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# Reinforced Concrete Design

# Reinforced Concrete Structure

Reinforced concrete is a combination of concrete and steel wherein the steel reinforcement provides the tensile strength lacking in the concrete. Steel reinforcing is also capable of resisting compression forces and is used in columns as well as in other situations.

## Advantages of Reinforced Concrete as a Structural Material

Reinforced concrete may be the most important material available for construction.

1. It has considerable compressive strength per unit cost compared with most other materials.

2. Reinforced concrete has great resistance to the actions of fire and water and in fact, is the best structural material available for situations where water is present. During fires of average intensity, members with a satisfactory cover of concrete over the reinforcing bars suffer only surface damage without failure.

3. Reinforced concrete structures are very rigid.

4. It is a low-maintenance material.

5. As compared with other materials, it has a very long service life. Under proper conditions, reinforced concrete structures can be used indefinitely without reduction of their load carrying abilities. This can be explained by the fact that the strength of concrete does not decrease with time but actually increases over a very long period, measured in years, because of the lengthy process of the solidification of the cement paste.

6. It is usually the only economical material available for footings, floor slabs, basement walls, piers, and similar applications.

7. A special feature of concrete is its ability to be cast into an extraordinary variety of shapes from simple slabs, beams and columns to great arches and shells.

8. In most areas, concrete takes advantage of inexpensive local materials (sand, gravel, and water) and requires relatively small amounts of cement and reinforcing steel, which may have to be shipped from other parts of the country.

9. A lower grade of skilled labor is required for erection as compared with other materials such as structural steel.

## Disadvantages of Reinforced Concrete as a Structural Material

To use concrete successfully, the designer must be completely familiar with its weak points as well as its strong ones. Among its disadvantages are the following:

1. Concrete has a very low tensile strength, requiring the use of tensile reinforcing.

2. Forms are required to hold the concrete in place until it hardens sufficiently. In addition, falsework or shoring may be necessary to keep the forms in place for roofs, walls, floors, and similar structures until the concrete members gain sufficient strength to support themselves. Formwork is very expensive. In the United States, its costs run from one-third to two-thirds of the total cost of a reinforced concrete structure, with average values of about 50%. It should be

obvious that when efforts are made to improve the economy of reinforced concrete structures, the major emphasis is on reducing formwork costs.

3. The low strength per unit of weight of concrete leads to heavy members. This becomes an increasingly important matter for long-span structures, where concrete's large dead weight has a great effect on bending moments. Lightweight aggregates can be used to reduce concrete weight, but the cost of the concrete is increased.

4. Similarly, the low strength per unit of volume of concrete means members will be relatively large, an important consideration for tall buildings and long-span structures.

5. The properties of concrete vary widely because of variations in its proportioning and mixing. Furthermore, the placing and curing of concrete is not as carefully controlled as is the production of other materials, such as structural steel and laminated wood.

**Compressive Strength:** The compressive strength of concrete, $fc'$ is determined by testing to failure 28 day old 6 inch diameter by 12 in. concrete cylinders at a specified rate of loading. For ordinary applications, 3000-psi and 4000-psi concretes are used, whereas for pre-stressed construction, 5000-psi and 6000-psi strengths are common.

**Modulus of Elasticity:** The modulus of elasticity of concrete $E_c$ (Young's modulus) is used in the design of concrete members, including design for deflections and of slender columns. The empirical equation provided in ACI 8.5.1 gives an approximate value of $E_c$

ACI Code states that the following expression can be used for calculating the modulus of elasticity of concretes

$$E_c = w_c^{1.5}\, 33\sqrt{f_3'}$$

In this expression, $E_c$ is the modulus of elasticity in psi, $w_c$ is the weight of the concrete in pounds per cubic foot and $f_c'$ is its specified 28-day compressive strength in psi.

**Shrinkage:** Shrinkage is defined as the decrease of hardened concrete volume with time. The decrease in volume can be attributed to changes in moisture content and chemical changes, which occur without the presence of external loading on the concrete. It is important to minimize and control shrinkage. When shrinkage strain is not controlled, cracks that can increase in size with time may form.

