

Engineering Mechanics

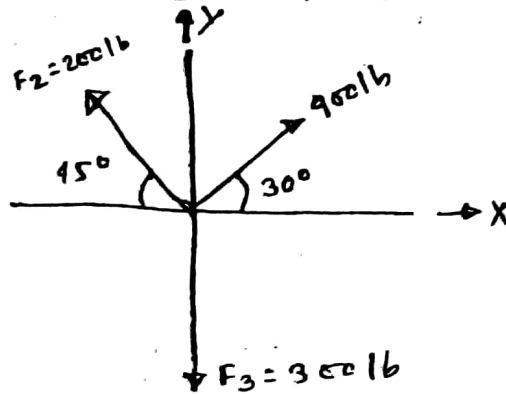
* Mechanics: It is that branch of physical science which treats of the effect of forces upon material bodies.

Two types — (i) solid mechanics

(ii) Fluid mechanics.

* Force: A force is defined as that physical quantity which tends to change the state of motion of a body.

From the following figure, determine the resultant of three forces.



Solⁿ:

$$\sum F_x = 400 \cos 30^\circ - 200 \cos 45^\circ = 209.99 \text{ lb}$$

$$\sum F_y = 400 \sin 30^\circ + 200 \sin 45^\circ - 300 = 41.42 \text{ lb}$$

$$\therefore R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = 209.13 \text{ lb}$$

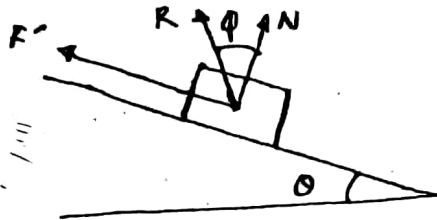
$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) = 11.42^\circ \quad \text{Ans}$$

* Free Body Diagram: A free body diagram is a diagram of a body or a group of bodies which is isolated from its environment and on which all external forces, such as weight, applied forces, reactions, etc. are acting.

* Centroid / Centre of Gravity: The point where the weight of a body is considered to be acting is called centre of gravity. The word centroid means the geometric centre of the object shape.

* Angle of Friction: The angle that the total plane reaction (R) makes with the line of action of normal force (N) is called angle of friction.

* Angle of Repose: The angle of inclination of the plane when motion of a body under the action of gravity is impending is called the angle of repose.



ϕ → Angle of Friction
 θ → Angle of Repose.

* Centroid for different shape.

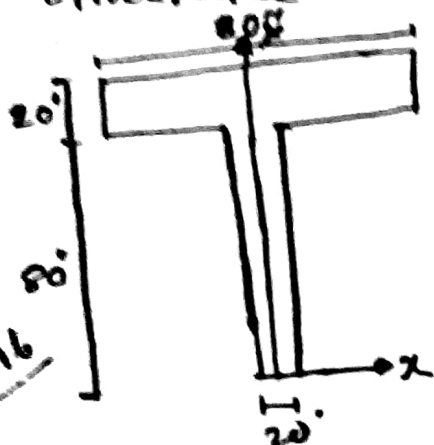
| Shape | Figure | Centroid |
|---------------------|--------|--|
| Area of Circle | | $\bar{x} = \frac{r \sin \beta}{\beta}$ |
| Sector of circle | - | $\bar{x} = \frac{2r \sin \beta}{3\beta}$ |
| Triangle | | $\bar{y} = \frac{h}{3}$ |
| Right circular cone | | $\bar{x} = \frac{3}{4} h$ |
| Quarter Circle | | $\bar{x} = \frac{4r}{3\pi}, \bar{y} = \frac{4r}{3\pi}$ |
| Semi circle | | $\bar{y} = \frac{4r}{3\pi}, \bar{x} = 0$ |

* The axis passing through the centroid is called Centroidal axis.

* Laws of friction:

- The coefficient of friction & the value of the frictional force are independent of the area of contact.
- The limiting frictional force for two bodies in contact is directly proportional to the normal force.
- The coefficient of friction is independent of the velocity.

* Find centroid, moment of inertia of the following structure.



Solⁿ: $\bar{x} = 0$ (Symmetry about y axis)

$$\bar{y} = \frac{20 \times 80 \times 40 + 20 \times 80 \times 90}{1600 + 1600} = 65''$$

$$I_x = \frac{20 \times 80^3}{12} + 20 \times 80 \times 25^2 + \frac{80 \times 20^3}{12} + 80 \times 20 \times 25^2 = 2.91 \times 10^6 \text{ in}^4 \text{ [about x-axis]}$$

$$I_y = \frac{80 \times 20^3}{12} + \frac{20 \times 80^3}{12} = 9.07 \times 10^5 \text{ in}^4 \text{ [about y-axis]}$$

Introduction:

* surveying: Surveying is the art of determining the relative positions of different objects on the surface of the earth by means of direct or indirect measurement of distance, direction and elevation and finally presenting them on a map to any suitable scale.

* Uses of Surveying:

- To prepare a topographical map which shows the hills, valleys, rivers, villages, towns, etc. of a country.
- To prepare a cadastral map showing the boundaries of fields, houses and other properties.
- To prepare an engineering map which shows the details of engineering works such as roads, railways, reservoirs, irrigation canals, etc.
- To prepare a military map showing the different strategic points important for the defence of a country.
- To prepare a contour map to find the best possible route for roads, railways, etc.

* Classification of Surveying:

A. Primary classification

1. plane Surveying → Earth curvature are not taken into consideration.
2. Geodetic Surveying → Earth curvature is taken into consideration.

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B. Secondary Classification

1. Based on instrument

- (a) Chain Surveying
- (b) Traverse "
- (c) plane table "
- (d) Theodolite "
- (e) Tacheometric "
- (f) photographic "

2. Based on methods

- (a) Triangulation surveying
- (b) Traverse "

3. Based on object

- (a) Geological surveying
- (b) Mine surveying
- (c) Military surveying

4. Based on field nature

- (a) Land surveying
- (b) Marine surveying
- (c) Astronomical "

* Different types of correction of surveying.

A. Tape correction

1. Temperature correction, $C_t = \alpha (T_m - T_0) * L$

2. pull correction, $C_p = \frac{P_m - P_0}{A.E} * L$

3. Slope correction, $C_h = \frac{h^2}{2L}$ (approx) [Always -ve]
 $= L(1 - \cos \theta)$ (exact)

4. Sag correction, $C_s = \frac{LW^2}{24 n^2 P_m}$ (Always negative)

where, α = co-efficient of thermal expansion.

L = Length of tape, T_m = Measurable temperature

T_0 = standard temperature

A = x-section of tape, E = Modulus of Elasticity

P_m = Measurable pull, P_0 = standard pull

l = length of tape betⁿ two support.

W = weight of tape, kg, n = no. of span

h = Diffⁿ of height betⁿ two support.

B. Chain Correction

1. Length correction, True Length, $TL = \frac{L'}{L} \times \text{measured length}$

2. Area Correction, True Area, $TA = \left(\frac{L'}{L}\right)^2 \times \text{measured Area}$

where, $L = \text{standard/True length of chain}$

$L' = \text{True length} \pm \text{error}$

* The distance between two points, measured with a 20 m chain, was recorded as 327 m. It was afterwards found that the chain was 3 cm too long. What was the true distance betⁿ the points?

Solⁿ: Here, $L = 20 \text{ m}$, $L' = 20 + \frac{3}{100} = 20.03 \text{ m}$ [$L' = L + e$]

$ML = 327 \text{ m}$

$$\begin{aligned}\therefore \text{True length of line, } TL &= \frac{L'}{L} \times ML \\ &= \frac{20.03}{20} \times 327 = 327.49 \text{ m}\end{aligned}$$

* The distance between two stations was 1200 m when measured with a 20 m chain. The same distance when measured with a 30 m chain was found 1195 m. If the 20 m chain was 0.05 m too long, what was the error in the 30 m chain?

Solⁿ: For 20 m chain

$$\begin{aligned}\text{True length, } TL &= \frac{L'}{L} \times ML = \frac{20 + 0.05}{20} \times 1200 \\ &= 1203 \text{ m}\end{aligned}$$

For 30 m chain, $L' = ?$, $TL = 1203 \text{ m}$, $ML = 1195 \text{ m}$

$$\therefore TL = \frac{L'}{L} \times ML \Rightarrow L' = \frac{1203 \times 30}{1195} = 30.20 \text{ m}$$

$\therefore \text{Amount of error} = L' - L = 0.2 \text{ m}$ Ans

* A line was measured by a 20m chain which was accurate before starting the day's work, After chaining 900m, the chain was found to be 0.6 decimal too long. After chaining a total distance of 1575m, the chain was found to be 14 cm too long. Find the true distance of the line?

Solⁿ: For 900m length.

$$L = 20\text{m}, L' = 20 + \frac{0 + 0.06}{2} = 20.03, ML = 900\text{m}.$$

$$\therefore TL_1 = \frac{L'}{L} * ML = \frac{20.03}{20} * 900 = 901.35\text{m}$$

For the next distance,

$$L = 20\text{m}, L' = 20 + \frac{0.06 + 0.14}{2} = 20.1\text{m}, ML = 1575 - 900 = 675\text{m}.$$

$$\therefore TL_2 = \frac{20.1}{20} * 675 = 678.375\text{m}.$$

$$\therefore \text{True distance, } TL_1 + TL_2 = 1579.725\text{m}$$

Ans.

* On a map drawn to scale of 50m to 1cm, a surveyor measured the distance between two stations as 3500m. But it was found that by mistake he had used a scale of 100m to ~~100~~ 1cm. Find the true distance between two stations.

Solⁿ: Distance betⁿ stations on map = $\frac{3500}{100} = 35\text{cm}$

As the actual scale 50m to 1cm.

$$\therefore \text{True distance} = 50 * 35 = 1750\text{m}$$

Ans

* A correct distance of 1000 m was measured with a 25 m chain which was actually 24.9 m long. Calculate the chain distance?

Solⁿ:

$$\text{Actual Chain distance} = \frac{24.9}{25} \times 1000 = 996 \text{ m} \quad \underline{\text{Ans}}$$

* A steel tape was exactly 30 m long at 20°C when supported throughout its length under a pull of 10 kg. A line was measured with this tape under a pull of 15 kg and at a mean temp. of 32°C and found to be 780 m long. The x-sectional area of the tape = 0.03 cm², and its total weight = 0.693 kg. α for steel = $11 \times 10^{-6} / ^\circ\text{C}$, $E = 2.1 \times 10^6 \text{ kg/cm}^2$. Compute the true length of the line if the tape was supported during measurement at every 30 m?

Solⁿ: Here, $L = 30 \text{ m}$, $T_0 = 20^\circ\text{C}$, $P_0 = 10 \text{ kg}$, $P_m = 15 \text{ kg}$, $T_m = 32^\circ\text{C}$
 $ML = 780 \text{ m}$, $A = 0.03 \text{ cm}^2$, $W = 0.693 \text{ kg}$, $\alpha = 11 \times 10^{-6} / ^\circ\text{C}$, $E = 2.1 \times 10^6 \text{ kg/cm}^2$

(i) Temperature correction,

$$C_t = \alpha (T_m - T_0) \times L$$

$$= 3.96 \times 10^{-3} \text{ m (+ve)}$$

(ii) pull correction,

$$C_p = \frac{P_m - P_0}{AE} \times L$$

$$= 0.00238 \text{ m (+ve)}$$

(iii) Sag correction,

$$C_s = \frac{LW^2}{24n^2 P_m^2} \quad [n=1]$$

$$= 0.00267 \text{ m (-ve)}$$

$$\therefore e = +0.00396 + 0.00238 - 0.00267$$

$$= 0.00367 \text{ m}$$

$$L' = L + e = 30.00367 \text{ m}$$

$$\therefore \text{True length} = \frac{L'}{L} \times ML$$

$$= \frac{30.00367}{30} \times 780 = 780.094 \text{ m}$$

"Chain Surveying"

* Chain Survey: Chain survey is the simplest type of survey in which the land area to be surveyed is divided into a no. of triangles.

* Chain Surveying is recommended when -

1. The ground surface is more or less level.
2. A small area is to be surveyed.
3. ~~The area is very large.~~
3. A small scale map is to be prepared and.
4. The formation of well-conditioned triangles is easy.

* 3 types of triangle.

- Well conditioned triangle - $30^\circ < \text{All angles} < 120^\circ$
- Ill " " - $30^\circ \nless \text{All angles} \nless 120^\circ$
- Ideal triangle - All angles 60°

* Survey station: Survey stations are the points at the beginning and the end of a chain line.

* Check line: Check lines/proof lines are the lines which are taken to check the accuracy of work.

* Tie lines: The line joining subsidiary or tie stations are known as tie line.

* Field Book: The notebook in which field measurements are noted is called field book. The size of the field book is 20 cm x 12 cm and opens lengthwise.

* Reconnaissance Survey: Before the commencement of any survey work, the area to be surveyed is examined by the surveyor, who then thinks about the possible arrangement of the framework of survey. This primary investigation of the area is called reconnaissance survey.

* Types of Chain:

1. The Engineer's Chain: It is 100ft long and consists of 100 links each one of 1 ft long. At every 10 links a tally is attached to facilitate the reading.
2. Gunter's Chain: It is 66 ft long and consists of 100 links each one of 7.92 inch.
3. Metre's Chain: Generally available in lengths 5, 10, 20 and 30 m.

* Write down the instruments used in Chain Survey.
Ans: Chain, Tape, Pegs, Arrows, Ranging Rods, Offset staff, optical square, etc.

Pegs: These are wooden blocks of conical shape used in fixing stations.

Arrows: They are steel wire 15" long pointed at one and the other end is looped for convenience of handling.

Ranging Rods: They are about 10ft long, $1\frac{1}{2}$ " dia round wooden poles painted with black and white alternate bands.

Offset staff: They are wooden rods 10ft long. Each foot is painted black and white alternately. They are used for measuring short lengths.

"Traverse Surveying"

13/01/19-14 * Traverse Surveying: It is a type of surveying in which the plot of land to be surveyed is enclosed by a series of straight lines making angle with each other. The length and angles of the lines are measured and plotted with all interior details to a suitable scale to produce a map.

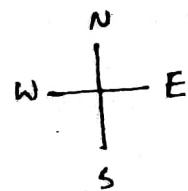
- * Types
- closed traverse = The traverse in which the point of the last line meet the starting point of the first line.
 - open traverse = When a sequence of connected lines extends along a general direction and does not return to the starting point, is known as open traverse.

* Bearing: Is the angle which the line makes with a reference / meridian and always measured in clockwise direction.

* Whole Circle Bearing: The magnetic bearing of a line measured clockwise from the north pole towards the line is termed as whole circle bearing (WCB).

* Reduced Bearing (RB): When the whole circle bearing of a line is converted to quadrantal bearing, it is termed as 'reduced bearing'. Its value lies betⁿ $0^\circ \sim 90^\circ$.

| <u>WCB</u> | <u>RB</u> | <u>Quadrant</u> |
|----------------------------|--------------------------|-----------------|
| $0 \sim 90^\circ$ | WCB | NE |
| $90^\circ \sim 180^\circ$ | $180^\circ - \text{WCB}$ | SE |
| $180^\circ \sim 270^\circ$ | $\text{WCB} - 180^\circ$ | SW |
| $270^\circ \sim 360^\circ$ | $360^\circ - \text{WCB}$ | NW |



* Convert the following whole circle bearing to quadrantal bearing.

(i) $22^{\circ}30'$ (ii) $170^{\circ}12'$ (iii) $211^{\circ}54'$ (iv) $327^{\circ}24'$

Solⁿ: (i) R.B = $N 22^{\circ}30' E$ (ii) R.B = $180^{\circ} - 170^{\circ}12' = 89^{\circ}48' E$

(iii) R.B = $211^{\circ}54' - 180^{\circ} = S 31^{\circ}54' W$ (iv) R.B = $360^{\circ} - 327^{\circ}24' = N 32^{\circ}36' W$

* Convert the following quadrantal bearing to w.c.B.

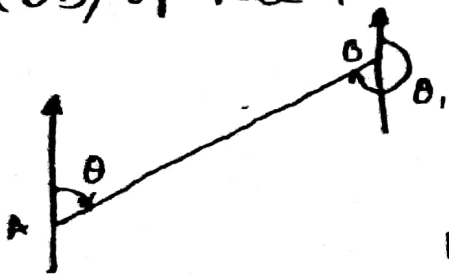
(i) $N 12^{\circ}24' E$ (ii) $S 31^{\circ}36' E$ (iii) $S 68^{\circ}6' W$ (iv) $N 5^{\circ}42' W$

Solⁿ: (i) WCB = $12^{\circ}24'$, (ii) WCB = $180^{\circ} - 31^{\circ}36' = 148^{\circ}24'$

(iii) WCB = $180^{\circ} + 68^{\circ}6' = 248^{\circ}6'$ (iv) WCB = $360^{\circ} - 5^{\circ}42' = 354^{\circ}18'$

* Fore and Back Bearing: The bearing of a line measured in the direction of the progress of survey is called the 'fore bearing' (FB) of the line.

The bearing of a line measured in the direction opposite to the survey is called back bearing (BB) of the line.



