0904124

SECTION A

Hydraulic jump

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Energy loss

$$h_L = E_1 - E_2$$

$$= \left(h_1 + \frac{u_1^2}{2g}\right) - \left(h_2 + \frac{u_2^2}{2g}\right)$$

$$= (h_1 - h_2) + \left(\frac{u_1^2}{2g} - \frac{u_2^2}{2g}\right)$$

$$(h_1 - h_2) + \left(\frac{Q^2}{2g b^2 h_1^2} - \frac{Q^2}{2g b^2 h_2^2}\right)$$

$$= (h_1 - h_2) + \frac{Q^2}{2g b^2}\left(\frac{1}{h_1^2} - \frac{1}{h_2^2}\right)$$

$$= (h_1 - h_2) + \frac{Q^2}{2g b^2}\left[\frac{(h_1 + h_2)(h_2 - h_1)}{h_1^2 h_2^2}\right] \quad \text{--- (1)}$$

for hydraulic jump,

$$F_1 = F_2$$

momentum of flow per unit time passing through the channel section per unit wt of water

$$\Rightarrow \frac{Q^2}{gA_1} + \bar{h}_1 A_1 = \frac{Q^2}{gA_2} + \bar{h}_2 A_2$$

$\bar{h} A$ = press force per unit wt of water

$\bar{h} = \frac{h}{2}$

$$\Rightarrow \frac{Q^2}{g(bh_1)} + \frac{h_1}{2}(bh_1) = \frac{Q^2}{g(bh_2)} + \frac{h_2}{2}(bh_2)$$

$$\Rightarrow \frac{Q^2}{gb}\left[\frac{1}{h_1} - \frac{1}{h_2}\right] = \frac{b}{2}(h_2^2 - h_1^2)$$

$$\Rightarrow \frac{Q^2}{gb}\left(\frac{h_2 - h_1}{h_1 h_2}\right) = \frac{b}{2}(h_2 + h_1)(h_2 - h_1)$$

$$\Rightarrow \frac{Q^2}{gb^2} = \frac{1}{2}\frac{(h_2 + h_1)\cancel{(h_2 - h_1)}}{\cancel{(h_2 - h_1)}}(h_1 h_2)$$

$$\Rightarrow \frac{Q^2}{gb^2} = \frac{1}{2} h_1 h_2 (h_1 + h_2)$$

$$\Rightarrow \frac{Q^2}{2gb^2} = \frac{1}{4} h_1 h_2 (h_1 + h_2) \quad \text{--- (2)}$$

from ① & ②

$$h_L = (h_1 - h_2) + \frac{1}{4} h_1 h_2 (h_1 + h_2) \frac{(h_1 + h_2)(h_2 - h_1)}{h_1^2 h_2^2}$$

$$= (h_1 - h_2) + \frac{(h_1 + h_2)^2 (h_2 - h_1)}{4 h_1 h_2}$$

$$= \frac{(h_2 - h_1)}{4 h_1 h_2} \left[(h_1 + h_2)^2 - 4 h_1 h_2 \right]$$

$$= \frac{h_2 - h_1}{4 h_1 h_2} (h_2 - h_1)^2$$

$$\boxed{h_L = \frac{(h_2 - h_1)^3}{4 h_1 h_2}}$$

$$F_1 = F_2$$

$$\Rightarrow \bar{h}_1 A_1 + \frac{Q^2}{g A_1} = \frac{Q^2}{g A_2} + \bar{h}_2 A_2$$

$$\Rightarrow \left(\frac{h_1}{2}\right)(b h_1) + \frac{b^2 q^2}{g (b h_1)} = \frac{b^2 q^2}{g (b h_2)} + \frac{h_2}{2}(b h_2)$$

$$\Rightarrow \frac{b q^2}{g h_1} + \frac{1}{2} b h_1^2 = \frac{b q^2}{g h_2} + \frac{1}{2} b h_2^2$$

$$\Rightarrow \frac{q^2}{g}\left(\frac{1}{h_1} - \frac{1}{h_2}\right) = \frac{1}{2}(h_2^2 - h_1^2)$$

$$\Rightarrow \frac{q^2}{g h_1 h_2}(h_2 - h_1) = \frac{1}{2}(h_2 - h_1)(h_2 + h_1)$$

$$\Rightarrow \frac{q^2}{g h_1 h_2} = \frac{1}{2}(h_2 + h_1) \quad \text{------} \quad ①$$

