Strength of materials

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RUET.14

Strength of Materials

Stress: Stress is defined as the strength of a material per unit area or unit strength. Mathematically stress may be defines as the force per unit area.

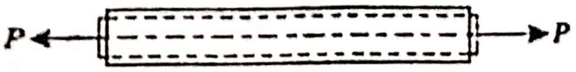
$$\sigma = \frac{P}{A}$$

$P =$ Load or force acting on the body
$A =$ Cross sectional area of the body

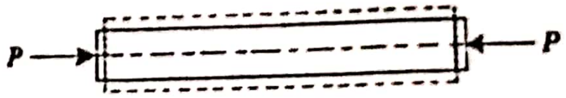
Though there are many types are stresses, the following two types of stresses are important,

1. Tensile stress 2. Compressive stress

When a section is subjected to two equal and opposite pulls and the body tends to increase its length, the dtress induced is called tensile stress. The corresponding strain is called tensile strain. When a section is subjected to two equal and opposite pushes and the body tends to shorten its length, the stress is called compressive stress. The corresponding strain is called compressive strain.



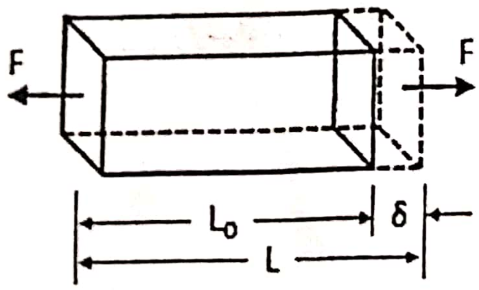
Tensile stress



Compressive stress

Strain: When an external force is applied on a body, there is some change occur in the dimension of the body. The ratio of this change of dimension in the body to its actual length is called strain.
Strain= (change in length) / (original length)

$$\text{Strain}, \varepsilon = \frac{\Delta L}{L} = \frac{L_f - L_0}{L}$$



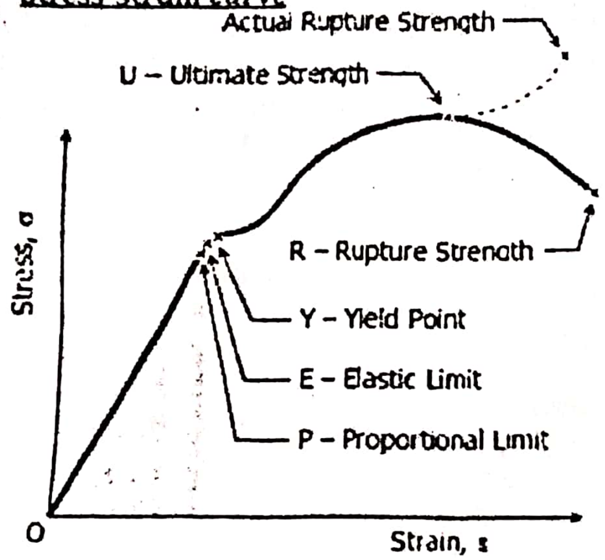
Hooke's law: It states "When a material is loaded, within its elastic limit, the stress is proportional to the strain". Mathematically

$$\frac{\text{Stress}}{\text{Strain}} = E = \text{constant}$$

$$\frac{\sigma}{\varepsilon} = E$$

$$\sigma = \varepsilon E$$

Stress-strain curve



Elasticity: The property of a material by virtue of which, it returns to its original dimension during unloading is called elasticity and the material is called elastic.

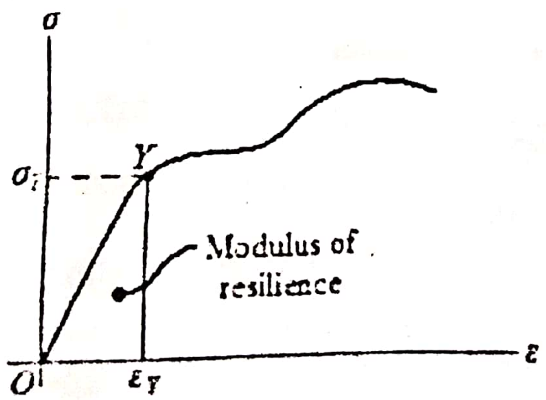
Plasticity: The characteristic of a material by which it undergoes inelastic strain beyond the strain at the elastic limit is known as plasticity.

Creep: The property by virtue of which a material undergoes additional deformation (over and above that due to applied load) with passage of time under sustained loading within elastic limit is called creep.

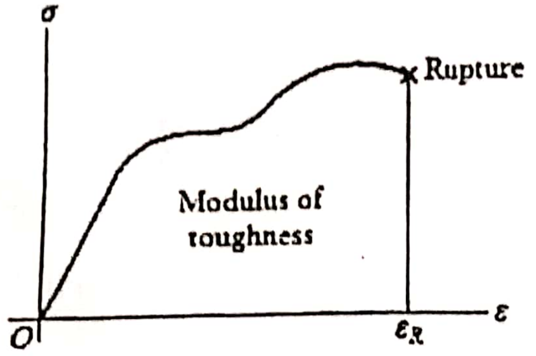
Relaxation: The decrease in stress in steel as a result of creep within steel under prolonged strain is called relaxation.

Fatigue: Deterioration of a material under repeated cycles of stress or strain resulting in progressive cracking that eventually produces fracture is called Fatigue.

Resilience: The resilience of the material is its ability to absorb energy without creating a permanent distortion. The area under stress strain curve within elastic limit is called modulus of resilience.



Toughness: Ability to absorb mechanical energy up to-failure is called toughness. Area under stress strain curve up to fracture is called modulus of toughness.

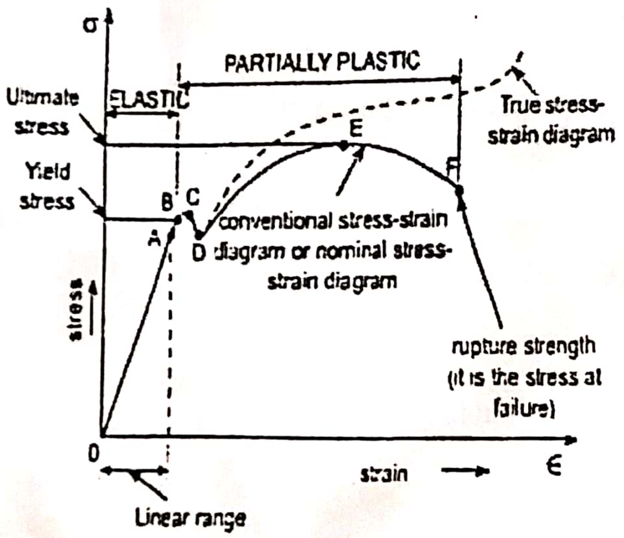


Question: What is hooks law? What is modulus of elasticity? Show the qualitative stress strain diagram of mild steel showing ultimate strength, upper yield strength, lower yield strength and fracture point? (RAJUK – 2014, Army – 2014, WASA – 2014, BUET M. Sc. –2013, 2017)

Solution:

The stress applied to a material is proportional to the strain on that material. For example, if a stress on a metal bar of ten newtons per square centimeter causes it to be compressed by four millimeters, then a stress of 20 newtons per square centimeter will cause the bar to be compressed by eight millimeters. Hooke's law generally holds only up to the elastic limit of stress for that material.

The modulus of elasticity (also known as the elastic modulus, the tensile modulus, or Young's modulus) is a number that measures an object or substance's resistance to being deformed elastically (i.e., non-permanently) when a force is applied to it.



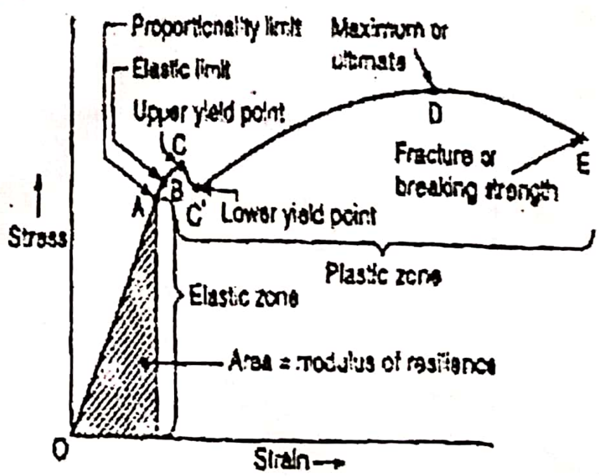


Fig: stress strain diagram of mild steel

Question: Define ductility. Why it is important in earthquake resistant structures? (Combined Bank – 2020)

Solution:

Ductility indicates the ability of a material to deform in the plastic range without breaking, which may be expressed as percent elongation or percent area reduction from a tensile test. Ductility describes the extent to which a material or structure can undergo large deformations without failing. The term is used in earthquake engineering to designate how well a building will endure large lateral displacements imposed by ground shaking. In the simplest of terms, a ductile structure will bend and not break, which greatly reduces the risk of a catastrophic failure.

A structure that is too stiff (often referred to as *brittle*) will be prone to failure under relatively small deformation demands. An example of a brittle structure is an unreinforced masonry building, which will tolerate very little displacement before the onset of damage and failure.

A ductile structure's ability to contort and dissipate energy during an earthquake is, therefore, also advantageous as it will keep deforming without reaching ultimate failure or collapse. An example of a ductile structure is a properly detailed steel frame with a degree of elasticity that will enable it to undergo large deformations before the onset of failure.

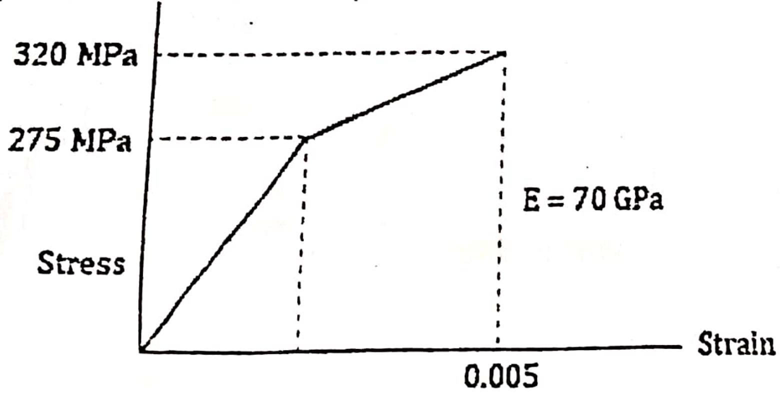
Question: A 6 inch long copper wire is stretched to a total length of 6.05 inches. What is the strain?

Solution:

The change in anything is the final dimension minus the initial dimension. Here, the change in length is the final length minus the initial length: $\Delta L = L_f - L_0 = 6.05 - 6.0 = 0.05$ in.

$$\text{Strain, } \varepsilon = \frac{\Delta L}{L} = \frac{0.05}{6} = 0.0083$$

Question: Find the modulus of resilience & toughness modulus from the following data. (Combined Bank – 2020)



Solution:

$$\text{Strain at 275 MPa, } \varepsilon_{yield} = \frac{\sigma_{yield}}{E} = \frac{275 \times 10^6}{70 \times 10^9} = 0.00392$$

$$\text{Modulus of resilience} = \frac{1}{2} \times \sigma_{yield} \times \varepsilon_{yield} = \frac{1}{2} \times 275 \times 10^6 \times 0.00392 = 539000 \text{ J/m}^3$$

$$\text{Modulus of toughness} = \frac{1}{2} \times 275 \times 0.00392 + \frac{275 + 320}{2} \times (0.005 - 0.00392) = 0.86 \text{ MPa}$$

Axial Deformation:

In the linear portion of the stress-strain diagram, the tress is proportional to strain. $\sigma = E\,\varepsilon$

Since $\sigma = \dfrac{P}{A}$ and $\varepsilon = \dfrac{\delta}{L}$

Then $\dfrac{P}{A} = E \times \dfrac{\delta}{L}$

$\delta = \dfrac{PL}{AE}$

Deformation of a body due to self – weight, $\delta = \dfrac{WL}{2AE}$

W = Total weight

Question: A hollow cylinder 2 m long has an outside diameter of 5 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder. Also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100 GPa

Solution:

Here, $L = 2\text{ m} = 2000\text{ mm}, P = 25\text{ kN} = 25000\text{ N}, E = 100\text{ GPa} = 100 \times 10^3\text{ N/mm}^2$

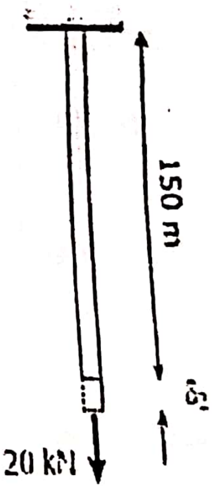
Area, $A = \dfrac{\pi}{4}(D^2 - d^2) = \dfrac{\pi}{4}(50^2 - 30^2) = 1257\text{ mm}^2$

Stress, $\sigma = \dfrac{P}{A} = \dfrac{25 \times 10^3}{A} = 19.9\text{ N/mm}^2$

Deformation, $\delta = \dfrac{PL}{AE} = \dfrac{25 \times 10^3 \times 2 \times 10^3}{1257 \times 100 \times 10^3} = 0.4\text{ mm}$

Question: A steel rod having a cross-sectional area of 300 mm² and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 KN at the lower end. If the unit mass of steel is 7850 kg/m³ and E = 200 × 10³ MN/m², find the total elongation of the rod.