$\theta = \text{FB of AB line}$

$\theta_1 = \text{BB of AB line}$

$$BB = FB \pm 180^{\circ}$$

(+) \rightarrow When FB is less than 180°

(-) \rightarrow When FB is more than 180°

* Find BB for the following fore bearings.

(i) $12^{\circ}24'$ (ii) $119^{\circ}48'$ (iii) $266^{\circ}30'$ (iv) $354^{\circ}18'$

(v) $N 18^{\circ} E$ (vi) $S 12^{\circ}24' E$ (vii) $S 59^{\circ}18' W$ (viii) $N 86^{\circ}12' W$

Solⁿ: (i) $BB = 12^{\circ}24' + 180^{\circ} = 192^{\circ}24'$ (ii) $BB = 119^{\circ}48' + 180^{\circ} = 299^{\circ}48'$

(iii) $BB = 266^{\circ}30' - 180^{\circ} = 86^{\circ}30'$ (iv) $BB = 354^{\circ}18' - 180^{\circ} = 174^{\circ}18'$

(v) $BB = S 18^{\circ} W$

(vi) $BB = N 12^{\circ}24' W$

\downarrow
 $18^{\circ} + 180^{\circ} = 198^{\circ}$
 N 18° E SW 57° 18'

\downarrow
 N - 12° 24' start $\rightarrow (90^{\circ} + 12^{\circ}24' + 180^{\circ}) - 360^{\circ}$
 27° NW 57° 36'

~~(vii) $BB = N 59^{\circ}18' W$~~

(vii) $BB = N 59^{\circ}18' E$

(viii) $BB = S 86^{\circ}12' E$

* The distance between two points x and y measured along a sloping surface is 12.4 chains. Calculate the horizontal projected distance when the angle of inclination is $10^{\circ}30'$. Also, find the same when the elevation of x and y above mean sea level are 740 and 840 respectively.

Solⁿ: $L = L_1 \cos \theta = 12.4 \times 100 \cos(10^{\circ}30') = 1220 \text{ ft.}$ Ans

Again, $L = \sqrt{1240^2 - (840 - 740)^2} = 1237 \text{ ft}$ Ans

* Some important Conversion:

- # 1 Furlong = 660 ft.
- # 1 mile = 8 Furlongs.
- # 1 mile = 5280 ft.
- # 1 Nautical mile = 6080 ft.
- # 1 inch = 2.54 cm
- # 1 Katha = 16 Chatak
= 720 Sq. ft.
- # 1 Bigha = 20 Kathas
- # 1 Gallon = 4.546 Litre
3.78 Litre

- # 1 yard = 0.9144 meter
- # 1 Fathom = 6 ft.
- # 1 Acre = 4840 Sq. Yd.
= 43560 Sq. ft.
- # 1 Sq. mile = 640 acres.
- # 1 Ganda = 2.25 Sq. ft.
- # 1 Chatak = 20 gandas.
- # 1 ton = 2240 lbs.

* Local Attraction: A magnetic needle indicates the north direction when freely suspended. But if the needle comes near some magnetic substances, such as iron ore, steel structures, etc. it is found to be deflected from its true direction. This disturbing influence of magnetic substances is called local attraction.

* Azimuth: The true bearing of a line is known as azimuth.

* What is the angular check of closed traverse?
Ans: The sum of interior angles should be equal to $(2N-4) \times 90^\circ$, where N is the number of sides of traverse.

* The FB of a line is $96^\circ 30'$ and BB is 276° . How will you adjust the bearings?

Solⁿ: FB = $96^\circ 30'$, BB = $276^\circ 30'$
 \therefore Adjusted BB = $\frac{276^\circ 30' + 276^\circ}{2} = 276^\circ 15'$

$$\text{Adjusted FB} = 270^{\circ}15' - 180^{\circ} = 90^{\circ}15'$$

Ans

* Closing Error: In a closed traverse the end point of the last line should meet the starting point. But while plotting, it will be observed that the last line does not meet the starting point of the first line. This discrepancy is known as closing error.

* write down the balancing rule of traverse.

i) Bowditch's rule

$$\text{Correction of Latitude} = \text{Total error in latitude} \times \frac{\text{Length of side}}{\text{perimeter}}$$

$$\therefore \text{departure} = \dots \dots \dots \text{departure} \times \dots$$

ii) Graphical method.

* A surveyor measured a distance 25302 m by a faulty chain. If the actual measurement is 25360 m, then how much correction is required?

Solⁿ: True length of line, $TL = \frac{L'}{L} \times \text{Measured Length}$

$$\Rightarrow 25360 = \frac{L'}{L} \times 25302$$

$$\Rightarrow L' = 1.00229L$$

$$\therefore \text{Correction required} = \frac{1.00229L - L}{L} \times 100\%$$

$$= 0.229\% \quad \underline{\text{Ans}}$$

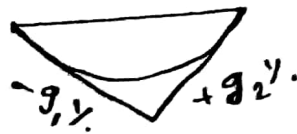
Curves & Curve Ranging

* Curve: A curve which is a circular ~~to~~ arc is required to connect two straight lengths and these must be tangential to the curve.

Curve $\left\{ \begin{array}{l} \rightarrow \text{Horizontal curve (simple, compound, Reverse, Transition curve)} \\ \rightarrow \text{vertical curve (sag, summit curve)} \end{array} \right.$



summit curve



sag curve

* Length of vertical curve, $L = \frac{g_1 - g_2}{r}$, $r \rightarrow$ rate of change of grade

* Two grades of +0.6% and -0.6% meet together, the rate of change of grade is 0.1% per 30. Find length of vertical curve.

Solⁿ:

$$\text{Length of vertical curve, } L = \frac{+0.6 - (-0.6)}{0.1/30} = 360 \text{ m.}$$

* Relationship between radius and degree of curve:

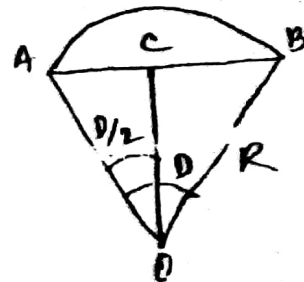
Let, AB is a unit chord, O the centre, R the radius and D the degree of curve.

$$\text{Length of AB} = 30 \text{ m}$$

$$OA = R, AC = 15 \text{ m}$$

From ΔOAC

$$\sin \frac{D}{2} = \frac{AC}{OA} = \frac{15}{R}$$



$$\Rightarrow R = \frac{15}{\sin \frac{\theta}{2}}$$

When D is very small, $\sin \frac{D}{2}$ may be taken as $\frac{D}{2}$ radians.

$$R = \frac{15}{\frac{D}{2} + \frac{\pi}{180}} = \frac{1719}{D} \quad \left[\begin{array}{l} \text{5.130} \\ \text{D} \end{array} \right] \text{ when } R \text{ in ft}$$

* Determine the degree of curve if given deflection angle of a simple curve is $25^\circ 30'$ and length of tangent 60m?

Solⁿ:

$$\text{Length of tangent} = R \tan \left(\frac{\theta}{2} \right)$$

$$\Rightarrow 60 = R \tan \left(\frac{25^\circ 30'}{2} \right)$$

$$\Rightarrow R = 265.16 \text{ m}$$

$$\text{Again, } R = \frac{1719}{D} \Rightarrow D = \frac{1719}{265.16} = 6^\circ 28' 58''$$

$$\text{* Correction for curvature} = 0.0785 D^2$$

$$\text{* Correction for curvature \& refraction} = 0.0673 D^2$$

* What is transition curve? When & why it is needed?

Ans: Transition curve: Transition curve is a curve which is provided to change the horizontal alignment from straight to circular curve in order to ensure safe running of vehicle. Its radius varies from infinity to a certain fixed value.

Objectives/why needed:

- for gradual introduction of centrifugal force.
- To introduce superelevation gradually.
- To introduce extra widening gradually.
- To provide comfort for the driver to control vehicle operation.

"Levelling"

- Army
- * Levelling: The art of determining the relative heights of different points on or below the surface of the earth is known as levelling.
 - * Datum surface or line: This is an imaginary level line from which the vertical distances of different points (above or below this line) are measured.
 - * Reduced level (RL): The vertical distance of a point above or below the datum line is known as the reduced level of that point.
 - * Line of collimation: It is an imaginary line passing through the intersection of the cross-hairs at the diaphragm and the optical centre of the object glass.
 - * Axis of the telescope: It is an imaginary line passing through the optical centre of the object glass and the optical centre of the eye-piece.
 - * Bench marks (BM): These are fixed points or marks of known RL determined with reference to the datum line.
 - * Mean sea level: It is the average elevation of the surface of the sea. In Bangladesh, the mean sea level at Cox's Bazar is taken as zero.
 - * Geoid: The surface of the earth at mean sea level is termed as Geoid.
 - * Diaphragm: It is a glass ring consisting of cross-hairs which is fitted in the telescope tube.

* Backsight reading (BS): This is the first staff reading taken in any set up of the instrument after the levelling has been perfectly done.

* Foresight reading (FS): It is the last staff reading in any set up of the instrument.

* Change point: This point indicates the shifting of the instrument.

* Parallax: The apparent movement of the image relative to the cross-hairs is known as parallax. This occurs due to imperfect focussing, when the image does not fall in the plane of the diaphragm.

1/10/20 - 14
The parallax may be tested by moving the eye up and down. If the focussing is not correct, the image moves up and down relative to the cross-hairs. The parallax may be eliminated by proper focussing the telescope.

* The following consecutive readings were taken with a dumpy level along a chain line at a common interval of 15 m. The first reading was at a chainage of 165 m where the RL is 98.065. The instrument was shifted after 4th & 9th reading.

3.15, 2.245, 1.125, 0.86, 3.125, 2.76, 1.835, 1.47, 1.965,
1.225, 2.39 and 3.035 m.

Make level book and find RL of all points.

| Station point | chainage | BS | IS | FS | Rise (+) | Fall (-) | RL | Remarks |
|---------------|----------|-------|-------|-------|----------|----------|---------|---------|
| | | | | | | | 98.085 | |
| 1 | 165 | 3.15 | | | | | 98.99 | |
| 2 | 180 | | 2.245 | | 0.905 | | 100.11 | |
| 3 | 195 | | 1.125 | | 1.12 | | 100.375 | C.P. |
| 4 | 210 | 3.125 | | 0.86 | 0.265 | | 100.74 | |
| 5 | 225 | | 2.76 | | 0.365 | | 101.665 | |
| 6 | 240 | | 1.835 | | 0.925 | | 102.03 | |
| 7 | 255 | | 1.470 | | 0.365 | | 101.535 | C.P. |
| 8 | 270 | 1.225 | | 1.965 | | 0.495 | 100.37 | |
| 9 | 285 | | 2.39 | | | 1.165 | 99.725 | |
| 10 | 300 | | | 3.035 | | 0.645 | | |
| | | 7.5 | | 5.86 | 3.945 | 2.305 | | |

Rise & Fall determination:

$$\begin{aligned}
 & 3.15 - 2.245 = 0.905, \quad 3.15 - 0.905 - 1.125 = 1.12, \quad 3.15 - 0.905 - 1.12 \\
 & - 0.86 = 0.265, \quad 3.125 - 2.76 = 0.365, \quad 3.125 - 0.365 - 1.835 = 0.925, \\
 & 3.125 - 0.925 - 0.925 - 1.47 = 0.365, \quad 3.125 - 0.365 - 0.925 - 0.365 - \\
 & 1.965 = -0.495, \quad 1.225 - 2.39 = -1.165, \quad 1.225 + 1.165 - 3.035 \\
 & = -0.645
 \end{aligned}$$

* Rise -> उन्नत (-), * Fall -> उन्नत (+).

Check: $\Sigma BS - \Sigma FS = 7.5 - 5.86 = +1.64$

$$\Sigma Rise - \Sigma Fall = 3.945 - 2.305 = +1.64$$

$$\text{last RL} - \text{1st RL} = 99.725 - 98.085 = +1.64$$

OK

* Rise -> data उन्नत BS > FS

* Rise indicate उन्नत RL - उन्नत उन्नत.

* Fall " " " " RL - उन्नत उन्नत.

* Calculate the missing data and apply boundary

Check.

| Station | BS | IS | FS | Rise (+) | Fall (-) | RL | Remarks |
|---------|--------|-------|-------|----------|----------|---------|---------|
| | | | | | | 249.26 | BM |
| 1 | 3.25 | | | | | | CP |
| 2 | 1.755 | | 4.0 | | 0.75 | 248.51 | |
| 3 | | 1.95 | | | 0.195 | 248.315 | CP |
| 4 | 3.84 | | 1.92 | 0.03 | | 248.945 | |
| 5 | | 2.34 | | 1.5 | | 249.845 | |
| 6 | | | | 1.0 | | 250.845 | |
| 7 | 1.85 | | 2.185 | | 0.845 | 250 | CP |
| 8 | | 1.575 | | 0.275 | | 250.275 | |
| 9 | | 3.545 | | | 1.97 | 248.305 | |
| 10 | 2.1 | | 1.895 | 1.65 | | 249.955 | CP |
| 11 | | | 1.35 | 0.75 | | 250.705 | |
| | 12.795 | | 11.95 | 5.205 | 3.76 | | |

$4 = 3.25 + 0.75$, $-0.195 = 1.755 - 1.95$, $0.03 = 1.755 +$
 $0.195 - 1.92$, $3.84 = 2.34 + 1.5$, $-0.845 = 3.84 - 1.5 - 1 - 2.185$,
 $0.275 = 1.85 - 1.575$, $-1.97 = 1.85 - 0.275 - 1.895 - 1.65$,
 $3.545 = 1.85 - 0.275 + 1.97$, $2.1 = 1.35 + 0.75$

Check: $\sum BS - \sum FS = +1.445$

$\sum Rise - \sum Fall = +1.445$

last RL - 1st RL = +1.445

OK

* The following observations were made during the testing of a dumpy level.

| Instrument at | Staff reading at | |
|---------------|------------------|-------|
| | A | B |
| A | 1.725 | 2.245 |
| B | 2.145 | 3.045 |

Distance between A & B = 250m

RL of A = 450m

RPGCL-17

- Is the instrument in adjustment?
- What should be the staff reading on A during the second set up of the instrument for the line of collimation to be exactly horizontal?
- To what reading should the line of collimation be adjusted when the instrument is at B?
- What should be the RL of B?

Solⁿ: When the instrument is at A:

$$\text{Apparent difference of level between AB} = (2.245 - 1.725) = 0.52 \text{ m (fall from A to B)}$$

When the instrument is at B:

$$\text{Apparent difference of level} = (3.045 - 2.145) = 0.9 \text{ m (rise from B to A)}$$

a) since the two apparent differences of level are not equal, the line of collimation is not in adjustment.

$$\text{True difference of level} = \frac{.52 + .9}{2} = 0.71 \text{ m}$$

b) In the second set up,

$$\text{True reading on B} = 3.045 \text{ m (As level is set up near B)}$$

$$\text{True reading on A} = 3.045 - .71 = 2.335 \text{ m}$$

$$\begin{aligned}
 \text{c) Collimation error} &= \text{observed reading at A} - \text{true reading at A} \\
 &= 2.145 - 2.335 \\
 &= -0.19 \text{ m}
 \end{aligned}$$

Error is negative, correction should be positive

$$\begin{aligned}
 \text{d) RL of B} &= \text{RL of A} - \text{true difference} \\
 &= 450 - 0.71 \\
 &= 449.29 \text{ m} \quad \text{Ans.}
 \end{aligned}$$

* In a two-peg test of a dumpy level, the following readings were taken -

| Instrument at | Staff reading on | |
|-----------------------|------------------|-------|
| | A | B |
| Mid-way between A & B | 1.585 | 1.225 |
| | 1.425 | 1.15 |

Distance between the pegs A & B is 100 m.

- With the instrument at A, what should be the staff reading on B for line of collimation to be exactly horizontal?
- Is the line of collimation inclined upwards/downwards?
- What is the amount of collimation error?

Solⁿ: When the instrument is mid-way between A & B:

$$\text{True difference of level} = 1.585 - 1.225$$

$$= 0.36 \text{ m (rise from A to B)}$$

a) When the instrument is at A:

$$\text{Correct staff reading on A} = 1.425 \text{ m (level near A)}$$

$$\text{correct staff reading on B} = 1.425 - 0.36$$

$$= 1.065 \text{ m}$$

- Since the observed staff reading on B is greater than the correct staff reading, the line of collimation is inclined upwards.

$$\begin{aligned}
 c) \text{ collimation error} &= \text{observed reading} - \text{correct reading} \\
 &= 1.15 - 1.065 \\
 &= + 0.085 \text{ m}
 \end{aligned}$$

\therefore Hence, error is positive, so, the correction should be negative.

