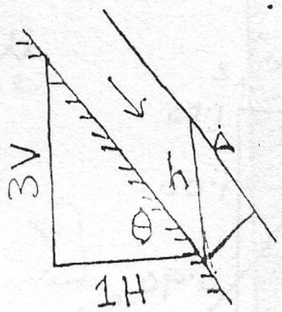
$$\therefore \alpha = \frac{\Sigma u^3 \Delta z}{U^3 h} = \frac{2.7509}{0.885^3 \times 1.15} = 3.450$$

$$\Sigma u^2 \Delta z = \frac{1}{2}\times 2^2 \times 0.05 + \frac{1}{2}(2.00^2+1.85^2)\times 0.10 + \frac{1}{2}(1.85^2+1.73^2)\times 0.15 + \frac{1}{2}(1.73^2+1.36^2)\times 0.15 + \frac{1}{2}(1.36^2+1.08^2)\times 0.15 + \frac{1}{2}(1.08^2+0.60^2)\times 0.15 + \frac{1}{2}\times 0.60^2\times 0.15 + \frac{1}{2}\times 0.30^2 \times 0.10 + \frac{1}{2}(0.30^2+0.55^2)\times 0.15 = 1.7107$$

$$\therefore \beta = \frac{\Sigma u^2 \Delta z}{U^2 h} = \frac{1.7107}{0.885^2 \times 1.15} = 1.906$$

1.9(a) Slope = 1H : 3V, $h = 1$ m

$\tan\theta = 3, \ \theta = 71.56^\circ, \ \cos\theta = 0.3162$



$\therefore p_{bed} = 1000 \times 9.81 \times 1 \times 0.3162^2$

$= 980.83 \ N/m^2$

$F = \frac{1}{2} \times 1000 \times 9.81 \times 1^2 \times 0.3162^3 = 155.07 \ N$

$O.M. = \frac{1}{6} \times 1000 \times 9.81 \times 1^3 \times 0.3162^4 = 16.35 \ m\text{-}N$

(b) The computed results are not correct.

$F_{wrong} = \frac{1}{2} \times 1000 \times 9.81 \times 1^2 = 4905 \ N$

$O.M._{wrong} = \frac{1}{6} \times 1000 \times 9.81 \times 1^3 = 1635 \ m\text{-}N$

The corrected results are (from Prob. 1.9a)

$F = 155.07 N$ and $O.M. = 16.35 \ m\text{-}N$

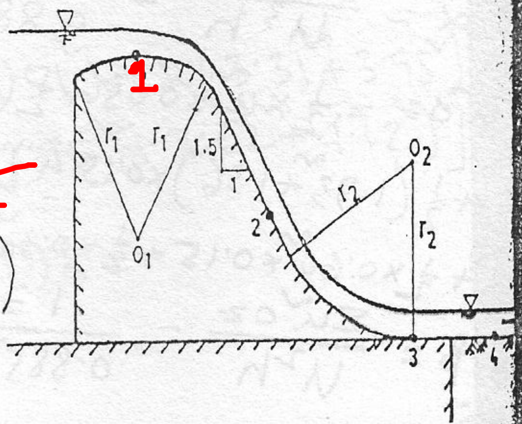
$\% \text{ error for } F = \frac{4905 - 155.07}{155.07} \times 100 = 3063.09$

$\% \text{ error for } O.M. = \frac{1635 - 16.35}{16.35} \times 100 = 9900.00$

(c) $r_1 = r_2 = 20$ m, $q = 6.5 \ m^2/s$

$h_1 = h_2 = h_3 = h_4 = 1$ m

$U = \frac{q}{h} = \frac{6.5}{1} = 6.5$ m/s



i) $p_1 = \gamma h \left(1 - \frac{U^2}{gr}\right)$ $\gamma h - \frac{\gamma}{g}\frac{h U^2}{r}$

$= 1000 \times 9.81 \times 1 \times \left(1 - \frac{6.5^2}{9.81 \times 20}\right)$

