7.1(i) $F_1 = F_2$ or, $\frac{Q^2}{gA_1} + \bar{Z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{Z}_2 A_2$

or, $\frac{Q^2}{g}\left(\frac{1}{A_1} - \frac{1}{A_2}\right) = \bar{Z}_2 A_2 - \bar{Z}_1 A_1 \Rightarrow \frac{Q^2}{g}\left(\frac{1}{bh_1} - \frac{1}{bh_2}\right) = \frac{bh_2^2}{2} - \frac{bh_1^2}{2}$

or, $\frac{Q^2}{g}\left(\frac{1}{bh_1} - \frac{1}{bh_2}\right) = \frac{b}{2}(h_2^2 - h_1^2) \Rightarrow \frac{Q^2}{gb^2}\left(\frac{h_2 - h_1}{h_1 h_2}\right) = \frac{(h_2-h_1)(h_2+h_1)}{2}$

or, $\frac{Q^2}{gb^2} = \frac{h_1 h_2}{2}(h_2 + h_1)$ (A)

or, $\frac{\cancel{b^2} h_1^2 U_1^2}{g\cancel{b^2}} = \frac{h_1 h_2}{2}(h_2 + h_1)$ $(\because Q = A_1 U_1 = bh_1 U_1)$

or, $\frac{U_1^2}{gh_1} = \frac{h_2}{2h_1^2}(h_2 + h_1)$ (Dividing by h_1^3)

or, $Fr_1^2 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1} + 1\right)$ $\left(\because Fr_1^2 = \frac{U_1^2}{gD_1} = \frac{U_1^2}{gh_1}\right)$

or, $2Fr_1^2 = \left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1} \Rightarrow \left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1} - 2Fr_1^2 = 0$

$\therefore \frac{h_2}{h_1} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1(-2Fr_1^2)}}{2 \cdot 1} = \frac{-1 + \sqrt{1 + 8Fr_1^2}}{2}$

$= \frac{1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$ (Ignoring − sign) (7.10)

$\Rightarrow$ Proved

(ii) From (A), using $Q = A_2 U_2 = bh_2 U_2$, we get

$\frac{\cancel{b^2} h_2^2 U_2^2}{g\cancel{b^2}} = \frac{h_1 h_2}{2}(h_2 + h_1) \Rightarrow \frac{U_2^2}{gh_2} = \frac{h_1}{2h_2^2}(h_1 + h_2)$ (Dividing by h_2^3)

or, $Fr_2^2 = \frac{1}{2}\frac{h_1}{h_2}\left(\frac{h_1}{h_2} + 1\right) \Rightarrow 2Fr_2^2 = \left(\frac{h_1}{h_2}\right)^2 + \frac{h_1}{h_2}$

or, $\left(\frac{h_1}{h_2}\right)^2 + \frac{h_1}{h_2} - 2Fr_2^2 = 0$

$\therefore \frac{h_1}{h_2} = \frac{1}{2}\left(\sqrt{1 + 8Fr_2^2} - 1\right)$ (As before ignoring − sign)

$\Rightarrow$ Proved (7.11)

7.1(iii) (contd.) $h_L = H_1 - H_2 = E_1 - E_2 = \left(h_1 + \frac{U_1^2}{2g}\right) - \left(h_2 + \frac{U_2^2}{2g}\right)$

$$= (h_1 - h_2) + \frac{Q^2}{2gb^2}\left(\frac{1}{h_1^2} - \frac{1}{h_2^2}\right)$$

$$= (h_1 - h_2) + \frac{Q^2}{2gb^2}\,\frac{(h_1+h_2)(h_2-h_1)}{h_1^2 h_2^2} \quad \text{(i)}$$

Again from $F_1 = F_2$ we have

$$\frac{Q^2}{gA_1} + \bar{Z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{Z}_2 A_2$$

Since $\bar{Z}_1 = h_1/2$ and $\bar{Z}_2 = h_2/2$

$$\frac{Q^2}{gbh_1} + \frac{h_1}{2} bh_1 = \frac{Q^2}{gbh_2} + \frac{h_2}{2} bh_2$$

$$\frac{Q^2}{gb}\left(\frac{1}{h_1} - \frac{1}{h_2}\right) = \frac{b}{2}\left(h_2^2 - h_1^2\right)$$

$$\frac{2Q^2}{gb^2} = h_1 h_2 (h_1 + h_2) \quad \text{(ii)}$$

From (i) and (ii) we get

$$h_L = (h_1 - h_2) + \frac{h_1 h_2}{4}(h_1+h_2)\cdot\frac{(h_1+h_2)(h_2-h_1)}{h_1^2 h_2^2}$$

$$= \frac{(h_2-h_1)}{4h_1h_2}\left[(h_1+h_2)^2 - 4h_1h_2\right]$$

$$= \frac{(h_2-h_1)}{4h_1h_2}(h_2-h_1)^2 = \frac{(h_2-h_1)^3}{4h_1h_2} \quad (7.15)$$

→ Proved

(iv) $E_1 = h_1 + \frac{u_1^2}{2g} \Rightarrow \frac{E_1}{h_1} = 1 + \frac{u_1^2}{2gh_1} = 1 + \frac{1}{2}Fr_1^2$

$\therefore \frac{E_1}{h_1} = \frac{2 + Fr_1^2}{2}$ (i)

$$\boxed{\begin{array}{l} A_1 u_1 = A_2 u_2 \\ h_1 u_1 = h_2 u_2 \end{array}} \quad \boxed{u_2 = u_1 \cdot \frac{h_1}{h_2}}$$

(vi)

$E_2 = h_2 + \frac{u_2^2}{2g} \Rightarrow \frac{E_2}{h_1} = \frac{h_2}{h_1} + \frac{u_2^2}{2gh_1}$

$$\frac{E_2}{h_1} = \frac{h_2}{h_1} + \frac{u_1^2}{2gh_1} \cdot \frac{h_1^2}{h_2^2} = \frac{h_2}{h_1} + \frac{Fr_1^2}{2} \cdot \left(\frac{h_1}{h_2}\right)^2$$

$$= \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) + \frac{Fr_1^2}{2}\left[\frac{1}{\frac{1}{2}(\sqrt{1+8Fr_1^2} - 1)}\right]^2$$

$$= \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) + 2Fr_1^2 \frac{(\sqrt{1+8Fr_1^2} + 1)^2}{(8Fr_1^2)^2}$$

$$= \frac{\frac{1}{2}\left[16Fr_1^2\sqrt{1+8Fr_1^2} - 16Fr_1^2 + 1 + 8Fr_1^2 + 2\sqrt{1+8Fr_1^2} + 1\right]}{16Fr_1^2}$$

$$= \frac{1}{2}\left[\frac{(\sqrt{1+8Fr_1^2})^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2}\right] \quad \text{(ii)}$$

From (i) & (ii)

$$\frac{E_2}{E_1} = \frac{(8Fr_1^2 + 1)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2 + Fr_1^2)} \Rightarrow \text{proved } (7.18)$$

(v) $h_j = h_2 - h_1$ $\qquad \frac{E_1}{h_1} = \frac{2 + Fr_1^2}{2}$ (i) (as before)

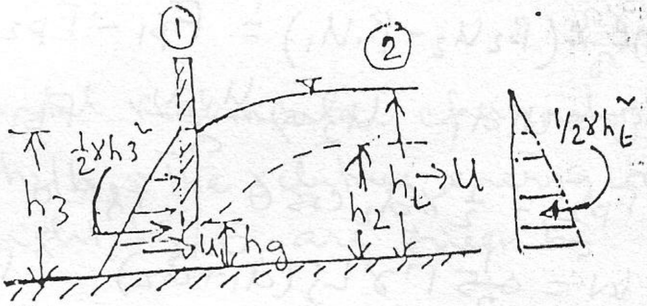
$$\therefore \frac{h_j}{h_1} = \frac{h_2}{h_1} - 1 = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) - 1$$

$$= \frac{\sqrt{1+8Fr_1^2} - 3}{2} \quad \text{(ii)}$$

$\therefore$ From (i) and (ii), $\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2 + Fr_1^2} \Rightarrow$ Proved (7.20)