Solution:



Elongation due to its own weight:

$P = W = \dfrac{7850 \times 9.81 \times 300 \times 150}{1000^2} = 3465.3825\text{ N}$

$L = 150 \times 1000 = 150000\text{ mm}, A = 300\text{ mm}^2, E = 200000\text{ MPa}$

$\delta_1 = \dfrac{PL}{2AE} = \dfrac{3465.3825 \times 150000}{2 \times 300 \times 200000} = 4.33\text{ mm}$

Elongation due to applied load:

$P = 20\text{ KN} = 20000\text{ N}, L = 150\text{ m} = 150\,000\text{ mm}, A = 300\text{ mm}^2, E = 200000\text{ MPa}$

$\delta_2 = \dfrac{PL}{AE} = \dfrac{20000 \times 150000}{300 \times 200000} = 50\text{ mm}$

Total elongation: $\delta = \delta_1 + \delta_2 = 4.33 + 50 = 54.33\text{ mm}$

Question: A steel wire 30 ft long, hanging vertically, supports a load of 500 lb. neglecting the weight of the wire, determine the required diameter if the stress is not to exceed 20 ksi and the total elongation is not to exceed 0.20 in. Assume $E = 29 \times 10^6$ psi.

Solution:

Based on maximum allowable stress, $\sigma = \frac{P}{A}$

$$20000 = \frac{500}{0.25\,\pi\, d^2}$$

$d = 0.1784$ in

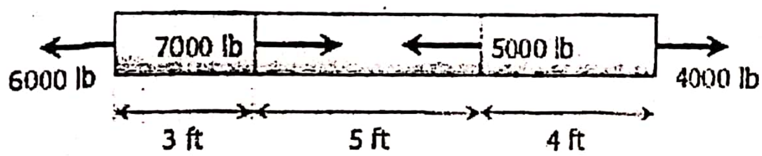
Based on maximum allowable deformation, $\sigma = \frac{P\,L}{A\,E}$

$$0.20 = \frac{500 \times 30 \times 12}{0.25\,\pi\, d^2 \times 29 \times 10^6}$$

$d = 0.1988$ in

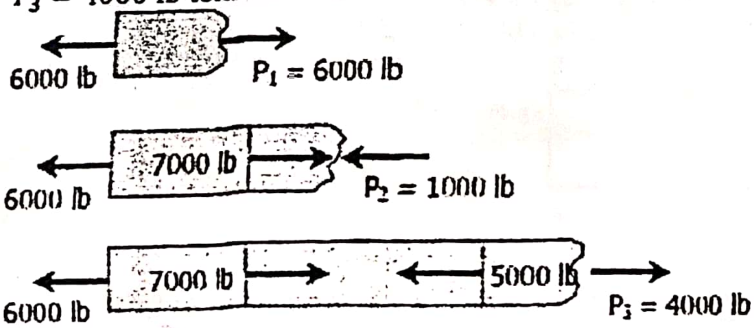
Use the bigger diameter, $d = 0.1988$ inch

Question: An aluminum bar having a cross-sectional area of 0.5 in² carries the axial loads applied at the positions shown in Fig. Compute the total change in length of the bar if $E = 10 \times 10^6$ psi. Assume the bar is suitably braced to prevent lateral buckling.



Solution:

$P_1 = 6000$ lb tension
$P_2 = 1000$ lb compression
$P_3 = 4000$ lb tension

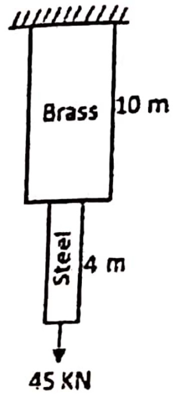


$$\delta = \frac{P\,L}{A\,E}$$

$$\delta = \delta 1 - \delta 2 + \delta 3$$

$$\delta = \frac{6000\ (3 \times 12)}{0.5\ (10 \times 10^6)} - \frac{1000\ (5 \times 12)}{0.5\ (10 \times 10^6)} + \frac{4000\ (4 \times 12)}{0.5\ (10 \times 10^6)} = 0.0696 \text{ in. (lengthening)}$$

Question: A box plate is constructed with brass and steel material which cross sectional area is brass 60 cm^2 and steel 40 cm^2 respectively. Length and unit weight of the brass box is 10m and 900 KN/m^3, steel box is 4m and 400 KN/m^3 also respectively. Determine maximum elongation of this composite bar if applied weight is 45 KN. Modulus of elasticity of brass is 1.1×10^8 KN/m^2 and modulus of elasticity of steel is 2×10^8 KN/m^2. (RRI – 2015)



Solution:

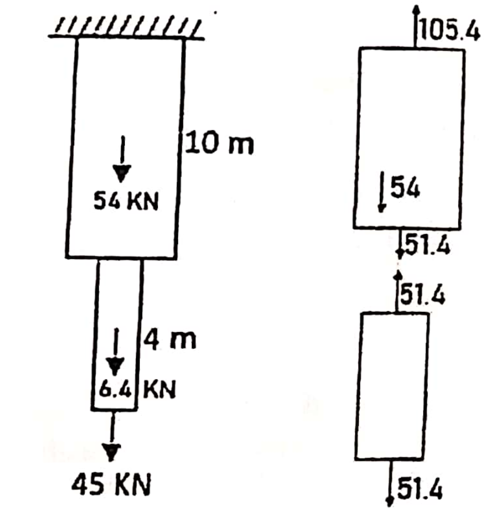
Weight of brass, $W_B = \gamma A_1 L_1 = 900 \times 0.006 \times 10 = 54$ KN

Weight of steel, $W_S = \gamma A_2 L_2 = 400 \times 0.004 \times 4 = 6.4$ KN

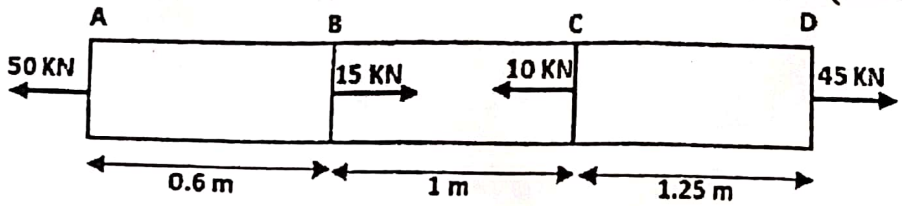
$$\Delta = \Sigma \frac{PL}{AE} = \frac{51.4 \times 4}{0.004 \times 2 \times 10^8} + \frac{105.4 \times 10}{0.006 \times 1.1 \times 10^8}$$

$\Delta = 2.57 \times 10^{-4} + 1.56 \times 10^{-3}$

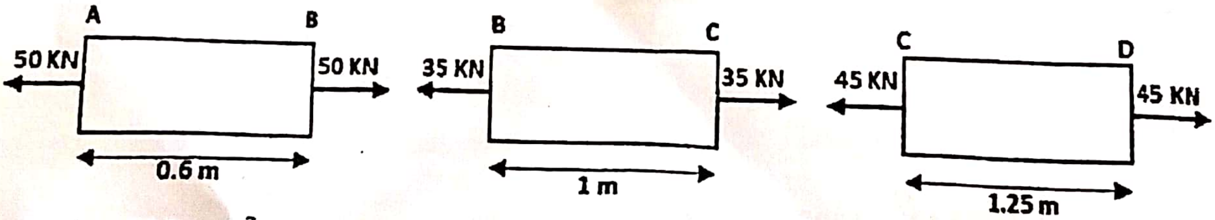
$\Delta = 0.001817$ m $= 1.817$ mm



Question: A steel bar of cross section 500 mm^2 is acted upon by the forces shown in Fig. Determine the total elongation of the bar. For steel, consider E = 200 GPa. (BEPZA – 2016)



Solution:



Given, $A = 500$ mm^2

$E = 200$ GPa $= 200$ KN/mm^2

Total elongation, $\Delta = \Delta_1 + \Delta_2 + \Delta_3 = \frac{P_1 L_1}{A E} + \frac{P_2 L_2}{A E} + \frac{P_3 L_3}{A E}$

$$\Delta = \frac{50 \text{ x } 0.6 \text{ x } 1000}{500 \text{ x } 200} + \frac{35 \text{ x } 1 \text{ x } 1000}{500 \text{ x } 200} + \frac{45 \text{ x } 1.25 \text{ x } 1000}{500 \text{ x } 200}$$

$$\Delta = 0.27 + 0.30 + 0.625 = 1.195 \text{ mm}$$

Shearing stress: If a plane is passed through a body, a force acting along this plane is called a shear force or shearing force. It will be denoted by F_s. The shear force, divided by the area over which it acts, is called the shear stress or shearing stress. It is denoted by τ. Thus

$$\tau = \frac{F_s}{A}$$

The ratio of the shear stress τ to the shear strain γ is called the shear modulus,

$$G = \frac{\tau}{\gamma}$$

G is also known as the modulus of rigidity.

Question: The shearing stress in a piece of structural steel is 100 MPa. If the modulus of rigidity G is 85 GPa, find the shearing strain γ

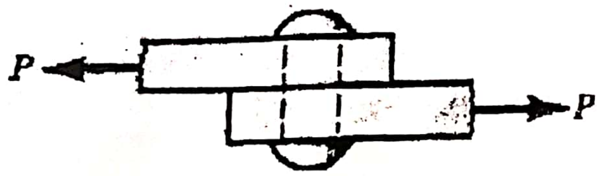
Solution:

$$\text{Shear modulus} = \frac{\text{Shear stress}}{\text{shear strain}}$$

$$G = \frac{\tau}{\gamma}$$

$$\gamma = \frac{\tau}{G} = \frac{100 \text{ x } 10^6}{85 \text{ x } 10^9} = 0.00117 \text{ rad}$$

Question: A single rivet is used to join two plates as shown in Fig. If the diameter of the rivet is 20 mm and the load P is 30 KN, what is the average shearing stress developed in the rivet?



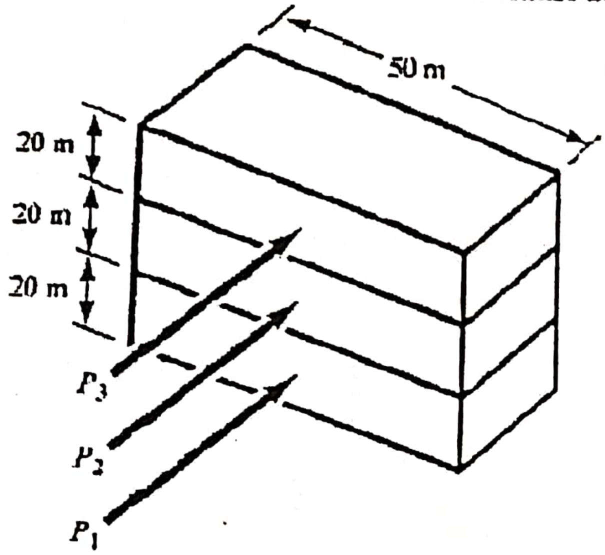
Solution:

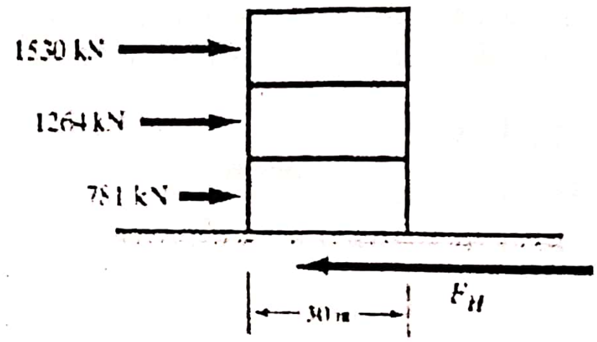
The average shear stress in the rivet is P/A where A is the cross-sectional area of the rivet.

$$\text{Shearing stress, } \tau = \frac{30000}{\left(\frac{\pi}{4}\right) \text{ x } 0.02^2}$$

$$\tau = 9.55 \text{ x } 107 \text{ N/m}^2 = 95.5 \text{ MPa}$$

Question: A building that is 60 m tall has essentially the rectangular configuration shown in Fig. Horizontal wind loads will act on the building exerting pressures on the vertical face that may be approximated as uniform within each of the three "layers" as shown. From empirical expressions for wind pressures at the midpoint of each of the three layers, we have a pressure of 781 N/m^2 on the lower layer, 1264 N/m^2 on the middle layer, and 1530 N/m^2 on the top layer. Determine the resisting shear that the foundation must develop to withstand this wind load.