* Instrument station staff reading

| | A | B |
|---|------|------|
| e | 6.72 | 5.86 |
| D | 4.96 | 4.18 |

If RL of B is 108.92 ft, calculate RL of A.

solⁿ:

True difference of level between A & B

$$\begin{aligned}
 &= \frac{(6.72 - 5.86) + (4.96 - 4.18)}{2} \\
 &= 0.82 \text{ ft}
 \end{aligned}$$

$$\therefore \text{RL of A} = 108.92 - 0.82 = 108.10 \text{ ft}$$

Ans

Contouring / Volume Computation

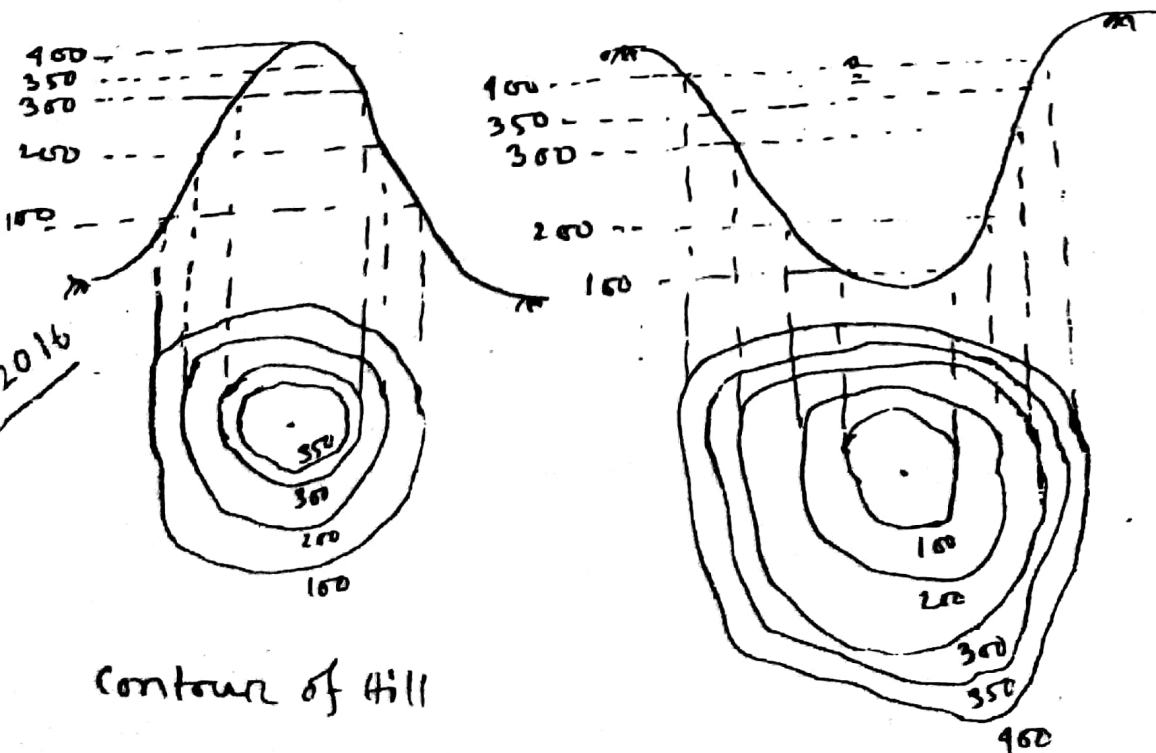
* Contour line: The line passing through points of equal reduced level (RL) is called contour line.

* Contour interval: The vertical distance between any two consecutive contours is called contour interval.

* uses of contour map:

- The nature of the ground surface of a country can be understood by contour map.
- A suitable site can be selected for any engineering project.
- The capacity of reservoir can be approximately computed.
- Quantities of earth work can be approximately computed.

* Draw contour for pond and hill. What are the basic difference betⁿ them?



Contour of Hill

Contour of pond

Barric difference: The contour lines are closer near the top and wide apart near the foot. On the other hand, the contour lines are closer near the bank and wide apart towards the centre.

* Volume Computation Rule:

Trapezoidal Rule:

$$\text{Volume, } V = \frac{D}{2} \{ A_1 + A_n + 2(A_2 + A_3 + \dots + A_{n-1}) \}$$

D = common distance between section.

Prismoidal Formula:

$$\text{Volume, } V = \frac{D}{3} \{ A_1 + A_n + 4(\text{sum of even section } 2, 4, 6, \dots) + 2(\text{sum of odd section } 3, 5, 7, \dots) \}$$

* Calculate the capacity of the proposed reservoir from the following data.

| | | | | | | | |
|---------------------------|------|-------|-------|--------|--------|--------|---------|
| Contour - | 305 | 310 | 315 | 320 | 325 | 330 | 335 |
| Area (ft ²) - | 2000 | 30000 | 80000 | 170000 | 280000 | 490000 | 1070000 |

Solⁿ:

Volume by trapezoidal rule, $V = \frac{5}{2} \left\{ 2000 + 1070000 + 2(30000 + 80000 + 170000 + 280000 + 490000) \right\} = 7930000 \text{ ft}^3$

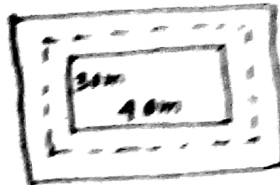
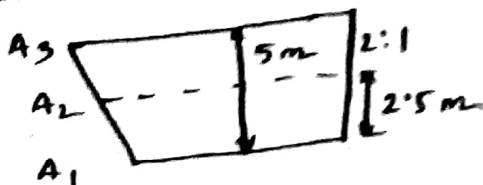
Contour interval.

By prismoidal rule, $V = \frac{5}{3} \left\{ 2000 + 1070000 + 4(30000 + 170000 + 490000) + 2(80000 + 280000) \right\} = 7586667 \text{ ft}^3$

Ans.

* An excavation is to be made for a reservoir 40 m long and 30 m wide at the bottom. The side slope of the excavation has to be 2:1. Calculate the volume of earth work if the depth of excavation is 5 m. Assume level ground at the site.

Solⁿ:



Bottom section: $L = 40 \text{ m}$, $B = 30 \text{ m}$, $A_1 = 40 \times 30 = 1200 \text{ m}^2$

Mid section: $L = b + 2sh = 40 + 2 \times 2 \times 2.5 = 50 \text{ m}$

$$B = 30 + 2 \times 2 \times 2.5 = 40 \text{ m}$$

$$A_2 = 50 \times 40 = 2000 \text{ m}^2$$

Top section: $L = b + 2sh = 40 + 2 \times 2 \times 5 = 60 \text{ m}$

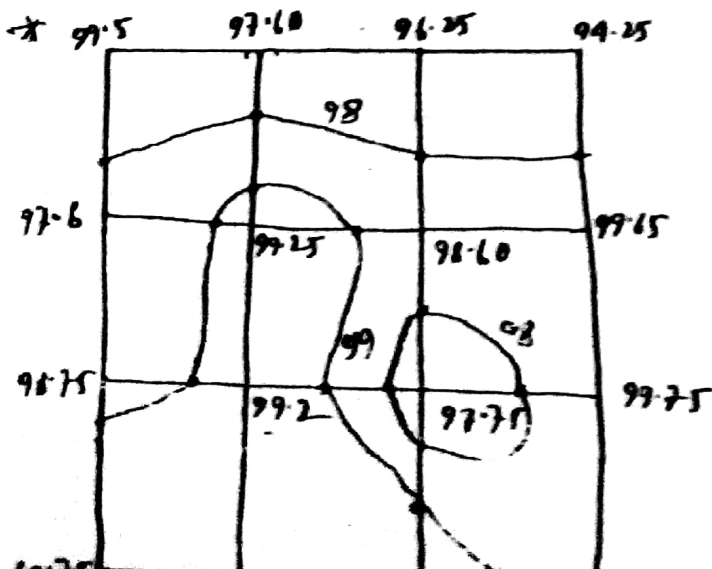
$$B = 30 + 2sh = 30 + 2 \times 2 \times 5 = 50 \text{ m}$$

$$A_3 = 60 \times 50 = 3000 \text{ m}^2$$

$$\text{vol}^m (\text{prismoidal}) = \frac{d}{3} (A_1 + A_3 + 4A_2)$$

$$= \frac{2.5}{3} (1200 + 3000 + 4 \times 2000)$$

$$= 10166.66 \text{ m}^3 \quad \underline{\text{Ans}}$$



"Computation of Area"

* Average ordinate rule:

$$\text{Area} = \frac{O_1 + O_2 + \dots + O_n}{n+1} \times L$$

where,

n = no. of division

$n+1$ = no. of ordinates.

L = length of base line.

* Trapezoidal Rule:

$$\text{Area} = \frac{d}{2} \{ O_1 + O_n + 2(O_2 + O_3 + \dots + O_{n-1}) \}$$

* Simpson's Rule:

$$\text{Area} = \frac{d}{3} \{ O_1 + O_n + 4(O_2 + O_4 + \dots) + 2(O_3 + O_5 + \dots) \}$$

* Comparison between Trapezoidal and Simpson's Rule.

| Trapezoidal Rule | Simpson's Rule |
|--|---|
| 1. The boundary between the ordinates is straight. | 1. parabola. |
| 2. It can be applied for any number of ordinates. | 2. The number of ordinates must be odd. |
| 3. It gives approximate result. | 3. More accurate result. |

* The following perpendicular offset were taken at 10m interval from a survey line to an irregular boundary line; 3.25, 5.60, 4.20, 6.65, 8.75, 6.20, 3.25, 4.20, 5.65. Calculate the area by (a) avg. ordinate rule (b) trapezoidal rule (c) Simpson's rule.

$n = 8$ (no. of division)

Solⁿ:

(a) Area = $\frac{\sum O}{n+1} \times L$

$$= \frac{3.25 + 5.60 + 4.20 + 6.65 + 8.75 + 6.20 + 3.25 + 4.20 + 5.65}{8+1} \times (10 \times 8)$$

$$= 421.94 \text{ m}^2$$

$$(b) \text{ Area} = \frac{10}{2} \{ 3.25 + 5.65 + 2(5.60 + 4.2 + 6.65 + 8.75 + 6.2 + 3.25 + 4.2) \}$$

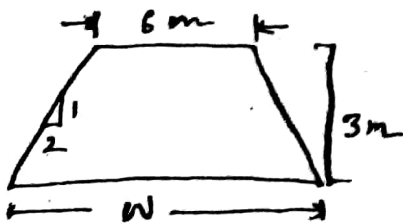
$$= 433 \text{ m}^2$$

$$(c) \text{ Area} = \frac{10}{3} \{ 3.25 + 5.65 + 4(5.60 + 6.65 + 6.20 + 4.2) + 2(4.2 + 8.75 + 3.25) \}$$

$$= 439.67 \text{ m}^2$$

Ans

* Determine the quantity of earthwork for 500m of embankment having the section below.



solⁿ: Here, $W = 6 + 3 \times \frac{2}{1} = 18 \text{ m}$

$$\therefore A = \frac{1}{2} (6 + 18) \times 3 = 36 \text{ m}^2$$

$$\therefore \text{vol}^m \text{ of earthwork} = 36 \times 500 = 18000 \text{ m}^3$$

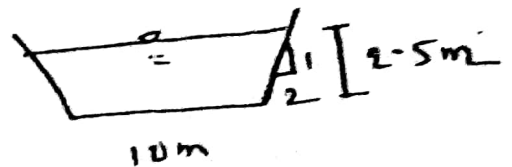
* Calculate the sectional area of an embankment $\frac{4m}{10m}$ wide, side slope 2:1, ground level is transverse direction to the centre line, central height of embankment is 2.5m.

solⁿ:

$$A = bh + sh^2$$

$$= 10 \times 2.5 + 2 \times 2.5^2$$

$$= 37.5 \text{ m}^2$$



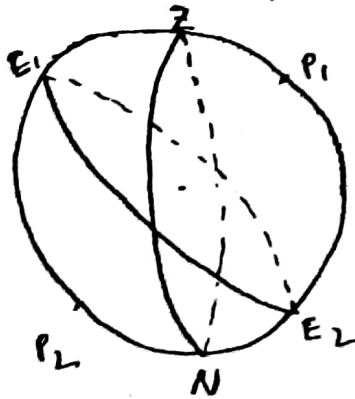
'Astronomical surveying'

Astronomical survey may be defined as that branch of surveying in which the meridian, azimuth, latitude, longitude, time, etc. of a place on the surface of the earth are determined by observation of some heavenly bodies.

* Celestial sphere: The imaginary sphere on which the stars appear to lie or to be studded is known as celestial sphere.

* Zenith: Zenith is the point on the celestial sphere above the surveyor's station. Denoted by Z.

* Nadir: Nadir is the point which lies on the celestial sphere vertically below the surveyor's station. It is denoted by N.



P - Pole
E - Equator
Z - Zenith
N - Nadir

Fig: Celestial Sphere.

* Comparison between Zenith & Nadir.

Ans: From definition above.

* An observer standing on the deck of a ship just sees a light-house. The top of the light house is 42 metres above the sea level and the height of the observer's eye is 6 metres above the sea level. Find the distance of the observer from light-house.

$$\begin{aligned} \text{Sol}^n: \quad d_1 &= 3.8553 \sqrt{42} & \left| \begin{array}{l} 42 = 42 \text{ m} \\ 6 = 6 \text{ m} \end{array} \right. \\ &= 24.985 \text{ km} \\ d_2 &= 3.8553 \sqrt{6} \\ &= 9.444 \text{ km} \end{aligned}$$

$$\begin{aligned} \therefore \text{Distance between observer \& light-house} \\ &= d_1 + d_2 \\ &= 34.429 \text{ km} \quad \underline{\text{Ans.}} \end{aligned}$$

Photogrammetry/photographic surveying

* Photogrammetry: It is a method of surveying in which plans or maps are prepared from photographs taken from suitable camera stations.

* Purposes:

- preparation of topographic maps.
- preparation of composite pictures of the ground.
- soil classification.
- Interpretation of geology.

* Types:

- Terrestrial / Ground photogrammetry.
- Aerial photogrammetry.
 - ↳ done by special camera.
- ↳ done by phototheodolite.

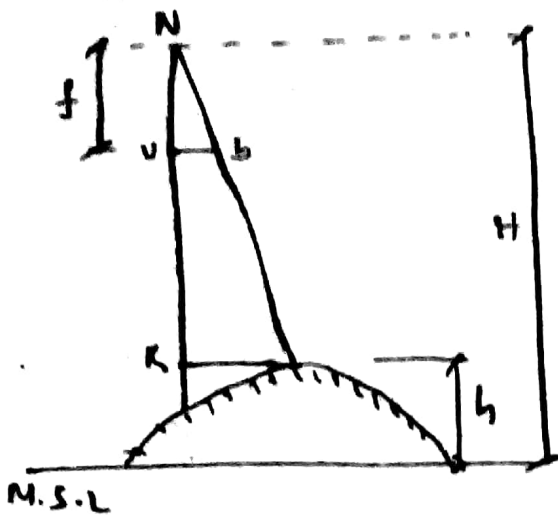
* vertical photograph: A photograph taken with the optical axis coinciding with the direction of gravity.

* Tilted photograph: A photograph taken with the optical axis unintentionally tilted from the vertical by a small amount (30°). It is called tilted photograph.

* Focal length: The distance from the front nodal point of the lens to the plane of the photograph is called focal length. Denoted by f .

* Flying Height: Distance between air station and mean sea level is flying height. Denoted by H .

* Scale of a vertical photograph:



From $\triangle NUV$ to $\triangle NKv$

$$\frac{vb}{Kv} = \frac{NV}{NK} = \frac{f}{H-h}$$

$$\Rightarrow s = \frac{f}{H-h} \Rightarrow \text{print scale}$$

$$\text{if } h=0, s = \frac{f}{H} \Rightarrow \text{datum scale.}$$

* A hilly area was photographed from a constant flying height of 3000m above MSL, $f = 150\text{ mm}$ and axis is truly vertical. Calculate the smallest and largest scale if the ground height varies from sea level to a height of 750m.

solⁿ: $H = 3000\text{ m}$, $f = 0.15\text{ m}$, $h = 0$, $h = 750\text{ m}$

Largest scale:

$$s = \frac{f}{H-h} = \frac{0.15}{3000-750} = 1:15000$$

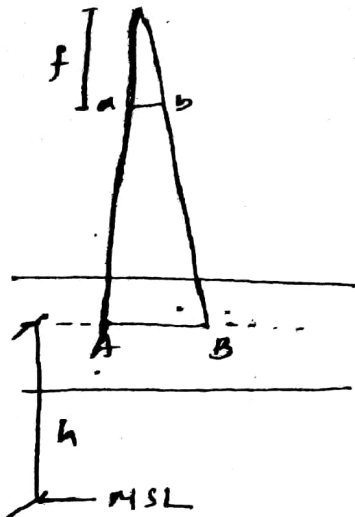
Smallest scale:

$$s = \frac{f}{H} = \frac{0.15}{3000} = 1:20000$$

Ans

* A straight length of highway AB appears to be 12.5 cm on a vertical air photograph of 15 cm focal length. Corresponding distance of the AB highway on a 1:30,000 topographic map is 6.5 cm. Assume the average elevation of the terrain as 1300m above MSL. Calculate the flying height of camera above MSL.

solⁿ: $f = 15\text{ cm}$, $ab = 12.5\text{ cm}$, $h = 1300\text{ m}$, $H = ?$



now,

$$\frac{\text{photo scale}}{\text{photo distance}} = \frac{\text{Map scale}}{\text{Map distance}}$$

$$\Rightarrow \frac{s}{ab} = \frac{1}{30000 \times 6.5}$$

$$\Rightarrow \frac{s}{12.5} = \frac{1}{30000 \times 6.5}$$

$$\Rightarrow s = 1: 5000$$

$$\therefore s = \frac{f}{H-h} \Rightarrow H = \frac{f}{s} + h$$

$$\Rightarrow H = \frac{15 \times 5000}{1} + 1300$$

$$= 76300 \text{ m} \quad \underline{\text{Ans}}$$

* Name five modern surveying equipments.