~~3~~

7.1 (contd)



(vi) $\frac{\gamma Q}{g}(u_t - u_g) = \frac{1}{2}\gamma h_3^2 - \frac{1}{2}\gamma h_t^2$

$$\frac{q}{g}\left(\frac{q}{h_t} - \frac{q}{h_g}\right) = \frac{1}{2}(h_3^2 - h_t^2) \qquad Fr = \frac{u_t}{\sqrt{g h_t}}$$

$$\frac{q^2}{g}\left[\frac{1}{h_t} - \frac{1}{h_g}\right] = \frac{1}{2}(h_3^2 - h_t^2) \qquad \therefore Fr^2 \cdot g = \frac{u_t^2}{h_t} = \frac{q^2}{h_t^3}$$

$$Fr_t^2 h_t^3\left[\frac{1}{h_t} - \frac{1}{h_g}\right] = \frac{1}{2}(h_3^2 - h_t^2)$$

$$Fr_t^2\left[h_t^2 - \frac{h_t^3}{h_g}\right] = \frac{1}{2}(h_3^2 - h_t^2) \qquad \therefore \frac{q^2}{g} = Fr_t^2 h_t^3$$

$$2Fr_t^2\left[h_t^2 - \frac{h_t^3}{h_g}\right] = h_3^2 - h_t^2$$

$$2Fr_t^2\left[1 - \frac{h_t}{h_g}\right] = \left(\frac{h_3}{h_t}\right)^2 - 1 \quad [\text{Dividing by } h_t^2]$$

$$\therefore \left(\frac{h_3}{h_t}\right)^2 = 1 + 2Fr_t^2\left(1 - \frac{h_t}{h_g}\right)$$

$$\therefore \frac{h_3}{h_t} = \sqrt{1 + 2Fr_t^2(1 - h_t/h_g)} \Rightarrow \text{Proved}$$

(7.21)

1 (iii) contd) $\frac{\gamma q}{g}(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W\sin\theta - F_f$

$q = U_1 d_1 \quad U_2 = \frac{U_1 d_1}{d_2} \quad F_{p1} = \frac{1}{2}\gamma d_1^2 \cos\theta$

$F_{p2} = \frac{1}{2}\gamma d_2^2 \cos\theta \quad F_f = 0 \quad \beta_1 = \beta_2 = 1$

$W = \cancel{0.5}\frac{1}{2}\Gamma\gamma L_j (d_1 + d_2) \quad Fr_1 = \frac{U_1}{\sqrt{g d_1}}$

$\therefore \frac{\gamma q}{g}(U_2 - U_1) = \frac{1}{2}\gamma\cos\theta(d_1^2 - d_2^2) + \frac{1}{2}\Gamma\gamma L_j (d_1+d_2)\sin\theta$

$\frac{q}{g}\left(\frac{U_1 d_1}{d_2} - U_1\right) = \frac{1}{2}\cos\theta(d_1^2 - d_2^2) + \frac{1}{2}\Gamma L_j (d_1+d_2)\sin\theta$

$\frac{U_1 d_1}{g}\left(\frac{U_1 d_1}{d_2} - U_1\right) = \cdots$

$\frac{U_1^2 d_1^2}{g d_2} - \frac{U_1^2 d_1}{g} = \cdots$

$\frac{Fr_1^2 \cdot d_1^3}{d_2} - Fr_1^2 d_1^2 = \frac{1}{2}(d_1+d_2)\left\{\cos\theta(d_1 - d_2) + \Gamma L_j \sin\theta\right\}$

$Fr_1^2\left(\frac{d_1^3}{d_2} - d_1^2\right) =$

$\therefore d_1^2\left(\frac{d_1}{d_2} - 1\right) = \frac{d_1+d_2}{2}\left\{\frac{\cos\theta(d_1-d_2) + \Gamma L_j \sin\theta}{Fr_1^2}\right\}$

$d_1^2\left(\frac{d_1 - d_2}{d_2}\right) =$

$\frac{d_1^2}{d_2} = \frac{d_1+d_2}{2}\left\{\frac{\cos\theta + \Gamma L_j \sin\theta/(d_1-d_2)}{Fr_1^2}\right\}$

$= \frac{d_1+d_2}{2}\left[\frac{\cos\theta - \frac{\Gamma L_j \sin\theta}{d_2 - d_1}}{Fr_1^2}\right] = \frac{d_1+d_2}{2G^2}$

where $G^2 = \frac{Fr_1^2}{\cos\theta - \frac{\Gamma L_j \sin\theta}{d_2 - d_1}}$

$\therefore -\frac{d_1^2}{d_2} + \frac{d_1+d_2}{2G^2} = 0 \Rightarrow -d_1^2(2G^2) + d_2(d_1+d_2) = 0$

$-2d_1^2 G^2 + d_1 d_2 + d_2^2 = 0 \Rightarrow d_2^2 + d_1 d_2 - 2d_1^2 G^2 = 0$

$\therefore d_2 = \frac{-d_1 \pm \sqrt{d_1^2 + 8d_1^2 G^2}}{2} \Rightarrow \frac{d_2}{d_1} = \frac{h_2^*}{h_1} = \frac{1}{2}\left(\sqrt{1+8G^2} - 1\right) \Rightarrow$ Proved

(considering + sign only) (7.26)

7.2(a) For jumps in horizontal rectangular channels, the sequent depth ratio h_2/h_1, the relative energy loss h_L/E_1 and relative height h_j/E_1 are given by

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right)$$

$$\frac{h_L}{E_1} = 1 - \frac{E_2}{E_1} = 1 - \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2+Fr_1^2)}$$

$$\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2}$$

The computed values of h_2/h_1, h_L/E_1 and h_j/E_1 for different values of Fr_1 are given below.

Fr_1	h_2/h_1	h_L/E_1	h_j/E_1
1	1	0	0
1.7	1.96	0.05	0.39
2.5	3.07	0.18	0.50
4.5	5.88	0.44	0.44
9	12.24	0.70	0.27
15	20.71	0.82	0.17

(b) $\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2}$ ⇒ Differenentiating w.r.t. Fr_1, we get

$$\frac{d(h_j/E_1)}{dFr_1} = \frac{(2+Fr_1^2)\left[\frac{1}{2}(1+8Fr_1^2)^{-\frac{1}{2}}\cdot 16Fr_1\right] - \left(\sqrt{1+8Fr_1^2}-3\right)2Fr_1}{(2+Fr_1^2)^2}$$

$= 0$ for maximum value of h_j/E_1

$$\therefore (2+Fr_1^2)\cdot 8Fr_1 \cdot \frac{1}{\sqrt{1+8Fr_1^2}} - \left(\sqrt{1+8Fr_1^2} - 3\right)2Fr_1 = 0$$

or, $8 + 4Fr_1^2 - (1+8Fr_1^2) + 3\sqrt{1+8Fr_1^2} = 0$

or, $7 - 4Fr_1^2 = -3\sqrt{1+8Fr_1^2}$

By trial, $Fr_1 = 2.77$

Using $Fr_1 = 2.77$, we get

$$h_j/E_1 = \frac{\sqrt{1+8\times2.77^2} - 3}{2+2.77^2}$$

$= 0.51$

Fr_1	R.H.S.	L.H.S.
2	−9	−17.27
3	−29	−15.63
2.7	−22.16	−23.11
2.77	−23.69	−23.69

7.3 (a) $U_1 = 6.1$ m/s $h_1 = 1$ m $b = 6.1$ m, rectangle

$$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{6.1}{\sqrt{9.81\times1}} = 1.95$$ $\boxed{Fr_1^2 = 3.793}$

i) $\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = \frac{1}{2}\left(\sqrt{1+8\times1.95^2} - 1\right) = 2.30$

