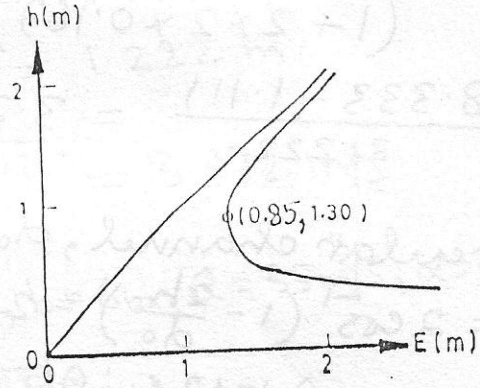
Chapter 3

3.1 (i) Rectangular channel, $b = 6\,m$, $Q = 15\,m^3/s$

$$E = h + \frac{u^2}{2g} = h + \frac{Q^2}{2gA^2} = h + \frac{Q^2}{2gb^2h^2}$$

$$= h + \frac{15^2}{2\times 9.81\times 6^2\times h^2} = h + \frac{0.3186}{h^2}$$

h	E	h	E
0	∞	1.0	1.32
0.2	8.16	1.2	1.42
0.4	2.39	1.5	1.64
0.6	1.48	2.0	2.08
0.7	1.35	5.0	5.02
0.8	1.30	10.0	10.00
0.9	1.29		

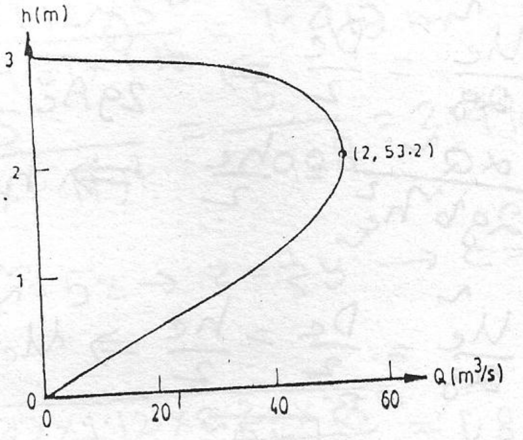


The specific energy curve is plotted as shown. From the plot, we see that $h_c = 0.85\,m$ and $E_c = 1.30\,m$.

(ii) Rectangular channel, $b = 6\,m$, $E = 3\,m$

$$Q^2 = 2gA^2(E-h)$$
$$= 2gb^2h^2(E-h)$$
$$= 2\times 9.81\times 6^2\times h^2(3-h)$$
$$= 706.32\,h^2(E-h)$$

h	Q	h	Q
0	0	1.6	50.31
0.2	8.89	1.8	52.40
0.4	17.14	2.0	53.15
0.6	24.70	2.2	52.29
0.8	31.53	2.4	49.40
1.0	37.59	2.6	43.70
1.2	42.78	2.8	33.27
1.4	47.06	2.9	24.37
		3.0	0



The discharge-depth curve is plotted as shown. From the plot we see that $h_c = 2\,m$ and $Q_{max} = 53.20\,m^3/s$.

.2 (i) Trapezoidal channel, $b = 6\,m$, $s = 2$, $Q = 20\,m^3/s$ 3.3(i

$h = 1\,m$ $h/b = 1/6 = 0.167$ (i

$$M = \frac{3[1+2s(h/b)]^2 - 2s(h/b)[1+s(h/b)]}{[1+2s(h/b)][1+s(h/b)]}$$

$$= \frac{3(1+2\times2\times0.167)^2 - 2\times2\times0.167\times(1+2\times0.167)}{(1+2\times2\times0.167)(1+2\times0.167)}$$ 3.4

$$= \frac{8.333-1.111}{2.222} = 3.25$$

(ii) Circular channel, $d_o = 3m$, $h = 1m$

$$\theta = 2\cos^{-1}\left(1-\frac{2h}{d_o}\right) = 2\cos^{-1}\left(1-\frac{2\times1}{3}\right) = 2.462\ \text{ra}$$ (i

$$M = \left(1-\cos\frac{\theta}{2}\right)\frac{12\sin\theta/2}{\theta-\sin\theta} - \frac{\cos\theta/2}{\sin^2\theta/2}$$

$$= \left(1-\cos\frac{2.462}{2}\right)\frac{12\sin\frac{2.462}{2}}{2.462-\sin 2.462} - \frac{\cos\frac{2.462}{2}}{\sin^2\frac{2.462}{2}}$$

$$= 0.6667\times6.171 - 0.375 = 3.74$$ (ii

3.3 i) $\alpha\frac{U_c^2}{2g} = \frac{D_c}{2} \Rightarrow \alpha\frac{Q^2}{2gA_c^2} = \frac{h_c}{2}$ (for a rectangular channel, $D_c = h_c$)

or, $\frac{\alpha Q^2}{2gb^2h_c^2} = \frac{h_c}{2} \Rightarrow h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{\alpha q^2}{g}}$

ii) $\alpha\frac{U_c^2}{2g} = \frac{D_c}{2} = \frac{h_c}{2} \Rightarrow U_c^2 = \frac{gh_c}{\alpha}$ $\therefore U_c = \sqrt{\frac{gh_c}{\alpha}}$ (i

But $h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} \Rightarrow h_c^3 = \frac{\alpha Q^2}{gb^2}$

$$\therefore U_c = \sqrt{\frac{gh_c}{\alpha}} = \sqrt[6]{\frac{g^3h_c^3}{\alpha^3}} = \sqrt[6]{\frac{g^3}{\alpha^3}\times\frac{\alpha Q^2}{gb^2}} = \sqrt[6]{\frac{g^2Q^2}{\alpha^2b^2}} = \sqrt[3]{\frac{gQ}{\alpha b}}$$

3.3(iii) $E_c = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2} = 1.5h_c$

(iv) $Q = A_c U_c = bh_c\sqrt{gh_c} = \sqrt{g}\,b\,h_c^{1.5} = 3.132\,b\,h_c^{1.5}$

$= \sqrt{g}\,b\left(\frac{E_c}{1.5}\right)^{1.5} = 0.544\sqrt{g}\,b\,E_c^{1.5}$

3.4(i) Wide channel, $q = 4\ m^2/s$, $\alpha = 1.12$

$h_c = \sqrt[3]{\frac{\alpha q^2}{g}} = \sqrt[3]{\frac{1.12\times 4^2}{9.81}} = 1.222\ m$

$U_c = \sqrt{\frac{gh_c}{\alpha}} = \sqrt{\frac{9.81\times 1.222}{1.12}} = 3.372\ m/s$

(ii) Rectangular channel, $b = 6m$, $Q = 35\ m^3/s$, $\alpha = 1.12$

$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{1.12\times 35^2}{9.81\times 6^2}} = 1.083\ m$

$U_c = \sqrt{\frac{gh_c}{\alpha}} = \sqrt{\frac{9.81\times 1.083}{1.12}} = 3.079\ m/s$

(iii) Triangular channel, $s=1$, $Q = 5\ m^3/s$, $\alpha = 1.12$

$h_c = \sqrt[5]{\frac{2\alpha Q^2}{gs^2}} = \sqrt[5]{\frac{2\times 1.12\times 5^2}{9.81\times 1^2}} = 1.417\ m$

$U_c = \frac{Q}{A_c} = \frac{Q}{sh^2} = \frac{5}{1\times 1.417^2} = \frac{5}{2.008} = 2.490\ m/s$

(iv) Parabolic channel, $y^2 = 5z \rightarrow z = \frac{1}{5}y^2 \rightarrow c = \frac{1}{5} = 0.20$

$Q = 25\ m^3/s$, $\alpha = 1.12$

$h_c = \sqrt[4]{\frac{27\alpha c Q^2}{32g}} = \sqrt[4]{\frac{27\times 1.12\times 0.20\times 25^2}{32\times 9.81}} = 1.863\ m$

$A_c = \frac{4h_c^{1.5}}{3\sqrt{c}} = \frac{4\times 1.863^{1.5}}{3\sqrt{0.20}} = 7.580\ m^2$

$\therefore U_c = Q/A_c = 25/7.580 = 3.298\ m/s$

3.5(a) Trapezoidal channel, $b = 6m$, $s = 2$, $Q = 30\ m^3/s$, $\alpha = 1.12$

(i) Trial-and-error method

$$Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{30}{\sqrt{9.81/1.12}} = 10.137$$

$A = (6+2h)h$
$B = 6+4h$
$D = A/B$, $Z = A\sqrt{D}$

h	A	B	D	Z	
1.00	8.00	10.00	0.800	7.155	
2.00	20.00	14.00	1.428	23.905	
1.22	10.297	10.88	0.946	10.017	
1.23	10.406	10.92	0.953	10.158	← Nearest

$\therefore h_c = 1.23\ m$ $\quad U_c = \frac{Q}{A_c} = \frac{30}{10.406} = 2.883\ m/s$