Ans: 1) EDM (Electro-Magnetic Distance Measurement)

2) Digital theodolites.

3) Digital levels.

4) GPS (Global Positioning System)

5) Total station equipment.

* A vertical photograph was taken at an altitude of 1200 metres above MSL. Determine the scale of the photograph for terrain lying at elevations of 80m and 300m i.f the focal length of the camera is 15 cm.

Solⁿ: For $h = 80 \text{ m}$, then scale, $s = \frac{f}{H-h}$

$$= \frac{15}{(1200 - 80) \times 100} = \frac{1}{7467}$$

For $h = 300 \text{ m}$, $s = \frac{15}{(1200 - 300) \times 100} = \frac{1}{6000}$

Ans

* A camera having focal length of 20 cm is used to take a vertical photograph to a terrain having an average elevation of 1500 m. What is the height above sea level at which an air craft must fly in order to get the scale of 1:8000.

Ans:
$$s = \frac{f}{H-h}$$

$$\Rightarrow \frac{1}{8000} = \frac{\frac{20}{100}}{H-1500} \Rightarrow H = 3100 \text{ m above MSL.}$$

Ans.

* A line AB 2000 m long lying at an elevation of 500 m measures 8.65 cm on a vertical photograph for which focal length is 20 cm. Determine the scale of the photograph in an area the average elevation of which is about 800 m.

Solⁿ:

we know,

$$s = \frac{f}{H-h} \Rightarrow \frac{8.65 \text{ cm}}{2000 \text{ m}} = \frac{20 \text{ cm}}{(H-500) \text{ m}}$$

$$\Rightarrow H = 5124 \text{ m.}$$

$$\text{Again, } s = \frac{f}{H-h} = \frac{20/100}{5124-800} = \frac{1}{21620}$$

Ans

* The distance from the principal point to an image on a photograph is 6.44 cm and the elevation of the object above the datum (sea level) is 250 m. What is the relief displacement of the point if the datum scale is 1/10000 and focal length is 20 cm?

Solⁿ: For datum scale,

$$s = \frac{f}{H}$$

$$\Rightarrow H = \frac{20}{\frac{1}{10000}} \Rightarrow H = 20000 \text{ m above MSL}$$

$$\therefore \text{relief displacement, } d = \frac{r h}{H}$$

$$= \frac{6.44 \times 250}{2000} = 0.805 \text{ cm}$$

Ans

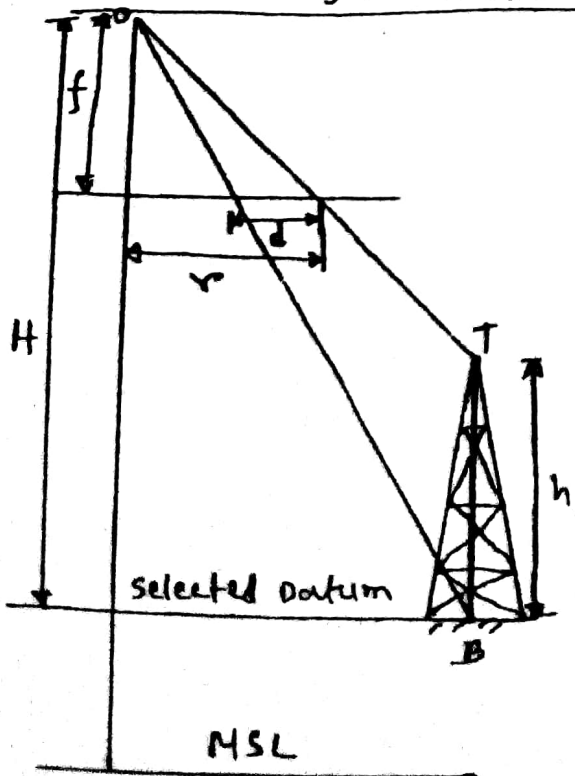
* A tower TB, 50m high, appears in a vertical photograph taken at a flight altitude of 2500m above MSL. The distance of the image of the top of the tower is 6.35 cm. Compute the displacement of the image of the top of the tower with respect to the image of its bottom. The elevation of the bottom of the tower is 1250 m.

Solⁿ: $H = \text{height of lens above the bottom of tower}$
 $= 2500 - 1250 = 1250 \text{ m.}$

$h = \text{height of tower above base} = 50 \text{ m}$

$$\therefore d = \frac{r h}{H} = \frac{6.35 \times 50}{1250} = 0.25 \text{ cm.}$$

Equations for relief displacement:



Datum scale, $S = \frac{f}{H}$

where, $H = \text{Height of lens above the bottom of tower.}$

\therefore relief displacement,

$$d = \frac{r h}{H}$$

where, $h = \text{Height of tower above base.}$

* A vertical photograph of a flat area having an avg. elevation of 250m above mean sea level was taken with a camera having a focal length of 20cm. A section line AB 250m long in the area, measures 8.50cm on the photograph. A tower TB in the area also appears on the photograph. The distance between the images of top and bottom of the tower measures 0.46cm on the photograph. The distance of the image of the top of the tower is 6.46cm. Find height of tower.

Solⁿ:

$$S = \frac{f}{H} = \frac{\text{Map distance}}{\text{Ground distance}}$$

$$\Rightarrow \frac{20}{H} = \frac{8.50}{250} \Rightarrow H = 588.2 \text{ m}$$

$$\therefore \text{height of tower, } h = \frac{dH}{r} = \frac{0.46 \times 588.2}{6.46}$$

$$= 41.89 \text{ m} \quad \underline{\underline{\text{Ans}}}$$

No. of photographs to cover an area:

1) If total area is given,

$$\text{NO. of photograph, } N = \frac{A}{a}$$

where, a = Ground area covered by each photograph

A = Total area covered by photograph.

$$a = L \times W, \quad L = (1 - P_1) S_l$$

$$W = (1 - P_2) S_w$$

where,

L = length of photograph in direction of flight

W = width of photograph normal to the direction of flight.

$S = \text{Scale of photograph} = \frac{H(m)}{f(\text{cm})}$ ($1\text{cm} = 5\text{metre}$)

$L = \text{Net ground distance corresponding to } l.$

$W = \text{Net ground distance corresponding to } w.$

$P_L = \text{percentage overlap between successive photograph.}$

$P_S = \text{side lap.}$

3) If instead of total area A , length and width of the ground area given —

$L_1 = \text{Dimension of the area parallel to flight direction.}$

$L_2 = \text{ " " " " Normal to " " }$

$N_1 = \text{No of photographs in each strip.}$

$N_2 = \text{No. of strip required.}$

$N = \text{Total No. of photograph.}$

$$N_1 = \frac{L_1}{L} + 1 = \frac{L_1}{(1-P_L)sl} + 1$$

$$N_2 = \frac{L_2}{W} + 1 = \frac{L_2}{(1-P_S)sw} + 1$$

~~$N = N_1 + N_2$~~ $N = N_1 \times N_2$

* The scale of an aerial photograph is $1\text{cm} = 100\text{m}$.

The photograph size is $20\text{cm} \times 20\text{cm}$. Determine the

no. of photographs required to cover an area of

100km^2 if the longitudinal lap is 60% and the side

lap is 30%.

Solⁿ; $l = 20\text{cm}, w = 20\text{cm}, P_L = 0.6, P_S = 0.3$

$$S = \frac{H}{f} = 100$$

$$L = (1 - P_L) SL \Rightarrow L = (1 - 0.6) * 100 * 20 = 800 \text{ m} \\ = 0.8 \text{ km}$$

$$W = (1 - P_S) SW \Rightarrow W = (1 - 0.3) * 100 * 20 = 1400 \text{ m} \\ = 1.4 \text{ km}$$

$$\therefore a = L \times W = 0.8 \times 1.4 = 1.12 \text{ km}^2$$

$$\therefore N = \frac{A}{a} = \frac{100}{1.12} = 90 \quad \underline{\text{Ans.}}$$

* The scale of an aerial photograph is 1 cm = 100 m. The photograph size is 20 cm x 20 cm. Determine the number of photographs required to cover an area 10 km x 10 km if the longitudinal lap is 60% and the side lap is 30%.

Solⁿ: $L_1 = 10 \text{ km}$, $L_2 = 10 \text{ km}$, $P_L = 0.6$, $P_S = 0.3$

$$N_1 = \frac{L_1}{(1 - P_L) SL} + 1 = \frac{10 \times 10^3}{(1 - 0.6) \times 100 \times 20} + 1 \approx 14$$

$$N_2 = \frac{L_2}{(1 - P_S) SW} + 1 = \frac{10 \times 10^3}{(1 - 0.3) \times 100 \times 20} + 1 \approx 9$$

$$N = N_1 * N_2 \\ = 14 * 9 = 126$$

Ans.

* The scale of an aerial photograph is $1\text{cm} = 100\text{m}$.
 The photograph size is $20\text{cm} \times 20\text{cm}$. Determine the
 no. of photographs required to cover an area of
 $8\text{ km} \times 12.5\text{ km}$ if the longitudinal lap is 60% & the
 side lap is 30% .

$$\text{Sol}^n: N_1 = \frac{12.5 \times 10^3}{(1 - 0.6) \times 100 \times 20} + 1 \approx 17$$

$$N_2 = \frac{8 \times 10^3}{(1 - 0.3) \times 100 \times 20} + 1 = 7$$

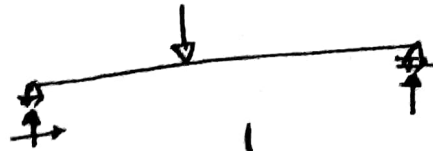
$$\therefore N = N_1 \times N_2 = 17 \times 7 = 119 \quad \underline{\text{Ans.}}$$

"Mechanics of Solid"

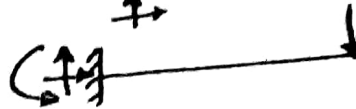
* Beam: A beam is a structural member which carries transverse loads. The supporting forces of the beams are called reactions.

* Types of beam:

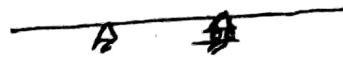
1. Simply supported beam



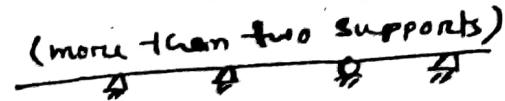
2. Cantilever beam



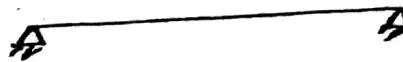
3. Over hanging beam



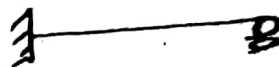
4. Continuous beam



5. Fixed beam



6. propped beam



* The fixed and propped beam have at least one more reactive elements than is absolutely necessary to support them.

* Types of load:

1) Concentrated / point load.

2) Uniformly Distributed load (UDL)

3) Uniformly varying load (UVL)

* More loads for structural Analysis.

- Dead load

- Live load

- Wind load.

- Earthquake load.

" Shear Force & Bending Moment "

* Shear/Shear Force: The shear force is the algebraic sum of all the external forces acting parallel to the section on either side of the section.

Sign:

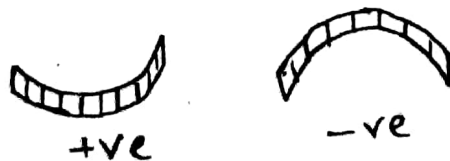


* Moment Ordinary Bending: Bending produced by forces acting on the beam is called ordinary bending.

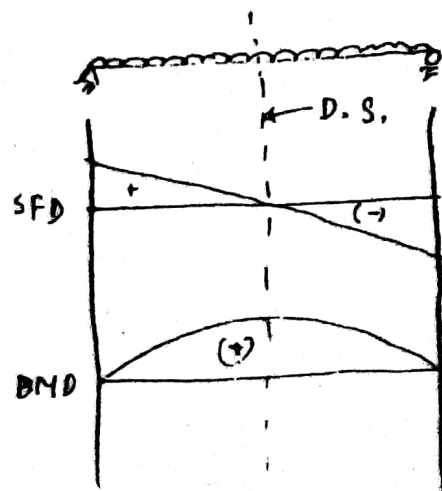
* pure Bending/moment: If couples are applied to the ends of the beam and no forces act on the bar, then the bending is termed as pure bending.

* Bending Moment: The bending moment at a section of a beam is the algebraic sum of the moments produced by all the external forces on one side of the section.

Sign:

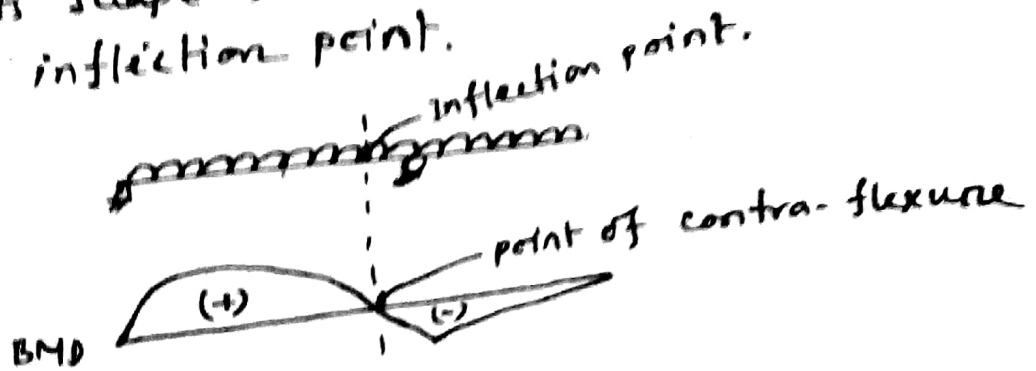


* Dangerous section: The section of zero shear and max^m bending moment where the bending stress ($\sigma = \frac{MY}{I}$) is max^m is called dangerous section.

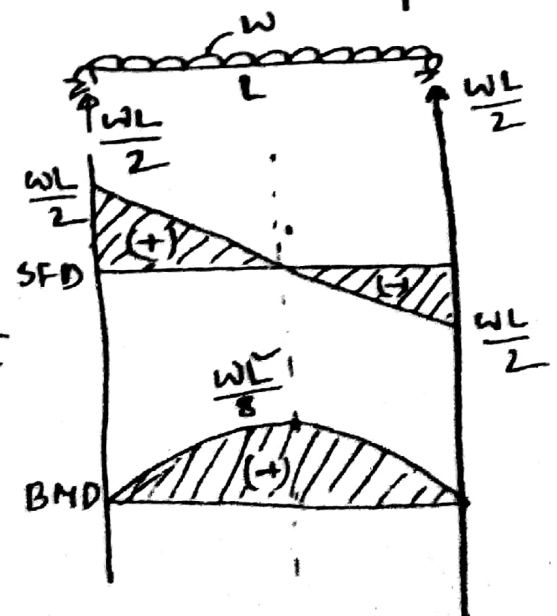
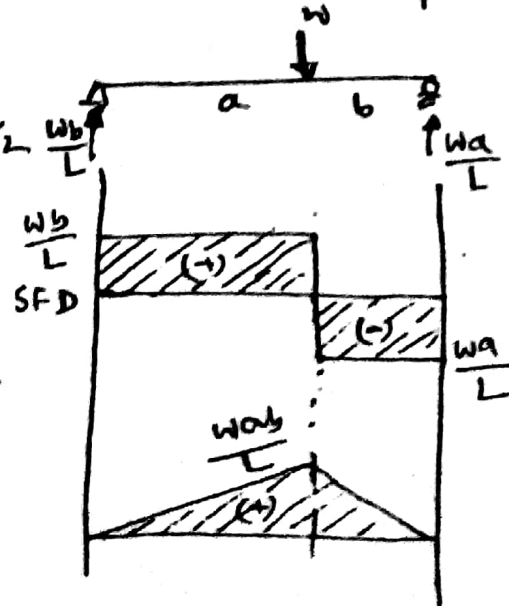
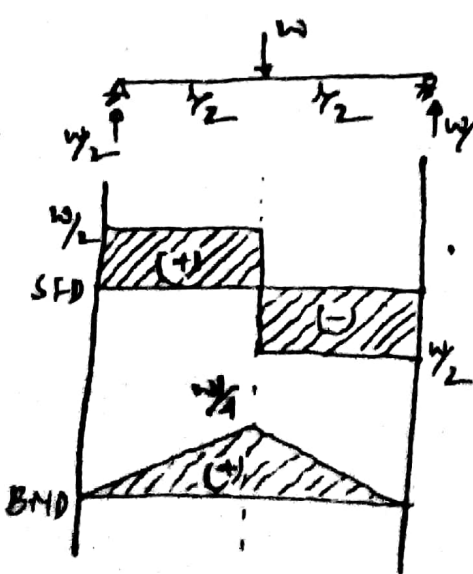
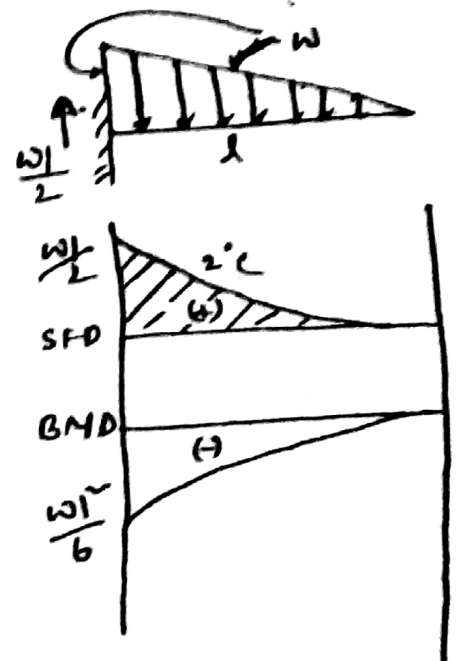
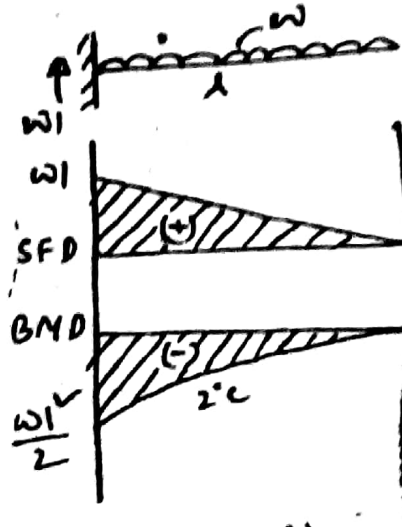
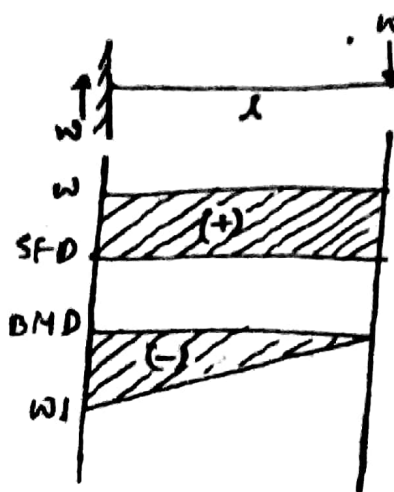


