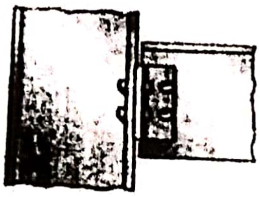
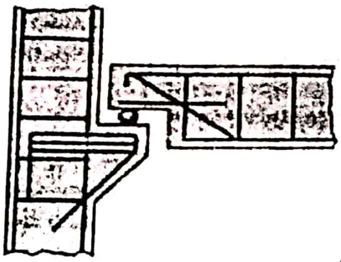
Structural Analysis

Structural Analysis

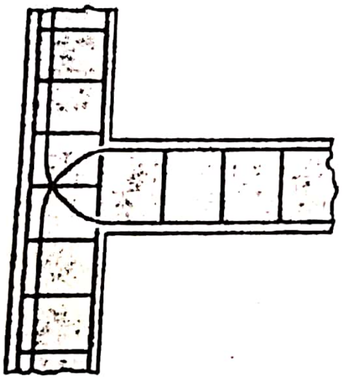
Support: Structural members are joined together in various ways depending on the intent of the designer. Support in a structure is a member which helps others member to resist loads. Supports in a structure transfers the load to the ground and provides stability to the structure supported on it. The three types of supports most often specified are the roller support, the pinned support and the fixed support.



Roller supports only resist perpendicular forces and they cannot resist parallel or horizontal forces and moment. It means, the roller support will move freely along the surface without resisting horizontal force. This type of support is provided at one end of bridge spans. The reason for providing roller support at one end is to allow contraction or expansion of bridge deck with respect to temperature differences in atmosphere.



Pinned support or hinged support can resists both vertical and horizontal forces but they cannot resist moment. It means hinged support is restrained against translation. The pinned support allows the rotation only in one direction and resists the rotation in any other direction. The best example where we can see the pinned support is the doors and windows of our houses and our knee joint. Here the rotation happens in one direction but the translation motion is restricted.



Fixed supports are restrained against both rotation and translation so they can resist vertical and horizontal forces as well as moment. The fixed support is also called as rigid support. It provides the greater stability to the structure as compared with all other supports.

Equations of Equilibrium

A structure or one of its members is in equilibrium when it maintains a balance of force and moment. In general this requires that the force and moment equations of equilibrium be satisfied along three independent axes, namely,

$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$

$\sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0$

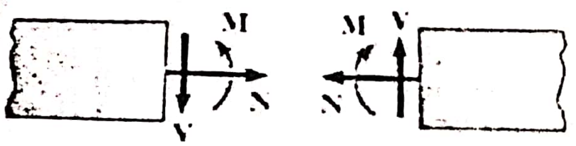


Figure: Internal loadings

For a plane structure lying in the $x\,y$ plane and subjected to a coplanar system of forces and couples, the necessary and sufficient conditions for equilibrium can be expressed as,

$\sum F_x = 0, \sum F_y = 0, \sum M_z = 0$

For space structure conditions for equilibrium can be expressed as,

$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$

$\sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0$

Statically Determinate structure: When all the forces in a structure is determined from equilibrium equations $(\sum f_x, \sum f_y, \sum M)$, the structure is known as statically determinate.

Statically Indeterminate structure: When the unknown forces in a structure are more than the available equilibrium equations that structure is known as statically indeterminate structure.

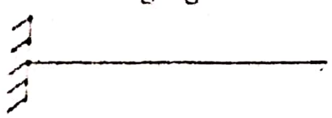
Statically indeterminate structures can't be analyzed using statics equations only; they require other material properties, such as deformations, in order to analyze them.

<u>Statically determinate beam</u>

→ Cantiliver beam

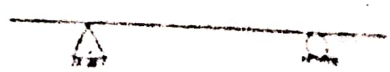
→ Simply supported beam

→ Over hanging beam



Cantilever beam

Simply supported beam



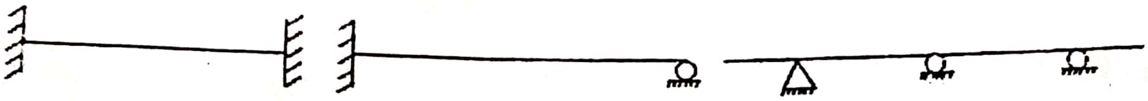
Overhanging beam

Statically indeterminate beam

→ Fixed beam

→ Propped cantilever

→ Continious beam



Fixed beam   Propped cantilever   Continuous beam

Determinacy

If a structure is supported by more than three reactions, then all the reactions cannot be determined from the three equations of equilibrium. Such structures are termed statically indeterminate externally. The reactions in excess of those necessary for equilibrium are called external redundants and the number of external redundants is referred to as the degree of external indeterminacy. Thus, if a structure has R reactions of $R > 3$, then the degree of external indeterminacy can be written as

$$E = R - 3$$

A structure is called statically determinate externally if all of its support reactions can be determined by solving the equations of equilibrium and condition.

If, $r = 3n$ → Statically determinate structure

If, $r > 3n$ → Statically indeterminate structure

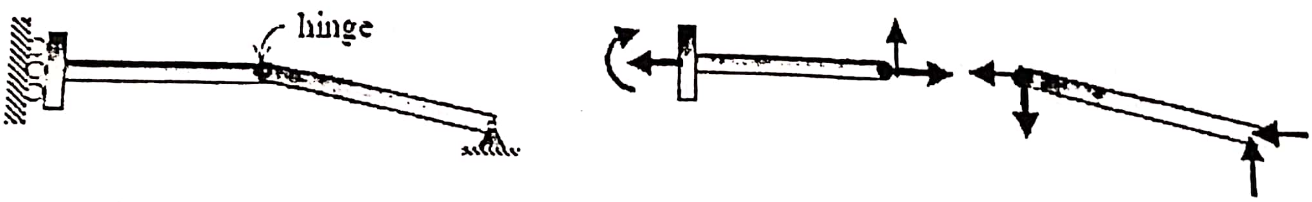
If, $r < 3n$ → Statically unstable

n = the total parts of structure members.

r = the total number of unknown reactive force and moment components

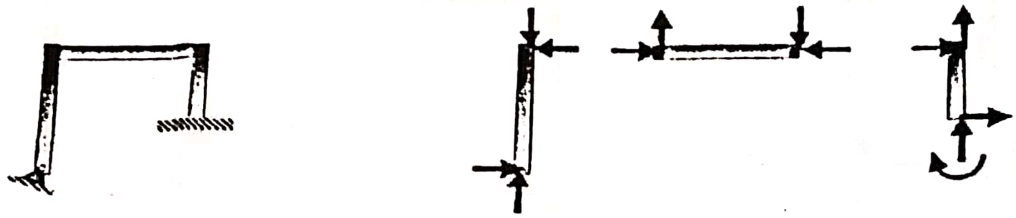
$r = 3$, $n = 1$, $3 = 3\,(1)$ statically determinate

$r = 5$, $n = 1$, $5 - 3\,(1) = 2$ statically indeterminate to the second degree

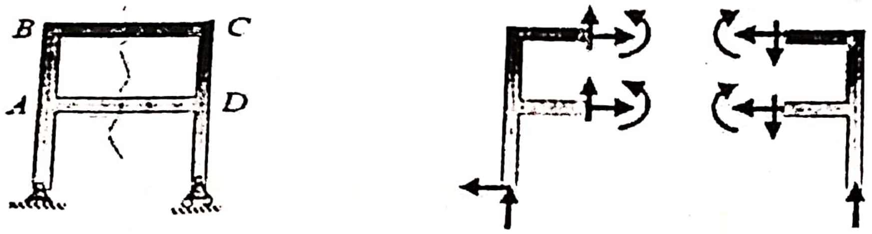


$r = 6$, $n = 2$, $6 = 3\,(2)$ statically determinate

$r = 10, n = 3, 10 - 3\,(3) = 1$ statically indeterminate to the first degree



$r = 9, n = 3, 9 = 3\,(3)$ statically determinate



$r = 9, n = 2, 9 - 6 = 3$ statically indeterminate to the third degree

Stability

A structure is considered to be internally stable or rigid, if it maintains its shape and remains a rigid body when detached from the supports. Conversely, a structure is termed internally unstable or non-rigid if it cannot maintain its shape and may undergo large displacements under small disturbances when not supported externally. If a structure is supported by fewer than three support reactions, the reactions are not sufficient to prevent all possible movements of the structure in its plane. Such a structure cannot remain in equilibrium under a general system of loads and is referred to as statically unstable externally.

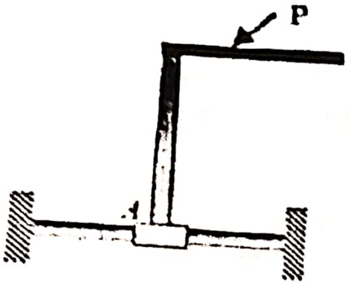
If, $r < 3n$ → Statically unstable externally
If, $r > 3n$ → Statically unstable if member reactions are concurrent or parallel.
n = the total parts of structure members.

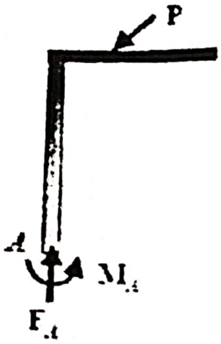
External Instability: Instability due to insufficient number or arrangement of external supports.

Internal Stability: Number and arrangement of members is such that the truss does not change its shape when detached from the supports.

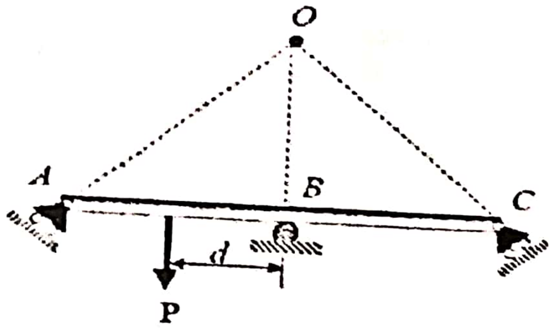
Condition for unstable structure.

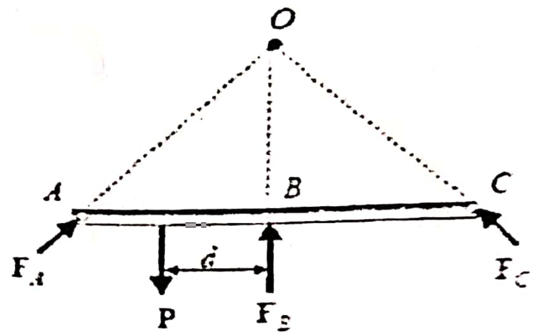
1. Reaction less than 3



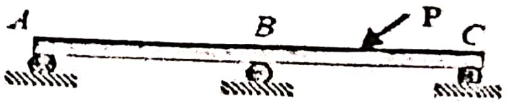


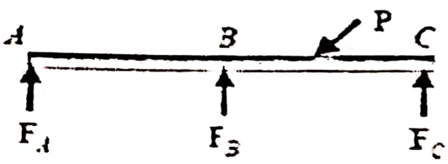
2. If all forces are concurrent



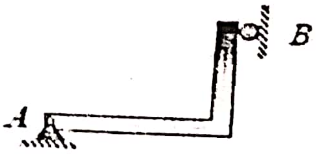


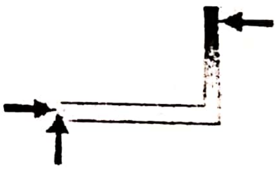
3. Reactions are parallel



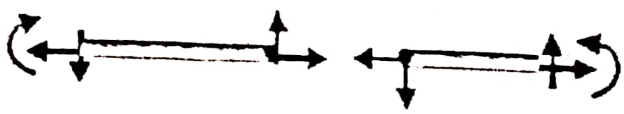


Some example:



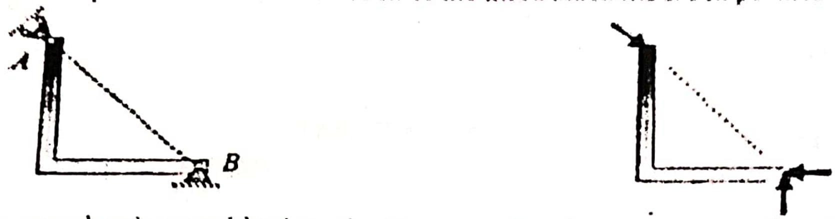


The member is stable since the reactions are non-concurrent and nonparallel. It is also statically determinate.

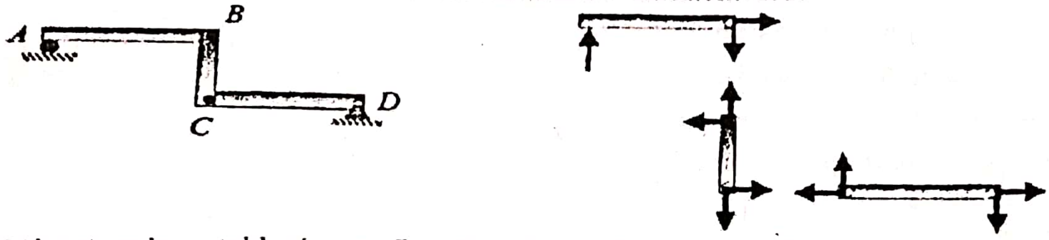


The compound beam is stable. It is also indeterminate to the second degree

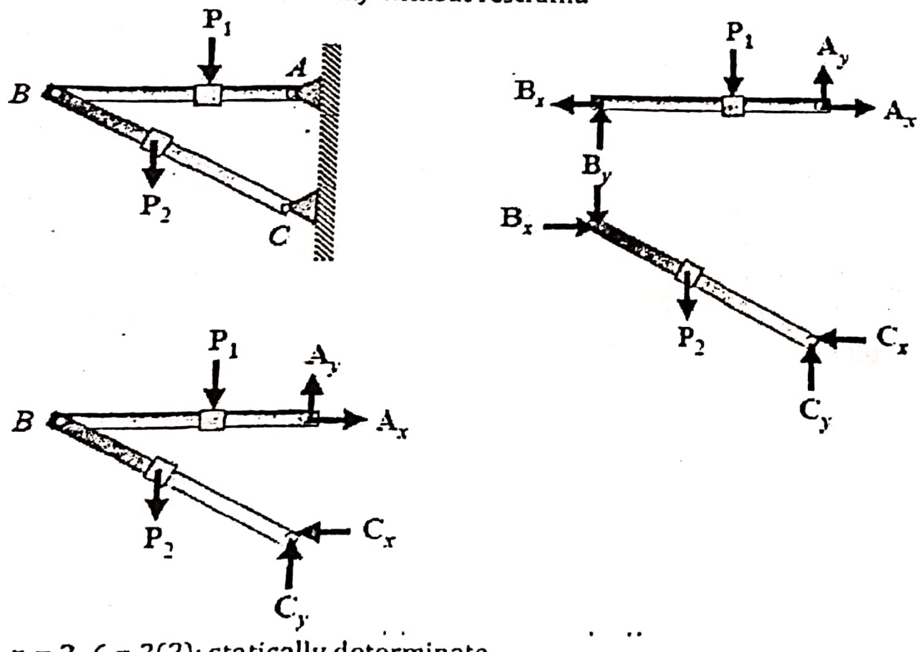
The compound beam is unstable since the three reactions are all parallel.



The member is unstable since the three reactions are concurrent at *B*.



The structure is unstable since $r = 7$, $n = 3$, so that, $r < 3n$, $7 < 9$. Also, this can be seen by inspection, since *AB* can move horizontally without restraint.



$r = 6$, $n = 2$, $6 = 3(2)$; statically determinate

Rigid structure (Plane Frame)

External indeterminacy = Reaction components − number of equations available

$$E = r - 3$$

Internal indeterminacy = 3 x closed frames

$$I = 3 \times a$$

Total indeterminacy = External indeterminacy + Internal indeterminacy

$T = E + I = r - 3 + 3a$

Or, Degree of static indeterminacy, $T = (3m + r) - 3j$

$3m + r < 3j$ → Statically unstable frame

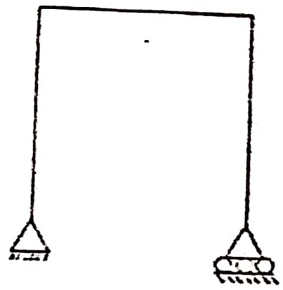
$3m + r = 3j$ → Statically determinate frame

$3m + r > 3j$ → Statically indeterminate frame

r = number of support reactions

m = number of members

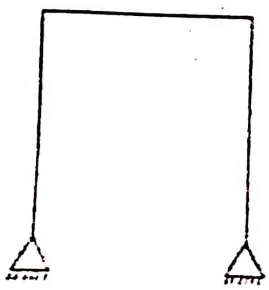
j = number of joints



$T = (3m + r) - 3j = (3 \times 3 + 3) - 3 \times 4 = 0$

$Or, T = E + I = r - 3 + 3a = 3 - 3 - 3 \times 0 = 0$

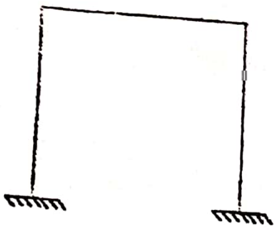
Statically determinate structure



$T = (3m + r) - 3j = (3 \times 3 + 4) - 3 \times 4 = 1$

$Or, T = E + I = r - 3 + 3a = 4 - 3 - 3 \times 0 = 1$

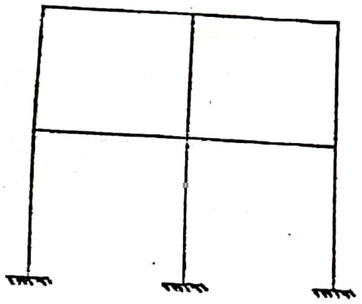
Statically indeterminate to 1st degree



$I = (3\,m + r) - 3\,j = (3 \times 3 + 6) - 3 \times 4 = 3$

$Or, T = E + I = r - 3 + 3\,a = 6 - 3 + 3 \times 0 = 3$

Statically indeterminate to 3rd degree



$I = (3\,m + r) - 3\,j = (3 \times 10 + 9) - 3 \times 9 = 12$

$Or, T = E + I = r - 3 + 3\,a = 9 - 3 + 3 \times 2 = 12$

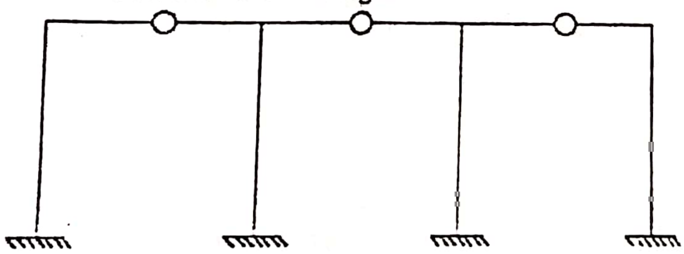
Statically indeterminate to 12th degree

Indeterminacy in rigid frames with internal hinges

(a) Hinges within a member

The introduction of an internal hinge in a rigid frame provides an additional equation of equilibrium at the hinge of $M = 0$. In effect, a moment release has been introduced in the member. Degree of static indeterminacy, $T = (3\,m + r) - (3\,j + h)$

h = number of internal hinges

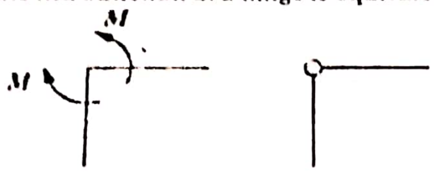


$T = (3\,m + r) - (3\,j + h) = (3 \times 7 + 12) - (3 \times 8 + 3) = 6$

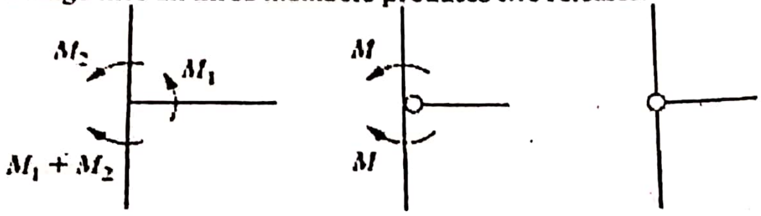
Statically indeterminate to 6th degree

(b) Hinges at a joint

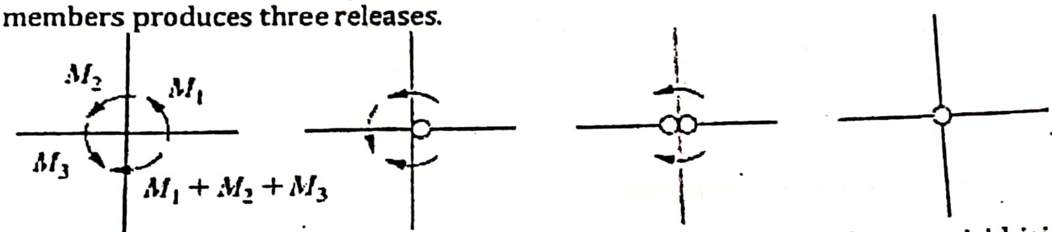
For two members meeting at a rigid joint there is one unknown moment, as shown in Figure, and the introduction of a hinge is equivalent to producing one release.



For three members meeting at a rigid joint there are two unknown moments, as shown in Figure, and the introduction of a hinge into one of the members produces one release; the introduction of a hinge into all three members produces two releases.

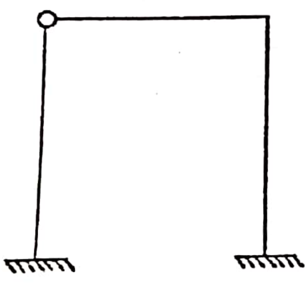


For four members meeting at a rigid joint there are three unknown moments, as shown in Figure; the introduction of a hinge into one of the members produces one release, the introduction of a hinge into two members produces two releases, and the introduction of a hinge into all four members produces three releases.



In general, the introduction of hinges into i of the n members meeting at a rigid joint produces i releases. The introduction of a hinge into all n members produces $(n - 1)$ releases.
Degree of static indeterminacy, $T = (3m + r) - (3j + C)$

Where C is the number of releases introduced.



Degree of static indeterminacy, $T = (3m + r) - (3j + C) = (3 \times 3 + 6) - (3 \times 4 + 1) = 2$

Statically indeterminate to 2nd degree

Rigid structure (Space frame)

External indeterminacy = Reaction components − number of equations available

$$E = r - 6$$

Internal indeterminacy = 6 x closed frames

$$I = 6 \times a$$

Total indeterminacy = External indeterminacy + Internal indeterminacy

$T = E + I = r - 6 + 6a$

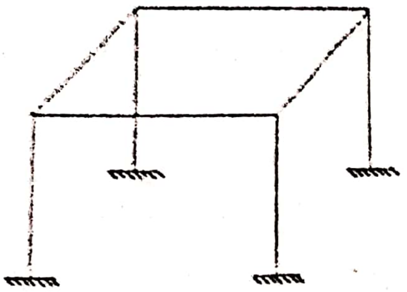
Degree of static indeterminacy, $T = (6m + r) - 6j$

For hinges within a member, $T = (6m + r) - (6j + h)$

For hinges at a joint, $T = (3m + r) - (3j + C)$

h = number of internal hinges
C = number of releases introduced

$C = 3(n - 1)$, n = members meeting at a rigid joint



Degree of static indeterminacy, $T = (6m + r) - 6j = (6 \times 8 + 24) - 6 \times 8 = 24$

Or, $T = E + I = r - 6 + 6a = 4 \times 6 - 6 + 6 \times 1 = 24$

Statically indeterminate to 24th degree

Plane truss

A truss is called statically determinate if the forces in all its members, as well as all the external reactions can be determined by using the equations of equilibrium.

External indeterminacy = Reaction components - number of equations available

$$E = r - 3$$

A simple truss is formed by enlarging the basic truss element, which contains three members and three joints, by adding two additional members for each additional joint, so the total number of members m in a simple truss is given by,

Hence, number of members for stability, $m = 3 + 2(j - 3) = 2j - 3$

Hence, internal indeterminacy, $I = m - (2j - 3)$

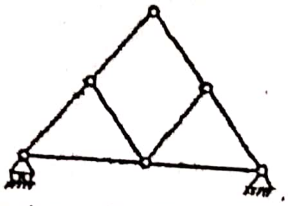
Total (Internal and external) indeterminacy, $T = E + I = r - 3 + m - (2j - 3) = m + r - 2j$

m = number of members

r = number of reaction components

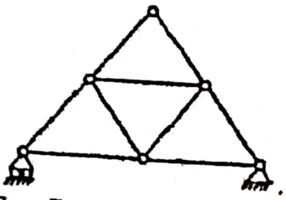
j = number of joints

Internal stability	
Case	Decisions
If, $m + r = 2j$	Truss is internally stable and statically determinate.
If, $m + r > 2j$	Truss is internally stable and statically indeterminate.
If, $m + r < 2j$	Truss is internally unstable.



$m + r = 8 + 3 = 11 < 2i = 2 \times 6 = 12$

$m + r < 2j$, Truss is internally unstable

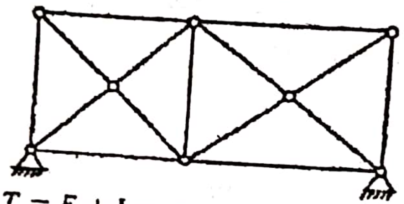


$T = E + I = m + r - 2j = 9 + 3 - 2 \times 6 = 0$

$E = r - 3 = 3 - 3 = 0$

$I = T - E = 0$

Statically determinate

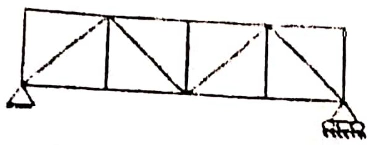


$T = E + I = m + r - 2j = 15 + 4 - 2 \times 8 = 3$

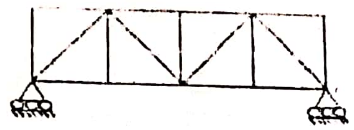
$E = r - 3 = 4 - 3 = 1$

$I = T - E = 2$

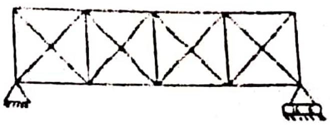
Statically indeterminate to 3rd degree



$m = 17, j = 10, r = 3$
$m + r = 2j$
Statically determinate



$m = 17, j = 10, r = 2$
$m + r < 2j$
Unstable



$m = 21, j = 10, r = 3$
$m + r > 2j$
Statically indeterminate

Space Truss

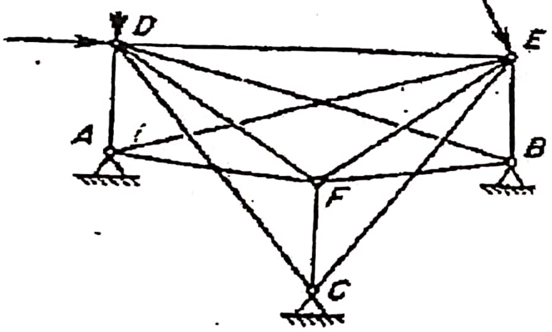
Minimum number of members required for stability, $m = 3j - 6$
Total indeterminacy, $T = E + I = m + r - 3j$

m = number of members

r = number of reaction components

j = number of joints

Case	Decisions
If, $m + r < 3j$	Truss is internally unstable.
If, $m + r = 3j$	statically determinate – check stability
If, $m + r > 3j$	statically indeterminate – check stability



$T = E + I = m + r - 3j = 12 + 9 - 3\text{x}6 = 3$

$E = r - 6 = 9 - 6 = 3$

$I = T - E = 0$

Statically indeterminate to 3rd degree

Types of loading

1. Concentrated load
2. Uniformly distributed load (UDL)
3. Uniformly varying load (UVL)
4. Couples

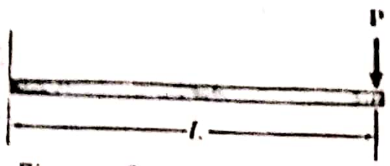


Figure: Concentrated load

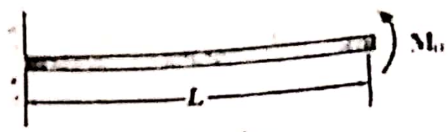


Figure: Couples

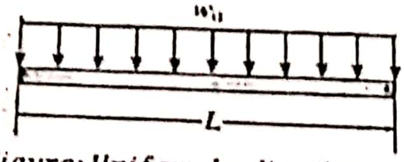


Figure: Uniformly distributed load

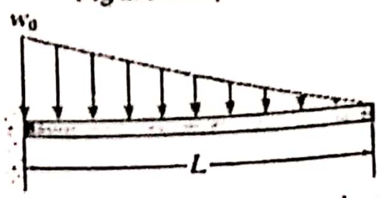


Figure: Uniformy varying load

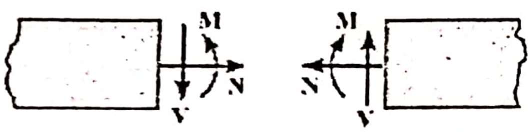
The shear force at the cross-section of a beam may be defined as the unbalanced vertical force to the right or left of the section. A shear force diagram (SFD) represents the magnitude and direction of the total shear force acting across any section of a beam.

The bending moment at the cross section of a beam may be defined as the algebraic sum of the moments of the forces to the right or left of the section.

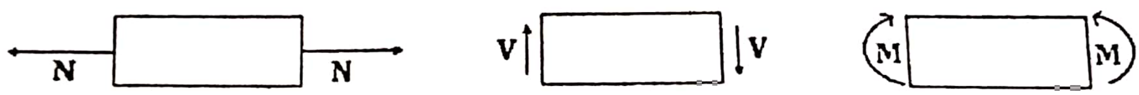
The axial force diagram represents the magnitude and type (compressive/tensile) force anywhere along the structure.

<u>Sign Convention of Axial force, Shear force and bending moment</u>

On the left-hand face of the cut member the normal force N acts to the right, the internal shear force V acts downward and the moment M acts counterclockwise. In accordance with Newton's third law, an equal but opposite normal force, shear force and bending moment must act on the right-hand face of the member at the section.

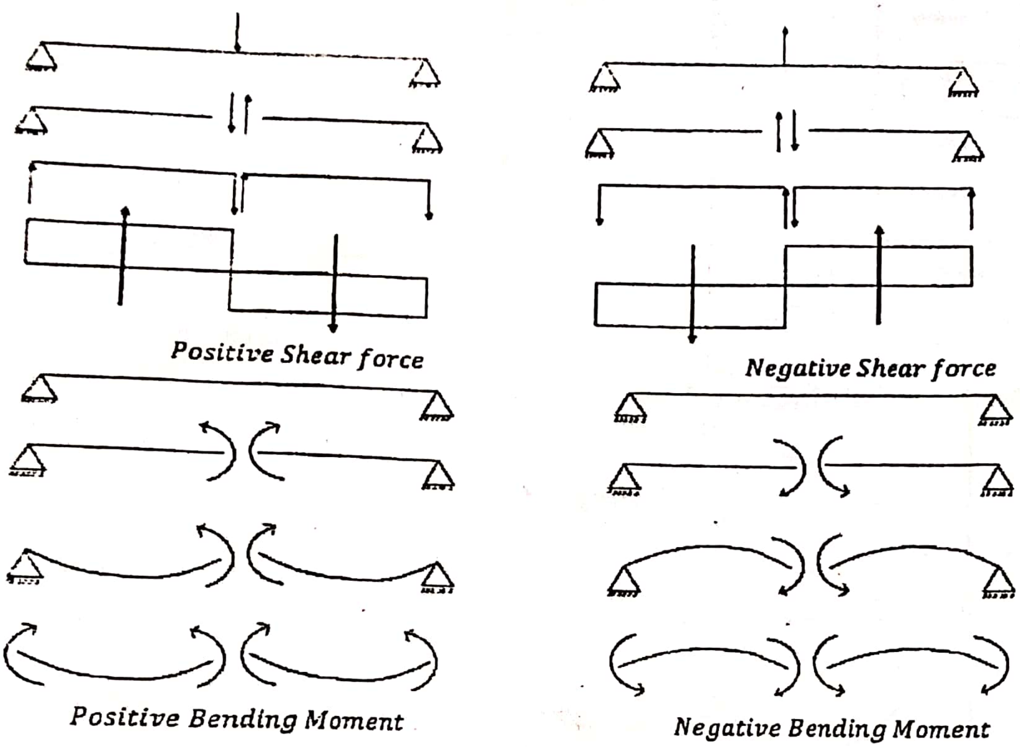


Positive internal axial force, shear force and bending moment at a section.



External forces causing positive axial force, shear force and bending moment.

Positive Axial Force (Tension) *Negative Axial Force (Compression)*



Tension is assumed positive for axial force, shear forces forming clockwise couple are considered positive while a bending moment causing sagging shape is taken as positive.