**Creep:** Creep is defined as the time-dependent increase in strain in excess of the elastic strain that is induced in a concrete member that is subjected to a sustained external load. Strain caused by creep is unlike strain caused by shrinkage because the latter occurs independent of load. Figure illustrates the increase in creep strain with time for pain concrete.

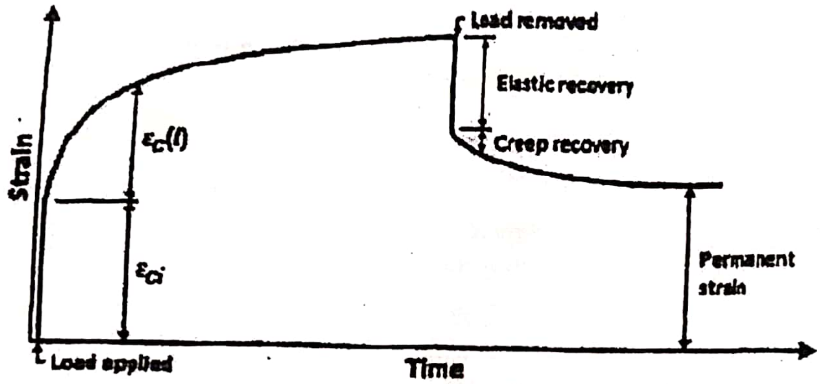


Figure: Creep strain of concrete over time

Creep strain is obtained by subtracting the elastic (instantaneous) strain from the total measured strain in a loaded specimen.

The gradual elimination of the working stress design method from the ACI Code primarily had to do with the following shortcomings of the method

- Inability to correctly account for the variability of loads and member resistances
- Inconsistent factor of safety in member design

The basic requirement for strength design is, Design strength ≥ required strength
$\varphi$ (nominal strength ) ≥ $U$
$M_u \leq \phi M_n$
$P_u \leq \phi P_n$
$V_u \leq \phi V_n$
$\varphi$ = strength reduction factor
$U$ = required strength

The strength design method is based on the following two fundamental conditions:

1. *Static equilibrium.* The compressive and tensile forces acting on any cross-section of a member are in equilibrium.

2. *Compatibility of strains.* The strain in a reinforcing bar that is embedded in concrete is equal to the strain in concrete at that level.

**ACI Load Combinations:** The magnitude of the load factor assigned to each nominal load effect is influenced primarily by the following:

1. The degree of accuracy to which the load effect can be determined.
2. The variation that might be expected in the load during the life of a structure.
3. The probability that different load types will occur at the same time.

**Question:** Advantages of reinforced concrete as a structural material. (SGFL – 2017)

**Solution:**

- It has a relatively high compressive strength.
- It has better resistance to fire than steel.
- It has a long service life.
- In some types of structures such as dams, piers and footings, it is the most economical structural material.
- It can be cast to take the shape required making it widely used in precast structural components. It yields rigid members with minimum apparent deflection.

**Question:** What is clear cover? Why it is provided? (CPGCBL – 2015, RRI – 2015)

**Solution:**

It may be defined as the distance between the top and bottom most reinforcement and the top (or bottom) most edge of the structural member like beam or column etc. In most of the general cases the minimum values of clear cover are 75 mm for Foundation, 40 mm for Column, 40 mm for Beam, 25 mm for shear wall, 15 mm for Slab and Stair. Clear cover is provided because of the following main reasons:

- To provide protection to reinforcement from corrosion.
- To provide fire resistance to reinforcement.
- To provide sufficient embedded depth so that reinforcement develops the requisite stress.

**Question:** What is WSD and USD? Write down their difference. (RAJUK – 2014)

**Solution:**

**Working Stress Design:** Stress condition of structural member under working load is given attention. Full strength of material is not utilized and is reduced by a factor. Stresses calculated are within the elastic limit. It is less economical than USD.