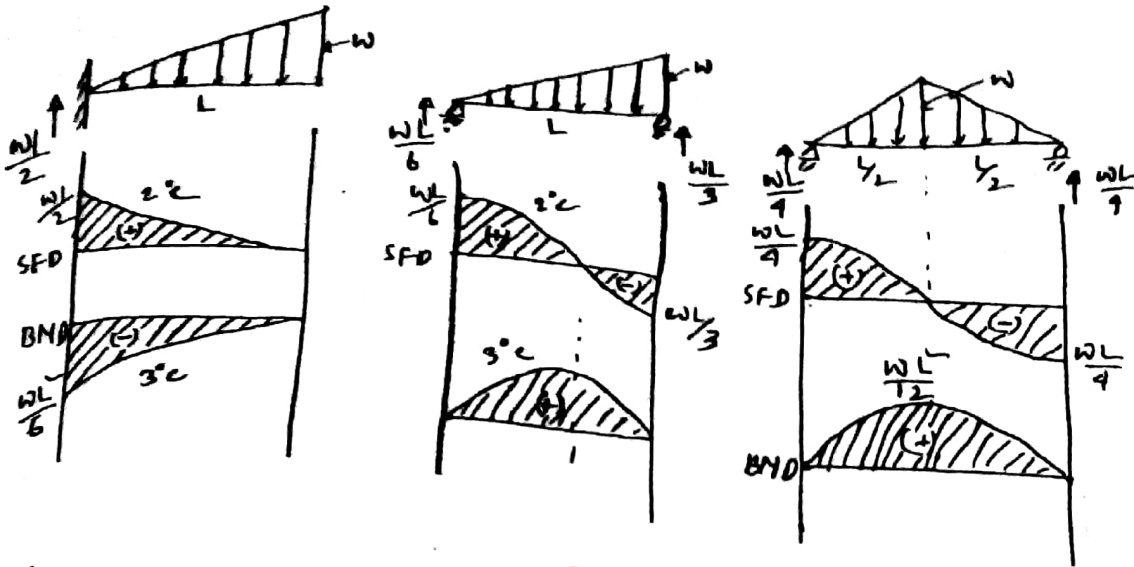
* point of contra-flexure: A point in a beam where the bending moment changes its sign from +ve to -ve or vice-versa, is called point of contra-flexure.

* Inflexion point: A point in a beam where the curvature changes its shape from concave up to concave down or vice-versa, is called inflexion point.

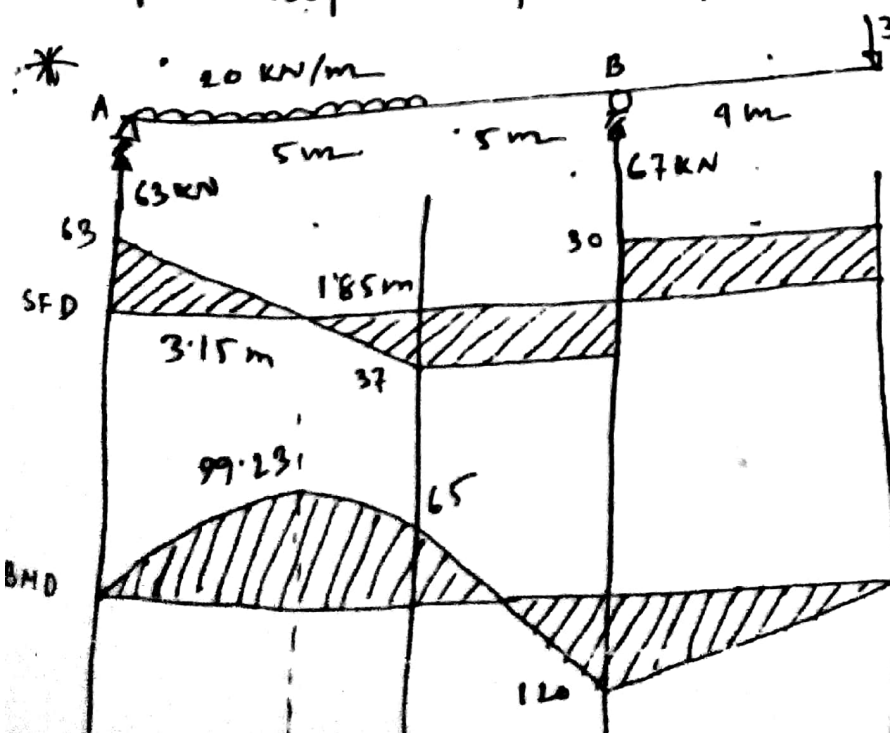
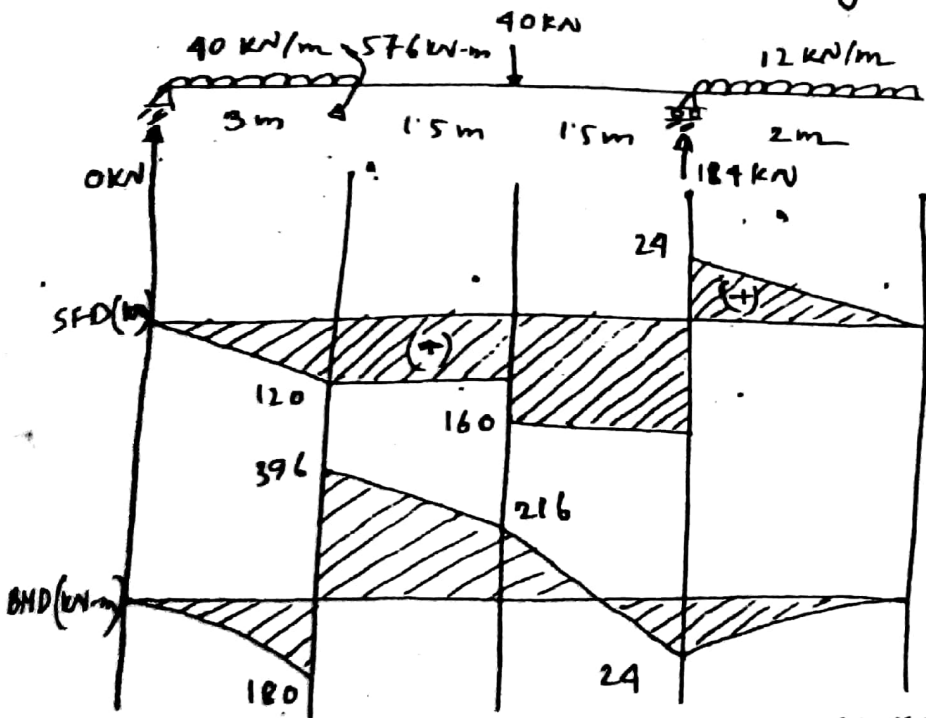


* SFD & BMD for viva.





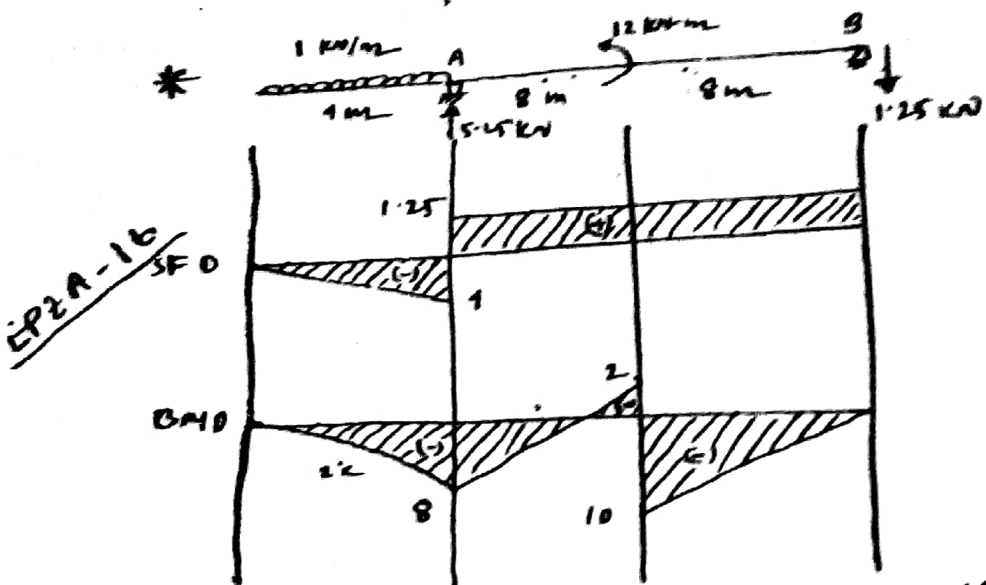
* Draw SFD & BMD of following beam.



$$R_A = \frac{100(5+2.5) - 120}{10} = 63 \text{ kN}$$

$$\frac{x}{13} = \frac{5-x}{37}$$

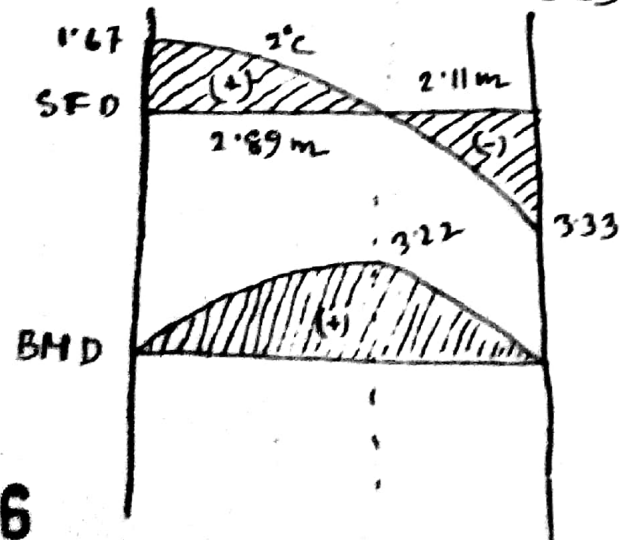
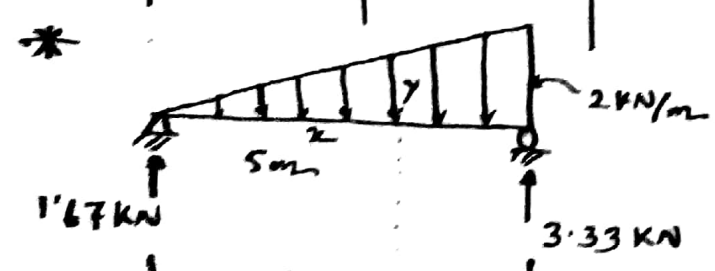
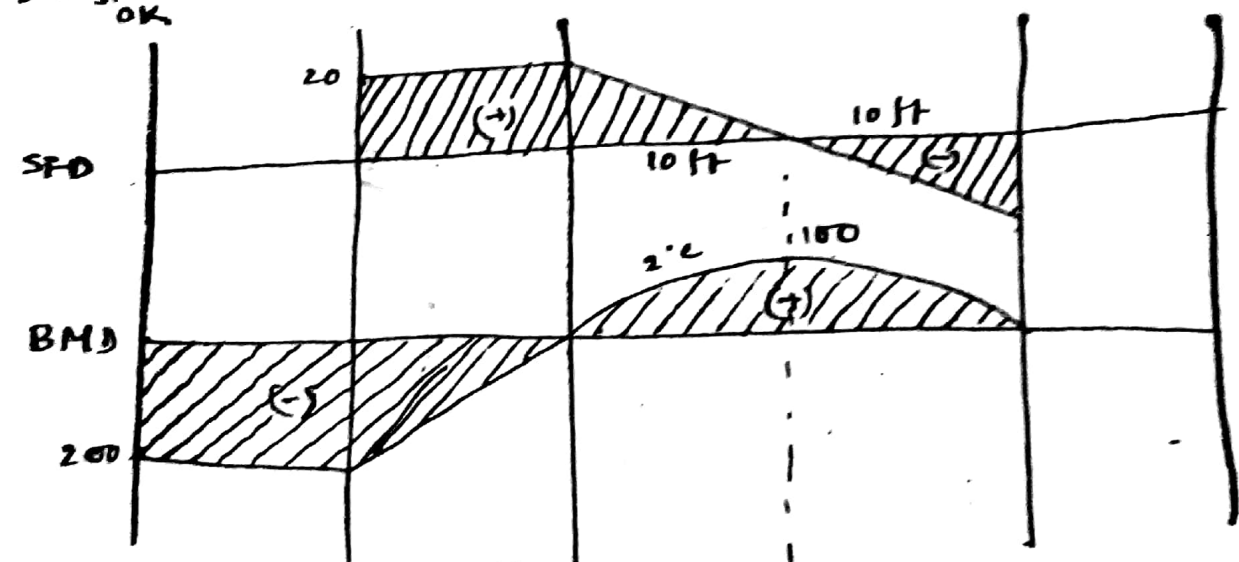
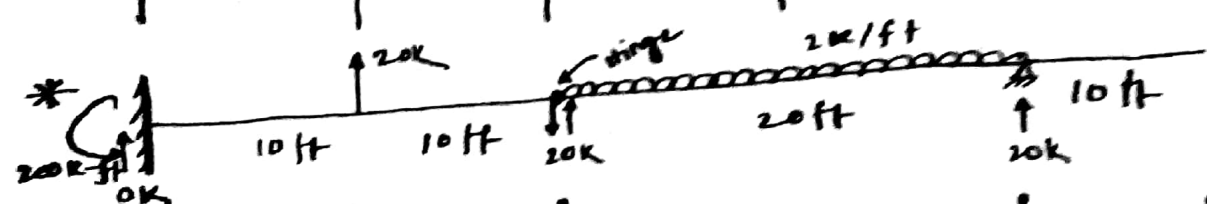
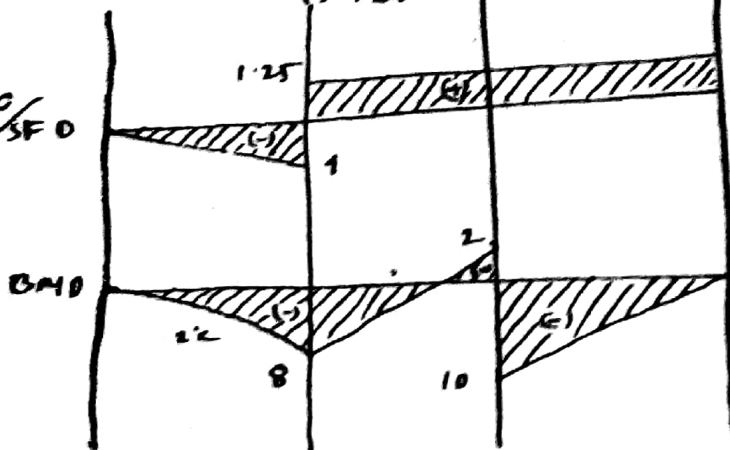
$$\Rightarrow x = 3.15 \text{ m}$$