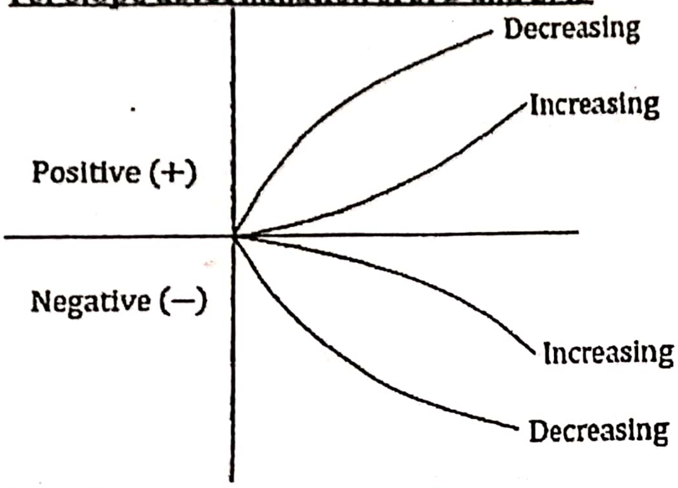
	Effects	Action
Loading	Shear force	Design shear reinforcement
Loading	Bending moment	Design flexural reinforcement

$w(x)$	Concentrated load	UDL	Triangular
$V(x)$	Straight line, No slope	Sloped straight line	Parabolic
$M(x)$	Sloped straight line	Parabolic	Cubic

Table: Nature of Load of SF and BM

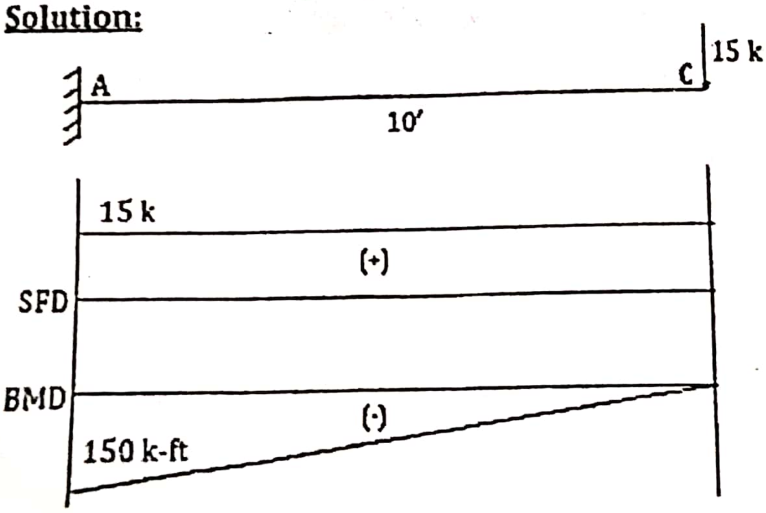
Load	Slope for shear force	Slope for bending Moment
P	Constant	Linear
Uniformly distributed load	Linear	Parabolic
Uniformly varying load	Parabolic	Cubic

For slope determination of SFD and BMD



Question: In a cantilever beam subjected to 15 kips load in free end. Determine the maximum bending moment and shear force. (BIFPCL-2015)

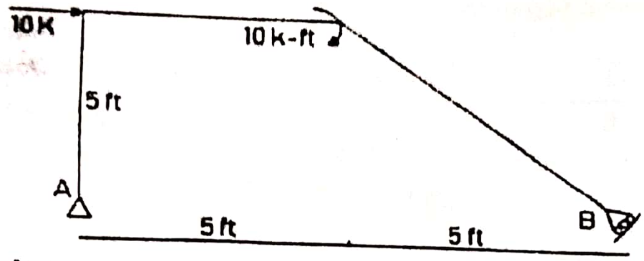
Solution:



Maximum bending moment = 150 kip-ft

Maximum shear force = 15 kip

Question: Find the reactions at supports for the following structure. (BWDB – 2018)



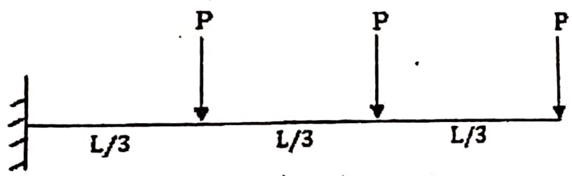
Solution:

$\sum M_A = 5 \times 10 + 10 - R_B \sin45 \times 10 = 0$
$R_B = 6\sqrt{2}$ k

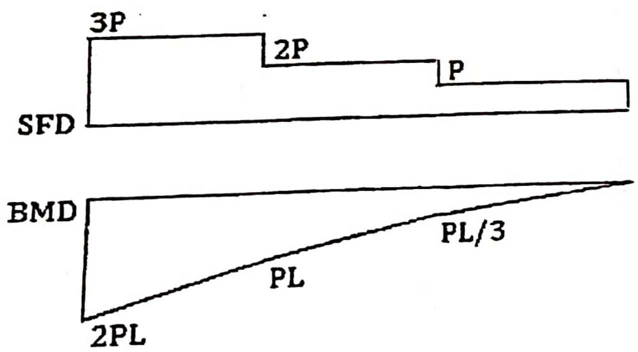
$\sum F_Y = 0$
$R_{AY} + R_B \sin45 = 0$
$R_{AY} = 6$ k

$\sum F_x = 0$
$R_{AX} + 10 - R_B \cos45 = 0$
$R_{AX} = -4$ k

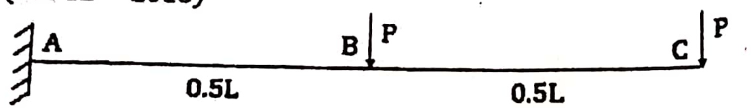
Question: Draw SFD and BMD for the following structure. (DNCC – 2016)



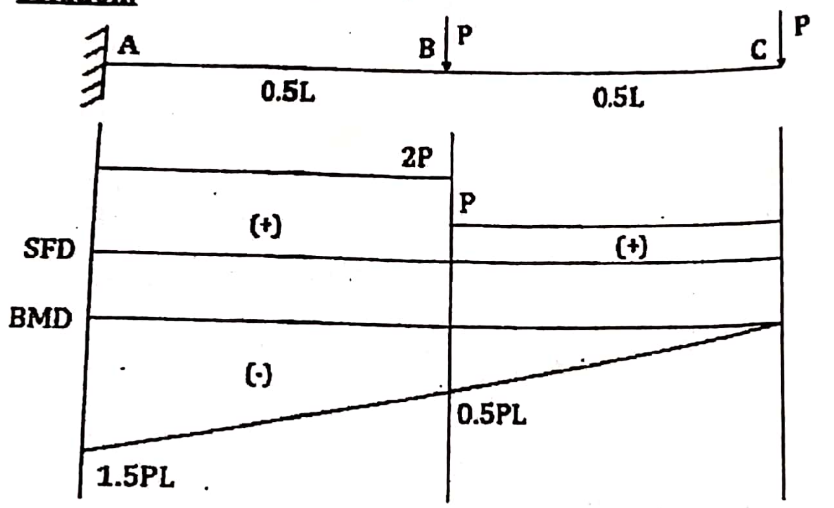
Solution:



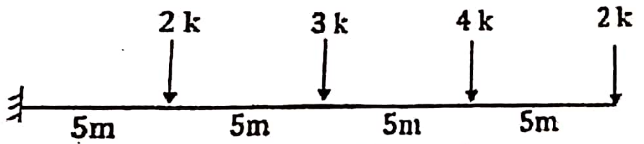
Question: Draw Shear force and bending moment diagram for the following beam. (BCPCL – 2016)



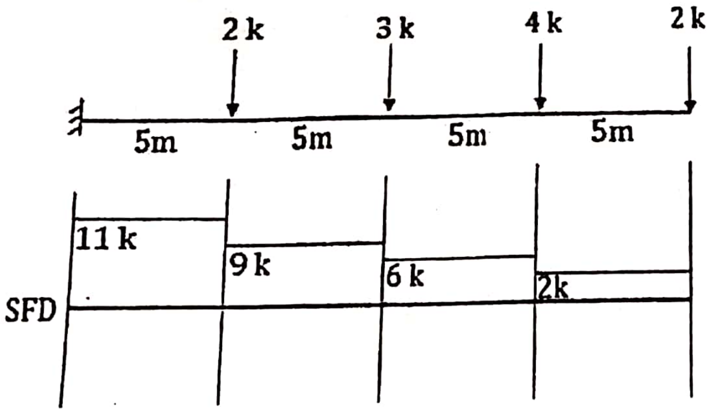
Solution:



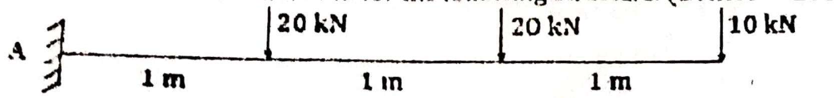
Question: Draw shear force diagram for the following beam. (NHA – 2020)



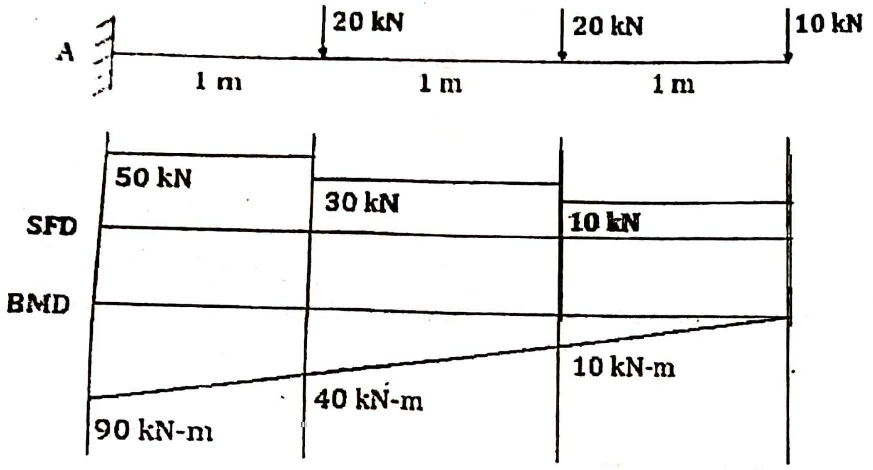
Solution:



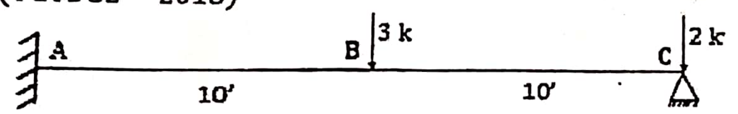
Question: Draw SFD and BMD for the following structure. (DMRTP – 2018)



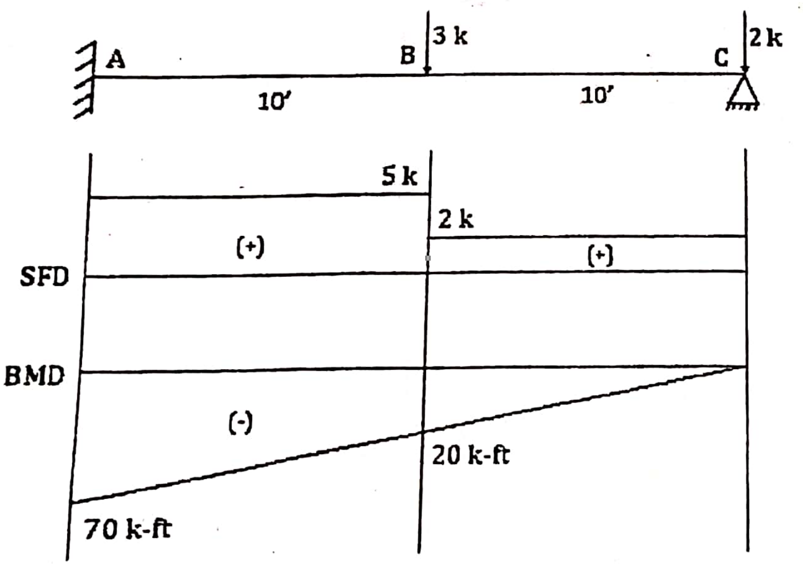
Solution:



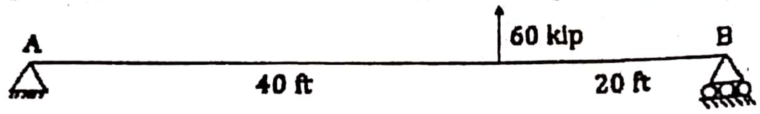
Question: Draw Shear force and bending moment diagram for the following beam. (TGTDCL – 2018)



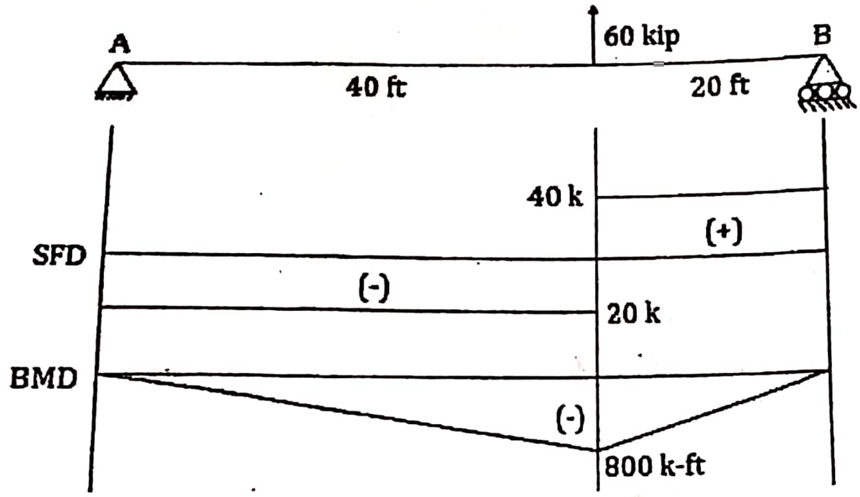
Solution:



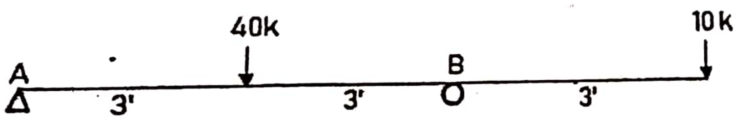
Question: Draw SFD and BMD for the following beam. (BPDB – 2016, CPGCBL – 2018)



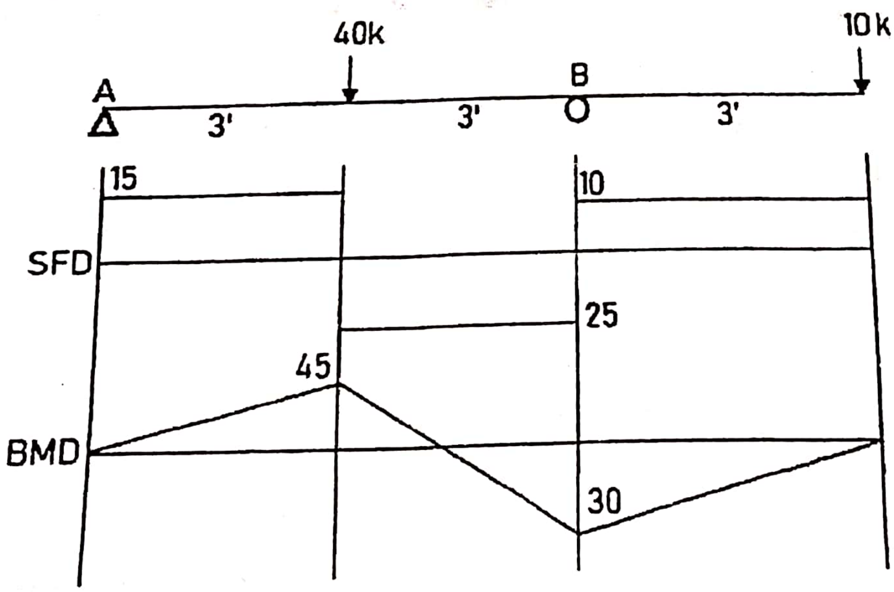
Solution:



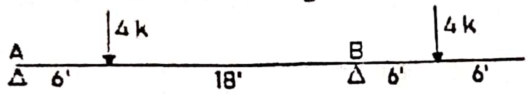
Question: Draw SFD and BMD for the following structure. (WRGCL – 2014)



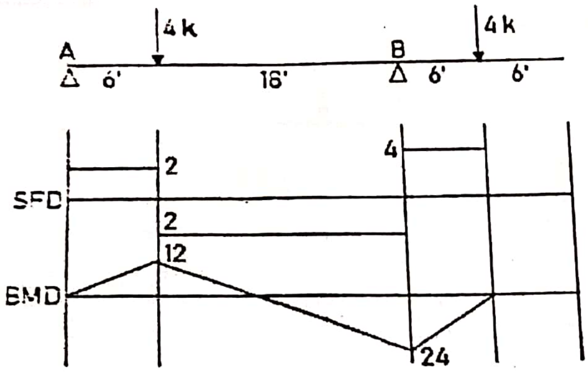
Solution:



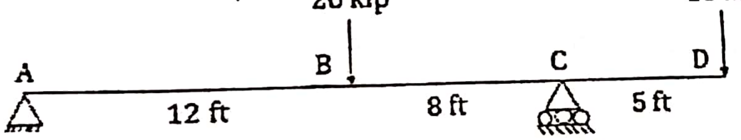
Question: Draw the bending moment diagram of the below structure. (BPDB – 2015)



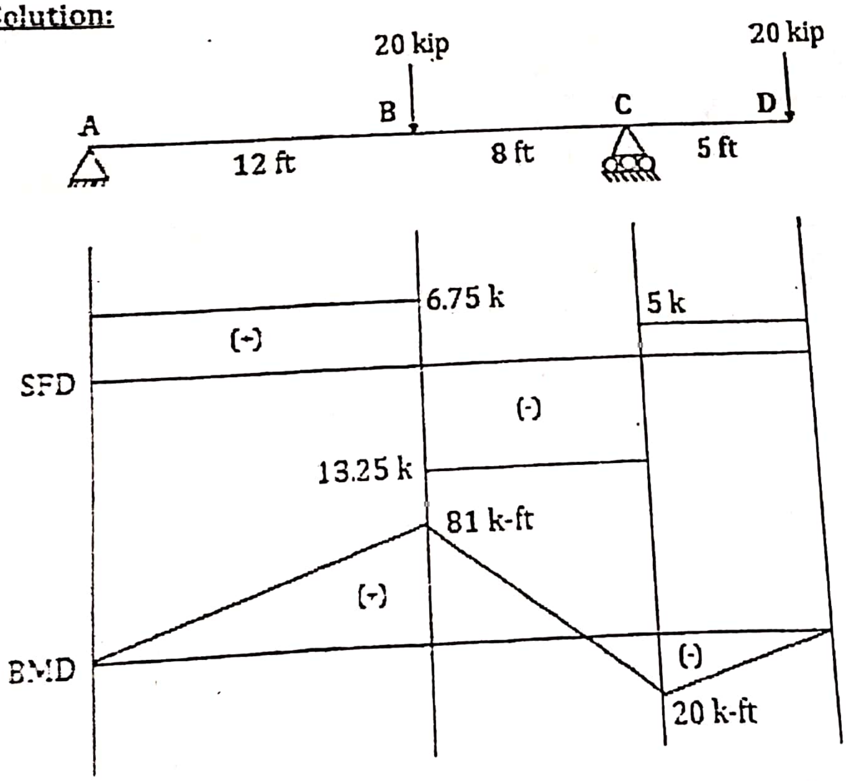
Solution:



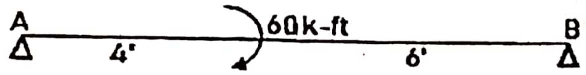
Question: Draw Shear force and bending moment diagram for the following beam. (EED – 2019)



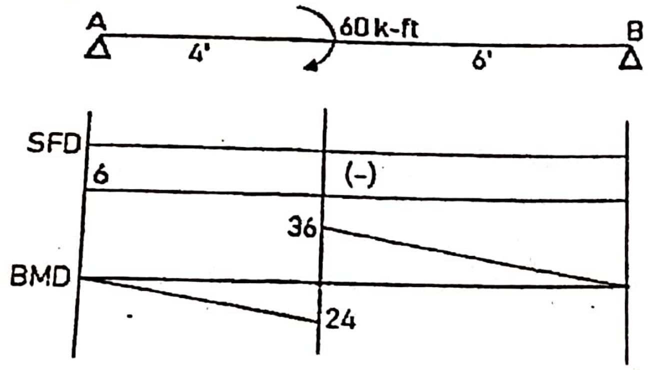
Solution:



Question: Draw shear force and bending moment diagram in the following beam. (PGCB – 2015, EGCB – 2015)

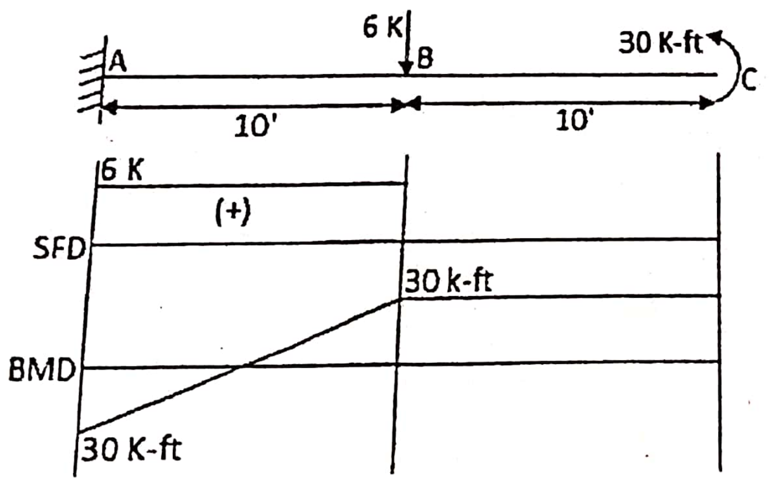


Solution:

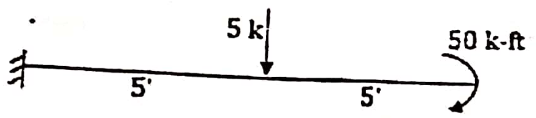


Question: Draw the bending moment diagram for the beam and show the main tensile reinforcement. (PGCL – 2014)

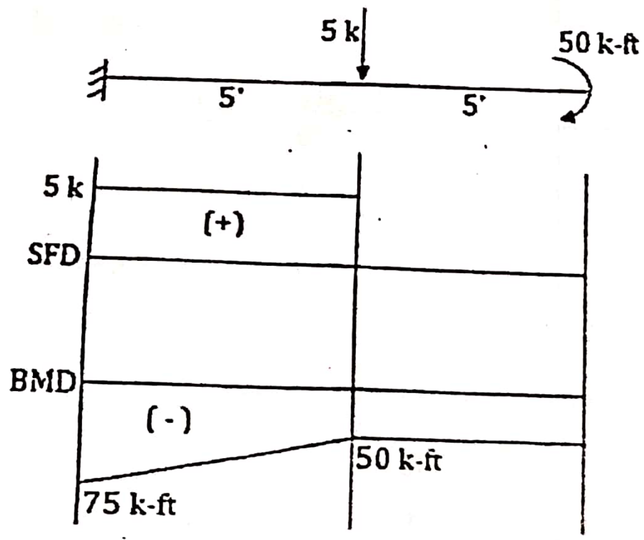
Solution:



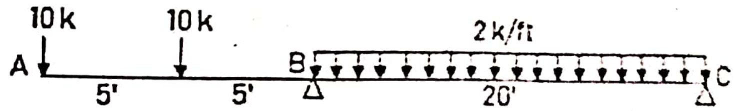
Question: Draw SFD and BMD for the following beam. (BUET M.Sc – 2019)



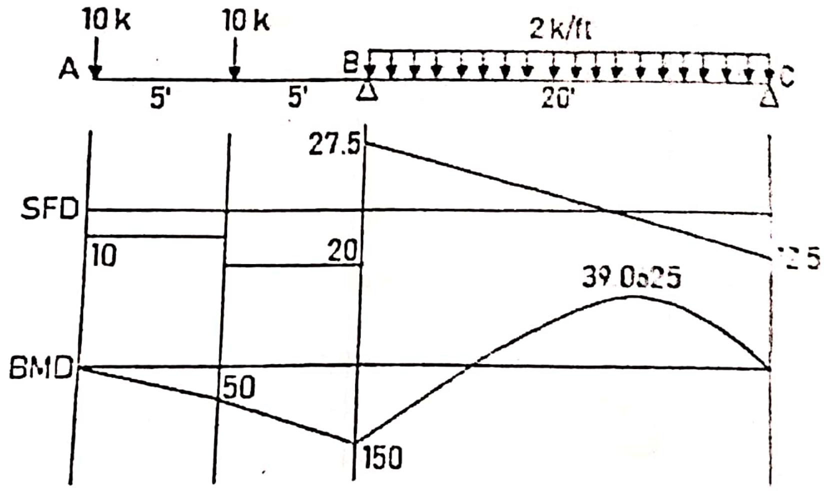
Solution:



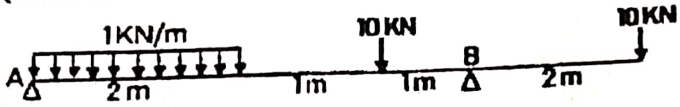
Question: Draw the SFD and BMD of the following beam. (TGTDCL – 2014)



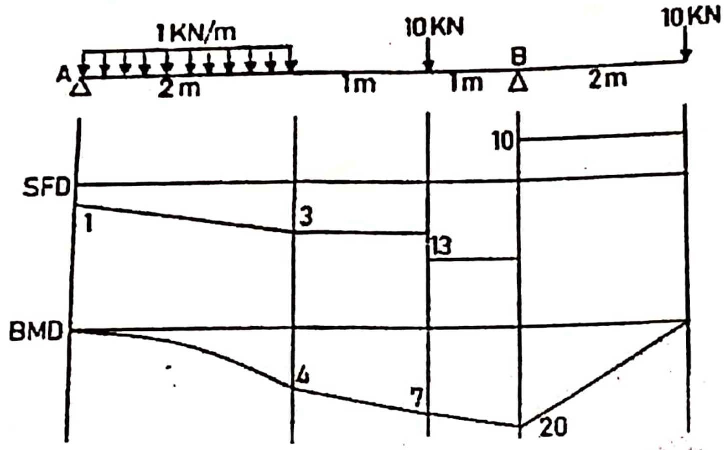
Solution:



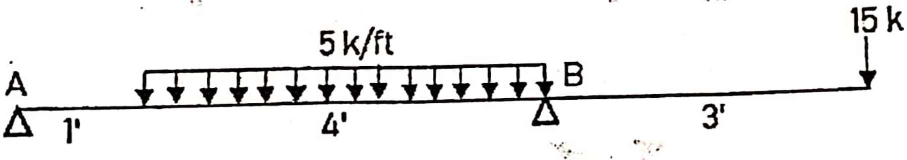
Question: Draw the shear force and bending moment diagram of the following beam. (CPGCBL – 2015, MES – 2015)



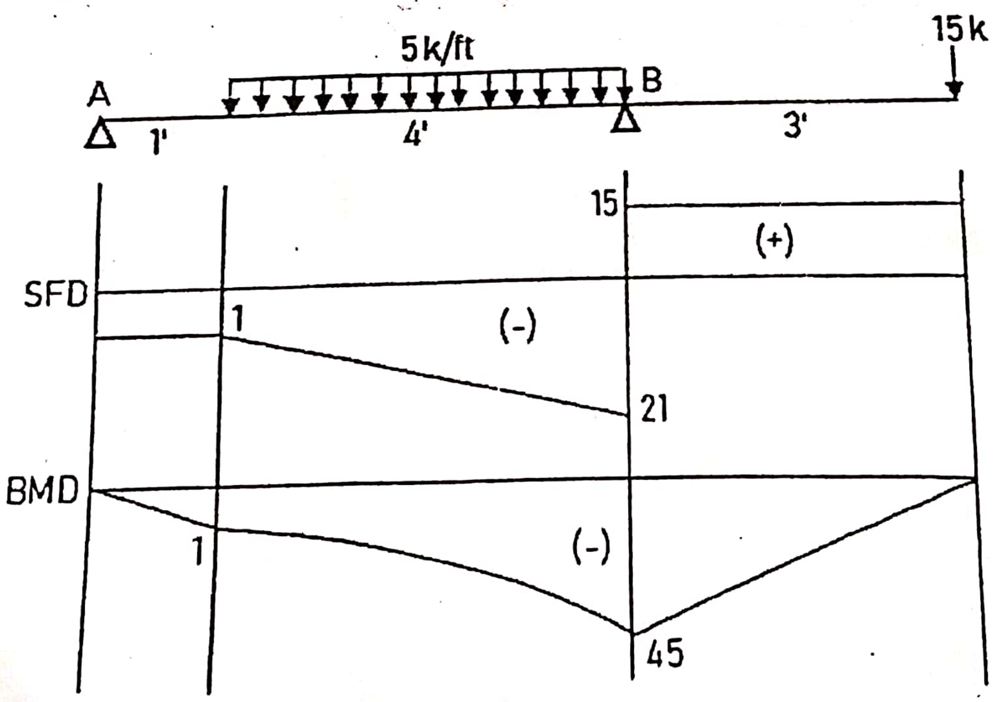
Solution:



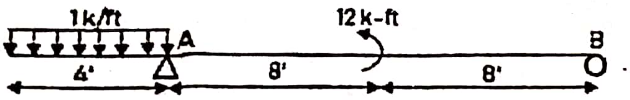
Question: Draw SFD and BMD of the following structure. (PMO – 2015)



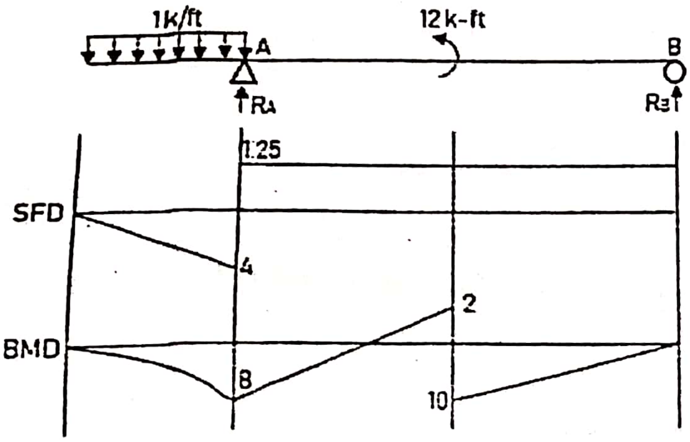
Solution:



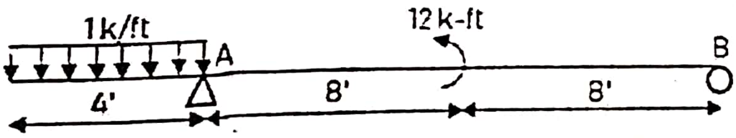
Question: Draw SFD and BMD for the following structure. (RRI – 2015)



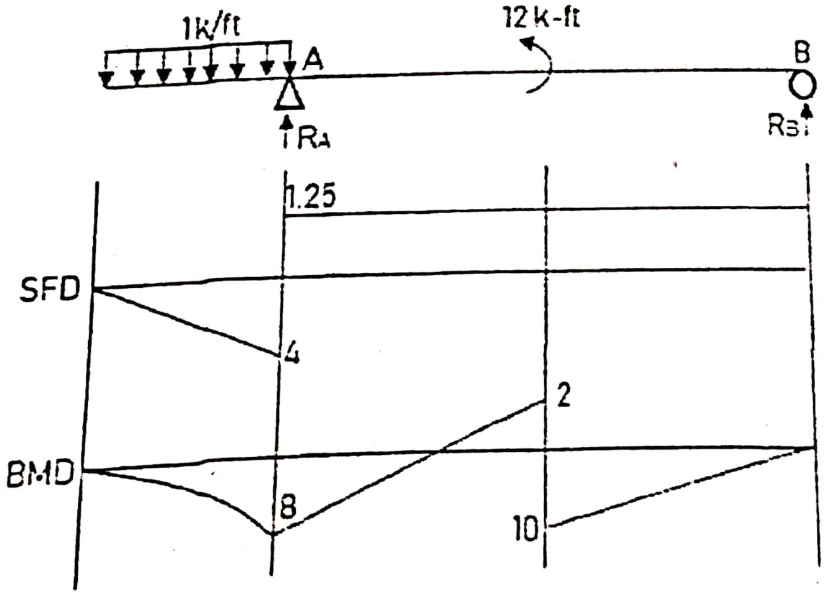
Solution:



Question: Draw SFD and BMD for the following section. (BEPZA – 2016)

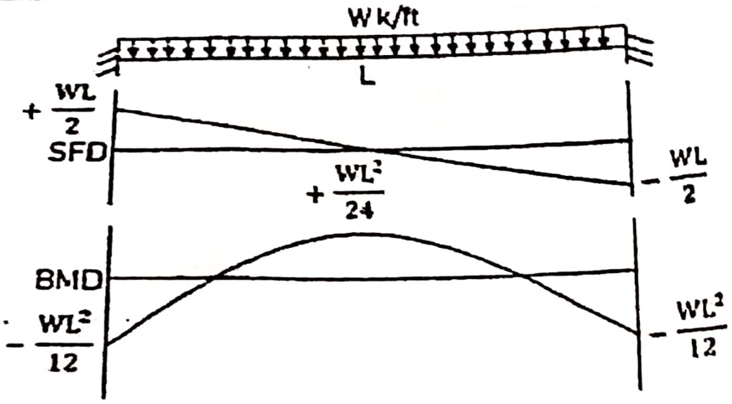


Solution:

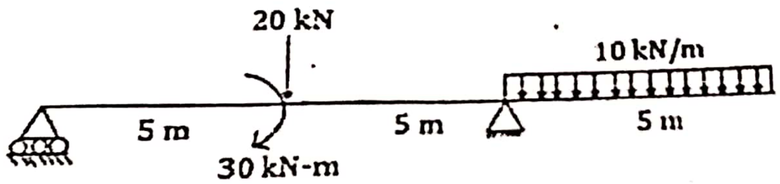


Question: Draw shear force and bending moment diagram of two end fixed beam having length 'L' and carrying uniform load 'w'. (RAJUK – 2016)

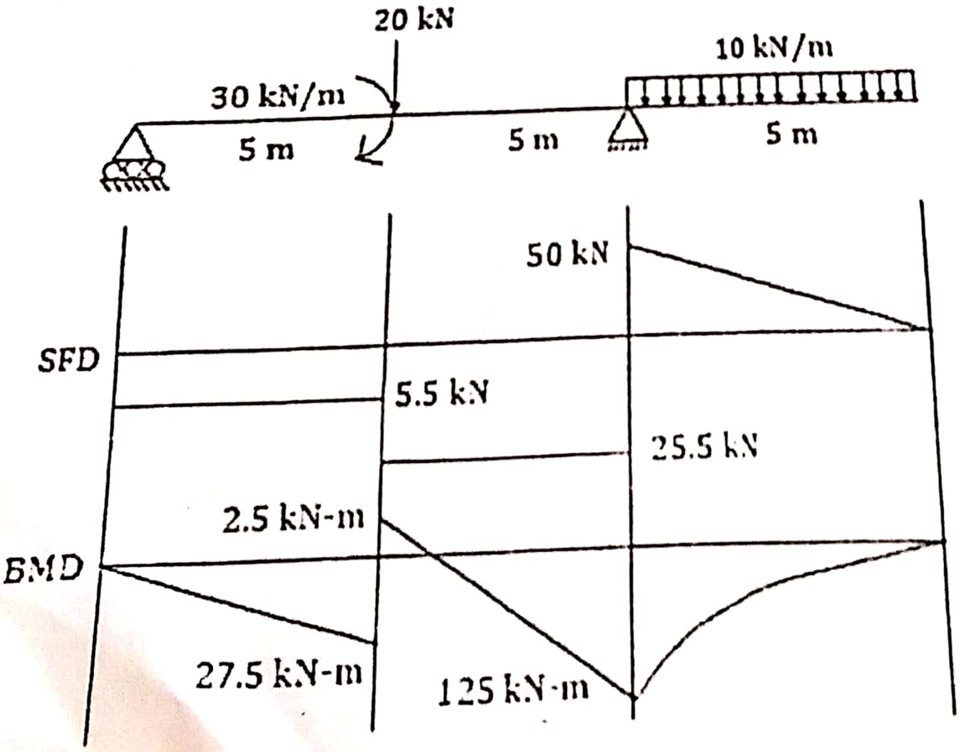
Solution:



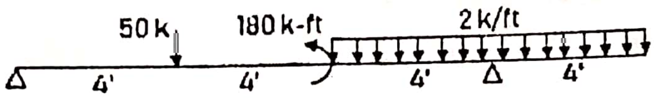
Question: Draw shear force and bending moment diagram for the following beam.



