Chapter 5 Design of channels

5.1 (a) Circular section

$$A = \frac{1}{8}(\omega - \sin\omega)\, d_0^2 \quad \therefore d_0^2 = 8A/(\omega - \sin\omega)$$

$$P = \frac{1}{2}\omega d_0 \quad \therefore P^2 = \frac{1}{4}\omega^2 d_0^2$$

$$\text{or, } P^2 = \frac{1}{4}\omega^2\, 8A/(\omega - \sin\omega) = \frac{2A\omega^2}{\omega - \sin\omega}$$

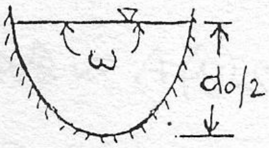
$$\therefore 2P\frac{dP}{d\omega} = 2A\,\frac{(\omega - \sin\omega)2\omega - \omega^2(1-\cos\omega)}{(\omega - \sin\omega)^2} = 0$$

$$\therefore (\omega - \sin\omega)2\omega - \omega^2(1 - \cos\omega) = 0$$

$$\text{or, } 2\omega^2 - 2\omega\sin\omega - \omega^2 + \omega^2\cos\omega = 0$$

$$\text{or, } \omega^2 - 2\omega\sin\omega + \omega^2\cos\omega = 0$$

ω	LHS
0	0
π	0
2π	$8\pi^2$
$\pi/2$	$\frac{\pi}{2}(\frac{\pi}{2}-2)$



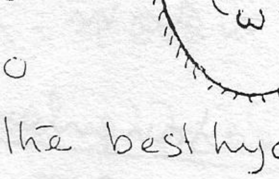
By trial, we get $\omega = \pi$, i.e. the best hydraulic circular section is a semi-circle.

(b) Triangular section $A = sh^2 \quad \therefore h^2 = A/s$

$$P = 2\sqrt{1+s^2}\,h \quad \therefore P^2 = 4(1+s^2)h^2 = 4(1+s^2)\frac{A}{s}$$

$$\text{or, } P^2 = 4\left(\frac{1}{s} + s\right)A \quad \therefore 2P\frac{dP}{ds} = 4\left(-\frac{1}{s^2} + 1\right)A = 0$$

$$\therefore -\frac{1}{s^2} + 1 = 0 \qquad \text{or, } s^2 = 1 \quad \therefore s = 1$$



Hence, the side slope is 1:1, i.e. $\phi/2 = \tan^{-1}(1) = 45^\circ \quad \therefore \phi = 90^\circ$

This means that the best hydraulic triangular section is one-half of a square.

5.1 (c) Parabolic section

$A = \frac{2}{3} Bh \qquad \therefore B = \frac{3A}{2h}$

$P = B + \frac{8}{3}\frac{h^2}{B} = \frac{3A}{2h} + \frac{16}{9}\frac{h^3}{A}$

$\therefore \frac{dP}{dh} = -\frac{3A}{2h^2} + \frac{16}{9A} \cdot 3h^2 = -\frac{3A}{2h^2} + \frac{16}{3A} h^2 = 0$

$\therefore h^4 = \frac{3A}{2} \cdot \frac{3A}{16} = \frac{9A^2}{32} = \frac{9}{32} \cdot \frac{4}{9} B^2 h^2$

$\therefore h^2 = \frac{1}{8} B^2 \quad \therefore B^2 = 8h^2 \Rightarrow B = 2\sqrt{2}h$ (Proved)

5.2 (a) $A = 40\ m^2$

i) Rectangular section

$A = 2h^2 = 40 \quad \therefore h^2 = 20 \qquad \therefore h = 4.472\,m$

$\therefore P = 4h = 4 \times 4.472 = \underline{17.889\ m}$

$R = A/P = 40/17.889 = 2.236\ m$

ii) Triangular section

$A = h^2 = 40 \qquad \therefore h = 6.325\,m$

$\therefore P = 2\sqrt{2}h = 2\sqrt{2} \times 6.325 = \underline{17.889\ m}$

$R = A/P = 40/17.889 = 2.236\ m$

iii) Trapezoidal section

$A = \sqrt{3} h^2 = 40 \qquad \therefore h = 4.806\,m$

5.2 iii) (Contd.)

$\therefore P = 2\sqrt{3}h = 2\sqrt{3} \times 4.806 = \underline{16.647}$ m

$R = A/P = 40/16.647 = 2.403$ m

iv) <u>Circular section</u>

$A = \frac{\pi}{2}h^2 = 40 \qquad \therefore h = 5.046$ m

$\therefore P = \pi h = \pi \times 5.046 = \underline{15.853 \text{ m}}$

$R = A/P = 40/15.853 = 2.523$ m

v) <u>Parabolic section</u>

$A = \frac{4}{3}\sqrt{2}h^2 = 40 \qquad \therefore h = 4.606$ m

$\therefore P = \frac{8}{3}\sqrt{2}h = \frac{8}{3}\sqrt{2} \times 4.606 = \underline{17.369}$ m

$R = A/P = 40/17.369 = 2.303$ m

Hence, for a given area, the circular section has the minimum wetted perimeter.

5.2 (b) For the best hydraulic rectangular section,

$A = 2h^2 \qquad \therefore h = \sqrt{A/2}$

$\therefore P = 4h = 4\sqrt{A/2} = 2\sqrt{2}\sqrt{A} \qquad$ (i)

For the best hydraulic triangular section,

$A = h^2 \qquad \therefore h = \sqrt{A}$

$\therefore P = 2\sqrt{2}h = 2\sqrt{2}.\sqrt{A} \qquad$ (ii)

Hence, for a given area A, the best hydraulic rectangular and triangular sections have the same wetted perimeter.

5.3 $Q = 15\ m^3/s$, $n = 0.013$, $S_0 = 0.001$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.013 \times 15}{\sqrt{0.001}} = 6.166$$

i) <u>Rectangular section</u> $A = 2h^2$ $R = h/2$

$\therefore 2h^2 \times (h/2)^{2/3} = 6.166$ $\therefore h^{8/3} = 4.894$

$\therefore h = 1.814\ m$ $\therefore P = 4h = 4 \times 1.814 = \underline{7.256\ m}$

ii) <u>Triangular section</u> $A = h^2$ $R = \frac{\sqrt{2}}{4} h$

$\therefore h^2 \times \left(\frac{\sqrt{2}}{4} h\right)^{2/3} = 6.166$ $\therefore h = 2.565\ m$

$\therefore P = 2\sqrt{2}h = 2\sqrt{2} \times 2.565 = \underline{7.256\ m}$

iii) <u>Trapezoidal section</u> $A = \sqrt{3}h^2$ $R = h/2$

$\therefore \sqrt{3}h^2 \times (h/2)^{2/3} = 6.166$ $\therefore h^{8/3} = 5.651$

$\therefore h = 1.914\ m$ $\therefore P = 2\sqrt{3}h = \underline{6.632\ m}$

iv) <u>Circular section</u> $A = \frac{\pi}{2} h^2$ $R = h/2$

$\therefore \frac{\pi}{2} h^2 \times (h/2)^{2/3} = 6.166$ $\therefore h^{8/3} = 6.231$

$\therefore h = 1.986\ m$ $\therefore P = \pi h = \underline{6.239\ m} \Longleftarrow$

v) <u>Parabolic section</u> $A = \frac{4}{3}\sqrt{2}h^2$ $R = h/2$

$\therefore \frac{4}{3}\sqrt{2}h^2 \times (h/2)^{2/3} = 6.166$ $\therefore h^{8/3} = 5.191$

$\therefore h = 1.859\ m$ $\therefore P = \frac{8}{3}\sqrt{2}h = \underline{6.999\ m}$

Hence, for a given discharge, the circular section has the minimum wetted perimeter.

5.4(a) $Q = 20\ m^3/s$, $S_0 = 1$ in 2500, $s = 1$, $n = 0.012$

For the best hydraulic trapezoidal section,

$R = h/2$

$b = 2(\sqrt{1+s^2} - s)h = 2(\sqrt{1+1^2} - 1)h = 0.828h$

$A = (b + sh)h = (0.828h + h)h = 1.828h^2$

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$\therefore 20 = \frac{1}{0.012} \times 1.828h^2 \times (h/2)^{2/3} \times \left(\frac{1}{2500}\right)^{1/2}$

$\therefore h^{8/3} = 10.42 \quad \therefore h = 2.408\ m$

$\therefore b = 0.828h = 0.828 \times 2.408 = 1.994\ m$

(b) $Q = 10\ m^3/s$, $s = 2$, $n = 0.015$, $h = 1\ m$

For the best hydraulic trapezoidal section

$R = h/2 = 1/2 = 0.5\ m$

$\therefore b = 2(\sqrt{1+s^2} - s)h = 2(\sqrt{1+2^2} - 2) \times 1 = 0.472\ m$

$A = (b + sh)h = (0.472 + 2 \times 1) \times 1 = 2.472\ m^2$

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$\therefore 10 = \frac{1}{0.015} \times 2.472 \times 0.5^{2/3} \times S_0^{1/2}$

$\therefore S_0 = \left(\frac{0.015 \times 10}{2.472 \times 0.5^{2/3}}\right)^2 = 9.278 \times 10^{-3}$

5.5 $n = 0.015$, $S_0 = 1$ in 2000, $s = 1.5$

(i) Triangular section, $Q = 40\,m^3/s$, rounded corner

$A = h^2(\phi + \cot\phi)$, $P = 2h(\phi + \cot\phi)$, $R = A/P = h/2$

$\cot\phi = s = 1.5 \quad \therefore \phi = 0.588$ rad

$A = h^2(0.588 + 1.5) = 2.088\,h^2$

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

$\therefore 40 = \frac{1}{0.015} \times 2.088\,h^2 \times (h/2)^{2/3} \times (1/2000)^{1/2}$

$\therefore h^{8/3} = 20.3996 \quad \therefore h = 3.098\,m$

(ii) Trapezoidal section, $Q = 80\,m^3/s$, rounded corner, $U_{max} = 2\,m/s$

$A = bh + h^2(\phi + \cot\phi) = Q/U_{max}$

$\therefore bh + 2.088\,h^2 = 80/2 = 40 \quad$ (i)