(ii) Bisection method $\quad A = (6+2h)h \quad B = 6+4h$

$$f(h) = 1 - \frac{\alpha Q^2 B}{gA^3} = 1 - \frac{1.12 \times 30^2 \times B}{9.81 \times A^3} = 1 - \frac{102.75B}{A^3}$$

hmin	hmax	hmean	A	B	f(h)	Root lies between
0	10	5	80.00	26.0	0.995	0 and 5
0	5	2.5	27.50	16.0	0.921	0 and 2.5
0	2.5	1.25	10.625	11.0	0.058	0 and 1.25
0	1.25	0.625	4.531	8.5	−8.711	0.625 & 1.25
0.625	1.25	0.9375	7.3828	9.75	−1.490	0.9375 & 1.25
0.9375	1.25	1.09375	8.955	10.375	−0.486	1.09375 & 1.25
1.0938	1.25	1.1719	9.778	10.688	−0.175	1.17 & 1.25
1.17	1.25	1.21	10.19	10.84	−0.057	1.21 & 1.25
1.21	1.25	1.23	10.41	10.92	0.004	1.21 & 1.23

3.5(a)(ii) Bisection method (contd.)

hmin	hmax	hmean	A	B	f(h)	Root lies betn
1.21	1.23	1.22	10.297	10.88	-0.023	1.22 and 1.23
1.22	1.23	1.225	10.351	10.90	-0.009	1.225 and 1.23

$\therefore$ The critical depth, $h_c = 1.23\,m$

$A_c = (6+4h)h = (6+4\times1.23)\times1.23 = 10.4058\,m^2$

$\therefore U_c = \dfrac{Q}{A_c} = \dfrac{30}{10.4058} = 2.883\ m/s$

iii) Newton-Raphson method

$A = (6+2h)h \qquad B = 6+4h \qquad dB/dh = 2s = 2\times2 = 4$

$f(h) = A^3 - \dfrac{\alpha Q^2 B}{g} = [(6+2h)h]^3 - \dfrac{1.12\times30^2\times(6+4h)}{9.81}$

$= [(6+2h)h]^3 - 102.75(6+4h)$

$f'(h) = 3A^2B - \dfrac{\alpha Q^2}{g}\dfrac{dB}{dh} = 3[(6+2h)h]^2(6+4h) - \dfrac{1.12\times30^2\times4}{9.81}$

$= 24(3+2h)[(3+2h)h]^2 - 411.9$

h	$f(h)$	$f'(h)$	$\Delta h = -\dfrac{f(h)}{f'(h)}$	$h = h+\Delta h$
1.00	-515.52	1508.99	0.34	1.34
1.34	406.26	4199.50	-0.10	1.24
1.24	36.49	3224.51	-0.01	1.23
1.23	4.69	3136.26	-0.001	$\therefore$ 1.23

$\therefore$ The critical depth, $h_c = 1.23\,m$

$A_c = (6+4h_c)h_c = (6+4\times1.23)\times1.23 = 10.4058\ m^2$

$\therefore U_c = Q/A_c = 30/10.4058 = 2.883\ m/s$

3.7(a) $E = h_1 + \frac{u_1^2}{2g} = h_2 + \frac{u_2^2}{2g}$ or, $h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$

$\therefore h_1 - h_2 = \frac{Q^2}{2gb^2}\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right) = \frac{Q^2}{2gb^2}\left(\frac{h_1^2 - h_2^2}{h_1^2 h_2^2}\right)$

or, $1 = \frac{Q^2}{2gb^2}\left(\frac{h_1 + h_2}{h_1^2 h_2^2}\right)$ (i)

Again, $h_c^3 = \frac{Q^2}{gb^2}$ $\therefore \frac{Q^2}{2gb^2} = \frac{h_c^3}{2}$ (ii)

$\therefore$ Combining (i) and (ii) we have

$1 = \frac{h_c^3}{2}\left(\frac{h_1 + h_2}{h_1^2 h_2^2}\right) \Rightarrow \frac{2h_1^2 h_2^2}{h_1 + h_2} = h_c^3$ (Proved)

(b) $h_1 = 1m$, $h_2 = 2m$, $b = 6m$

$h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$

or, $1 + \frac{Q^2}{2gb^2 \times 1^2} = 2 + \frac{Q^2}{2gb^2 \times 2^2}$

or, $2 - 1 = \frac{Q^2}{2gb^2}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow 1 = \frac{3Q^2}{8gb^2}$

$\therefore Q = b\sqrt{\frac{8g}{3}} = 6 \times \sqrt{\frac{8 \times 9.81}{3}} = 30.688 \ m^3/s$

$\therefore E = h_1 + \frac{Q^2}{2gb^2h_1^2} = 1 + \frac{30.688^2}{2 \times 9.81 \times 6^2 \times 1^2} = 2.333m$

$h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{30.688^2}{9.81 \times 6^2}} = 1.387 \ m$

3.8 Maximum discharge is carried by a channel for a given specific energy when the flow is critical. So, we have $E_c = 2$ m.

i) <u>Rectangle</u> $b = 6$m

$$E_c = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2} = 1.5 h_c$$

$$\therefore h_c = \frac{E_c}{1.5} = \frac{2}{1.5} = 1.333 \text{ m}$$

$$A_c = bh_c = 6 \times 1.333 = 8 \text{ m}^2$$

$$U_c = \sqrt{gD_c} = \sqrt{gh_c} = \sqrt{9.81 \times 1.33} = 3.617 \text{ m/s}$$

$$\therefore Q = A_c U_c = 8 \times 3.617 = 28.933 \text{ m}^3/\text{s}$$

ii) <u>Triangle</u> $s = 1$

$$E_c = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{A_c}{2B_c} = h_c + \frac{sh_c^2}{2 \times 2sh_c}$$

$$= h_c + \frac{h_c}{4} = 1.25 h_c$$

$$\therefore h_c = \frac{E_c}{1.25} = \frac{2}{1.25} = 1.6 \text{ m}$$

$$A_c = sh_c^2 = 1 \times 1.6^2 = 2.56 \text{ m}^2$$

$$B_c = 2sh_c = 2 \times 1 \times 1.6 = 3.2 \text{ m}$$

$$D_c = A_c / B_c = 2.56 / 3.20 = 0.8 \text{ m}$$

$$U_c = \sqrt{gD_c} = \sqrt{9.81 \times 8} = 2.801 \text{ m/s}$$

$$\therefore Q = A_c U_c = 2.56 \times 2.801 = 7.172 \text{ m}^3/\text{s}$$

iii) <u>Parabola</u> $y^2 = 4z \Rightarrow z = \frac{1}{4} y^2 \quad \therefore c = \frac{1}{4} = 0.25$

$$E_c = h_c + \alpha \frac{U_c^2}{g} = h_c + \frac{D_c}{2} = h_c + \frac{A_c}{2B_c} = h_c + \frac{2h_c}{3 \times 2h_c}$$

$$\left[\because A_c = \tfrac{2}{3} B_c h_c\right]$$

$$= h_c + \frac{h_c}{3} = \frac{4}{3} h_c$$

$$\therefore h_c = \frac{3}{4} E_c = \frac{3}{4} \times 2 = 1.5 \text{ m/s}$$

3.8 (iii) (contd)

$$A_c = \frac{4}{3\sqrt{c}} h_c^{1.5} = \frac{4}{3\times\sqrt{0.25}} 1.5^{1.5} = 4.900\ m^2$$

$$D_c = \frac{A_c}{B_c} = \frac{2}{3} h_c = \frac{2}{3}\times 1.5 = 1\ m$$

$$U_c = \sqrt{gD_c} = \sqrt{9.81\times 1} = 3.132\ m/s$$

$$\therefore Q = A_c U_c = 4.900\times 3.132 = 15.344\ m^3/s$$

iv) Trapezoid $\quad b = 6m \quad s = 2$

$$E_c = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{A_c}{2B_c} = h_c + \frac{(6+2h_c)h_c}{2(6+4h_c)} = 2$$

By trial, $h_c = 1.45m$

$\therefore A_c = (6+2\times 1.45)\times 1.45 = 12.905\ m^2$, $B_c = 6+4\times 1.45 = 11.80m$