$\therefore h_2 = 2.30\times1 = 2.30$m

ii) Since $Fr_1 = 1.95$, the jump is a weak jump.

iii) $h_j = h_2 - h_1 = 2.30 - 1.00 = 1.30$m

iv) $L_j/h_2 = 4.2$ $\therefore L_j = 4.2\times2.30 = 9.66$m

Also, $L_j = 9.75h_1(Fr_1-1)^{1.01} = 9.75\times1\times0.95^{1.01} = 9.28$m

v) $h_L = \frac{(h_2-h_1)^3}{4h_1h_2} = \frac{(2.30-1)^3}{4\times1\times2.30} = 0.238 \frac{\text{m-kg}}{\text{kg}}$

$\therefore$ hp dissipated $= \frac{7.22\times Q h_L}{550} = \frac{7.22\times1000\times6.1\times1\times6.1\times0.238}{550}$

$= 116.2$

vi) $E_1 = h_1 + \frac{U_1^2}{2g} = 1 + \frac{6.1^2}{2\times9.81} = 2.8965$

$E_2 = h_2 + \frac{U_2^2}{2g} = 2.30 + \frac{2.652^2}{2\times9.81} = 2.6585$

$\therefore$ Efficiency $= E_2/E_1 = 2.6585/2.8965 = 0.9178$
$= 91.78\%$

$$U_2 = \frac{Q}{A_2} = \frac{bh_1U_1}{bh_2} = \frac{h_1U_1}{h_2} = \frac{1\times6.1}{2.30} = 2.652 \text{ m/s}$$

or $\frac{E_2}{E_1} = \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2+Fr_1^2)} = \frac{(1+8\times3.793)^{3/2} - 4\times3.793 + 1}{8\times3.793(2+3.793)} = 0.9178 = 91.78\%$

7.3(b) $h_1 = 0.50\text{ m}$ $\quad U_1 = 15.50\text{ m/s}$

$$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{15.50}{\sqrt{9.81\times0.5}} = 7.0$$

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) = \frac{1}{2}\left(\sqrt{1+8\times7^2}-1\right) = 9.41$$

$\therefore$ TW depth reqd., $h_t = h_2 = 9.41\times0.5 = 4.71\text{ m}$

Since $Fr_1 = 7.0$, the jump is a steady jump.

Height of jump, $h_j = h_2 - h_1 = 4.71 - 0.50 = 4.21\text{ m}$

$L_j = 6.1h_2 = 6.1\times4.71 = 28.7\text{ m}$

$$h_L = \frac{(h_2-h_1)^3}{4h_1h_2} = \frac{(4.71-0.50)^3}{4\times4.71\times0.50} = 7.92\text{ m-kg/kg}$$

$$E_1 = h_1 + \alpha\frac{U_1^2}{2g} = 0.5 + \frac{1\times15.50^2}{2\times9.81} = 12.75\text{ m-kg/kg}$$

$\therefore$ $h_L/E_1 = 7.92/12.75 = 0.621 = 62.1\%$

~~The depth and velocity at the foot of an overflow spillway are 0.50 m and 15.50 m/s, respectively. What TW depth is needed to form a hydraulic jump? If a jump is formed, determine the type of jump, the height of the jump, the length of jump and the energy loss in the jump as a percentage of the initial energy.~~

7.4(a) $h_1 = 0.25\text{ m} \quad U_1 = 12.75\text{ m/s}$

$$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{12.75}{\sqrt{9.81\times0.25}} = 8.1415$$

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) = 11.0247$$

$$\therefore h_2 = 11.0247\times0.25 = 2.756\text{ m}$$

$$q = h_1U_1 = 0.25\times12.75 = 3.1875\text{ m}^2/\text{s}$$

$$U_2 = \frac{h_1U_1}{h_2} = 1.1566\text{ m/s}$$

$$h_L = \frac{(h_2-h_1)^3}{4h_1h_2} = 5.710\text{ m-kg/kg}$$

$$Fr_2 = \frac{U_2}{\sqrt{gh_2}} = 0.2224$$

(b) $h_1 = 0.25\text{ m} \quad h_2 = 2.76\text{ m}$

$$\frac{h_2}{h_1} = 11.04 = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right)$$

$$\therefore Fr_1 = 8.15$$

$$Fr_1 = \frac{U_1}{\sqrt{gh_1}} \quad \therefore U_1 = Fr_1\sqrt{gh_1} = 12.767\text{ m/s}$$

Other results follow.

(c) $h_2 = 2.76\text{ m} \quad U_2 = 1.16\text{ m/s}$

$$Fr_2 = \frac{U_2}{\sqrt{gh_2}} = 0.2229$$

$$\frac{h_1}{h_2} = \frac{1}{2}\left(\sqrt{1+8Fr_2^2}-1\right) = 0.09109$$

$$\therefore h_1 = 0.09109\times2.76 = 0.2514\text{ m}$$

$$U_1 = \frac{h_2U_2}{h_1} = 12.734\text{ m/s}$$

(viii)

7.4(d) $u_1 = 12.75$ m/s $\quad u_2 = 1.16$ m/s

$h_1 u_1 = h_2 u_2 \quad \frac{h_2}{h_1} = \frac{u_1}{u_2} = \frac{12.75}{1.16} = 10.99$

$\therefore 10.99 = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) \Rightarrow Fr_1 = 8.1179$

Same as (b)

e) $Fr_1 = 8.19 \quad h_L = 5.71$ m-kg

$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) = 11.02 \Rightarrow h_2 = 11.02 h_1$

$h_L = 5.71 = \frac{(h_2-h_1)^3}{4h_1h_2} = \frac{(11.02h_1-h_1)^3}{4\times 11.02h_1\times h_1} = 22.83h_1$

$\therefore h_1 = 5.71/22.83 = 0.25$ m

$\therefore h_2 = 11.02\times .25 = 2.756$ m

$Fr_1 = 8.19 = \frac{u_1}{\sqrt{gh_1}} \quad \therefore u_1 = 8.19\times\sqrt{9.81\times .25}$ = 1
$= 12.7937$ m/s.

f) $Fr_2 = \cancel{8.19}\ 0.22 \quad h_L = 5.71$

$\frac{h_1}{h_2} = \frac{1}{2}\left(\sqrt{1+8Fr_2^2}-1\right) = 0.0888972$

$\therefore h_2 = 11.2489 h_1$

same as (e)

g) $q = 3.19$ m²/s. $h_L = 5.71$m

Trial i) Assume h_1
ii) Compute u_1, Fr_1, h_2 & h_L
iii) Check for h_L

h_1	u_1	Fr_1	h_2	h_L	Remarks
0.20	15.95	11.387	3.122	9.991	>
0.30	10.63	6.198	2.484	3.495	<
0.25	12.76	8.148	2.758	5.719	≃ OK

5 $h_1 = 1m$ trapezoidal $b = 5m$ $s = 1$ $Q = 30 m^3/s$

$A_1 = (b + s.h_1)h_1 = 6 m^2$

$B_1 = b + 2s h_1 = 7m$

$D_1 = A_1/B_1 = 0.857 m$

$U_1 = Q/A_1 = 30/6 = 5 m/s$

$$Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{5}{\sqrt{9.81 \times 0.857}} = 1.724$$

$$\bar{z}_1 = \frac{h_1}{6}\left(\frac{3b + 2sh_1}{b + s.h_1}\right) = \frac{1}{6}\left(\frac{3 \times 5 + 2.1.1}{5 + 1.1}\right) = \frac{17}{36} = 0.472m$$

$$\therefore F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{30^2}{9.81 \times 6} + 0.472 \times 6 = 18.12385$$

h_2	A_2	$\bar{z}_2$	F_2	Remarks
3.00	24	1.3125	35.32	>
2.00	14	0.905	19.22	>
1.90	13.11	0.863	18.31	≃
1.89	13.0221	0.859	18.22	
1.88	12.9344	0.854	18.14	Closest