$q = bq_1$
$= b(h_1 u_1)$

$$\Rightarrow \frac{q^2}{g h_1} = \frac{h_2}{2}(h_1 + h_2)$$

$$\Rightarrow \frac{u_1^2 h_1^2}{g h_1} = \frac{h_2}{2}(h_1 + h_2)$$

$$\Rightarrow \frac{u_1^2 h_1}{g} = \frac{h_2}{2}(h_1 + h_2)$$

$$\Rightarrow \frac{u_1^2}{g} = \frac{h_2}{2h_1}(h_1 + h_2)$$

$$\Rightarrow \frac{u_1^2}{g h_1} = \frac{h_2}{2h_1^2}(h_1 + h_2)$$

$$\Rightarrow F_1^2 = \frac{h_2}{2h_1^2}\, h_1 \left(1 + \frac{h_2}{h_1}\right)$$

$$\Rightarrow F_1^2 = \frac{1}{2}\,\frac{h_2}{h_1}\left(1 + \frac{h_2}{h_1}\right)$$

$$\Rightarrow \left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - 2F_1^2 = 0$$

$$\frac{h_2}{h_1} = \frac{-1 \pm \sqrt{1 - 4(-2F_1^2)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 + 8F_1^2}}{2}$$

$$\boxed{\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1 + 8F_1^2} - 1\right)}$$

similarly,
from (i),

$$\frac{q^2}{g h_1 h_2} = \frac{1}{2}(h_1 + h_2)$$

$$\Rightarrow \frac{q^2}{g h_2} = \frac{h_1}{2}(h_1 + h_2)$$

(4)

$$\Rightarrow \frac{u_2^2 h_2^2}{g h_2} = \frac{h_1}{2}(h_1+h_2)$$

$$\Rightarrow \frac{u_2^2 h_2}{g} = \frac{h_1}{2}(h_1+h_2)$$

$$\Rightarrow \frac{u_2^2}{g} = \frac{h_1}{2h_2}(h_1+h_2)$$

$$\Rightarrow \frac{u_2^2}{g h_2} = \frac{h_1}{2h_2^2}(h_1+h_2)$$

$$\Rightarrow F_2^2 = \frac{h_1}{2h_2}\left(1+\frac{h_1}{h_2}\right)$$

$$\Rightarrow \left(\frac{h_1}{h_2}\right)^2 + \left(\frac{h_1}{h_2}\right) - 2F_2^2 = 0$$

$$\boxed{\frac{h_1}{h_2} = \frac{1}{2}\left(\sqrt{1+8F_2^2} - 1\right)}$$

**

$$h_L = E_1 - E_2$$

$$= \left(h_1 + \frac{u_1^2}{2g}\right) - \left(h_2 + \frac{u_2^2}{2g}\right)$$

$$= (h_1 - h_2) + \left(\frac{u_1^2}{2g} - \frac{u_2^2}{2g}\right)$$

$$Q = b h_1 u_1 \Rightarrow h_1 = \frac{Q}{b u_1}$$

$$= \left(\frac{Q}{b u_1} - \frac{Q}{b u_2}\right) + \left(\frac{u_1^2}{2g} - \frac{u_2^2}{2g}\right)$$

$$= \frac{Q}{b}\left(\frac{u_2 - u_1}{u_1 u_2}\right) + \frac{1}{2g}\left(u_1^2 - u_2^2\right)$$

$$= \left(\frac{u_1-u_2}{2g}\right)\left[(u_1+u_2) - \frac{2g\,Q}{b}\,\frac{1}{u_1u_2}\right] \text{ ----- (i)}$$

$E_1 = E_2$

$$\Rightarrow \frac{Q^2}{gA_1} + \bar{h}_1A_1 = \frac{Q^2}{gA_2} + \bar{h}_2A_2$$

$$\Rightarrow \frac{Q(\cancel{A_1}u_1)}{g\cancel{A_1}} + \left(\frac{h_1}{2}\right)A_1 = \frac{Q(\cancel{A_2}u_2)}{g\cancel{A_2}} + \left(\frac{h_2}{2}\right)A_2$$

$$\Rightarrow \frac{Qu_1}{g} + \frac{1}{2}\left(\frac{Q}{bu_1}\right)\left(\frac{Q}{u_1}\right) = \frac{Qu_2}{g} + \frac{1}{2}\left(\frac{Q}{bu_2}\right)\left(\frac{Q}{u_2}\right)$$

$$\Rightarrow \frac{1}{2}\,\frac{Q^2}{bu_1^2} - \frac{1}{2}\,\frac{Q^2}{bu_2^2} = \frac{Q}{g}(u_2-u_1)$$

$$\Rightarrow \frac{Q^2}{2b}\left[\frac{1}{u_1^2} - \frac{1}{u_2^2}\right] = \frac{Q}{g}(u_2-u_1)$$