$= 9810 \, (1 - 0.215) \ N/m^2$

$= 7{,}697.50 \ N/m^2$

1.9(c)(ii) $p_2 = \gamma h \cos^2\theta$ $\qquad \tan\theta = \frac{1.5}{1} = 1.5$

$= 9810 \times 1 \times \cos^2 56.31^\circ \qquad \therefore \theta = \tan^{-1} 1.5 = 56.31^\circ$

$= 3{,}018.46\ N/m^2$

iii) $p_3 = \gamma h \left(1 + \frac{u^2}{gr}\right)$ $\quad \gamma h + \rho h \frac{u^2}{r} = \gamma h + \frac{\gamma h u^2}{g\, r}$

$= 9810 \times 1 \times \left(1 + \frac{6.5^2}{9.81 \times 20}\right)$

$= 9810\,(1 + 0.215) = 11{,}922.50\ N/m^2$

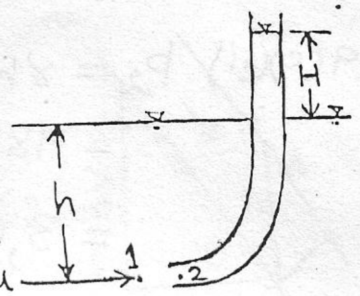
iv) $p_4 = \gamma h = 9810 \times 1\ N/m^2$

$= 9{,}810\ N/m^2$

Note that at point 1, the centrifugal force acts against the hydrostatic force. Point 2 lies on the slope of the spillway and hence the effect of the slope is to be included. At point 3, the centrifugal force adds to the hydrostatic force. Point 4 lies on the horizontal and the pressure due to hydrostatic head h only.

Chapter 2

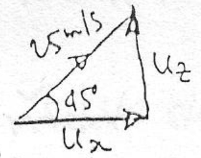
2.1 Applying the energy equation between points 1 and 2 and neglecting the energy loss, we get

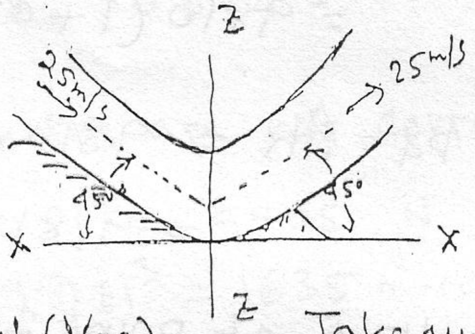


$$0 + h + \frac{u^2}{2g} = 0 + (h+H) + 0$$

or, $\frac{u^2}{2g} = H$ $\therefore$ $u = \sqrt{2gH}$ (Proved)

2.2





$u_z = 25 \sin 45° = 25/\sqrt{2}$ m/s

$u_x = 25 \cos 45° = 25/\sqrt{2}$ m/s

Take unit width

From $V^2 = u^2 - 2gh$, we get ($V=0$)

$$h = \frac{u_z^2}{2g} = \frac{25^2}{2\times2\times9.81} = 15.93 \text{ m}$$

Force in the Z-Z direction (i.e. vertical force)

= mass × change in velocity

= ρQ (change in velocity)

= $1000\times0.5\times1\times25\times[25/\sqrt{2} - (-25/\sqrt{2})]$

= 441941N (acting downward) per unit width

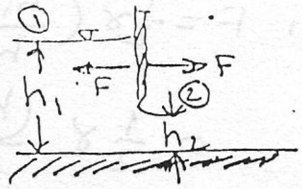
[$\rho Q \cdot 2u_z = \frac{kgm}{m^3}\times\frac{m^3}{s}\times\frac{m}{s} = kgm\text{-}m/s^2 = \text{force}$]

Force in the X-X direction (i.e. horizontal force)