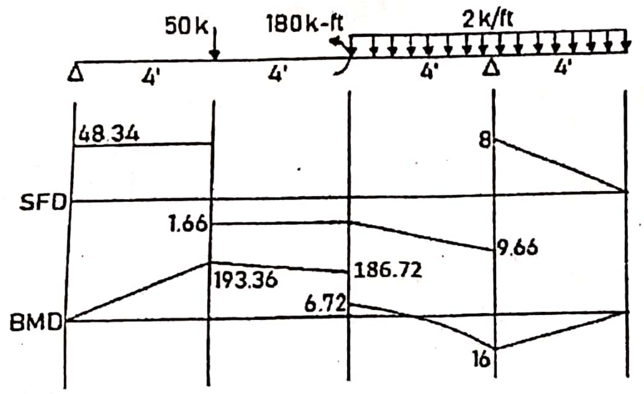
Solution:



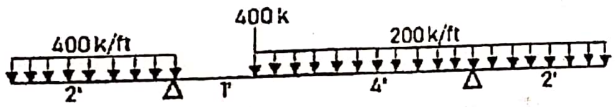
Question: Draw SFD and BMD for the following beam. (WASA – 2017)



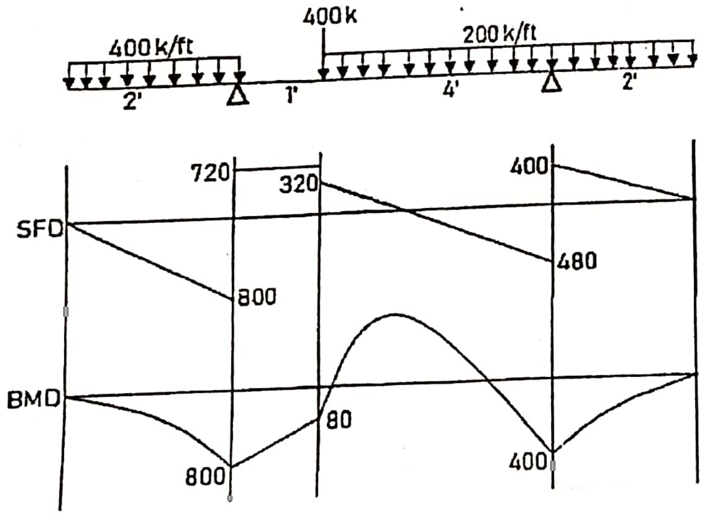
Solution:



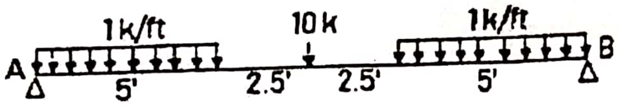
Question: Draw SFD and BMD of the following figure (AB – 2017)



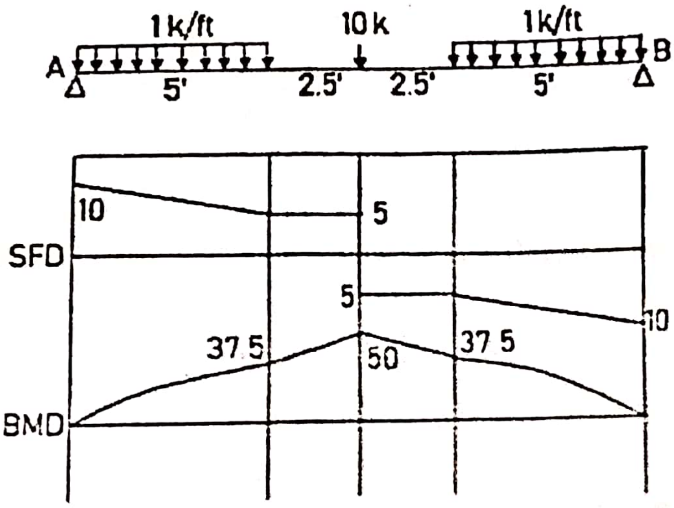
Solution



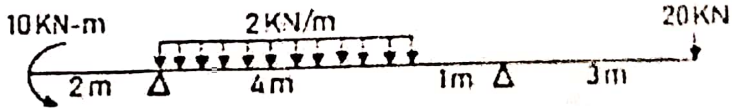
Question: Draw SFD and BMD for the following beam. (BWDB – 2013)



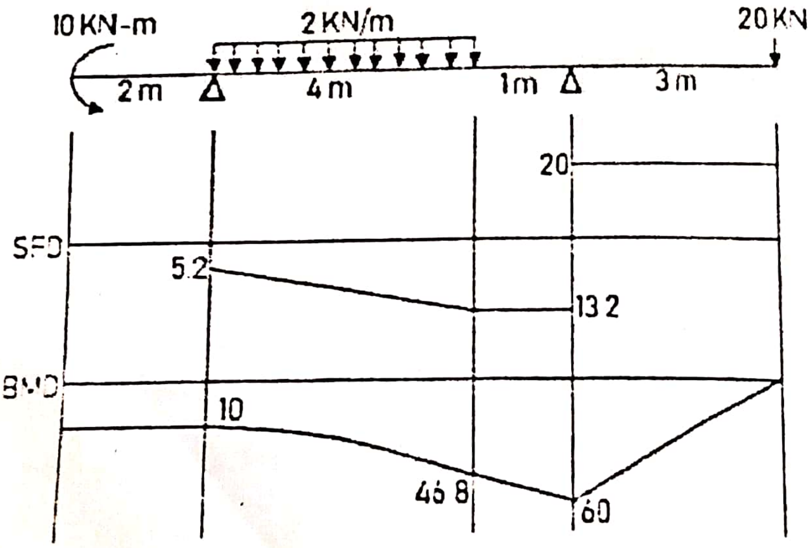
Solution:



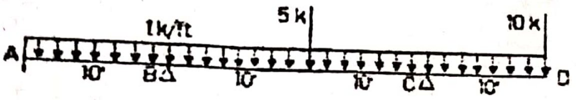
Question: Draw SFD and BMD for the following beam (BKB – 2018)



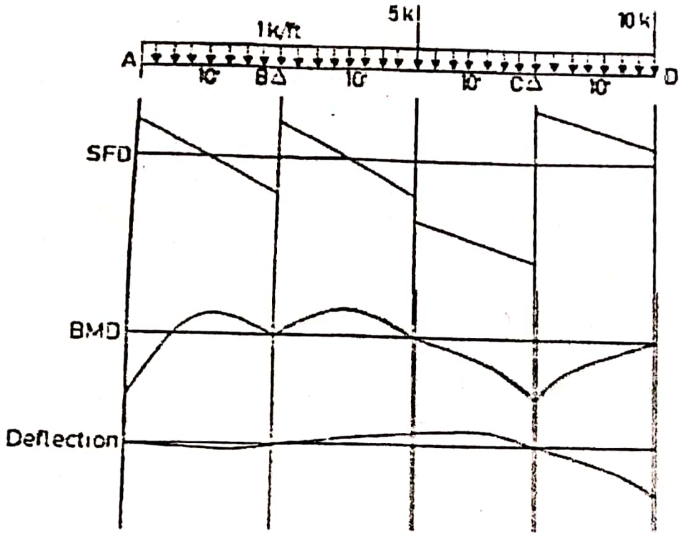
Solution:



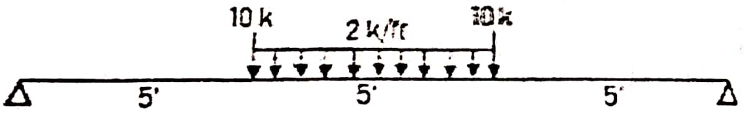
Question: Draw the SFD, BMD and deflected shape of the following continuous beam (Qualitative) (HBRI – 2019)



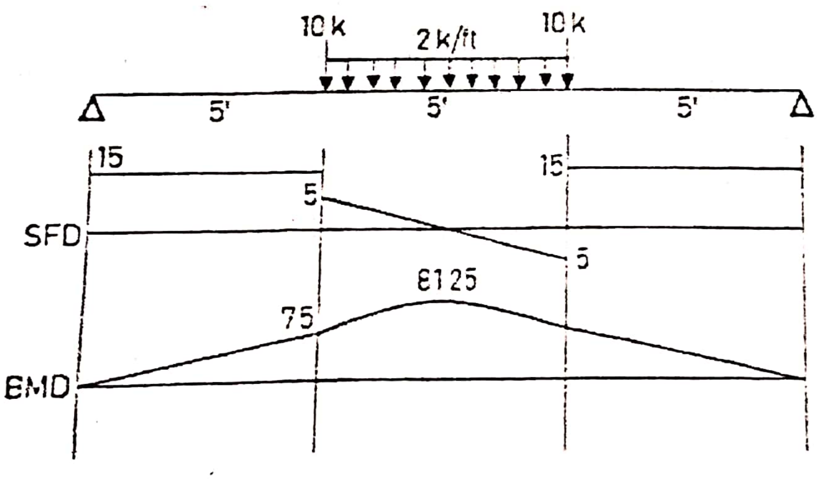
Solution:



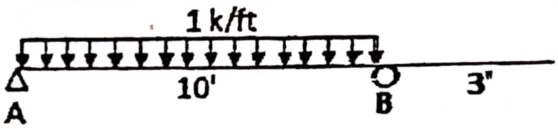
Question: Draw SFD and BMD for the following figure. (HBRI – 2019)



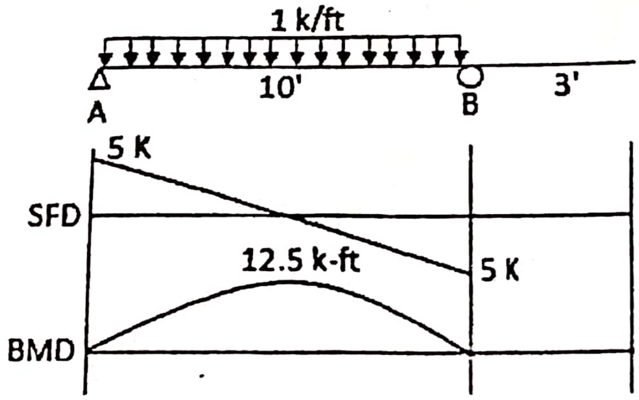
Solution:



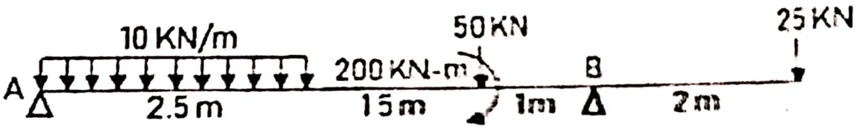
Question: Draw SFD and BMD for the following beam (DESCO – 2019. SGFL – 2022)



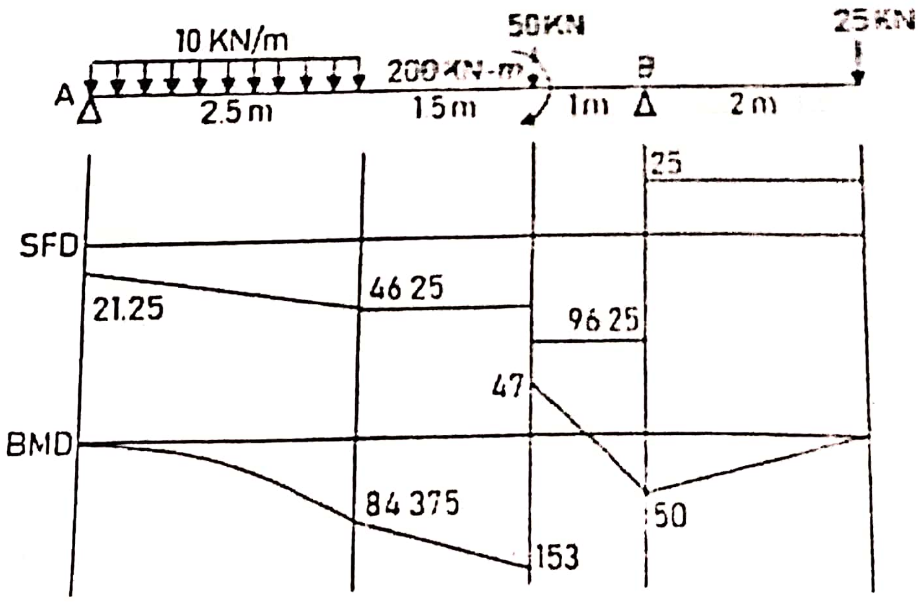
Solution:



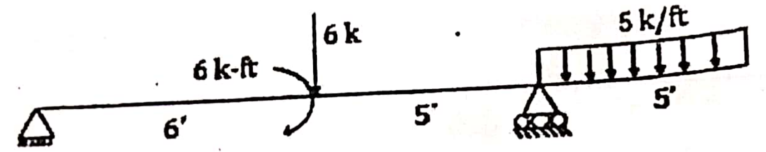
Question: Draw SFD and BMD of the following beam. (ISTT – 2015)



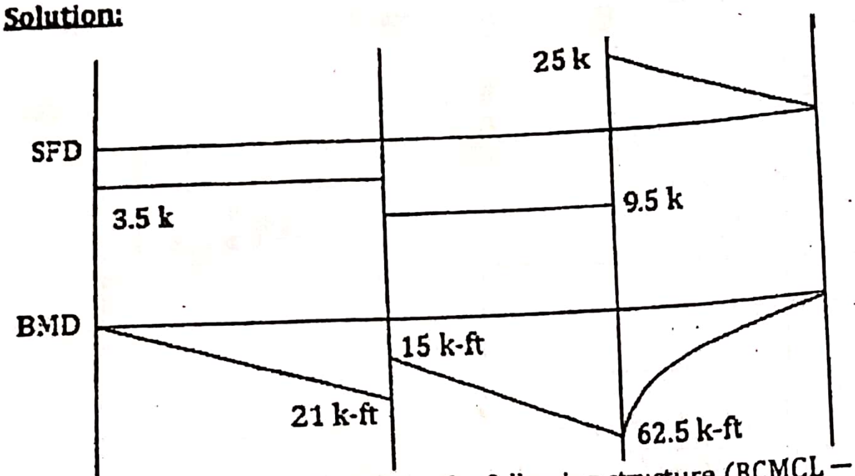
Solution:



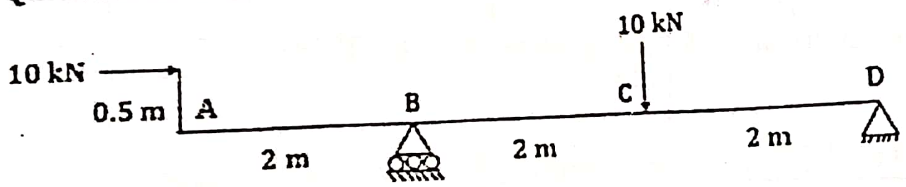
Question: Draw SFD and BMD for the following beam. (BEPZA – 2019)



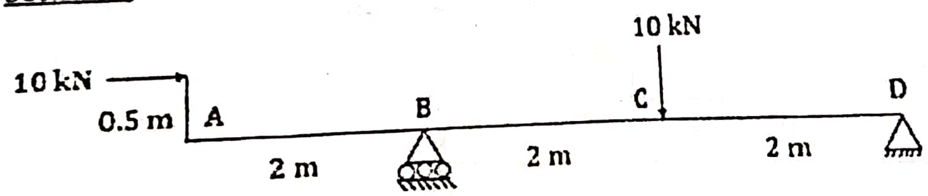
Solution:

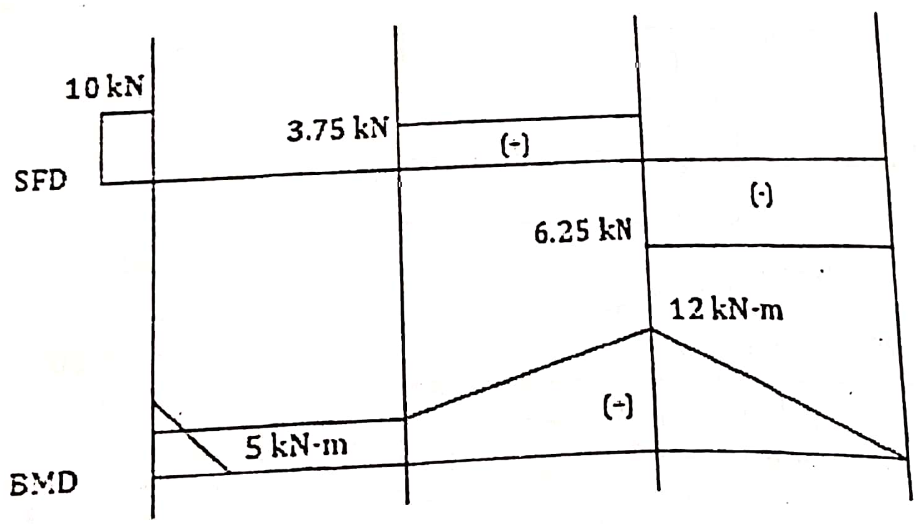


Question: Draw SFD and BMD for the following structure (BCMCL – 2020)

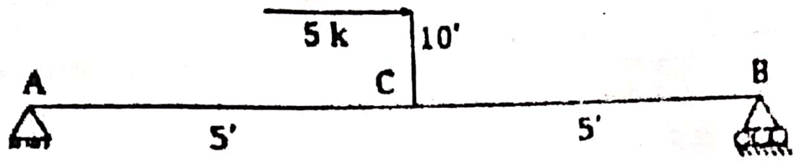


Solution:

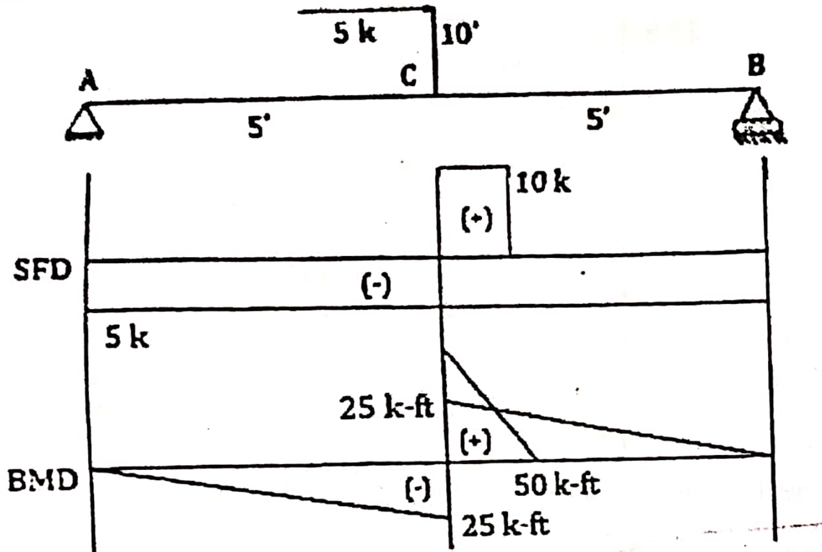




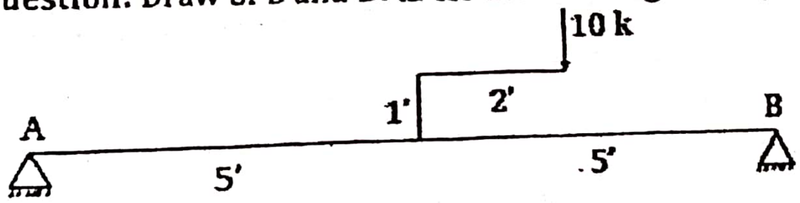
Question: Draw shear force and bending moment diagram for the following frame. (BIWTA – 2019)



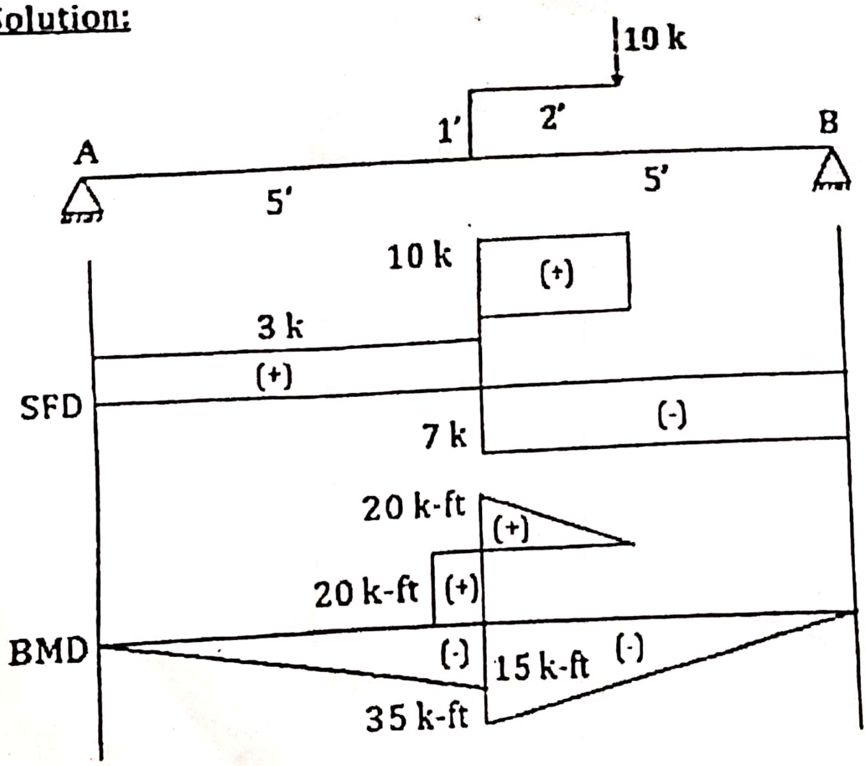
Solution:



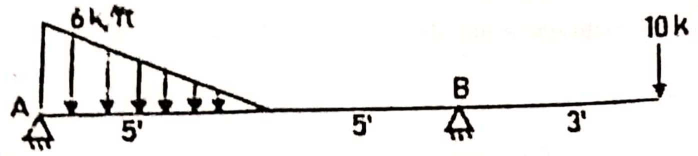
Question: Draw SFD and BMD for the following beam. (NPCBL – 2019)



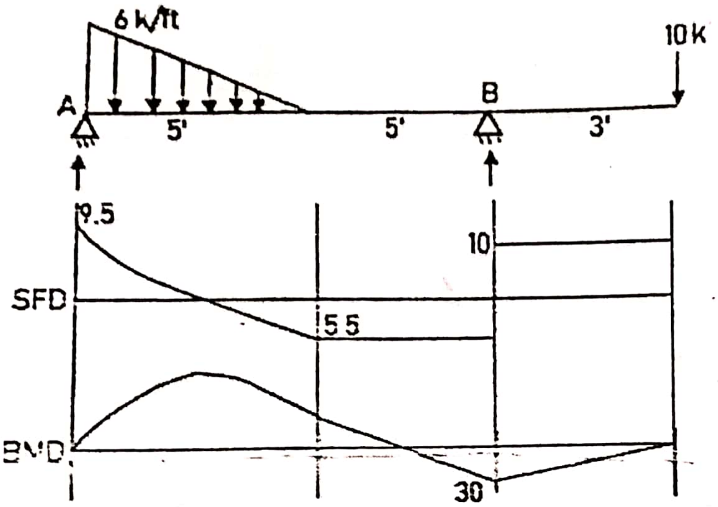
Solution:



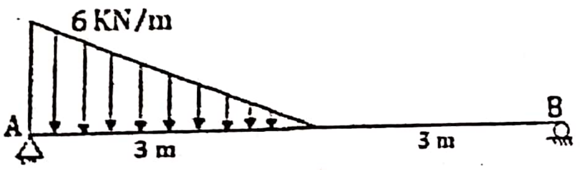
Question: Draw SFD and BMD for the following beam (RAJUK – 2014)



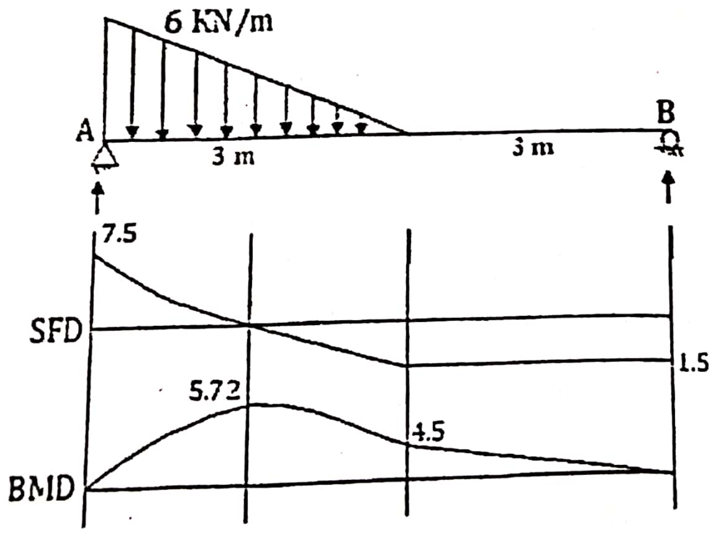
Solution:



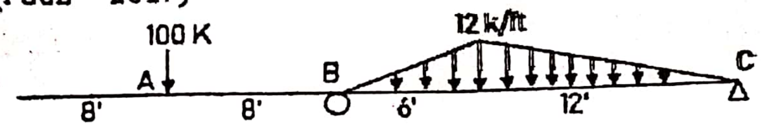
Question: Draw SFD and BMD for the following beam. (NPCBL – 2017)



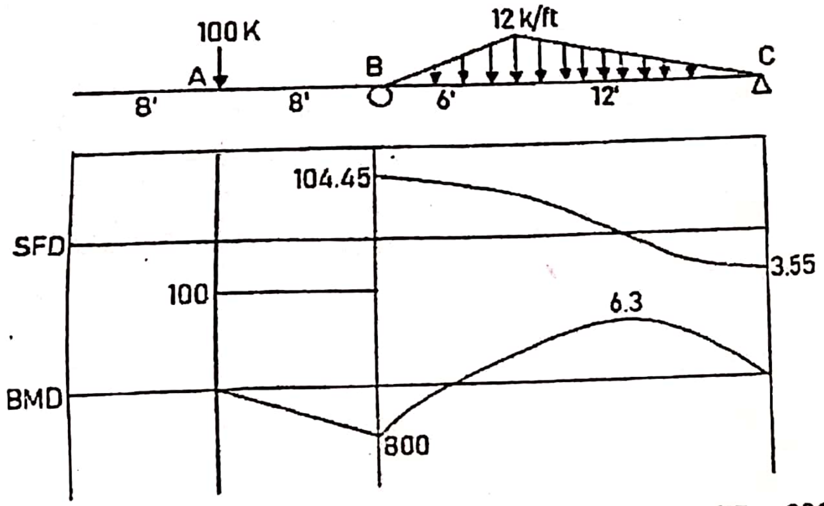
Solution:



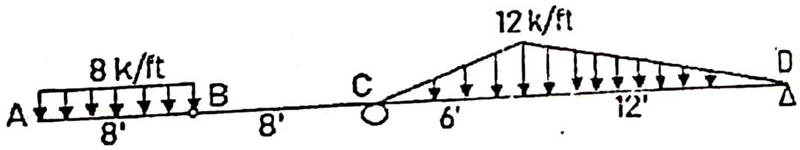
Question: Draw shear force diagram and bending moment diagram for the following beam. (PGCL – 2017)



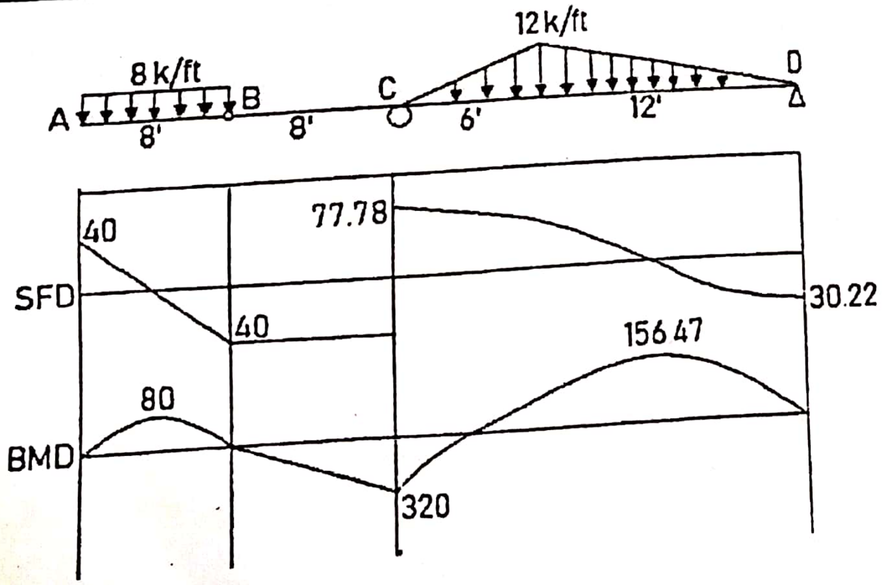
Solution:



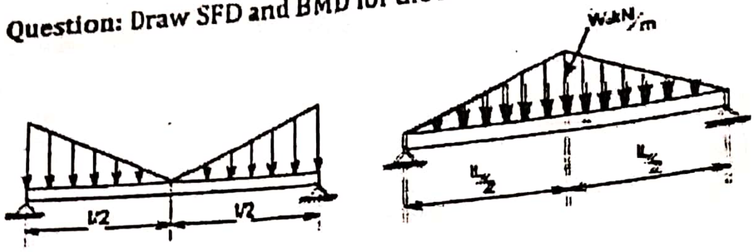
Question: Draw SFD and BMD of the following beam. (JB – 2017)



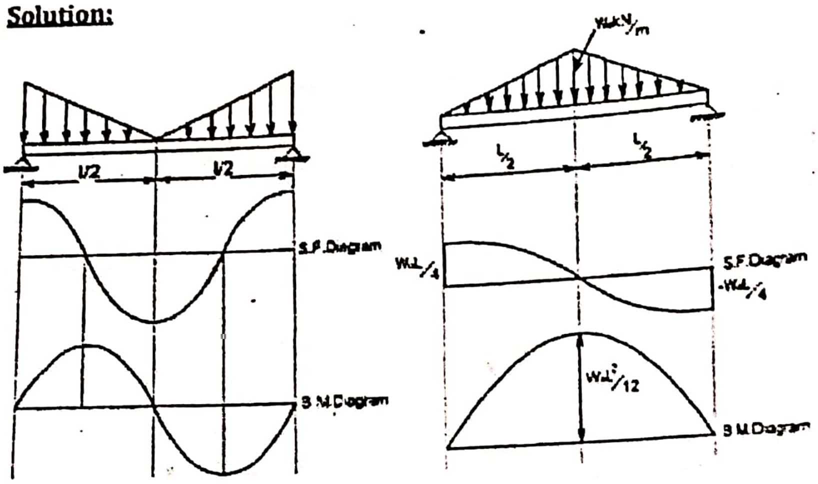
Solution:



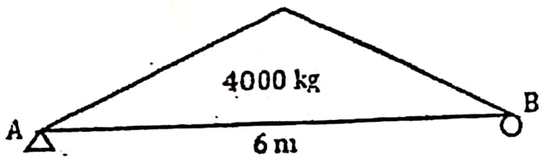
Question: Draw SFD and BMD for the following beam. (BREB – 2018)



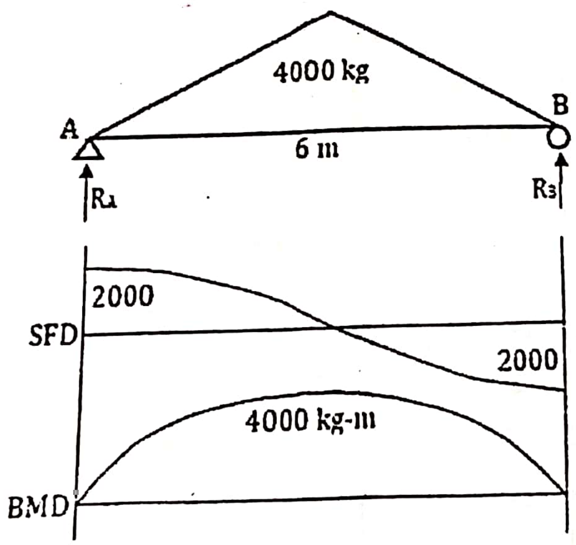
Solution:



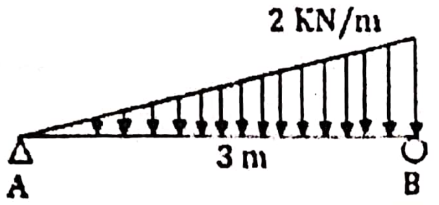
Question: Draw Shear force and bending moment diagram of the following figure (BB AD – 2018)



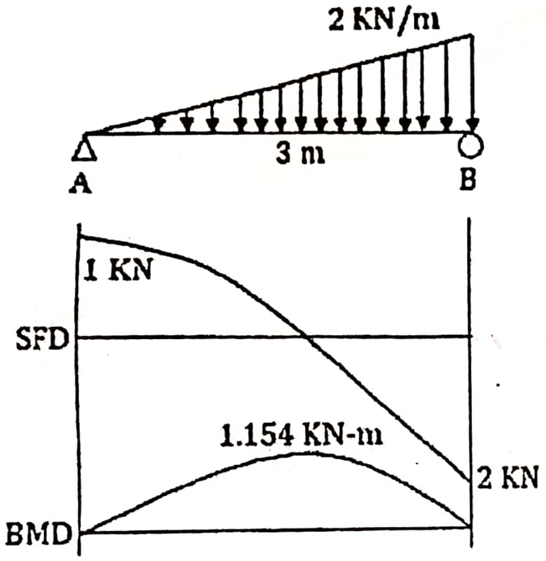
Solution:



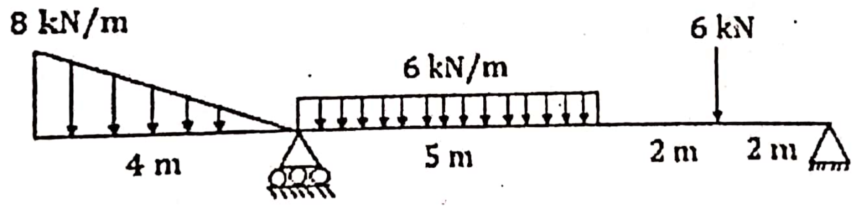
Question: Draw SFD and BMD for the following beam. (52 BMA)



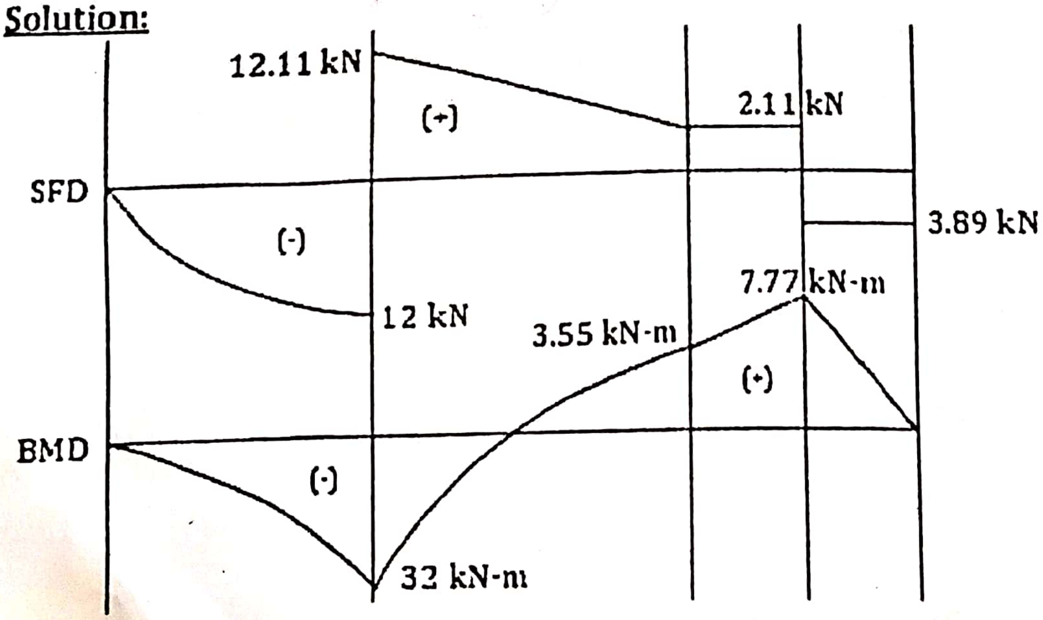
Solution:



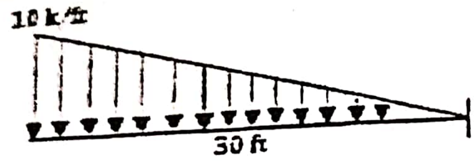
Question: Draw SFD, BMD of the following Beam (LGED – 2019)



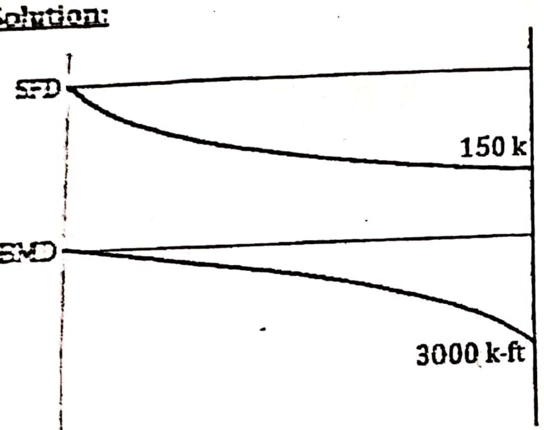
Solution:



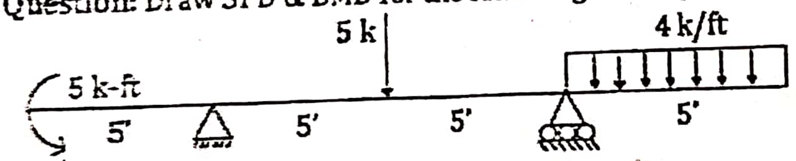
Question: Draw shear force and bending moment of the following loaded structure. (CUET M. Sc – 2020)



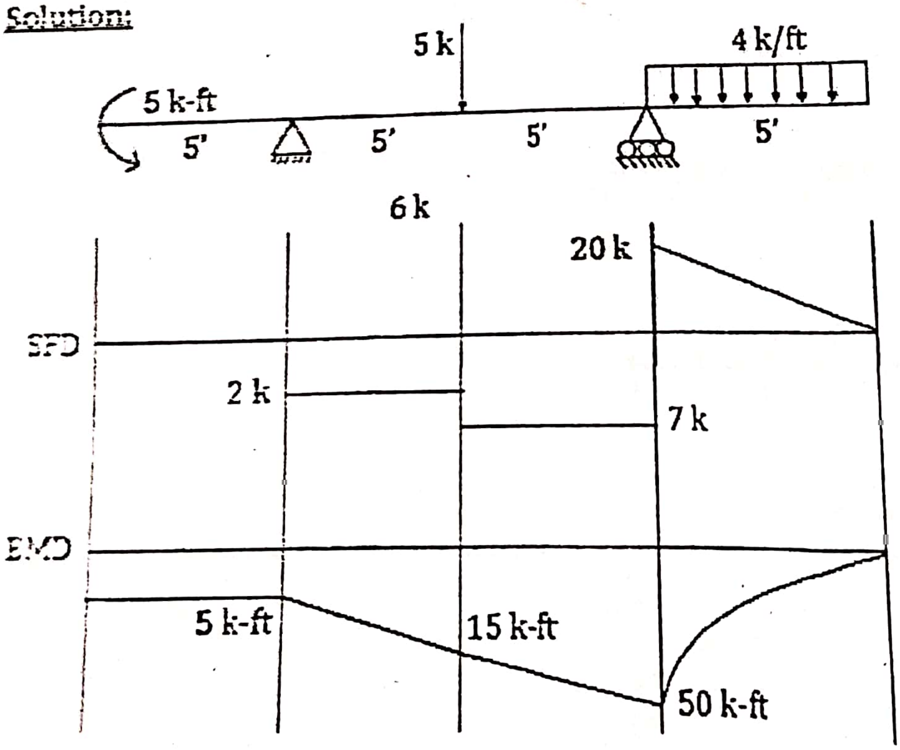
Solution:



Question: Draw SFD & BMD for the following beam. (SGFL – 2021)

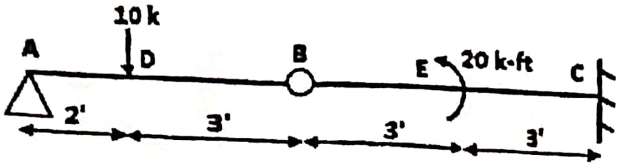


Solution:

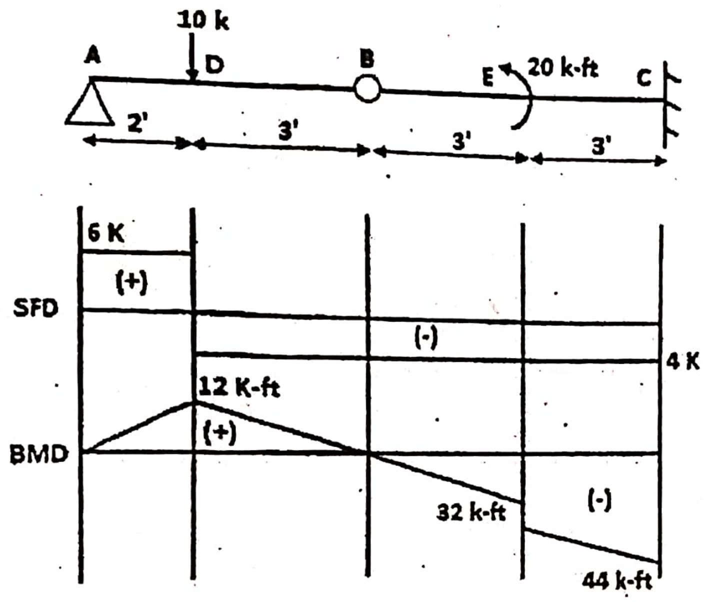


Ques

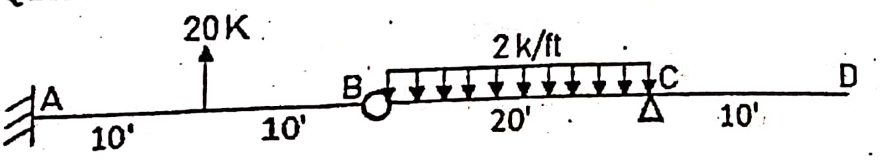
Question: Draw SFD and BMD for the following beam. (PGCB – 2018, DPDC – 2019)