**Ultimate Strength Design:** Focuses on the strength capacity of the members at conditions corresponds to failure. Material properties are assessed and used in design. Loads are multiplied by a factor. More economical.

| USD | WSD |
|---|---|
| It considers Design Critical Combination of load | It considers Design Carrying load |
| It considers elastic behavior of materials | It considers Plastic behavior of materials |
| Material strength to be used for member design | Modular Ratio is used for member design |
| It is primarily based on strength concept of concrete | It is based on the linear theory or elastic theory |
| Stability of structure is more than WSD | Stability of structure is less than USD |
| Low cost design method | High cost design method. |

## Beam Analysis and Design

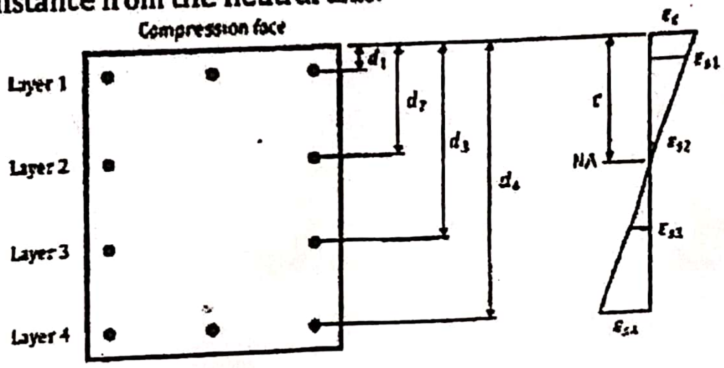
**Question: What is beam?**

**A beam is a structural member used for bearing loads. It is typically used for resisting vertical loads, shear forces and bending moments.**

### Assumption in RCC beam design

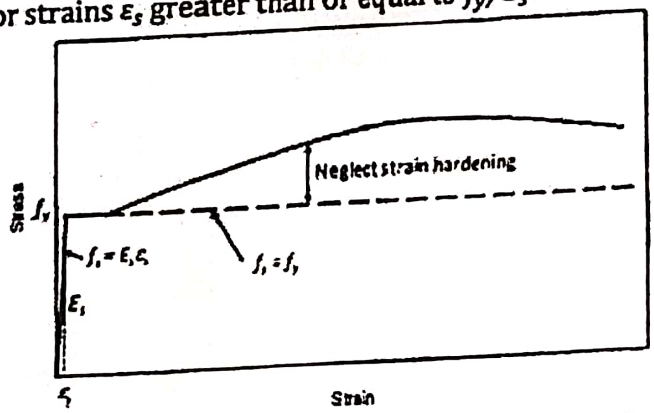
**Reinforced concrete sections are heterogeneous (nonhomogeneous), because they are made of two different materials, concrete and steel. Therefore, proportioning structural members by ultimate-strength design is based on the following assumptions:**

**1. The strains in the reinforcement and the concrete shall be assumed directly proportional to the distance from the neutral axis.**




**2. The maximum usable strain at the extreme concrete compression fiber is 0.0030.**

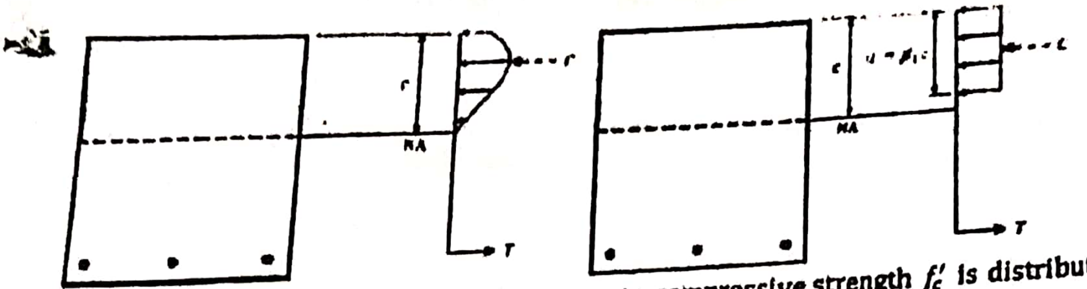
**3. The stress in the reinforcement $f_s$ below its specified yield strength $f_y$ is equal to the modulus of elasticity of the steel $E_s$ times the steel strain $\varepsilon_s$. The stress in the reinforcement is equal to $f_y$ for strains $\varepsilon_s$ greater than or equal to $f_y/E_s$.**