$$R_B = \frac{-4 \times 2 - 12}{16} = -1.25 \text{ kN}$$

$$R_A = 41.25 = 5.25 \text{ kN}$$

EP2A-16
SFD



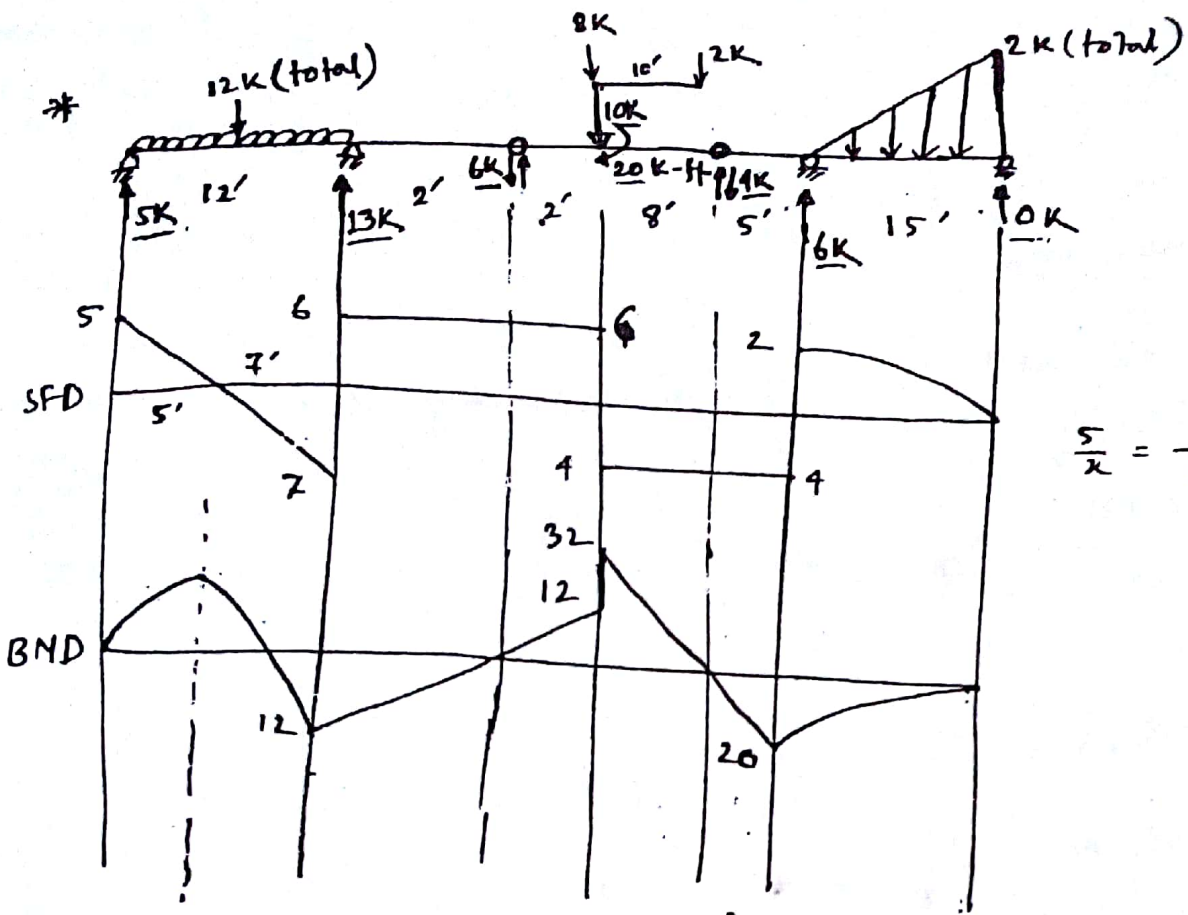
$$y = \frac{2}{5}x$$

$$\Rightarrow 1.67 - \frac{1}{2}xy = 0$$

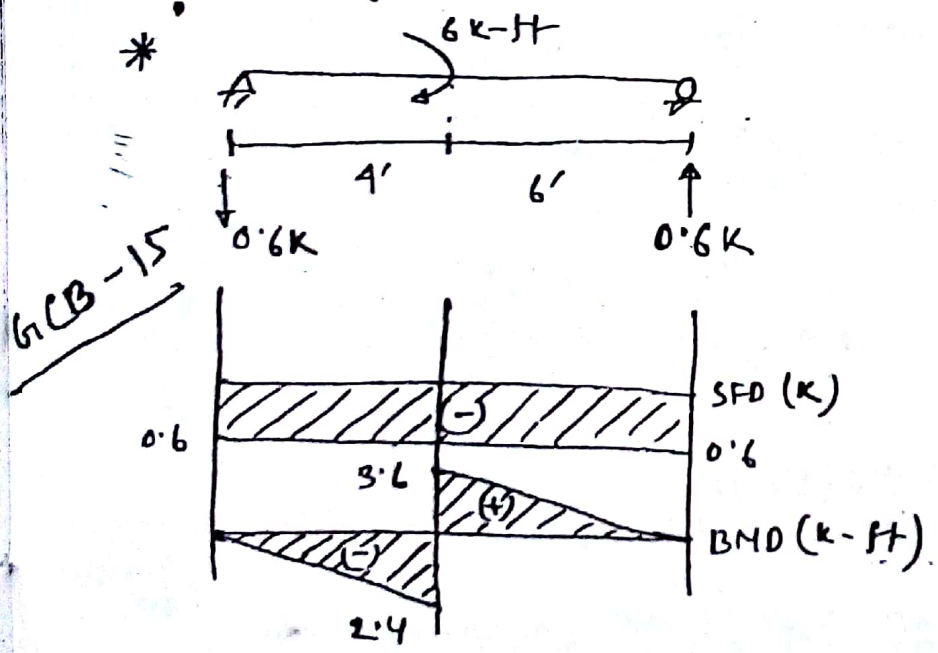
$$\Rightarrow 1.67 - \frac{1}{2}x \times \frac{2}{5}x = 0$$

$$\Rightarrow x = 2.89 \text{ m}$$

$$\therefore y =$$



$$\frac{5}{x} = \frac{7}{12-x}$$

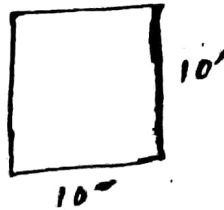
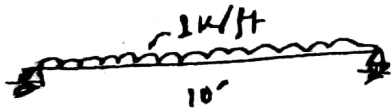


* Shear Flow:

$$q = \tau b = \frac{VQ}{Ib} \cdot b = \frac{VQ}{I}$$

↙ Shear Flow.

* find out the max^m bending and shearing stresses for the following structures.



Solⁿ:

$$\text{Max}^m \text{ shear, } V = \frac{wL}{2} = \frac{1 \times 10}{2} = 5 \text{ K}$$

$$\text{Max}^m \text{ moment, } M = \frac{wL^2}{8} = \frac{1 \times 10^2}{8} = 12.5 \text{ K-ft.}$$

$$Q = A'\bar{y} = (b \times \frac{d}{2}) \times \frac{d}{4} = 10 \times \frac{10}{2} \times \frac{10}{4} = 125 \text{ in}^3$$

$$I = \frac{10 \times 10^3}{12} = 833.33 \text{ in}^4, \quad b = 10''$$

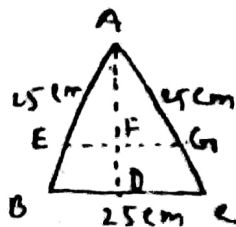
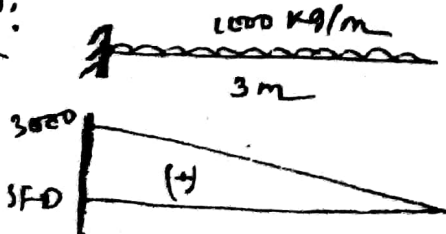
$$e = \frac{10}{2} = 5''$$

$$\therefore \sigma_{\text{max}} = \frac{Mc}{I} = \frac{12.5 \times 12 \times 5}{833.33} = 0.9 \text{ K/in} = 130 \text{ K/ft}$$

$$\tau_{\text{max}} = \frac{VQ}{Ib} = \frac{5 \times 125}{833.33 \times 10} = 0.075 \text{ K/in} = 10.8 \text{ K/ft}$$

* Determine the maximum shear stress at 25 cm from the apex of equivalent triangle of a 3m span of a cantilever beam carrying 1000 kg/m.

Solⁿ:



$$AD = \sqrt{25^2 - 12.5^2} = 21.65 \text{ cm}$$

$$AF = \frac{2}{3} AD = 14.43 \text{ cm}$$

$$\frac{EG}{AF} = \frac{BC}{AD}$$

$$\Rightarrow EG = \frac{25}{21.65} \times 14.43 = 16.67 \text{ cm}$$

$$V = 3000 \text{ kg}, \quad b = 16.67 \text{ cm}, \quad I = \frac{25 \times 21.65^3}{36} = 7097.11 \text{ cm}^4$$

$$Q = A'\bar{Y} = \text{Area of AEG} \times \frac{1}{3} \text{ of AF}$$

$$= \frac{1}{2} \times 16.67 \times 19.93 \times \frac{1}{3} \times 19.93 = 578.51 \text{ cm}^3$$

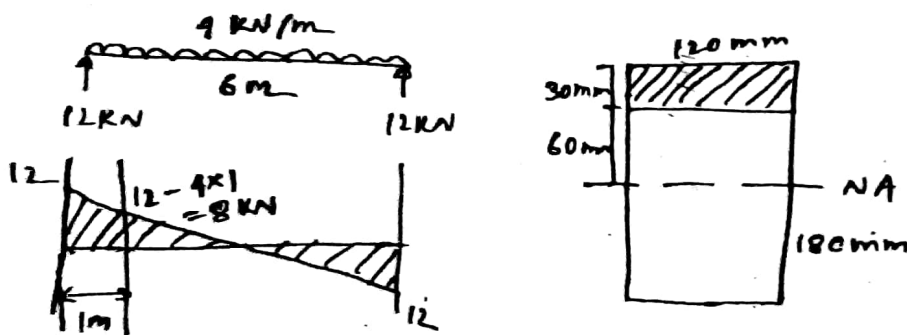
$$\therefore \tau_{\max} = \frac{VQ}{Ib} = \frac{3000 \times 578.51}{7097.11 \times 16.67} = 14.77 \text{ kg/cm}^2 \quad \underline{\text{Ans}}$$

$$\sigma_{\max} = \frac{MC}{I} = \frac{\frac{1}{2} \times 3 \times 3000 \times 19.93}{7097.11} \times 100 = 922 \text{ kg/cm}^2$$

↑
top

* Determine the shear stress at a distance 1m from left support for 30 mm from the top fiber.

Solⁿ:

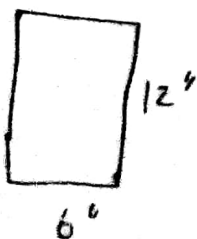


$$V = 8 \text{ kN}, \quad b = 120 \text{ mm} = 0.12 \text{ m}, \quad I = \frac{0.12 \times 0.18^3}{12} = 5.83 \times 10^{-5} \text{ m}^4$$

$$Q = A'\bar{Y} = 0.03 \times 0.12 \times (0.06 + 0.015) = 0.00027 \text{ m}^3$$

$$\therefore \tau = \frac{VQ}{Ib} = \frac{8 \times 0.00027}{5.83 \times 10^{-5} \times 0.12} = 308.75 \text{ kN/m}^2 \quad \underline{\text{Ans}}$$

* Find the max^m shear stress subjected to vertical shear force 48 kips.



Solⁿ:

$$\tau = \frac{VQ}{Ib}$$

$$= \frac{48 \times 108}{864 \times 6}$$

$$= 1 \text{ k/in}^2$$

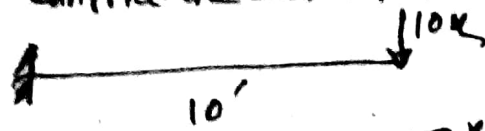
Ans

$$Q = \left(6 \times \frac{12}{2}\right) \times \frac{12}{4}$$

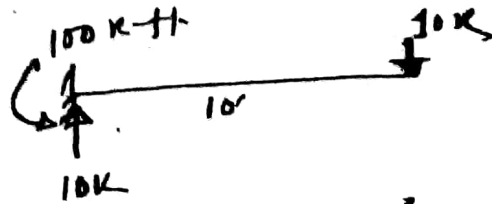
$$= 108 \text{ in}^3$$

$$I = \frac{6 \times 12^3}{12} = 864 \text{ in}^4$$

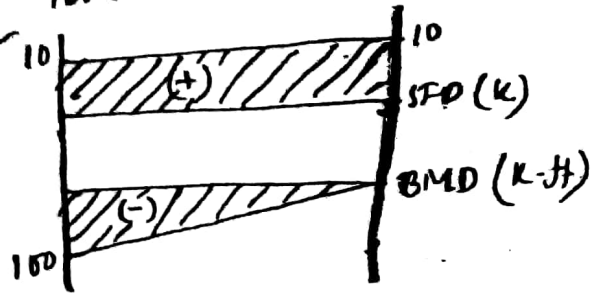
* Determine the flexure and shear stress of the following cantilever beam, if the section is 12 ft x 12 ft



Solⁿ: $M = 100 \text{ k-ft}$
 $V = 10 \text{ k}$

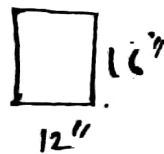
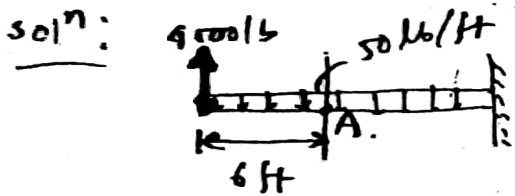


Stress $\sigma = \frac{MC}{I} = \frac{100 \times 5}{\frac{12 \times 12^3}{12}} = 600 \text{ k/ft}^2$
 Shear stress $\tau = \frac{VQ}{Ib} = \frac{10 \times (1 \times \frac{1}{2}) \times \frac{1}{4}}{\frac{12 \times 12^3}{12} \times 1} = 15 \text{ k/ft}^2$



Ans.

* A 12" by 16" wooden cantilever beam weighing 50 lb/ft carries an upward counterforce of 4000 lb at the free end. Determine max^m bending stress at a section 6 ft from the free end.



$$I = \frac{(12 \times 16^3)}{12} / 12^4 = 0.1975 \text{ ft}^4$$

$$y = 8" = 0.6667 \text{ ft}$$

$$M_A + 4000 \times 6 - 50 \times 6 \times \frac{6}{2} = 0$$

$$\Rightarrow M_A = -23100 \text{ lb-ft} = 23100 \text{ lb-ft} (\curvearrowright)$$

$$\sigma_A = \frac{My}{I} = \frac{23100 \times 0.6667}{0.1975} = 77978.58 \text{ lb/ft}^2 = 542 \text{ PSI}$$

Ans.

* A cantilever beam 3m long subjected to a uniformly distributed load of 30 kN/m. The allowable working stress in either tension/compression is 150 MPa. If the cross section is to be rectangular. Determine the dimension. Height is twice of width.

solⁿ:

$$M = \frac{wL^2}{2} = \frac{30 \times 3^2}{2} = 135 \text{ KN-m}$$

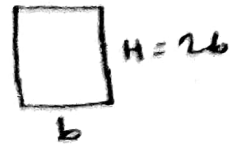
$$I = \frac{b \times h^3}{12} = \frac{2b^4}{12} = \frac{2b^4}{3}$$

$$c = \frac{2b}{2} = b$$

$$\sigma = \frac{Mc}{I}$$

$$\Rightarrow 150 \times 10^6 = \frac{135 \times 10^3 \times b}{\frac{2b^4}{3}}$$

$$\Rightarrow b = 0.1105 \text{ m} = 110.5 \text{ mm}$$



$$\therefore, b = 110.5 \text{ mm}$$

$$H = 2b$$

$$= 221 \text{ mm}$$

\therefore size 110.5 mm x 221 mm

Ans.

Stress - strain

* stress: resisting force per unit area.

$$\sigma = \frac{P}{A}$$

* Tension test of mild steel:

• Universal Testing Machine (UTM) is used.

* Engineering stress: It is the ratio of the applied force and original cross sectional area of the specimen.

* True stress: It is the ratio of the applied force and the corresponding reduced area of the cross section of the specimen.

* strain (ϵ): This is defined as the change per unit of length in a linear dimension of a body. $\epsilon = \frac{\delta}{L}$

* Deformation (δ): It is the term used to indicate the change in the form of the body.

* Elasticity: It is the property of a material by virtue of which deformations caused by stress disappear upon the removal of stress.

* plasticity: It is the property by virtue of which a material can undergo permanent deformation without rupture.

permanent set: It is defined as the amount of plastic deformation that remains even after the removal of stress.

proportional limit (σ_{pl}): It may be defined as the max^m stress that a material is capable of developing without deviation from a straight line between stress and strain. This is valid upto Hooke's law.

Elastic Limit: It is defined as the max^m stress that a material is capable of developing without a permanent set remaining upon complete release of stress.

Modulus of Elasticity (E): It is the ratio of stress to corresponding strain within the proportional limit.

Modulus of Rigidity (G): It is the ratio of stress to corresponding strain under shearing stress.

$$G = \frac{E}{2(1+\mu)}$$

Modulus of Resilience (U_R): The energy stored per unit volume at the elastic limit is the modulus of resilience. It is the area under the stress-strain diagram up to elastic limit.