From the Manning formula $U = \frac{1}{n} R^{2/3} S_0^{1/2}$, we get

$2 = \frac{1}{0.015} \times R^{2/3} \times (1/2000)^{1/2} \Rightarrow R = 1.554\,m$

$\therefore P = A/R = 40/1.554 = 25.74\,m$

or, $b + 2h(\phi + \cot\phi) = 25.74$

or, $b + 2h(0.588 + 1.5) = 25.74 \Rightarrow b + 4.176h = 25.74$ (ii)

From (ii), using $b = 25.74 - 4.176h$ in (i), we get

$h(25.74 - 4.176) + 2.088h^2 = 0$

or, $2.088h^2 - 25.74h + 40 = 0$

$\therefore h = \frac{25.74 \pm \sqrt{25.74^2 - 4 \times 2.088 \times 40}}{2 \times 2.088} = 10.50, 1.824$

When $h = 10.50\,m$, $b = -18.108\,m$

When $h = 1.824\,m$, $b = 18.128\,m$

We accept $h = 1.824\,m$, $b = 18.129\,m$ Ans

5.6 $Q = 30\ m^3/s,\ d_{50} = 2.5\ cm,\ d_{75} = 3\ cm,\ n = 0.025,\ \theta = 32^\circ,$
Trapezoidal section, $s = 2,\ S_0 = 0.001$

i) <u>Using the method of Lane</u>

$d_{75} = 3/2.54 = 1.18$ inch, $\quad s = 2 \therefore \phi = \tan^{-1}\frac{1}{2} = 26.56^\circ$

Assume $b/h_n = 5$.

The maximum unit tractive force on sides (Fig. 5.4(a)) $= 0.77\gamma h_n S_0 = 0.77 \times 9810 \times h_n \times 0.001 = 7.55 h_n$

$$K = \sqrt{1 - \sin^2\phi/\sin^2\theta} = \sqrt{1 - \sin^2 26.56^\circ/\sin^2 32^\circ} = 0.537$$

$$\tau_b = 0.40 d_{75} = 0.40 \times 1.18 = 0.47\ lb/ft^2 = 22.62\ N/m^2 \quad (1\ lb/ft^2 = 47.86\ N/m^2)$$

$$\tau_s = K\tau_b = 0.537 \times 22.62 = 12.15\ N/m^2$$

$\therefore 7.55 h_n = 12.15 \quad \therefore h_n = 1.61\ m$

$\therefore b = 5h_n = 5 \times 1.61 = 8.06\ m$

$\therefore A = (b + sh_n)h_n = (8.06 + 2 \times 1.61) \times 1.61 = 18.19\ m^2$

$P = b + 2\sqrt{1+s^2}\,h = 8.06 + 2\sqrt{1+2^2} \times 1.61 = 15.27\ m$

$R = A/P = 18.19/15.27 = 1.19\ m$

$\therefore Q = (1/0.025) \times 18.19 \times 1.19^{2/3} \times 0.001^{1/2} = 25.85\ m^3/s$

which is less than the design discharge of $30\ m^3/s$. Computation for other values of b/h_n is as follows.

b/h_n	h_n	b	A	P	R	Q
5.9	1.61	9.50	20.53	16.72	1.228	29.77
6.0	1.61	9.67	20.79	16.88	1.231	30.21

Hence, we accept $b/h_n = 6.0$, $h_n = 1.61\ m$ and $b = 9.67\ m$.

With $s = 2$ and $b/h_n = 6.0$, the maximum unit tractive force on bottom (Fig. 5.4(b)) is $0.99\gamma h_n S_0 = 0.99 \times 9810 \times 1.61 \times 0.001 = 15.64\ N/m^2$ which is less than $22.62\ N/m^2$, the permissible traction force on bottom. Hence, the design is acceptable.

5.6(ii) Using modified Yalin - Karahan curve

$$R_1^* = \left(\frac{\rho_s-\rho}{\rho}\right)^{1/3} \times \frac{g^{1/3} d_{50}}{\nu^{2/3}} = 1.65^{1/3} \times \frac{9.81^{1/3} \times 0.025}{(10^{-6})^{2/3}} = 632.40$$

The corresponding value of τ_c^* from Fig. 5.9 is 0.045.

$\therefore \tau_b = \tau_c = \tau_c^* \gamma (\rho_s - 1) d_{50} = 0.045 \times 9810 \times 1.65 \times 0.025 = 18.21$ N/m^2

$\therefore \tau_s = K\tau_b = 0.537 \times 18.21 = 9.78$ N/m^2

$\therefore 7.55 h_n = 9.78 \quad \therefore h_n = 1.30$ m

$\therefore b = 5h_n = 6.47$ m

$\therefore A = (6.47 + 2 \times 1.30) \times 1.30 = 11.74\ m^2 \quad | \quad R = A/P = 0.956$ m

$P = 6.47 + 2\sqrt{1+2^2} \times 1.30 = 12.26$ m

$\therefore Q = (1/0.025) \times 11.74 \times 0.956^{2/3} \times 0.001^{1/2} = 14.11\ m^3/s$

which is far less than the design discharge of 30 m^3/s.

Computation for other values of b/h_n is as follows.

b/h_n	h_n	b	A	P	R	Q
10.00	1.30	12.95	20.12	18.74	1.074	26.69
12.00	1.30	15.59	23.48	21.33	1.101	31.66
11.40	1.30	14.76	22.47	20.55	1.093	30.17

Hence, we accept $b/h_n = 11.40$, $h_n = 1.30$ m and $b = 14.76$ m.

With $s = 2$ and $b/h_n = 11.40$, the maximum unit traction force on level bottom (Fig. 5.4(b)) is $\gamma h_n S_0$

$= 9810 \times 1.30 \times 0.001 = 12.75\ N/m^2$ which is less than $18.21\ N/m^2$, the permissible traction force on bottom. Hence, the design is acceptable.

5.7 Lacey method, $d = 1.5$ mm, $Q = 25\ m^3/s$, $s = 0.5$

$$f_s = 1.76\sqrt{d} = 1.76\sqrt{1.5} = 2.156$$

$$S_o = \frac{f_s^{5/3}}{3340\,Q^{1/6}} = \frac{2.156^{5/3}}{3340 \times 25^{1/6}} = 6.30 \times 10^{-4}$$

$$R = 0.47\,(Q/f_s)^{1/3} = 0.47\,(25/2.156)^{1/3} = 1.064\ m$$

$$P = 4.75\sqrt{Q} = 4.75 \times \sqrt{25} = 23.75\ m$$

$$A = PR = 23.75 \times 1.064 = 25.27\ m^2$$

$$\therefore\ P = 23.75 = b + 2\sqrt{1+0.5^2}\,h = b + 2.236h \quad \text{(i)}$$

$$A = 25.27 = (b + 0.5h)h \quad \text{(ii)}$$

From (i) we have, $b = 23.75 - 2.236h$

$$\therefore \text{ From (ii), } \quad 25.27 = (23.75 - 2.236h + 0.5h)h$$

$$= 23.75h - 1.736h^2$$

$$\therefore\ 1.736h^2 - 23.75h + 25.27 = 0$$

$$\therefore\ h = \frac{23.75 \pm \sqrt{23.75^2 - 4 \times 1.736 \times 25.27}}{2 \times 1.736}$$

$$= 12.518,\ 1.163$$

when $h = 12.518$ m, $b = -4.240$ m

when $h = 1.163$ m, $b = 21.150$ m

Hence we accept $h = 1.163$ m and

$b = 21.150$ m.

Chapter 6 Gradually Varied Flow

Solutions to the Problems

6.1 $H = z_b + h + \alpha \frac{U^2}{2g} \Rightarrow \frac{dH}{dx} = \frac{dz_b}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\alpha \frac{U^2}{2g}\right)$

$$-S_f = -S_o + \frac{dh}{dx} + \frac{d}{dh}\left(\alpha \frac{Q^2}{2gA^2}\right)\frac{dh}{dx}$$

$$\frac{dh}{dx} + \left(\frac{\alpha Q^2}{2g}\right)(-2)A^{-3}\frac{dA}{dh}\frac{dh}{dx} = S_o - S_f$$

$$\frac{dh}{dx} - \frac{\alpha Q^2}{gA^3}\frac{d}{dh}(bh)\frac{dh}{dx} = S_o - S_f$$

$$\frac{dh}{dx} - \frac{\alpha Q^2}{gA^3}\left(b + h\frac{db}{dh}\right)\frac{dh}{dx} = S_o - S_f$$

$$\frac{dh}{dx} - \frac{\alpha Q^2 b}{gA^3}\frac{dh}{dx} = S_o - S_f + \frac{\alpha Q^2 h}{gA^3}\frac{db}{dx}$$

$$\frac{dh}{dx}\left[1 - \frac{\alpha Q^2 b}{gA^3}\right] = S_o - S_f + \frac{\alpha Q^2 h}{gA^3}\frac{db}{dx}$$

$$\therefore \frac{dh}{dx} = \frac{S_o - S_f + \frac{\alpha Q^2 h}{gA^3}\frac{db}{dx}}{1 - \alpha Q^2 b / gA^3} \Rightarrow \text{Proved}$$