**4. The tensile strength of concrete is neglected in the axial and flexural calculations of reinforced concrete.**

**5. Plane cross-sections continue to be plane after bending.**

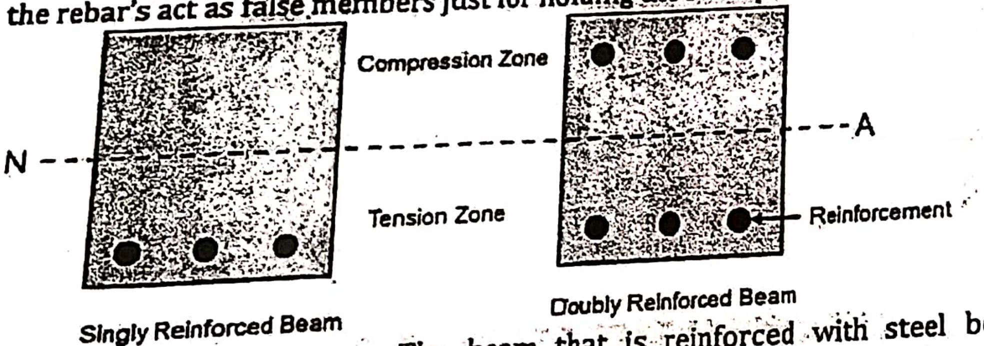
**6. The requirements of ACI 10.2.6 are satisfied by an equivalent rectangular concrete stress distribution, which is defined in ACI 10.2.7.**




A uniform stress equal to 85% of the concrete compressive strength $f_c'$ is distributed over the depth *a*, which is equal to the factor $\beta_1$ times the depth to the neutral axis *c*.

**Question: Why is the difference between singly and double reinforced beams?**

**Singly Reinforced Beam:** The beam that is longitudinally reinforced only in tension zone, it is known as singly reinforced beam. In Such beams, the ultimate bending moment and the tension due to bending are carried by the reinforcement, while the compression is carried by the concrete. Practically, it is not possible to provide reinforcement only in the tension zone, because we need to tie the stirrups. Therefore two rebar are utilized in the compression zone to tie the stirrups and the rebar's act as false members just for holding the stirrups.




**Doubly Reinforced Beam:** The beam that is reinforced with steel both in tension and compression zone, it is known as doubly reinforced beam. This type of beam is mainly provided when the depth of the beam is restricted. If a beam with limited depth is reinforced on the tension side only it might not have sufficient resistance to oppose the bending moment.

**Question: What is balanced, under-reinforced and over-reinforced beam?**

Balanced Beam Section: Reinforced concrete beam sections in which the tension steel also reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called balanced sections.

Under-Reinforced Beam Section: Reinforced concrete beam sections in which the steel reaches yield strain at loads lower than the load at which the concrete reaches failure strain are called under-reinforced sections.

Over-Reinforced Beam Sections: Reinforced concrete beam sections in which the failure strain in concrete is reached earlier than the yield strain of steel is reached, are called over-reinforced beam sections.