= mass × change in velocity

= $\rho Q(u_x - u_x) = 0$

2·3 (a) Consider unit width of the channel. The force exerted on the fluid by the gate is F which is equal and opposite to that exerted by the fluid on the gate. Assume that (i) section 1 is sufficiently u/s from the gate, (ii) section 2 is at the vena contracta, (iii) $\alpha_1 = \alpha_2 = 1$, (iv) $\beta_1 = \beta_2 = 1$, (v) energy losses between the sections is negligible, and (vi) the bottom friction force F_f between the sections is negligible. Then writing the momentum equation between sections 1 and 2



$$\rho q(U_2 - U_1) = F_{p1} - F_{p2} - F$$

$$\text{or, } \rho q\left(\frac{q}{h_2} - \frac{q}{h_1}\right) = \tfrac{1}{2}\gamma h_1^2 - \tfrac{1}{2}\gamma h_2^2 - F$$

$$\text{or, } F = \tfrac{1}{2}\gamma h_1^2 - \tfrac{1}{2}\gamma h_2^2 - \rho q^2\left(\frac{1}{h_2} - \frac{1}{h_1}\right)$$

$$= \tfrac{1}{2}\gamma h_1^2 - \tfrac{1}{2}\gamma h_2^2 - \rho q^2\left(\frac{h_1 - h_2}{h_1 h_2}\right)$$

$$= \tfrac{1}{2}\gamma\left[(h_1^2 - h_2^2) - \frac{2q^2}{g}\left(\frac{h_1 - h_2}{h_1 h_2}\right)\right]$$

$$= \tfrac{1}{2}\gamma\,\frac{(h_1 - h_2)}{h_1 h_2}\left[h_1 h_2(h_1 + h_2) - \frac{2q^2}{g}\right] \quad (1)$$

The energy equation between sections 1 + 2 gives

$$h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g} \quad \text{or} \quad h_1 + \frac{q^2}{2g h_1^2} = h_2 + \frac{q^2}{2g h_2^2}$$

$$\text{or, } \frac{q^2}{2g}\left[\frac{1}{h_2^2} - \frac{1}{h_1^2}\right] = h_1 - h_2$$

$$\text{or, } \frac{q^2}{2g}\left[\frac{h_1 + h_2}{h_1 h_2}\right] = 1 \quad \text{or} \quad \frac{q^2}{g} = \frac{2 h_1 h_2}{h_1 + h_2} \quad (2)$$

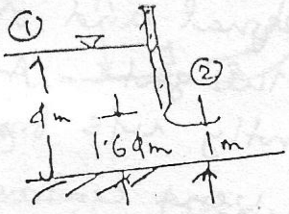
Combining Eqs (1) and (2)

$$F = \tfrac{1}{2}\gamma\left(\frac{h_1 - h_2}{h_1 h_2}\right)\left[h_1 h_2(h_1 + h_2) - \frac{4 h_1 h_2}{h_1 + h_2}\right]$$

or, $F = \frac{1}{2}\gamma\left(\frac{h_1-h_2}{h_1h_2}\right)\left[\frac{h_1h_2(h_1+h_2) - 4h_1h_2}{h_1+h_2}\right]$

$= \frac{1}{2}\gamma \frac{(h_1-h_2)^3}{h_1+h_2}$ (3)

The force on the gate is equal and opposite to F.



1.3 (b)(i) $h_1 = 4m$, $h_2 = 1m$, $b = 6m$

Applying the continuity equation between sections 1 and 2

$A_1U_1 = A_2U_2$ or, $bh_1U_1 = bh_2U_2$ or $4U_1 = U_2$ ①

Applying the energy equation between sections ① & ②

$h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g}$ or, $4 + \frac{U_1^2}{2g} = 1 + \frac{U_2^2}{2g}$

or, $4 + \frac{U_1^2}{2g} = 1 + \frac{16U_1^2}{2g}$ (using (1))

or, $15\frac{U_1^2}{2g} = 4-1 = 3$ $\therefore U_1 = \sqrt{\frac{2\times 9.81\times 3}{15}} = 1.98$ m/s