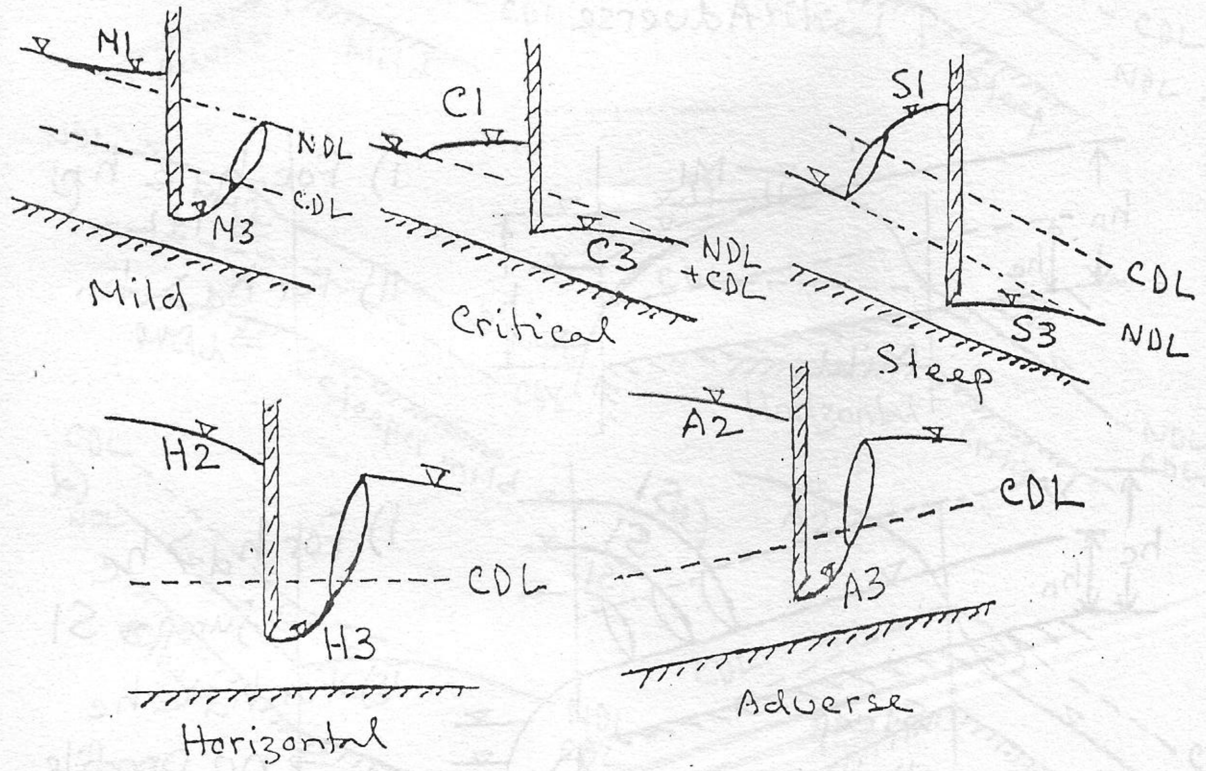
6.2 $\frac{dE}{dx} = \frac{dh}{dx} + \frac{d}{dx}\left(\alpha \frac{U^2}{2g}\right) = \frac{dh}{dx} - Fr^2 \frac{dh}{dx}$

$$= \frac{dh}{dx}(1 - Fr^2)$$

6.2 (contd.)

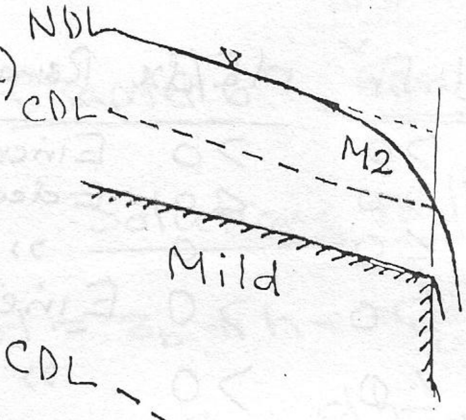
No.	Profile	dh/dx	Fr	$1-Fr^2$	dE/dx	Remarks
1	M1	>0	<1	>0	>0	E increases
2	M2	<0	<1	>0	<0	E decreases
3	M3	>0	>1	<0	0	"
4	S1	>0	<1	>0	>0	E increases
5	S2	<0	>1	<0	>0	"
6	S3	>0	>1	<0	<0	E decreases
7	C1	>0	<1	>0	>0	E increases
8	C3	>0	>1	<0	<0	E decreases
9	H2	<0	<1	>0	<0	"
10	H3	>0	>1	<0	<0	"
11	A2	<0	<1	>0	<0	"
12	A3	>0	>1	<0	<0	"

6.3

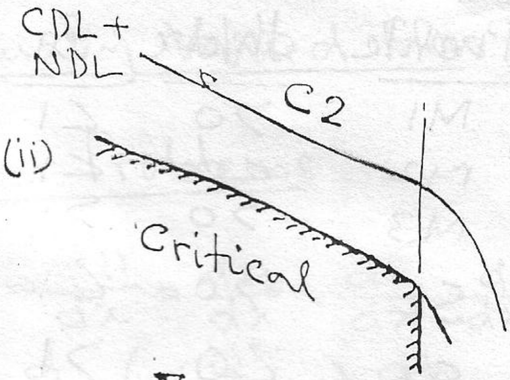


6.4(a)

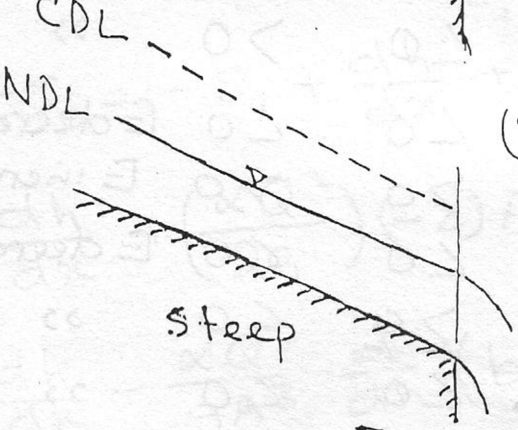
(i)



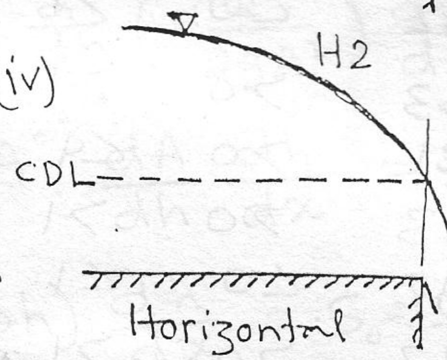
(ii)



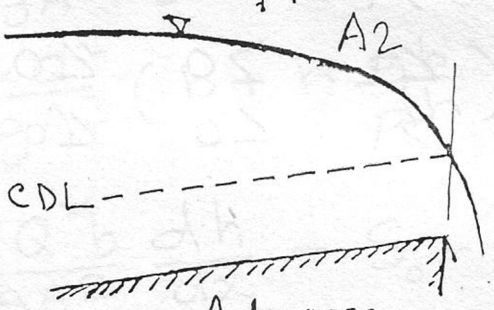
(iii)



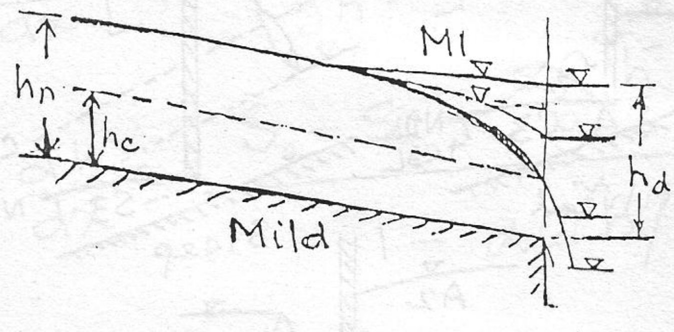
(iv)



(v)



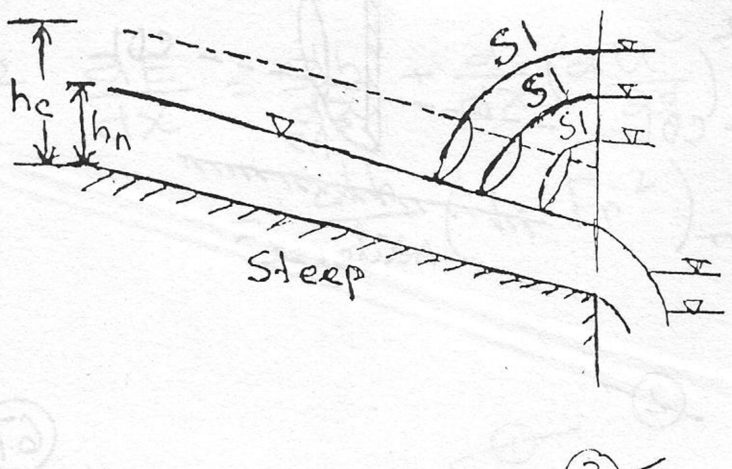
(b)



i) For $h_d < h_n$
$\Rightarrow$ M2

ii) For $h_d > h_n$
$\Rightarrow$ M1

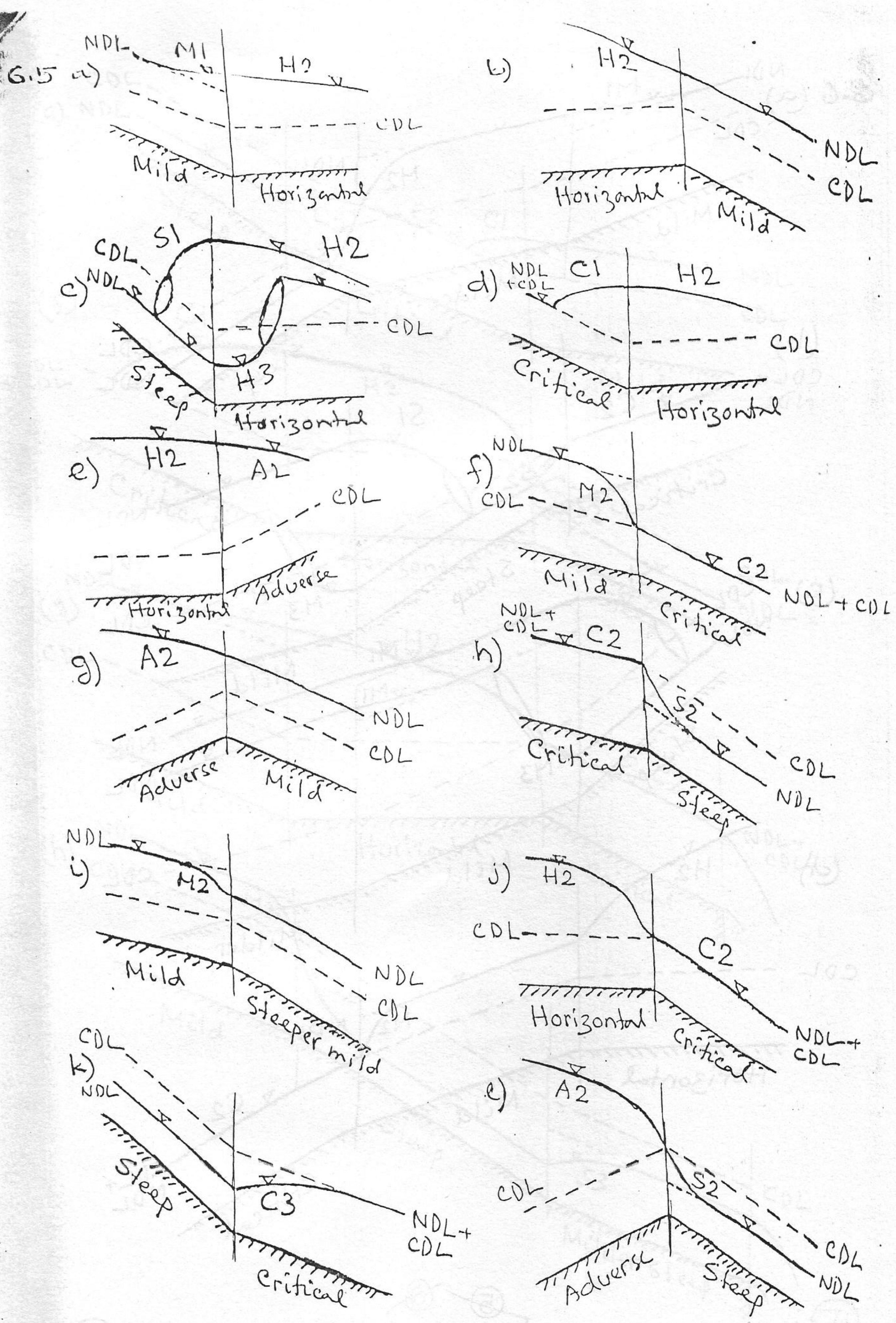
(c)