$D_c = A_c/B_c = 12.905/11.80 = 1.094m \quad U_c = \sqrt{gD_c} = \sqrt{9.81\times 1.094}$
$= 3.275\ m/s$

$$\therefore Q = A_c U_c = 12.905\times 3.275 = 42.270\ m^3/s$$

v) Circle $\quad d_o = 3m$ $\qquad A_c = \frac{1}{8}(\theta - \sin\theta)d_o^2$, $B_c = d_o \sin\theta/2$

$$E_c = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{A_c}{2B_c} = h_c + \frac{1}{16}\left(\frac{\theta - \sin\theta}{\sin\theta/2}\right) d_o$$

$$\left.\begin{aligned} \therefore 2 &= h_c + \frac{3}{16}\left(\frac{\theta-\sin\theta}{\sin\theta/2}\right) \\ h_c &= \frac{d_o}{2}\left(1-\cos\frac{\theta}{2}\right) = 1.5\left(1-\cos\frac{\theta}{2}\right) \end{aligned}\right] \begin{aligned} &\text{By trial, } \theta = 3.063\ rad \\ &\therefore h_c = 1.44m \end{aligned}$$

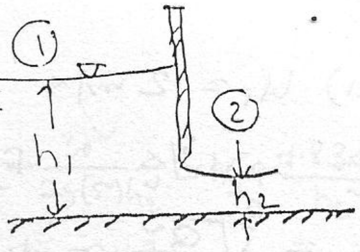
$$A_c = \frac{1}{8}(\theta - \sin\theta) d_o^2 = \frac{1}{8}(3.063 - \sin 3.063)\times 3^2 = 3.358\ m^2$$

$$B_c = d_o \sin\theta/2 = 3\times \sin\frac{3.063}{2} = 2.998\ m$$

$D_c = A_c/B_c = 3.358/2.998 = 1.120m$, $U_c = \sqrt{gD_c} = \sqrt{9.81\times 1.120}$
$= 3.315\ m/s$

$$\therefore Q = A_c U_c = 3.358\times 3.315 = 11.13\ m^3/s$$

3.9 Rectangle, $b = 6\,m$, $h_1 = 2\,m$, $Q = 30.67\ m^3/s$



$$u_1 = \frac{Q}{A_1} = \frac{Q}{bh_1} = \frac{30.67}{6\times 2} = 2.556\ m/s$$

Applying the energy equation between sections 1 and 2, neglecting the frictional head loss and taking the channel bed as datum, we get

$$0 + h_1 + \frac{u_1^2}{2g} = 0 + h_2 + \frac{u_2^2}{2g}$$

$$\text{or, } 2 + \frac{2.556^2}{2\times 9.81} = h_2 + \frac{Q^2}{2gb^2h_2^2}$$

$$\text{or, } 2.333 = h_2 + \frac{30.67^2}{2\times 9.81\times 6^2\times h_2^2} = h_2 + \frac{1.332}{h_2^2}$$

By trial, we get $h_2 = 1\,m$ and $h_2 = 2\,m$.

When $h_2 = 1\,m$, $u_2 = 5.112\ m/s$, $Fr_2 = \frac{5.112}{\sqrt{9.81\times 1}} = 1.63 > 1$

When $h_2 = 2\,m$, $u_2 = 2.556\ m/s$, $Fr_2 = \frac{2.556}{\sqrt{9.81\times 2}} = 0.56 < 1$

Since the sluice gate acts as a control, the downstream flow must be supercritical.

$\therefore h_2 = 1\,m$ (Ans)

3.10 When the flow is critical in a rectangular channel,

$$E_c = \frac{3}{2}h_c = \frac{3}{2}\sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \frac{3}{2}\sqrt[3]{\frac{\alpha b^2 h_1^2 u_1^2}{gb^2}} = \frac{3}{2}\sqrt[3]{\frac{\alpha u_1^2 h_1^2}{g}}$$

$(\because Q = A_1 u_1 = bh_1 u_1)$

$$\therefore \Delta z_c = E_1 - E_c = h_1 + \frac{\alpha u_1^2}{2g} - \frac{3}{2}\sqrt[3]{\frac{\alpha u_1^2 h_1^2}{g}}$$

$$= h_1\left(1 + \frac{\alpha u_1^2}{2gh_1} - \frac{3}{2}\sqrt[3]{\frac{\alpha u_1^2}{gh_1}}\right)$$

$$= h_1\left(1 + 0.5\,Fr_1^2 - \frac{3}{2}Fr_1^{0.67}\right) \quad \left(\because Fr_1 = \frac{u_1}{\sqrt{gh_1/\alpha_1}}\right)$$

(Proved)

3.11 a) $U_1 = 2\ m/s \quad h_1 = 2.5\ m \quad b = 6\ m$

$Q = 30\ m^3/s \qquad Fr_1 = U_1/\sqrt{gh_1} = 0.40 < 1$

$h_c = \sqrt[3]{\dfrac{Q^2}{gb^2}} = 1.3659\ m \qquad E_c = 1.5h_c = 2.049\ m$

$E_1 = h_1 + \dfrac{U_1^2}{2g} = 2.7039\ m$

$\therefore \Delta z_c = 2.7039 - 2.0490 = 0.655\ m$

b) I) $\Delta z = 0.40\ m \quad E_2 = E_1 - \Delta z = 2.7039 - 0.4 = 2.3039\ m$

$\therefore 2.3039 = h_2 + \dfrac{Q^2}{2gb^2h_2^2} = h_2 + \dfrac{30^2}{2g \times 6^2 \times h_2^2} = h_2 + \dfrac{1.2742}{h_2^2}$

By trial, $h_2 = 1.98\ m,\ 0.98\ m \quad \therefore h_2 = 1.98\ m$

$\therefore$ drop in w/l $= 2.5 - 1.98 - 0.40 = 0.12\ m$

II) $\Delta z = 0.80\ m > \Delta z_c = 0.655\ m$. Assuming that the dis-charge in the channel does not change,

$h_c = 1.3659\ m \quad E_c = 2.049\ m$

$E_1' = E_c + \Delta z = 2.849\ m$

$\therefore 2.849 = h_1' + \dfrac{1.2742}{h_1'^2}$

$\therefore h_1' = 2.67\ m$

Drop $= 2.67 - 1.36 - 0.80 = 0.51\ m$

E_1 is maintained

$E_2 = E_c' = E_1 - \Delta z = 1.9039\ m$

$h_c' = \frac{2}{3} E_c = 1.269\ m$

$Q' = \sqrt{g}\, b\, h_c'^{1.5} = 26.87\ m^3/s$

Drop $= h_1 - h_c' - \Delta z = 0.431\ m$

III) $\Delta z = -0.40\ m \quad E_2 = E_1 - \Delta z = 2.7039 - (-0.40) = 3.1039\ m$

$\therefore 3.1039 = h_2 + \dfrac{1.2742}{h_2^2}$

By trial, $h_2 = 2.96\ m \qquad$ rise $= h_2 - h_1 - \Delta z = 0.06\ m$

3.12 a) $U_1 = 2\ m/s \quad h_1 = 2.5\ m \quad b_1 = 6\ m \quad Q = 30\ m^3/s$

$E_1 = 2.7039\ m = E_c \qquad \therefore h_c = \frac{2}{3} E_c = 1.8026\ m$

$Q^2 = gb_c^2h_c^3 \Rightarrow b_c = \dfrac{Q}{\sqrt{g}\, h_c^{1.5}} = 3.96\ m$

3.12(b) i) $b_2 = 5\,m \qquad E_1 = E_2 = 2.7039 = h_2 + \frac{30^2}{2g(5^2)h_2^2} = h_2 + \frac{1.83486}{h_2^2}$

$\therefore h_2 = 2.38\,m \qquad \text{Drop} = y_1 - y_2 = 0.12\,m$

ii) $b_2 = 3\,m < b_c = 3.9\,m$

Assuming that the discharge in the channel does not change,

$$h_c = \sqrt[3]{\frac{30^2}{9.81 \times 3^2}} = 2.168\,m$$

$$E_1' = E_c = 1.5 h_c = 3.252\,m$$

$\therefore 3.252 = h_1' + \frac{30^2}{2g(6^2)h_1'^2}$

$= h_1' + \frac{1.2742}{h_1'^2}$

$\therefore h_1' = 3.12\,m$

$\text{Drop} = h_1' - h_c = 0.95\,m$

~~E1 maintained~~

$\cancel{E_c = E_1' \quad E_2 = 2.7039}$

$\cancel{h_c = \tfrac{2}{3}E_c = 1.8026\,m}$

$\cancel{Q_1' = \sqrt{g}\, b_2 h_c^{1.5} = \sqrt{9.81} \times 3 \times 1.8026^{1.5}}$

$\cancel{= 22.79\ m^3/s}$

$\cancel{\text{Drop} = h_1 - h_2 = 0.70\,m}$

iii) $b_2 = 8\,m \qquad E_1 = 2.7039 = E_2 = h_2 + \frac{30^2}{2g \cdot 8^2 h_2^2} = h_2 + \frac{0.7167}{h_2^2}$

By trial, $h_2 = 2.60\,m \qquad \text{Rise} = 2.60 - 2.50 = 0.10\,m$

3.13 $u_1 = 2\,m/s \quad b_1 = 6\,m \quad h_1 = 2.5\,m \quad Q = 30\,m^3/s \quad \Delta z = 0.8\,m$

$$h_c = \sqrt[3]{\frac{30^2}{9.81 \times 6^2}} = 1.3659\,m \qquad E_c = 1.5 h_c = 2.0488\,m$$

$$E_1 = h_1 + \frac{u_1^2}{2g} = 2.7039\,m$$

The d/s specific energy considering only the step would be

$$E_2 = E_1 - \Delta z = 2.7039 - 0.80 = 1.9039\,m$$

Since $E_2 < E_c$, flow is not possible as specified without an expansion in width. If the d/s flow occurs at critical depth, then the expansion reqd. is a minimum. The d/s condition is thus

$$E_c = E_1 - \Delta z = 1.9039\,m$$

4

3.13 (Contd.)