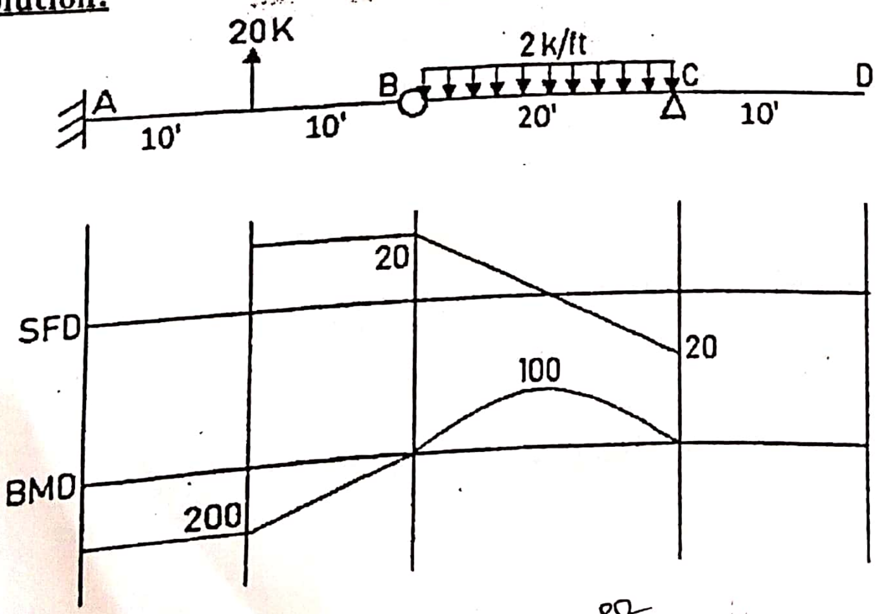
Solution:



Question: Draw SFD and BMD of the following structure. (DSCC – 2016)

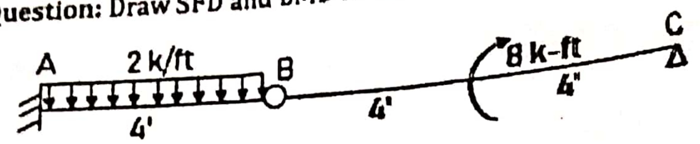


Solution:

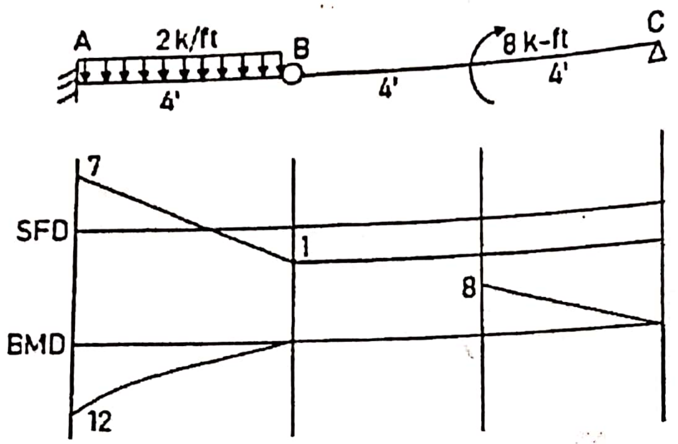


88

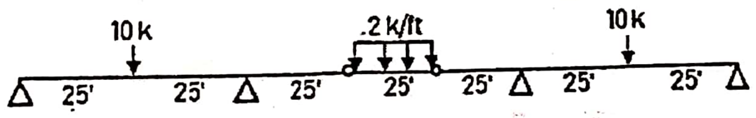
Question: Draw SFD and BMD for the following beam. (RPGCL – 2017)



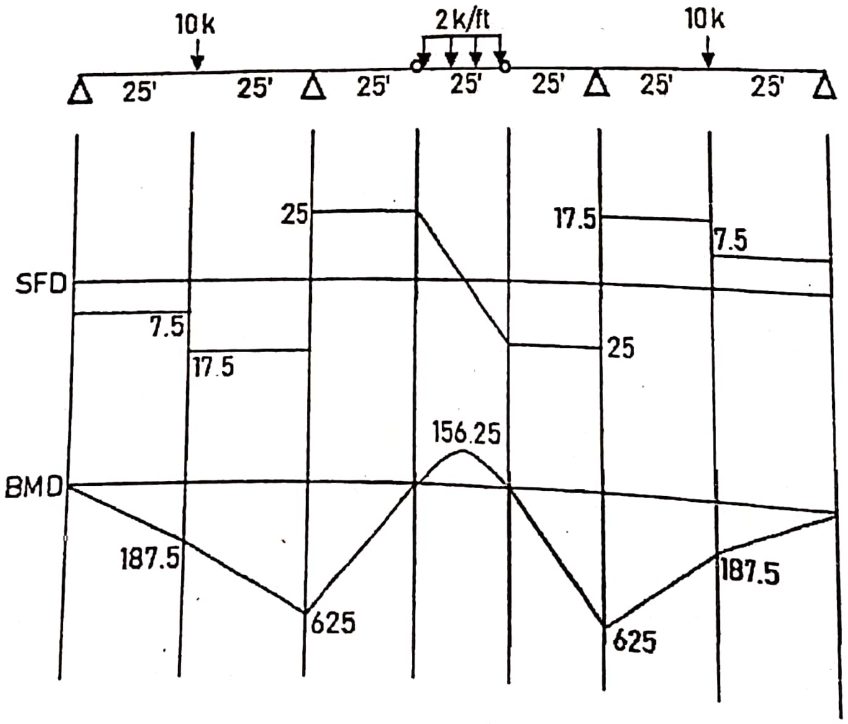
Solution:



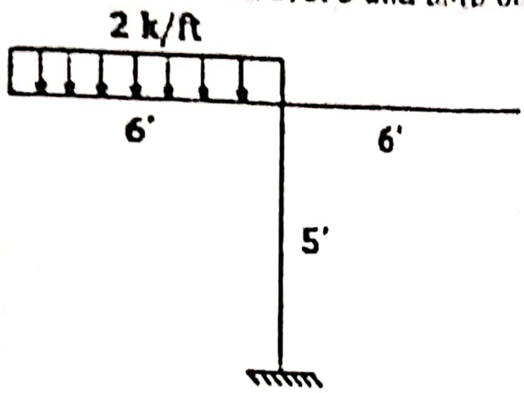
Question: Draw SFD and BMD for the below structure. (BWDB – 2014)



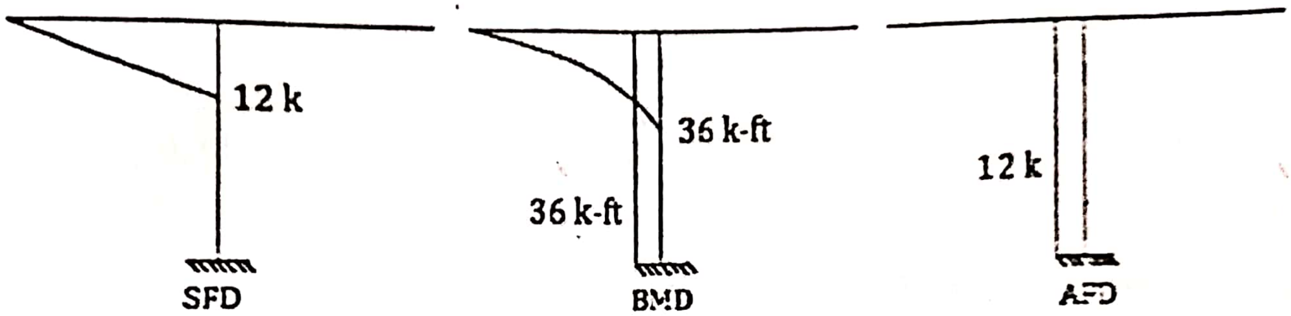
Solution:



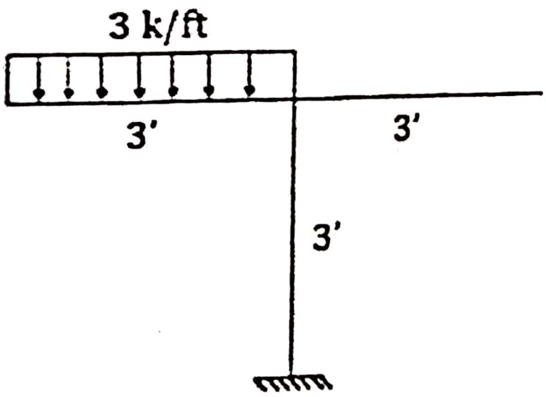
Question: Draw AFD, SFD and BMD of the below frame. (DESCO – 2015)



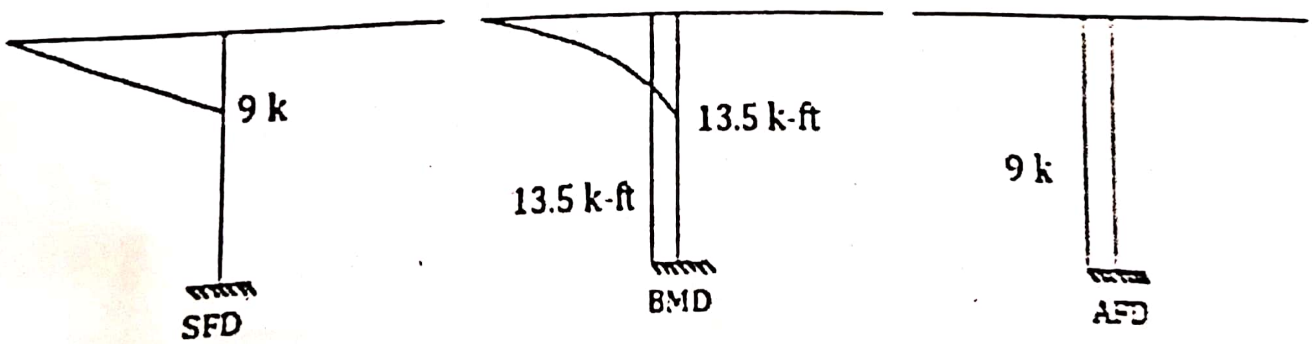
Solution:



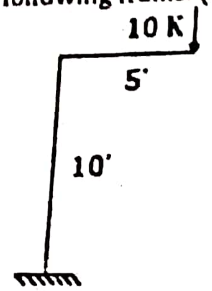
Question: Draw SFD, BMD and axial force diagram for the structure. (SGFCL – 2017)



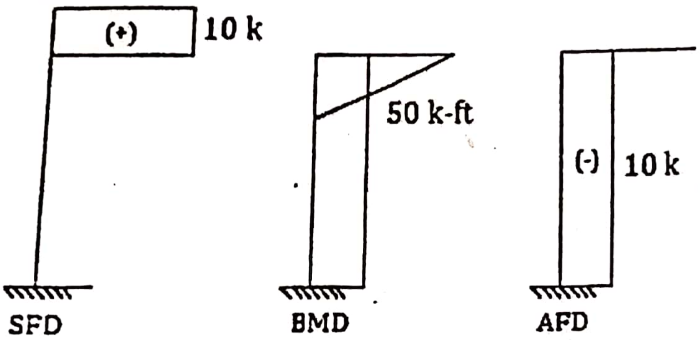
Solution:



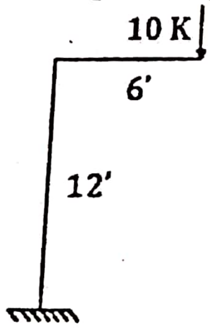
Question: Draw Axial load diagram, bending moment diagram and shear force diagram for the following frame. (BADC – 2020)



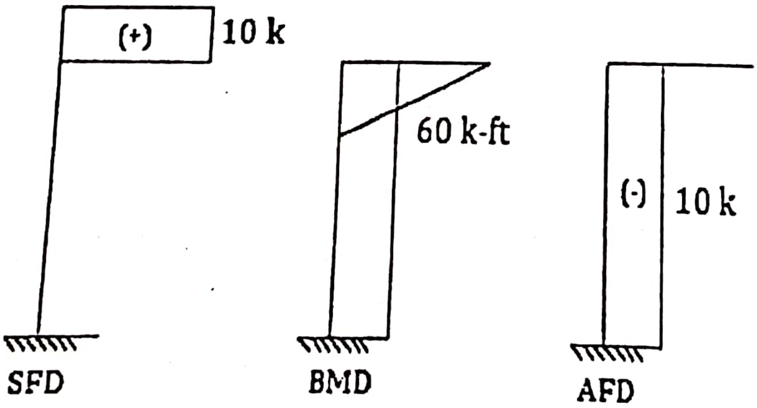
Solution:



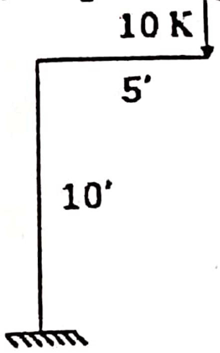
Question: Draw Axial load diagram, bending moment diagram and shear force diagram for the following frame. (BCIC – 2019)



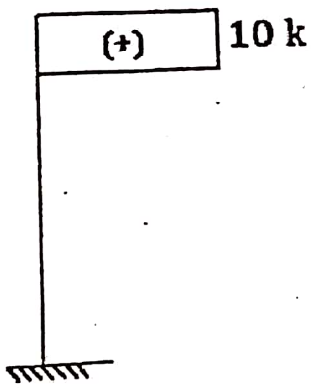
Solution:



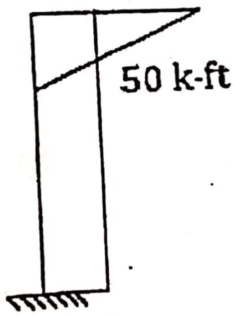
Question: Draw Axial load diagram, bending moment diagram and shear force diagram for the following frame. (BWDB – 2019)



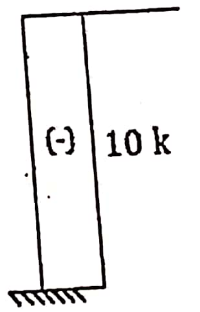
Solution:



SFD

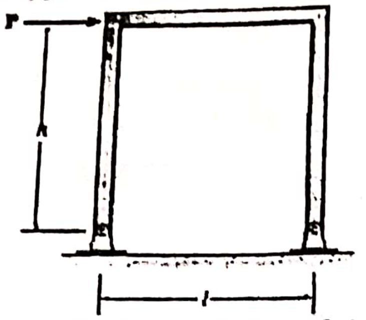


BMD



AFD

Frames: Portal frames are frequently used over the entrance of a bridge* and as a main stiffening element in building design in order to transfer horizontal forces applied at the top of the frame to the foundation. On bridges, these frames resist the forces caused by wind, earthquake, and unbalanced traffic loading on the bridge deck. Portals can be pin supported, fixed supported, or supported by partial fixity.



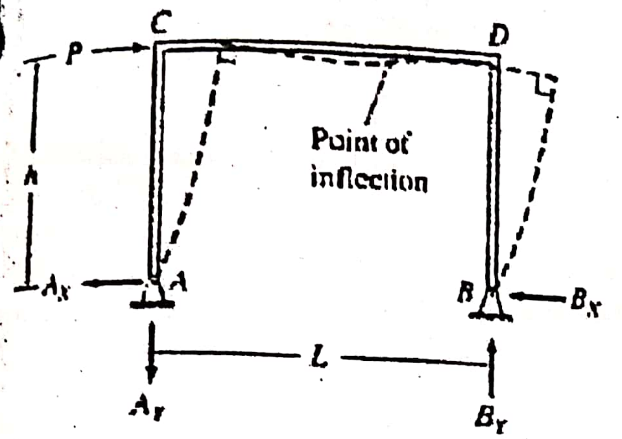
Approximate analysis of indeterminate structures is sometimes performed by making assumptions about the distribution of forces among the members and/or reactions of the structures. The number of such assumptions required for the analysis of a structure is equal to the degree of indeterminacy of the structure, with each assumption providing an independent equation relating the unknown member forces and/or reactions.

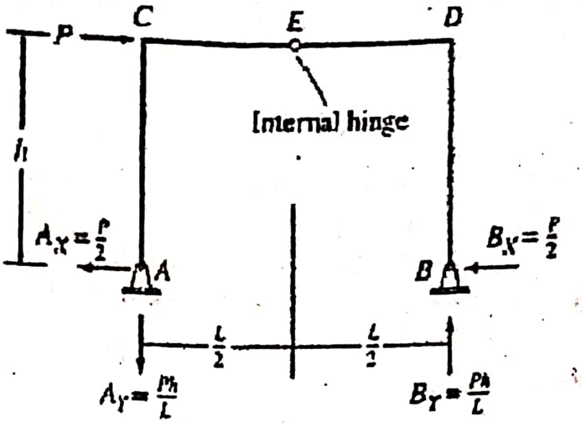
1. An inflection point is located at the middle of each member of the frame.
2. On each story of the frame, interior columns carry twice as much shear as exterior columns.

Assumptions about the Location of Points of Inflection

In the first approach, a qualitative deflected shape of the indeterminate structure is sketched and used to assume the location of the points of inflection—that is, the points where the curvature of the elastic curve changes signs, or becomes zero. Since the bending moments must be zero at the points of inflection, internal hinges are inserted in the indeterminate structure at the assumed locations of inflection points to obtain a simplified determinate structure. Each of the internal hinges provides one equation of condition, so the number of inflection points assumed should be equal to the degree of indeterminacy of the structure.

Pin Supported: A typical pin-supported portal frame is shown in Fig. Since four unknowns exist at the supports but only three equilibrium equations are available for solution, this structure is statically indeterminate to the first degree. Consequently, only one assumption must be made to reduce the frame to one that is statically determinate. The elastic deflection of the portal is shown in Fig. This diagram indicates that a point of inflection, that is, where the moment changes from positive bending to negative bending, is located approximately at the girder's midpoint. Since the moment in the girder is zero at this point, we can assume a hinge exists there and then proceed to determine the reactions at the supports using statics. If this is done, it is found that the horizontal reactions (shear) at the base of each column are equal and the other reactions are those indicated in Fig. Furthermore, the moment diagrams for this frame are indicated in Fig.





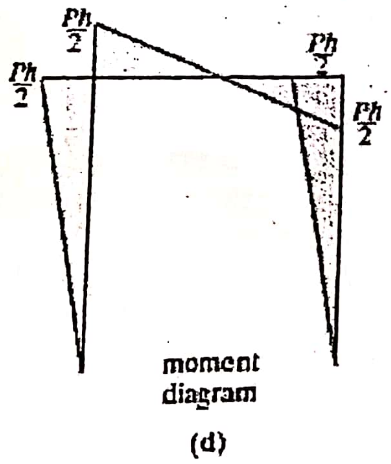
The four reactions of the frame can now be determined by applying the three equilibrium equations, $\sum F_x$, $\sum F_y$ and $\sum M$ and one equation of condition, $\sum M_E^{AE} = 0$ or $\sum M_E^{BE} = 0$ to determinate frame.

$$\circlearrowleft + \sum M_B = 0 \qquad A_Y(L) - Ph = 0 \qquad A_Y = \frac{Ph}{L}\downarrow$$

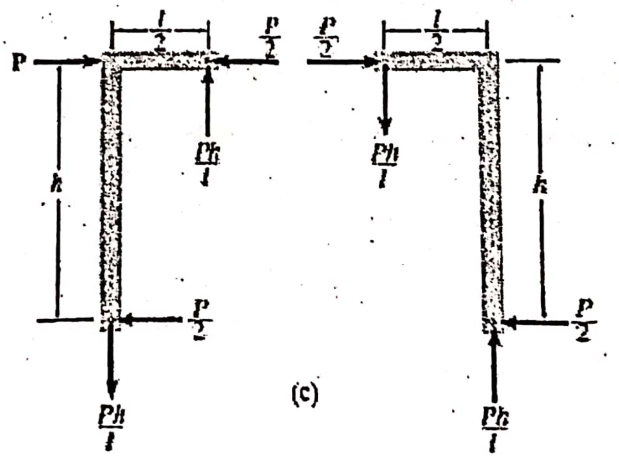
$$+\uparrow \sum F_Y = 0 \qquad -\frac{Ph}{L} + B_Y = 0 \qquad B_Y = \frac{Ph}{L}\uparrow$$

$$\circlearrowleft + \sum M_E^{BE} = 0 \qquad \frac{Ph}{L}\left(\frac{L}{2}\right) - B_X(h) = 0 \qquad B_X = \frac{P}{2}\leftarrow$$

$$+\rightarrow \sum F_X = 0 \qquad P - A_X - \frac{P}{2} = 0 \qquad A_X = \frac{P}{2}\leftarrow$$

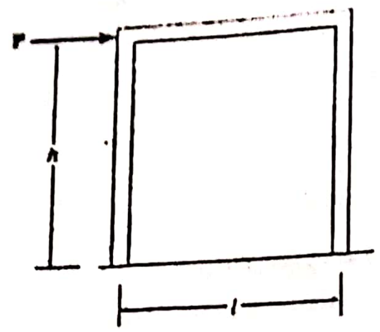


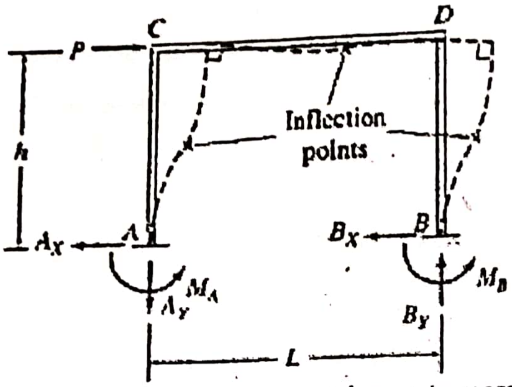
(d)

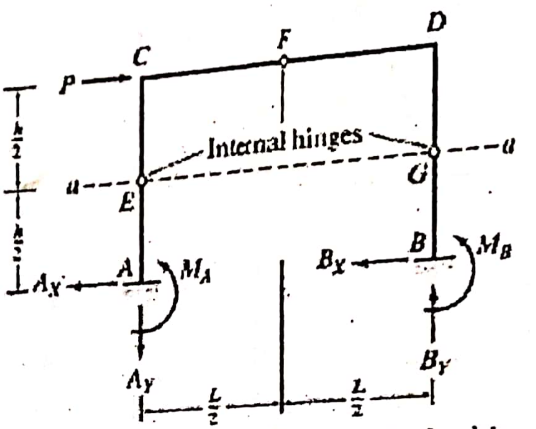


(c)

Fixed Supported: Portals with two fixed supports are statically indeterminate to the third degree since there are a total of six unknowns at the supports. If the vertical members have equal lengths and cross-sectional areas, the frame will deflect as shown in Figure. For this case we will assume points of inflection occur at the midpoints of all three members and therefore hinges are placed at these points. The reactions and moment diagrams for each member can therefore be determined by dismembering the frame at the hinges and applying the equations of equilibrium to each of the four parts.







To determine the six reactions, we pass a horizontal section aa through the hinges E and G, as shown in Fig, and apply the equations of equilibrium (and condition, if any) to the three portions of the frame. Applying the three equilibrium equations and one equation of condition to the portion $ECDG$, we compute the forces at the internal hinges E and G to be,

$$+\circlearrowleft \sum M_G = 0 \qquad E_Y(L) - P\left(\frac{h}{2}\right) = 0 \qquad E_Y = \frac{Ph}{2L}\downarrow$$

$$+\uparrow \sum F_Y = 0 \qquad -\frac{Ph}{2L} + G_Y = 0 \qquad G_Y = \frac{Ph}{2L}\uparrow$$

$$+\circlearrowleft \sum M_F^{EF} = 0 \qquad \frac{Ph}{2L}\left(\frac{L}{2}\right) - E_X\left(\frac{h}{2}\right) = 0 \qquad E_X = \frac{P}{2}\leftarrow$$

$$+\rightarrow \sum F_X = 0 \qquad P - \frac{P}{2} - G_X = 0 \qquad G_X = \frac{P}{2}\leftarrow$$

The reactions at supports A and B can now be determined by considering the equilibrium of portions AE and BG, respectively. For portion AE

$$+\rightarrow \sum F_X = 0 \qquad A_X = \frac{P}{2}\leftarrow$$

$$+\uparrow \sum F_Y = 0 \qquad A_Y = \frac{Ph}{2L}\downarrow$$

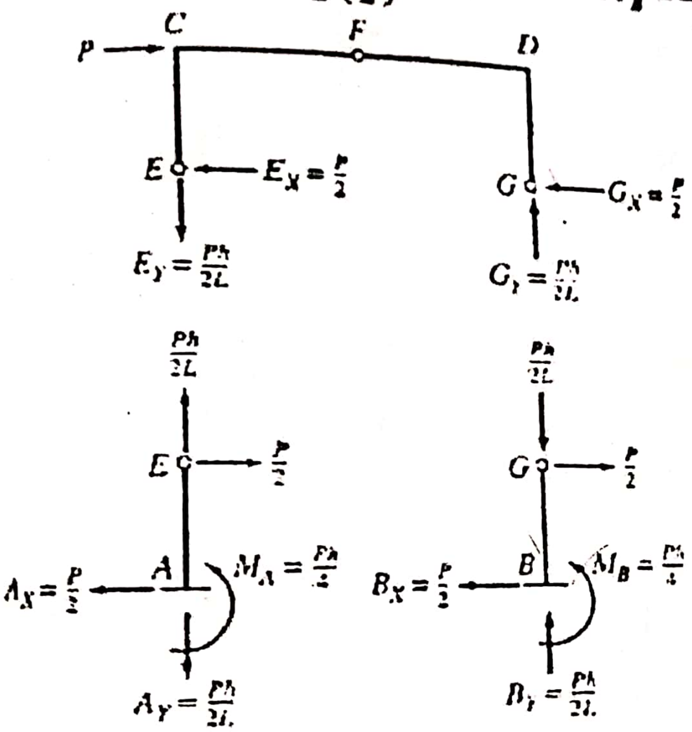
$$+\circlearrowleft \sum M_A = 0 \qquad -\frac{P}{2}\left(\frac{h}{2}\right) + M_A = 0 \qquad M_A = \frac{Ph}{4}\circlearrowleft$$

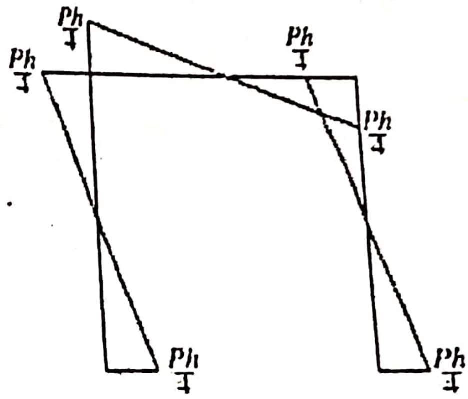
Similarly, for portion BG

$$\xrightarrow{+}\sum F_x = 0 \qquad B_x = \frac{P}{2}\leftarrow$$

$$+\uparrow\sum F_y = 0 \qquad B_y = \frac{Ph}{2L}\uparrow$$

$$+\circlearrowleft\sum M_B = 0 \qquad -\frac{P}{2}\left(\frac{h}{2}\right) + M_B = 0 \qquad M_B = \frac{Ph}{4}$$





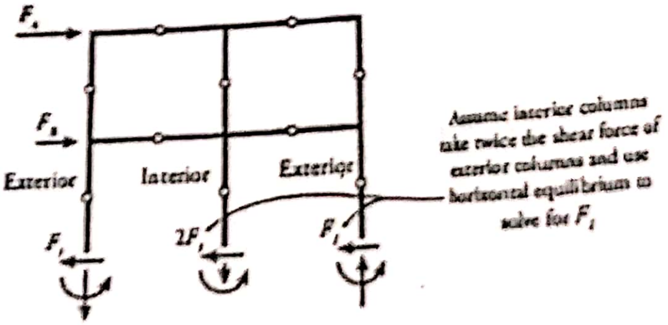
moment diagram

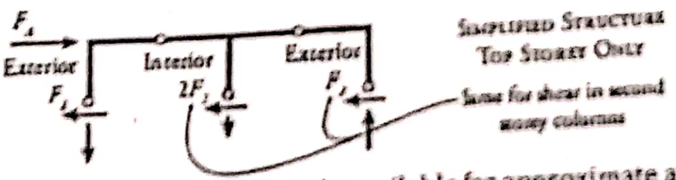
Question: Mention the assumptions made in the portal method of structural analysis for frames subjected to lateral loads. Illustrate with sketches. (32th BCS)

Solution:

There are three major assumption that applied in the approximate portal method of analysis. These assumptions are as follow:

1. An inflection point is located at mid-height of each column.
2. An inflection point is located at the centre of each beam
3. The horizontal shear is divided among all the columns on the basis that each interior column takes twice as much as exterior column.





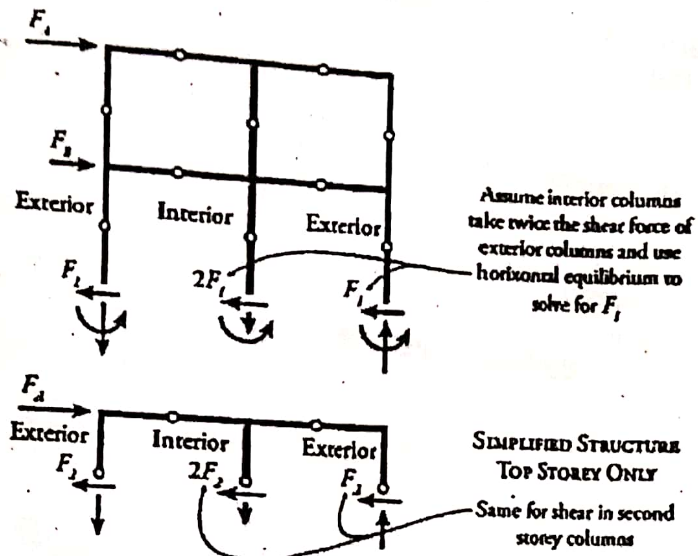
Question: List the methods available for approximate analysis of building frame subjected to lateral loads. Mention the assumptions for each method. (34th BCS)

Solution:

The behavior of a structure subjected to horizontal forces depends on its height to width ratio. The deformation in low-rise structures, where the height is smaller than its width, is characterized predominantly by shear deformations. In high rise building, where height is several times greater than its lateral dimensions, is dominated by bending action. To analyze the structures subjected to horizontal loading we have two methods: Portal method and Cantilever method. In portal method for analyzing fixed-supported building frames requires the following assumptions:

1. A hinge is placed at the center of each girder, since this is assumed to be a point of zero moment.
2. A hinge is placed at the center of each column, since this is assumed to be a point of zero moment.
3. At a given floor level the shear at the interior column hinges is twice that at the exterior column hinges, since the frame is considered to be a superposition of portals.

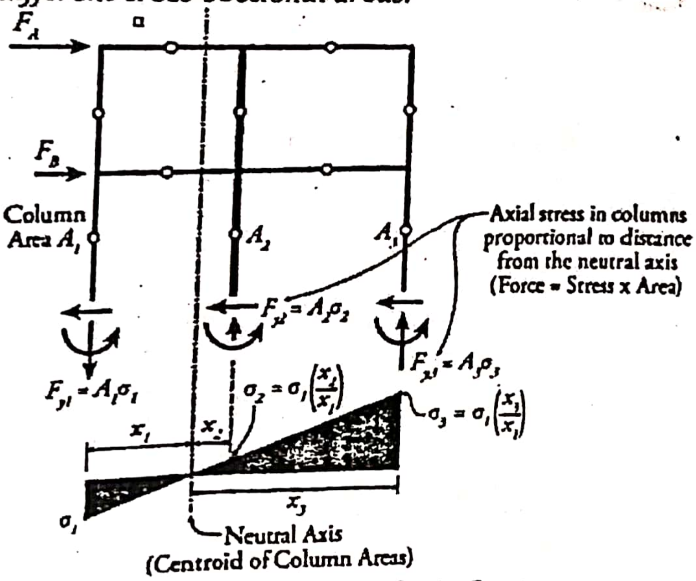
The portal method is most suitable for buildings having low elevation and uniform framing.



In cantilever method, the following assumptions apply to a fixed-supported frame.

1. A hinge is placed at the center of each girder, since this is assumed to be a point of zero moment.
2. A hinge is placed at the center of each column, since this is assumed to be a point of zero moment.
3. The axial stress in a column is proportional to its distance from the centroid of the cross-sectional areas of the columns at a given floor level. Since stress equals force per area, then in the special case of the columns having equal cross-sectional areas, the force in a column is also proportional to its distance from the centroid of the column areas.

The cantilever method is therefore appropriate if the frame is tall and slender, or has columns with different cross-sectional areas.

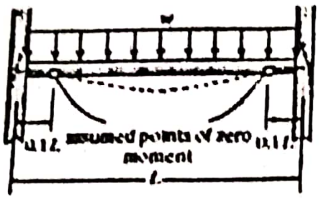


Question: What do you mean by inflection point? Write down the assumptions needed for analysis of building frame subjected to vertical load. (33th BCS)

Solution:

These are the points at which the deflection curve of the body changes the sign of curvature. In other terms, we can say that the inflection points are the points where the shear force changes its direction from *+ve* to *–ve* and vice versa. In the approximate analysis of a rectangular frame subjected to vertical loads the following assumptions are made for each girder of the frame:

1. The inflection points are located at one-tenth of the span from each end of the girder.
2. The girder axial force is zero.



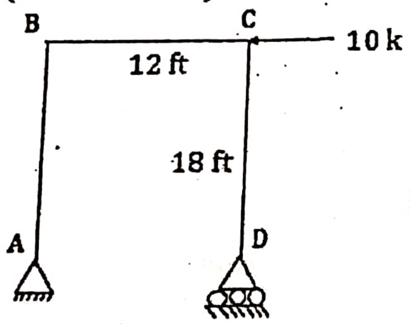
approximate case

Question: How does the effect of earthquake load differ from wind load when imposed to structures? (34th BCS)

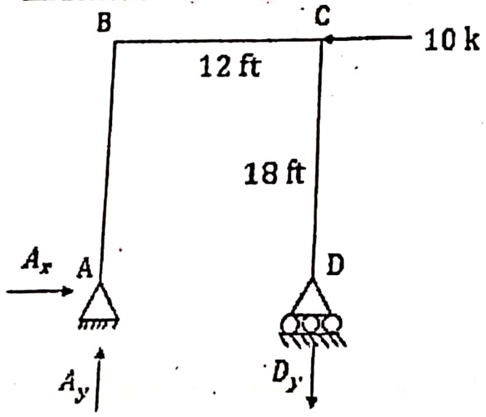
Solution:

The lateral load resisting systems for earthquake loads are similar to those for wind loads. Both are designed as if they are horizontally applied to the structural system. The wind load is considered to be more of a constant force while the earthquake load is almost instantaneous. The wind load is an external force, the magnitude of which depends upon the height of the building, the velocity of the wind and the amount of surface area that the wind "attacks." The magnitude earthquake load depends up the mass of the structure, the stiffness of the structural system and the acceleration of the surface of the earth. It can be seen that the application of these two types of loads is very different.

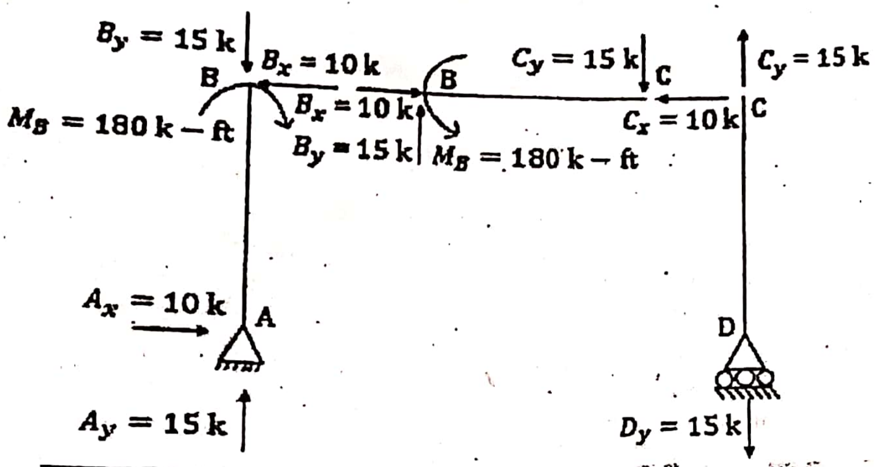
Question: Draw the shear force and bending moment diagram of the following portal frame. (DMTCL – 2019)

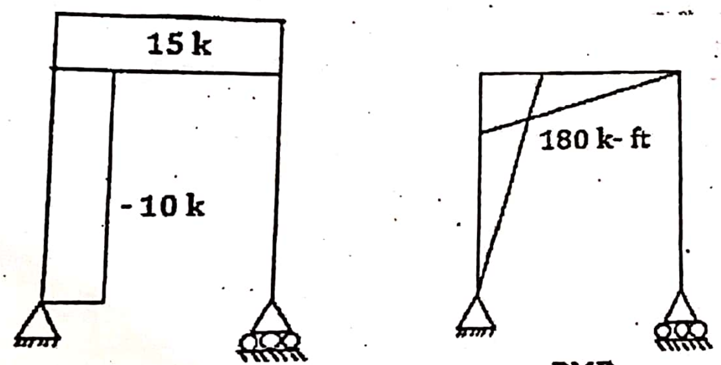


Solution:



$\sum M_D = A_y \times 12 - 10 \times 18 = 0$
$A_y = 15\ k$
$\sum F_y = A_y - D_y = 0$
$D_y = 15\ k$
$\sum F_x = A_x - 10 = 0$
$A_x = 10\ k$

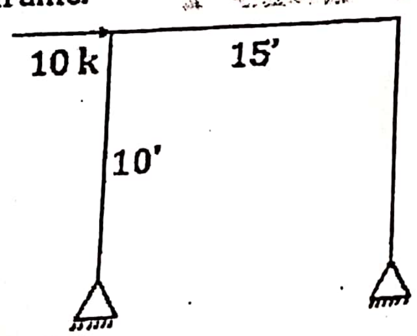




SFD

BMD

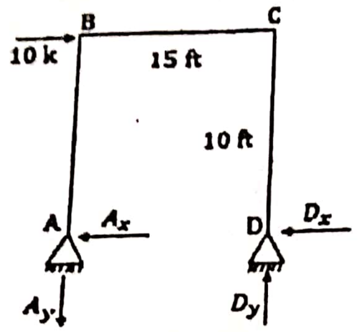
Question: Draw the shear force, axial force and bending moment diagram of the following portal frame.