$$\Rightarrow \frac{Q}{2b}\left[\frac{u_2^2-u_1^2}{u_1^2u_2^2}\right] = \frac{1}{g}(u_2-u_1)$$

$$\Rightarrow \frac{Q}{2b}\,\frac{(u_2+u_1)}{(u_1u_2)^2} = \frac{1}{g}$$

$$\Rightarrow \frac{gQ}{b} = \frac{2u_1^2u_2^2}{(u_1+u_2)} \text{ ----- (ii)}$$

From (i) & (ii)

$$h_L = \frac{u_1-u_2}{2g}\left[(u_1+u_2) - \frac{4u_1^2u_2^2}{(u_1+u_2)}\,\frac{1}{u_1u_2}\right]$$

$$= \frac{u_1-u_2}{2g}\left[(u_1+u_2) - \frac{4u_1u_2}{u_1+u_2}\right]$$

$$= \frac{u_1-u_2}{2g}\,\frac{(u_1-u_2)^2}{(u_1+u_2)}$$

$$\boxed{h_L = \frac{(u_1-u_2)^3}{2g(u_1u_2)}}$$

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$$E_1 = h_1 + \frac{u_1^2}{2g}$$

$$\Rightarrow E_1 = h_1\left(1+\frac{u_1^2}{2gh_1}\right)$$

$$\Rightarrow \frac{E_1}{h_1} = \left(1+\frac{u_1^2}{2gh_1}\right)$$

$$= 1+\left(\frac{u_1^2}{gh_1}\right)\frac{1}{2}$$

$$= 1+\frac{1}{2}F_{r_1}^2$$

$$= \frac{2+F_{r_1}^2}{2}$$

$$E_2 = h_2 + \frac{u_2^2}{2g}$$

$$\Rightarrow \frac{E_2}{h_1} = \frac{h_2}{h_1} + \frac{u_2^2}{2gh_1}$$

$u_1h_1 = u_2h_2$

$\Rightarrow u_2 = \left(u_1\frac{h_1}{h_2}\right)$

$$= \left(\frac{h_2}{h_1}\right) + \frac{1}{2gh_1} * u_1^2\left(\frac{h_1}{h_2}\right)^2$$

$$= \left(\frac{h_2}{h_1}\right) + \frac{u_1^2}{gh_1}\,\frac{1}{2}\left(\frac{h_1}{h_2}\right)^2$$

$$= \left(\frac{h_2}{h_1}\right) + \frac{1}{2}F_{r_1}^2\left(\frac{h_1}{h_2}\right)^2$$

$$= \frac{1}{2}\left(\sqrt{1+8F_{r_1}^2}-1\right) + \frac{F_{r_1}^2}{2}\,\frac{4}{\left(\sqrt{1+8F_{r_1}^2}-1\right)^2}$$

$$= \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right) + 2F_1^2 \frac{\left(\sqrt{1+8F_1^2}+1\right)^2}{\left\{\left(\sqrt{1+8F_1^2}-1\right)\left(\sqrt{1+8F_1^2}+1\right)\right\}^2}$$

$$= \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right) + 2F_1^2 \frac{\left(\sqrt{1+8F_1^2}+1\right)^2}{\left(\cancel{1}+8F_1^2-\cancel{1}\right)^2}$$

$$= \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right) + \frac{\left(\sqrt{1+8F_1^2}+1\right)^2}{32F_1^2}$$

$$= \frac{16F_1^2\left(\sqrt{1+8F_1^2}-1\right) + 1+8F_1^2+1+2\sqrt{1+8F_1^2}}{32F_1^2}$$

$$= \frac{16F_1^2\sqrt{1+8F_1^2} - 16F_1^2 + 2+8F_1^2 + 2\sqrt{1+8F_1^2}}{32F_1^2}$$

$$= \frac{\sqrt{1+8F_1^2}\left(2+16F_1^2\right) - 8F_1^2 + 2}{32F_1^2}$$

$$= \frac{2\left(1+8F_1^2\right)^{3/2} - 8F_1^2+2}{32F_1^2}$$

$$= \frac{\left(1+8F_1^2\right)^{3/2} - 4F_1^2+1}{16F_1^2}$$

$$\frac{E_2}{E_1} = \frac{\left(1+8F_1^2\right)^{3/2} - 4F_1^2+1}{16F_1^2} \times \frac{2}{\left(2+F_1^2\right)}$$

$$\Rightarrow \boxed{\frac{E_2}{E_1} = \frac{\left(1+8F_1^2\right)^{3/2} - 4F_1^2+1}{8F_1^2\left(2+F_1^2\right)}}$$