Then $h_c = \frac{2}{3} E_c = 1.2693$ m

and $U_c = \sqrt{g h_c} = 3.5287$ m

Therefore the downstream width is

$$b_2 = \frac{Q}{h_c U_c} = \frac{30}{1.2693 \times 3.5287} = 6.698 \text{ m}$$

Hence the minimum expansion required is

$(6.698 - 6.0) = 0.698$ m $\simeq 0.70$ m.

3.14 $U_1 = 2$ m/s, $b_1 = 6$m, $h_1 = 2.5$m, $Q = 30 m^3/s$, $b_2 = 5$m

$E_1 = 2.7039$ m, $h_{c2} = \sqrt[3]{\frac{Q^2}{g b_2^2}} = \sqrt[3]{\frac{30^2}{9.81 \times 5^2}} = 1.592$ m

$\therefore E_{c2} = E_2 = 1.5 h_{c2} = 1.5 \times 1.592 = 2.3137$ m

Now, $E_1 = E_2 + \Delta z \Rightarrow \Delta z = E_1 - E_c = 2.7039 - 2.3137 = 0.3902$ m

Hence, the channel bed is to be raised by 0.39m to produce critical flow at section 2.

3.15 $b = 10$ km, $Q = 1,00,000 m^3/s$, $h = 10$ m

$U = Q/A = Q/bh = 100000/(10000 \times 10) = 1$ m/s

$$E = h + \frac{U^2}{2g} = 10 + \frac{1^2}{2 \times 9.81} = 10.051 \text{ m}$$

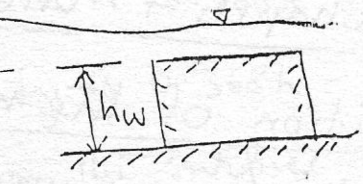
The length of the bridge will be minimum when the width of the river is minimum. The width of the river is minimum when the flow at the bridge section is critical (without affecting the upstream flow).

$E = E_c = 10.051 \Rightarrow h_c = \frac{2}{3} E = \frac{2}{3} \times 10.051 = 6.701$ m

$$Q = \sqrt{g}\, b_c h_c^{1.5} \Rightarrow b_c = \frac{100000}{\sqrt{9.81} \times 6.701^{1.5}} = 1840 \text{ m}$$

3.16(a) Rectangular channel,

$b = 1m, h_1 = 0.40m, h_w = 0.60m$



Initially, we neglect the velocity of approach. Then, the discharge is given by

$$Q = 1.705 b h_1^{1.5} = 1.705 \times 1 \times 0.40^{1.5} = 0.431 m^3/s$$

A more accurate discharge is obtained by including the velocity of approach and using the equation

$$Q = 1.705\, b\left(h_1 + \frac{U_1^2}{2g}\right)^{1.5}$$

as follows.

Assumed Q	A_1	U_1	$\frac{U_1^2}{2g}$	$h_1+\frac{U_1^2}{2g}$	Computed Q
0.431	1	0.431	0.0095	0.4095	0.4468
0.4468	1	0.4468	0.0102	0.4102	0.4479
0.4479	1	0.4479	0.0102	0.4102	0.4479

$$\therefore Q = 0.448 \; m^3/s$$

(b) $b = 0.30m, \; h_1 = 0.50m$

$$\therefore Q = 1.705 b h_1^{1.5} = 1.705 \times 0.30 \times 0.50^{1.5} = 0.181 m^3/s$$

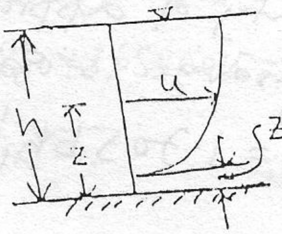
or $\quad h_c = \frac{2}{3} h_1 = \frac{2}{3} \times 0.50 = 0.333m$

$$\therefore Q = 3.13 b h_c^{1.5} = 3.13 \times 0.30 \times 0.333^{1.5} = 0.181 m^3/s$$

Chapter 4 Uniform Flow

..1 Position of the mean velocity

$$\frac{u}{u_*} = \frac{1}{K} \ln z/z_0 \Rightarrow u = 2.5 u_* \ln z/z_0 \quad \text{(i)} \quad (K = 0.4)$$



$$\bar{U} = \frac{2.5 u_*}{h - z_0} \int_{z_0}^{h} \ln(z/z_0)\, dz$$

$$= \frac{2.5 u_*}{h - z_0} \int_{z_0}^{h} (\ln z - \ln z_0)\, dz$$

$$= \frac{2.5 u_*}{h - z_0} \Big[z \ln z - z - z \ln z_0 \Big]_{z_0}^{h} \quad \left[\because \int \ln x\, dx = x \ln x - x\right]$$

$$= \frac{2.5 u_*}{h - z_0} \Big[z \ln \frac{z}{z_0} - z \Big]_{z_0}^{h}$$

$$= \frac{2.5 u_*}{h - z_0} \Big[h \ln \frac{h}{z_0} - h - z_0 \cancel{\ln \frac{z_0}{z_0}}^{\,0} + z_0 \Big]$$

$$= 2.5 u_* \Big[\frac{h}{h - z_0} \ln \frac{h}{z_0} - \frac{h - z_0}{h - z_0} \Big]$$

Since $h >> z_0$, we ~~take~~ take $h - z_0 \simeq h$.

$$\therefore \bar{U} = 2.5 u_* \Big[\ln \frac{h}{z_0} - 1 \Big]$$

$$= 2.5 u_* \ln h/z_0 - 2.5 u_* \quad \text{(ii)}$$

From (i) and (ii), we obtain

$$\bar{U} - u = 2.5 u_* \ln h/z_0 - 2.5 u_* - 2.5 u_* \ln z/z_0$$

$$= 2.5 u_* \ln h/z - 2.5 u_* \quad \text{(iii)}$$

$$= 2.5 u_* (\ln h/z - 1) \quad \text{(iv)}$$

The position of the mean velocity is obtained by using $u = \bar{U}$, i.e. $\bar{U} - u = 0$ in (iv). Hence,

$\ln \frac{h}{z} = 1 = \ln e$ i.e. $\frac{h}{z} = e$ $\therefore z = e^{-1} h = 0.368h$

Hence, the position of mean velocity is $0.368h$ from the channel bottom or $0.632h$ from the surface.

i) <u>Velocity at $0.6h$</u> $(z = 0.4h)$

From (iii) we obtain, $u = \bar{u} - 2.5u_* \ln h/z + 2.5u_*$

$$\therefore u_{0.6h} = \bar{u} - 2.5u_* \ln \frac{h}{0.4h} + 2.5u_*$$

$$= \bar{u} - 2.29u_* + 2.5u_*$$

$$= \bar{u} + 0.21u_*$$

$$\simeq \bar{u} \quad (\because u_* << \bar{u})$$

ii) <u>Average of the velocities at 0.2 and 0.8 depths</u>

$$u_{0.2h} = \bar{u} - 2.5u_* \ln \frac{h}{0.8h} + 2.5u_*$$

$$= \bar{u} - 0.558u_* + 2.5u_*$$

$$u_{0.8h} = \bar{u} - 2.5u_* \ln \frac{h}{0.2h} + 2.5u_*$$

$$= \bar{u} - 4.023u_* + 2.5u_*$$

$$\therefore \frac{u_{0.2h} + u_{0.8h}}{2} = \bar{u} - 2.29u_* + 2.5u_*$$

$$= \bar{u} + 0.21u_* \simeq \bar{u} \quad (\because u_* << \bar{u})$$

Thus, the velocity at 0.6 depth and the average of the velocities at 0.2 and 0.8 depths from the free surface are approximately equal to the mean velocity in the vertical.