$\therefore h_2 = 1.88 m$

$$E_1 = h_1 + \frac{U_1^2}{2g} = 1 + \frac{5^2}{2 \times 9.81} = 2.27421 m$$

$$U_2 = \frac{30}{12.9344} = 2.3193 m/s$$

$$E_2 = h_2 + \frac{U_2^2}{2g} = 1.88 + \frac{2.3193^2}{2 \times 9.81} = 2.1542 m$$

$\therefore h_L = E_1 - E_2 = 0.12 m$

7·6 Horizontal triangular, $s = 2$, $Q = 20\ m^3/s$, $h_1 = 1m$

$A_1 = sh_1^2 = 2m^2$ $\quad B_1 = 2sh_1 = 4m$ $\quad D_1 = 0.5\ m$

$U_1 = 10\ m/s$ $\quad Fr_1 = \frac{U_1}{\sqrt{gD_1}} = \frac{10}{\sqrt{9.81\times0.5}} = 4.515$

$\bar{z}_1 = h_1/3 = 0.333$

$F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{20^2}{9.81\times2^2} + 0.333\times2 = 21.05$

h_2	A_2	$\bar{z}_2$	F_2
2·00	8·00	0·667	10·433
3·00	18·00	1·000	20·265
3·10	19·22	1·033	21·982
3·05	18·605	1·017	21·107

$\therefore h_2 = 3.05\ m$

7·7 Horizontal parabolic $y^2 = 4z$ $\quad Q = 10\ m^3/s$ $\quad h_1 = 0.50\ m$

$h_1 = 0.50\ m$ $\quad y_1 = (4\times0.50)^{1/2} = 1.4142\ m$ $\quad B_1 = 2y_1 = 2.828\ m$

$A_1 = \frac{2}{3}B_1h_1 = 0.942\ m^2$ $\quad D_1 = 0.333\ m$ $\quad U_1 = 10.616\ m/s$

$Fr_1 = 5.873$ $\quad \bar{z}_1 = 2h_1/5 = 0.20\ m$

$F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{10^2}{9.81\times0.942} + 0.20\times0.942 = 11.010$

h_2	y_2	B_2	A_2	$\bar{z}_2$	F_2
2·00	2·828	5·657	7·542	0·800	7·385
3·00	3·464	6·928	13·856	1·200	17·363
2·50	3·162	6·325	10·541	1·000	11·508
2·45	3·130	6·261	10·226	0·980	11·019

$\therefore h_2 = 2.45\ m$

7.8(a) Rectangle, $b = 1.2\,m$, $\theta = 3°$, $Q = 0.14\,m^3/s$, $h_1 = 0.02\,m$

$A_1 = bh_1 = 0.024\,m^2$ $U_1 = 5.833\,m/s$ $Fr_1 = 13.16$

$k_1 = 10^{0.0270} = 1.205$ $G^2 = k_1^2 Fr_1^2 = 251.5$

$h_2^*/h_1 = d_2/d_1 = \frac{1}{2}(\sqrt{1+8G^2} - 1) = 21.93$

$\therefore h_2^* = 21.93 \times 0.02 = 0.438\,m$

(b). Rectangle, $b = 6\,m$, $\theta = 3.5°$, $Q = 0.75\,m^3/s$, $h_1 = 0.02\,m$

$h_t = 0.45\,m$

$A_1 = 0.12\,m^2$ $U_1 = 6.25\,m/s$ $Fr_1 = 14.11$

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr^2} - 1\right) = 19.46 \quad \therefore h_2 = 0.3892\,m$$

Since $h_t > h_2$, the depth h_2^* must be calculated.

$k_1 = 10^{0.0270} = 1.243$ $G^2 = k_1^2 Fr_1^2 = 307.65$

$$\therefore \frac{d_2}{d_1} = \frac{h_2^*}{h_1} = \frac{1}{2}\left(\sqrt{1+8G^2} - 1\right) = 24.3$$

$\therefore h_2^* = 0.486\,m$

Since $h_2^* > h_t$, i.e. $0.486 > 0.45$ (h_t is less), hence the jump is of B type.

7.9 Rectangle, $b = 6\,m$, $\theta = 5°$, $Q = 20\,m^3/s$, $h_t = 3.20\,m$

i) $d_1 = 0.20\,m$

$A_1 = bd_1 = 1.2\,m^2$ $U_1 = Q/A_1 = 16.67\,m/s$

$Fr_1 = U_1/\sqrt{gD_1} = U_1/\sqrt{gd_1} = 11.90$

The sequent depth for a jump corresponding to d_1 in a horizontal channel is given by

$$h_2 = \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = \frac{0.2}{2}\left(\sqrt{1+8\times 11.9^2} - 1\right) = 3.27\,m$$

Since $h_t < h_2$, jump type A occurs, i.e. the jump is on the horizontal floor.

7.9 (ii) $d_1 = 0.30\,m$

$A_1 = 6 \times .3 = 1.8\,m^2$ $\quad U_1 = \frac{20}{1.8} = 11.11\,m/s$

$$Fr_1 = \frac{U_1}{\sqrt{g d_1}} = \frac{11.11}{\sqrt{9.81 \times .3}} = 6.476$$

$$\therefore h_2 = \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = 2.60\,m$$

Since $h_t (= 3.20\,m) > h_2$, ~~jump occurs~~ we need to compute h_2^*. $\quad U_2 = \frac{20}{3.61 \times 6} = 0.92\,m/s$

$$k_1 = 10^{.027\theta} = 10^{.027 \times 5} = 1.36$$

$$G_1^2 = k_1^2 Fr_1^2 = (1.36 \times 6.476)^2 = 78.09$$

$$\therefore d_2 = \frac{d_1}{2}\left(\sqrt{1+8G_1^2} - 1\right) = 3.60\,m$$

$$\therefore h_2^* = \frac{d_2}{\cos 5^\circ} = 3.61\,m$$

Since $h_2^* > h_t > h_2$, B type jump occurs.

7.9 (iii) $d_1 = 0.40\,m$ $\quad A_1 = 6 \times .4 = 2.4\,m^2$ $\quad U_1 = 20/2.4 = 8.33\,m/s$

$$Fr_1 = \frac{U_1}{\sqrt{g d_1}} = \frac{8.33}{\sqrt{9.81 \times 0.4}} = 4.205$$

$$h_2 = \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = 2.187\,m$$

Since $h_t > h_2$, we need to compute h_2^*.

$$k_1 = 1.36$$

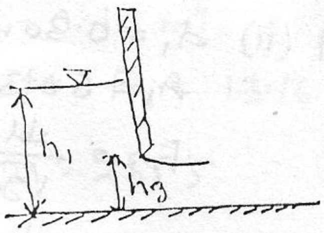
$$G^2 = k_1^2 Fr_1^2 = (1.36 \times 4.205)^2 = 32.707$$

$$\therefore d_2 = \frac{d_1}{2}\left(\sqrt{1+8G^2} - 1\right) = 3.04\,m$$ $\quad U_2 = \frac{Q}{b h_2} = \frac{20}{6 \times 3.05} = 1.09\,m/s$

$$\therefore h_2^* = \frac{d_2}{\cos\theta} = \frac{3.04}{\cos 5^\circ} = 3.05\,m$$

Since $h_t > h_2^* > h_2$; D type jump occurs, i.e. the jump occurs entirely the sloping channel.