Solution:

$DOSI = (3m + r) - 3j = (3 \times 3 + 4) - 3 \times 4 = 1$

The structure is statically indeterminate to the 1st degree. For this case we will assume horizontal reactions are same.



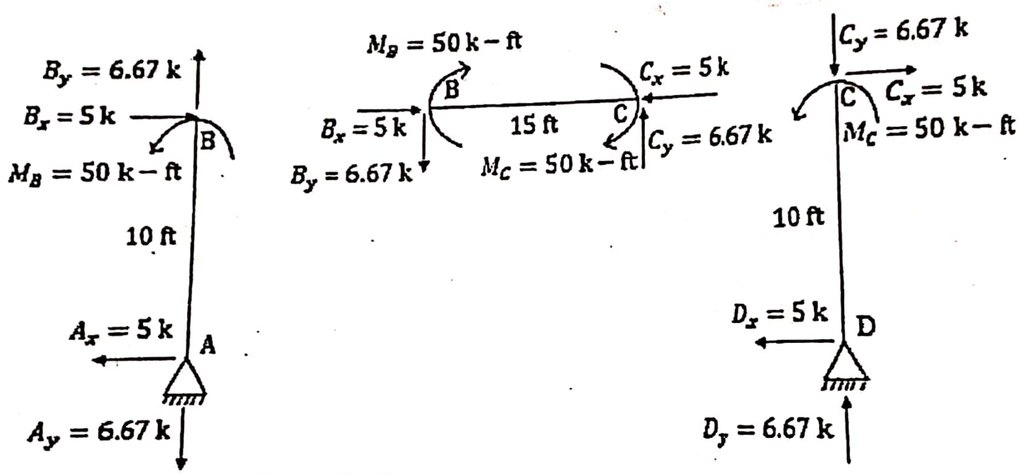
$A_x = D_x = 5\text{ k}$

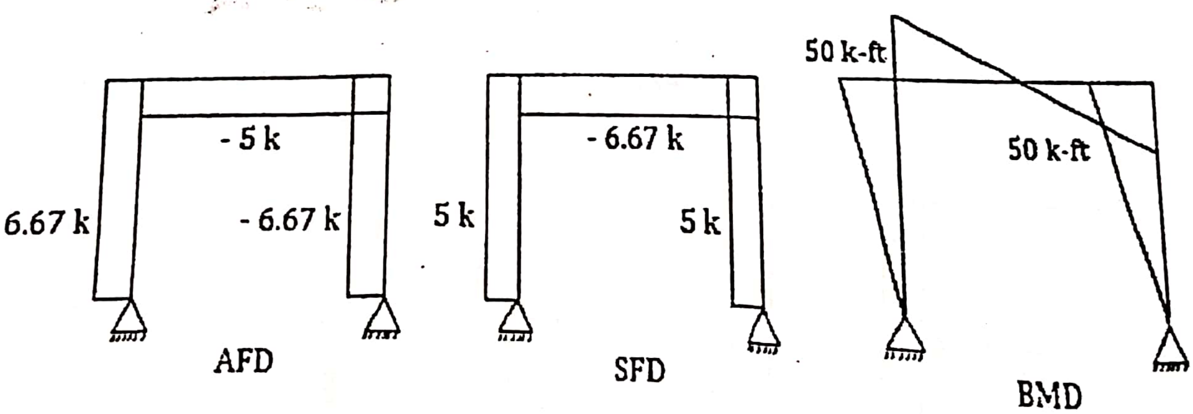
$\sum M_D = -A_y \times 15 + 10 \times 10 = 0$

$A_y = 6.67\text{ kip}$

$\sum F_y = -A_y + D_y = 0$

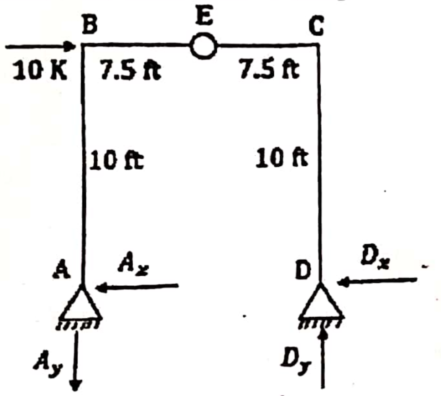
$D_y = 6.67\text{ kip}$





2nd method

The structure is statically indeterminate to the first degree. For this case we will assume points of inflection occur at the midpoints of girder and therefore hinges are placed at these points.



$\sum M_D = -A_y \times 15 + 10 \times 10 = 0$

$A_y = 6.67$ kip

$\sum F_y = -A_y + D_y = 0$

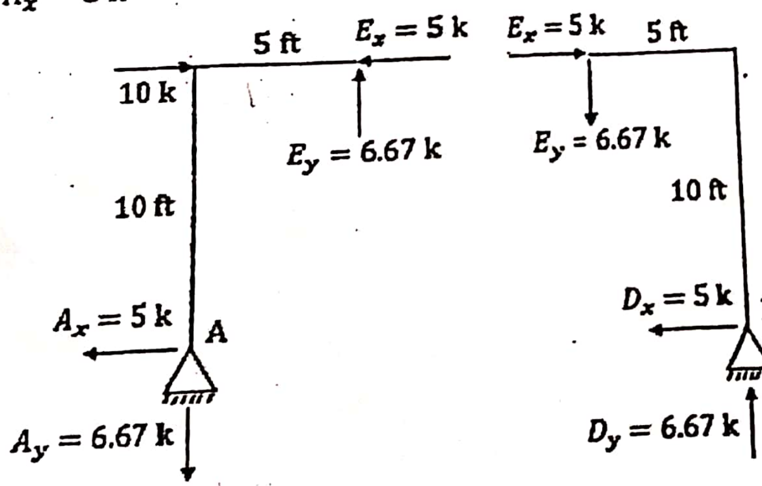
$D_y = 6.67$ kip

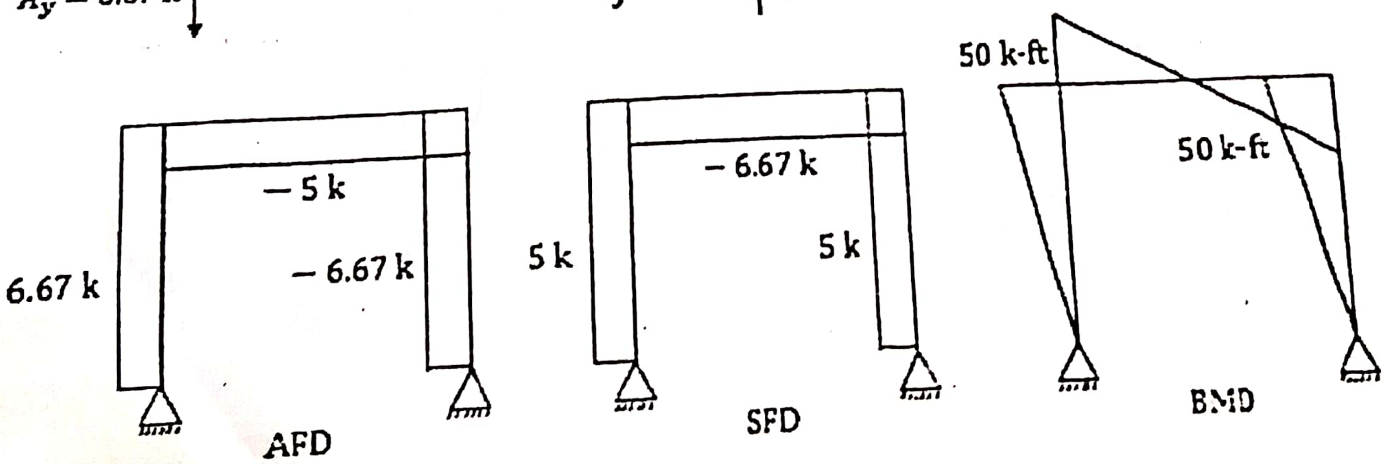
$\sum M_E^{DE} = D_x \times 10 - 6.67 \times 7.5 = 0$

$D_x = 5$ k

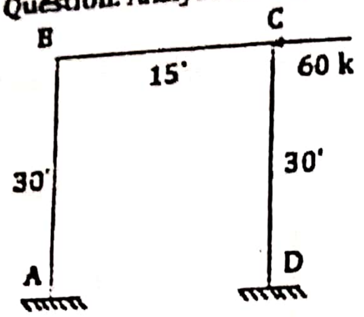
$\sum F_x = A_x + D_x - 10 = 0$

$A_x = 5$ k





Question: Analyze the following frame by portal method and draw SFD and BMD. (34th BCS)



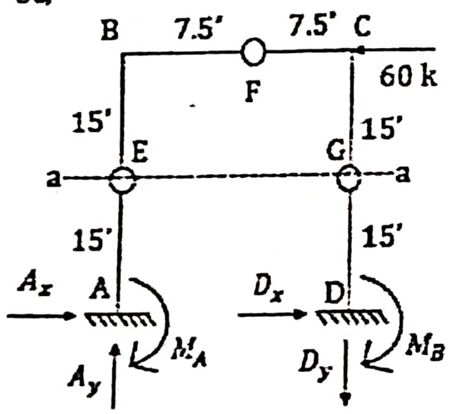
Solution:

$DOSI = (3m + r) - 3j = (3 \times 3 + 6) - 3 \times 4 = 3$

The structure is statically indeterminate to the third degree. For this case we will assume points of inflection occur at the midpoints of all three members and therefore hinges are placed at these points.

$DOSI = (3m + r) - (3j + h) = (3 \times 3 + 6) - (3 \times 4 + 3) = 0$

Now the structure is determinate. To determine the six reactions, we pass a horizontal section *aa* through the hinges *E* and *G*, as shown in Fig. Applying the three equilibrium equations and one equation of condition to the portion *EBCG*, we compute the forces at the internal hinges *E* and *G* to be,

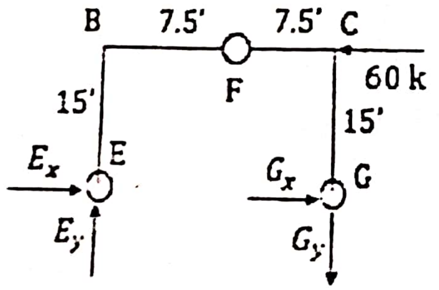


$\sum M_G = E_y \times 15 - 60 \times 15 = 0$

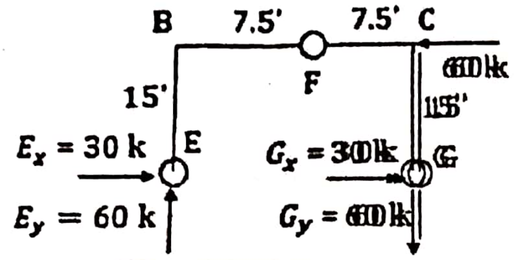
$E_y = 60$ k

$\sum F_y = E_y - G_y = 0$

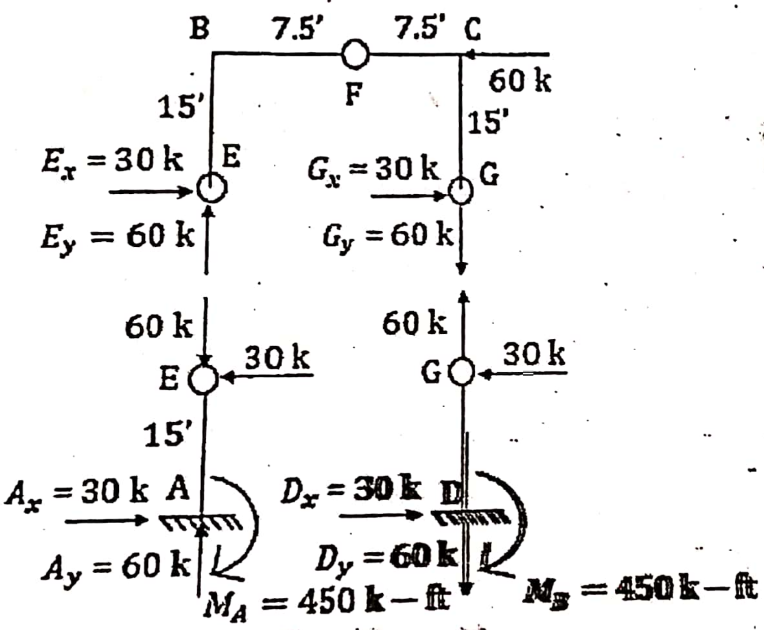
$G_y = 60$ k



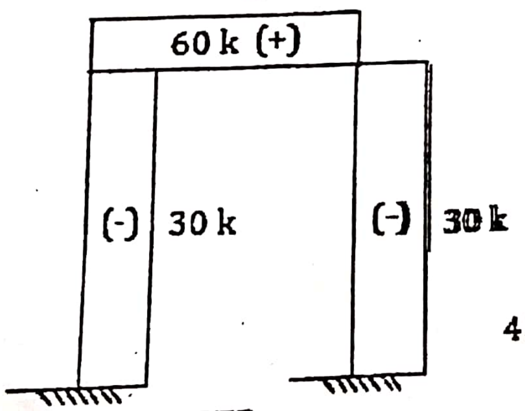
$$\sum M_F^{EF} = 60 \times 7.5 - E_x \times 15 = 0$$
$$E_x = 30 \text{ k}$$
$$\sum F_x = E_x + G_x - 60 = 0$$
$$G_x = 30 \text{ k}$$



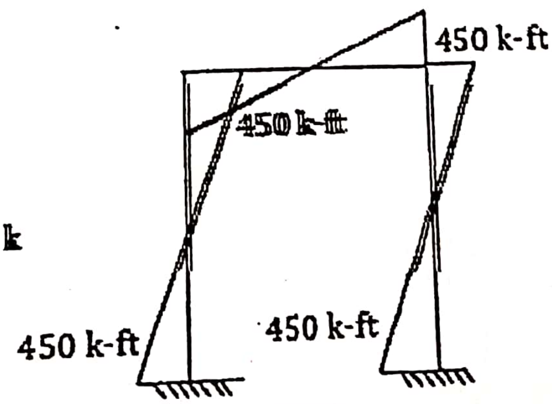
The reactions at supports A and D can now be determined by considering the equilibrium of portions AE and DG, respectively.



Now Draw SFD and BMD.

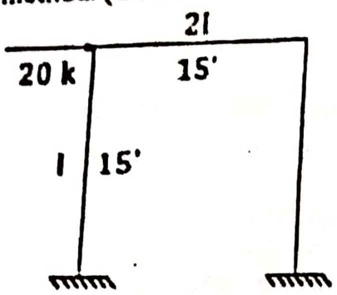


SFD

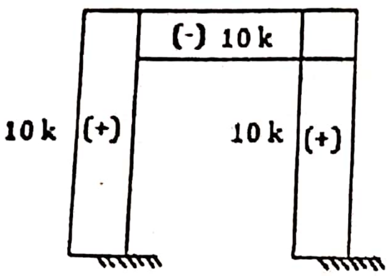


BMD

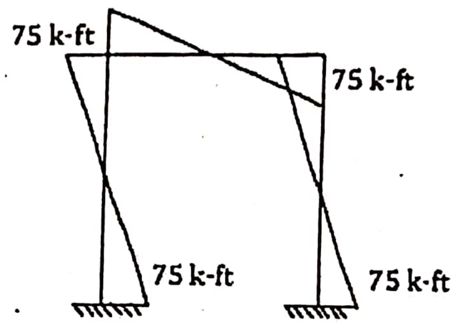
Question: Find the shear force and bending moment diagram for the given portal frame by portal method. (GTCL – 2016)



Solution:

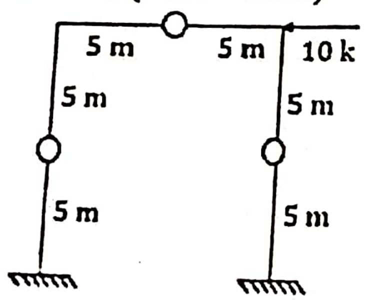


SFD

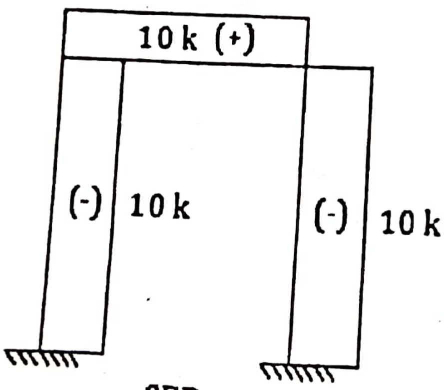


BMD

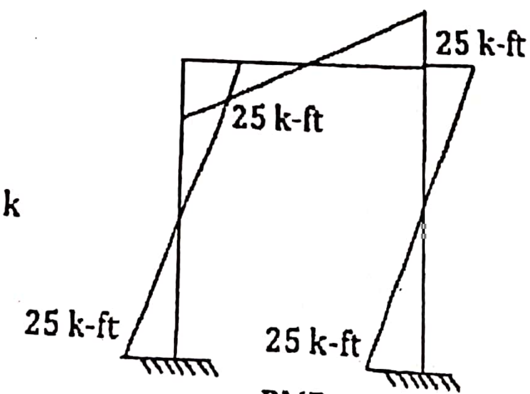
Question: Draw the shear force and bending moment diagram of the following frame by portal method. (PGCB – 2019)



Solution:

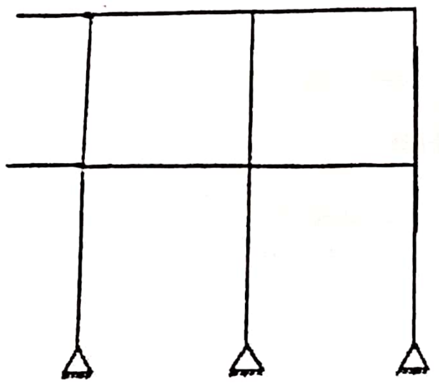


SFD

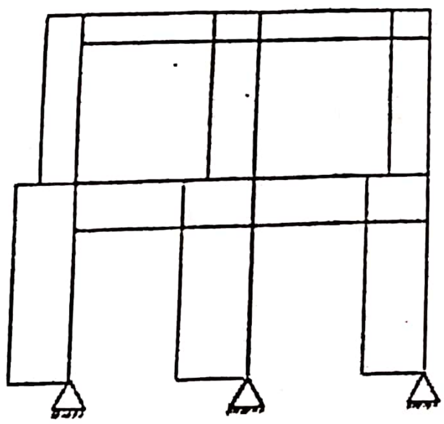


BMD

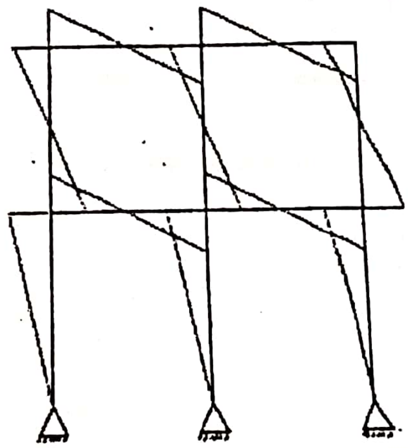
- **Question:** Draw qualitative SFD and BMD of the frame for the lateral loads (BCMCL – 2020)



Solution:



SFD



BMD

Truss: A truss is an assemblage of straight members connected at their ends by flexible connections to form a rigid configuration. Because of their light weight and high strength, trusses are widely used, and their applications range from supporting bridges and roofs of buildings.

Truss Assumptions

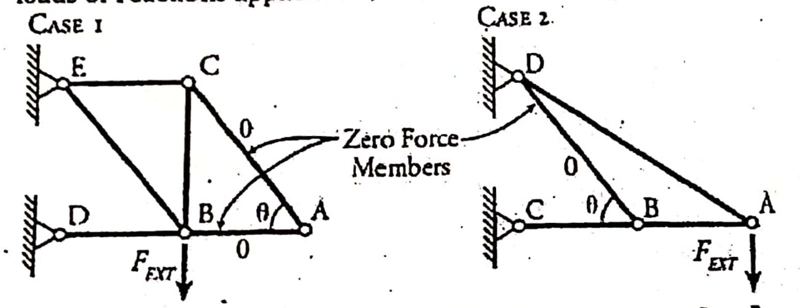
There are four main assumptions made in the analysis of truss.

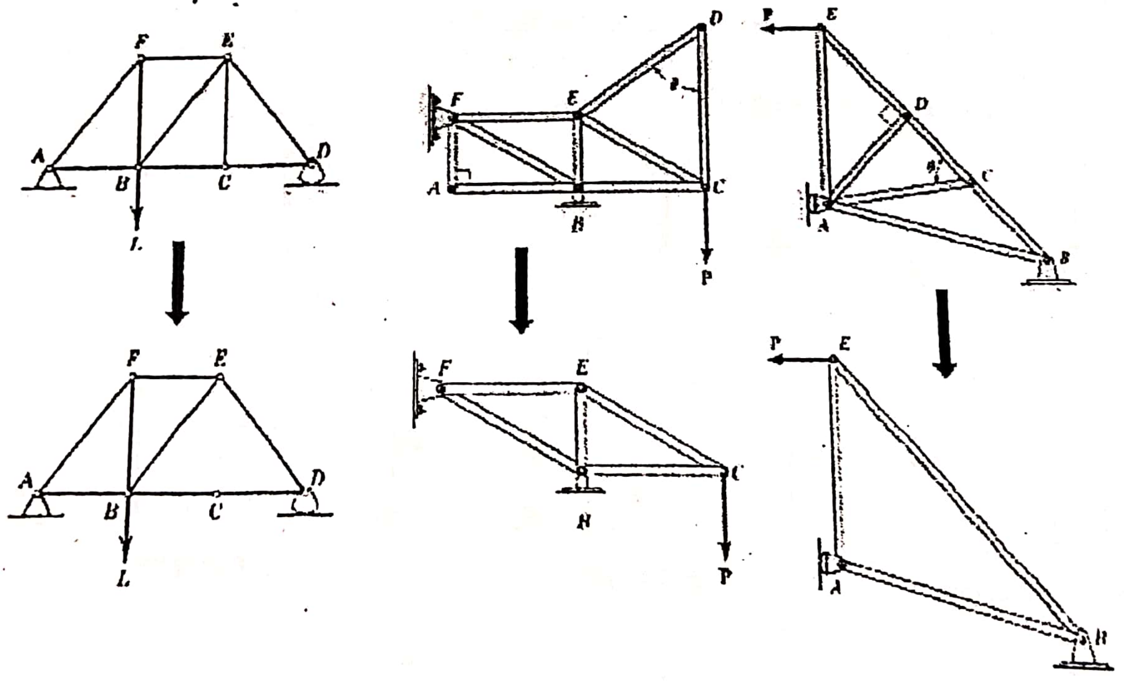
→ Truss members are connected together at their ends only.
→ Trusses are connected together by frictionless pins.
→ The truss structure is loaded only at the joints.
→ The weights of the members may be neglected.

Truss analysis using the method of joints is greatly simplified if one is able to first determine those members that support no loading. These zero-force members may be necessary for the stability of the truss during construction and to provide support if the applied loading is changed.

Case 1: If only two non-collinear members are connected to a joint that has no external loads or reactions applied to it, then the force in both members is zero.

Case 2: If three members, two of which are collinear, are connected to a joint that has no external loads or reactions applied to it, then the force in the member that is not collinear is zero.





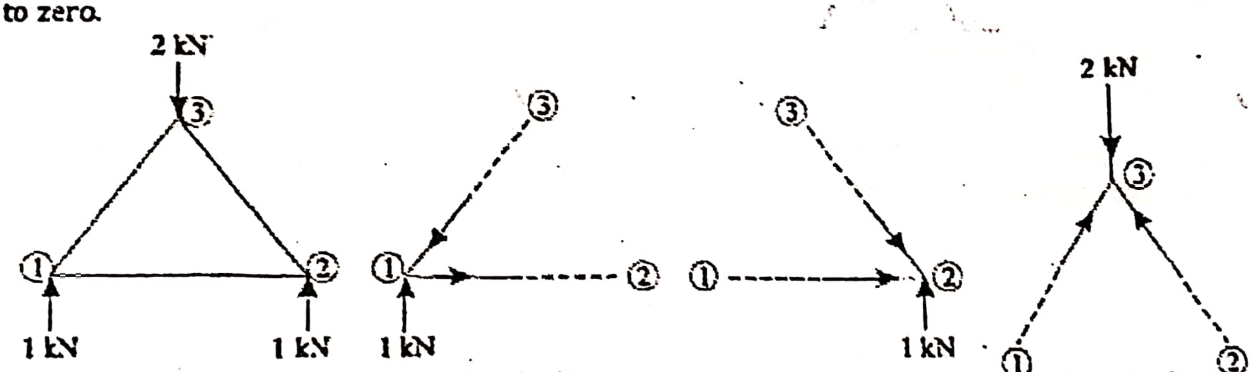
Question: Write down three assumption of truss analysis. (PGCB – 2019)

Solution:

1. Truss members are connected together at their ends only.
2. Truss are connected together by frictionless pins.
3. The truss structure is loaded only at the joints.
4. The weights of the members may be neglected.

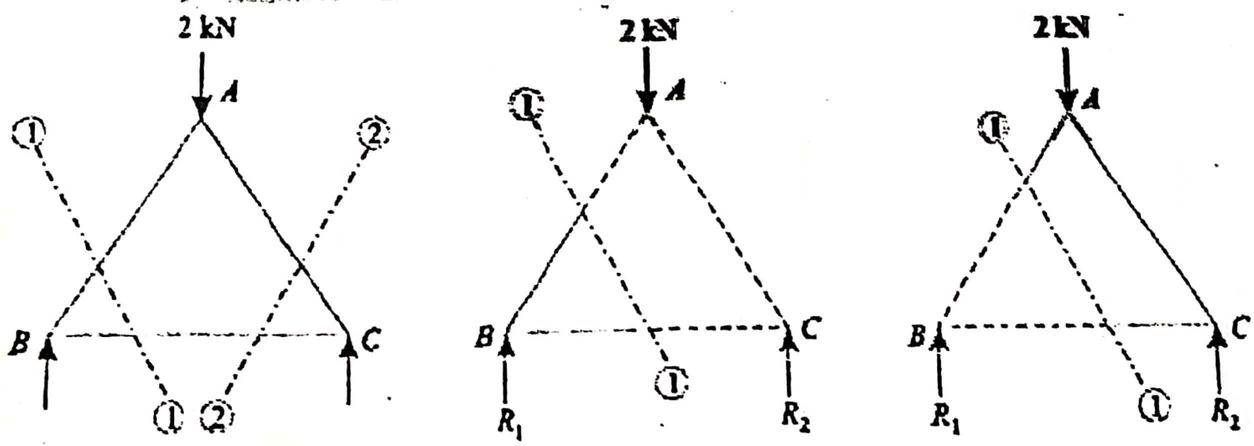
Analytical methods for the forces: The following two analytical methods for finding out the forces, in the members of a perfect frame, are important from the subject point of view:
1. Method of joints.
2. Method of sections.

Method of joints: In this method, each and every joint is treated as a free body in equilibrium as shown in Fig. (*a*), (*b*), (*c*) and (*d*). The unknown forces are then determined by equilibrium equations viz $\Sigma V = 0$ and $\Sigma H = 0$. *i.e.*, Sum of all the vertical forces and horizontal forces is equated to zero.



(*a*) Space diagram (*b*) Joint (1) (c) Joint (2) (*d*) Joint (3)

Method of sections: This method is particularly convenient, when the forces in a few members of a frame are required to be found out. In this method, a section line is passed through the member or members, in which the forces are required to be found out as shown in figure. A part of the structure on any one side of the section line is then treated as a free body in equilibrium under the action of external forces as shown in figure.

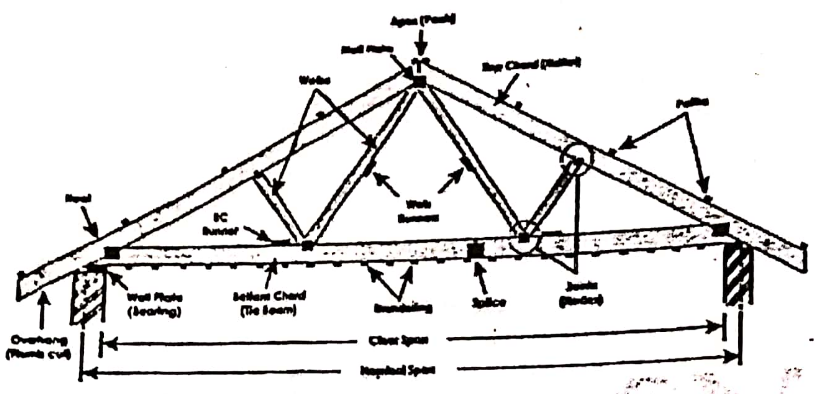


(*a*) Space diagram (*b*) Left part (c) Right part

The unknown forces are then found out by the application of equilibrium or the principles of statics *i.e.*, $\Sigma M = 0$.

Question: Draw typical cross section of a truss with different portion. (PMO – 2015)

Solution:



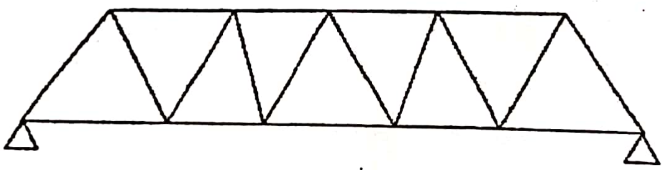
Question: Write down Five commonly used structural system of building in Bangladesh. (PGCL – 2017)

Solution:

Five commonly used structural system of buildings are given below

- Tensile structures
- Rigid frame
- Braced frame / Shear walled frame
- Truss structures
- Outrigger structures
- Tube structures

Question: Determine the determinacy and stability of the truss. (BUET M.Sc – 2019)



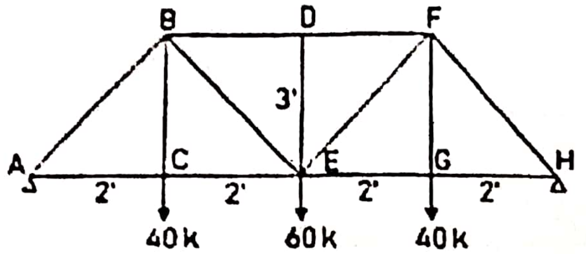
Solution:

Here, Number of member, $m = 19$
Number of joints, $j = 11$
Number of reactions, $r = 4$

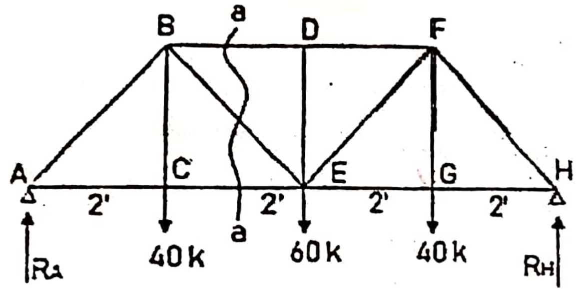
Now, $m + r = 23 > 2j = 22$

So the truss is indeterminate (1st degree) and stable

Question: Determine the member force of BD and BE member for the following truss. (RRI – 2015)



Solution:



$\Sigma M_A = 0$

$-R_H \times 8 + 40 \times 2 + 60 \times 4 + 40 \times 6 = 0$

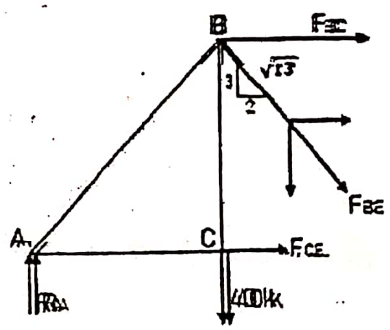
$R_H = 70\text{ k}$

$\Sigma F_Y = 0$

$R_H + R_A - 40 - 60 - 40 = 0$

$R_A = 70\text{ k}$

From section a-a



$\Sigma M_B = 0$

$R_A \times 2 - F_{CE} \times 3 = 0$

$F_{CE} = \dfrac{70 \times 2}{3} = 46.66\text{ k}$

$\Sigma F_Y = 0$

$70 - F_{BE} \times \dfrac{3}{\sqrt{13}} - 40 = 0$

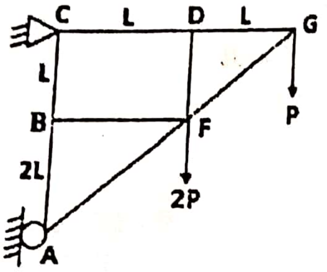
$F_{BE} = 36.05\text{ k}$

$\Sigma F_X = 0$

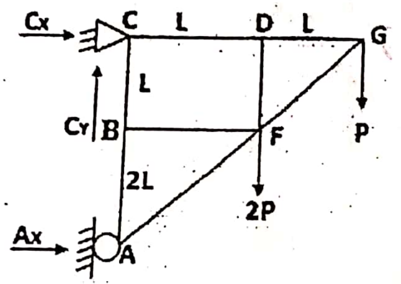
$F_{BD} + F_{CE} + \dfrac{2}{\sqrt{3}} F_{BE} = 0$

$F_{BD} = -66.66\text{ k}$

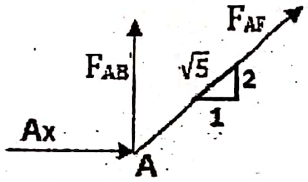
Question: Calculate member force AF for the below truss. (BEPZA – 2016)



Solution:



At joint A:



$\sum M_C = 0$

$2P \times L + P \times 2L - A_x \times 3L = 0$

$A_x = \frac{4}{3}P$

$\sum F_x = 0$

$\frac{4}{3}P + \frac{1}{\sqrt{5}}F_{AF} = 0$

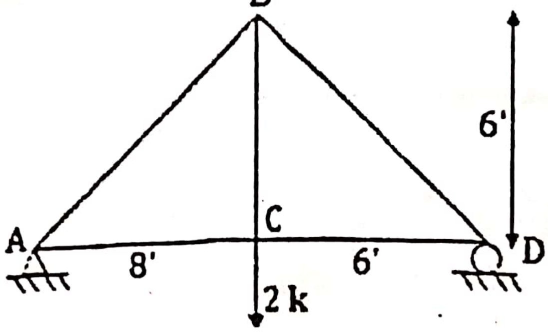
$F_{AF} = -\frac{4\sqrt{5}}{3}P$

$\sum F_y = 0$

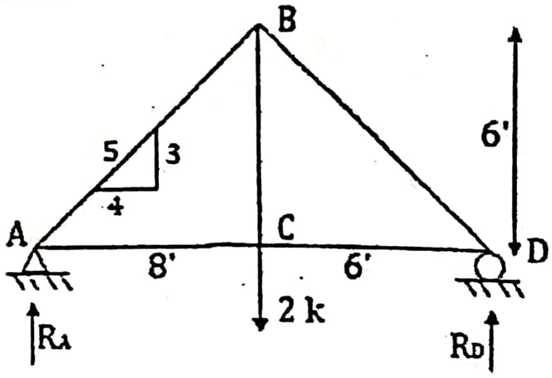
$F_{AB} + \frac{2}{\sqrt{5}}F_{AF} = 0$

$F_{AB} = \frac{8}{3}P$

Question: Find the axial force for the member AB and AC (TGTDCL – 2018)



<u>Solution:</u>



$\Sigma M_D = 0$

$R_A \times 14 - 2 \times 6 = 0$

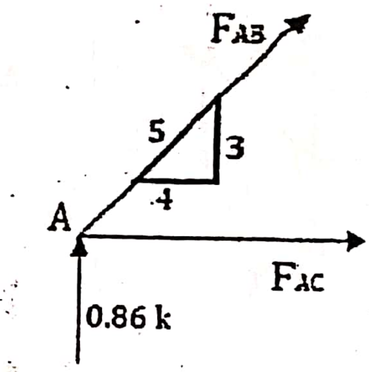
$R_A = 0.86\ k$

$\Sigma F_Y = 0$

$R_D + 0.86 - 2 = 0$

$R_D = 1.14\ k$

<u>At joint A</u>



$\Sigma F_Y = 0$

$0.86 + \frac{3}{5} F_{AB} = 0$

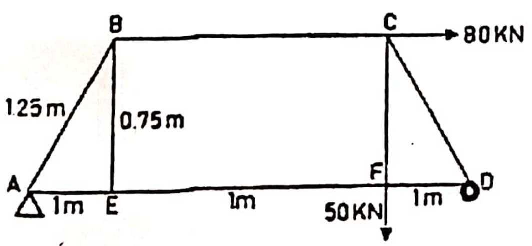
$F_{AB} = -1.43\ k\ (C)$

$\Sigma F_X = 0$

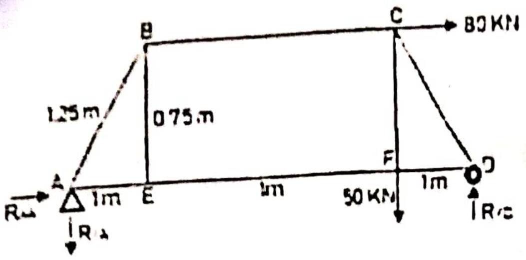
$F_{AC} + \frac{4}{5} F_{AB} = 0$

$F_{AC} = 1.144\ k\ (T)$

Question: Find the member force AB, BC, AE. (HBFC – 2018)