2(a) Trapezoid, $b = 6m$, $s = 1.5$, $h_n = 2m$, $n = 0.025$
$S_0 = 0.0001$, Flow is uniform

$$A = (b + sh)h = (6 + 1.5 \times 2) \times 2 = 18 \ m^2$$

$$P = b + 2\sqrt{1+s^2}\,h = 6 + 2\sqrt{1+1.5^2} \times 2 = 13.21 m$$

$$R = A/P = 18/13.21 = 1.36 m$$

i) $$Q = \frac{1}{n} A R^{2/3} S_o^{1/2} = \frac{1}{0.025} \times 18 \times 1.36^{2/3} \times 0.0001^{1/2}$$
$$= 8.838 \ m^3/s$$

ii) $$C = \frac{1}{n} R^{1/6} = \frac{1}{0.025} \times 1.36^{1/6} = 42.1 \ m^{1/2}/s$$

$$f = \frac{8g}{C^2} = \frac{8 \times 9.81}{42.1^2} = 0.044$$

$$\tau_o = \gamma R S_o = \rho g R S_o = 1000 \times 9.81 \times 1.36 \times 0.0001$$
$$= 1.334 \ N/m^2$$

$$u_* = \sqrt{gRS_o} = \sqrt{9.81 \times 1.36 \times 0.0001} = 0.0365 \ m/s$$

iii) To compute ks, assume that the surface is hydraulically rough.

$$\therefore \frac{U}{u_*} = 5.75 \log \frac{12.2R}{ks} \qquad U = \frac{Q}{A} = \frac{8.838}{18} = 0.491 m/s$$

$$\text{or, } \frac{0.491}{0.0365} = 5.75 \log \frac{12.2 \times 1.36}{ks}$$

$$\text{or, } 2.339 = \log \frac{12.2 \times 1.36}{ks}$$

$$\text{or, } \frac{12.2 \times 136}{ks} = 10^{2.339} = 218.518$$

$$\therefore ks = \frac{12.2 \times 13.6}{218.518} = 0.0759 \ m$$

$\therefore \dfrac{k_s u_*}{\nu} = \dfrac{0.0759 \times 0.0365}{10^{-6}} = 2771.4 > 70$

Hence, the surface is hydraulically rough and the Manning formula is applicable.

(b) $A = 33,500\ m^2$, $Q = 56,200\ m^3/s$, $B = 3820\ m$, $S_0 = 4\ cm/km$

$= 4\times10^{-2}/10^{3} = 4\times10^{-5} = 0.00004$

Since the river is wide, $R = D = h = \dfrac{A}{B} = \dfrac{33,500}{3820}$

$u = Q/A = 56,200/33,500 = 1.68\ m/s$ $\quad = 8.77\ m$

i) $Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$\therefore n = \dfrac{A R^{2/3} S_0^{1/2}}{Q} = \dfrac{33500 \times 8.77^{2/3} \times 0.00004^{1/2}}{56,200} = 0.016$

$C = \frac{1}{n} R^{1/6} = \dfrac{1}{0.016} \times 8.77^{1/6} = 89.57\ m^{1/2}/s$

$f = \dfrac{8g}{C^2} = \dfrac{8\times9.81}{89.57^2} = 0.010$

$u^* = \sqrt{gRS_0} = \sqrt{9.81\times8.77\times0.00004} = 0.059\ m/s$

$\tau_0 = \gamma R S_0 = \rho g R S_0 = 1000\times9.81\times8.77\times0.00004 = 3.44\ N/m^2$

ii) $\dfrac{u}{u_*} = 5.75 \log \dfrac{12.2R}{k_s} \Rightarrow \dfrac{1.68}{0.059} = 5.75 \log \dfrac{12.2\times8.77}{k_s}$

or, $\log \dfrac{12.2\times8.77}{k_s} = 4.952 \Rightarrow \dfrac{12.2\times8.77}{k_s} = 10^{4.952} = 89,536$

$\therefore k_s = \dfrac{12.2\times8.77}{89,536} = 0.001195\ m$

Now, $\dfrac{k_s u_*}{\nu} = \dfrac{0.001195\times0.059}{10^{-6}} = 70.5 > 70$

Hence, the surface is hydraulically rough.

(4)

4.3(a) For a rough channel, Eqs. (4.15) and (4.17) with $\kappa = 0.4$ gives

$$u_z = \frac{u_*}{\kappa}\ln\frac{z}{z_0} = 2.5\,u_*\ln\frac{z}{0.033k_s} \overset{\times 2.3}{=} 5.75\,u_*\log\frac{30z}{k_s}$$

$$\therefore u_{0.2} = 5.75\,u_*\log\frac{24h}{k_s} \quad \text{(i)}$$

$$u_{0.8} = 5.75\,u_*\log\frac{6h}{k_s} \quad \text{(ii)}$$

$$\therefore \frac{u_{0.2}}{u_{0.8}} = \frac{\log\frac{24h}{k_s}}{\log\frac{6h}{k_s}} \Rightarrow r = \frac{\log 24 + \log h/k_s}{\log 6 + \log h/k_s} = \frac{1.380 + \log h/k_s}{0.778 + \log h/k_s}$$

$$\therefore \log\frac{h}{k_s} = \frac{1.380 - 0.778r}{r-1} \quad \text{(iii)}$$

Using $R = h$ for a wide channel, Eq. (4.20) can be written as

$$\frac{U}{u_*} = 5.75\log\frac{12.2h}{k_s} = 5.75\log\frac{h}{k_s} + 6.25 \quad \text{(iv)}$$

5.75*log(12.2)

Using (iii) in (iv) and simplifying, we get

$$\frac{U}{u_*} = 5.75\,\frac{1.380 - 0.778r}{r-1} + 6.25 = \frac{1.78(r+0.95)}{r-1} \quad \text{(v)}$$

Combination of Eqs. (4.9), (4.24) and (4.29) with $S_f = S_0$ and $R = h$ gives

$$\frac{U}{u_*} = \frac{C}{\sqrt{g}} = \frac{R^{1/6}}{n\sqrt{g}} = \frac{h^{1/6}}{3.13n} \quad \text{(vi)}$$

Equating the right-hand sides of (v) and (vi), we get

$$n = \frac{(r-1)\,h^{1/6}}{5.57(r+0.95)} \quad \text{(Proved)}$$

~~5~~

4.3 (b) $u_{0.2} = 1.25\ m/s$, $u_{0.8} = 1\ m/s$, $r = \frac{u_{0.2}}{u_{0.8}} = \frac{1.25}{1} = 1.25$, $h = 10\ m$

(i) $$n = \frac{(r-1)h^{1/6}}{5.57(r+0.95)} = \frac{(1.25-1)\times 10^{1/6}}{5.57(1.25+0.95)} = 0.030$$

(ii) The velocity variation along a vertical in a wide channel is given by

$$u_z = \frac{u_*}{\kappa} \ln \frac{z}{z_0} = 2.5\, u_* \ln z/z_0 \qquad (i)$$

$$\therefore u_{0.2} = \frac{u_*}{\kappa} \ln \frac{8}{z_0} = \frac{u_*}{\kappa} \ln 8 - \frac{u_*}{\kappa} \ln z_0 \qquad (ii)$$

$$u_{0.8} = \frac{u_*}{\kappa} \ln \frac{2}{z_0} = \frac{u_*}{\kappa} \ln 2 - \frac{u_*}{\kappa} \ln z_0 \qquad (iii)$$

Subtracting (iii) from (ii) and rearranging, we get

$$u_* = \frac{\kappa(u_{0.2} - u_{0.8})}{\ln(8/2)} = \frac{0.4(1.25-1)}{\ln 4} = 0.072\ m/s$$

Also from (ii) we have

$$z_0 = \frac{8}{\exp(\kappa u_{0.2}/u_*)} = \frac{8}{\exp(0.4 \times 1.25/0.072)} = 0.00771\ m$$

[Check: $z_0 = 0.033\, k_s$ $\therefore k_s = z_0/0.033 = 0.00771/0.033 = 0.231\ m$

$$\therefore \frac{k_s u_*}{\nu} = \frac{0.231 \times 0.072}{10^{-6}} = 16653.6 >> 70$$

Hence the surface is hydraulically rough.]