8.1 $b = 6\,m,\ h_g = 1\,m,\ h_1 = 4\,m$



i) <u>Free flow</u>

<u>Using Eq. (8.3)</u> Take $C_c = 0.61$

$$C_d = \frac{C_c}{\sqrt{1 + C_c h_g / h_1}} = \frac{0.61}{\sqrt{1 + 0.61 \times 1/4}} = 0.568$$

$\boxed{h_c = 1.37\,m}$

$$\therefore Q = C_d\, b\, h_g \sqrt{2 g h_1} = 0.568 \times 6 \times 1 \sqrt{2 \times 9.81 \times 4} = 30.20\ m^3/s$$

<u>Using Eq. (8.6)</u> $\Delta h = h_1 - C_c h_g = 4 - 0.61 \times 1 = 3.39\,m$

$h_g / h_1 = 1/4 = 0.25$ $C_d = 0.605$ (Fig. 8.2)

$$Q = C_d\, b\, h_g \sqrt{2 g \Delta h} = 0.605 \times 6 \times 1 \times \sqrt{2 \times 9.81 \times 3.39} = 29.60\ m^3/s$$

ii) <u>Submerged flow</u> $h_t = 3.25\,m$

$$\frac{h_g C_d}{h_t} = \frac{1 \times 0.605}{3.25} = 0.186$$

$$\frac{h_3}{h_g C_d} = 2(1 - 0.186) + \sqrt{4(1-0.186)^2 + (1/0.186)^2 - 4\left(\frac{4}{1 \times 0.605} - \frac{4}{3.25}\right)}$$

$$= 4.795$$

$$\therefore h_3 = 4.795 \times 1 \times 0.605 = 2.90\,m$$

$$\therefore Q = 0.605 \times 6 \times 1 \times \sqrt{2 \times 9.81 \times (4 - 2.90)} = 16.85\ m^3/s$$

iii) <u>Depth of submergence by Eq. (8.9)</u>

$$h_t = 3.25\,m,\quad U_t = \frac{Q}{b h_t} = \frac{16.85}{6 \times 3.25} = 0.86\ m/s$$

$$Fr_t = \frac{U_t}{\sqrt{g h_t}} = \frac{0.86}{\sqrt{9.81 \times 3.25}} = 0.153$$

$$\therefore \frac{h_3}{3.25} = \left[1 + 2 \times 0.153^2 \left(1 - \frac{3.25}{1}\right)\right]^{1/2} = 0.945$$

$$\therefore h_3 = 0.945 \times 3.25 = 3.07\,m$$

(1)

8.2(a) $q = 1.30\ m^2/s$, $h_1 = 4m$, Free flow, Unit width $(b = 1m)$

$y_c = 0.56m$

Assume $h_g = 0.50\ m$, $h_g/h_1 = 0.50/4 = 0.125$

From Fig. 8.2, we get $C_d = 0.60$

$\therefore q = 0.60 \times 0.50 \times \sqrt{2 \times 9.81 \times (4 - 0.60 \times 0.50)} = 2.55\ m^2/s > 1.30\ m^2/s$

$\therefore$ Reduce the gate opening h_g.

Now, assume $h_g = 0.20m$, $h_g/h_1 = 0.05$, $C_d = 0.60$

$\therefore q = 0.60 \times 0.20 \times \sqrt{2 \times 9.81 \times (4 - 0.60 \times 0.20)} = 1.05\ m^2/s < 1.30\ m^2/s$

$\therefore$ Increase the gate opening h_g.

Next assume $h_g = 0.25m$, $h_g/h_1 = 0.0625$, $C_d = 0.60$

$\therefore q = 0.60 \times 0.25 \times \sqrt{2 \times 9.81 \times (4 - 0.60 \times 0.25)} = 1.30\ m^2/s$

which is exactly the given discharge,

$\therefore h_g = 0.25m \Leftarrow$

(b) Submerged flow, $h_t = 3.20\ m$, $q = 1.30\ m^2/s$, $h_g = 0.25m$

Assume $h_1 = 5m$, $\dfrac{h_g C_d}{h_t} = \dfrac{0.25 \times 0.60}{3.20} = 0.047$

$$\therefore \frac{h_3}{h_g C_d} = 2(1 - 0.047) + \sqrt{4(1-0.047)^2 + \left(\frac{1}{0.047}\right)^2 - 4\left(\frac{5}{0.25 \times 0.60} - \frac{5}{3.20}\right)}$$

$$= 1.906 + \sqrt{3.634 + 455.111 - 127.083} = 20.118$$

$\therefore h_3 = 20.111 \times 0.25 \times 0.60 = 3.018\ m$

$\therefore q = 0.60 \times 0.25 \times \sqrt{2 \times 9.81 \times (5 - 3.018)} = 0.935\ m^2/s < 1.30\ m^2/s$

Hence, the upstream depth h_1 has to be increased.

Now, assume $h_1 = 6m$

$$\therefore \frac{h_3}{h_g C_d} = 1.906 + \sqrt{3.634 + 455.111 - 152.50} = 19.406$$

$\therefore h_3 = 19.406 \times 0.25 \times 0.60 = 2.911 m$

$\therefore q = 0.60 \times 0.25 \times \sqrt{2 \times 9.81 \times (6 - 2.911)} = 1.17\ m^2/s < 1.30\ m^2/s$

2

8.2(b)(Contd.)

Next assume $h_1 = 6.5\,m$

$$\frac{h_3}{h_g C_d} = 1.906 + \sqrt{3.634 + 455.11 - 165.208} = 19.036$$

$$h_3 = 19.036 \times 0.25 \times 0.60 = 2.855\,m$$

$$q = 0.60 \times 0.25 \times \sqrt{2 \times 9.81 \times (6.5 - 2.855)} = 1.27\,m^2/s < 1.30\,m^2/s$$

Next assume $h_1 = 6.7\,m$

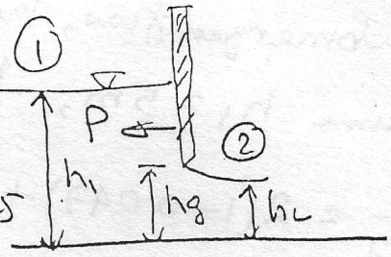
$$\frac{h_3}{h_g C_d} = 1.906 + \sqrt{3.634 + 455.11 - 130.29} = 18.890$$

$$h_3 = 18.890 \times 0.25 \times 0.60 = 2.833\,m$$

$$\therefore q = 0.60 \times 0.25 \times \sqrt{2 \times 9.81 \times (6.7 - 2.833)} = 1.306\,m^2/s \simeq 1.30\,m^2/s$$

$\therefore h_1 = 6.70\,m$ and $h_3 = 2.83\,m$

8.2(c) Free flow condition



$q = 1.30\,m^2/s$, $h_1 = 4\,m$, $C_c = 0.61$

$h_g = 0.25\,m$, $h_2 = 0.25 \times 0.61 = 0.1525\,m$

$$u_1 = \frac{q}{h_1} = \frac{1.30}{4} = 0.325\,m/s$$

$$u_2 = \frac{q}{h_2} = \frac{1.30}{0.1525} = 8.5246\,m/s$$

Now applying the momentum equation between sections 1 and 2 and taking unit width of the channel (i.e. $b = 1\,m$), we get

$$\rho q (u_2 - u_1) = \frac{1}{2}\rho g h_1^2 - \frac{1}{2}\rho g h_2^2 - P$$

8·2 (c) (contd.)

$\therefore P = \frac{1}{2}\rho g(h_1^2 - h_2^2) - \rho q(U_2 - U_1)$

$= \frac{1}{2}\times 1000\times 9.81(4^2 - 0.1525^2) - 1000\times 1.30(8.5246 - 0.325)$

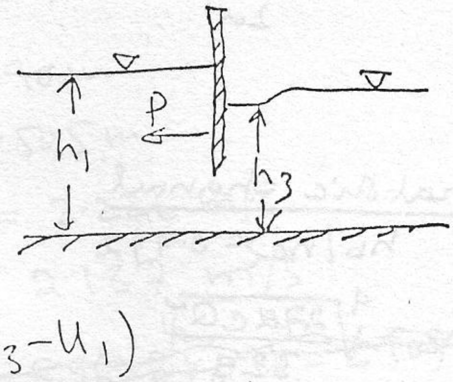
$= 78,365.93 - 10,659.47$

$= 67,706.46$ N

The force on the gate is equal and opposite to P.