<u>Solution:</u>



$\sum M_A = 0$

$80 \times 0.75 + 50 \times 2 - R_D \times 3 = 0$

$R_D = 53.33KN$

$\sum F_Y = 0$

$R_D - R_{YA} - 50 = 0$

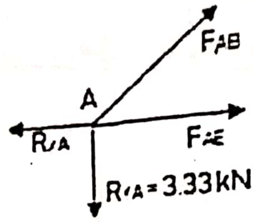
$R_{YA} = 3.33KN$

$\sum F_X = 0$

$R_{XA} - 80 = 0$

$R_{XA} = -80$ KN (T)

Taking the joint A



$\sum F_Y = 0$

$\frac{0.75}{1.25} F_{AB} - 3.33 = 0$

$F_{AB} = 5.55KN$ (T)

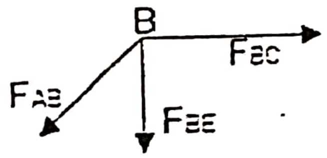
$\sum F_X = 0$

$F_{AE} + \frac{1}{1.25} F_{AB} - R_{XA} = 0$

$F_{AE} + \frac{1}{1.25} \times 5.55 - 80 = 0$

$F_{AE} = 75.56$ KN

Taking joint B

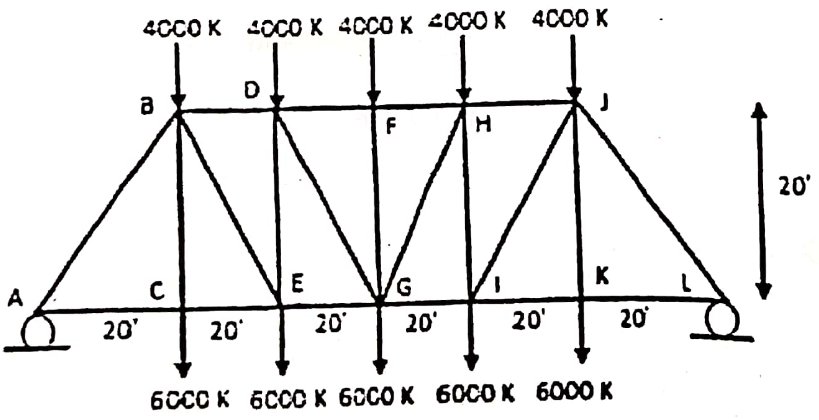


$\sum F_X = 0$

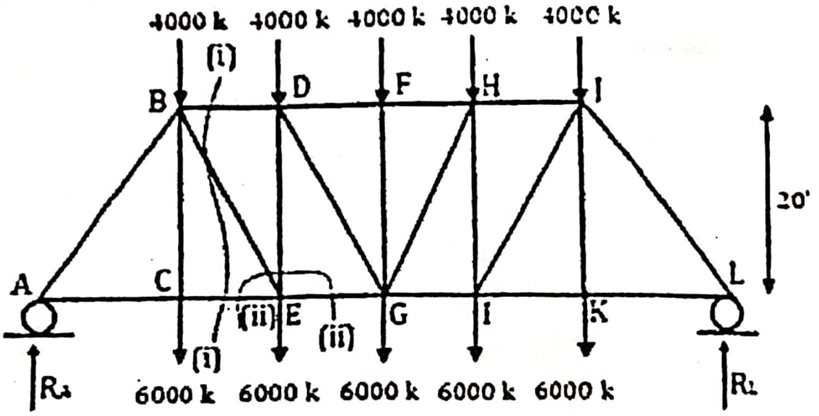
$F_{BC} - \frac{1}{1.25} F_{AB} = 0$

$F_{BC} = 4.44KN$

Question: Determine the member forces of DE and BE (GTCL – 2018)



Solution:



$\Sigma M_A = 0$

$10000 \times (20 + 40 + 60 + 80 + 100) - R_L \times 120 = 0$

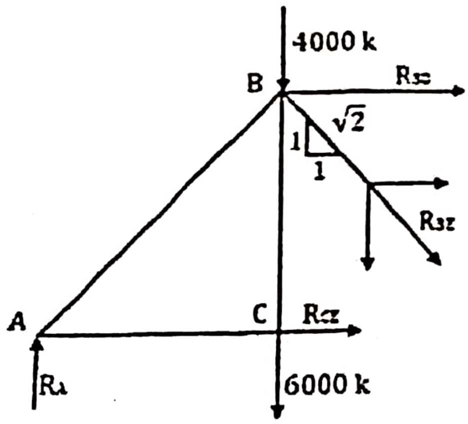
$R_L = 25000$ kip

$\Sigma F_Y = 0$

$25{,}000 - (4000 + 6000) \times 5 + R_A = 0$

$R_A = 25000$ kip

Taking left position of section (i) - (i)



$\Sigma M_B = 0$

$25{,}000 \times 20 - R_{CE} \times 20 = 0$

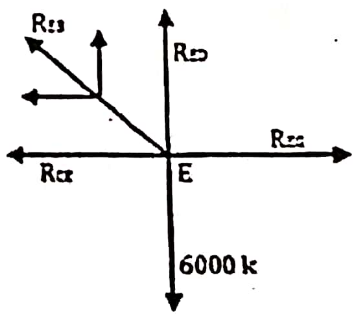
$R_{CE} = 25000$ kip

$\Sigma F_Y = 0$

$25{,}000 - \frac{1}{\sqrt{2}} R_{BE} - 6000 - 4000 = 0$

$R_{BE} = 21213.20$ kip

Taking section (ii) - (ii)



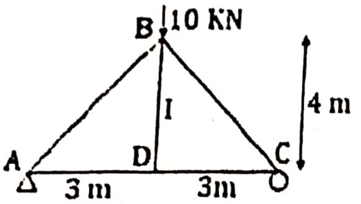
$\Sigma F_Y = 0$

$\frac{1}{\sqrt{2}} R_{EB} + R_{ED} - 6000 = 0$

$\frac{1}{\sqrt{2}} \times 21213.2 - 6000 + R_{ED} = 0$

$R_{ED} = 24000$ kip

Question: Find out the stress at member I and the members at the left hinge support from the following truss. (DESCO – 2019)



Solution:

$\Sigma M_A = 0$ (Clockwise +)

$10 \times 3 - R_C \times 6 = 0$

$R_C = 5$ KN

$\Sigma F_Y = 0$

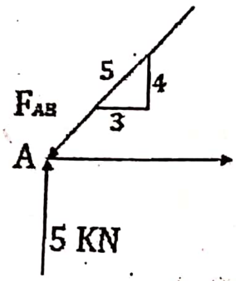
$R_{AY} + R_{CY} = 10$

$R_{AY} = 5$ KN

$\Sigma F_X = 0$

$R_{AX} = 0$

At joint A:

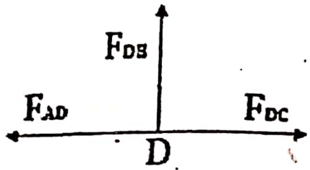


$\Sigma F_Y = 0$

$5 - \frac{4}{5} F_{AB} = 0$

$F_{AB} = 6.25$ KN

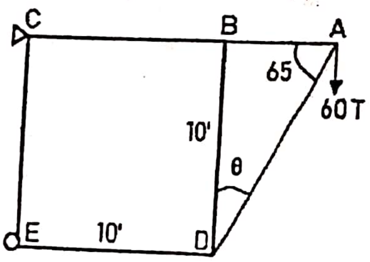
At joint D:



$\Sigma F_Y = 0$

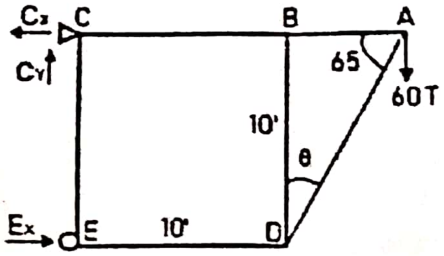
$F_{DB} = 0$

Question: As shown in figure below, in the truss, F = 60 tons and θ = 25°. Find the external reactions components at C and the force member AB and AD. (TGTDCL – 2014)



65

- **Solution:**



$\tan 25 = \frac{AB}{10}$

AB = 4.67'

$\sum F_y = 0$

$C_y - 60 = 0$

$C_y = 60$ T

$\sum M_E = 0$

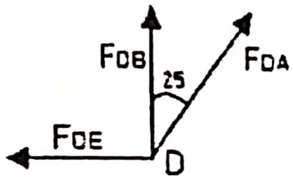
$-C_x \times 10 + 6 \times (10 + 4.67) = 0$

$C_x = 88.02$ T

$\sum F_x = 0$

$E_x = 88.02$ T

At joint D

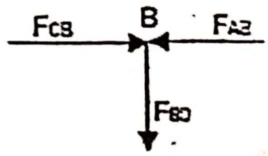


$\sum F_x = 0$

$F_{AD}\ \text{Sin}25 - F_{DE} = 0$

$F_{AD} = 208.27$ T (T)

At joint B

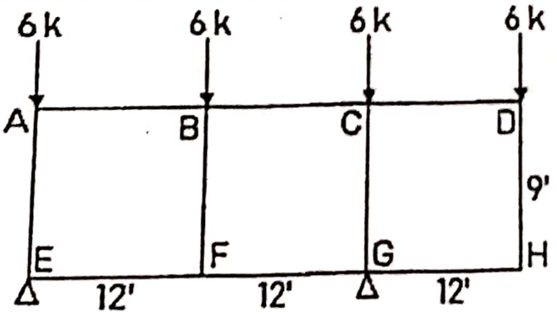


$\sum F_x = 0$

$F_{CB} - F_{AB} = 0$

$F_{AB} = 88.02$ T (C)

Question: Find the bar forces of the member BC, CF and CG. (BPDB – 2015)



Solution:

$\sum M_E = 0$

$6 \times 12 + 6 \times 24 + 6 \times 36 - R_G \times 24 = 0$

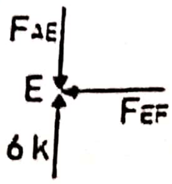
$R_G = 18$ k

$\sum F_y = 0$

$R_E + R_G - 6 - 6 - 6 - 6 = 0$

$R_E = 6$ k

At joint E:



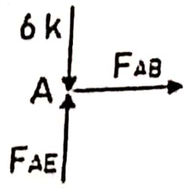
$\sum F_y = 0$

$F_{AE} = 6\,k$

$\sum F_x = 0$

$F_{EF} = 0$

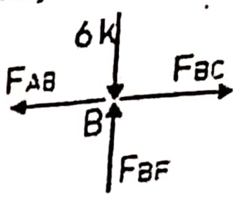
At joint A:



$\sum F_x = 0$

$F_{AB} = 0$

At joint B:



$\sum F_x = 0$

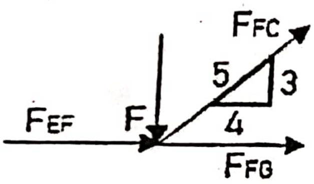
$F_{AB} = F_{BC}$

$F_{BC} = 0$

$\sum F_y = 0$

$F_{BF} = 6\,k$

At joint F:



$\sum F_y = 0$

$\frac{3}{5}F_{AE} - 6 = 0$

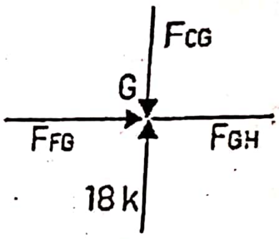
$F_{FC} = 10\,k$

$\sum F_x = 0$

$\frac{4}{5}F_{FC} + F_{EF} + F_{FG} = 0$

$F_{FG} = -8\,k$

At joint G:



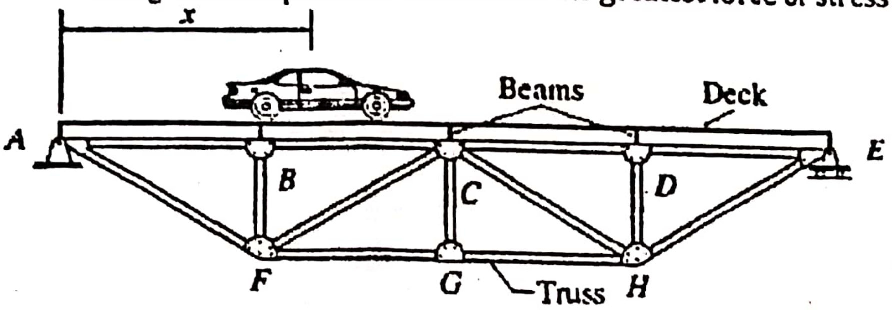
$\sum F_y = 0$

$18 - F_{CG} = 0$

$F_{CG} = 18\,k$

Influence Line

Consider the bridge in Figure. As the car moves across the bridge, the forces in the truss members change with the position of the car and the maximum force in each member will be at a different car location. The design of each member must be based on the maximum probable load each member will experience. Therefore, the truss analysis for each member would involve determining the load position that causes the greatest force or stress in each member.



Bridge Truss Structure Subjected to a Variable Position Load

Structural analysis for variable loads consists of two steps:
1. Determining the positions of the loads at which the response function is maximum.
2. Computing the maximum value of the response function.

Response Function = support reaction, axial force, shear force or bending moment.

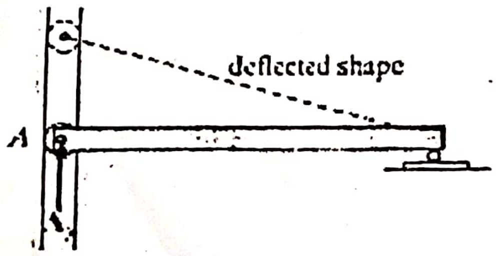
An influence line is a graph of a response function of a structure as a function of the position of a downward unit load moving across the structure. Influence lines for statically determinate structures are always piecewise linear.

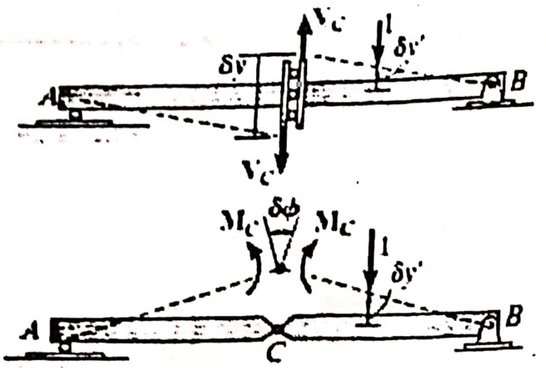
Muller-Breslau Principle

The influence line for a response function is given by the deflected shape of the released structure due to a unit displacement (or rotation) at the location in the direction of the response function. A released structure is obtained by removing the displacement constraint corresponding to the response function of interest from the original structure.

This principle is valid only for the influence lines for response functions involving forces and moments (reactions, shears, bending moments or forces in truss members) and it does not apply to the influence lines for deflections.

Releases:
Support reaction – remove translational support restraint.
Internal shear - introduce an internal glide support to allow differential displacement movement.
Bending moment - introduce an internal hinge to allow differential rotation movement.



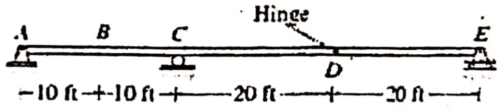


Sign convention

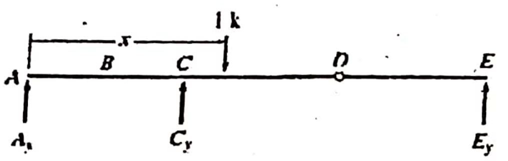
If the IL for a vertical force reaction at a point on a beam is to be constructed, consider the reaction to be positive at the point when it acts upward on the beam.

If a shear or moment IL is to be drawn for a point, take the shear or moment at the point as *+ve* according to the same sign convention used for drawing shear and moment diagram.

Question: Draw the influence lines for the vertical reactions at supports *A*, *C*, and *E*, the shear just to the right of support *C*, and the bending moment at point *B* of the beam shown in Fig.

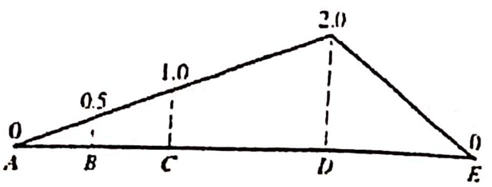


(a)

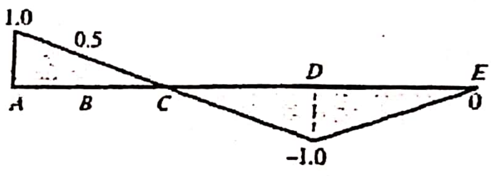


(b)

(c) Influence Line for E_y (k/k)

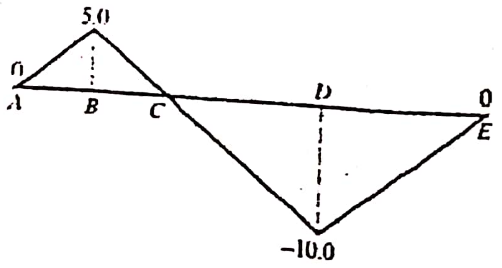


(d) Influence Line for C_y (k/k)



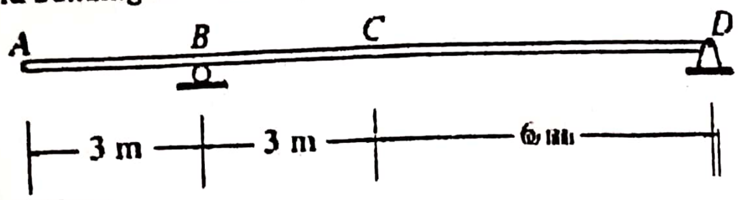
(e) Influence Line for A_y (k/k)

(f) Influence Line for $S_{C,R}$ (k/k)

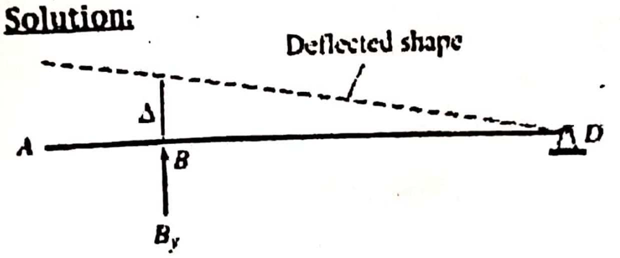


(g) Influence Line for M_B (k-ft/k)

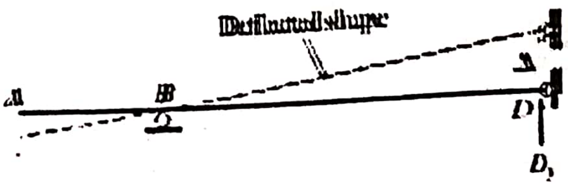
Question: Draw the influence lines for the vertical reactions at supports B and D and the shear and bending moment at point C of the beam shown in Fig.



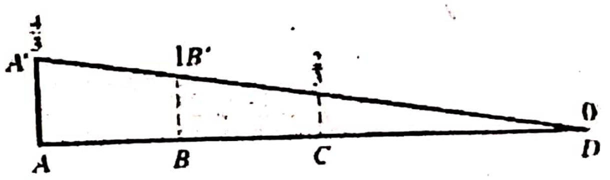
Solution:



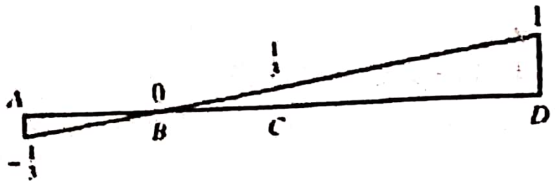
Released structure for B_y



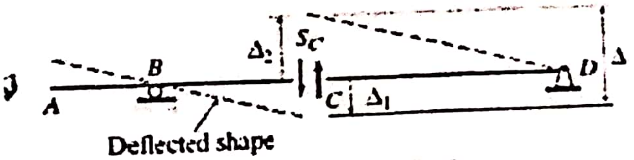
Released structure for D_y



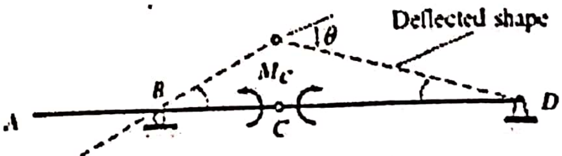
Influence line for B_y (kN/kN)



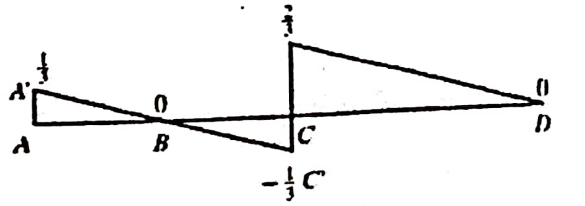
Influence line for D_y (kN/kN)



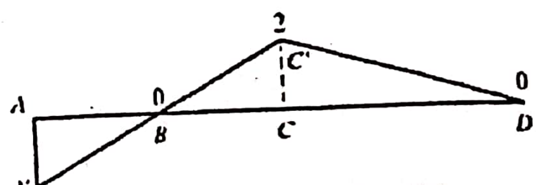
Released structure for S_C



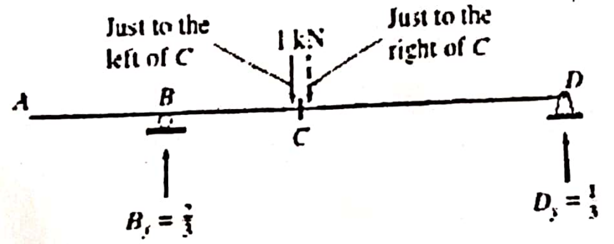
Released structure for M_C

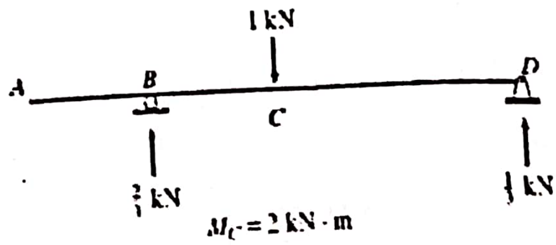


Influence line for S_C (kN/kN)



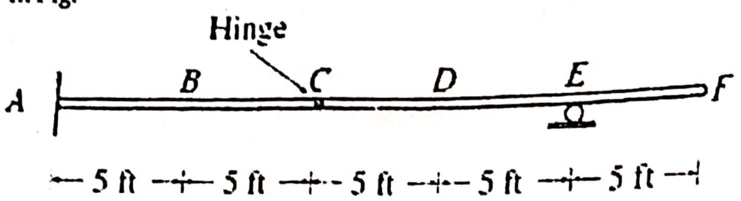
Influence line for M_C (kN · m/kN)



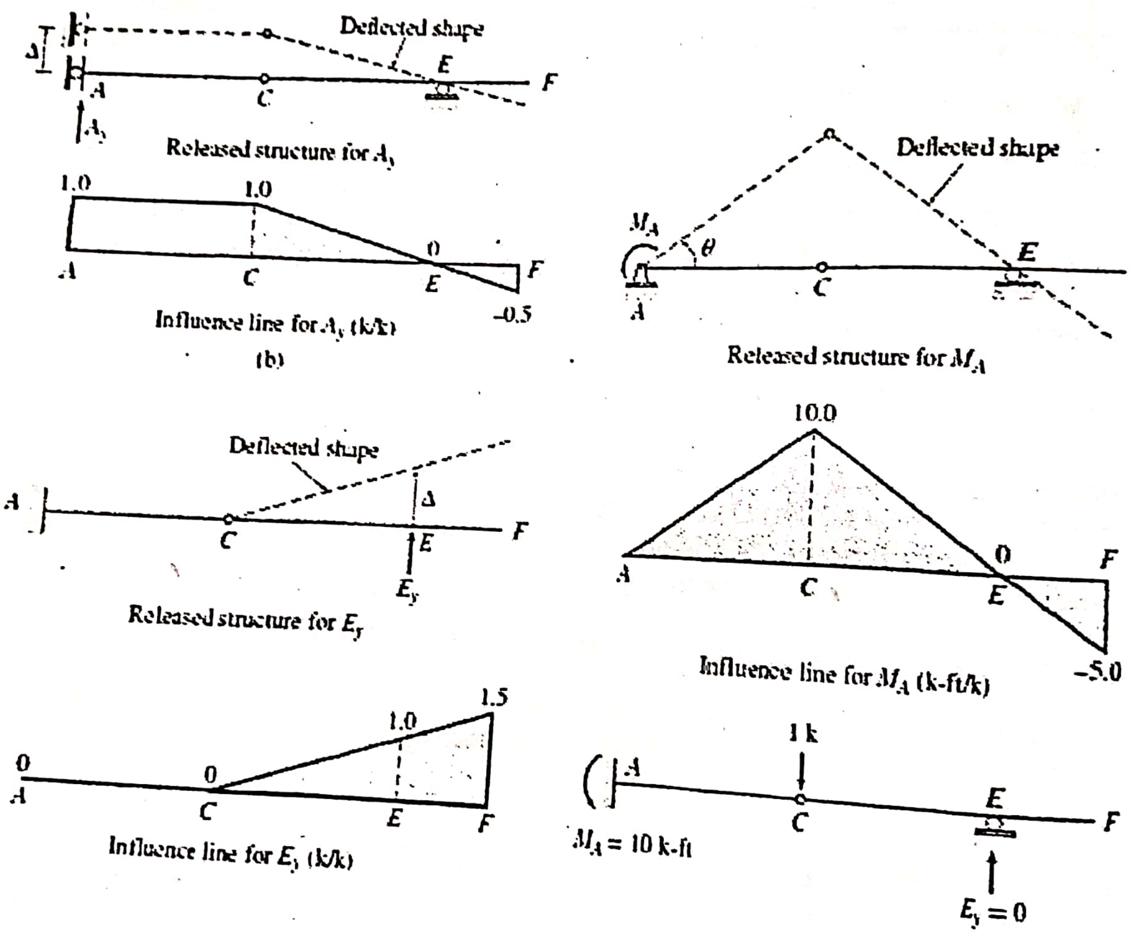


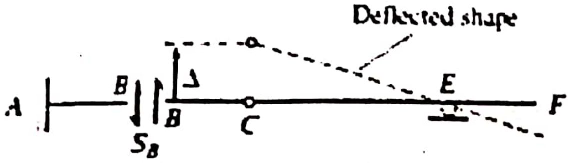
$$S_C = \begin{cases} -\frac{1}{3}\text{ kN} & \text{when 1 kN is at just to the left of } C \\ +\frac{2}{3}\text{ kN} & \text{when 1 kN is at just to the right of } C \end{cases}$$

Question: Draw the influence lines for the vertical reactions at support... moment at support *A*, the shear at point *B*, and the bending moment at point *D* in Fig.

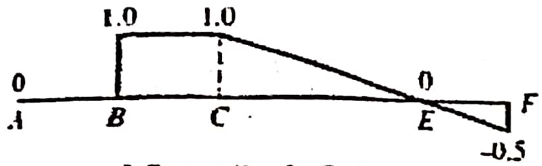


Solution:

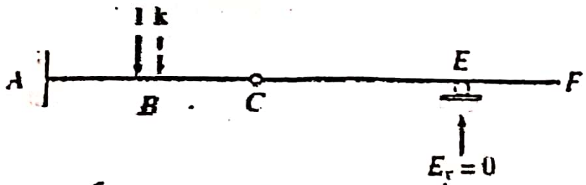




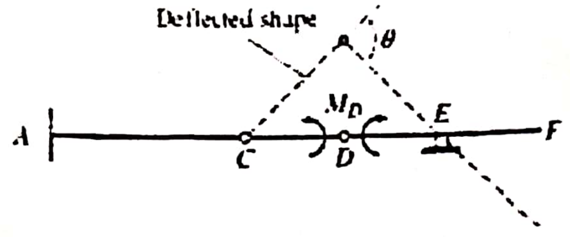
Released structure for S_B



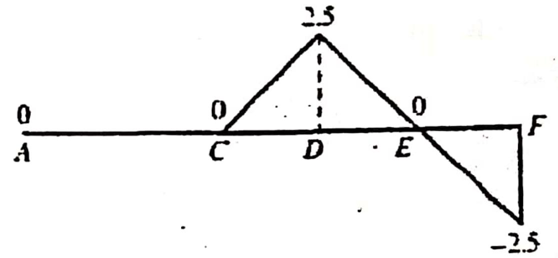
Influence line for S_B (k/k)



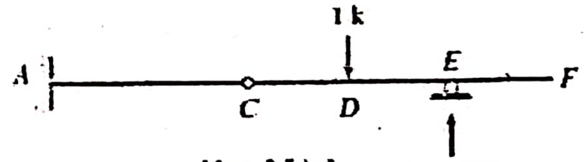
$$S_B = \begin{cases} 0 \text{ when 1 k is at just to the left of } B \\ 1 \text{ k when 1 k is at just to the right of } B \end{cases}$$



Released structure for M_D



Influence line for M_D (k-ft/k)



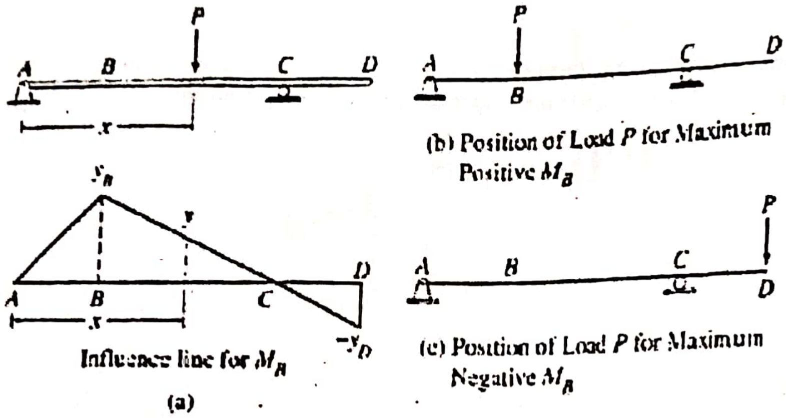
$M_D = 2.5$ k-ft $\quad E_y = 0.5$ k

In many practical applications, it is necessary to determine only the general shape of the influence lines but not the numerical values of the ordinates. Such an influence line diagram is known as a qualitative influence line diagram. An influence line diagram with numerical values of its ordinates is known as a quantitative influence line diagram.

Response at a particular location due to a single moving concentrated load

The value of a response function **due to any single concentrated load can be obtained by** multiplying the magnitude of **the load by the ordinate of the response function influence line at the** position of the load.

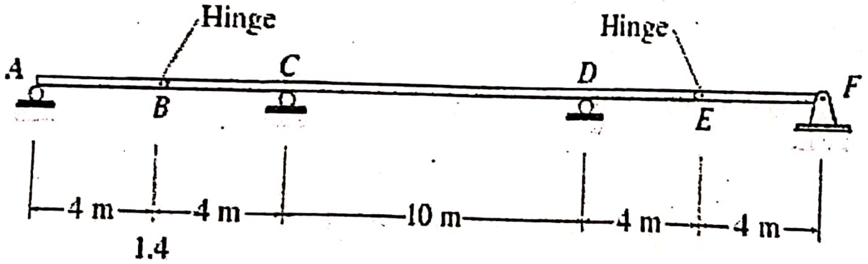
To determine the maximum positive value of a response function due to a single moving concentrated load, the load must be placed at the location of the maximum positive ordinate of the response function influence line, whereas to determine the maximum negative value of the response function, the load must be placed at the location of the maximum negative ordinate of the influence line.
Suppose that we wish to determine the bending moment at B when the load P is located at a distance x. M_B = P y

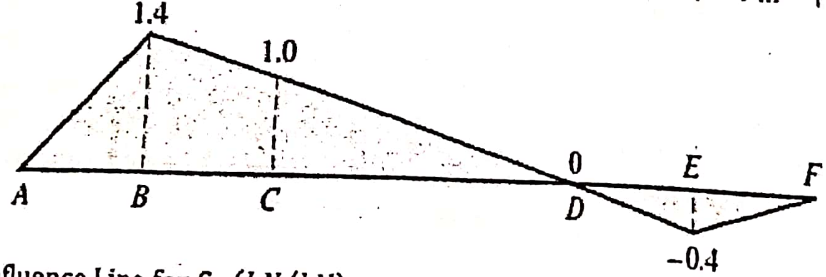


Maximum Positive bending moment at B
Place the load P at point B
Maximum bending moment at B, $M_B = P\,y_B$

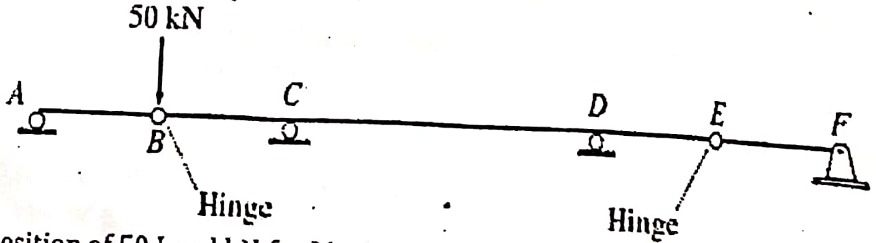
Maximum Negative bending moment at B
Place the load P at point D
Maximum bending moment at B, $M_B = P\,y_D$

Question: For the beam shown in Fig, determine the maximum upward reaction at support C due to a 50 kN concentrated live load.





Influence Line for C_y (kN/kN)

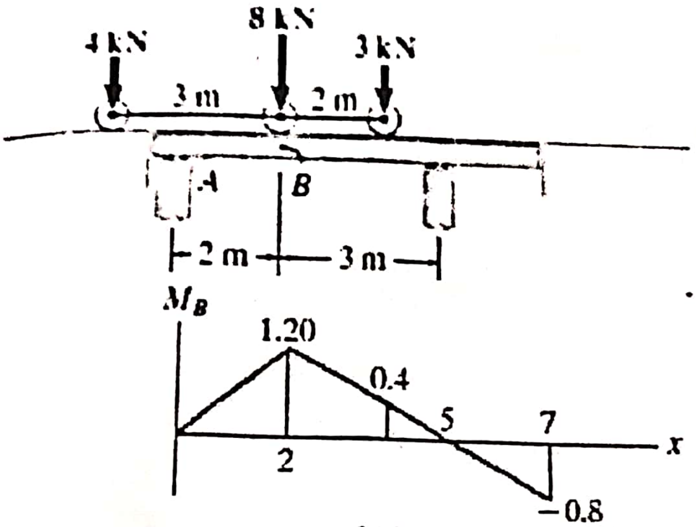


Position of 50 Load kN for Maximum Upward C_y

Maximum positive value of C_y due to the 50 kN concentrated live load, we place the load at B, where the maximum positive ordinate (1.4 kN/kN) of the influence line occurs.

Maximum upward reaction at C

$C_y = 50\ (+1.4) = +70\ \text{kN} = 70\ \text{kN}\uparrow$



influence line for M_B

$M_B = 8 \times 1.20 + 3 \times 0.4 = 10.8\ \text{kN}-\text{m}$

Question: Define influence line. Why is it drawn? (40th BCS)

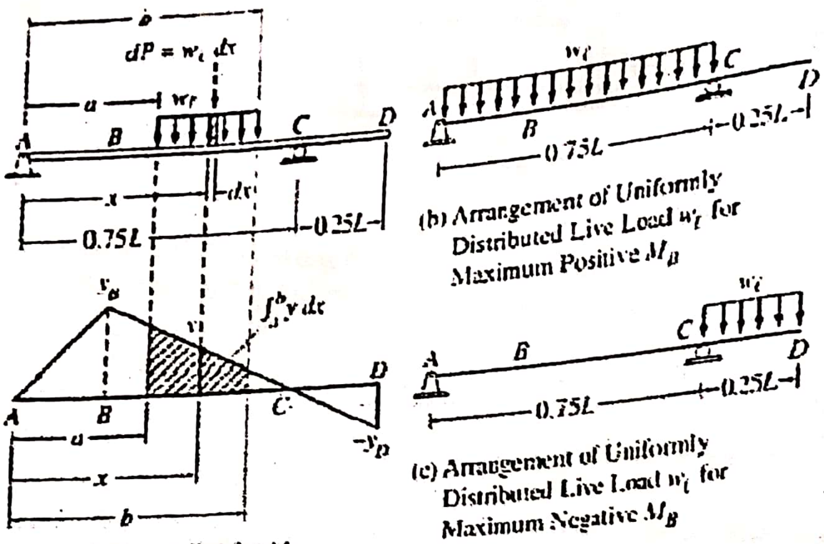
Solution:

An Influence line represents the variation of structural response function (support reaction, shear force, bending moment or deflection etc.) due to change in the position of a concentrated load on the span (such as bridges, cranes, conveyor etc.). A plot of the influence line vs. the load position is known as influence line diagram. The influence line represents the variation of either the reaction, shear force, moment or deflection at a specific point in a member as a concentrated force that moves over the member. When the diagram is formulated one can easily determine the most critical live load position for which the structure must be designed for live load.