i) For $h_d > h_c$
$\Rightarrow$ jump + S1

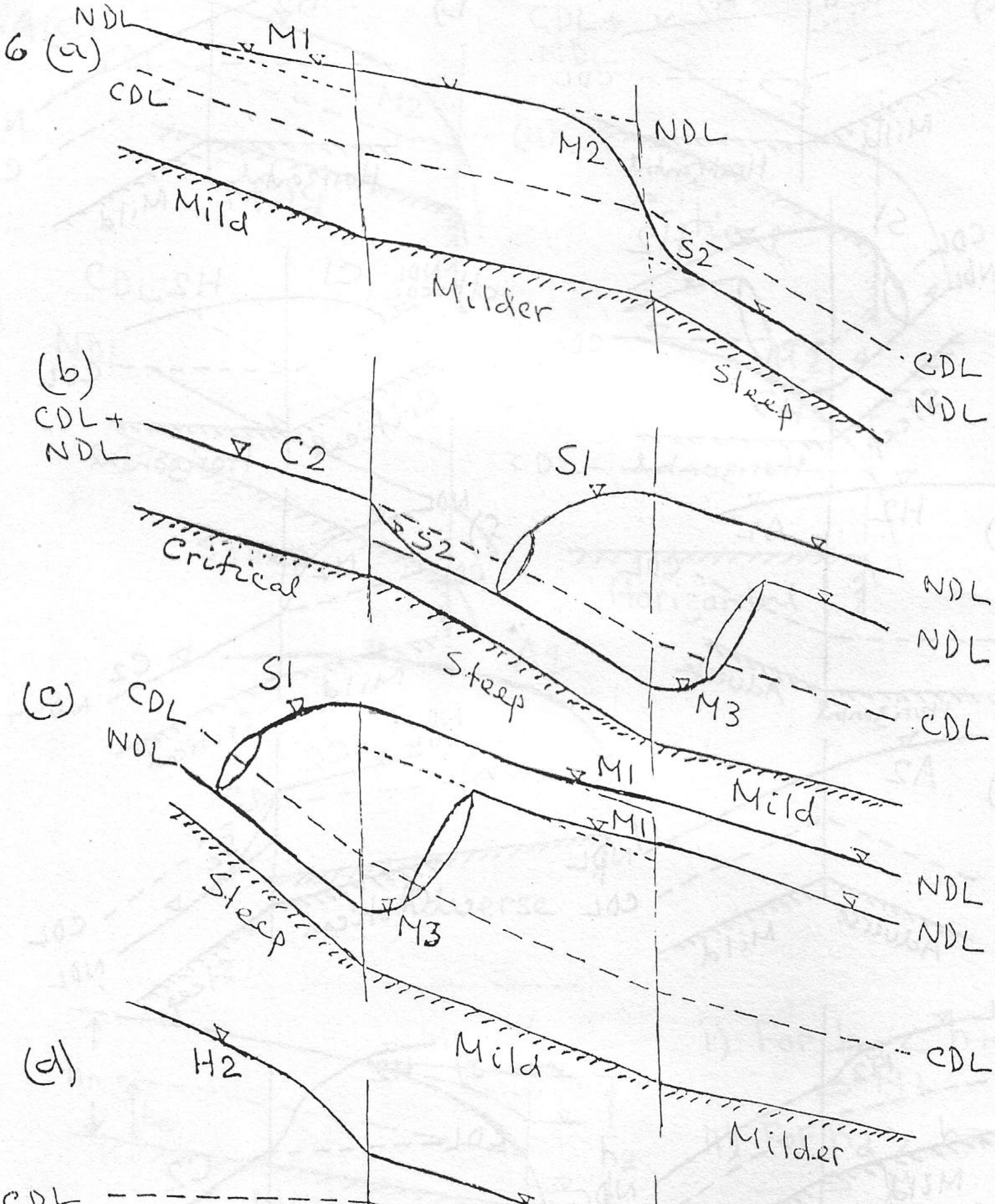
ii) For $h_d < h_c$
$\Rightarrow$ no profile

6.5

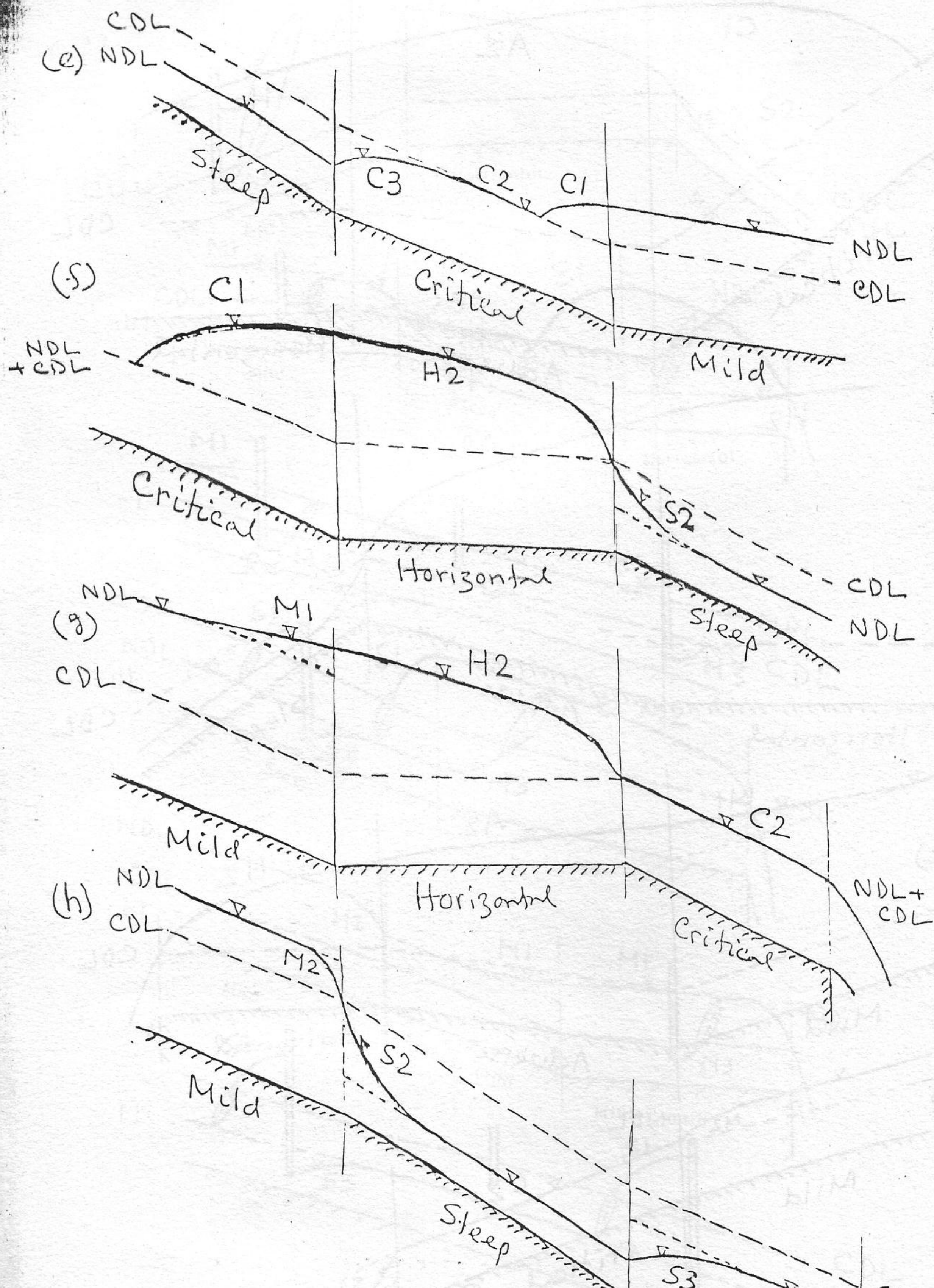


a)
NDL
M1
H2
CDL
Mild
Horizontal
b)
H2
NDL
CDL
Horizontal
Mild
c)
S1
H2
CDL
NDL
CDL
H3
Steep
Horizontal
d)
NDL +CDL
C1
H2
CDL
Critical
Horizontal
e)
H2
A2
CDL
Horizontal
Adverse
f)
NDL
CDL
M2
C2
NDL + CDL
Mild
Critical
g)
A2
NDL
CDL
Adverse
Mild
h)
NDL+ CDL
C2
S2
CDL
NDL
Critical
Steep
i)
NDL
M2
NDL
CDL
Mild
Steeper mild
j)
H2
CDL
C2
NDL+ CDL
Horizontal
Critical
k)
CDL
NDL
C3
NDL+ CDL
Steep
Critical
l)
A2
S2
CDL
CDL
NDL
Adverse
Steep