Using the values of u_* and z_0 in (i), we get the variation of velocity in the vertical as

$$u_z = 2.5 \times 0.072 \ln(z/0.00771)$$
$$= 0.18 \ln(129.67z) \quad \text{when } z \geq z_0.$$

4(a) $h = 1m$, Use Manning formula

i) Rectangle, $b = 6m$, $h = 1m$, $h/b = 1/6 = 0.167$

$$\therefore N = \frac{2}{3}\left(5 - \frac{4\times 0.167}{1+2\times 0.167}\right) = 3$$

ii) Trapezoid, $b = 6m$, $s = 2$, $h = 1m$, $h/b = 0.167$

$$\therefore N = \frac{10}{3}\,\frac{1+2\times 2\times 0.167}{1+2\times 0.167} - \frac{8}{3}\cdot\frac{\sqrt{5}\times 0.167}{1+2\sqrt{5}\times 0.167}$$

$$= 4.167 - 0.569 = 3.598$$

iii) Parabolic, $y^2 = 4z$, $z = \frac{1}{4}y^2$, $c = \frac{1}{4}$, $h = 1m$

$$\therefore N = 3 + \frac{4}{3+2\times 1/4\times 1} = 4.143$$

iv) Circle, $d_0 = 2m$, $h = 1m$

$$\theta = 2\cos^{-1}\left(1 - \frac{2h}{d_0}\right) = 2\cos^{-1}\left(1 - \frac{2\times 1}{2}\right) = 3.1416 \text{ rad}$$

$$\therefore N = \frac{16\times 1}{3\times 2}\left[\frac{5\sin\frac{3.1416}{2}}{3.1416 - \sin 3.1416} - \frac{1}{3.1416\sin\frac{3.1416}{2}}\right]$$

$$= \frac{8}{3}(1.592 - 0.318) = 3.395$$

4.4(b) $h = 1m$, Use Chezy formula

$$\text{i) } N = 3 - \frac{2\times 0.167}{1+2\times 0.167} = 2.75$$

$$\text{ii) } N = 3\,\frac{1+2\times 2\times 0.167}{1+2\times 0.167} - \frac{2\sqrt{5}\times 0.167}{1+2\sqrt{5}\times 0.167} = 3.323$$

$$\text{iii) } N = 3 + \frac{3}{3+2\times 1\times 1/4} = 3.857$$

$$\text{iv) } N = \frac{4\times 1}{2}\left[\frac{6\sin\frac{3.1416}{2}}{3.1416 - \sin 3.1416} - \frac{1}{3.1416\times\sin\frac{3.1416}{2}}\right]$$

$$= 2(1.910 - 0.381) = 3.058$$

4.5 (a) $n = 0.025 \quad S_0 = 0.0025 \quad q = 3\,m^2/s$

$$h_n = \left(\frac{nq}{\sqrt{S}}\right)^{3/5} = \left(\frac{0.025\times 3}{\sqrt{0.0025}}\right)^{3/5} = 1.275\,m$$

$$U_n = \frac{q}{h_n} = \frac{3}{1.275} = 2.35\ m/s$$

(b) wide $\quad S = 0.006 \quad C = 50\ m^{1/2}/s \quad q = 4\ m^3/s/m$

$$h_n = \left(\frac{q}{C^2 S}\right)^{1/3} = \left(\frac{4}{50^2\times 0.006}\right)^{1/3} = 1.0217\ m$$

$$U_n = \frac{q}{h_n} = \frac{4}{1.0217} = 3.915\ m/s$$

4.6 (a) V $\quad s = 1 \quad S_0 = 0.001 \quad n = 0.015 \quad h_n = 1\,m$

$A = sh^2 = 1\times 1^2 = 1\ m^2$

$P = 2\sqrt{1+s^2} = 2\sqrt{2} = 2.828\ m$

$R = A/P = 0.35355\ m$

$$\therefore Q = \frac{1}{n} A R^{2/3} S^{1/2} = \frac{1}{.015}\times 1\times 0.35355^{2/3}\times 0.001^{1/2}$$

$= 1.054\ m^3/s$

(b) $s = 1,\ n = 0.025,\ Q = 5\ m^3/s \quad S_0 = 0.0025$

$$AR^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{0.025\times 5}{\sqrt{0.0025}} = 2.5$$

$A = sh^2 = h^2$

$P = 2\sqrt{1+s^2}\,h$

$= 2\sqrt{2}\,h$

$R = h/2\sqrt{2}$

$$\therefore h^2\left(\frac{h}{2\sqrt{2}}\right)^{2/3} = 2.5$$

or, $h^{8/3} = 2.5\times(2\sqrt{2})^{2/3} = 5.00$

$\therefore h_n = 1.8285\ m \qquad U_n = \frac{5}{1.8285^2} = 1.4953\ m/s$

4.7 $U = 1\ m/s \quad S_o = 0.0016 \quad n = 0.020$

$$U = \frac{1}{n} R^{2/3} S_o^{1/2} \Rightarrow 1 = \frac{1}{0.020} \times R^{2/3} \times 0.0016^{1/2}$$

$$\therefore R^{2/3} = \frac{0.020}{0.04} = 0.5 \Rightarrow R = 0.5^{3/2} = 0.35\,m$$

i) Rectangle $b = 6\,m$

$$R = 0.35 = A/P \Rightarrow A = 0.35P$$

or, $6h = 0.35(6+2h) \Rightarrow 6h = 2.10 + 0.70h$

or, $5.30h = 2.10 \quad \therefore h_n = \frac{2.10}{5.30} = 0.40\,m$

ii) Trapezoid $s = 2$ As in (i), $A = 0.35P$

$$\therefore (6+2h)h = 0.35(6+2\sqrt{5}h) \Rightarrow h^2 + 2.21h - 1.06 = 0$$

$$\therefore h_n = \frac{-2.21 \pm \sqrt{2.21^2 - 4\times1\times(-1.06)}}{2\times1} = 0.41\,m \text{ (+ sign only)}$$

iii) Triangle $s = 1$

$$R = A/P = sh^2/2\sqrt{2}h = h/2\sqrt{2} = 0.35$$

$$\therefore h_n = 0.35 \times 2\sqrt{2} = 1\,m$$

iv) Parabola $y^2 = 4z$ $(B = 2y \text{ with } h = z)$

h	y	B	A	P	R
0.57	1.510	3.020	1.148	3.286	0.349
0.58	1.523	3.046	1.178	3.319	0.355

$\therefore h_n = 0.57\,m$

v) Circle $d_o = 2\,m$ $\quad A = \frac{1}{8}(\omega - \sin\omega)d_o^2,\ P = \frac{1}{2}\omega d_o$

$$\therefore R = \frac{1}{4}\left(1 - \frac{\sin\omega}{\omega}\right)d_o = 0.35 \Rightarrow 1 - \frac{\sin\omega}{\omega} = \frac{0.35\times4}{2} = 0.70$$

$$\therefore \sin\omega/\omega = 1 - 0.70 = 0.30 \Rightarrow \sin\omega - 0.30\omega = 0$$

By trial, $\omega = 2.36^c \quad \therefore h_n = \frac{d_o}{2}\left(1 - \cos\frac{\omega}{2}\right) = 0.62\,m$

4

4.8 $h_n = 1\,m \quad S_0 = 0.0001 \quad n = 0.015 \quad Q_n = ?$

i) $Q = \frac{1}{.015} \times (6\times1) \times \left(\frac{6\times1}{6+2\times1}\right)^{2/3} \times 0.0001^{1/2} = 3.301\ m^3/s$

ii) $A = (6+2\times1)\times1 = 8\,m^2 \quad P = 6+2\sqrt{2}\times1 = 8.829\ m$

$R = A/P = 0.906\,m$

$Q = \frac{1}{.015} \times 8 \times 0.906^{2/3} \times .0001^{1/2} = 4.999\ m^3/s$

iii) $s = 1.5 \quad A = s h^2 = 1.5\times1^2 = 1.5\,m^2 \quad P = 2\sqrt{1+1.5^2}\times1$
$= \cancel{6.5\,m}\ 3.60555\,m$

$R = A/P = 0.416\,m$

$\therefore Q = \frac{1}{.015} \times 1.5 \times .416^{2/3} \times .0001^{1/2} = 0.557\ m^3/s$

iv) $c = \frac{1}{4} \quad A = \frac{4}{3\sqrt{c}} h^{3/2} = \frac{8}{3} h^{3/2} = 2.667\ m^2$

$B = \frac{2}{\sqrt{c}} h^{1/2} = 4\,m \quad P = \frac{4}{2}\left[\sqrt{1+1^2} + 1\times\ln\left(1+\sqrt{1+1^2}\right)\right]$
$= 4.591\,m$

$Q = \frac{1}{.015} \times 2.667 \times \left(\frac{2.667}{4.591}\right)^{2/3} \times .0001^{1/2} = 1.238\ m^3/s$

v) $d_0 = 1.5\,m \quad \omega = 2\cos^{-1}\left(1 - \frac{2\times1}{1.5}\right) = 3.821\ rad$

$A = \frac{1}{8}(\omega - \sin\omega)d_0^2 = \frac{1}{8}(3.821 - \sin 3.821)1.5^2 = 1.2515\ m^2$

$P = \frac{1}{2}\omega d_0 = \frac{1}{2}\times 3.821\times1.5 = 2.866\,m, \quad R = 0.4367\ m$

$Q = \frac{1}{.015} \times 1.2515 \times 0.4367^{2/3} \times 0.0001^{1/2} = 0.480\ m^3/s$

4.9 (a) $\sqcup \quad n = .025 \quad S_0 = 0.0001 \quad Q = 15\ m^3/s \quad h_n = 1.5\,m \quad b = ?$