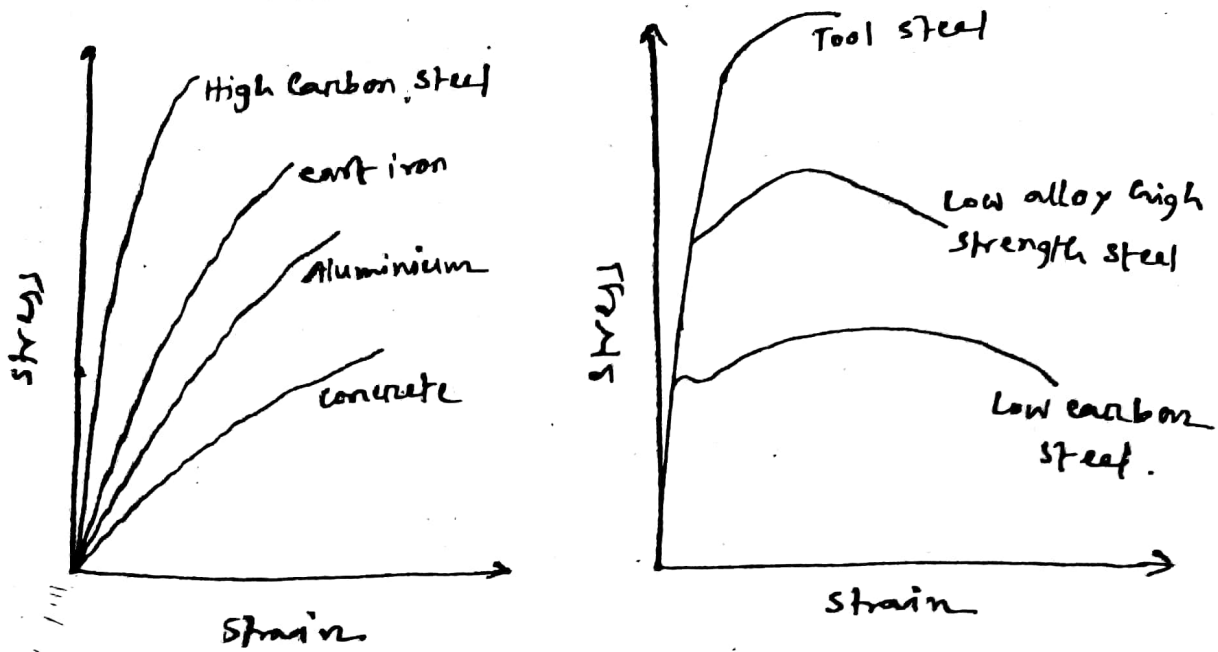
$$U_R = \frac{\sigma_{pl}^2}{2E}$$

Toughness: ability to retain energy upto fracture.

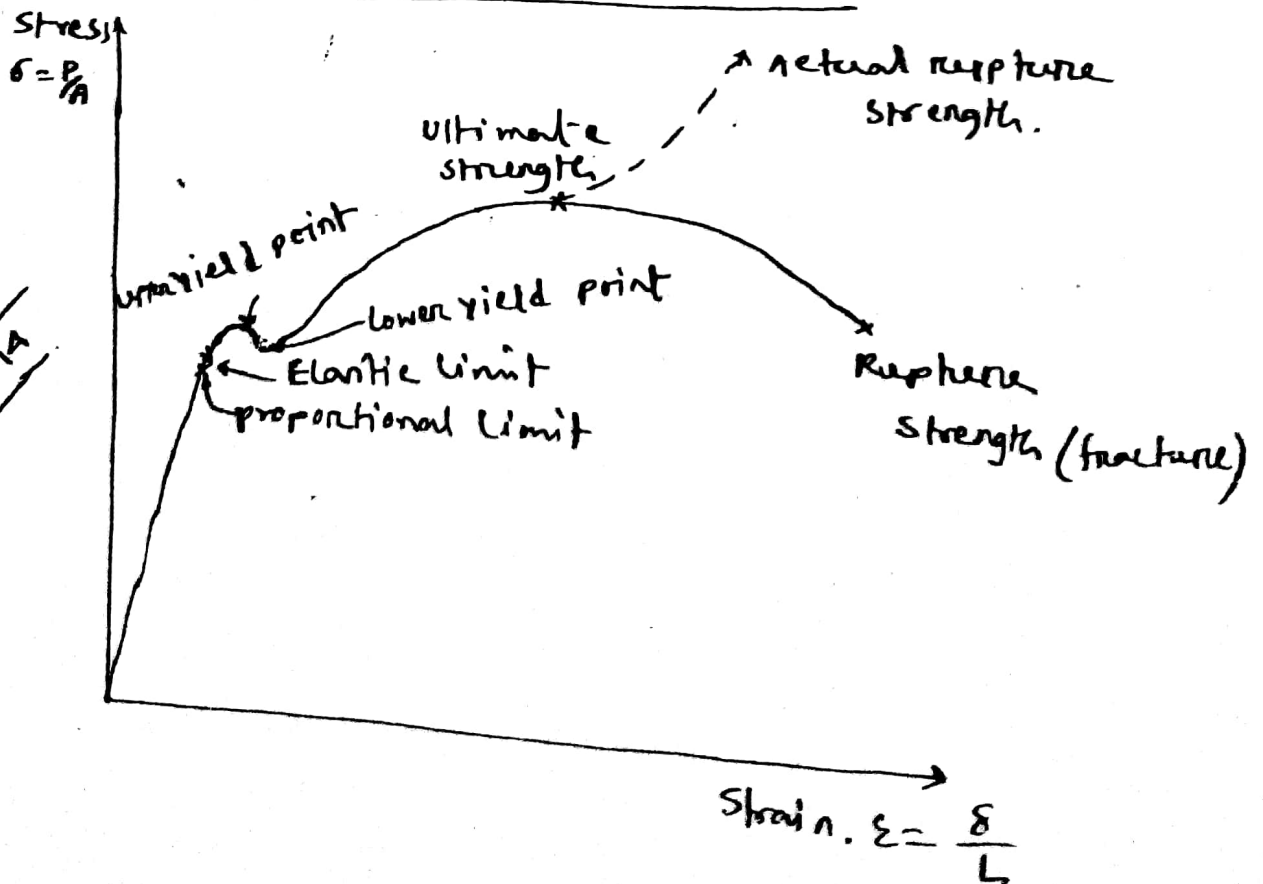
poisson ratio = $\frac{\text{lateral strain}}{\text{axial strain}}$

Yield point (σ_y): It is defined as the stress at which there occurs a marked increase in strain without increase in stress.

Comparative stress-strain diagram:



stress-strain diagram of mild steel:



Mughna-17
DWASA-14

Failure pattern of mild steel.



Cup-cone



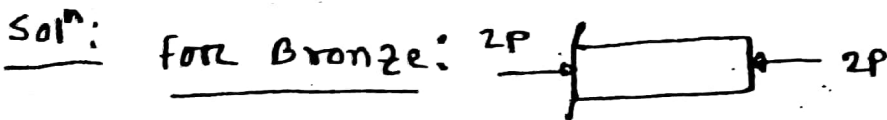
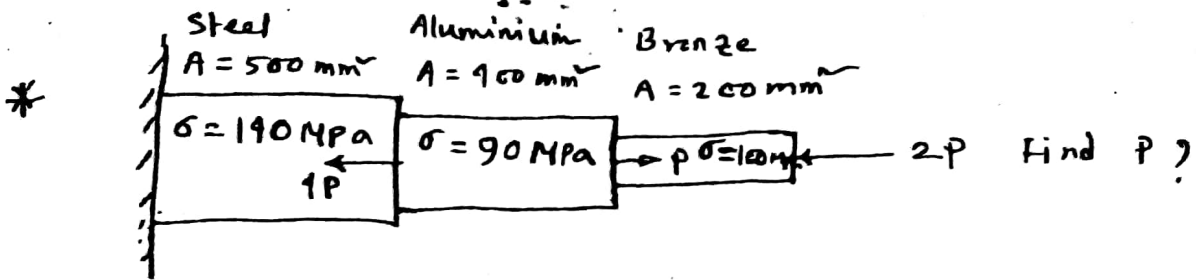
Fibrous



Shear



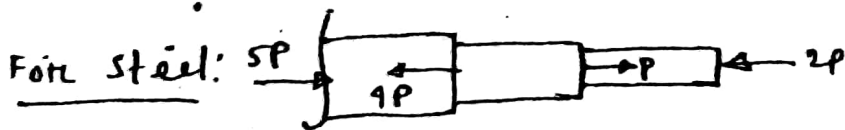
Granular.



$$\Rightarrow \sigma = \frac{2P}{A} \Rightarrow P = \frac{100 \times 200}{2} \Rightarrow P = 10000 \text{ N}$$



$$\Rightarrow \sigma = \frac{P}{A} \Rightarrow P = 90 \times 400 = 36000 \text{ N}$$



$$\Rightarrow \sigma = \frac{5P}{A} \Rightarrow P = \frac{140 \times 500}{5} \Rightarrow P = 14000 \text{ N}$$

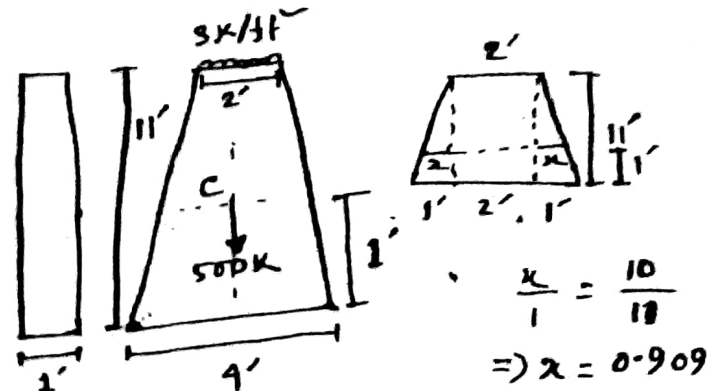
∴ The safest value of P is 10000 N. Ans

* Find out the stress at point c (1' above the base of the pile).

Solⁿ:

Length (above 1') $L = 2 \times 0.909 + 2$
 $= 3.82 \text{ ft.}$

Area (above 1') $A = 3.82 \times 1$
 $= 3.82 \text{ ft}^2$



~~Total load = 500 + 3.82 \times 9 =~~

Total load act = $500 + 3 \times (2 \times 1) = 506 \text{ kips.}$

\therefore stress at 1' above the base, $\delta = \frac{506}{3.82} = 132.46 \text{ k/ft}^2$

Ans

* Determine maximum shear stress for the following structure.

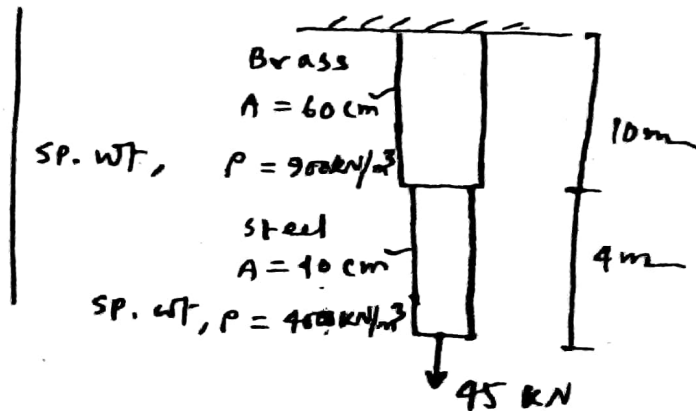
Solⁿ:

Wt. of Brass, $W_B = P A L$

$W_B = 900 \times 60 \times 10^{-4} \times 10$
 $= 54 \text{ kN}$

Wt. of steel.

$W_S = 400 \times 40 \times 10^{-4} \times 4$
 $= 64 \text{ kN}$

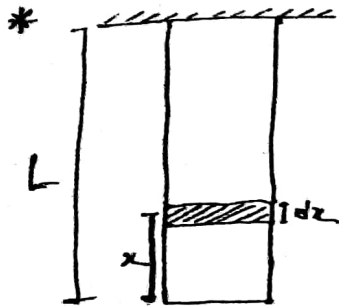


\therefore Total force = $45 + W_B + W_S = 163 \text{ kN.}$

stress on Brass = $\frac{45 + 54}{60} = 1.65 \text{ kN/cm}^2$

• steel = $\frac{45 + 64}{40} = 2.73 \text{ kN/cm}^2$

• Max^m shear stress = $1.65 + 2.73$
 $= 4.38 \text{ kN/cm}^2$



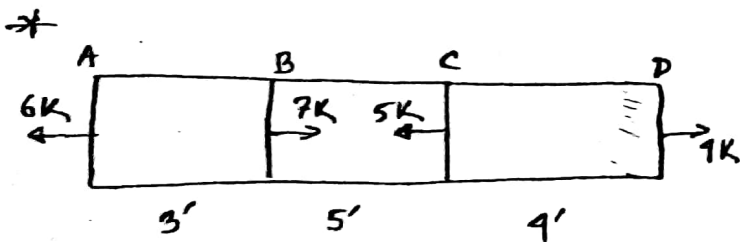
x-Sectional Area = A
 Modulus of elasticity = E
 unit weight = γ

Find out the increase in length due to its own weight.

Solⁿ: Total weight, $P = \gamma V \Rightarrow P = \gamma A x$

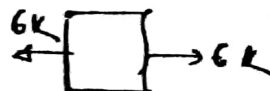
$$\therefore d\delta = \frac{PL}{AE} = \frac{\gamma A x dx}{AE} = \frac{\gamma x dx}{E}$$


$$\therefore \delta = \int_0^L \frac{\gamma x dx}{E} = \left[\frac{\gamma x^2}{2E} \right]_0^L = \frac{\gamma L^2}{2E} \quad \underline{\text{Ans}}$$

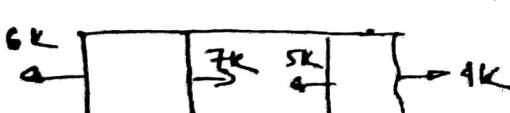


$A = 0.5 \text{ in}^2$
 $E = 10 \times 10^6 \text{ psi}$
 $\delta_{\text{total}} = ?$

Solⁿ:

$$\delta_{AB} = \frac{PL}{AE} = \frac{6 \times 10^3 \times 3 \times 12}{0.5 \times 10 \times 10^6} = +0.0132$$


$$\delta_{BC} = \frac{1 \times 10^3 \times 5 \times 12}{0.5 \times 10 \times 10^6} = -0.012$$


$$\delta_{CD} = \frac{4 \times 10^3 \times 4 \times 12}{0.5 \times 10 \times 10^6} = +0.0384$$


$$\delta_{\text{total}} = 0.0132 - 0.012 + 0.0384$$

$$= 0.0696 \text{ in}$$

Ans

* The vertical movement of a point on the neutral surface of a horizontal beam due to applied load is called the deflection of beam of that point.

Methods:

- Double integration method.
- Area-moment Method.
- Conjugate beam method
- Strain energy method.
- Method of Superposition.

Equation: (Double integration Method)

① Differential eqⁿ of elastic curve beam

$$EI \frac{d^2y}{dx^2} = M$$

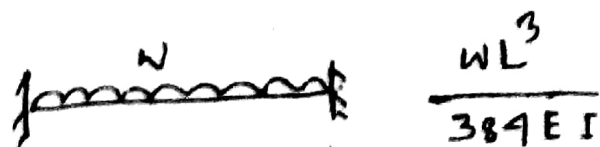
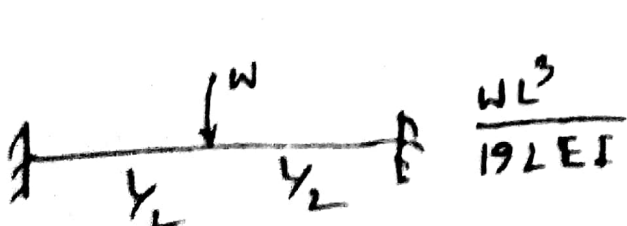
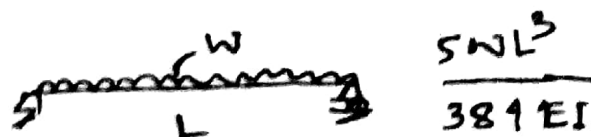
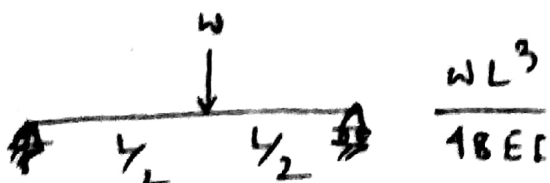
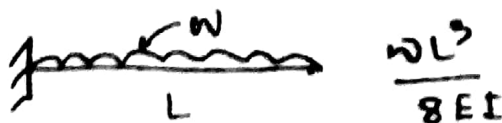
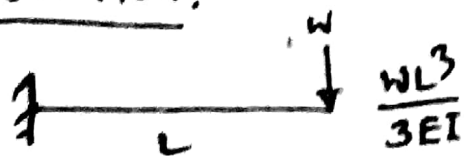
② Equation for slope (By integrating eqⁿ 1)

$$EI \frac{dy}{dx} = \int M dx + C_1$$

③ Equation for deflection (Integrating eqⁿ 2)

$$EIY = \iint M dx + C_1x + C_2$$

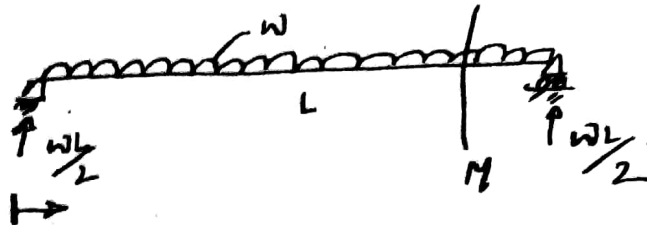
Deflection:



Double integration method:

* Find out the max^m deflection for the following structure.