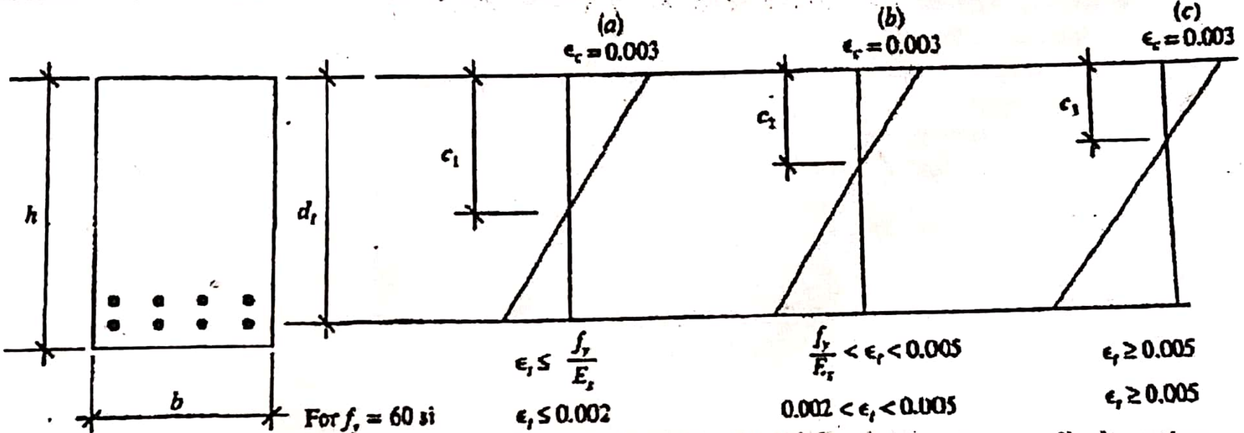
**Question: What is balanced steel ratio? In a well-designed RC beam, actual steel ratio is well below the balanced steel ratio? Why? (30th BCS)**

**Solution:**

Reinforced concrete beam sections in which the tension steel also reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called balanced sections.

In Under-reinforced beam failure initiated by yielding of the steel, typically is gradual. Distress is obvious from observing the large deflections and widening of concrete cracks associated with yielding of the steel reinforcement and measures can be taken to avoid total collapse. That is why to design a beam which fail by yielding of steel, reinforcement ratio $\rho$ is chose less the balance reinforcement ration $\rho_b$.

**Strain limits on tension and tension controlled section**




(a) compression controlled section (b) transition region (c) tensioned controlled section

**Question: Why strength reduction factor is used in reinforced concrete beam?**

The nominal strength of a section M for flexural members, calculated in accordance with the requirements of the ACI Code provisions must be multiplied by the strength reduction factor φ which is always less than 1. Strength reduction factors are commonly referred to as resistance factors or φ-factors and play a key role in the determination of the design strength of a reinforced concrete member. The main purposes of these factors are as follows:

1. To account for the understrength of a member due to variations in material strengths and dimension and inaccuracies in the design equations.

2. To reflect the importance of a member. The failure of a column in a structure is usually considered to be more detrimental than failure of a beam.

3. To reflect the degree of ductility and required reliability under the applied loads.

Strength Reduction Factor [illegible]

| | | |
|---|---|---|
| Tension-controlled sections | | 0.9 |
| Compression controlled sections | Members with spiral reinforcement | 0.75 |
| | Other reinforced members | 0.65 |
| Shear and torsion | | 0.75 |
| Bearing on concrete | | 0.65 |
| Strut and tie models | | 0.75 |

**Table: ACI Strength Reduction Factors**

**For the transition region, may be determined by linear interpolation between 0.65 and 0.9 for grade 60 steel. The linear equations are as follows:**