Response at a particular location due to a uniformly distributed live load

The value of a response function due to a uniformly distributed load applied over a portion of the structure can be obtained by multiplying the load intensity by the net area under the corresponding portion of the response function influence line.

To determine the maximum positive (or negative) value of a response function due to a uniformly distributed live load, the load must be placed over those portions of the structure where the ordinates of the response function influence line are positive (or negative).



Influence line for M_B

Maximum positive bending moment at B

$M_B = w_l$ (area under the influence line between A and C)

$M_B = w_l \times \frac{1}{2} \times 0.75L \times y_B = 0.375\ w_l\ y_B\ L$

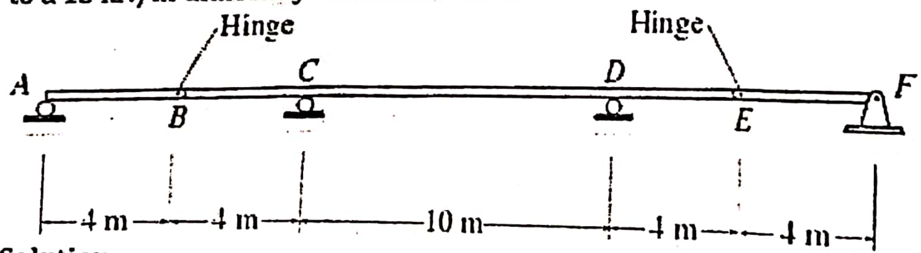
Maximum negative bending moment at B

$M_B = w_l$ (area under the influence line between C and D)

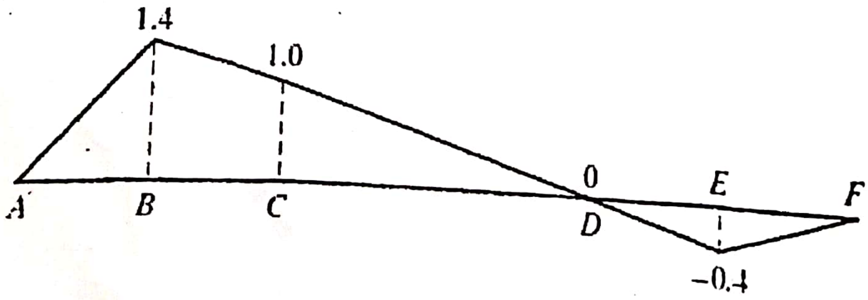
$M_B = w_l \times \frac{1}{2} \times 0.25L \times (-y_D) = 0.375\ w_l\ y_D\ L$

To determine the maximum positive (or negative) value of a response function due to a uniformly distributed live load, the load must be placed over those portions of the structure where the ordinates of the response function influence line are positive (or negative).

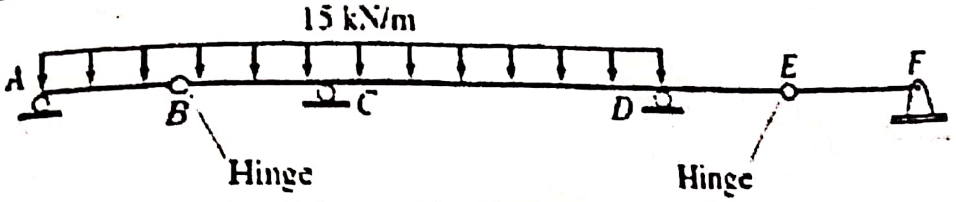
Question: For the beam shown in Fig, determine the maximum upward reaction at support C due to a 15 kN/m uniformly distributed live load.



Solution:



Influence Line for C_y (kN/kN)



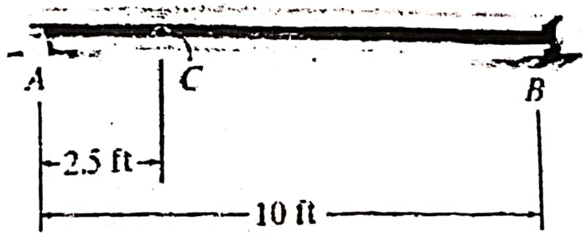
Arrangement of 15 kN/m Load for Maximum Upward C_y

The ordinates of the influence line for C_y are positive between points A and D. Therefore, to obtain the maximum positive value of C_y, place the 15 kN/m uniformly distributed live load over the portion AD of the beam.

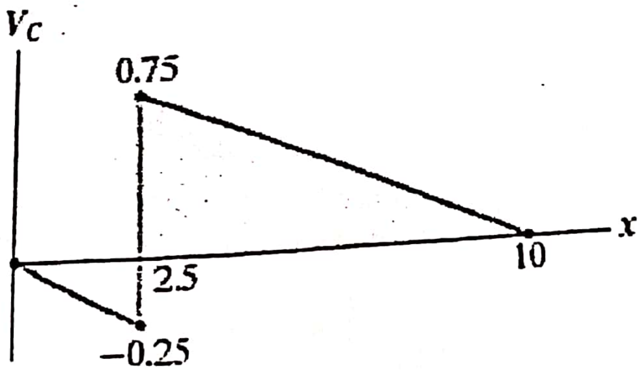
Maximum upward reaction at C

$$C_y = 15\left[\frac{1}{2} \times (+1.4) \times 18\right] = +189 \text{ kN} = 189 \text{ kN} \uparrow$$

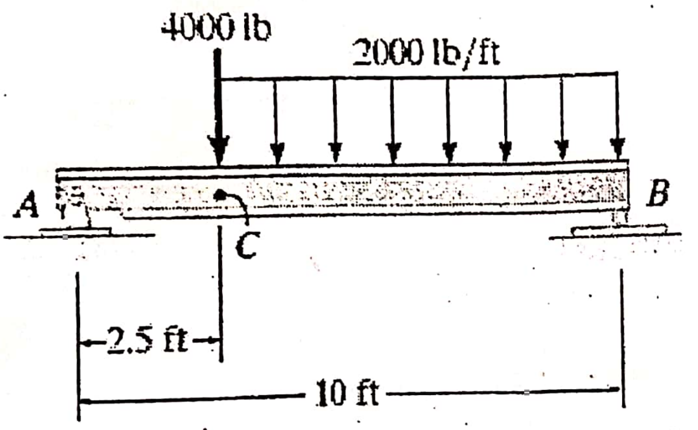
Question: Determine the maximum positive shear that can be developed at point C in the beam shown in Figure due to a concentrated moving load of 4000 lb and a uniform moving load of 2000 lb/ft.



Solution:



influence line for V_C



Maximum positive shear at C due to 4000 lb concentrated load

$V_c = 0.75 \times 4000 = 3000$ lb

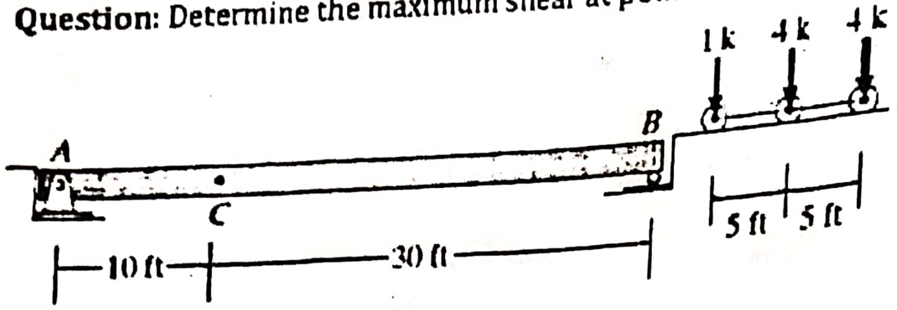
Maximum positive shear at C due to 2000 lb/ft uniform load

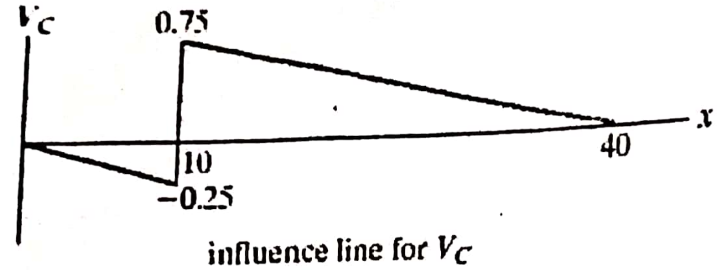
$$V_c = 2000\left[\frac{1}{2}(10 - 2.5)0.75\right] = 5625 \text{ lb}$$

Total maximum shear at C,

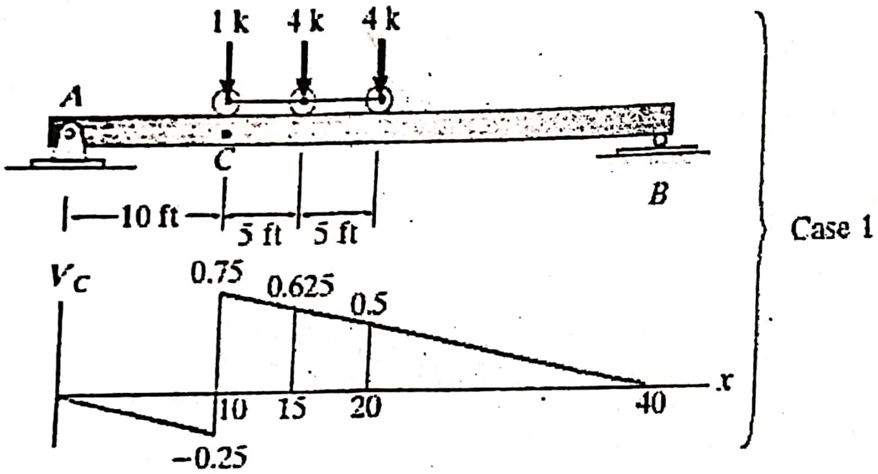
$V_{c(max)} = 3000 + 5625 = 8625$ lb

Question: Determine the maximum shear at point C for the following series loading.

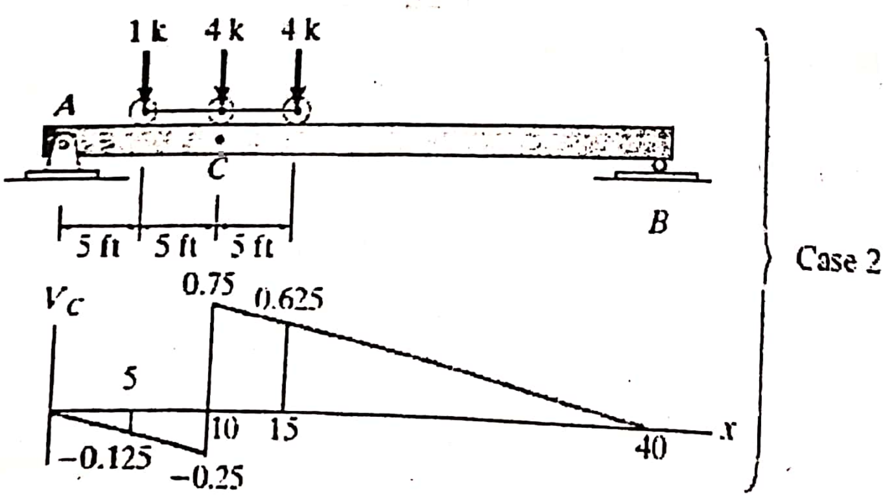




influence line for V_C

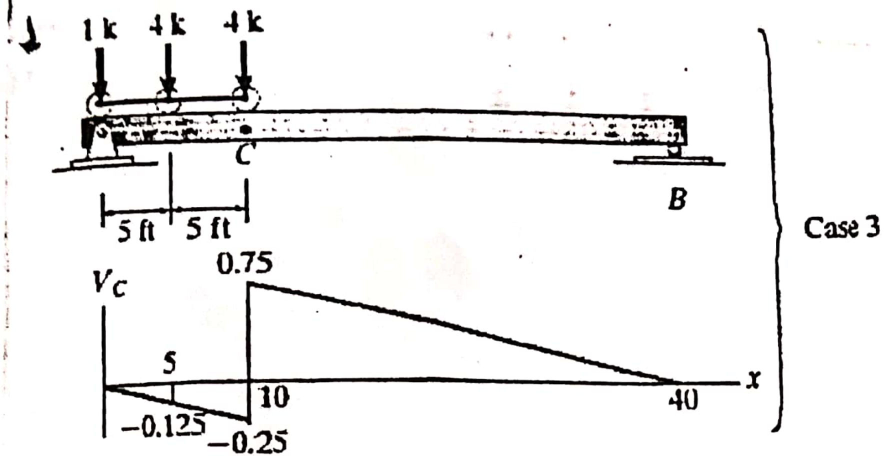


$V_c = 1 \times 0.75 + 4 \times 0.625 + 4 \times 0.5 = 5.25\ \text{k}$



$V_c = -1 \times 0.125 + 4 \times 0.75 + 4 \times 0.625 = 5.375\ \text{k}$

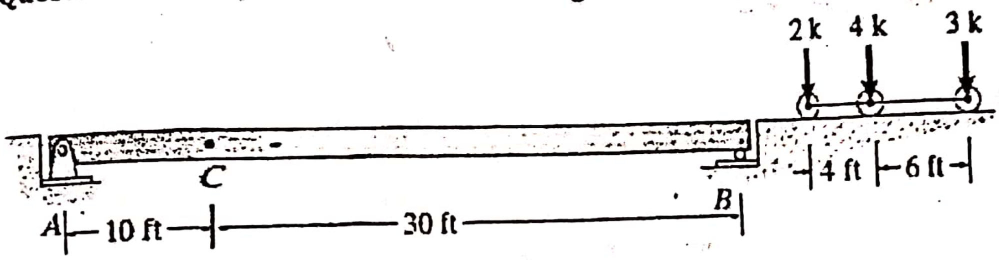
DESIGN INT...



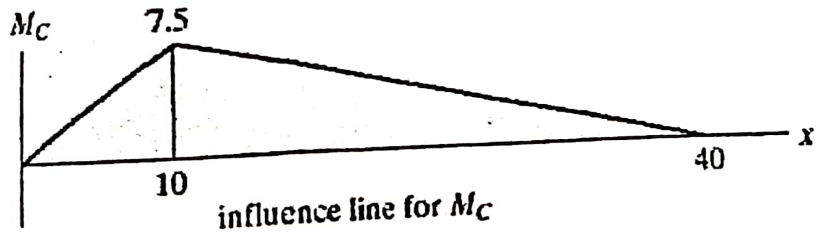
$V_c = 1 \times 0 - 4 \times 0.125 + 4 \times 0.5 = 2.5$ k

The maximum positive shear at point C = 5.375 k

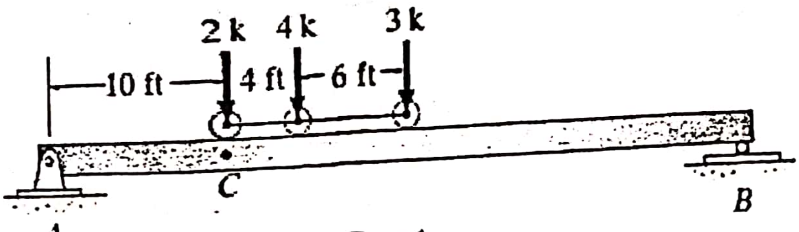
Question: Determine the maximum bending moment at point C for the following series loading.



Solution:

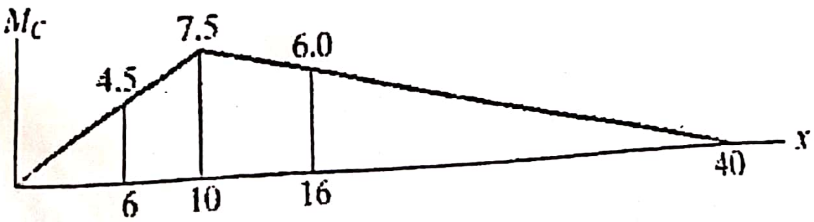


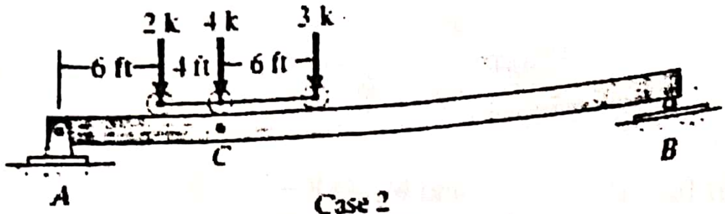
influence line for M_C



Case 1

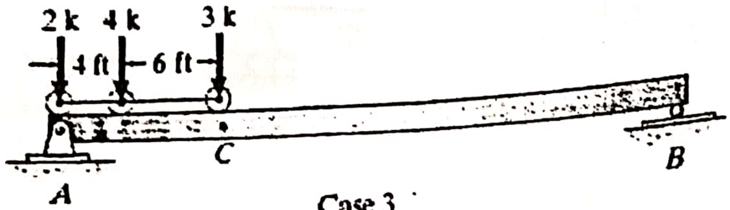
$M_C = 2 \times 7.5 + 4 \times 6.5 + 3 \times 5 = 56.0$ k − ft





Case 2

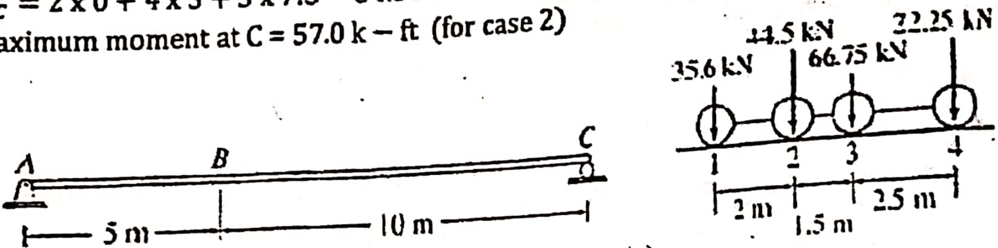
$M_C = 2 \times 4.5 + 4 \times 7.5 + 3 \times 6 = 57.0\ \text{k-ft}$



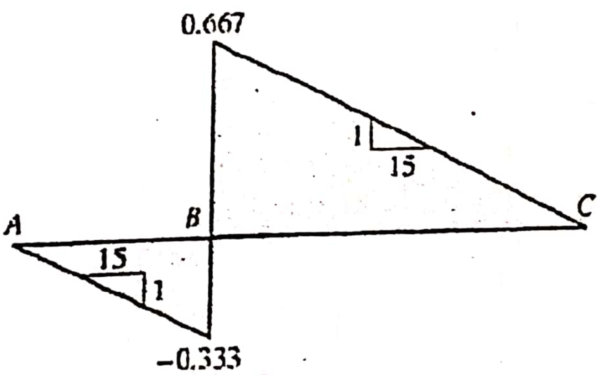
Case 3

$M_C = 2 \times 0 + 4 \times 3 + 3 \times 7.5 = 34.50\ \text{k-ft}$

Maximum moment at C = 57.0 k – ft (for case 2)

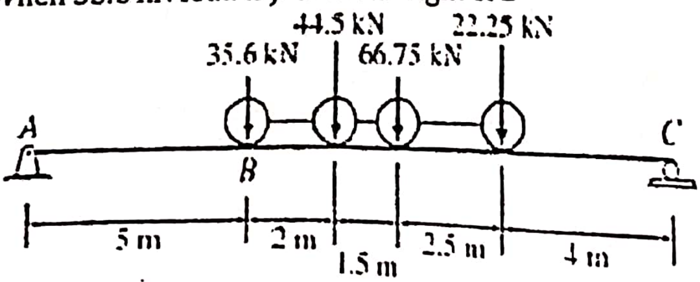


(a)



(b) Influence Line for S_B (kN/kN)

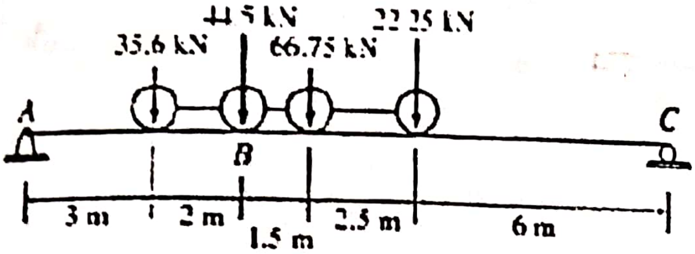
When 35.6 kN load is just to the right of B



(c) Loading Position 1

$$S_B = 35.6 \times 10 \times \frac{1}{15} + 44.5 \times 8 \times \frac{1}{15} + 66.75 \times 6.5 \times \frac{1}{15} + 22.25 \times 4 \times \frac{1}{15} = 83.32\ \text{kN}$$

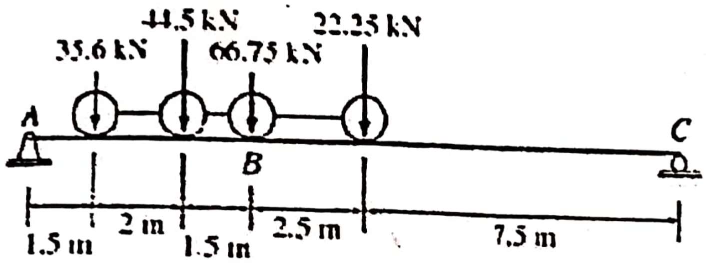
When 44.5 kN load is just to the right of B



(d) Loading Position 2

$$S_B = -35.6 \times 3 \times \frac{1}{15} + 44.5 \times 10 \times \frac{1}{15} + 66.75 \times 8.5 \times \frac{1}{15} + 22.25 \times 6 \times \frac{1}{15} = 69.28 \text{ kN}$$

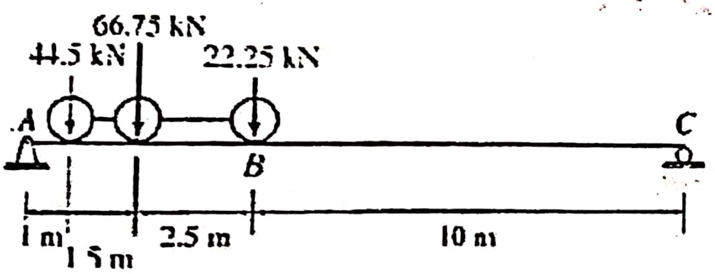
When 66.75 kN load is just to the right of B



(e) Loading Position 3

$$S_B = -35.6 \times 1.5 \times \frac{1}{15} - 44.5 \times 3.5 \times \frac{1}{15} + 66.75 \times 10 \times \frac{1}{15} + 22.25 \times 7.5 \times \frac{1}{15} = 41.68 \text{ kN}$$

When 22.25 kN load is just to the right of B



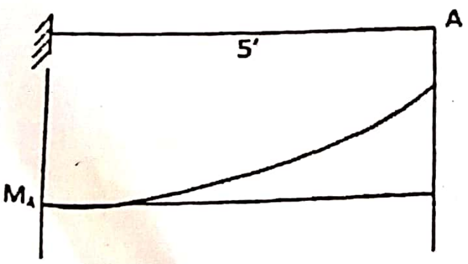
(f) Loading Position 4

$$S_B = -44.5 \times 1 \times \frac{1}{15} - 66.75 \times 2.5 \times \frac{1}{15} + 22.25 \times 10 \times \frac{1}{15} = 0.742 \text{ kN}$$

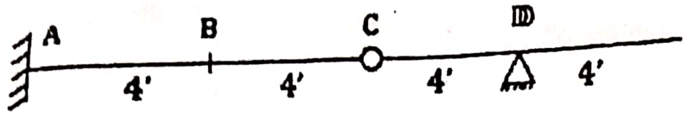
Maximum positive shear at B occurs for the first loading position.
Maximum positive S_B = 82.32 kN

Question: Draw IL for moment at point A in below figure. (RRI – 2015)

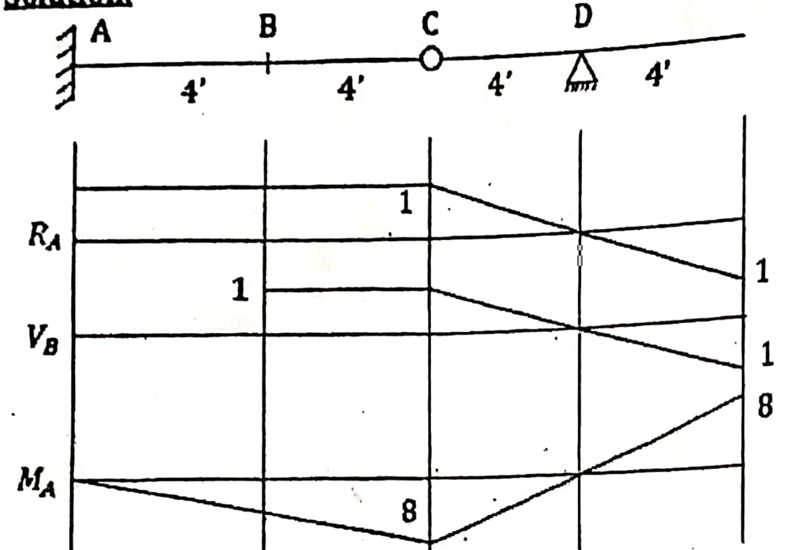
Solution:



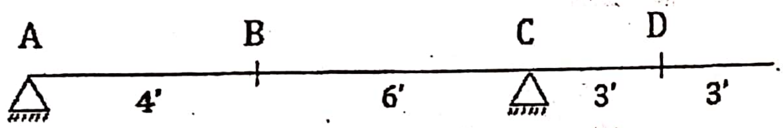
Question: Draw IL diagram for R_A, V_B and M_A. (WASA – 2017)



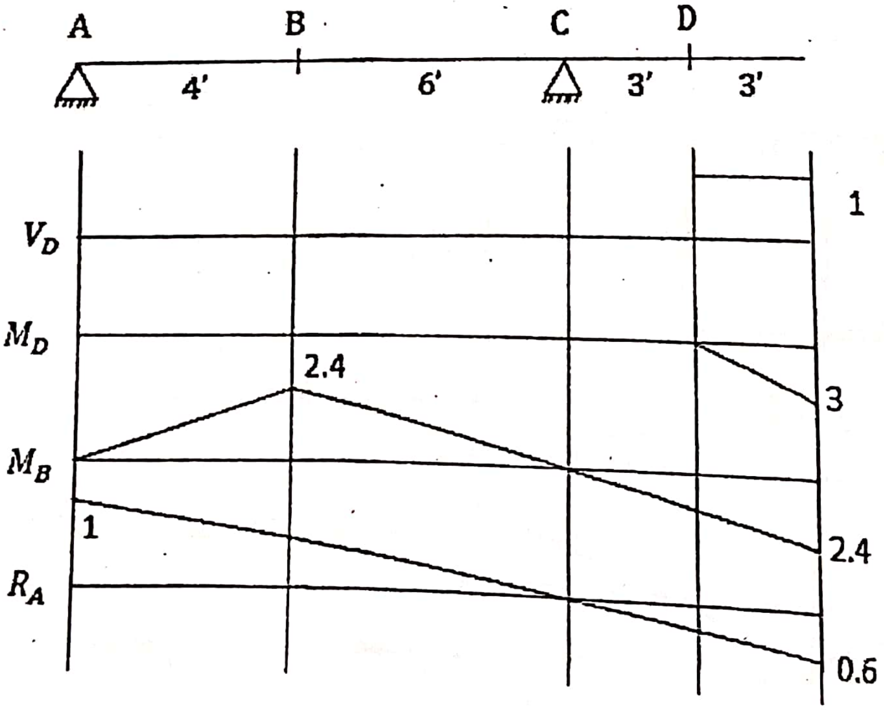
Solution:



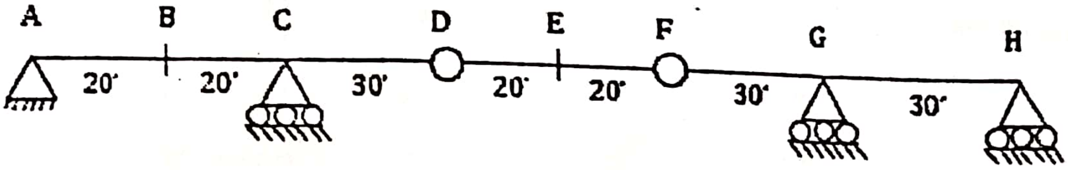
Question: Draw influence line for shear at D (V_D), bending moments at D (M_D) and B (M_B) and support reaction at A (R_A) for the beam in the figure below. (PGCL – 2017, JB – 2017)



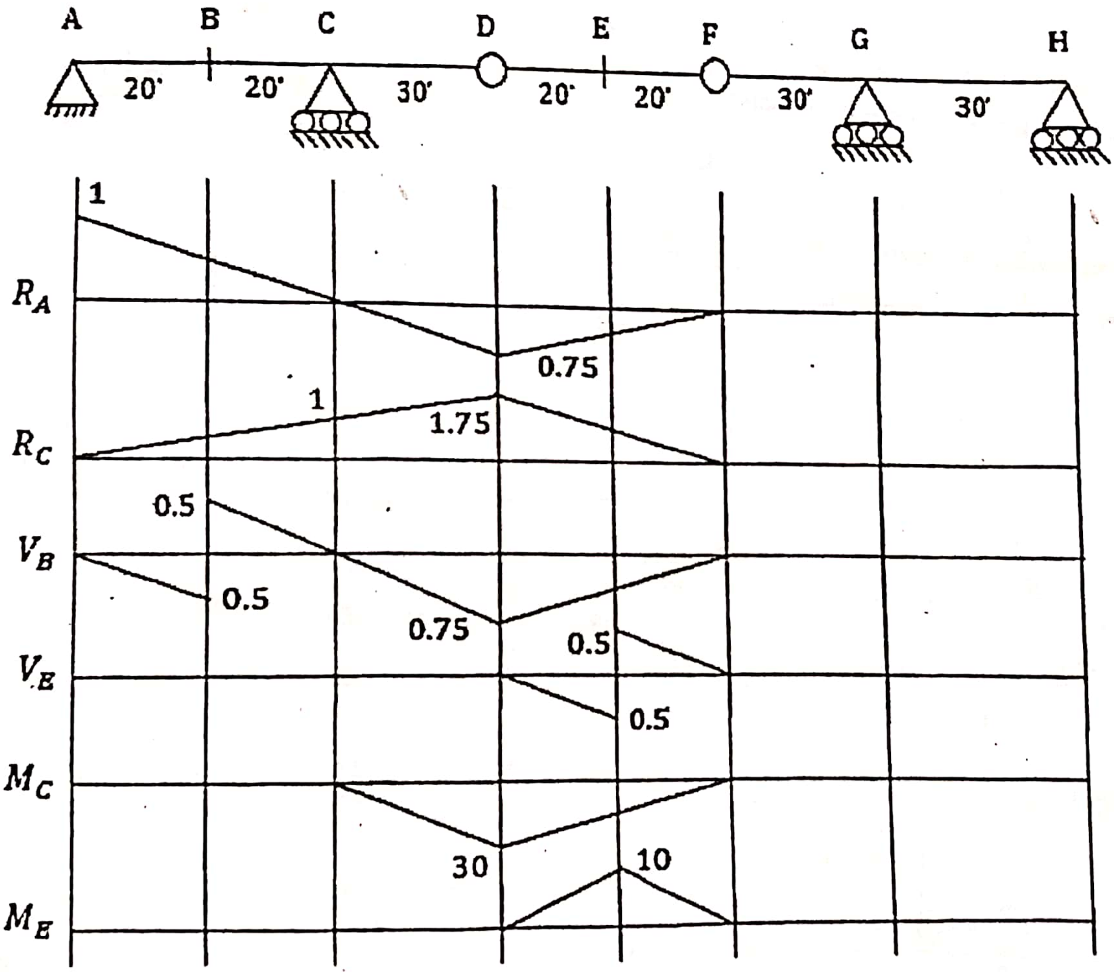
Solution:



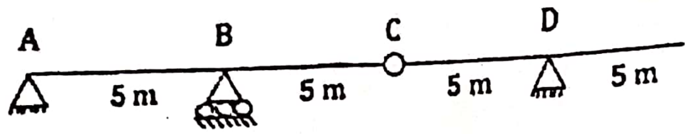
Question: Draw influence line diagram for R_A, R_C, V_B, V_E, M_C and M_E. (AB – 2017)



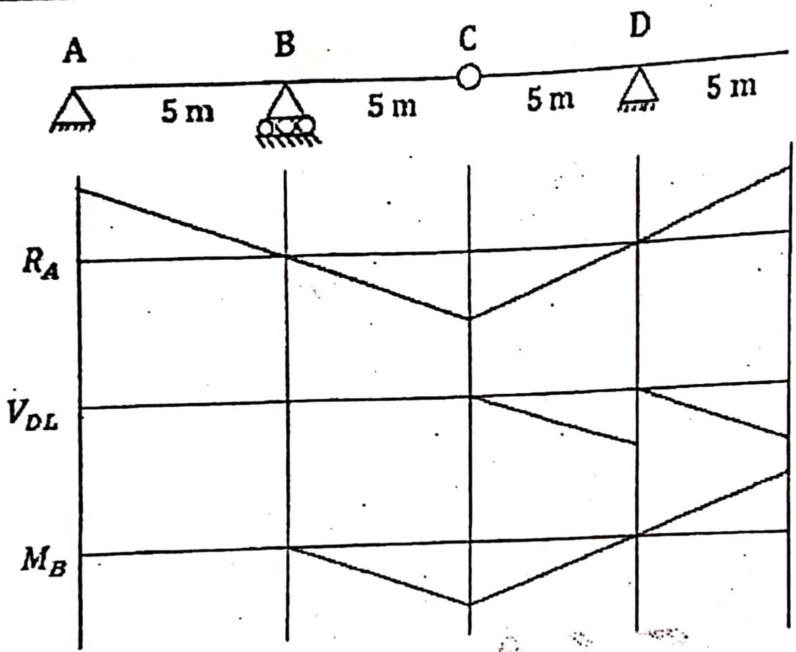
Solution:



Question: Draw IL for R_A, V_{DL} and M_B of the below structure: (SGFL – 2017)

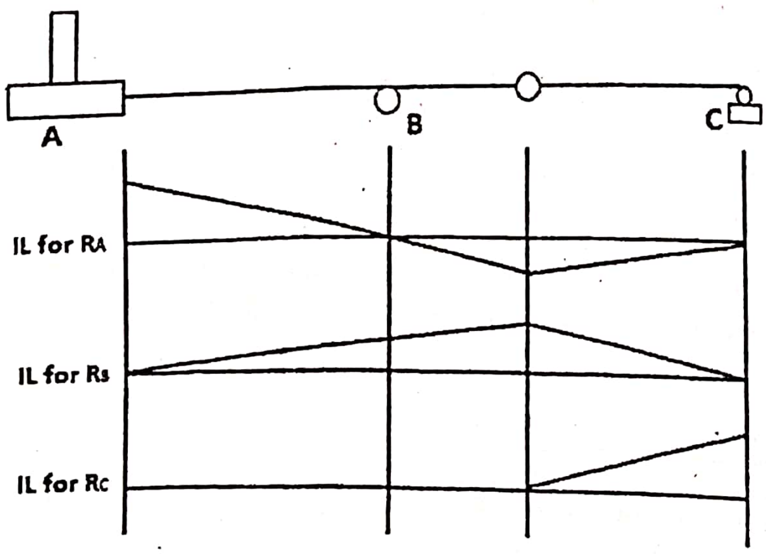


Solution:

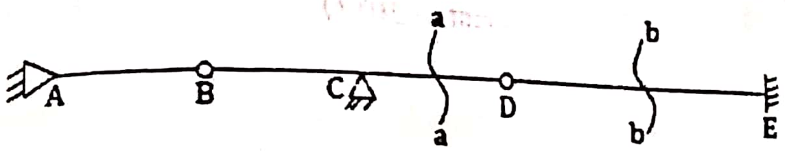


Question: Draw the influence line diagram for R_A, R_B and R_C (DMRTP – 2018)

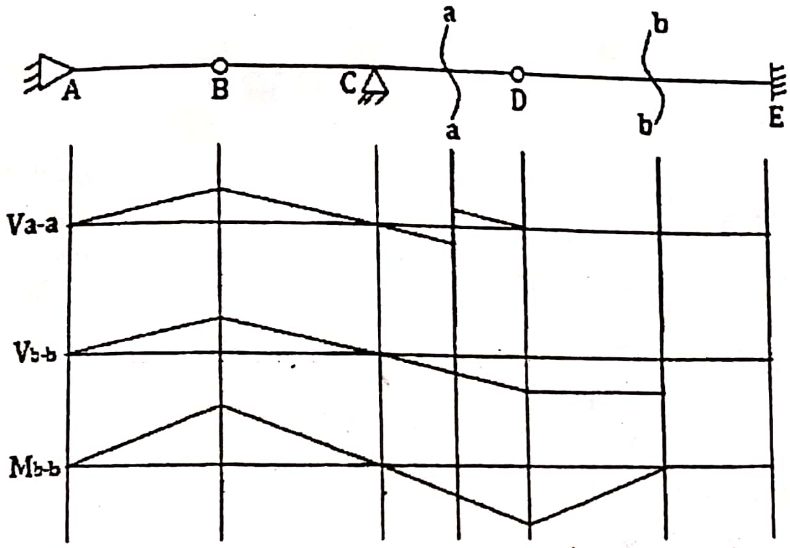
Solution:



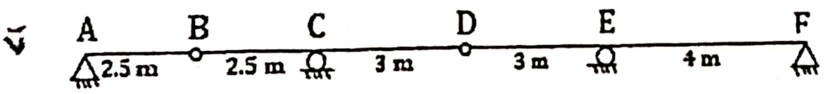
Question: Draw influence line diagram for V_{a-a}, V_{b-b} and M_{b-b} of following figure. (PGCB – 2018)



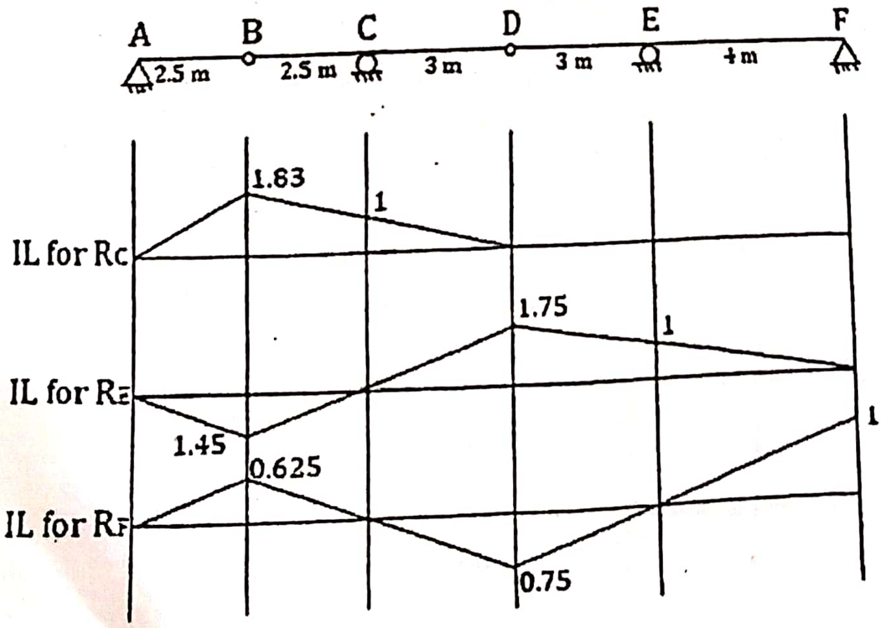
Solution:



Question: Draw the influence line diagram R_C, R_E, R_F (HBFC – 2018)

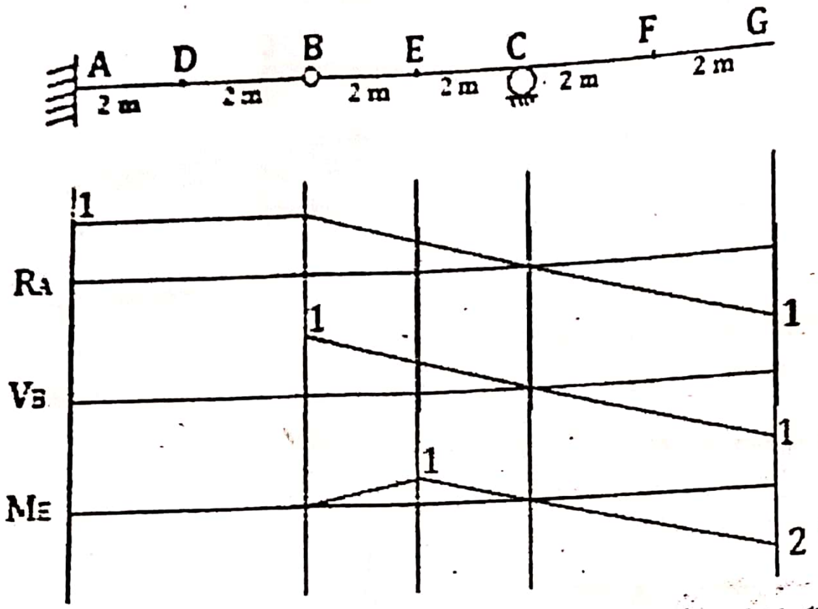


Solution:

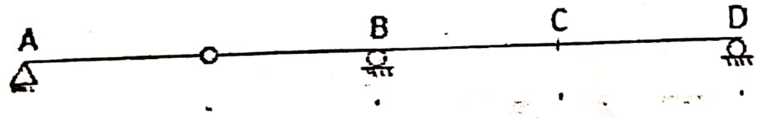


Question: Draw IL for R_A, V_B and M_E (50 BMA)

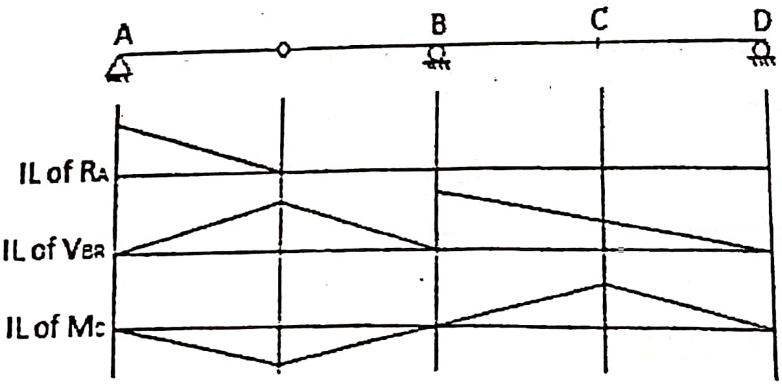
Solution:



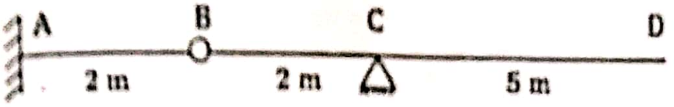
Question: Draw the influence line diagram for R_A, V_{BR} and M_C (BIWTA – 2019)



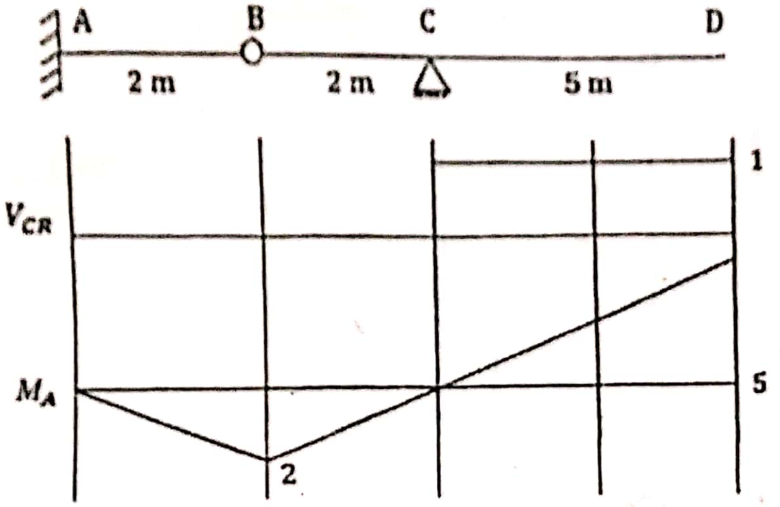
Solution:



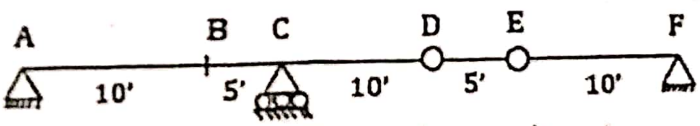
Question: Draw IL for Vc (Right) and MA (BCMCL – 2020)



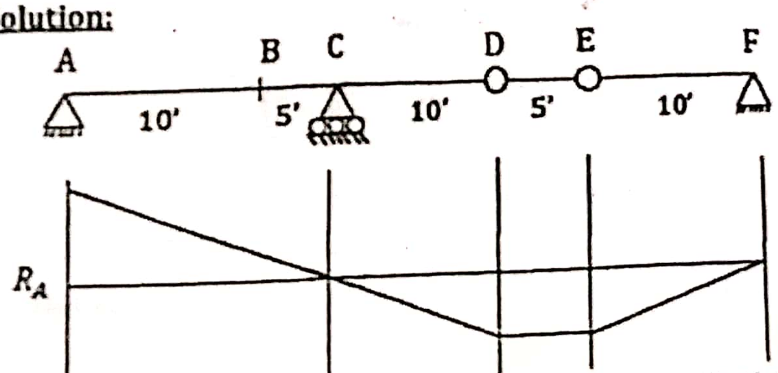
Solution:



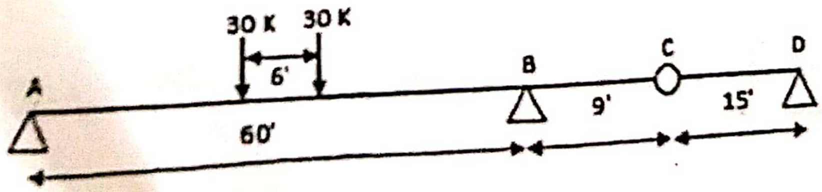
Question: Draw the IL for reaction at point A. (TGTDCL – 2021)



Solution:

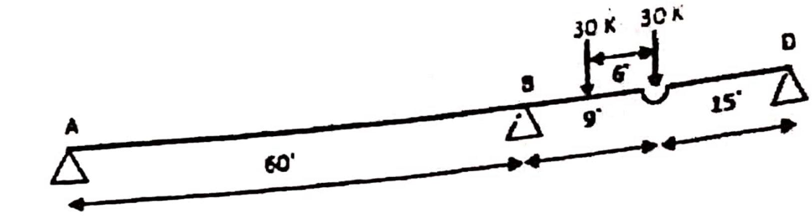


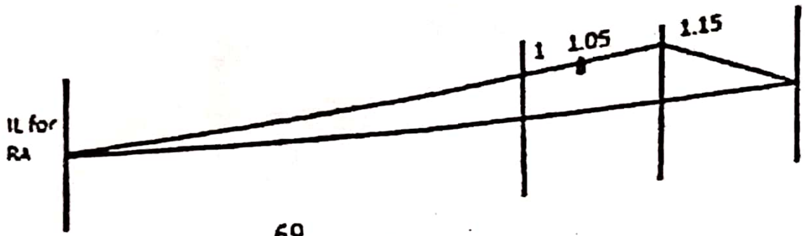
Question: A group of moving load consists of a pair of 30 kips load acting 6' constant apart on a beam shown below. Determine the maximum reaction at support B. (GTCL – 2018)



Solution:

Call: 01633905761



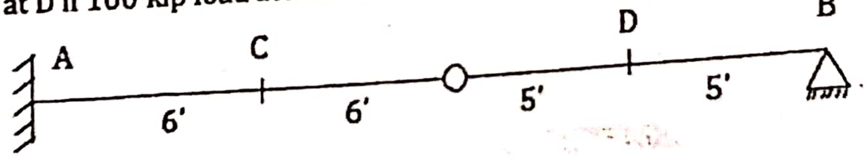


$$\text{Co-efficient at C} = \frac{69}{60} = 1.15$$

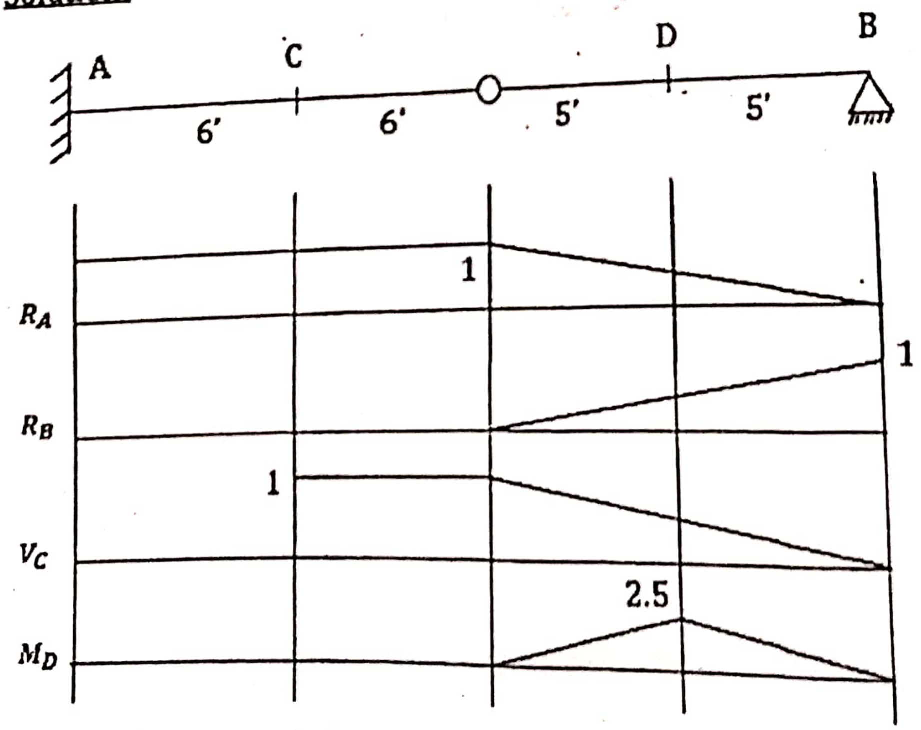
$$\text{Co-efficient at left load} = \frac{63}{60} = 1.05$$

$RB, max = 1.15 \times 30 + 1.05 \times 30 = 66$ kip

Question: Draw influence line of reaction at A & B, shear at C & moment at D. What will be moment at D if 100 kip load act on the hinge.



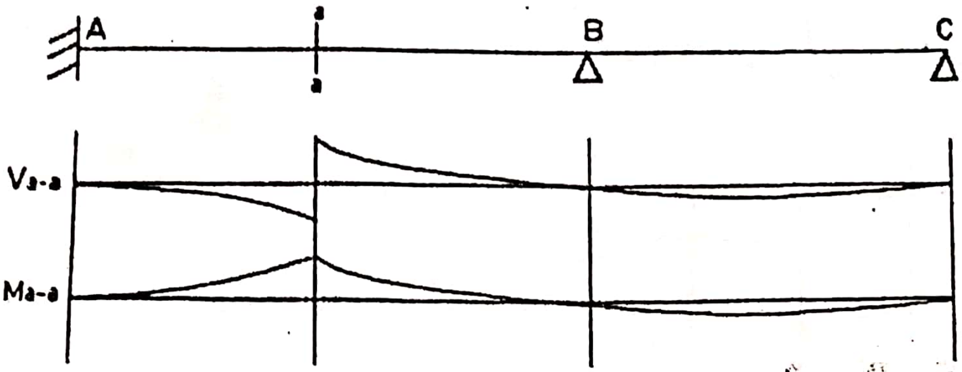
Solution:



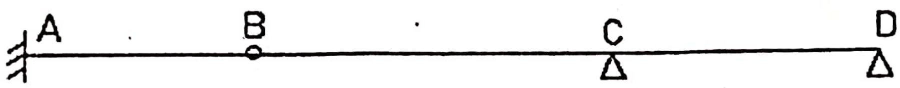
If 100 kip load act on the hinge the moment at D will be zero.

Question: Draw IL (qualitative) on shear and moment in a - a area. (RPGCL – 2017)

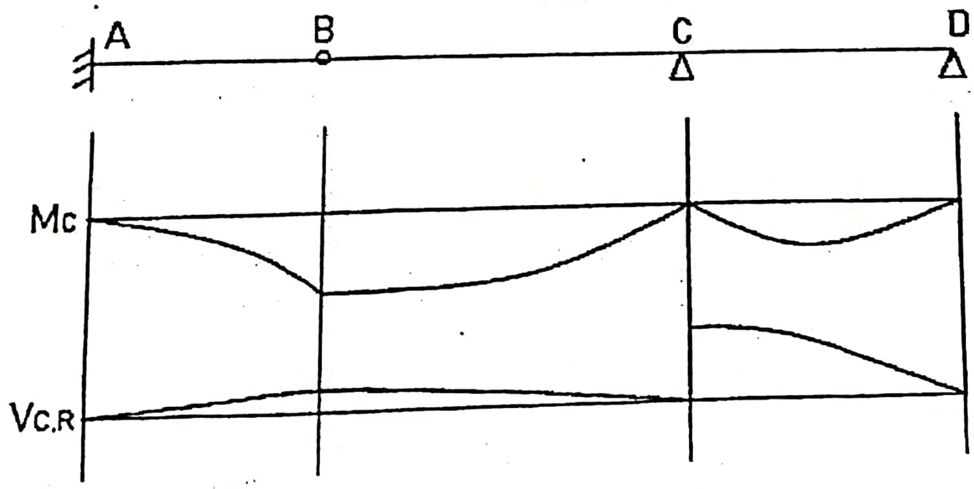
Solution:



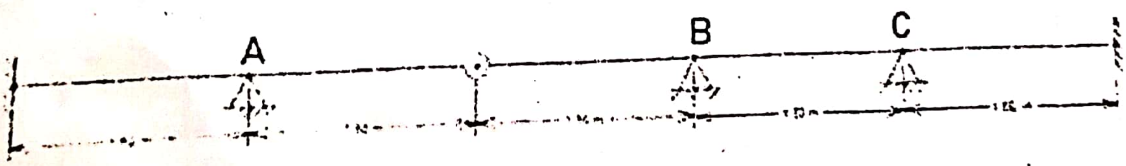
Question: Draw influence line diagram for Mc and Vc at right for the following beam. (BUET M. Sc – 2018)



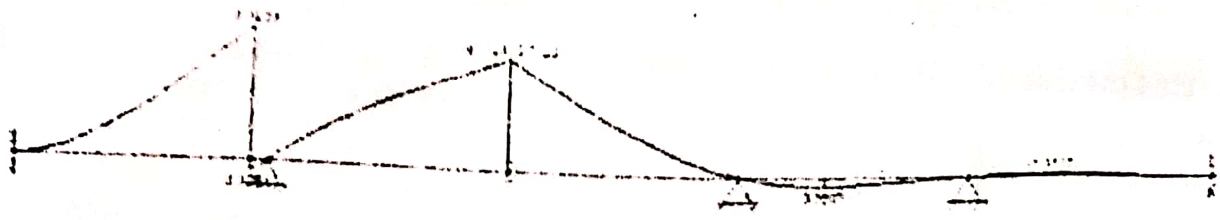
Solution:



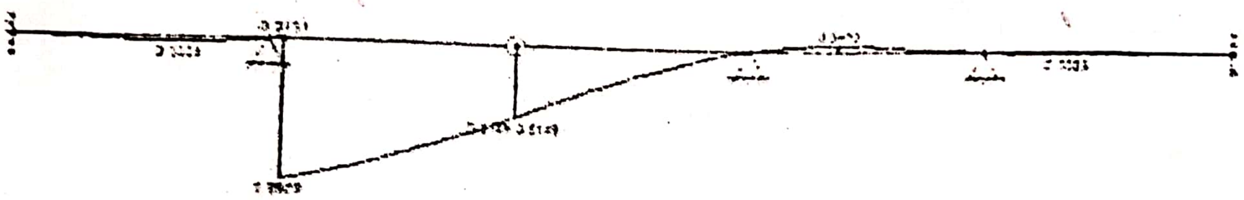
Question: Draw IL for shear at A, B and C (BKB – 2018)



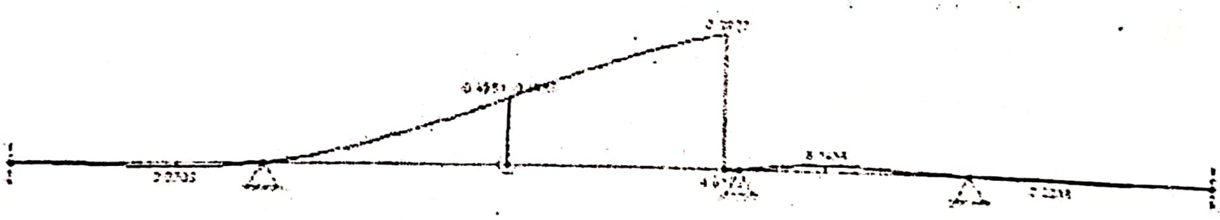
Solution:



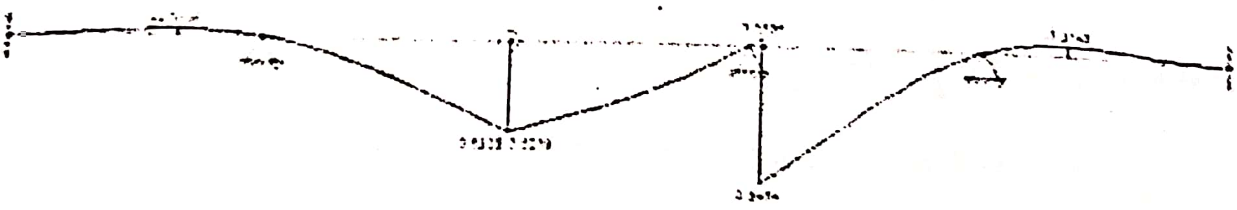
Shear A (left)



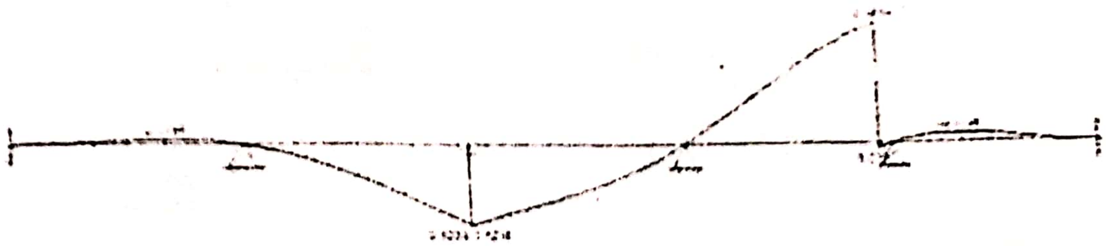
Shear A (Right)



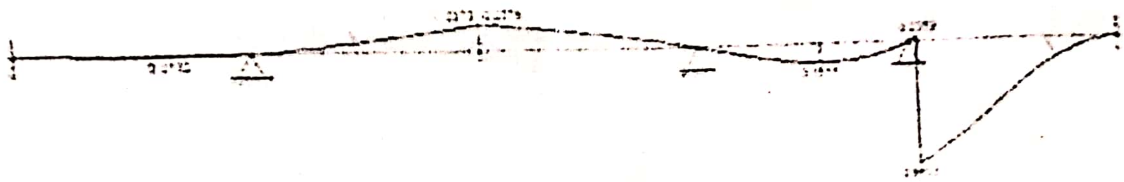
Shear B (left)



Shear B (Right)



Shear C (left)

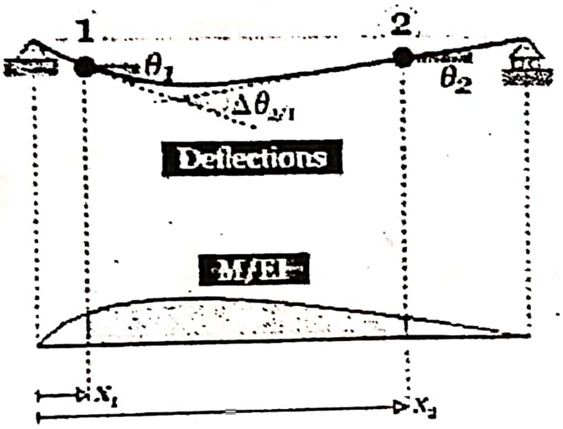


Shear C (Right)

Deflection of beam by moment area method

The moment area method is based on two theorems, also called 'moment area theorems' or **'Mohr's theorems'**. The first one correlates the slope change between any two points of the beam, while the second one is related with the deflection at a point of the beam. The two theorems will be presented after the following schematic that will be used as a reference. It illustrates a simple beam, deflected by some random loading, and the corresponding bending moment diagram.

Theorem 1: *The change of slope between any two points of the elastic curve is equal to the area of the bending moment diagram, between these two points, divided by EI.*

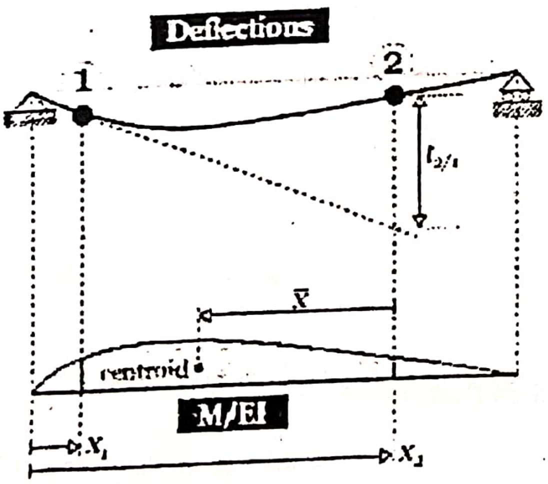


First moment area theorem,

$$\theta_{2/1} = \int_{X_1}^{X_2} \frac{M}{EI} dx$$

$$\text{Slope of B relative to A} = \frac{1}{EI} \text{ x area of B.M. diagram between A and B}$$

Theorem 2: The deviation of the elastic curve at any point, from the slope line, projected from another point, is equal to the first moment of area, about the first point, of the bending moment diagram, between the two points, divided by EI.

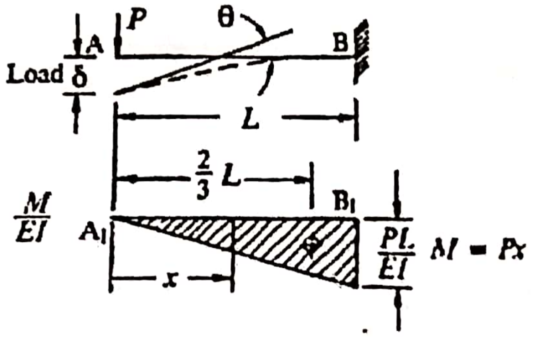


Second moment area theorem,

$$t_{2/1} = \bar{X}\int_{x_1}^{x_2} \frac{M}{EI}dx$$

Deflections at A relative to B $= \frac{1}{EI}$ x first moment of area of B. M. diagram between A and B about A

Question: Find the deflection for the following beam.



First, by integration,

$$\text{Slope at } A = \theta = \int_A^B \frac{M}{EI}dx = \int_A^B \frac{Px}{EI}dx = \left[\frac{Px^2}{EI}\right]_0^L = \frac{PL^2}{2EI}$$

$$\text{Deflection at } A = \delta = \int_A^B \frac{Mx}{EI}dx = \int_A^B \frac{Px^2}{EI}dx = \left[\frac{Px^3}{3EI}\right]_0^L = \frac{PL^3}{3EI}$$

Second, by graphical Area moment method

Slope at A $= \frac{1}{EI}$ x area of B. M. diagram between A and B

$$\text{Slope at A} = \frac{PL}{EI}L\frac{1}{2} = \frac{PL^2}{2EI}$$

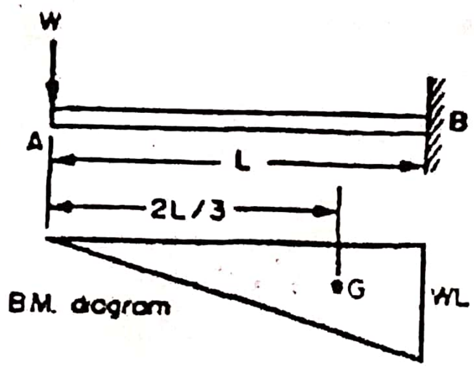
Deflections at A relative to B $= \frac{1}{EI}$ x first moment of area of B. M. diagram between A and B about A

$$\text{Deflections at A relative to B} = \frac{PL}{EI}\frac{L}{2}\frac{2L}{3} = \frac{PL^3}{3EI}$$

Slope at A $= \frac{1}{EI}$ x area of B. M. diagram between A and B

Deflections of A relative to B $= \frac{1}{EI}$ x first moment of area of B. M. diagram between A and B about A

Question: Find the deflection for the following beam.



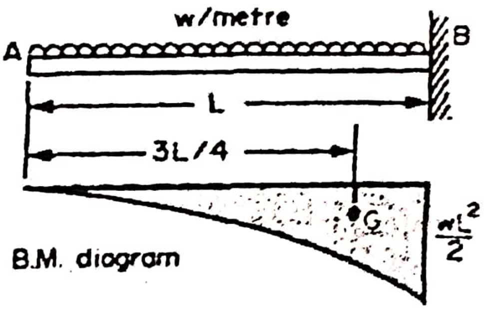
$$\text{Slope at A} = \frac{1}{EI} \text{ x area of B. M. diagram between A and B}$$

$$\text{Slope at A} = \frac{1}{EI}\left[\frac{L}{2}WL\right] = \frac{WL^2}{2EI}$$

$$\text{Deflections at A relative to B} = \frac{1}{EI} \text{ x first moment of area of B. M. diagram between A and B about A}$$

$$\text{Deflections at A relative to B} = \frac{1}{EI}\left[\left(\frac{L}{2}WL\right)\frac{2L}{3}\right] = \frac{WL^3}{3EI}$$

Question: Find the deflection for the following beam.



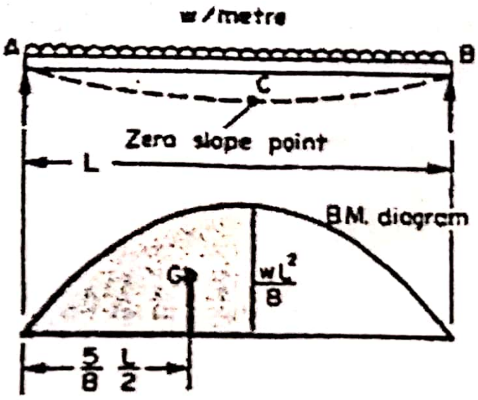
$$\text{Slope at A} = \frac{1}{EI} \text{ x area of B. M. diagram between A and B}$$

$$\text{Slope at A} = \frac{1}{EI}\left[\frac{1}{3}L\,\frac{wL^2}{2}\right] = \frac{wL^3}{6EI}$$

$$\text{Deflections at A relative to B} = \frac{1}{EI} \text{ x first moment of area of B. M. diagram between A and B about A}$$

$$\text{Deflections at A relative to B} = \frac{1}{EI}\left[\left(\frac{1}{3}L\,\frac{wL^2}{2}\right)\frac{3L}{4}\right] = \frac{wL^4}{8EI}.$$

Question: Find the deflection for the following beam.



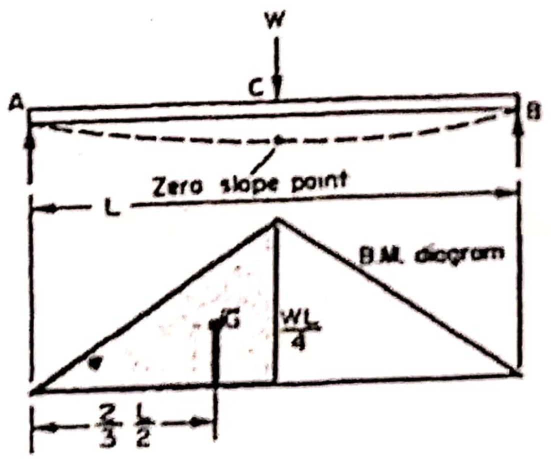
Slope at A $= \frac{1}{EI}$ x area of B.M. diagram between A and C

$$\text{Slope at A} = \frac{1}{EI}\left[\frac{3}{3}\frac{wL^2}{8}\frac{L}{2}\right] = \frac{wL^3}{24EI}$$

Deflections at A relative to C $= \frac{1}{EI}$ x first moment of area of B.M. diagram between A and C about A

$$\text{Deflections at A relative to C} = \frac{1}{EI}\left[\left(\frac{3}{3}\frac{wL^2}{8}\frac{L}{2}\right)\frac{5L}{16}\right] = \frac{5wL^4}{384EI}$$

Question: Find the deflection for the following beam.



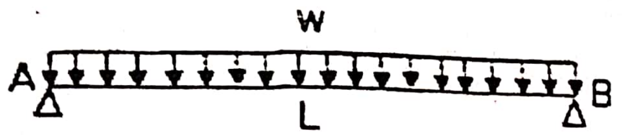
Slope at A $= \frac{1}{EI}$ x area of B.M. diagram between A and C

$$\text{Slope at A} = \frac{1}{EI}\left[\frac{1}{2}\frac{L}{2}\frac{WL}{4}\right] = \frac{WL^2}{16EI}$$

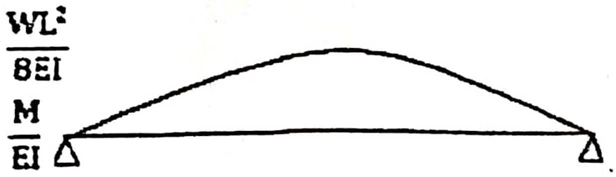
Deflections at A relative to C $= \frac{1}{EI}$ x first moment of area of B.M. diagram between A and C about A

$$\text{Deflections at A relative to C} = \frac{1}{EI}\left[\left(\frac{1}{2}\frac{L}{2}\frac{WL}{4}\right)\frac{2}{3}\frac{L}{2}\right] = \frac{WL^3}{48EI}$$

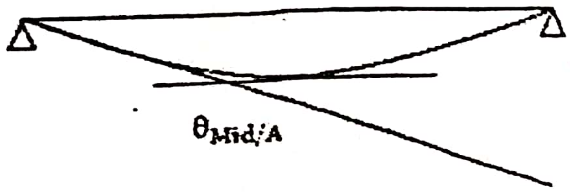
Question: Find the mid span deflection of the following beam. EI constant. (DPDC – 2014)



Solution:



Slope CA:

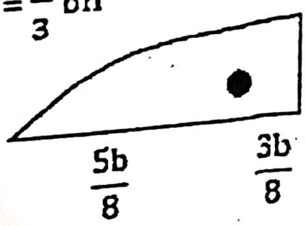


$$\theta_{mid/A} = \int_0^{L/2} \frac{M}{EI} dx = \frac{2}{3}\left(\frac{L}{2}\right)\left(\frac{W L^2}{8EI}\right) = \frac{W L^3}{24 EI}$$

$$\theta_A = \frac{W L^3}{24 EI}$$

Deflection at mid-span:

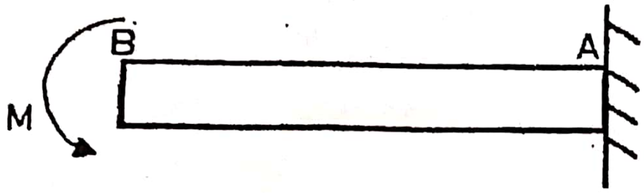
$A = \frac{2}{3} bh$



$$\theta_{mid/A} = X_A \int_0^{L/2} \frac{M}{EI} dx = \frac{5}{8}\left(\frac{L}{2}\right)\left(\frac{W L^3}{24EI}\right) = \frac{5 W L^4}{384 EI}$$

$$\Delta_{mid} = \frac{5 W L^4}{384 EI}$$

Question: Using moment area method find deflection in a 10′ cantilever beam with fixed support. A moment value of M = 32 k-ft free end shown in figure, where E = 2 x 10^6 kg/cm^2 and I = 10^5 cm^4? (DESCO – 2015)



Solution:

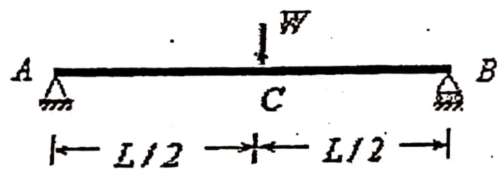
Given, $E = 2 \times 10^6\ kg/cm^2, I = 10^5\ cm^4,$
$M = 32\ k-ft = 975.61\ k-cm, L = 10' = 304.9\ cm$

$$\theta_{B/A} = \int_0^L \frac{M}{EI} dx = L\left(\frac{M}{EI}\right) = \frac{ML}{EI}$$

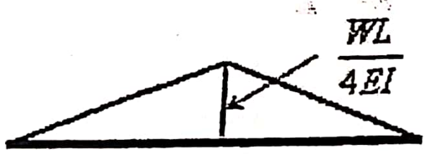
Deflection at end, $t_{B/A} = \bar{X}\int_0^{L/2} \frac{M}{EI} dx = \left(\frac{L}{2}\right)\frac{ML}{EI} = \frac{ML^2}{EI}$

$$\Delta = \frac{975.61 \times 304.9^2}{2 \times 10^6 \times 10^5} = 5.43 \times 10^{-5}\ cm$$

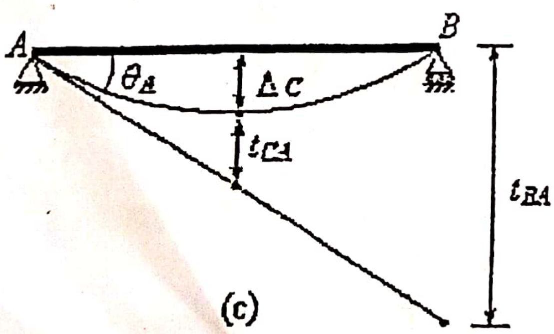
Question: Using moment area method determine the deflection of the following beam. (BCPCL – 2016)



Solution:



Slope CA:



$$\theta_{mid/A} = \int_0^{L/2} \frac{M}{EI} dx = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{WL}{4EI}\right) = \frac{W L^2}{16EI}$$

$$t_{mid/A} = X_A \int_0^{L/2} \frac{M}{EI} dx = \frac{2}{3}\left(\frac{L}{2}\right)\left(\frac{W L^2}{16EI}\right) = \frac{W L^3}{48EI}$$