Solution:

The horizontal forces acting on these layers are found to be

$P_1 = 20 \times 50 \times 781 = 781000\ N = 781\ KN$

$P_2 = 20 \times 50 \times 1264 = 1264000\ N = 1264\ KN$

$P_3 = 20 \times 50 \times 1350 = 1530000\ N = 1530\ KN$

Now, $\sum F_H = 1530 + 1264 + 781 - F_H = 0$

$F_H = 3575\ KN$

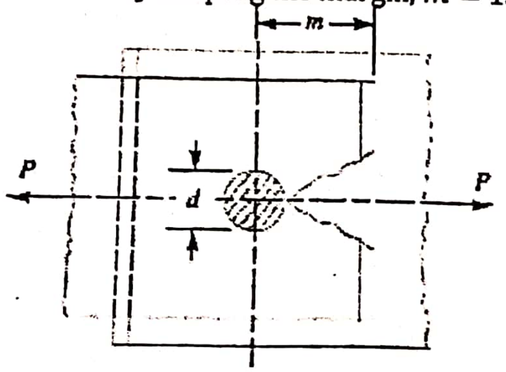
Shearing stress, $\tau = \dfrac{F_H}{A} = \dfrac{3575}{30 \times 50} = 2.38 = 2.38\ KN/m^2$

Question: Describe different types of failure modes in rivets joints. (EGCB – 2020)

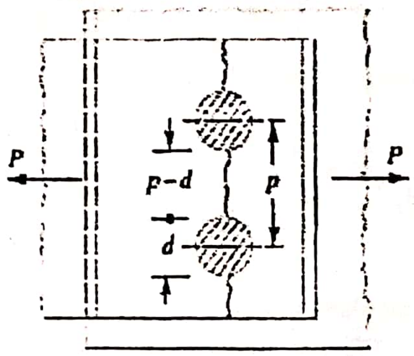
Solution:

Riveted joint may fail in various ways:

Tearing of plate at an edge: A joint may fail due to tearing of the plate at an edge. And this can be avoided by keeping the margin, $m = 1.5\ d$, where d is the diameter of the rivet hole.



Tearing of the plate at an edge.



Tearing of the plate across the rows of rivets.

Tearing of plate across the rows of rivets: Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

Shearing of rivets: The plates which are connected by the rivets exert tensile stress on the rivets, and when the rivets are unable to resist the stress, they shear off.

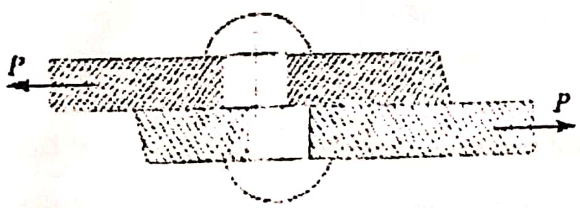


Figure: Shearing of rivets

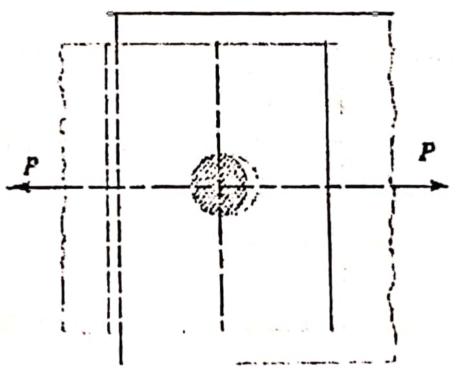


Figure: Crushing of a rivet

Crushing of rivets: Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose.

Shear Failure: The bolt could shear off, preserving the integrity of the plates; this is the least expensive type of failure because bolts are cheaper and easier to replace than plates.

Tensile load, $P_S = \tau_{all} A_B$

For more than one bolt, $P_S = \tau_{all} A_B N$

For more than one shear plane, $P_S = n \tau_{all} A_B N$

Where, $A_B = \frac{\pi}{4} d^2$ (d = diameter of bolt)

N = Number of bolts

n = Number of shear planes

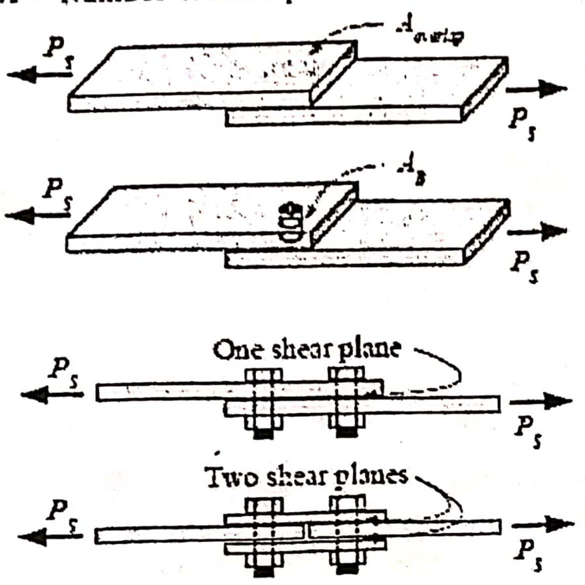


Figure: Shear Failure

Allowable Shear Stress for Bolts		
Bolt Specification	τ_{all}	τ_{all}
A307 low-carbon steel	12.0 ksi	82.5 MPa
A325N, threads in the shear plane	24.0 ksi	165 MPa
A325X, threads excluded from the shear plane	30.0 ksi	207 MPa
A490N, threads in the shear plane	30.0 ksi	207 MPa
A490X, threads excluded from the shear plane	37.5 ksi	260 MPa

Bearing Failure: The bolt could crush the plate where it bears against the plate.

If one or more of the plates is crushed by the bolt, then the area that is crushed equals the bolt diameter, *d*, times the plate thickness, *t*. Multiply this area by the bearing strength of the plate, and we have the allowable bearing load for a joint with one bolt: $P_P = d\, t\, \sigma_{P-all}$.

The allowable bearing load for a joint with multiple bolts is $P_P = d\, t\, \sigma_{P-all} N$.

If the main plate is thicker than the sum of the thicknesses of two splice plates, then use the sum of the splice plate thicknesses for *t*.

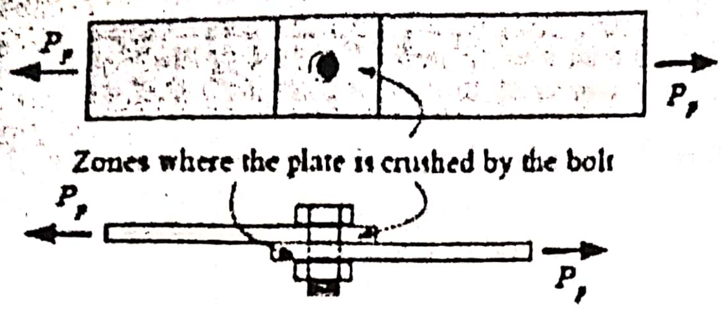
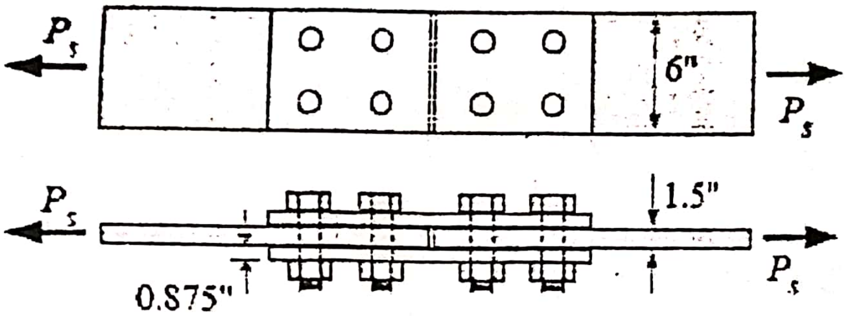


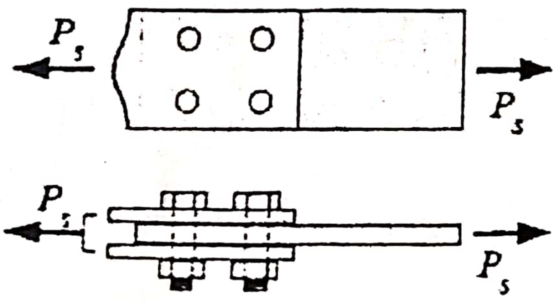
Figure: Bearing Failure

Question: Two A-36 steel plates are joined with two splice plates and eight 7/8 in. diameter A325 bolts. The plates are 6 in. wide and 1.5 in. thick; the splice plates are the same width, and 7/8 in. thick. Bolt threads are excluded from the shear plane. Calculate the load that the joint can support in order to resist shear failure.



Solution:

From the Bolt Shear Strength table, A325 bolts with threads excluded from the shear plane have a shear strength of 30 ksi. The problem is symmetrical, so either the four bolts in the left plate will fail in shear first, or the four bolts in the right plate will fail first. Therefore, we can erase half of the diagram, and focus on four bolts, and $N = 4$.



The load is carried by two shear planes per bolt, so $n = 2$.

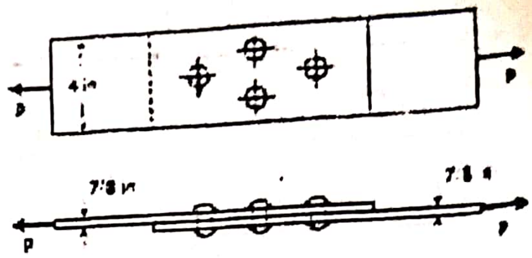
$$P_S = n\,\tau_{all}\,A_B\,N = 2 \times 30 \times \frac{\pi}{4} \times (0.875)^2 \times 4 = 144 \text{ kip}$$

Question: The lap joint shown in figure is fastened by four 0.75 in. diameter rivets. Calculate the maximum safe load P that can be applied if the shearing stress in the rivets is limited to 14 ksi and the bearing stress in the plates is limited to 18 ksi. Assume the applied load is uniformly distributed among the four rivets.

Solution:

Considering shearing of rivets, $P_S = n\,\tau_{all}\,A_B\,N$

$P_S = 4 \text{ x } 14 \text{ x } \frac{\pi}{4} \text{ x } (0.75)^2 = 24.74 \text{ kip}$

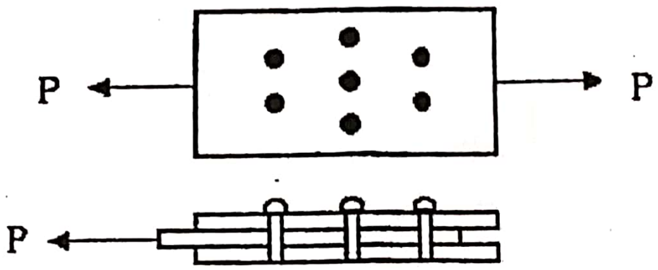


Considering bearing of rivets, $P_P = d\ t\ \sigma_{P-all}\ N$

$P_P = 0.75 \text{ x } \frac{7}{8} \text{ x } 18 \text{ x } 4 = 47.25 \text{ kip}$

Hence, Maximum value of tensile load = 24.74 kip

Question: What will be the maximum value of tensile load "P" for the bolted connection? All plates are 0.75" and bolt diameter 0.50 inch. Shearing stress in the rivets is limited to 20 ksi and the bearing stress in the plates is limited to 30 ksi. (RPGCL – 2017)



Solution:

Considering shearing of rivets, $P_S = n\ \tau_{all}\ A_B\ N$

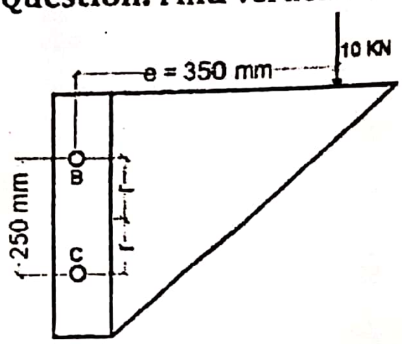
$P_S = 2 \text{ x } 20 \text{ x } \frac{\pi}{4} \text{ x } 0.52 \text{ x } 7 = 54.97 \text{ kip}$

Considering bearing of rivets, $P_P = d\ t\ \sigma_{P-all}\ N$

$P_P = 0.75 \text{ x } 0.5 \text{ x } 30 \text{ x } 7 = 78.75 \text{ kip}$

Hence, Maximum value of tensile load = 54.97 kip

Question: Find vertical and horizontal force at B and C rivet. (NPCBL – 2017)



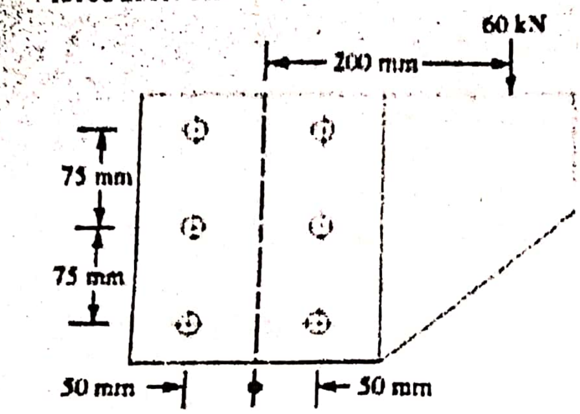
Solution:

P = Vertical force on one bolt $= \frac{P}{n} = \frac{10}{2} = 5 \text{ kN}$

F = Horizontal force due to moment $(P \text{ x } e)$

$$F = \frac{P\ e\ r}{\Sigma\ r^2} = \frac{10 \text{ x } 350 \text{ x } 125}{125^2 + 125^2} = 14 \text{ kN}$$

Question: A bracket is riveted to a column by 6 rivets of equal size as shown in Fig. It carries a load of 60 kN at a distance of 200 mm from the centre of the column. Find vertical and horizontal force at rivets.