**For spiral members,** $\varphi = 0.75 + (\varepsilon_t - 0.002) \times \left(\frac{150}{3}\right)$

**for other members,** $\varphi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right)$

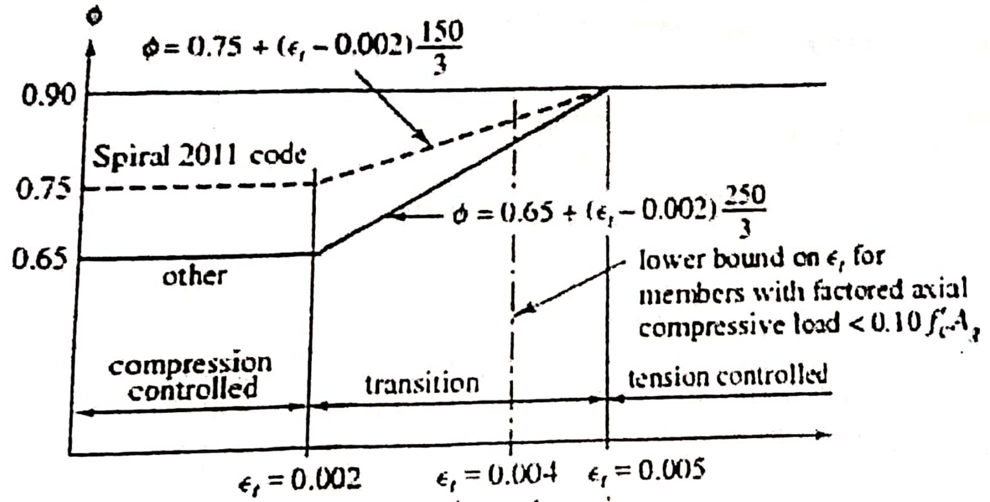


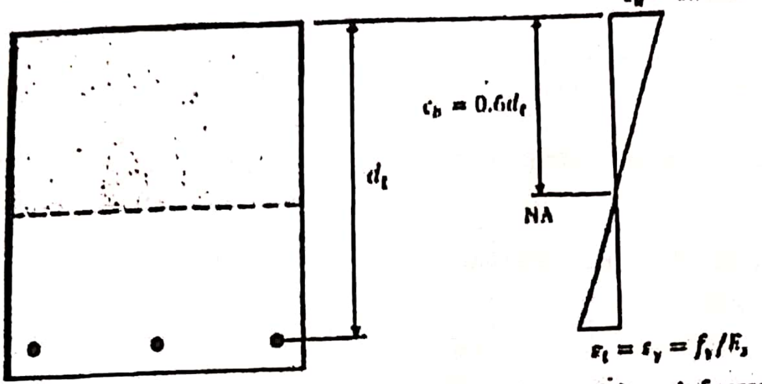

**Figure: Variation of $\varphi$ with net tensile strain $\varepsilon_t$**

Members with $\varepsilon_t < 0.002$ are *compression controlled* and are referred to as **brittle members.** Obviously, such members must be avoided. Members whose computed tensile strains are equal to or greater than 0.0050 at the same time the concrete strain is 0.003 are to be referred to as *tension-controlled sections*. For such members, the steel will yield before the compression side crushes and deflections will be large, giving users warning of impending failure. Furthermore, members with $\varepsilon_t \geq 0.005$ are considered to be fully ductile.

Reinforced concrete beams must have a tension strain of at least 0.004. When $\varepsilon_t$ values fall between 0.002 and 0.005, they are said to be in the transition range between tension-controlled and compression controlled sections. *You must clearly understand that the use of flexural members in this range is usually uneconomical, and it is probably better, if the situation permits, to increase member depths and/or decrease steel percentages until $\epsilon_t$ is equal to or larger than 0.005.*

**Balanced Strain Conditions**

A balanced strain condition exists at a cross-section of a reinforced concrete member when the strain in the tension reinforcement reach its yield strain at the same time the extreme compression concrete fibers attain a failure strain equal to 0.003.