[When the formula is used $P = \frac{1}{2}\rho g\frac{(h_1-h_2)^3}{h_1+h_2} = \frac{1}{2}\times 1000\times 9.81\frac{(4-.1525)^3}{4+.1525} = 67,276.79$ N]

Submerged condition



$q = 1.3\,m^2/s$, $h_1 = 6.70\,m$

$h_3 = 2.83\,m$, $U_1 = \frac{1.30}{6.70} = 0.194$ m/s

$U_2 = \frac{1.30}{2.83} = 0.459$ m/s

$P = \frac{1}{2}\rho g(h_1^2 - h_3^2) - \rho q(U_3 - U_1)$

$= \frac{1}{2}\times 1000\times 9.81(6.70^2 - 2.83^2) - 1000\times 1.30(0.459 - 0.194)$

$= 180,901.80 - 344.50 = 180,557.30$ N

[For free flow and hydrostatic pressure distribution

$P = \frac{1}{2}\gamma(h_1 - h_g)^2 = \frac{1}{2}\times 9810\times(4^2 - 0.25^2)$

$= 68,976.56$ N

Hydrostatic > Vena contracta > gate

For free flow when the gate is taken as section 2

$P = \frac{1}{2}\times 9810\times\frac{(4-0.25)^3}{4+0.25} = 60,816.67$ N]

8.3. Rectangular channel

$h_b/h_c = 0.715 \quad \therefore h_c = h_b/0.715$

$h_c = \sqrt[3]{\dfrac{\alpha Q^2}{g b^2}} \quad \therefore h_c^3 = \dfrac{\alpha Q^2}{g b^2}$

$\therefore Q^2 = \dfrac{g}{\alpha} b^2 h_c^3 \quad \therefore Q = \sqrt{\dfrac{g}{\alpha}}\, b\, h_c^{1.5} = \sqrt{\dfrac{g}{\alpha}}\, b \left(\dfrac{h_b}{0.715}\right)^{1.5}$

$= 5.1805\, b\, h_b^{1.5} \quad (8.11)$

Triangular channel

$h_b/h_c = 0.795 \quad \therefore h_c = \dfrac{h_b}{0.795}$

$h_c = \sqrt[5]{\dfrac{2\alpha Q^2}{g s^2}} \quad \therefore h_c^5 = \dfrac{2\alpha Q^2}{g s^2}$

$\therefore Q^2 = \dfrac{g s^2 h_c^5}{2\alpha} \quad \therefore Q = \sqrt{\dfrac{g}{2\alpha}}\, s\, h_c^{2.5} = \sqrt{\dfrac{g}{2\alpha}}\, s \left(\dfrac{h_b}{0.795}\right)^{2.5}$

$= 3.930\, s\, h_b^{2.5} \quad (8.12)$

Parabolic channel

$h_b/h_c = 0.772 \quad \therefore h_c = h_b/0.772$

$h_c = \sqrt[4]{\dfrac{27\alpha c Q^2}{32 g}} \quad \therefore h_c^4 = \dfrac{27\alpha c Q^2}{32 g}$

$\therefore Q^2 = \dfrac{32 g}{27 \alpha c} h_c^4 \quad \therefore Q = \sqrt{\dfrac{32 g}{27 \alpha}}\, \dfrac{1}{\sqrt{c}}\, h_c^2 = \sqrt{\dfrac{32 g}{27 \alpha}}\, c^{-\frac{1}{2}} \left(\dfrac{h_b}{.772}\right)^2$

$= 5.721\, c^{-\frac{1}{2}}\, h_b^2 \quad (8.13)$

5

8.4 $h_b = 0.50\,m$

i) Rectangle, $b = 6m$

$$Q = 5.181 b h_b^{1.5} = 5.181 \times 6 \times 0.50^{1.5} = 10.99\ m^3/s$$

ii) Triangle $s = 2$

$$Q = 3.93\, s\, h_b^{2.5} = 3.93 \times 2 \times 0.50^{2.5} = 1.39\ m^3/s$$

iii) Parabola $x^2 = 4z \Rightarrow z = \frac{1}{4}x^2 \Rightarrow c = \frac{1}{4} = 0.25$

$$Q = 5.72\, c^{-1/2} h_b^2 = 5.72 \times 0.25^{-1/2} \times 0.50^2 = 2.86\ m^3/s$$

iv) Circle $d_0 = 2m$

$h_b/h_c = 0.725 \quad \therefore h_c = \frac{0.50}{0.725} = 0.69\ m$

$\theta_c = 2\cos^{-1}\left(1 - \frac{2 \times 0.69}{2}\right) = 2.51$ rad

$A_c = \frac{1}{8}(\theta - \sin\theta) d_0^2 = \frac{1}{8}(2.51 - \sin 2.51) \times 2^2 = 0.96\,m^2$

$B_c = d_0 \sin\theta/2 = 2 \sin\frac{2.51}{2} = 1.90\ m$

$D_c = A_c/B_c = 0.96/1.90 = 0.505\ m$

$u_c = \sqrt{g D_c} = \sqrt{9.81 \times 0.505} = 2.226\ m/s$

$\therefore Q = A_c u_c = 0.96 \times 2.226 = 2.137\ m^3/s$

~~[Approximate $Q = 5.96 d_0^{1/2} h_b^2 = 5.96 \times 2^{1/2} \times 0.50^2 = 2.107\ m^3/s$]~~

v) Trapezoid $b = 6m,\ s = 2$

In a horizontal trapezoidal channel, the ratio h_b/h_c varies from 0.715 to 0.795. We first take $h_b/h_c = 0.75$. Then, $h_c = 0.5/0.75 = 0.667\ m$

$\therefore \frac{s h_c}{b} = \frac{2 \times 0.667}{6} = 0.222$

$\therefore$ from Fig. 8.5, $h_b/h_c = 0.73$. $\therefore h_c = \frac{0.5}{0.73} = 0.685\ m$

$\therefore s h_c/b = 2 \times 0.685/6 = 0.228$

(v)
8·4 (Contd.)

Again, from Fig. 8.5, $h_b/h_c = 0.73$, $h_c = 0.50/0.73 = 0.685$ m

Therefore, the value of h_c may be taken as 0.685 m

$\therefore A_c = (b + s h_c) h_c = (6 + 2 \times 0.685) \times 0.685 = 5.048$ m

$B_c = b + 2 s h_c = 6 + 2 \times 2 \times 0.685 = 8.74$ m

$D_c = A_c / B_c = 5.048/8.74 = 0.578$ m

$\therefore U_c = \sqrt{g D_c} = \sqrt{9.81 \times 0.578} = 2.380$ m/s

$\therefore Q = A_c U_c = 5.048 \times 2.380 = 12.20\ m^3/s$

8·5 $h_b = 0.50$ m, $S_o = 0.0060$, $n = 0.020$

i) Rectangular channel, $b = 6$m

~~When the slope is 0.006m~~ Let us take $Q = 10.99\ m^3/s$ from Prob. 8.5 (i) to determine the critical slope. Then

$$h_c = \sqrt[3]{\frac{Q^2}{g b^2}} = \sqrt[3]{\frac{10.99^2}{9.81 \times 6^2}} = 0.70 \text{ m}$$

and applying the Manning formula, the critical slope is

$$10.99 = \frac{1}{0.020} \times (6 \times 0.70) \times \left(\frac{6 \times 70}{6 + 2 \times 0.70}\right)^{2/3} S_c^{1/2}$$

or, $S_c = 0.0058$

$\therefore S_o/S_c = 0.0060/0.0058 = 1.03$

Hence, from Fig. 8.5, $h_b/h_c = 0.68$

$\therefore h_c = 0.50/0.68 = 0.74$ m

$\therefore Q = \sqrt{g}\, b\, h_c^{1.5} = \sqrt{9.81} \times 6 \times 0.74^{1.5} = 11.85\ m^3/s$

(7)

$$\therefore\ 11.85 = \frac{1}{0.020} \times (6\times 0.79) \times \left(\frac{6\times 0.79}{6+2\times 0.79}\right)^{2/3} S_c^{1/2}$$

$$\therefore\ S_c = 0.0057$$

which is very close to $S_c = 0.0058$ previously obtained. Hence

$$Q = 11.85\ m^3/s$$

8.5(ii) <u>Triangular channel</u>, $s = 2$

~~When the slope is 0.0060~~ Let us take $Q = 1.39\,m^3/s$ from Problem 8.5(ii) to determine the critical slope.