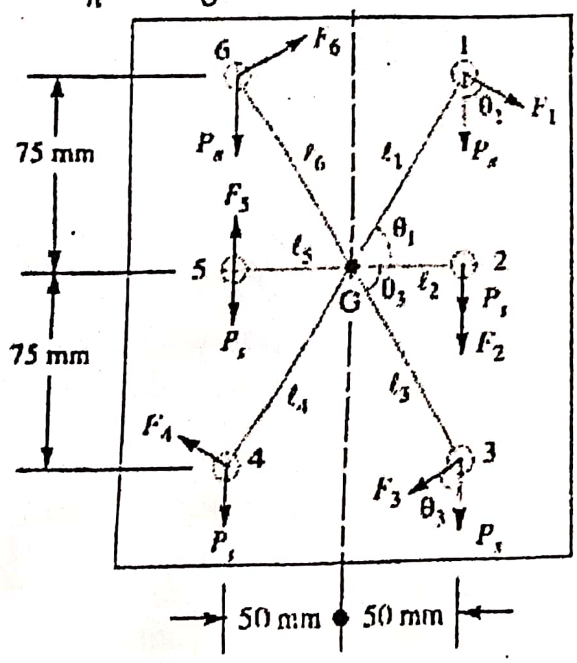


Solution:

Given, $n = 6, P = 60\text{ kN} = 60 \times 10^3\text{ N}, e = 200\text{ mm}$

Direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{60 \times 10^3}{6} = 10000\text{ N}$$



Let F_1, F_2, F_3, F_4, F_5 and F_6 be the secondary shear load on the rivets 1, 2, 3, 4, 5 and 6 at distances l_1, l_2, l_3, l_4, l_5 and l_6 from the centre of gravity (G) of the rivet system. From the symmetry of the figure, we find that

$$l_1 = l_2 = l_3 = l_4 = \sqrt{75^2 + 50^2} = 90.1\text{ mm}$$

$$l_1 = l_2 = 50\text{ mm}$$

F_1 = Horizontal force due to moment ($P \times e$)

$$F_1 = \frac{P\,e\,l_1}{\Sigma\, l^2} = \frac{P\,e\,l_1}{(l_1^2 + l_2^2 + l_3^2 + l_4^2 + l_5^2 + l_6^2)} = \frac{60 \times 10^3 \times 200 \times 90.1}{4 \times 90.1^2 + 2 \times 50^2} = 28846\ N$$

$$F_2 = \frac{P\,e\,l_2}{\Sigma\, l^2} = \frac{60 \times 10^3 \times 200 \times 50}{4 \times 90.1^2 + 2 \times 50^2} = 16008\ N$$

$$F_3 = \frac{P\,e\,l_3}{\Sigma\, l^2} = \frac{60 \times 10^3 \times 200 \times 90.1}{4 \times 90.1^2 + 2 \times 50^2} = 28846\ N$$

$$F_4 = \frac{P\,e\,l_4}{\Sigma\, l^2} = \frac{60 \times 10^3 \times 200 \times 90.1}{4 \times 90.1^2 + 2 \times 50^2} = 28846\ N$$

$$F_5 = \frac{P\,e\,l_5}{\Sigma\, l^2} = \frac{60 \times 10^3 \times 200 \times 50}{4 \times 90.1^2 + 2 \times 50^2} = 16008\ N$$

$$F_6 = \frac{P\,e\,l_6}{\Sigma\, l^2} = \frac{60 \times 10^3 \times 200 \times 90.1}{4 \times 90.1^2 + 2 \times 50^2} = 28846\ N$$

Question: Determine the number of bolts to transit a dead load force of 25 kips and a live load force of 75 kips through two L8 x 8 x 1 connected to a gusset plate (1 inch thick). All materials are A36 steel. The bolts are 0.5 inch diameter A35 steel in a bearing type connection with threads excluded from shear plane. Use three bolts across the web of the channel and consider $f_y = 30$ ksi. Apply ASD method. (JB – 2017)

<u>Solution:</u>

Design force $= DL + LL = 25 + 75 = 100$ k

For 0.5 inch bolt, $A = \frac{\pi}{4} \times 0.5^2 = 0.196\ in^2$

$F_v = 30$ (Threads excluded from shear plane)

$R_v = 0.196 \times 30 = 5.88$ k/shear surface

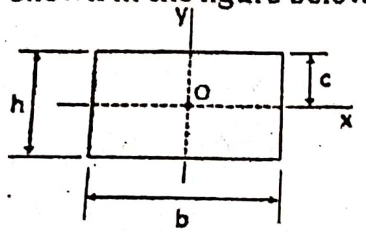
Two shear surface per bolts.

Number of bolts required $= \frac{100}{2 \times 5.88} = 8.5 =$ use 9

Centroid: The centroid of a shape represents the point about which the area of the section is evenly distributed. If the area is doubly symmetric about two orthogonal axes, the centroid lies at the intersection of those axes.

$$x_c = \frac{\sum_i A_i\, x_{c.i}}{\sum_i A_i} \qquad y_c = \frac{\sum_i A_i\, y_{c.i}}{\sum_i A_i}$$

Centroidal Distance: The centroidal distance, C is the distance from the centroid of a cross section to the extreme fiber. The centroidal distance in the y direction for a rectangular cross section is shown in the figure below:



Area Moment of Inertia: The second moment of area, more commonly known as the moment of inertia I of a cross section is an indication of a structural member's ability to resist bending. I_x and I_y are the moments of inertia about the x and y axes respectively and are calculated by:

$$I_x = \int y^2\, dA \qquad I_y = \int x^2\, dA$$

Where x and y are the coordinates of element dA with respect to the axis of interest.

Parallel Axis Theorem: If the moment of inertia of a cross section about a centroidal axis is known, then the parallel axis theorem can be used to calculate the moment of inertia about any parallel axis:

$$I_{parallel} = I_c + A\, d^2$$

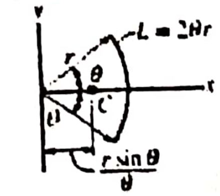
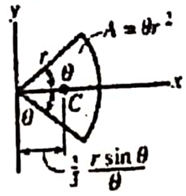
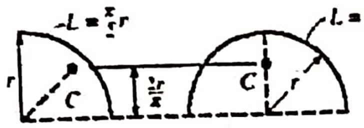
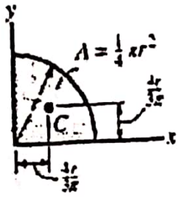
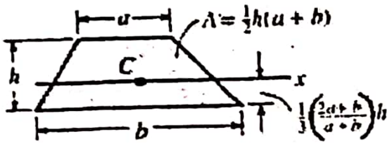
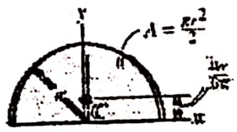
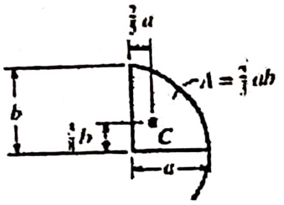
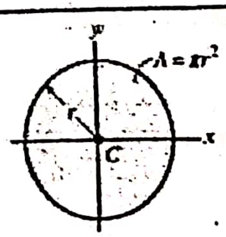
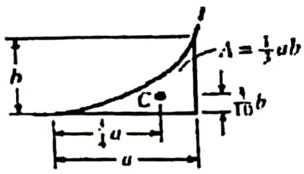
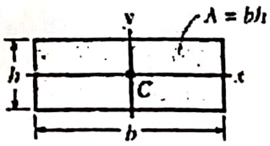
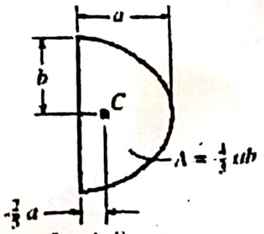
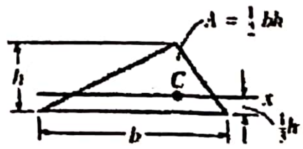
Where I_c the moment of inertia about the centroidal axis, d is the distance between the centroidal axis and the parallel axis, and A is the area of the cross section.

Section Modulus: The maximum bending stress in a beam is calculated as $\sigma_b = M\,C/I_c$, where C is the distance from the neutral axis to the extreme fiber, ${}_c$ is the centroidal moment of inertia, and M is the bending moment. The section modulus combines the C and I_c terms in the bending stress equation:

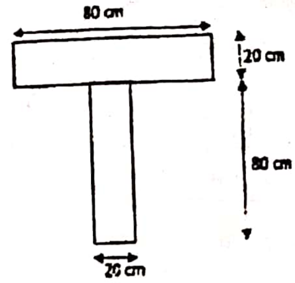
$$S = I_c/C$$

Using the section modulus, the bending stress is calculated as $\sigma_b = M/S$

Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia
 Circular arc segment	 Circular sector area	$I_x = \frac{1}{4} r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_y = \frac{1}{4} r^4(\theta + \frac{1}{2}\sin 2\theta)$
 Quarter and semicircle arcs	 Quarter circle area	$I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
 Trapezoidal area	 Semicircular area	$I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
 Semiparabolic area	 Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
 Exparabolic area	 Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
 Parabolic area	 Triangular area	$I_x = \frac{1}{36}bh^3$

Question: Calculate Y and I for following section. (BEPZA – 2016, RRI – 2015)



Solution:

$\bar{x} = 0$ [about y axis]

$$\bar{y} = \frac{80 \times 20 \times 90 + 20 \times 80 \times 40}{80 \times 20 + 20 \times 80} = 65 \text{ cm [about x axis]}$$

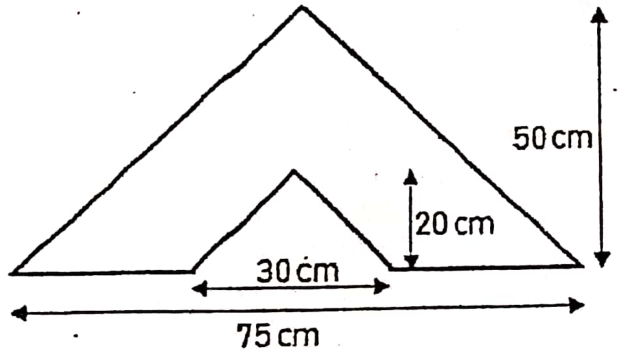
$$I_{xx} = \Sigma \frac{b h^3}{12} + A d^2$$

$$I_{xx} = \left[\frac{80 \times 20^3}{12} + 80 \times 20 \times (65 - 90)^2\right] + \left[\frac{20 \times 80^3}{12} + 20 \times 80 \times (65 - 40)^2\right]$$

$$I_{xx} = 1.05 \times 10^6 + 1.85 \times 10^6 = 2.90 \times 10^6 \text{ cm}^4 \text{ [about x axis]}$$

$$I_{yy} = \frac{b h^3}{12} = \frac{20 \times 80^3}{12} + \frac{80 \times 20^3}{12} = 9.06 \times 10^5 \text{ cm}^4 \text{ [about y axis]}$$

Question: Determine the moment of inertia of the following figure. (BUET M. sc – 2018)



Solution:

Moment of inertia of a triangular section about the base, $I_x = \frac{b h^3}{36}$

$$\bar{y} = \frac{A_2 y_2 - A_1 y_1}{A_2 - A_1} = \frac{\left(0.5 \times 75 \times 50 \times \frac{50}{3}\right) - \left(0.5 \times 20 \times 300 \times \frac{20}{3}\right)}{(0.5 \times 75 \times 50) - (0.5 \times 20 \times 300)} = 18.57 \text{ cm}$$

$$I_x = (I_{x2} + A_2 d_2^2) - (I_{x1} + A_1 d_1^2)$$

$$I_x = \left[\frac{75 \times 50^3}{36} + 0.5 \times 75 \times 50 \times \left(\frac{50}{3} - 18.57\right)^2\right] + \left[\frac{30 \times 20^3}{36} + 0.5 \times 30 \times 20 \times \left(18.57 - \frac{20}{3}\right)^2\right]$$

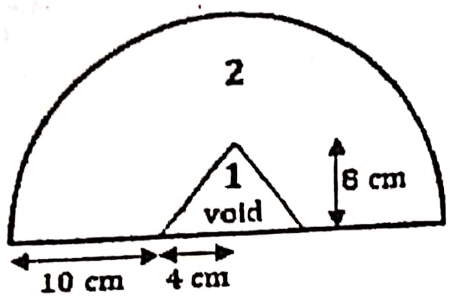
$$I_x = 218035.72 \text{ cm}^4$$

Second method:

Moment of inertia of a triangular section about the base,

$$I_{base} = \frac{B H^3}{12} - \frac{b h^3}{12} = \frac{75 \times 50^3}{12} - \frac{30 \times 20^3}{12} = 781250 - 20000 = 761250 \text{ cm}^4$$

Question: Determine I_x and I_y for the half circle. (BUET M.Sc – 2019)



Solution:

Radius of the half circle, $R = 10 + 4 = 14$ cm

Diameter of the half circle, $D = 28$ cm

$A_2 = 0.5 \times \pi \times r^2 = 307.87\ \text{cm}^2$

$$y_2 = \frac{4R}{3\pi} = \frac{4 \times 14}{3\pi} = 5.94\ \text{cm}$$

$A_1 = 0.5 \times 8 \times 8 = 32\ \text{cm}^2$

$$y_1 = \frac{h}{3} = \frac{8}{3} = 2.67\ \text{cm}$$

$$\bar{y} = \frac{A_2\, y_2 - A_1\, y_1}{A_2 - A_1} = \frac{(307.87 \times 5.94) - (32 \times 2.67)}{307.87 - 32} = 6.32\ \text{cm}$$

$\bar{x} = 0$

$I_x = (I_{x2} + A_2\, d_2^2) - (I_{x1} + A_1\, d_1^2)$

$$I_x = \left[\frac{\pi \times 28^4}{128} + 307.87 \times (6.32 - 5.94)2\right] - \left[\frac{8 \times 8^3}{36} + 32 \times (6.32 - 2.67)2\right]$$