$AR^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{0.025\times6}{\sqrt{0.0001}} = 15$

b	A	P	R	$AR^{2/3}$
9.21	13.815	12.21	1.131	15.0

By trial, $b = 9.21\ m$

(b) $s = 1.5 \quad n = 0.020 \quad S = 0.0002 \quad Q = 50\ m^3/s \quad h_n = 2\,m$

$AR^{2/3} = \frac{nQ}{\sqrt{S}} = \frac{0.020\times25}{\sqrt{0.0002}} = 35.355$

b	A	P	R	$AR^{2/3}$
10.37	26.84	17.581	1.5210	35.365
10.36	26.72	17.591	1.5265	35.334

By trial, $b = 10.37\ m$

4.10 $n = 0.025$ $S_0 = 0.0025$

i) Rectangular channel, $b = 8m$, $Q = 22\ m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 22}{\sqrt{0.0025}} = 11.00$$

h	A (=bh)	P (=b+2h)	R (=A/P)	$AR^{2/3}$
1	8.00	10.00	0.800	6.894
2	16.00	12.00	1.333	19.383
1.36	10.88	10.72	1.015	10.928
1.37	10.96	10.74	1.020	11.109

$$\therefore h_n = 1.36m, \quad U_n = \frac{Q}{A_n} = \frac{22}{10.88} = 2.022\ m/s$$

ii) Trapezoidal channel, $b = 6m$, $s = 2$, $Q = 30\ m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 30}{\sqrt{0.0025}} = 15.00$$

$A = (b + sh)h$
$= (6 + 2h)h$
$P = b + 2\sqrt{1+s^2}\,h$
$= 6 + 2\sqrt{5}\,h$

h	A	P	R	$AR^{2/3}$
1	8.00	10.472	0.764	6.685
2	20.00	14.944	1.338	24.288
1.55	14.105	12.932	1.091	14.946
1.56	14.227	12.966	1.096	15.127

$\therefore h_n = 1.55m$

$$U_n = \frac{Q}{A_n} = \frac{30}{14.105} = 2.127\ m/s$$

4.10 iii) Parabolic channel, $y^2 = 5z$, $Q = 15\ m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 15}{\sqrt{0.0025}} = 7.50 \qquad z = h,\ B = 2y$$

h	y	B	A	P	R	$AR^{2/3}$
1	2.236	4.447	2.981	4.984	0.598	2.166
2	3.162	6.325	8.433	7.742	1.089	8.927
1.83	3.025	6.050	7.381	7.304	1.011	7.433
1.84	3.033	6.066	7.441	7.330	1.015	7.516

$$\therefore h_n = 1.84\,m \qquad U_n = \frac{Q}{A_n} = \frac{15}{7.441} = 2.016\ m/s$$

iv) Circular channel, $d_o = 2\,m$, $Q = 3\ m^3/s$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 3}{\sqrt{0.0025}} = 1.5 \qquad A = \frac{1}{8}(\omega - \sin\omega)\,d_o^2,\quad P = \frac{1}{2}\omega d_o$$

ω (rad)	A	P	R	$AR^{2/3}$
1	0.079	1.00	0.079	0.015
2	0.595	2.00	0.273	0.229
3	1.429	3.00	0.476	0.871
4	2.378	4.00	0.595	1.681
3.75	2.162	3.75	0.577	1.500

$$\therefore h_n = \frac{d_o}{2}\left(1 - \cos\frac{\omega}{2}\right) = \frac{2}{2}\left(1 - \cos\frac{3.75}{2}\right) = 1.30\,m$$

$$U_n = \frac{Q}{A_n} = \frac{3}{2.162} = 1.387\ m/s$$

4.11(a) Trapezoidal channel, $b = 6m$, $s = 1.5$, $n = 0.025$

$\alpha = 1$

i) $h_n = 1m$, $Q = 20 m^3/s$

$A = (b+sh)h = (6+1.5\times1)\times1 = 7.5 m^2$

$P = b + 2\sqrt{1+s^2}\,h = 6 + 2\sqrt{1+1.5^2}\times1 = 9.6055 m$

$R = A/P = 7.5/9.6055 = 0.7808 m$

$$\therefore S_n = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025\times20}{7.5\times0.7808^{2/3}}\right)^2 = 0.0062$$

ii) $Q = 20 m^3/s$ $\quad Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{20}{\sqrt{9.81/1}} = 6.3855$

h	A	B	D	Z = A√D
1.00	7.50	9.00	0.833	6.8465
0.95	7.05	8.85	0.797	6.2974
0.96	7.14	8.88	0.804	6.4056

$\therefore h_c = 0.96m = h_n$, $A = 7.14 m^2$

$P = 6 + 2\sqrt{1+1.5^2}\times0.96 = 9.46 m$ $\quad R = \frac{A}{P} = \frac{7.14}{9.46} = 0.755 m$

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025\times20}{7.14\times0.755^{2/3}}\right)^2 = 0.0071$$

iii) $h_n = 1m = h_c$, $A = 7.5 m^2$, $B = b + 2sh = 6 + 2\times1.5\times1 = 9 m$

$D = A/B = 7.5/9.0 = 0.833 m$, $R = 0.7808 m$

$U = \sqrt{gD/\alpha} = \sqrt{9.81\times0.833/1} = 2.859 m/s$

$Q = AU = 7.5\times2.859 = 21.444 m^3/s$

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025\times21.444}{7.5\times0.7808^{2/3}}\right)^2 = 0.0071$$

4.11(b) Trapezoidal channel, $b = 6m$, $s = 1.5$, $n = 0.025$, $\alpha = 1.12$

i) $h_n = 1m$, $Q = 20\ m^3/s$

$A = (6 + 1.5 \times 1) \times 1 = 7.5\ m^2$

$P = 6 + 2\sqrt{1+1.5^2} \times 1 = 9.6055\ m$

$R = A/P = 7.5/9.6055 = 0.7808\ m$

$$\therefore S_n = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025 \times 20}{7.5 \times 0.7808^{2/3}}\right)^2 = 0.0062$$

ii) $Q = 20\ m^3/s$ $\quad Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{20}{\sqrt{9.81/1.12}} = 6.758$

h	A	B	D	$Z = A\sqrt{D}$
1.00	7.50	9.00	0.833	6.847
0.98	7.32	8.94	0.819	6.624
0.99	7.41	8.97	0.826	6.735

$\therefore h_c = 0.99m = h_n$, $A = 7.41\ m^2$

$P = 6 + 2\sqrt{5} \times 0.99 = 10.427\ m$, $R = A/P = 0.711\ m$

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025 \times 20}{7.41 \times 0.711^{2/3}}\right)^2 = 0.0072$$

iii) $h_n = 1m = h_c$, $A = 7.5\ m^2$, $B = 6 + 2 \times 1.5 \times 1 = 9m$

$D = A/B = 0.833m$, $R = 0.7808\ m$

$U = \sqrt{gD/\alpha} = \sqrt{9.81 \times 0.833/1.12} = 2.701\ m/s$

$Q = AU = 7.5 \times 2.701 = 20.26\ m^3/s$

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025 \times 20.26}{7.5 \times 0.7808^{2/3}}\right)^2 = 0.0063$$

~~14~~

4.12 The critical slope is the slope for which the flow is both uniform and critical, i.e. $h_n = h_c$, $U_n = U_c$ and so on. It is obtained by using a uniform flow formula in which the critical flow condition is incorporated. Now, using the Manning formula, we can write

$$U_n = \frac{1}{n} R_n^{2/3} S_c^{1/2}$$

$$\text{or,} \quad S_c = \frac{n^2 U_n^2}{R_n^{4/3}} \quad \text{(i)}$$

But $U_n = U_c$ and U_c is obtained using the critical condition as

$$U_c = \sqrt{gD_c} = \sqrt{gD_n}$$

$$\text{or,} \quad U_n^2 = U_c^2 = gD_n \quad \text{(ii)}$$

Hence, combining (i) and (ii) we obtain

$$S_c = \frac{n^2 \cdot gD_n}{R_n^{4/3}} = \frac{gn^2 D_n}{R_n^{4/3}} \quad \text{(Proved)}$$