Solⁿ:



$$M = \frac{wL}{2}x - \frac{wx^2}{2}$$

$$\therefore EI \frac{d^2y}{dx^2} = M = \frac{wL}{2}x - \frac{wx^2}{2} \quad \text{--- (1)}$$

$$EI \frac{dy}{dx} = \frac{wL}{2} \frac{x^2}{2} - \frac{w}{2} \frac{x^3}{3} + c_1 \quad \text{--- (2)}$$

$$EI y = \frac{wL}{4} \frac{x^3}{3} - \frac{w}{6} \frac{x^4}{4} + c_1 x + c_2 \quad \text{--- (3)}$$

At $x=0, y=0$

$\therefore c_2 = 0$

At $x=L, y=0$

$$0 = \frac{wL^4}{12} - \frac{wL^4}{24} + c_1 L$$

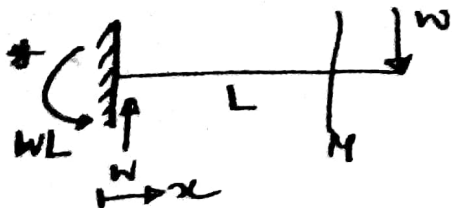
$$\Rightarrow c_1 = -\frac{wL^3}{24}$$

$$\therefore EI y = \frac{wL}{12} x^3 - \frac{w}{24} x^4 + -\frac{wL^3}{24} x$$

At $x = \frac{L}{2}, y = y_{\max}$

$$EI y_{\max} = \frac{wL^4}{96} - \frac{wL^4}{384} - \frac{wL^4}{48} = -\frac{5wL^4}{384}$$

$$\therefore y_{\max} = -\frac{5}{384} \frac{wL^4}{EI} \quad \left(- \text{ sign indicates downward deflection} \right)$$



find y_{max} .

Soln:

$$M = wx - WL$$

$$\therefore EI \frac{d^2y}{dx^2} = M = wx - WL \quad \text{--- (1)}$$

$$EI \frac{dy}{dx} = w \cdot \frac{x^2}{2} - WLx + c_1 \quad \text{--- (2)}$$

$$EI y = \frac{w}{2} \frac{x^3}{3} - WL \frac{x^2}{2} + c_1 x + c_2 \quad \text{--- (3)}$$

At $x=0, y=0$

$$c_2 = 0$$

At $x=0, \frac{dy}{dx} = 0$

$$c_1 = 0$$

$$\therefore EI y = \frac{w}{6} x^3 - \frac{WL}{2} x^2$$

At $x=L, y = y_{max}$.

$$\therefore EI y_{max} = \frac{wL^3}{6} - \frac{WL^3}{2} = -\frac{2wL^3}{6} = -\frac{wL^3}{3}$$

$$\therefore y_{max} = -\frac{wL^3}{3EI} \quad \underline{\underline{\text{Ans}}}$$

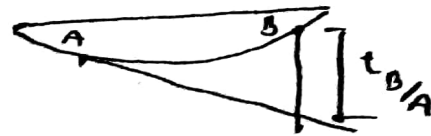
Area-Moment Method:

Theorem-1: The change in slope between two tangents at any two points on the elastic curve is equal to the area of the $\frac{M}{EI}$ diagram between these two points.

$$\theta_{(B/A)} = (\text{Area})_{AB}$$

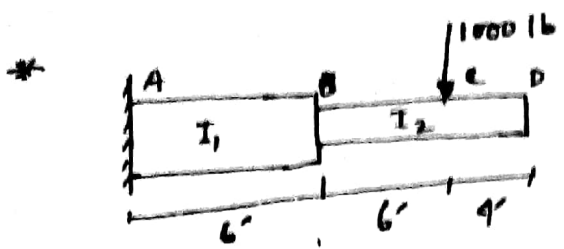
Theorem-2: The tangential variation of any point w.r. to the tangent drawn at any other point on the elastic curve is equal to the first moment of the area of the $\frac{M}{EI}$ diagram between those two points, evaluated from the former point.

$$t_{B/A} = \bar{x}_B (\text{Area})_{AB}$$



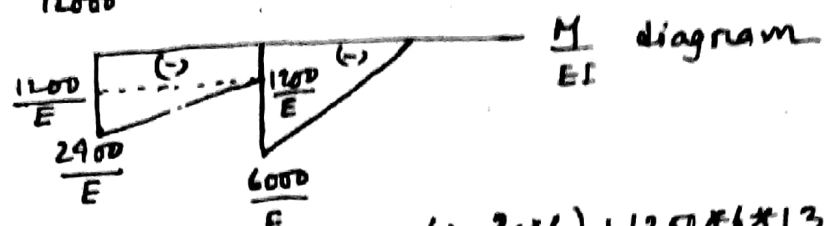
| Type of load | Cantilever beam | Moment dia | Area | \bar{x} |
|--------------|-----------------|------------|-----------------|----------------|
| Couple | | | bh | $\frac{1}{2}b$ |
| Concentrated | | | $\frac{1}{2}bh$ | $\frac{1}{3}b$ |
| U.D.L | | | $\frac{1}{3}bh$ | $\frac{1}{4}b$ |
| U.V.L | | | $\frac{1}{4}bh$ | $\frac{1}{5}b$ |

Note: Cantilever Beam पर $\frac{M}{EI}$ diagram के area जहाँ $\frac{M}{EI}$ deflection के sign है, वहाँ tangential variation के sign भी है।



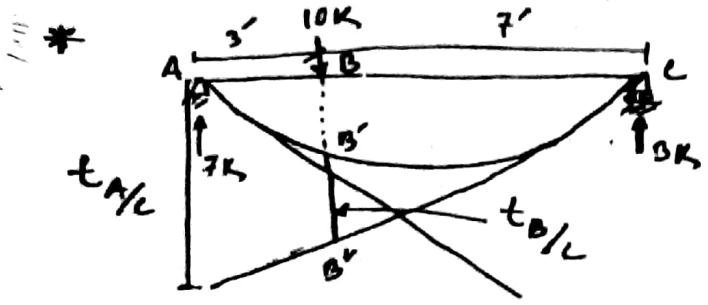
$$\begin{aligned} I_1 &= 5 \text{ in}^4 \\ I_2 &= 1 \text{ in}^4 \\ E &= 10^7 \text{ psi} \end{aligned} \quad \left| \quad \delta_D = ?$$

solⁿ:

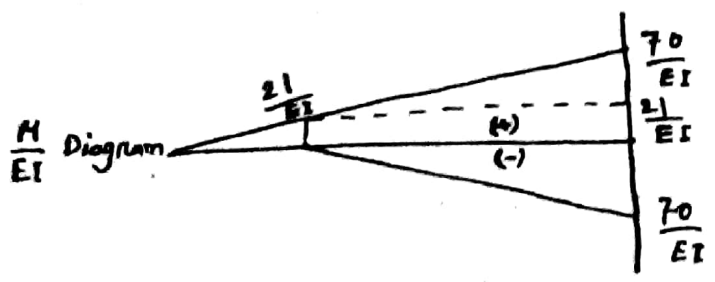


$$\delta_D = - \frac{\frac{1}{2} \times 6000 \times 6 \times (1 + \frac{2}{3} \times 6) + 1200 \times 6 \times 13 + \frac{1}{2} \times 6 \times 1200 \times (10 + \frac{2}{3} \times 6)}{E}$$

$$= -0.0288 \text{ in} \quad \underline{\text{Ans}}$$



$$\begin{aligned} \delta_B &= ? \\ \theta_A &= ? \end{aligned}$$



$$\begin{aligned} \frac{70}{10} &= \frac{y}{3} \\ \Rightarrow y &= 21 \end{aligned}$$

$$\theta_{A/C} = \frac{\frac{1}{2} \times 10 \times 70 \times \frac{2}{3} \times 10 - \frac{1}{2} \times 7 \times 70 \times (3 + \frac{1}{3} \times 7)}{EI} = \frac{455}{EI}$$

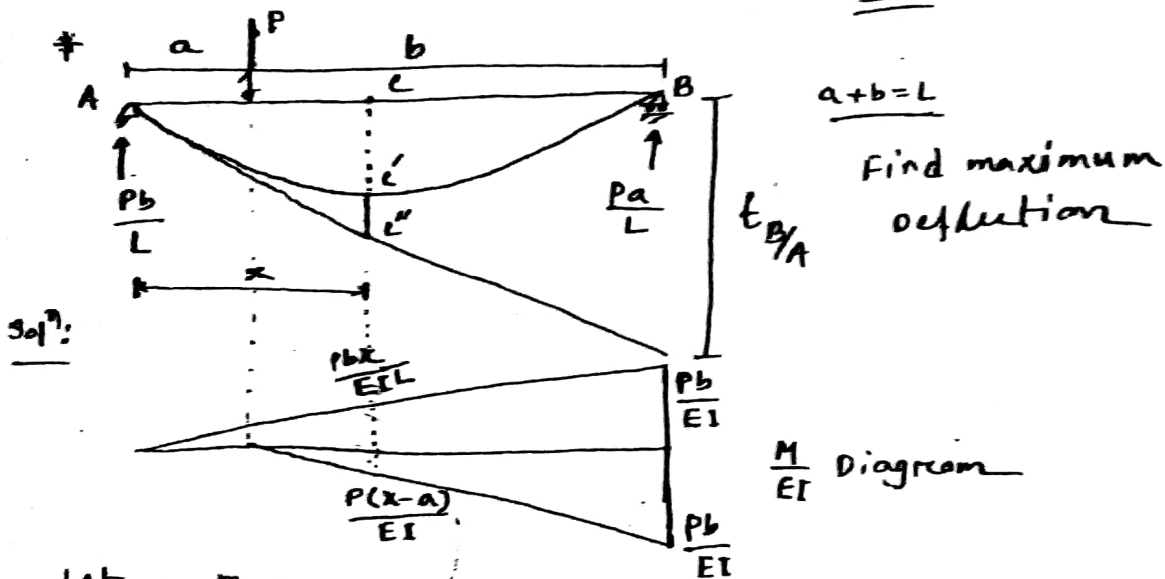
$$\begin{aligned} \theta_{B/A} = \theta_{B/C} &= \frac{\frac{1}{2} \times 7 \times 49 \times \frac{2}{3} \times 7 + 21 \times 7 \times \frac{7}{2} - \frac{1}{2} \times 7 \times 70 \times \frac{2}{3} \times 7}{EI} \\ &= \frac{171.5}{EI} \end{aligned}$$

From similar triangle -

$$\frac{t_{A/L}}{10} = \frac{BB''}{7} \Rightarrow BB'' = \frac{318.5}{EI}$$

$$\therefore \delta_B = BB'' - B'B'' = \frac{147}{EI} \quad \underline{\underline{\text{Ans}}}$$

$$\therefore \tan \theta_A = \theta_A = \frac{BB''}{3} = \frac{106.17}{EI} \quad \underline{\underline{\text{Ans}}}$$



Let max^m deflection occur at point c.

$$t_{B/A} = \frac{\frac{1}{2} * L * \frac{Pb}{EI} * \frac{1}{3} * L - \frac{1}{2} * b * \frac{Pb}{EI} * \frac{1}{3} * b}{EI}$$

$$= \frac{\frac{1}{6} P b L^2 - \frac{1}{6} P b^3}{EI}$$

$$t_{C/A} = \frac{\frac{1}{2} * x * \frac{Pbx}{L} - \frac{1}{2} * (x-a) * \frac{P(x-a)}{1}}{2EI}$$

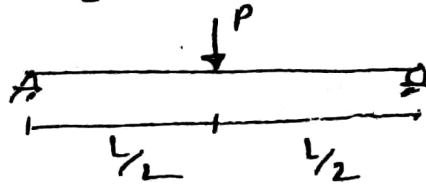
$$= \frac{\frac{Pbx^2}{L} - \frac{P(x-a)^2}{1}}{2EI} = \underline{\underline{c c''}}$$

From similar triangle

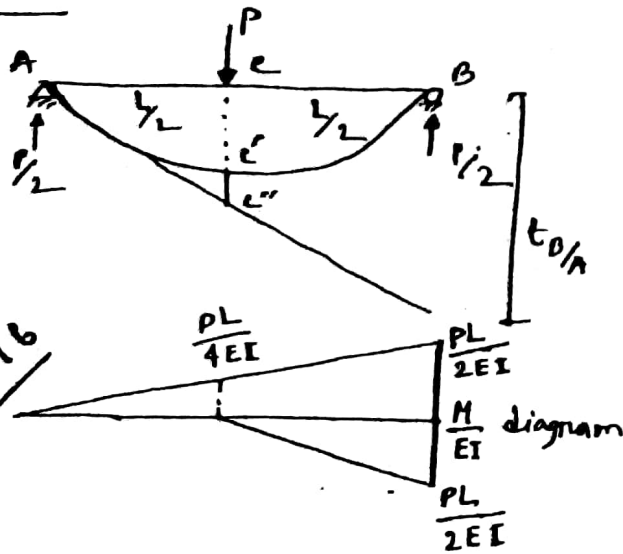
$$\frac{t_{B/A}}{L} = \frac{c c''}{x} \Rightarrow c c'' = ?$$

$$\therefore \text{Deflection at } c, c c' = c c'' - c' c'' = ?$$

* Using moment area method, determine the deflection of the following beam -



Soln:



$$t_{B/A} = \frac{\frac{1}{2} \times L \times \frac{PL}{4EI} \times \frac{L}{2} + \frac{1}{2} \times \frac{L}{2} \times \frac{PL}{4EI} \times \frac{L}{6}}{1}$$

$$= \frac{PL^3}{12EI} - \frac{PL^3}{18EI}$$

$$= \frac{3PL^3 - 2PL^3}{36EI} = \frac{PL^3}{18EI}$$

$$t_{C/A} = c'c'' = \frac{1}{2} \times \frac{L}{2} \times \frac{PL}{4EI} \times \frac{L}{6}$$

$$= \frac{PL^3}{96EI}$$

From similar triangle

$$\frac{t_{B/A}}{L} = \frac{c'c''}{L/2} \Rightarrow c'c'' = \frac{PL^3}{2 \times 18EI} = \frac{PL^3}{36EI}$$

$$\therefore \delta_c = c'c' = c'c'' - c'e''$$

$$= \frac{PL^3}{36EI} - \frac{PL^3}{96EI} = \frac{2PL^3}{96EI} = \frac{PL^3}{48EI} \text{ Ans}$$

* Euler Formula:

$$P = \frac{n^2 EI \pi^2}{L^2}$$

$$\Rightarrow P = \frac{E I \pi^2}{L^2}$$

$$n=1 \quad n=2$$

* Find the shortest length for a steel column with pinned ends having a x-sectional area of 60 by 100 mm for which the elastic Euler's formula applies. Let $E = 200 \text{ GPa}$, Assume the proportional limit to be 250 MPa .

Soln:

Here now,

$$P = \frac{n^2 EI \pi^2}{L^2}$$

$$\Rightarrow L = \sqrt{\frac{1^2 \times 200 \times 10^3 \times 1.8 \times 10^6 \pi^2}{1.5 \times 10^6}}$$

$$= 1539 \text{ m} \quad \underline{\text{Ans}}$$

$n=1$

$$P = \sigma_{pl} \times A$$

$$= 250 \times 60 \times 100$$

$$= 1.5 \times 10^6 \text{ N}$$

$$I = \frac{100 \times 60^3}{12} = 1.8 \times 10^6 \text{ mm}^4$$

$$L_e = L$$

* A $50 \times 100 \text{ mm}$ timber is used as a column with fixed ends. Determine the min^m length required at which Euler's formula can be used if $E = 10 \text{ GPa}$ and proportional limit is 30 MPa . What central/working/safe load can be carried with a F.S. of 2 if the length is 2.5 m .

Soln:

$$P = \frac{n^2 EI \pi^2}{(0.5L)^2}$$

$$\Rightarrow L = \sqrt{\frac{1^2 \times 10 \times 10^3 \times 1.04 \times 10^6 \pi^2}{0.5^2 \times 150 \times 10^3}}$$

$$= 1654 \text{ mm} \quad \underline{\text{Ans}}$$

$n=1$

$$I = \frac{100 \times 50^3}{12} = 1.04 \times 10^6 \text{ mm}^4$$

$$P = \sigma_{pl} \times A$$

$$= 30 \times 50 \times 100$$

$$= 150 \times 10^3 \text{ N}$$

$$L_e = 0.5L$$

75

$$\begin{aligned} \therefore \text{Working load, } P &= \frac{\text{Load}}{F.S} = \frac{\sqrt{EI}}{2 * (.5L)^3} \\ &= \frac{\sqrt{10 * 10^3 * 1.04 * 10^6}}{2 * (.5 * 2500)^3} = 32896 \text{ N} \end{aligned} \quad \underline{\text{Ans}}$$