$I_x = 14590.32\ \text{cm}^4$

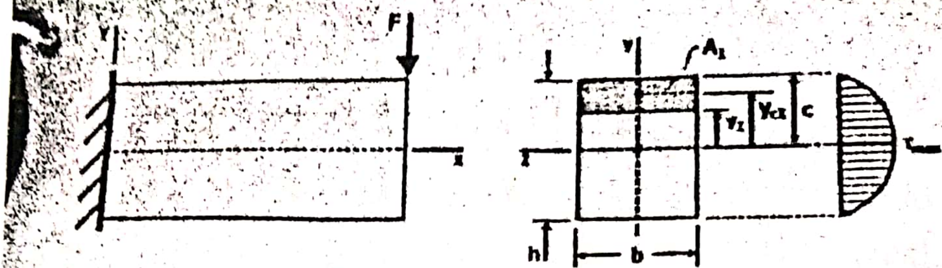
$I_y = I_{y2} - I_{y1}$

$$I_y = \frac{\pi \times 28^4}{128} - \frac{8 \times 8^3}{36}$$

$I_y = 14972.18\ \text{cm}^4$

Shear stress: Normal stress is a result of load applied perpendicular to a member. Shear stress however results when a load is applied parallel to an area. The shear force, V, along the length of the beam can be determined from the shear diagram. The shear force at any location along the beam can then be used to calculate the shear stress over the beam's cross section at that location. The shear stress is zero at the free surfaces (the top and bottom of the beam), and it is maximum at the centroid.

$$\text{Shear stress, } \tau = \frac{V\,Q}{I\,b}$$

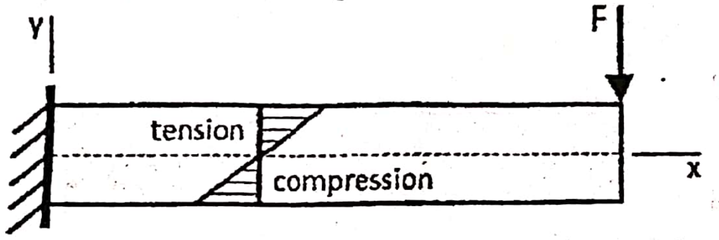


The maximum shear stress for rectangular beam, $\tau = 1.5\,\frac{V}{A}$

Flexural stress: When a member is being loaded similar to that in figure one bending stress (or flexure stress) will result. Stress produced by the bending force in the body is what we call flexural stress. Bending stress is maximum at top or bottom most fibre and it is zero at neutral axis.

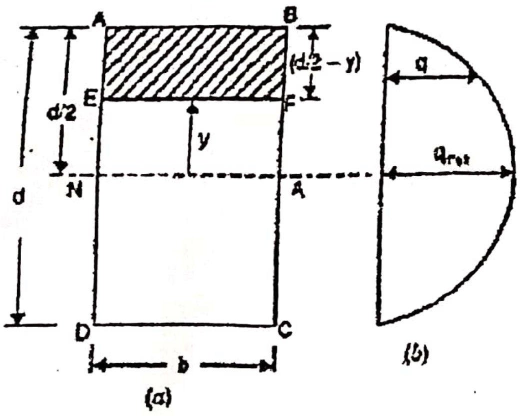
The bending moment, M, along the length of the beam can be determined from the moment diagram. The bending moment at any location along the beam can then be used to calculate the bending stress over the beam's cross section at that location. The bending moment varies over the height of the cross section according to the flexure formula below.

Flexural stress, $\sigma = -\frac{M\,Y}{I}$



Where M is the bending moment at the location of interest along the beam's length, I is the centroidal moment of inertia of the beam's cross section, and y is the distance from the beam's neutral axis to the point of interest along the height of the cross section. The negative sign indicates that a positive moment will result in a compressive stress above the neutral axis.

Question: Show that shear stress of a rectangular beam along the section in neutral axis is 1.5 times of average shear stress. (Combined Bank – 2020)



Solution:

Figure shows a rectangular section of a beam of width b and depth d. Let F is the shear force acting at the section. Consider a level EF at a distance y from the neutral axis.

The shear stress at this level, $\tau = \frac{A\bar{y}}{b \times I}$

Where, A = area of the section above (shaded area $ABFE$) $= \left(\frac{d}{2} - y\right) b$

$\bar{y}$ = Distance of the C.G of area A from neutral axis

$$\bar{y} = y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2} = \frac{y}{2} + \frac{d}{4} = \frac{1}{2}\left(y + \frac{d}{2}\right)$$

b = Actual width of the section at the level EF

I = $M.O.I$ of the whole section at the level EF

$$\text{Now, } \tau = \frac{F\left(\frac{d}{2} - y\right) \times b \times \frac{1}{2}\left(y + \frac{d}{2}\right)}{b \times I} = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

we see that τ increases as y decreases, the variation of τ with respect to y is parabola.

At the top edge, $y = \frac{d}{2}, \tau = \frac{F}{2I}\left[\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right] = \frac{F}{2I} \times 0 = 0$

At the neutral axis, $y = 0, \tau = \frac{F}{2I}\left(\frac{d^2}{4} - 0\right) = \frac{F}{2I} \times \frac{d^2}{4} = \frac{F d^2}{8I}$

$$\tau = \frac{F d}{8 \times \frac{b d^3}{12}} = \frac{12}{8} \times \frac{F}{b d} = 1.5 \frac{F}{b d}$$

Average shear stress, $\tau_{avg} = \frac{\text{Shear force}}{\text{Area of section}} = \frac{F}{b d}$

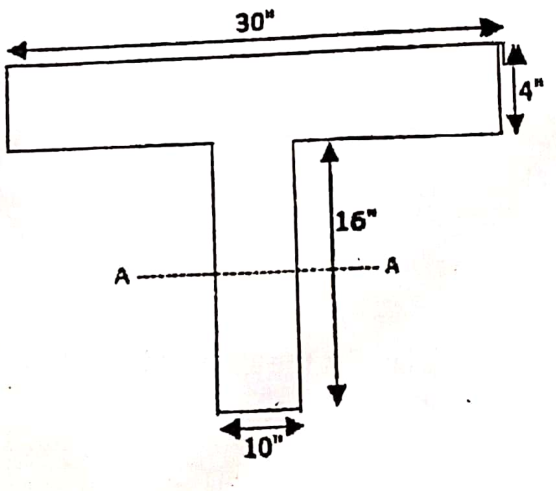
So, $\tau = 1.5\ \tau_{avg}$

Which gives the shear stress at the neutral axis where $y = 0$, this stress is also the maximum shear stress.

$\tau_{max} = 1.5\ \tau_{avg}$

Question: Find the compressive stress of the bottom fiber of a T-beam subjected to a bending moment of 100 kip-ft about its centroidal axis A-A. (WRGCL – 2014)

Solution:



B2

$\bar{x} = 0$ [about y axis]

$$\bar{y} = \frac{30 \times 4 \times 18 + 10 \times 16 \times 8}{30 \times 4 + 10 \times 16} = 12.286 \text{ cm [about x axis]}$$

$$I_{xx} = \Sigma \frac{b\,h^3}{12} + A\,d^2$$

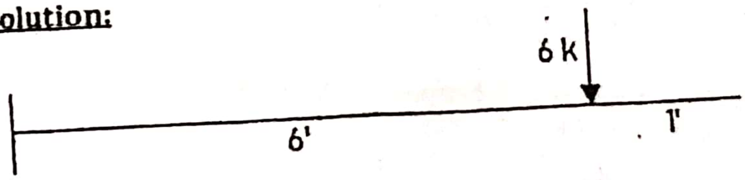
$$I_{xx} = \left[\frac{10 \times 16^3}{12} + 10 \times 16 \times (12.286 - 8)^2\right] + \left[\frac{30 \times 4^3}{12} + 30 \times 4 \times (18 - 12.286)^2\right]$$

$$I_{xx} = 3413.33 + 2939.16 + 160 + 3917.97 = 10430.46 \text{ cm}^4 \text{ [about x axis]}$$

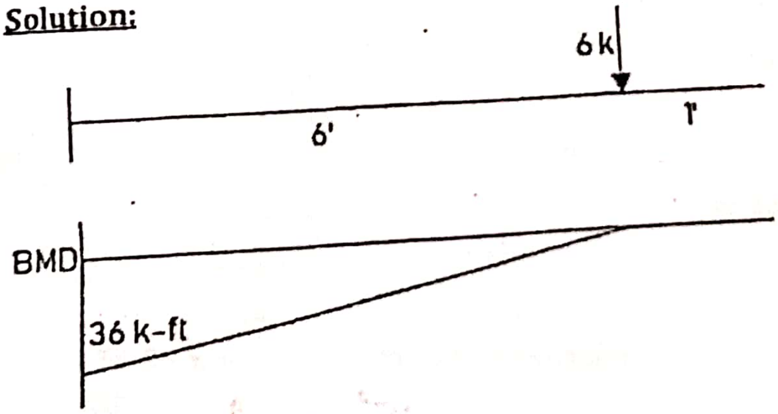
$$f = \frac{M\,y}{I} = -\frac{100 \times 12 \times 12.286}{10430.46} = -1.413 \text{ ksi}$$

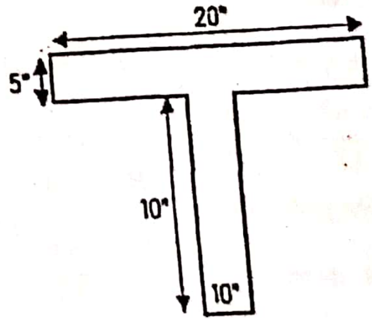
Question: Compute the maximum tensile stress of the beam (Consider a section of T with web dimension is 10 in x 10 in and flange dimension of 20 in x 5 in). (WASA – 2014)

Solution:



Solution:





$\bar{x} = 0$ [about y axis]

$$\bar{y} = \frac{20 \times 5 \times 12.5 + 10 \times 10 \times 5}{20 \times 5 + 10 \times 10} = 8.75 \text{ in [about x axis]}$$

$$I_{xx} = \Sigma \frac{b\,h^3}{12} + A\,d^2$$

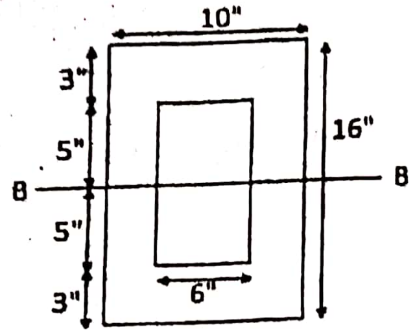
$$I_{xx} = \left[\frac{20 \times 5^3}{12} + 20 \times 5 \times (8.75 - 12.5)^2\right] + \left[\frac{10 \times 10^3}{12} + 10 \times 10 \times (8.75 - 5)^2\right]$$

$$I_{xx} = 3854.16 \text{ cm}^4 \text{ [about x axis]}$$

$$C = \bar{y} = 8.75 \text{ in}$$

$$f = \frac{M\,C}{I} = \frac{36 \times 12 \times 8.75}{3854.16} = 0.98 \text{ ksi}$$

Question: Find the maximum flexural compressive stress in a box section of 10 x 16 inches subjected to bending moment of 100 k-ft about its axis B-B (PGCL – 2014)



Solution:

$$I = \frac{b\,h^3}{12} - \frac{b_1\,h_1^3}{12} = \frac{10 \times 16^3}{12} - \frac{6 \times 10^3}{12} = 2913.33 \text{ in}^4$$

Maximum flexural compressive stress, $f = \frac{M\,C}{I} = \frac{100 \times 12 \times 8}{2913.33} = 3.29$ ksi

Question: Determine the bending stress and shear stress in a beam having 12" x 12" size and maximum bending moment 12 kip-ft and shear force is 6 kip. (BIFPCL – 2015)

Solution:

$$Q = A\,y = \left(b \times \frac{d}{2}\right)\frac{d}{4} = (12 \times 6)\,3 = 216 \text{ in}^3$$

$$I = \frac{b\,h^3}{12} = \frac{12 \times 12^3}{12} = 1728 \text{ in}^4$$

Bending stress, $S_b = \frac{M\,y}{I} = \frac{12 \times 12 \times 6}{1728} = 0.5$ ksi

Shear stress, $S_s = \frac{V\,Q}{I\,b} = \frac{6 \times 216}{1728 \times 12} = 0.063$ ksi

Question: Find the shear stress of section 12" x 22" subjected to a critical shear force 48 kips. (PGCB – 2015, DNCC – 2016, BPDB – 2016)

Solution:

$$Q = A\,y'$$

$$A = b\left(\frac{d}{2} - y\right) \text{ and } y' = y + \frac{1}{2}\left(\frac{d}{2} - y\right)$$

When $y = 0$, shear stress will be maximum

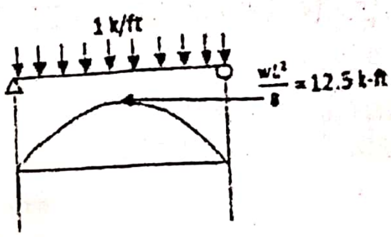
$$A = 12\left(\frac{22}{2} - 0\right) = 132 \text{ in}^2 \text{ and } y' = 0 + \frac{1}{2}\left(\frac{22}{2} - 0\right) = 5.5 \text{ in}$$

$$Q = 132 \times 5.5 = 726 \text{ in}^3$$

$$I = \frac{b\,h^3}{12} = \frac{12 \times 22^3}{12} = 10648 \text{ in}^4$$

Shear stress, $S_s = \frac{V\,Q}{I\,b} = \frac{48 \times 726}{10648 \times 12} = 0.273$ ksi