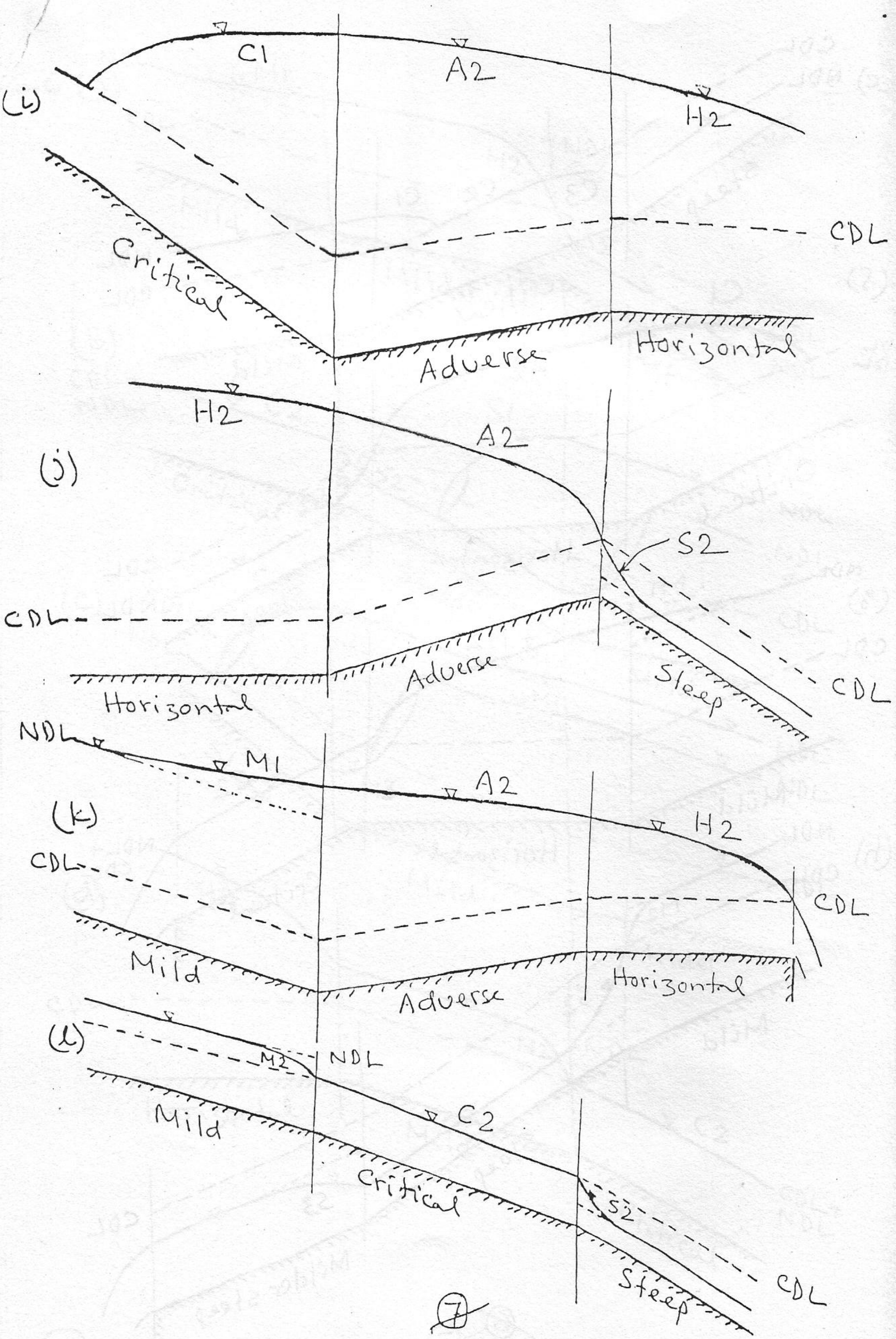
6.6

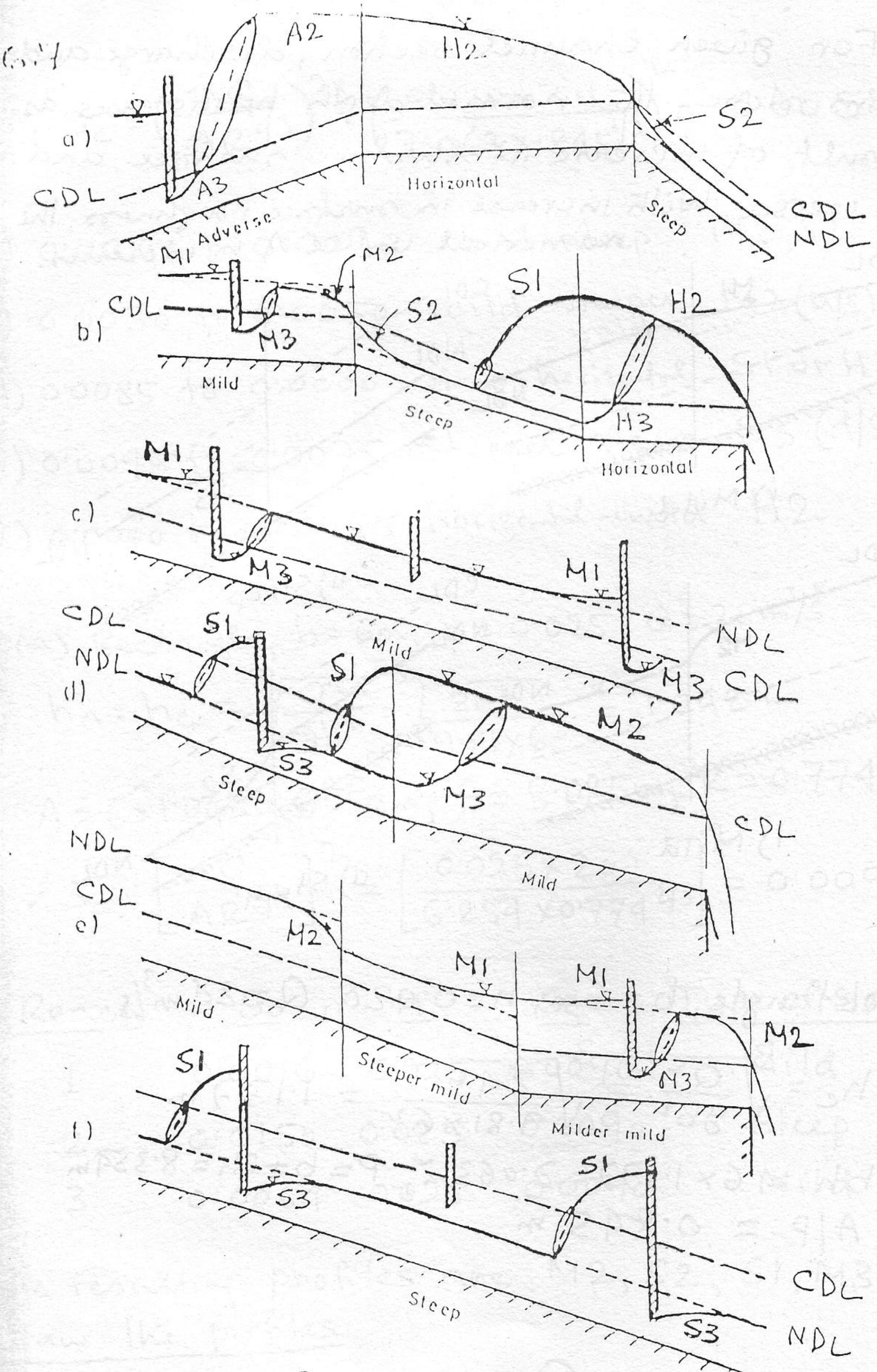


(a)
NDL
M1
CDL
Mild
NDL
M2
S2
Milder
Steep
CDL
NDL
(b)
CDL+
NDL
C2
Critical
S2
S1
Steep
NDL
NDL
M3
CDL
(c)
S1
CDL
NDL
M1
Mild
M1
NDL
NDL
Steep
M3
CDL
(d)
H2
Mild
Milder
CDL
M2
NDL
Horizontal
Mild
C2
CDL+
NDL
Critical



(e)
CDL
NDL
Steep
C3
C2
C1
Critical
Mild
NDL
CDL
(f)
C1
NDL + CDL
H2
Critical
Horizontal
S2
Steep
CDL
NDL
(g)
NDL
M1
H2
CDL
Mild
Horizontal
C2
Critical
NDL + CDL
(h)
NDL
CDL
M2
S2
Mild
Steep
S3
Milder steep
CDL

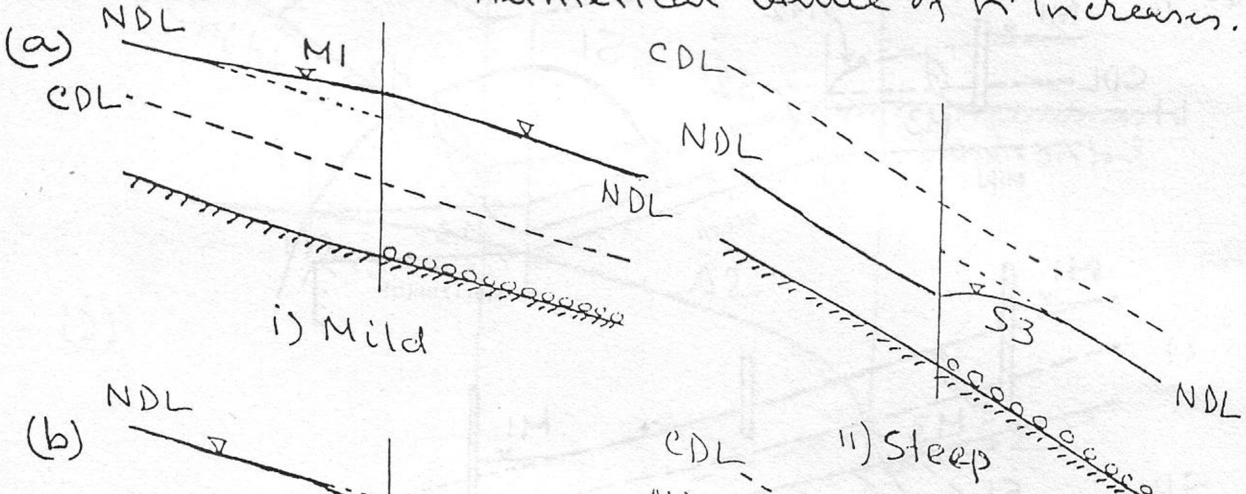

(i)
C1
A2
H2
CDL
Critical
Adverse
Horizontal
(j)
H2
A2
S2
CDL
Horizontal
Adverse
Steep
CDL
(k)
NDL
M1
A2
H2
CDL
CDL
Mild
Adverse
Horizontal
(l)
M2
NDL
Mild
C2
Critical
S2
Steep
CDL