Efficiency

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$h_j = h_2 - h_1$

$= h_1\left(\frac{h_2}{h_1} - 1\right)$

$= \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 1 - \frac{2}{2}\right) = \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - \frac{3}{2}\right)$

$\frac{h_j}{E_1} = \frac{\frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 3\right)}{(2+Fr_1^2)}$

$\frac{h_j}{E_1} =$

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$h_j = h_2 - h_1$, Height of the jump

$= h_1\left(\frac{h_2}{h_1} - 1\right)$

$$\frac{h_j}{h_1} = \left(\frac{h_2}{h_1} - 1\right)$$

$$= \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) - 1$$

$$= \frac{\sqrt{1+8Fr_1^2} - 3}{2}$$

$\therefore \frac{h_j}{\frac{2E_1}{2+Fr_1^2}} = \frac{\sqrt{1+8Fr_1^2} - 3}{2}$

$\Rightarrow \frac{E_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2}$

$$\frac{E_1}{h_1} = \frac{2+Fr_1^2}{2}$$

$\Rightarrow h_1 = \frac{2E_1}{2+Fr_1^2}$

$\Rightarrow$

$$\boxed{\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2}}$$

, relative height

$\frac{h_1}{E_1}$ = relative initial depth, $\frac{h_2}{E_1}$ = relative sequent depth

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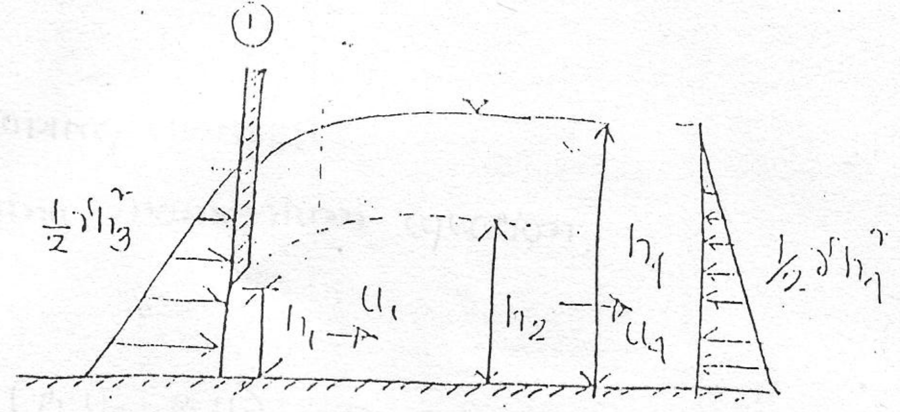


momentum equation.

$$\frac{\gamma Q}{g}(u_4-u_1) = \frac{1}{2}\gamma h_3^2 - \frac{1}{2}\gamma h_4^2$$

$$\Rightarrow \frac{\gamma q}{g}(u_4-u_1) = \frac{\gamma}{2}\left(h_3^2-h_4^2\right)$$

$$\Rightarrow \frac{q}{g}\left(\frac{q}{h_4}-\frac{q}{h_1}\right) = \frac{1}{2}\left(h_3^2-h_4^2\right)$$

$$\Rightarrow \frac{q^2}{g}\left(\frac{1}{h_4}-\frac{1}{h_1}\right) = \frac{1}{2}\left(h_3^2-h_4^2\right)$$

$$\Rightarrow F_4^2 h_4^3\left(\frac{1}{h_4}-\frac{1}{h_1}\right) = \frac{1}{2}\left(h_3^2-h_4^2\right)$$

$$\Rightarrow F_4^2\left(h_4^2-\frac{h_4^3}{h_1}\right) = \frac{1}{2}\left(h_3^2-h_4^2\right)$$

$$\Rightarrow 2F_4^2 h_4^2\left(1-\frac{h_4}{h_1}\right) = \frac{h_4^2}{2}\left(\frac{h_3^2}{h_4^2}-1\right)$$

$$\Rightarrow 2F_4^2\left(1-\frac{h_4}{h_1}\right) = \left(\frac{h_3}{h_4}\right)^2-1$$

$$\Rightarrow \left(\frac{h_3}{h_4}\right)^2 = 1+2F_4^2\left(1-\frac{h_4}{h_1}\right)$$

$$\Rightarrow \boxed{\frac{h_3}{h_4} = \sqrt{1+2F_4^2\left(1-\frac{h_4}{h_1}\right)}}$$

$b = 1\,m$

$Q = bq$
$= 1(q)$
$= q$

$u_4 = \frac{q}{h_4}$

$F_4 = \frac{u_4}{\sqrt{g h_4}}$

$\Rightarrow F_4^2 = \frac{u_4^2}{g h_4}$
$= \frac{q^2}{g h_4^3}$

$\Rightarrow \frac{q^2}{g} = F_4^2 h_4^3$

** FOR SLOPING CHANNEL

applying momentum equation.

$$\frac{\gamma q}{g}(\beta_2 u_2 - \beta_1 u_1) = F_1 - F_2 + H\sin\theta - F_f$$

here.