Question: A 10' simply supported beam, applied distributed load 1 kip/ft above the beam. If size of beam 10" x 10", find the maximum bending stress and maximum shear stress of that beam. (RRI – 2015)



Solution:

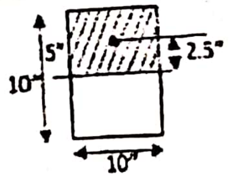
Here, $V = \frac{wL}{2} = \frac{1 \times 10}{2} = 5$ kips

$Q = A\,y'$

$A = b\left(\frac{d}{2} - y\right)$ and $y' = y + \frac{1}{2}\left(\frac{d}{2} - y\right)$

When $y = 0$, shear stress will be maximum

$A = 10\left(\frac{10}{2} - 0\right) = 50\ \text{in}^2$ and $y' = 0 + \frac{1}{2}\left(\frac{10}{2} - 0\right) = 2.5$ in



$Q = 50 \times 2.5 = 125\ \text{in}^3$

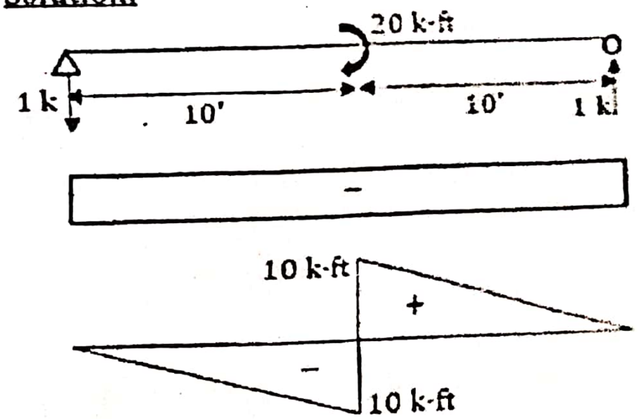
$I = \frac{bh^3}{12} = \frac{10 \times 10^3}{12} = 833.33\ \text{in}^4$

Bending stress, $S_b = \frac{My}{I} = \frac{12.5 \times 12 \times 5}{833.33} = 0.90$ ksi

Shear stress, $S_s = \frac{VQ}{Ib} = \frac{5 \times 125}{833.33 \times 10} = 0.075$ ksi

Question: Draw SFD and BMD and Maximum shears tress for 12" x 12" beam. (WARPO – 2017)

Solution:



$Q = A\,y'$

$A = b\left(\frac{d}{2} - y\right)$ and $y' = y + \frac{1}{2}\left(\frac{d}{2} - y\right)$

When $y = 0$, shear stress will be maximum

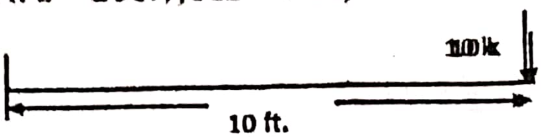
$A = 12\left(\frac{12}{2} - 0\right) = 72\ \text{in}^2$ and $y' = 0 + \frac{1}{2}\left(\frac{12}{2} - 0\right) = 3$ in

$Q = 72 \times 3 = 216\ \text{in}^3$

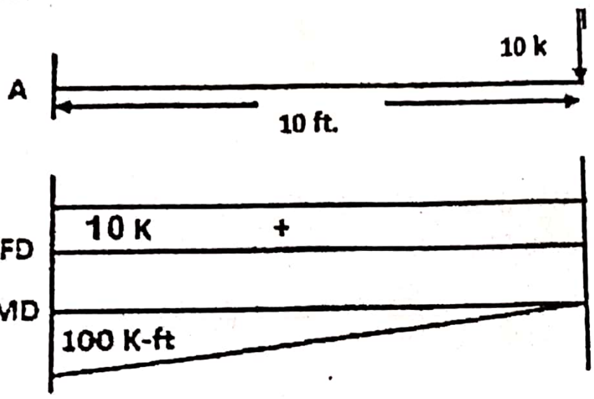
$I = \frac{bh^3}{12} = \frac{12 \times 12^3}{12} = 1728\ \text{in}^4$

Shear stress, $S_s = \frac{VQ}{Ib} = \frac{1 \times 216}{1728 \times 12} = 0.0104$ ksi

Question: Calculate the shear stress and bending stress of the following beam of section 12" x 12" (MPL – 2017, JOCL – 2018)



Solution:



Here, $V = 5$ kips, $M = 100$ k – ft

$Q = A\,y'$

$A = b\left(\frac{d}{2} - y\right)$ and $y' = y + \frac{1}{2}\left(\frac{d}{2} - y\right)$

When $y = 0$, shear stress will be maximum

$A = 12\left(\frac{12}{2} - 0\right) = 72 \text{ in}^2$ and $y' = 0 + \frac{1}{2}\left(\frac{12}{2} - 0\right) = 3$ in

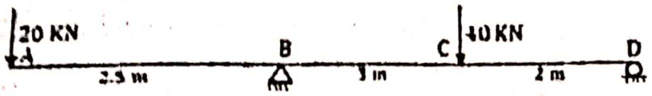
$Q = 72 \times 3 = 216 \text{ in}^3$

$I = \frac{b\,h^3}{12} = \frac{12 \times 12^3}{12} = 1728 \text{ in}^4$

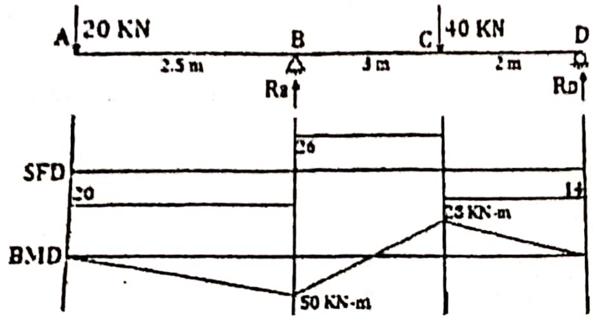
Bending stress, $S_b = \frac{M\,y}{I} = \frac{100 \times 12 \times 6}{1728} = 4.166$ ksi

Shear stress, $S_s = \frac{V\,Q}{I\,b} = \frac{10 \times 216}{1728 \times 12} = 0.104$ ksi

Question: For the timber beam and loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending if the beam size is 250 mm x 160 mm. (HBFC – 2018)



Solution:



For the above BMD diagram, $M_{max} = 50 \text{ kN} - \text{m}$

Now, $S = \dfrac{b\,h^2}{6} = \dfrac{1}{6} \times 0.08 \times 0.25^2 = 833.33 \times 10^{-6}\ \text{m}^3$

$$\sigma_{max} = \frac{M}{S} = \frac{50 \times 10^3}{833.33 \times 10^{-6}} = 60 \times 10^6\ \text{kN/m}^2$$

Maximum normal stress in the beam = 60000 MPa

Question: A 100 mm by 100 mm beam is subjected to bending moment about both principal axis of 1 KN-m and 2 KN-m. Find maximum bending stress. (SGFL – 2017)

Solution:

$$I_x = I_y = \frac{100 \times 100^3}{12} = 8.33 \times 10^6\ \text{mm}^4$$

$M_x = 1\ \text{kN} - \text{m} = 1 \times 10^6\ \text{N} - \text{mm}$

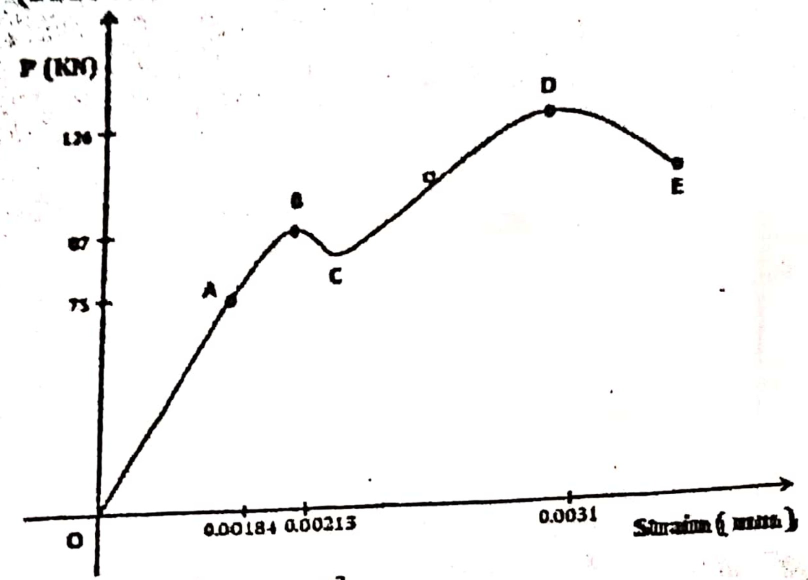
$M_y = 2\ \text{kN} - \text{m} = 2 \times 10^6\ \text{N} - \text{mm}$

$$\sigma_{xx} = \frac{M_x\,C}{I_x} = \frac{10^6 \times 50}{8.33 \times 10^6} = 6\ \text{MPa}$$

$$\sigma_{yy} = \frac{M_y\,C}{I_y} = \frac{2 \times 10^6 \times 50}{8.33 \times 10^6} = 12\ \text{MPa}$$

$$\sigma_{max} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2} = \frac{6 + 12}{2} + \sqrt{\left(\frac{6 - 12}{2}\right)^2} = 18\ \text{Mpa}$$

Question: A 16 mm diameter steel bar is 600 mm length. Determine the following mechanical properties from the diagram: (a) modulus of elasticity (b) yield point (c) ultimate strength. (BUET M. Sc – 2019)



Area, $A = \frac{\pi d^2}{4} = \frac{\pi \times 16^2}{4} = 201.06\ mm^2$

Length, $L = 600$ mm

Strain = Elongation/Length

Stress = Load/Area

Proportional limit $= \frac{75 \times 1000}{201.06} = 373.02$ MPa

Modulus of Elasticity, E = slope of stress-strain diagram within proportional limit

$$E = \frac{373.02}{0.00184} = 202729.87 \text{ MPa} = 202.72 \text{ GPa}$$

Yield point $= \frac{87 \times 1000}{201.06} = 431.54$ MPa

Ultimate strength $= \frac{126 \times 1000}{201.06} = 626.67$ **MPa**

Question: Applied load on a rebar is 50 kip and diameter of rod 25.4 mm. Determine the grade of rebar. (BIFPCL – 2015)

<u>Solution:</u>

Given, $F = 50$ kip, $A = \frac{\pi d^2}{4} = \frac{\pi}{4}\left(\frac{25.4}{25.4}\right)^2 = 0.785\ in^2$

Ultimate strength, $f_y = \frac{F}{A} = \frac{50}{0.785} = 63.7$ **ksi**

For 60 grade steel, minimum $f_y = 60$ **ksi**

So this rebar is 60 grades steel.

Question: A hollow steel pipe which inside diameter 10 mm, wall thickness 2 mm and length of the pipe is 15 mm. Determine the weight of the steel pipe in kg if unit weight of the steel pipe is 7850 kg/m³. (BIFPCL – 2015)

Solution:

Area of total steel $= \pi R^2 h - \pi r^2 h = \pi h (R^2 - r^2)$

$$A = \pi \times 15 \left[\left(\frac{10+2}{2}\right)^2 - \left(\frac{10}{2}\right)^2\right] = 518.36\ mm^3 = 518.36 \times 10^{-9}\ m^3$$

Weight of steel $= 518.36 \times 10^{-9} \times 7850 = 0.00407$ kg

Question: 2" x 2" cube load test. Fill the box. (BWDB – 2018)

Observed load	Actual load	Compressive strength	Avg. compressive strength
36.7 KN			
37.3 KN		2209 psi	

Solution:

Observed load	Actual load	Compressive strength	Avg. compressive strength
36.7 KN	38.54 KN	2166 psi	2188 psi
37.3 KN	39.30 KN	2209 psi	

Actual load on specimen two $= 2209 \times 2 \times 2 = 8836\ lb = \frac{8836}{224.8} = 39.30$ kN

Load ratio $= \frac{39.3}{37.3} = 1.05$

Actual load on specimen one $= 36.7 \times 1.05 = 38.54$ kN

Compressive load on specimen one $= \frac{38.54 \times 224.8}{2 \times 2} = 2166.04$ psi

Question: The thickness of a brick wall is 250 mm, 2 m in height. Find the horizontal distributed wind load for which the base shear is just negative. The unit weight of the masonry is given 1950 kg/m³. (GTCL – 2016)

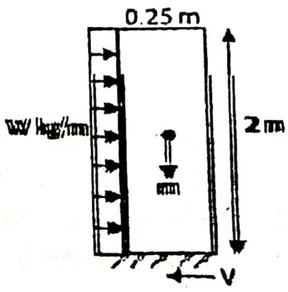
Solution:

Considering w kg/m uniformly distributed wind load acting on the wall.