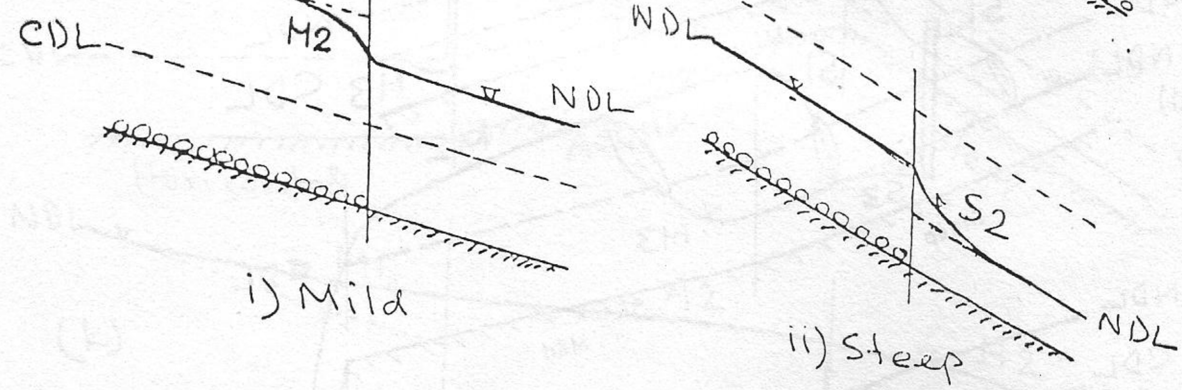
Profiles for Problem 6·7

6.8 For given channel section, discharge and bottom slope, the normal depth h_n increases as a result of increase in surface surface and vice versa. With increase in surface roughness, the numerical value of n increases.

(a)



(b)



6.9 Rectangle, $b = 6\,m$, $n = 0.020$, $Q = 24\,m^3/s$

$$h_n = h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{24^2}{9.81 \times 6^2}} = 1.177\,m$$

$A = bh = 6 \times 1.177 = 7.063\,m^2$, $P = b + 2h = 8.354\,m$

$R = A/P = 0.845\,m$

9

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.020 \times 24}{7.063 \times .845^{2/3}}\right)^2 = 0.0058$$

i) 0.0040 to 0.0090 mild-steep M2, S2

ii) 0.0030 to 0.0050 mild-steeper mild M2 (u/s)

iii) 0.0085 to 0.0000 steep-horizontal S1 or H3

iv) 0.0095 to 0.0075 Steeper-steep S3 (d/s)

v) 0.0000 to 0.0095 horizontal-mild H2

6.10(a) Rectangle, $b = 6m$, $n = 0.025$, $Q = 20\ m^3/s$

$$h_n = h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{20^2}{9.81 \times 6^2}} = 1.042\ m$$

$A = 6 \times 1.042 = 6.254m$, $P = 8.085m$, $R = 0.774\ m$

$$\therefore S_c = \left[\frac{nQ}{AR^{2/3}}\right]^2 = \left[\frac{0.025 \times 20}{6.254 \times 0.774^{2/3}}\right]^2 = 0.0090$$

Reach	S_o	n	S_c	Type of slope
1	0.0016	0.025	0.0090	Mild
2	0.0150	0.025	0.0090	Steep
3	0.0064	0.025	0.0090	Mild

The resulting profiles are M2, S2, S1, M3.

Draw the profiles.

~~10~~

·10(b) $n = 0.015$ for the middle reach.

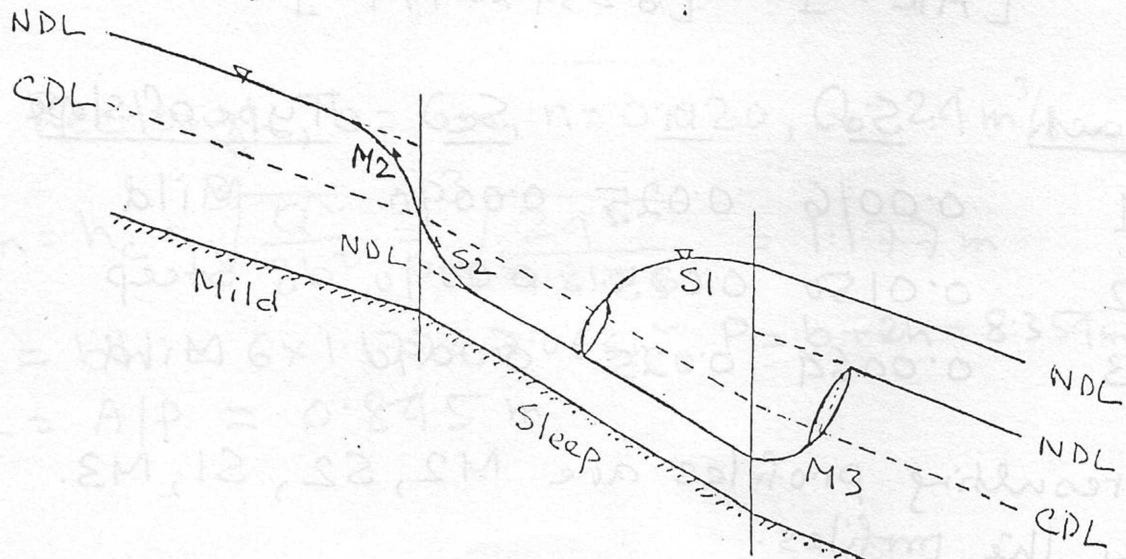
Reach	S_o	n	S_c	Type of slope
1	0.0016	0.025	0.0090	Mild
2	0.0150	0.015	0.0032	Steep
3	0.0064	0.025	0.0090	Mild

The resulting profiles are M2, S2, S1, M3. Draw the profiles.

(c) $n = 0.020, 0.0150$ and 0.025 for the three reaches.

Reach	S_o	n	S_c	Type of slope
1	0.0016	0.020	0.0058	Mild
2	0.0150	0.015	0.0032	Steep
3	0.0064	0.025	0.0090	Mild

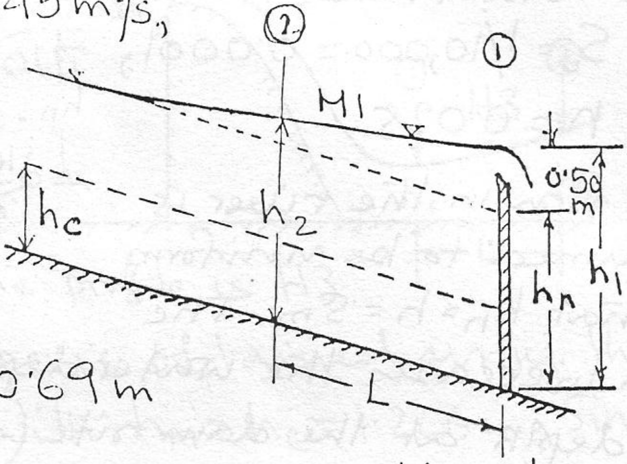
The resulting profiles are M2, S2, S1, M3.



6.11(a) Wide channel, $C = 45\ m^{1/2}/s$, $S_0 = 0.0001$, $q = 1.8\ m^2/s$

$$h_n = \sqrt[3]{\frac{q^2}{C^2 S_0}} = \sqrt[3]{\frac{1.8^2}{45^2 \times 0.0001}}$$

$$= 2.52\ m$$



$$h_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{1.8^2}{9.81}} = 0.69\ m$$

Since $h_n > h_c$, the slope is mild. Also, $h_1 = 2.52 + 0.50 = 3.02\ m$ and $h_2 = 2.80\ m$. Since h_1 or $h_2 > h_n > h_c$, the profile is M1.

$u_1 = h_1/h_n = 3.02/2.52 = 1.20$, $u_2 = h_2/h_n = 2.80/2.52 = 1.11$

$$\phi = \frac{1}{6} \ln \frac{u_1^2 + u_1 + 1}{(u_1 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2u+1}$$

$$\therefore \phi_1 = \frac{1}{6} \ln \frac{1.20^2 + 1.20 + 1}{(1.20 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2 \times 1.20 + 1} = 0.479$$

$$\phi_2 = \frac{1}{6} \ln \frac{1.11^2 + 1.11 + 1}{(1.11 - 1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2 \times 1.11 - 1} = 0.652$$

$$\therefore L = \frac{h_n}{S_0} \left[(u_2 - u_1) - \left(1 - \frac{h_c^3}{h_n^3}\right)(\phi_2 - \phi_1) \right]$$

$$= \frac{2.52}{0.0001} \left[(1.11 - 1.20) - \left(1 - \frac{0.69^3}{2.52^3}\right)(0.652 - 0.479) \right]$$

$$= 2.52 \times 10^4 [-0.09 - 0.979 \times 0.173]$$

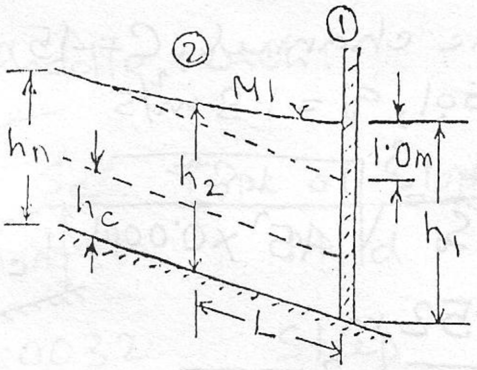
$$= -6484.22\ m$$

~~12~~

5.11(b) Wide river, $h = 5\,m$,

$S_0 = 1/10{,}000 = 0.0001$,

$n = 0.025$



The flow in the river is assumed to be uniform so that $h_n = h = 5\,m$. The dam increases the water depth by 1 m, so that the depth at the dam site (section 1) is $h_1 = 5+1 = 6\,m$.

$$q = \frac{1}{n} h_n^{5/3} S_0^{1/2} = \frac{1}{0.025} \times 5^{5/3} \times 0.0001^{1/2} = 5.85\ m^2/s$$

$$\therefore h_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{5.85^2}{9.81}} = 1.52\,m$$

Since $h_n > h_c$, the slope is mild. Also, $h_2 = 1.1 \times h_n = 5.50\,m$. Hence, h_1 or $h_2 > h_n > h_c$ and the profile is M1.