$q = u_1 d_1$

$u_1 d_1 = u_2 d_2$

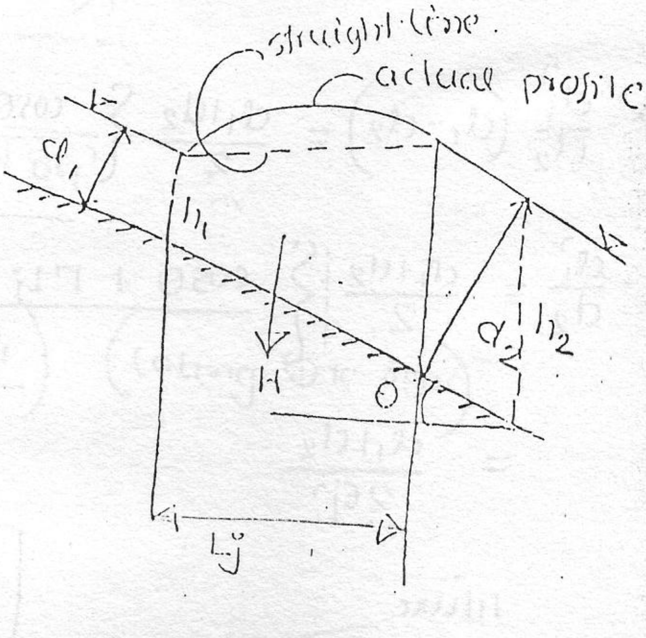
$\Rightarrow u_2 = \frac{u_1 d_1}{d_2}$

$F_1 = \frac{1}{2}\gamma d_1^2 \cos\theta$

$F_2 = \frac{1}{2}\gamma d_2^2 \cos\theta$

$F_f = 0$

$\beta_1 = \beta_2 = 1$



$$H = \frac{1}{2}\gamma \Gamma L_j (d_1 + d_2)$$

Γ = correction due to difference between actual profile and st line

$$\frac{q}{g}(u_2 - u_1) = \frac{1}{2}\cos\theta\,(d_1^2 - d_2^2) + \frac{1}{2}\Gamma L_j (d_1 + d_2)\sin\theta$$

$$\Rightarrow \frac{u_1 d_1}{g}\left(\frac{u_1 d_1}{d_2} - u_1\right) = \frac{1}{2}(d_1 + d_2)\left[\cos\theta\,(d_1 - d_2) + \Gamma L_j \sin\theta\right]$$

$$\Rightarrow \frac{u_1^2 d_1^2}{g d_2} - \frac{u_1^2 d_1}{g} = \frac{1}{2}(d_1+d_2)\left[\cos\theta\,(d_1-d_2) + \Gamma L_j \sin\theta\right]$$

$$\Rightarrow \frac{Fr_1^2\, d_1^3}{d_2} - Fr_1^2\, d_1^2 = \frac{d_1+d_2}{2}\left[\cos\theta\,(d_1-d_2) + \Gamma L_j \sin\theta\right]$$

$$\Rightarrow Fr_1^2\left(\frac{d_1^3}{d_2} - d_1^2\right) = \frac{d_1+d_2}{2}\left[\cos\theta\,(d_1-d_2) + \Gamma L_j \sin\theta\right]$$

$$\Rightarrow d_1^2\left(\frac{d_1}{d_2} - 1\right) = \frac{d_1+d_2}{2}\left\{\frac{\cos\theta\,(d_1-d_2) + \Gamma L_j \sin\theta}{Fr_1^2}\right\}$$

$$\Rightarrow \frac{d_1^2}{d_2}(d_1-d_2) = \frac{d_1+d_2}{2}\left\{\frac{\cos\theta\,(d_1-d_2) + \Gamma L_j \sin\theta}{Fr_1^2}\right\}$$

$$\Rightarrow \frac{d_1^2}{d_2} = \frac{d_1+d_2}{2}\left\{\frac{\cos\theta + \Gamma L_j \sin\theta/(d_1-d_2)}{Fr_1^2}\right\}$$

$$= \frac{d_1+d_2}{2G^2}$$

Here.