For 1 m wall length,

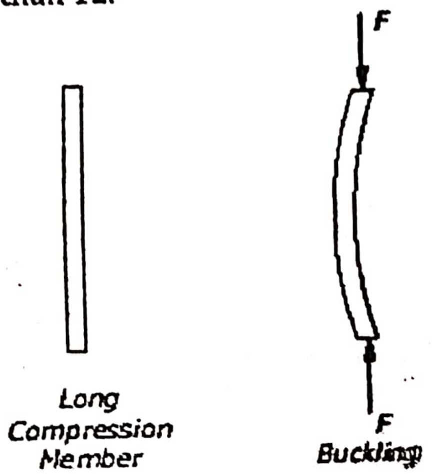
$$\sum M_0 = (2 \times 0.25 \times 1) \times 1950 \times \frac{0.25}{2} - w \times 2 \times 1 = 0$$

$\therefore w = 60.94$ kg/m

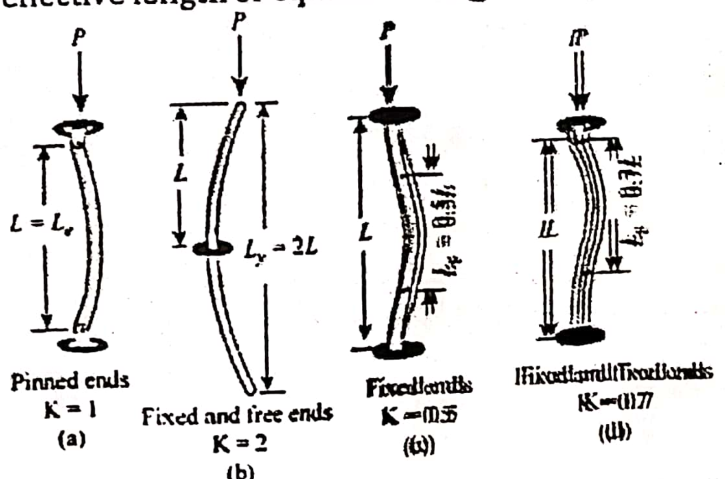


Slender column & Effective length

Slender columns can be defined as columns with small cross sections compared to their lengths. Generally, slender columns have lower strength when compared to short columns, for a constant cross section, increasing the length causes a reduction in the strength. Slender column fails buckling at loads considerably less than those required to cause failure by crushing. A long or slender column is the one whose ratio of effective length to its least lateral dimension is greater than 12.



Effective length of the column: The distance between adjacent points of infection is called effective length or equivalent length. It is generally represent by L_e.



Slenderness ratio: It is the ratio of unsupported length of the column to the minimum radius of gyration of the cross-sectional area of the column.

$$\lambda = \frac{L_e}{r_{min}}$$

Buckling and Buckling Load (Crippling load or critical load): A long column when subjected to direct load deflects in lateral direction which is known as buckling. The maximum value of force or load which causes the buckling of the column is known as the buckling load or crippling load or critical load. It is generally represented by P_{cr}.

Euler Critical Load

The Euler critical load, $P_{cr} = \frac{\pi^2 E I}{L^2}$

Where I is the smallest moment of inertia for the cross-sectional area of the column. For example, if the column is a wide-flange beam, $I_y < I_x$, so use I_y.

Critical buckling stress, $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E I}{L^2 A} = \frac{\pi^2 E r^2}{L^2} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$

The quantity L/r is called the slenderness ratio of the column; the larger this value, the slenderer the column.

Considering effective length factor

The Euler critical load, $P_{cr} = \frac{\pi^2 E I}{(KL)^2}$

Critical buckling stress, $\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$

Question: A steel bar of rectangular cross section 40 mm x 50 mm and pinned at each end is subject to axial compression. If the proportional limit of the material is 230 MPa and E = 200 GPa, determine the minimum length for which Euler's equation may be used to determine the buckling load.

Solution:

Moment of inertia, $I = \frac{b h^3}{12} = \frac{50 \times 40^3}{12} = 2.67 \times 10^5 \text{ m}^4$

Least radius of gyration, $r = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.67 \times 10^5}{40 \times 50}} = 11.5 \text{ mm} = 0.0115 \text{ m}$

Critical buckling stress, $\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$

$$230 \times 10^6 = \frac{\pi^2 (200 \times 10^9)}{\left(\frac{L}{0.0115}\right)^2}$$

$L = 1.135$ m

Question: Consider again a rectangular steel bar 40 mm x 50 mm in cross section, pinned at each end and subject to axial compression. The bar is 2 m long and E = 200 GPa. Determine the buckling load using Euler's formula.

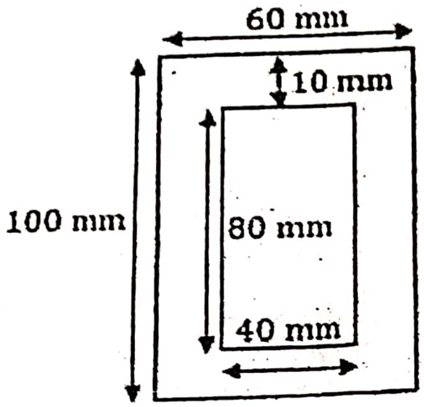
Solution:

Moment of inertia, $I = \frac{b\,h^3}{12} = \frac{50 \times 40^3}{12} = 2.67 \times 10^5\ mm^4 = 2.67 \times 10^{-7}\ m^4$

The Euler critical load, $P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 (200 \times 10^9) \times (2.67 \times 10^{-7})}{2^2} = 132 \times 10^3\ N = 132\ KN$

Critical buckling stress, $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{132 \times 10^3}{0.04 \times 0.05} = 66 \times 106\ \text{Pa} = 66\ \text{MPa}$

Question: A steel box section is 100 mm x 60 mm x 10 mm having a length of 3 m, find slenderness ratio. (51 BMA)



Solution:

Slenderness ratio is the ratio of the effective length of a column (L_e) and least radius of gyration (r) about the axis under consideration.

$$\text{Slenderness ratio} = \frac{\text{effective length}}{\text{least radius of gyration}} = \frac{L_e}{r}$$

$$\text{Monet of inertia, } I = \frac{60 \times 100^3}{12} - \frac{40 \times 80^3}{12} = 3293333.333\ mm^4$$

$$\text{Area, } A = 100 \times 60 - 80 \times 40 = 2800\ mm^2$$

$$\text{Slenderness ratio, } r = \sqrt{\frac{I}{A}} = \sqrt{\frac{3293333.33}{2800}} = 39.29\ mm$$

$$\text{Slenderness ratio} = \frac{3 \times 1000}{39.29} = 76.35$$

Question: Determine Euler buckling stress of a 12 ft long column having sectional properties r_x = 4.25 in & r_y = 2.1 in. Assume, pinned-pinned end for strong axis buckling and fixed-pinned end for weak axis buckling. Given E = 29000 ksi. (DPDC – 2019)

Solution:

When column is pinned – pinned at its top and base for strong axis,

Critical buckling stress, $\sigma_{cr} = \dfrac{\pi^2 E}{\left(\frac{kL}{r_x}\right)^2} = \dfrac{\pi^2 \times 29000}{\left(\frac{1 \times 12 \times 12}{4.25}\right)^2} = 249.31$ ksi

When column is fixed – pinned at its top and base for weak axis,

Critical buckling stress, $\sigma_{cr} = \dfrac{\pi^2 E}{\left(\frac{kL}{r_y}\right)^2} = \dfrac{\pi^2 \times 29000}{\left(\frac{0.7 \times 12 \times 12}{2.1}\right)^2} = 124.22$ ksi

So Critical buckling stress, σ_{cr} = 124.22 ksi (controls)

Tortional stresses and strains

$$\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$$

τ = shear stress developed in the outermost layer of the shaft in N/mm2
R = Radius of circular shaft in mm
G = Modulus of rigidity or torsional rigidity of the shaft material
L = Length of shaft in mm
φ = Angle of twist in the shaft
T = Torque in N – mm
J = Polar moment of inertia of circular shaft in mm4

Strength of a solid shaft

$$T = \frac{\pi}{16} \times \tau \times D^3$$

T = Torque in N – mm
τ = shear stress developed in the outermost layer of the shaft in N/mm2
D = Diameter of circular shaft in mm

Strength of a hollow shaft, $T = \dfrac{\pi}{16} \times \tau \times \left(\dfrac{D^4 - d^4}{D}\right)$

d = internal diameter of the shaft in mm

Power transmitted by a shaft

Work done per second $= \dfrac{2\pi N T}{60}$ kN – m

Power transmitted, $P = \dfrac{2\pi N T}{60}$ kW

N = No. of revolutions per minute
T = Average torque in KN – m

Question: Find the maximum torque that can be applied safely to a shaft of diameter 300 mm. The permissible angle of torque is 1.5° in a length of 7.5 m & the shearing stress not exceeding 42 N/mm². Take modules of rigidity G = 84.4 N/mm² (HBFC – 2018)

Solution:

Polar moment of inertia, $J = \frac{\pi D^4}{32} = \frac{\pi \times 300^4}{32} = 7.95 \times 108 \text{ mm}^4$

Angle of twist, $\theta = \frac{\pi}{180} \times 1.5^0 = 0.0262$

We know, $\frac{T}{J} = \frac{G\theta}{L}$

$$T = \frac{\theta G J}{L} = \frac{0.026 \times 84.4 \times 7.95 \times 10^8}{(7.5 \times 1000)} = 2.345 \times 10^5 \text{ N} - \text{mm}$$

Question: A hollow shaft of external and internal diameters as 100 mm and 40 mm is transmitting power at 120 r.p.m. Find the power the shaft can transmit, if the shearing stress is not to exceed 50 MPa.

Solution:

Here, $D = 100$ mm, $d = 40$ mm, $N = 120$ and $\tau = 50$ N/mm²

Torque the shaft can transmit, $T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$

$$T = \frac{\pi}{16} \times 50 \times \left(\frac{100^4 - 40^4}{100}\right) = 9.56 \times 106 \text{ N} - \text{mm} = 9.56 \text{ kN} - \text{m}$$

Power the shaft can transmit, $P = \frac{2\pi N T}{60} = \frac{2\pi \times 120 \times 9.56}{60} = 120 \text{ kW}$

Question: Find the angle of twist per meter length of a hollow shaft of 100 mm external and 60 mm internal diameter, if the shear stress is not to exceed 35 MPa. Take G = 85 GPa.

Solution:

Given, $L = 1 \text{ m} = 1000 \text{ mm}, D = 100 \text{ mm}, d = 60 \text{ mm}, \tau = 35 \text{ N/mm}^2, G = 85 \times 103 \text{ N/mm}^2$

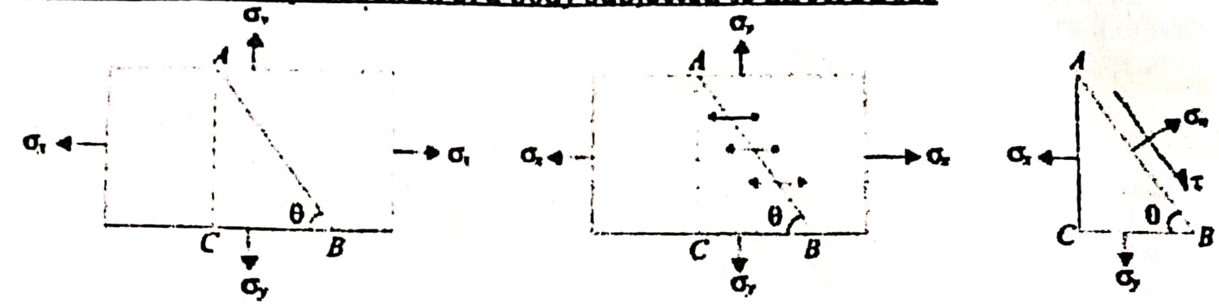
$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times 35 \times \left(\frac{100^4 - 60^4}{100}\right) = 5.98 \times 106 \text{ N} - \text{mm}$$

$$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(100^4 - 60^4) = 8.55 \times 106 \text{ mm}^4$$

We know, $\frac{T}{J} = \frac{G\theta}{L}$

$$\theta = \frac{TL}{JG} = \frac{5.98 \times 10^6 \times 1000}{8.55 \times 10^6 \times 85 \times 10^3} = 0.008 \text{ rad} = 0.5^0$$

Stresses on an oblique section of a body subjected to direct stress



σ_x = tensile stress along x − x axis (also termed as major tensile stress)
σ_y = tensile stress along y − y axis (also termed as minor tensile stress)
φ = Angle which the oblique section AB makes with x − x axis in the clockwise direction.

Normal stress across the section AB, $\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$

Shear stress across the section AB, $\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$

Maximum shear stress, $\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$

Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$

Question: The stresses at point a machine component are 150 Mpa and 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of major tensile stress. Also find the magnitude of the maximum shear stress in the component.