$u_1 = h_1/h_n = 6/5 = 1.20$ and $u_2 = h_2/h_n = 5.5/5 = 1.10$.

$$\phi_1 = \frac{1}{6} \ln \frac{1.20^2 + 1.20 + 1}{(1.20-1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2 \times 1.20 + 1} = 0.479$$

$$\phi_2 = \frac{1}{6} \ln \frac{1.10^2 + 1.10 + 1}{(1.10-1)^2} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2 \times 1.10 + 1} = 0.681$$

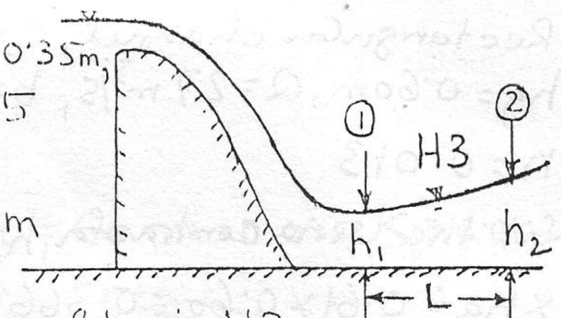
$$\therefore L = \frac{h_n}{S_0} \left[(u_2 - u_1) - \left(1 - \frac{h_c^3}{h_n^3}\right)(\phi_2 - \phi_1) \right]$$

$$= \frac{5}{0.0001} \left[(1.10 - 1.20) - \left(1 - \frac{1.52^3}{5^3}\right)(0.681 - 0.479) \right]$$

$$= 50{,}000 \times [-0.10 - 0.9719 \times 0.202]$$

$$= 50{,}000 \times 0.296 = -14816.25\,m$$

6.12 (a) $q = 4.25\ m^2/s$, $h_1 = 0.35m$, $h_2 = 0.65m$, $n = 0.015$



$$h_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{4.25^2}{9.81}} = 1.226\ m$$

Since h_1 or $h_2 < h_c$, the profile is H3.

The critical slope is obtained using the Manning formula with $h_n = h_c = 1.226\ m$. Since the channel is wide, we write

$$q = \frac{1}{n} h_n^{5/3} S_c^{1/2}$$

$$\therefore S_c = \left(\frac{nq}{\sqrt{S_0}}\right)^2 = \left(\frac{0.015 \times 4.25}{1.226^{5/3}}\right)^2 = 2.0625 \times 10^{-3}$$

For a wide channel, $M = 3$ and when the Manning formula is used, $N = 3.33$. Therefore, $N - M + 1 = 1.33$ and $N + 1 = 4.33$. Also, $p_1 = h_1/h_c = 0.35/1.226 = 0.2855$ and $p_2 = h_2/h_c = 0.65/1.226 = 0.5302$.

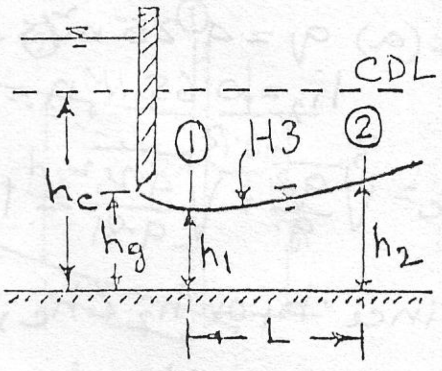
$$\therefore L = \frac{h_c}{S_c}\left[\frac{p_2^{N-M+1} - p_1^{N-M+1}}{N-M+1} - \frac{p_2^{N+1} - p_1^{N+1}}{N+1}\right]$$

$$= \frac{1.226}{2.0625 \times 10^{-3}}\left[\frac{0.5302^{1.33} - 0.2855^{1.33}}{1.33} - \frac{0.5302^{4.33} - 0.2855^{4.33}}{4.33}\right]$$

$$= 594.424 \times (0.1814 - 0.0138)$$

$$= 594.424 \times 0.1676 = 99.63\ m$$

5.12 (b) Rectangular channel, $C_c = 0.61$, $h_g = 0.60\,m$, $Q = 27\,m^3/s$, $b = 6\,m$, $n = 0.013$



Depth at the vena contracta, h_1
$= C_c \times h_g = 0.61 \times 0.60 = 0.366\,m$

$$h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{27^2}{9.81 \times 6^2}} = 1.273\,m$$

$h_2 = 0.50\,m$

Since h_1 or $h_2 < h_c$, the profile is H3.

The critical slope is obtained using the Manning formula with $h_n = h_c = 1.273\,m$. Then,
$A = 6 \times 1.273 = 7.638\,m^2$, $P = 6 + 2 \times 1.273 = 8.546\,m$
and $R = A/P = 7.638/8.546 = 0.894\,m$ so that

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.013 \times 27}{7.638 \times 0.894^{2/3}}\right)^2 = 2.453 \times 10^{-3}$$

For a rectangular channel, $M = 3$. The value of N is obtained for the average depth, $\bar{h} = \frac{h_1 + h_L}{2}$
$= \frac{0.366 + 0.5}{2} = 0.433\,m$ so that $\bar{h}/b = 0.072$.

$$\therefore N = \frac{2}{3}\left[5 - \frac{4(h/b)}{1 + 2(h/b)}\right] = \frac{2}{3}\left[5 - \frac{4 \times 0.072}{1 + 2 \times 0.072}\right] = 3.165$$

Therefore, $N - M + 1 = 1.165$, $N + 1 = 4.165$, $p_1 = h_1/h_c = 0.366/1.273 = 0.2875$ and $p_2 = h_2/h_c = 0.50/1.273 = 0.3928$.

$$\therefore L = \frac{h_c}{S_c}\left[\frac{p_2^{N-M+1} - p_1^{N-M+1}}{N-M+1} - \frac{p_2^{N+1} - p_1^{N+1}}{N+1}\right]$$

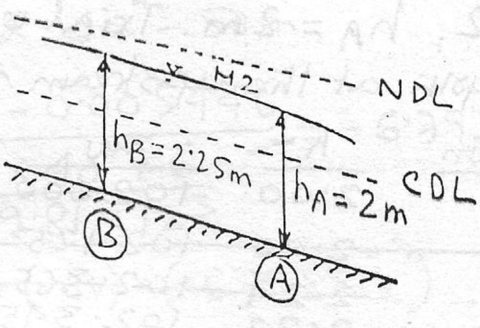
$$= \frac{1.273}{2.453 \times 10^{-3}}\left[\frac{0.3928^{1.165} - 0.2875^{1.165}}{1.165} - \frac{0.3928^{4.165} - 0.2875^{4.165}}{4.165}\right]$$

$$= 518.96 \times (0.08806 - 0.00356) = 43.85\,m$$

6.13 Rectangular channel, $b = 6m$, $n = 0.025$, $S_o = 0.0025$, $Q = 40\ m^3/s$, $h_A = 2m$

$$h_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{40^2}{9.81 \times 6^2}} = 1.65m$$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 40}{\sqrt{0.0025}} = 20.00$$



h	A	P	R	$AR^{2/3}$
2.65	15.90	11.30	1.407	19.96
2.66	15.96	11.32	1.410	20.07

$\therefore h_n = 2.65m$

Since $h_n > h_c$, the slope is mild. Since $h_n > h_A > h_c$, the profile is M2.

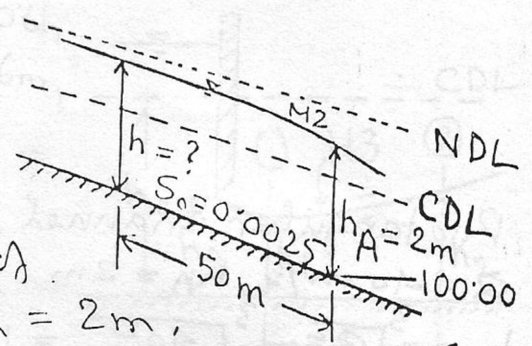
a) Suppose, the depth at section B, $h_B = 2.25m$. Since M2 is a drawdown profile and $h_B > h_A$, section B is located upstream of section A. Since the M2 profile represents subcritical flow, we have to start computation at section A and proceed towards B. Since the distance AB is required, Using the direct step method, the computation is shown below.

h	A	P	R	U	$\alpha U^2/2g$
2.00	12.00	10.00	1.200	3.333	0.5663
2.25	13.50	10.50	1.286	2.963	0.4475

E	ΔE	S_f	$\bar{S_f}$	$S_o - \bar{S_f}$	Δx
2.5663	–	0.005946	–	–	
2.6975	0.1313	0.003924	0.004685	−0.002185	−60.09

$\therefore$ The section B where the depth is 2.25m is located 60.09m upstream from section A.

5.13 (b) Rectangular channel, $b = 6m$, $n = 0.025$, $S_0 = 0.0025$, $Q = 40 m^3/s$, $h_n = 2.65m$, $h_c = 1.65m$, Mild slope, Profile M2, $h_A = 2m$. Trial value of depth at the upstream section = 2m.