$$G^2 = \frac{Fr_1^2}{\cos\theta + \Gamma L_j \sin\theta/(d_1-d_2)}$$

$$= \frac{Fr_1^2}{\cos\theta - \Gamma L_j \sin\theta/(d_2-d_1)}$$

$$\frac{d_1^2}{d_2} = \frac{d_1+d_2}{2G^2}$$

$$\Rightarrow \frac{d_1^2}{d_2} - \frac{d_1+d_2}{2G^2} = 0$$

$$\Rightarrow -\frac{d_1^2}{d_2} + \frac{d_1 + d_2}{2G^2} = 0$$

$$\Rightarrow -d_1^2(2G^2) + d_2(d_1 + d_2) = 0$$

$$\Rightarrow -d_1^2(2G^2) + d_1 d_2 + d_2^2 = 0$$

$$\Rightarrow d_2^2 + d_1 d_2 - 2G^2 d_1^2 = 0$$

$$d_2 = \frac{-d_1 \pm \sqrt{d_1^2 + 8 d_1^2 G^2}}{2}$$

$$= \frac{-d_1 \pm d_1\sqrt{1 + 8G^2}}{2}$$

$$= d_1\left(\frac{\sqrt{1+8G^2} - 1}{2}\right) \quad \text{(taking (+)ve sign)}$$

$$\boxed{\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1+8G^2} - 1\right)}$$

<u>Prove</u>: $\dfrac{E_L}{E_1} = \dfrac{\Delta E}{E_1} = \dfrac{\left(\sqrt{1+8F_1^2}-3\right)^3}{8\left(2+F_1^2\right)\left(\sqrt{1+8F_1^2}-1\right)}$

<u>Sol^n</u>

$$\frac{E_L}{y_1} = \frac{(y_2-y_1)^3}{4y_1^2 y_2} \qquad \left[\because E_L = \frac{(y_2-y_1)^3}{4y_1 y_2}\right]$$

$$= \frac{\left(\frac{y_2}{y_1}-1\right)^3}{4y_2/y_1} \qquad [\text{dividing by } y_1^3]$$

$$= \frac{\left\{\frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right)-1\right\}^3}{4 * \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right)}$$

$$= \frac{\left(\sqrt{1+8F_1^2}-3\right)^3}{16\left(\sqrt{1+8F_1^2}-1\right)} \quad -(1)$$

Again, $\dfrac{E_1}{y_1} = 1 + \dfrac{v_1^2}{2gy_1} = 1 + \dfrac{1}{2}F_1^2 \quad -(2)$

$$\therefore \frac{E_L}{E_1} = (1) \div (2) = \frac{\left(\sqrt{1+8F_1^2}-3\right)^3}{8\left(2+F_1^2\right)\left(\sqrt{1+8F_1^2}-1\right)} \quad [\text{Proved}]$$

Hydraulic jump

PROBLEMS

7/2/09

i) $\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right)$

ii) $\frac{h_1}{h_2} = \frac{1}{2}\left(\sqrt{1+8Fr_2^2} - 1\right)$

iii) $\Delta E = \frac{(h_2-h_1)^3}{4h_1h_2}$ (Relative loss) $\frac{h_L}{E_1} = 1 - \frac{E_2}{E_1}$

iv) $\frac{E_1}{h_1} = \frac{1}{2}\left(Fr_1^2 + 2\right)$

v) $\frac{E_2}{h_2} = \frac{1}{2} \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1)}{8Fr_1^2}$

vi) $\frac{E_2}{E_1} = \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2\,(2\cdot Fr_1^2 + 2)}$ (Efficiency of the jump)

vii) $\frac{h_j}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 3\right)$ (Relative height)

viii) $\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{Fr_1^2 + 2}$

7.2 (a)

We know,

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right)$$

$$\frac{h_L}{E_1} = 1 - \frac{E_2}{E_1} = 1 - \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2\,(Fr_1^2+2)}$$