Then

$$h_c = \sqrt[5]{\frac{2Q^2}{gs^2}} = \sqrt[5]{\frac{2\times 1.39^2}{9.81\times 2^2}} = 0.63\,m$$

$\therefore$ Using the Manning formula

$$1.39 = \frac{1}{0.020} \times (2\times 0.63^2) \times \left(\frac{2\times 0.63^2}{2\sqrt{5}\times 0.63}\right)^{2/3} \times S_c^{1/2}$$

$$\therefore\ S_c = 0.0066$$

$$\therefore\ S_0/S_c = 0.0060/0.0066 = 0.91$$

Then from Fig. 8.5, we have $h_b/h_c = 0.77$

$$\therefore\ h_c = h_b/0.77 = 0.50/0.77 = 0.65\,m$$

$$\therefore\ Q = \sqrt{\frac{g}{2}}\, s\, h_c^{2.5} = \sqrt{\frac{9.81}{2}} \times 2\times 0.65^{2.5} = 1.51\ m^3/s$$

$$\therefore\ 1.51 = \frac{1}{0.020} \times (2\times 0.65^2) \times \left(\frac{2\times 0.65^2}{2\sqrt{5}\times 0.65}\right)^{2/3} S_c^{1/2}$$

$$\therefore\ S_c = 0.0058$$

(8)

which is exactly the same value previously obtained. Hence we take

$$Q = 1.51 \ m^3/s$$

8.5(iii) Trapezoidal channel, $b = 6 m$, $s = 2$

Let us take $Q = 12.20 \ m^3/s$ from Problem 8.5(v) to determine the critical slope so that $h_c = 0.685 m$ and $A_c = 5.048 m$. Then

$$12.20 = \frac{1}{0.020} \times 5.048 \times \left(\frac{5.048}{6 + 2\sqrt{5} \times 0.685} \right)^{2/3} \times S_c^{1/2}$$

$$\therefore S_c = 0.0057$$

$$\therefore S_o/S_c = 0.0060/0.0057 = 1.18$$

From Fig. 8.5 with $sh_c/b = 2 \times 0.685/6 = 0.23$ and $S_o/S_c = 1.18$, we get $h_o/h_c = 0.68$

$$\therefore h_c = h_o/0.68 = 0.50/0.68 = 0.74$$

$$\therefore A_c = (6 + 2 \times 0.74) \times 0.74 = 5.53 \ m$$

$$B_c = b + 2sh_c = 6 + 2 \times 2 \times 0.74 = 8.96 \ m$$

$$D_c = A_c/B_c = 5.53/8.96 = 0.62 \ m$$

$$U_c = \sqrt{gD_c} = \sqrt{9.81 \times .62} = 2.46 \ m/s$$

$$\therefore Q = A_c U_c = 5.53 \times 2.46 = 13.62 \ m^3/s$$

$$\therefore 13.62 = \frac{1}{0.020} \times 5.53 \times \left(\frac{5.53}{6 + 2\sqrt{5} \times 0.74} \right)^{2/3} \times S_c^{1/2}$$

$$\therefore S_c = 0.0048$$

$$\therefore S_o/S_c = 0.0060/0.0048 = 1.25$$

8.5 (iii) (contd.)

$\therefore$ From Fig. 8.5, $h_b/h_c = 0.67$

$\therefore h_c = h_b/0.67 = 0.50/0.67 = 0.75\,m$

$\therefore A_c = (6+2\times0.75)\times0.75 = 5.625\,m^2$

$B_c = 6+2\times2\times0.75 = 9.0\,m$

$D_c = A_c/B_c = 5.625/9 = 0.625\,m$

$U_c = \sqrt{9.81\times0.625} = 2.48\,m/s$

$\therefore Q = A_cU_c = 5.625\times2.48 = 13.93\,m^3/s$

8.6 $p = 1m$, $L = 2m$, $h_1 = 0.80\,m$, free flow, suppressed

$h_1/L = 0.80/2 = 0.40$ $\quad h_1/(h_1+p) = 0.80/(0.80+1.0) = 0.44$

From Fig. 8.7, $f = 1.02$ $\therefore C_d = 0.848\times1.02 = 0.865$

$\therefore q = \left(\frac{2}{3}\right)^{1.5} C_d\sqrt{g}\,h_1^{1.5} = 1.705\times0.865\times h_1^{1.5}$

$= 1.4747\,h_1^{1.5}$

h_1	U_1	$U_1^2/2g$	H_1	q
0.80	—	—	0.80	1.055
0.80	0.5862	0.0175	0.8175	1.090
0.80	0.6056	0.0187	0.8187	1.092
0.80	0.6069	0.0188	0.8188	1.093

Hence, the discharge/unit width, $q = 1.093\,m^2/s$.

(i) when $h_2 = 0.60\,m$, $h_2/h_1 = 0.60/0.80 = 0.75 < 0.80$

Hence the discharge does not change.

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8.6 (contd)

(ii) When $h_2 = 0.75 m$, $h_2/h_1 = 0.75/0.80 = 0.9375 > 0.80$

Hence the flow becomes submerged. From Table 8.3, $C_s = 0.67$

$\therefore q = 1.093 \times 0.67 = 0.732\ m^2/s$

8.7 Suppressed weir, Rectangular channel, $b = 6 m$, $L = 2.5 m$, $p = 1.50 m$

(i) $Q = 6\ m^3/s$ $\therefore q = Q/b = 6/6 = 1\ m^2/s$

First assume $C_d = 0.848$ for a pure broad-crested weir. Then

$$1 = 1.705 \times 0.848 \times h_1^{1.5} \Rightarrow h_1 = 0.78 m$$

h_1	h_1/L	$h_1/(h_1+p)$	C_d	$U_1^2/2g$	H_1	q
0.78	0.31	0.34	0.848	—	0.780	0.996
				0.010	0.790	1.015
0.77	0.31	0.34	0.848	—	0.770	0.977
				0.009	0.779	0.994

Hence the depth upstream, $h_1 = 0.77 m$

(ii) $Q = 10\ m^3/s$ $q = Q/b = 10/6 = 1.667\ m^2/s$

First assume $C_d = 0.848$. Then

$$1.667 = 1.705 \times 0.848 \times h_1^{1.5}$$

which gives $h_1 = 1.10 m$.

8.7 (Contd.) $L = 2.5\,m,\ p = 1.50\,m$

h_1	h_1/L	$h_1/(h_1+p)$	C_d	$u_1^2/2g$	H_1	q	Q
1.08	0.432	0.419	0.868	—	1.08	1.661	9.966
				0.021	1.10	1.710	10.260
1.07	0.428	0.416	0.864	—	1.07	1.632	9.794
				0.021	1.09	1.677	10.066

Hence, the depth of flow upstream, $h_1 = 1.07\,m$.

8.8 Rectangular channel, $b = 10\,m$, $Q = 15\,m^3/s$

Assume a pure broad-crested weir with vertical faces and sharp upstream corner so that $C_d = 0.848$. Then

$$15 = 1.705 \times 0.848 \times 10 \times h_1^{1.5}$$

which gives $h_1 = 1.0248\,m$.