Station	h	zw	A	U	$\tilde{U}/2g$	H_1 (1)
0	2.00	102.000	12.00	3.333	0.5663	102.5663
~~50~~	~~2.00~~	~~102.125~~	~~12.00~~	~~3.333~~	~~0.5663~~	~~102.6913~~ ⇒
~~50~~	~~2.24~~	~~102.365~~	~~13.44~~	~~2.976~~	~~0.4515~~	~~102.8165~~
50	2.22	102.345	13.22	3.003	0.4596	102.8046

	R	Sf	$\bar{S}_f$	Δx	hf	he	H_1 (2)
			—	−50	—	0	102.5663
00	1.200	0.005446					
~~00~~	~~1.200~~	~~0.005446~~	~~0.005446~~	~~−50~~	~~−0.2723~~	~~0~~	~~102.8386~~
~~48~~	~~1.282~~	~~0.003973~~	~~0.004710~~	~~−50~~	~~−0.2355~~	~~0~~	~~102.8018~~
44	1.266	0.004144	0.004795	−50	−0.2398	0	102.8061

ial improvement

$(h_2) = 102.6913 - 102.8386 = -0.1473 m$, $B_2 = 6m$, $D_2 = 12/6 = 2m$, $\tilde{Fr}_2 = \tilde{U}_2/gD_2 = \frac{3.333^2}{9.81\times2} = 0.5663$, $N_2 = \frac{2}{3}\left[5 - \frac{4(h_2/b)}{1+2(h_2/b)}\right]$

$= \frac{2}{3}\left[5 - \frac{4\times0.333}{1+2\times0.333}\right] = 2.8$ $(\because h_2/B = 2/6 = 0.333)$

$\therefore \Delta h \text{ (or } \Delta zw) = -\frac{F(h_2)}{1 - \tilde{Fr}_2 - N_2 S_{f2}\Delta x/2h_2}$

$= -\frac{-0.1473}{1 - 0.5663 - 2.80\times0.005446\times(-50)/(2\times2)} = +0.2359 m$

$\therefore h_2^* = h_2 + \Delta h = 2.00 + 0.2359 = \underline{2.24 m}$

2. $F = 102.8165 - 102.8018 = 0.0147$
$B_2 = 6m$, $D_2 = 2.24m$, $\tilde{Fr}_2 = 0.4030$
$N_2 = 2.7633$
$\Delta h = -0.0204$
$\therefore h_2^* = 2.24 - 0.02 = \underline{2.22 m}$

3. $F = 102.8046 - 102.8061$
$= -0.0015$
$B_2 = 6m$, $D_2 = 2.20 m$
$\tilde{Fr}_2 = 0.4172$, $N_2 = 2.77$
$\Delta h = +0.0022 m$
which is small.

~~17~~

6.13(c) $S_0 = 0.0025$, $Q = 40\ m^3/s$, $n = 0.025$

(i) Euler method

$h_1 = 2\,m$, $A_1 = 12.00\,m^2$, $P_1 = 10.00\,m$, $R_1 = 1.20\,m$, $S_{f1} = 0.005446$, $B_1 = 6\,m$ and $\Delta x = 50\,m$

$$\left(\frac{dh}{dx}\right)_1 = \frac{S_0 - S_f}{1 - \frac{\alpha Q^2 B}{gA^3}} = \frac{0.0025 - 0.005446}{1 - \frac{1 \times 40^2 \times 6}{9.81 \times 12^3}} = -6.793 \times 10^{-3}$$

$$\therefore h_2 = h_1 + \Delta x \left(\frac{dh}{dx}\right)_1 = 2.00 + (-50) \times (-6.793 \times 10^{-3})$$

$$= 2.3396\,m$$

(ii) Modified Euler method

$$h_{1\frac{1}{2}} = h_1 + \frac{\Delta x}{2}\left(\frac{dh}{dx}\right)_1 = 2.00 + 0.5 \times (-50) \times (-6.793 \times 10^{-3})$$

$$= 2.1698\,m$$

For this depth, $A = 13.0189\,m^2$, $P = 10.3396\,m$, $R = 1.259\,m$, $S_f = 4.3393 \times 10^{-3}$ and $B = 6\,m$

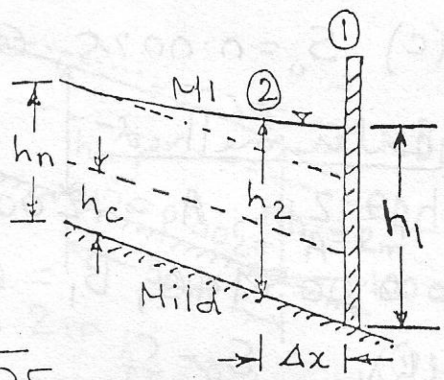
$$\therefore \left(\frac{dh}{dx}\right)_{1\frac{1}{2}} = \frac{S_0 - S_f}{1 - \frac{\alpha Q^2 B}{gA^3}} = \frac{0.0025 - 4.3393 \times 10^{-3}}{1 - \frac{1 \times 40^2 \times 6}{9.81 \times 13.0189^3}}$$

$$= -3.3051 \times 10^{-3}$$

$$\therefore h_2 = h_1 + \Delta x \left(\frac{dh}{dx}\right)_{1\frac{1}{2}} = 2.00 + (-50) \times (-3.3051 \times 10^{-3})$$

$$= 2.1653\,m$$

6.14 Trapezoidal channel, $b = 5m$, $s = 2$, $n = 0.020$, $S_o = 0.002$, $Q = 48.67\ m^3/s$, $h_1 = 5m$



$$Z_c = \frac{Q}{\sqrt{g}} = \frac{48.67}{\sqrt{9.81}} = 15.539$$

h	A	B	D	Z
1.68	14.041	11.720	1.198	15.375
1.69	14.162	11.760	1.204	15.591

$\therefore h_c = 1.69m$

$$AR^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.020 \times 48.67}{\sqrt{0.002}} = 21.766$$

h	A	P	R	$AR^{2/3}$
2.01	18.130	13.989	1.296	21.552
2.02	18.261	14.034	1.301	21.765

$\therefore h_n = 2.02m$

Since $h_n > h_c$, the slope is mild.
Since $h_1 > h_n > h_c$, the profile is M1. Since the M1 profile represents subcritical flow, we have to start the computation at the dam site and proceed upstream.

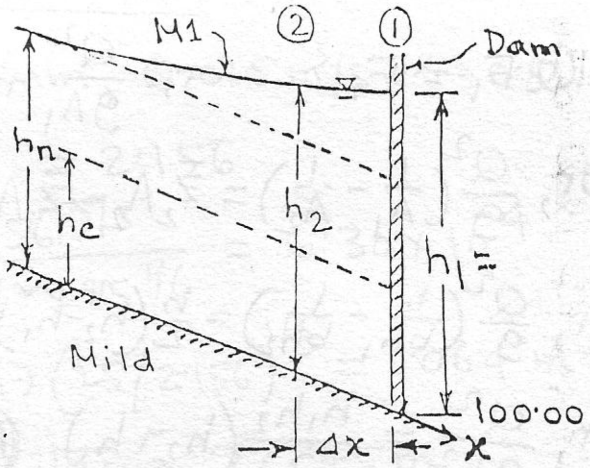
a) Direct step method $\quad h_2 = 4.90m \quad \Delta x = ?$

h	A	P	R	U	$U^2/2g$	E
5.00	75.00	27.36	2.741	0.6489	0.02146	5.02146
4.90	72.52	26.91	2.695	0.6711	0.02296	4.9230

ΔE	S_f	$\bar{S}_f$	$S_o - \bar{S}_f$	Δx
–	4.38×10^{-5}	–	–	–
0.0985	4.80×10^{-5}	4.59×10^{-5}	0.0019541	– 50.41

Hence, the depth 4.90m is 50.41m upstream from the dam.

6·14 (b) Trapezoidal channel, $b = 5m$, $s = 2$, $n = 0·020$, $S_0 = 0·002$, $Q = 48·67\ m^3/s$, $h_1 = 5m$, $h_c = 1·69m$, $h_n = 2·02m$, Slope is mild, Profile is M1. The trial value of depth $h_2 = 5·00m$.



The computation using the standard step method is as follows.

Station	h	z_w	A	U	$U^2/2g$	H_1 ①
0	5·00	105·00	75·00	0·649	0·0215	105·0215
~~50~~	~~5·00~~	~~105·10~~	~~75·00~~	~~0·649~~	~~0·0215~~	~~105·1215~~
50	4·90	105·00	72·52	0·672	0·0230	105·0230

P	R	S_f	$\bar{S}_f$	Δx	h_f	h_e	H_1 ②
27·36	2·741	$4·39 \times 10^{-5}$	—	—	—	0	105·0215
~~27·36~~	~~2·741~~	~~$4·39 \times 10^{-5}$~~	~~$4·39 \times 10^{-5}$~~	~~−50~~	~~$-2·195 \times 10^{-3}$~~	~~0~~	~~105·0237~~
26·91	2·694	$4·80 \times 10^{-5}$	$4·59 \times 10^{-5}$	−50	$-2·300 \times 10^{-3}$	0	105·0238

<u>Improved trial value of h_2</u>

$F(h_2) = 105.1215 - 105·0231 = 0·0978$

$B_2 = b + 2sh_2 = 5 + 2 \times 2 \times 5 = 25m \quad D_2 = A_2/B_2 = 75/25 = 3\ m$

$Fr_2^2 = U_2^2/gD_2 = 0·649^2/(9·81 \times 3) = 0·0143$

$N_2 = \frac{2h_2}{3A_2}\left(5B_2 - 2R_2\frac{dP}{dh}\right) = \frac{2 \times 5}{3 \times 75}(5 \times 25 - 2 \times 2·741 \times 2\sqrt{5}) = 4·466$

$$\therefore \Delta h \text{ (or } \Delta z_w) = -\frac{F(h_2)}{1 - Fr_2^2 - N_2 S_{f2} \Delta x / 2h_2}$$

$$= -\frac{0·0978}{1 - 0·0143 - 4·466 \times 4·39 \times 10^{-5} \times (-50)/(2 \times 5)}$$

$$= -0·10\ m$$

$\therefore h_2^* = h_2 + \Delta h_2 = 5·00 - 0·10 = 4·90\ m$