$$\frac{h_j}{E_1} = \left(\sqrt{1+8Fr_1^2} - 3\right)/(Fr_1^2+2)$$

When, $Fr_1 = 1$

$$\frac{h_2}{h_1} = 1$$

$$\frac{h_L}{E_1} = 0$$

$$\frac{h_j}{E_1} = 0$$

When, $Fr_1 = 1.7$

$$\frac{h_2}{h_1} = 1.96$$

$$\frac{h_L}{E_1} = 0.05$$

$$\frac{h_j}{E_1} = 0.39$$

When, $Fr_1 = 2.5$

$$\frac{h_2}{h_1} = 3.07$$

$$\frac{h_L}{E_1} = 0.18$$

$$\frac{h_j}{E_1} = 0.5$$

When, $Fr_1 = 4.5$

$$\frac{h_2}{h_1} = 5.88$$

$$\frac{h_L}{E_1} = 0.42$$

$$\frac{h_j}{E_1} = 0.44$$

When, $Fr_1 = 9$

$$\frac{h_2}{h_1} = 12.24$$

$$\frac{h_L}{E_1} = 0.7$$

$$\frac{h_j}{E_1} = 0.27$$

When, $Fr_1 = 15$,

$$\frac{h_2}{h_1} = 20.71$$

$$\frac{h_L}{E_1} = 0.82$$

$$\frac{h_j}{E_1} = 0.17$$

7.3 (a) Given, $u_1 = 6.1\ ms^{-1}$

$h_1 = 1\ m$

$b = 6.1\ m$

i) $Fr_1^2 = \dfrac{u_1^2}{gh_1} = \dfrac{6.1^2}{9.81 \times 1} = 3.793$

Now, $\dfrac{h_2}{h_1} = \dfrac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right)$

$$\Rightarrow \cancel{\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8\times 3.793^2}-1\right)}$$

$$\Rightarrow \frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8\times 3.793}-1\right)$$

$$\Rightarrow \frac{h_2}{h_1} = 2.3\ m$$

$\therefore h_2 = 2.3\ m$ Ans.

) As the Froude number is $\sqrt{3.793} = 1.95$ is in between 1.7 and 2.5 the jump is weak jump.

) From fig 7.3, for $Fr_1 = 1.95$, $\frac{Lj}{h_2} = 4.2$

$\therefore Lj = 4.2 \times 2.3 = 9.66\ m$

Or by Silvester formula.

$Lj = 9.75\, h_1 (Fr_1 - 1)^{1.01} = 9.75 \times 1 (1.95-1)^{1.01}$

$\therefore Lj = 9.26\ m$

) $h_L = \frac{(h_2-h_1)^3}{4h_1h_2} = \frac{(2.3-1)^3}{4.1.2.3} = 0.24\ m$

$Q = u_1 A = 6.1 \times 1 \times 6.1$

$\therefore Q = 37.21\ m^3/sec$

$\therefore$ The horsepower dissipation in the jump

$$= \frac{\gamma Q h_L}{746} = \frac{9810 \times 37.21 \times 0.24}{746}$$

$$= 117.43$$

vi) $\frac{E_2}{E_1} = \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(Fr_1^2+2)}$

$= \frac{(1+8\times 3.793)^{3/2} - 4\times 3.793 + 1}{8\times 3.793(3.793+2)}$

$= 0.9176$,

$\therefore$ Efficiency $= 91.76\%$.

7.3 b) Given,

$h_1 = 0.5\ m$

$u_1 = 15.5\ ms^{-1}$

i) $Fr_1 = \frac{u_1}{\sqrt{gh_1}} = \frac{15.5}{\sqrt{9.81\times 0.5}}$

$\therefore Fr_1 = 7$.

$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^2} - 1) = \frac{1}{2}(\sqrt{1+8\times 7^2} - 1)$

$\Rightarrow h_2 = 9.41 \times 0.5$

$\therefore h_2 = 4.7\ m$.

ii) As $Fr_1 = 7$, it is steady jump.

iii) Height of the jump,

$h_j = h_2 - h_1 = 4.7 - 0.5$

$\therefore h_j = 4.2\ m$.

iv) The length of jump. from fig. 7.3,

$\frac{L_j}{h_2} = 6.1\ m$

$\therefore L_j = 6.1 \times 4.7$

$\therefore L_j = 28.67$.

v) % Energy loss $= \left(1 - \frac{E_2}{E_1}\right) \times 100\%$

$$= \left(1 - \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(Fr_1^2+2)}\right) \times 100$$

$$= \left(1 - \frac{(1+8\times7^2)^{3/2} - 4\times7^2 + 1}{8\times7^2(7^2+2)}\right) \times 100$$

$$= 62\%$$

7.5 Given,

$h_1 = 1\,m$

$b = 5\,m$

$s = 1$

$Q = 30\,m^3/s$

$A_1 = (b+sh_1)h_1$

$= (5+1.1)1$

$= 6\,m^2$

$u_1 = \frac{Q}{A_1} = \frac{30}{6}$

$\therefore u_1 = 5\,m/s$

$Fr_1 = \frac{u_1}{\sqrt{gh_1}} = \frac{5}{\sqrt{9.81\times1}} = 1.6$

$\bar{z}_1 = \frac{h_1}{6}\left(\frac{3b+2sh_1}{b+sh_1}\right) = \frac{1}{6}\left(\frac{3\times5+2\times1\times1}{5+1\times1}\right)$