Solution:

Normal stress on the inclined plane, $\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$

$$\sigma_n = \frac{150 + 50}{2} - \frac{150 - 50}{2} \cos(2 \times 55^\circ) = 117.1 \text{ MPa}$$

Shear stress on the inclined plane, $\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$

$$\tau = \frac{150 - 50}{2} \sin(2 \times 55^\circ) = 47 \text{ MPa}$$

Resultant stress on the inclined plane, $\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$

$$\sigma_R = \sqrt{117.1^2 + 47^2} = 126.2 \text{ MPa}$$

Maximum shear stress in the component, $\tau_{max} = \pm \frac{\sigma_x - \sigma_y}{2}$

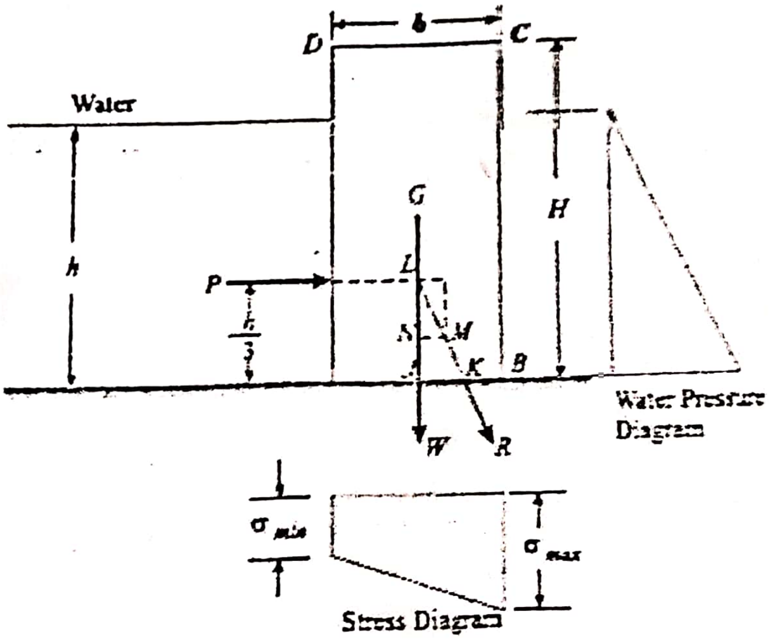
$$\tau_{max} = \pm \frac{150 - 50}{2} = \pm 50 \text{ MPa}$$

Question: What is gravity dam? Why need gravity dam? (ARMY – 2014)

__Solution:__

A gravity dam is a massive sized dam fabricated from concrete and designed to hold back large volumes of water. By using concrete, the weight of the dam is actually able to resist the horizontal thrust of water pushing against it. This is why it is called a gravity dam. Gravity essentially holds the dam down to the ground, stopping water from toppling it over.

Rectangular Dams: Consider a unit length of a rectangular dam, retaining water on one of its vertical sides as shown in figure.



b = width of the dam
H = height of the dam
w = specific weight of the water
h = height of water retained by the dam

Total pressure per unit length of the dam, $P = \frac{wh^2}{2}$

Maximum stress, $\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

Maximum stress, $\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

W = weight of the dam
e = ecentricity

Question: What are the forces acting on gravity dam? (PGCB—2017)

Solution:

In the design of a dam, the first step is the determination of various forces which acts on the structure and study their nature. Depending upon the situation, the dam is subjected to the following forces:

- Water pressure
- Earthquake forces
- Silt pressure
- Wave pressure
- Ice pressure
- Uplift pressure
- The stabilizing force is the weight of the dam itself.

Question: A concrete dam of rectangular section 15 m high and 6 m wide contains water up to a height of 13 m. Find (a) total pressure per meter length of the dam (b) point, where the resultant cuts the base (c) maximum and minimum intensities of stress at the base. Assume weight of water and concrete as 10 and 25 kN/m³ respectively.

Solution:

(a) total pressure per meter length of the dam

Total pressure, $P = \frac{w h^2}{2} = \frac{10 \times 13^2}{2} = 845\ kN$

(b) point where the resultant cuts the base

Weight of the concrete per meter length, $W = \rho b H = 25 \times 6 \times 15 = 2250\ kN$

Horizontal distance between the centre of gravity of the dam section and the point where the resultant cuts the base, $x = \frac{P}{W} \times \frac{h}{3} = \frac{845}{2250} \times \frac{13}{3} = 1.63\ m$

(c) Maximum and minimum intensities of stress at the base.

Eccentricity, e = 1.63 m

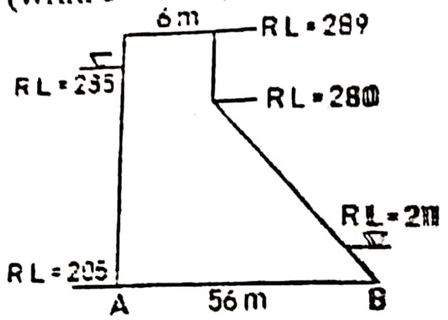
Maximum stress, $\sigma_{max} = \frac{W}{b}(1 + \frac{6e}{b})$

$\sigma_{max} = \frac{2250}{6}(1 + \frac{6 \times 1.63}{6}) = 986.25\ kN/m^2$ (Compression)

Maximum stress, $\sigma_{min} = \frac{W}{b}(1 - \frac{6e}{b})$

$\sigma_{min} = \frac{2250}{6}(1 - \frac{6 \times 1.63}{6}) = -236.25\ kN/m^2$ (Tension)

Question: Figure shows the section of a gravity dam built of concrete Calculate the uplift pressure. (WARPO – 2017)



Solution:

Uplift pressure at A = (285 – 205) x 9.81 = 784.8 KN/m^2
Uplift pressure at B = (211 – 205) x 9.81 = 58.86 KN/m^2

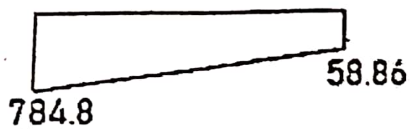


Figure: Pressure diagram

Uplift pressure = 0.5 x (784.8 + 58.86) x 56 = 23622.48 KN/m

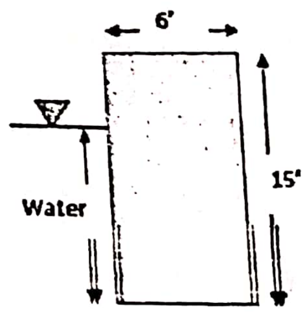
Question: A wall resisting water pressure as the moment from the wall is 3 times moment from the water pressure. Find the height of the water level. (LGD – 2018)

Solution:

Water pressure, $P = \gamma h = 62.4$ h lb/ft^2

Water force $= \frac{1}{2} P h = 0.5 \times 62.4 \times h \times h = 31.2 h^2$ lb/ft.

Moment, Mw $= \frac{h}{3} \times 31.2 h^2 = 10.4$ h3 lb – ft/ft.



Weight of wall, $W_c = 15 \times 6 \times 1 \times 150 = 13500$ lb/ft.

Moment at A, $Mc = 13500 \times 3 = 40500$ lb – ft/ft.

Now, $\frac{M_C}{M_W} = \frac{40500}{10.4 h^3} = 3$

$\therefore h = 10.9$ ft

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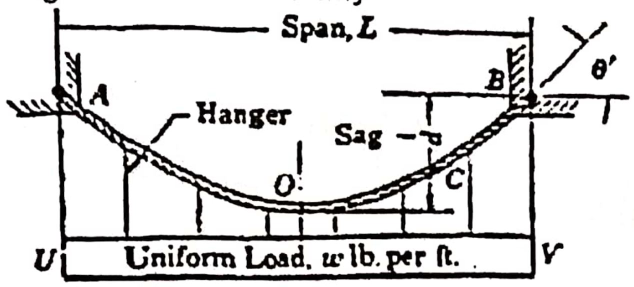
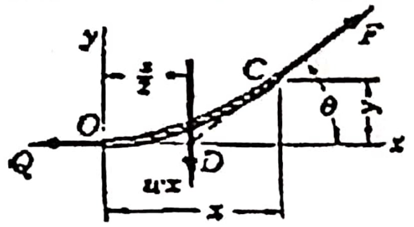
Flexible cords: When the load on the cord (or cable) is uniform horizontally, the cord is curved to form a parabola. An approximation of this situation is suggested by following Fig. where the uniform load UV may be considered as the road bed of a suspension bridge supported from the cable *AOB* by the hangers. In order that the curve AOB should be a parabola, an infinite number of hangers would be necessary.



To prove that AOB is a parabola, choose as a free body a section of the cord *OC*.



$\Sigma F_x = F\cos\theta - Q = 0$

$Q = F\cos\theta$

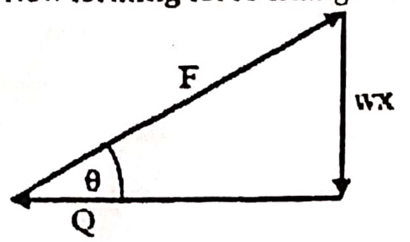
$\Sigma F_y = F\sin\theta - wx = 0$

$wx = F\sin\theta$

$\Sigma M_c = Q\,y - \dfrac{w\,x^2}{2}$

$y = \dfrac{w\,x^2}{2\,Q}$ Which is an equation of parabola

Now forming force triangle of the forces Q, wx and F



$F^2 = Q^2 + (wx)^2$

$F = \sqrt{Q^2 + (wx)^2}$

From the above equation we see that the tension, F increases as x increases. The maximum tension F' occurs at the supports A and B. At B, $x = L/2$, hence the maximum tension is

$$F' = \left[Q^2 + \left(\frac{wL}{2}\right)^2\right]^{1/2}$$

The value of Q may be found from equation for any simultaneous values of x and y. Assume $y =$ d, the sag, and $x = L/2$, the half span. we have,

$$Q = \frac{w\,x^2}{2\,y} = \frac{w\,L^2}{8\,h}$$

$$\tan\theta = \frac{dy}{dx} = \frac{w\,L^2}{8\,d}$$

At the supports, $\tan\theta' = \frac{w\,x}{Q} = \frac{w\,L}{2\,Q} = \frac{4\,d}{L}$

Question: A cable is suspended between two points on the same level with a span of 500 ft. and a sag of 50 ft. If the load is 2 lb per ft. uniformly distributed horizontally, find (a) the tension in the cable at the low point (b) the maximum tension (c) the slope of the cable at the supports.

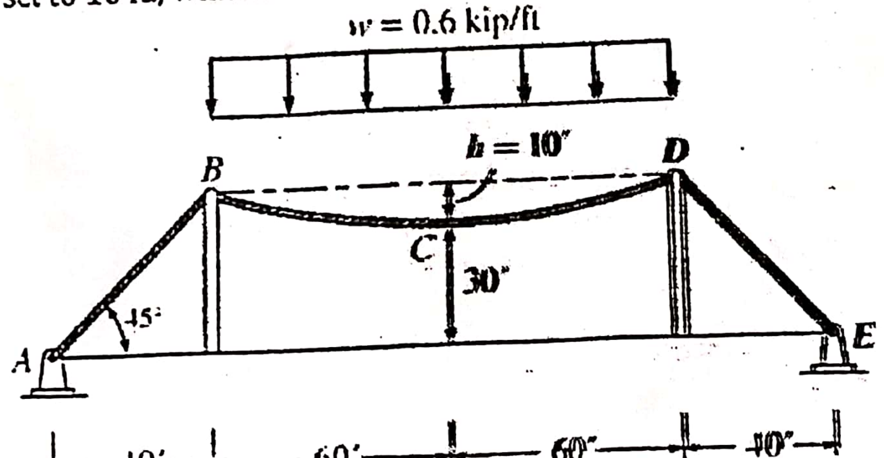
Solution:

Tension in the cable, $Q = \frac{w\,L^2}{8\,d} = \frac{20 \times 500^2}{8 \times 50} = 1250$ lb

Maximum tension, $F' = \left[Q^2 + \left(\frac{w\,L}{2}\right)^2\right]^{1/2} = \left[1250^2 + \left(\frac{2 \times 500}{2}\right)^2\right]^{1/2} = 1346$ lb

$\theta' = \tan^{-1}\frac{4\,d}{L} = \frac{4 \times 50}{500} = 21.8^0$

Question: A cable-supported roof carries a uniform load as shown. If the cable sag at mid-span is set to 10 ft., what is the maximum tension in the cable between points B and D. (BWDB – 2020)



Solution:

Tension in the cord at C, $Q = \frac{w\,L^2}{8\,h} = \frac{0.6 \times 120^2}{8 \times 10} = 108$ kip

Maximum tension, $F' = \left[Q^2 + \left(\frac{w\,L}{2}\right)^2\right]^{1/2} = \left[108^2 + \left(\frac{0.6 \times 120}{2}\right)^2\right]^{1/2} = 113.84$ kip

Question: A cable is suspended with its ends at the same elevation and 200 ft apart. The load is uniformly distributed horizontally. When the sag is 5 % of the span, the maximum tension is 2040 lb. What is the load in pounds per foot?

Solution:

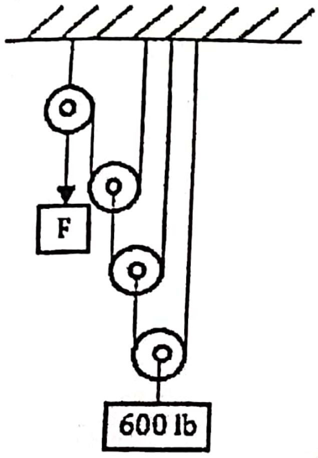
$$F' = \sqrt{Q^2 + \frac{w^2 L^2}{4}}$$

$$F'^2 = \left(\frac{w L^2}{8 d}\right)^2 + \frac{w^2 L^2}{4}$$

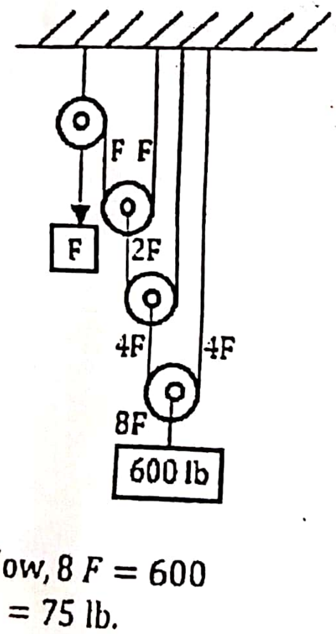
$$4161600 = 250000 \times w^2 + 10000 \times w^2$$

$$w = 4 \text{ lb/ft}$$

Question: In the system of sheaves shown in figure, what force F will hold a weight 600 lb in equilibrium? There are no frictional losses at the axes. (CUET M. Sc – 2020)



Solution:



Now, $8F = 600$

$F = 75$ lb.

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