$\therefore Q = A_1U_1 = bh_1U_1 = 6\times 4\times 1.98 = 47.54\ m^3/s$

ii) $Q = 47.54\ m^3/s$ $U_1 = 1.98$ m/s $U_2 = 7.92$ m/s

$\rho Q(U_2-U_1) = \frac{1}{2}\gamma b h_1^2 - \frac{1}{2}\gamma b h_2^2 - F$

or, $F = \frac{1}{2}\gamma b h_1^2 - \frac{1}{2}\gamma b h_2^2 - \rho Q(U_2-U_1)$

$= \frac{1}{2}\times 1000\times 9.81\times 6(4^2-1^2) - 1000\times 47.54(7.92-1.98)$

$= 441,450 - 282,387.6 = 159,062.4$ N $= 159.06$ kN

When the formula is used

$F = \frac{1}{2}\gamma b\frac{(h_1-h_2)^3}{h_1+h_2} = \frac{1}{2}\times 1000\times 9.81\times 6\ \frac{(4-1)^3}{4+1} = 158922$ N

(3)

3 (contd.) Assuming that the pressure distribution is hydrostatic

$h_g = \frac{h_2}{C_c} = \frac{1}{0.61}$

$= 1.64\,m$

$F = \frac{1}{2}\gamma b (h_1 - h_g)^2$

$= \frac{1}{2} \times 1000 \times 9.81 \times 6 (4 - 1.64)^2$

$= 163913\ N \quad = 163.91\ kN$

Also

Intensity of pressure on the C.G. of the gate

$= \gamma \bar{h}$

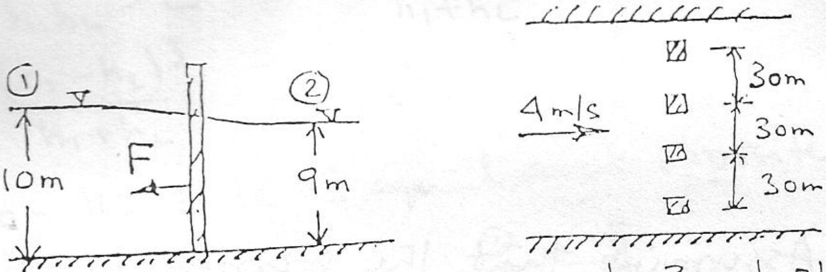
$\therefore F = \gamma \bar{h} A = \rho g \bar{h} A = 1000 \times 9.81 \times (4 - 1.64)/2 \times (4 - 1.64) \times 6$

$= 11575.8 \times 14.16 = 163913.3\ N = 163.91\ kN$

The force on the gate is equal and opposite to F.

4

2.4



$b = 30m, h_1 = 10m,$
$h_2 = 9m, U_1 = 4 m/s$

$q = h_1 U_1 = 10 \times 4 = 40 \ m^2/s$

$U_2 = q/h_2 = 40/9 = 4.44 \ m/s$

Since the piers are placed @ 30m c/c, we consider a width of 30m and apply the steady 1-D momentum eqn. assuming that $\beta_1 = \beta_2 = 1$.

$$\rho Q(U_2 - U_1) = F_{p1} - F_{p2} + \cancel{W \sin\theta}^{\,0} - \cancel{F_f}^{\,0} - F$$

(F is the force on a pier)

$$\therefore F = F_{p1} - F_{p2} - \rho Q (U_2 - U_1)$$

$$= \tfrac{1}{2}\gamma b(h_1^2 - h_2^2) - \rho b q (U_2 - U_1)$$

$$= \tfrac{1}{2} \times 1000 \times 9.81 \times 30 (10^2 - 9^2) - 1000 \times 40 \times 30 \times (4.44 - 4)$$

$$= 1000 \times 30 \left(\tfrac{1}{2} \times 9.81 \times 19 - 40 \times 0.44\right)$$

$$= 1000 \times 30 \times 75.595 = 2268 \times 10^3 \ N$$

$$= 2268 \ kN$$

$\cancel{5}$