$\therefore \bar{z}_1 = 0.47\,m$

$F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{30^2}{9.81\times6}$

$\therefore F_1 = 18.11$

At the downstream section

$$F_2 = \frac{Q^2}{g A_2} + \bar{z}_2 A_2$$

The condition which must be satisfied to cause a hydraulic jump between sections at upstream and downstream is

$$F_1 = F_2$$

The value of h_2 which satisfies this condition is determined by trial and error,

h_2	$A_2 = (b+5h_2)h_2$	$\bar{z}_2 = \frac{h_2}{6}\left(\frac{3b+25h_2}{b+5h_2}\right)$	$F_2 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$
1.5	9.75	0.692	16.156
2	14	0.9	19.15
1.75	11.81	0.8	17.21
1.9	13.11	0.86	18.27

Hence the downstream depth required to produce a hydraulic jump, $h_2 = 1.9$ m.

Now, upstream energy

$$E_1 = h_1 + \frac{u_1^2}{2g} = 1 + \frac{5^2}{2\times 9.81} = 2.27 \text{ m}$$

$$E_2 = h_2 + \frac{u_2^2}{2g}$$

$$u_2 = \frac{30}{13.11} = 2.29 \text{ m s}^{-1}$$

$$\therefore E_2 = 1.9 + \frac{2.29^2}{2\times 9.81}$$

$\therefore E_2 = 2.16$ m

$\therefore$ Energy lost in the jump $= (2.27 - 2.16)$ m

$= 0.11$ m.

7.6 Given,

$s = 2$

$Q = 20\ m^3/s$

$h_1 = 1$ m.

$\therefore A_1 = sh_1^2 = 2 \times 1^2$

$\therefore A_1 = 2\ m^2$.

$\therefore U_1 = \frac{Q}{A_1} = \frac{20}{2} = 10\ ms^{-1}$

$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{10}{\sqrt{9.81 \times 1}}$

$\therefore Fr_1 = 3.19$

$\therefore \bar{z}_1 = \frac{h_1}{3} = \frac{1}{3} = 0.33$ m.

$\therefore F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{20^2}{9.81 \times 2} + 0.33 \times 2$

$\therefore F_1 = 21.05$

At downstream section,

$F_2 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$

The condition which must be satisfied to cause a hydraulic jump between sections of upstream and downstream is,

$F_1 = F_2$.

The value of h_2 which satisfies this condition is determined by trial and error.

h_2 (m)	$A = 5h_2^2$ (m^2)	$\bar{z}_2 = \frac{h_2}{3}$ (m)	$F_2 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$
3	18	1	20.26
3.1	19.22	1.033	21.97
3.05	18.61	1.0167	20.8

So, the downstream depth required to prod a hydraulic jump is 3.05 m.

7.7 Given,

$Q = 10\ m^3/s$,

$h_1 = 0.5\ m$

$x^2 = 4z$

$\therefore z = \frac{1}{4} x^2$

$\therefore c = \frac{1}{4}$

$\therefore A_1 = \frac{4 h_1^{3/2}}{3\sqrt{c}} = \frac{4 \times 0.5^{3/2}}{3 \times \sqrt{1/4}}$ | $\bar{z}_1 = \frac{2h_1}{5} = \frac{2 \times 0.5}{5} = 0.2$

$\therefore A_1 = 0.943\ m^2$

$\therefore F_1 = \frac{Q^2}{gA_1} + \bar{z}_1 A_1$

$\Rightarrow F_1 = \frac{10^2}{9.81 \times 0.943} + 0.2 \times 0.943$

$\therefore F_1 = \cancel{10.99}\ 11$

So, for producing hydraulic jump.

$F_1 = F_2$

To find F_2, trial-error Process is developed.

h_2	$A_2 = \frac{4h_2^{3/2}}{3\sqrt{c}}$	$\bar{z}_2 = \frac{2h_2}{5}$	$F_2 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$
2	7'54	0'8	7'38.
2'5	10'54	1	11'51
2'4	9'91	0'96	10'54.
2'45	10'22	0'98	11'01

So, the downstream depth required to Produce a hydraulic jump is 2'45 m.

7'8 a) Given $b = ~~ten~~\ 1'2\ m$

$\theta = 30$

$Q = 0'14\ m^3/s$.

$h_1 = 0'02\ m$.