Now, for a wide channel, $R = D = h$. Hence

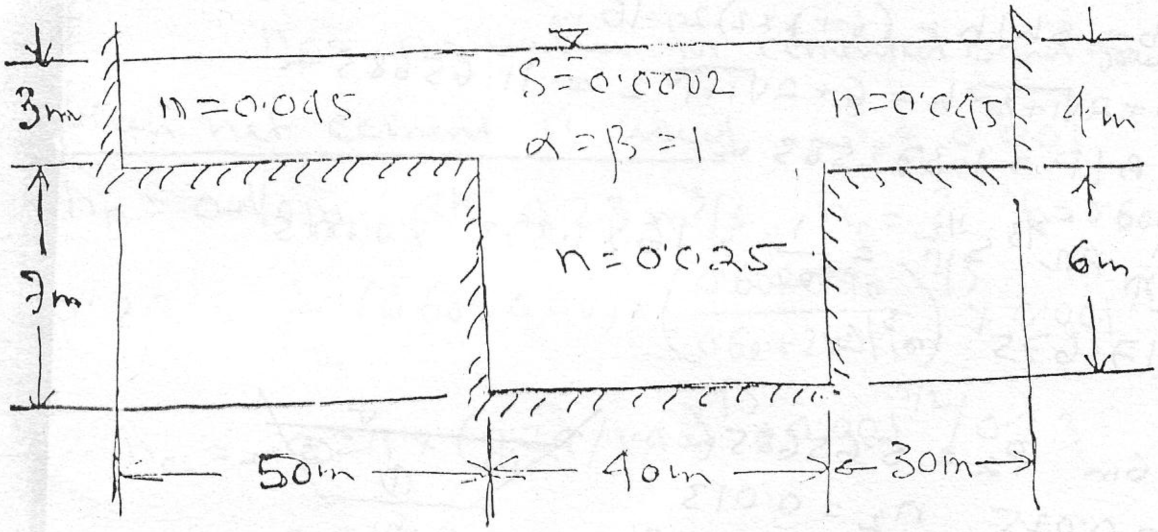
$$S_c = \frac{gn^2 \cdot h_n}{h_n^{4/3}} = \frac{gn^2}{h_n^{1/3}} \quad \text{(Proved)}$$

But the critical depth in a wide channel is given by

$$h_c = \sqrt[3]{\frac{q^2}{g}} = h_n \quad \therefore h_n^{1/3} = \left(\frac{q^2}{g}\right)^{1/9} = \frac{q^{2/9}}{g^{1/9}}$$

$$\therefore S_c = \frac{gn^2}{q^{2/9}/g^{1/9}} = \frac{g^{10/9} n^2}{q^{2/9}} = \frac{n^2 g^{10/9}}{q^{2/9}} \quad \text{(Proved)}$$

4.13



Section	A	P	R	Q	K
Main	400	53	7.547	870.61	61562
Left	150	53	2.830	94.32	6669
Right	120	34	3.529	87.42	6182
Σ	670	140		1052.35	74413

$$U = \frac{Q}{A} = \frac{1052.35}{670} = 1.571 \text{ m/s}$$

$$n = AR^{2/3} S_o^{1/2}/Q = 670 \times (670/140)^{2/3} \times .0002^{1/2} / 1052.35$$

$$= 0.0256 \simeq 0.026$$

$$\alpha = \frac{1 \times 61562^3/400^2 + 1 \times 6669^3/150^2 + 1 \times 6182^3/120^2}{74413^3/670^2}$$

$$= 1.621$$

$$\beta = \frac{1 \times 61562^2/400 + 1 \times 6669^2/150 + 1 \times 6182^2/120}{74413^2/670}$$

$$= 1.221$$

4 $n = 0.025$ $b = 6\,m$ $s = 1$ $h_n = 2\,m$ $S_0 = 0.0005$

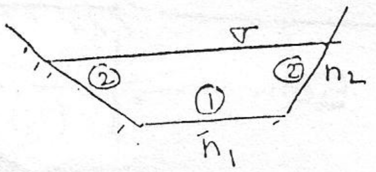
$A = (b + sh)h = (6 + 1 \times 2)2 = 16\ m^2$

$P = b + 2\sqrt{1+s^2}\,h = 6 + 2\sqrt{1+1^2}\,2 = 11.65685\ m$

$R = A/P = 1.372583\ m$

$Q = \frac{1}{n} A R^{2/3} S^{1/2} = \frac{1}{.025} \times 16 \times 1.373^{2/3} \times 0.0005^{1/2}$

$= 17.675\ m^3/s$



i) $P_1 = 6\,m$ $P_2 = 5.65685\,m$

$n_1 = 0.025$ $n_2 = 0.013$

$$n = \left[\frac{P_1 n_1^{1.5} + P_2 n_2^{1.5}}{P}\right]^{2/3}$$

$$= \left(\frac{6 \times .025^{1.5} + 5.65685 \times .013^{1.5}}{11.65685}\right)^{2/3} = 0.01965$$

$$\therefore Q = \frac{17.675 \times 0.025}{0.01965} = 22.49\ m^3/s$$

ii) $P_1 = 6\,m$ $P_2 = 5.65685\,m$, $n_1 = 0.013$ $n_2 = 0.025$

$$n = \left(\frac{6 \times 0.013^{1.5} + 5.65685 \times .025^{1.5}}{11.65685}\right)^{2/3} = 0.0192966$$

$$Q = \frac{17.675 \times 0.025}{0.0192966} = 22.895\ m^3/s$$

iii) $n = 0.013$

$$Q = \frac{17.675 \times 0.025}{0.013} = 33.99\ m^3/s$$

4.15 Let n_c = n-value for neat cement

n_s = n-value for cemented sand grains

<u>When net cement is used</u> $S_0 = 0.001$

$h_n = 0.40\,m,\ Q = 0.23\,m^3/s,\ n_c = ?\quad b = 0.60\,m$

$$0.23 = \frac{1}{n_c}(0.60\times0.40)\times\left(\frac{0.60\times0.40}{0.60+2\times0.40}\right)^{2/3}\times0.001^{1/2}$$

$$\therefore n_c = 0.24\times(0.24/1.40)^{2/3}\times0.001^{1/2}/0.23$$

$$= 0.0102$$

<u>When cemented sand grains are used</u>

$h_n = 0.35\,m,\ Q = 0.12\,m^3/s,\ n_s = ?$

$$0.12 = \frac{1}{n_s}\times(0.60\times0.35)\times\left(\frac{0.60\times0.35}{0.60+2\times0.35}\right)^{2/3}\times0.001^{1/2}$$

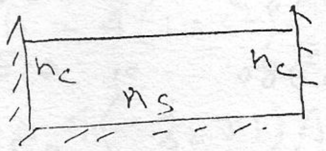
$$\therefore n_s = 0.21\times(0.21/1.30)^{2/3}\times0.001^{1/2}/0.12 = 0.0164$$

<u>Bed is roughened and walls are smooth</u>

$h_n = 0.45\,m,\ b = 0.6\,m,\ S_0 = 0.001$

$P_c = 2\times0.45 = 0.90\,m,\ P_s = 0.60\,m$

$P = P_c + P_s = 0.90 + 0.60 = 1.50\,m$



$$n = \left[\frac{P_c n_c^{1.5} + P_s n_s^{1.5}}{P}\right]^{2/3} = \left[\frac{0.90\times0.0102^{1.5} + 0.60\times0.0164^{1.5}}{1.50}\right]^{2/3}$$

$$= 0.01286$$

$$\therefore Q = \frac{1}{0.01286}\times(0.6\times0.45)\times\left(\frac{0.6\times0.45}{0.6+2\times.45}\right)^{2/3}\times0.001^{1/2}$$

$$= 0.21166\,m^3/s = 0.21\,m^3/s$$

4.16 Compute the flood discharge through a river reach of 1000 m using the following data:

$A_u = 12{,}000\ m^2$, $P_u = 2150\ m$, $n_u = 0.030$, $\alpha_u = 1.15$

$A_d = 10{,}500\ m^2$, $P_d = 2050\ m$, $n_d = 0.030$, $\alpha_d = 1.18$

The fall of water surface in the reach is 0.85 m

Solution $L = 1000\ m$ $F = 0.85\ m$ $h_e = 0$

$$K_u = \frac{1}{.030} \times 12000 \times \left(\frac{12000}{2150}\right)^{2/3} = 1258602$$

$$K_d = \frac{1}{.030} \times 10{,}500 \times \left(\frac{10{,}500}{2050}\right)^{2/3} = 1039980$$

$$K = \sqrt{1258602 \times 1039980} = 1144081$$

$$h_e = 0. \quad \therefore h_f = F + \left(\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}\right)$$

Approximation	Assumed Q	F	$\alpha_u \frac{U_u^2}{2g}$	$\alpha_d \frac{U_d^2}{2g}$	h_f	S_f	Computed Q
1st	—	0.85	—	—	0.85	.00085	33353
2nd	33355	"	0.4529	0.6069	0.6960	.000696	30183
3rd	30183	"	0.3708	0.4970	0.7238	.0007238	30780
4th	30780	"	0.3856	0.5168	0.7188	.0007188	30670
5th	30670	"	0.3829	0.5131	0.7198	.0007198	30695
6th	30695	"	0.3835	0.5140	0.7195	.0007195	30690
7th	30690	"	0.3834	0.5138	0.7196	.0007196	30690

$\therefore Q = 30{,}690\ m^3/s$