Now, $0.10 \le \frac{h_1}{L} \le 0.35 \quad \therefore \frac{1}{0.10} \geq \frac{L}{h_1} \geq \frac{1}{0.35}$

or, $10 \geq \frac{L}{h_1} \geq 2.857$ or, $10h_1 \geq L \geq 2.857h_1$

or, $10 \times 1.0248 \geq L \geq 2.857 \times 1.0248$

or, $10.248 \geq L \geq 2.928$

Let us use $L = 3.5\,m$

Again, $\frac{h_1}{h_1+p} \le 0.35$ or, $h_1 \le 0.35(h_1+p)$

or, $0.35(h_1+p) \geq h_1$ or, $0.35p \geq 0.65h_1$

8.8 (contd.)

$\therefore p \geq \frac{0.65}{0.35} h_1$ or, $p \geq \frac{0.65}{0.35} \times 1.0248$ or, $p \geq 1.90$ m

Use $p = 2.50$ m

$\therefore L = 3.5$ m, $p = 2.50$ m, $h_1 = 1.025$ m

$\therefore \frac{h_1}{L} = 0.292$ $\quad \frac{h_1}{h_1+p} = 0.290$ $\quad \therefore C_d = 0.848$

h_1	$u_1^2/2g$	H_1	Q
1.00	—	1.00	14.46
1.00	0.0087	1.0087	14.65
1.02	—	1.02	14.894
1.02	0.0091	1.0291	15.094

Use $L = 3.50$ m and $p = 2.50$ m which will give $Q = 15$ m^3/s under a head of 1.02m and C_d will be 0.848.

8.9 Rectangular channel, $b = B = 2$m, $p = 0.50$m, $h_1 = 0.50$m

$C_d = 0.611 + 0.08 h_1/p = 0.611 + 0.08 \times 0.50/0.50 = 0.691$

$Q = \frac{2}{3} C_d \sqrt{2g}\, b h_1^{1.5} = \frac{2}{3} \times 0.691 \times \sqrt{2 \times 9.81} \times 2 \times 0.50^{1.5}$

$= 1.443$ m^3/s

8.10 $b = 2.5$m, $B = 2$m, $p = 0.50$m, $h_1 = 0.50$m

$B/b = 2/2.5 = 0.80$

8.10 (Contd..)

From Table 8.4, $K_B = 0.0043$, $K_1 = 0.597$, $K_2 = 0.0450$

$\therefore C_{de} = K_1 + K_1 \frac{h_1}{p} = 0.597 + 0.0450 \times \frac{0.50}{0.50} = 0.642$

$B_e = B + K_b = 2.0 + 0.0043 = 2.0043$

$h_{1e} = h_1 + 0.001 = 0.50 + 0.001 = 0.501$

$\therefore Q = \frac{2}{3} C_{de} \sqrt{2g}\, B_e\, h_{1e}^{1.5}$

$= \frac{2}{3} \times 0.642 \times \sqrt{2 \times 9.81} \times 2.0043 \times 0.501^{1.5}$

$= 1.347\ m^3/s$

8.11 Rectangular channel, $b = 2m$, $h_1 + p = 0.75m$, $Q = 0.50\ m^3/s$

A trial-and-error procedure is needed to compute the height of the weir p. First assume that $C_d = 0.65$.

$\therefore 0.50 = \frac{2}{3} \times 0.65 \times \sqrt{2 \times 9.81} \times 2 \times h_1^{1.5}$

which gives $h_1 = 0.257\ m$

$\therefore p = 0.75 - 0.257 = 0.493\ m$

$\therefore h_1/p = 0.257/0.493 = 0.521$

$\therefore C_d = 0.611 + 0.08 \times 0.521 = 0.652 \simeq 0.65$

Hence, use $p = 0.49\ m$.

.12 $b = 2m$, $B = 1.6m$, $h_1 + p = 0.75m$ $Q = 0.50\ m^3/s$

$B/b = 1.6/2 = 0.8$

From Table 8.4, $K_B = 0.0043$, $K_1 = 0.597$, $K_2 = 0.0450$

8.12 (Contd.)

$B_e = B + K_B = 1.60 + 0.043 = 1.643$

Assume $C_{de} = 0.64$

$\therefore 0.50 = \frac{2}{3} \times 0.64 \times \sqrt{2 \times 9.81} \times 1.643 \times h_{1e}^{1.5}$

which gives $h_{1e} = 0.296 m$ so that $h_1 = h_{1e} - 0.001$
$= 0.296 - 0.001 = 0.295 m$ and $p = 0.75 - 0.295 = 0.454 m$

$\therefore h_1/p = 0.295/0.454 = 0.650$

$\therefore C_{de} = K_1 + K_1 \frac{h_1}{p} = 0.597 + 0.0450 \times 0.650 = 0.626$

Using this value of C_d, we have

$0.50 = \frac{2}{3} \times 0.626 \times \sqrt{2 \times 9.81} \times 1.643 \times h_{1e}^{1.5}$

which gives $h_{1e} = 0.300 m \quad \therefore h_1 = 0.299 m$

$\therefore p = 0.75 - 0.299 = 0.451 m$

$\therefore h_1/p = 0.299/0.451 = 0.663$

$\therefore C_{de} = 0.597 + 0.0450 \times 0.663 = 0.627 \simeq 0.626$

Hence, we accept $p = 0.45 m$.

8.13 $b = 2m, h_1 = 1.6m, h_2 = 1.3m, p = 1m, n = 1.5$

$C_d = 0.611 + 0.08 \frac{h_1}{p} = 0.611 + 0.08 \times \frac{1.6}{1} = 0.739$

$Q = \frac{2}{3} C_d \sqrt{2g} \cdot b h_1^{1.5} = \frac{2}{3} \times 0.739 \times \sqrt{2 \times 9.81} \times 2 \times 1.6^{1.5} = 8.8$

$\therefore Q_s = Q \left[1 - (h_2/h_1)^n\right]^{0.385} = 8.833 \left[1 - \left(\frac{1.3}{1.6}\right)^{1.5}\right]^{0.385}$

$= 5.318 m^3/s$

8.14 From Eq. (8.32), when $p=0$, $C_d = 1.06\left(1+\frac{p}{h_1}\right)^{1.5} = 1.06$. When $p=0$, the weir becomes a free overfall and $C_c = h_b/h_c$ (Fig. 8.3). At the section where the flow is critical, $U_1 = U_c$, $h_1 = h_c$ and $F_r^2 = \frac{U_c^2}{gh_c} = \frac{U_1^2}{gh_1} = 1$. Hence, from Eq. (8.29), $C_d = C_c\left[\left(1+\frac{1}{2}\right)^{1.5} - \left(\frac{1}{2}\right)^{1.5}\right] = 1.48\,C_c$.

$\therefore C_c = C_d/1.48 = 1.06/1.48 = 0.715$

$\therefore h_b/h_c = C_c = 0.715$ (Proved)

8.15 $B = 0.65$ m

(i) $h_c = 0.45$ m

The discharge through the Venturi flume under free flow condition is given by

$$Q = B\sqrt{g}\,h_c^{1.5} = 0.65\times\sqrt{9.81}\times 0.45^{1.5} = 0.614\ m^3/s$$

(ii) $b = 0.75$ m, $h_1 = 0.60$ m, $h_2 = 0.50$ m, $A_1 = bh_1 = 0.75\times 0.60 = 0.45\ m^2$, $A_2 = Bh_2 = 0.65\times 0.50 = 0.325\ m^2$, $r = A_2/A_1 = 0.325/0.45 = 0.722$. Hence, the discharge is

$$Q = A_2\sqrt{2g(h_1-h_2)}/\sqrt{1-r^2}$$
$$= 0.325\times\sqrt{2\times 9.81\times(0.60-0.50)}/\sqrt{1-0.722^2}$$
$$= 0.218\ m^3/s$$

8.16 $B = 5$ ft. $h_1 = 1.32$ m $= 1.32\times 3.28 = 4.3296$ ft

$$\therefore Q = 4Bh_1^{1.522B^{0.026}} = 4\times 5\times 4.3296^{1.522\times 5^{0.026}}$$
$$= 204.69\ ft^3/sec = 5.80\ m^3/s$$