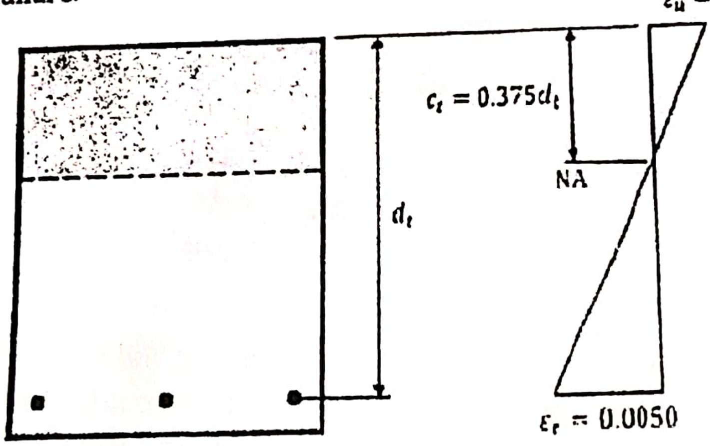

For Grade 60 reinforcement, the yield strain $\varepsilon_y$ of the reinforcement to be taken as 0.0020

**Compression-controlled Sections**

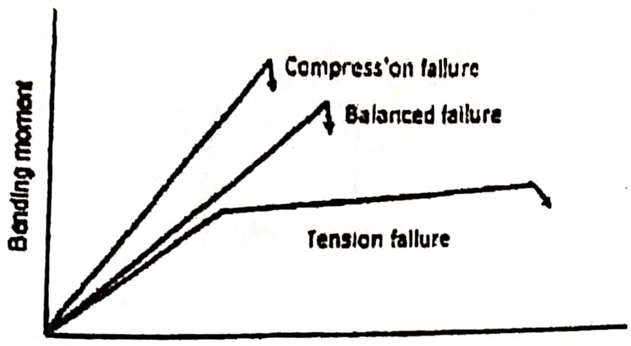
A section is compression-controlled if the net tensile strain in the extreme tension steel $\varepsilon_t$ is less than or equal to the compression-controlled strain limit when the concrete in compression reaches its assumed strain limit $\varepsilon_u$ of 0.0030. A brittle type of failure is generally expected in compression-controlled sections. This type of failure occurs suddenly with little or no warning.

**Tension-controlled Sections**

A tension-controlled section is defined as a section where $\varepsilon_t \geq 0.0050$ when the concrete in compression reaches its assumed strain limit $\varepsilon_u$ of 0.0030. Flexural members, such as beams, usually have tension-controlled sections. Unlike compression-controlled sections, tension-controlled sections are ductile and generally exhibit significant deflections and cracking before failure.




Strain distribution at the tension-controlled limit.




The idealized moment–curvature diagram for a reinforced concrete beam with one layer of tension reinforcement.

The Code requires that all flexural members and members with a factored axial compressive force less than $0.1f_c'Ag$ (typically, beams and one-way slabs) have properties that ensure that tension failure occurs.

## Single Layer of Tension Reinforcement

### Equivalent compressive stress distribution

The value of the stress block depth factor $\beta_1$ is:

$$\beta_1 = \begin{cases} 0.85 & \text{for } 2500 \text{ psi} < f_c' \leq 4000 \text{ psi} \\ 0.85 - 0.005\left(\frac{fc' - 4000}{1000}\right) & \text{for } 4000 \text{ psi} < f_c' \leq 8000 \text{ psi} \\ 0.65 & \text{for } f_c' > 8000 \text{ psi} \end{cases}$$

For concretes with $f_c' > 4000$ psi, $\beta_1$ can be determined with the following formula:

$$\beta_1 = 0.85 - 0.005\left(\frac{f_c' - 4000}{1000}\right) \geq 0.65$$

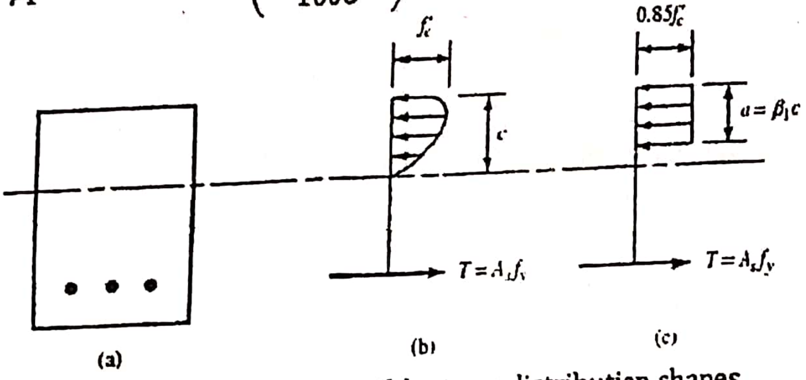
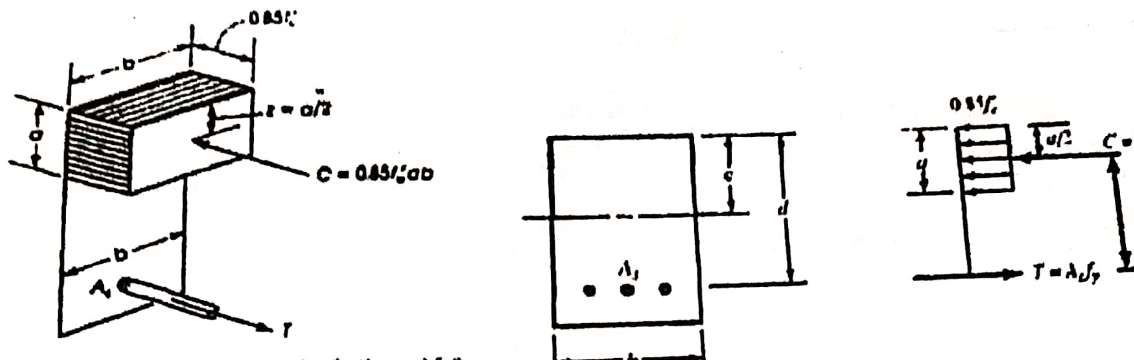



Figure: Some possible stress distribution shapes.

Equating the horizontal forces C and T and solving for $a$, we obtain,

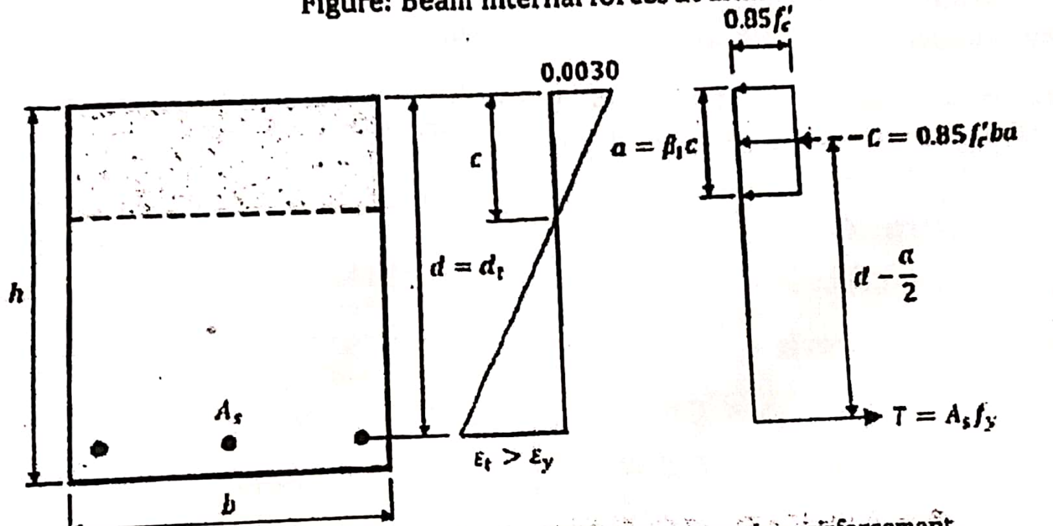
$$0.85\, f_c'\, a\, b = A_s\, f_y$$

$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b}$$




Actual and equivalent stress distributions at failure.

Figure: Beam internal forces at ultimate conditions.




The strain and stress distributions in a rectangular beam with tension reinforcement.

Because the reinforcing steel is limited to an amount such that it will yield well before the concrete reaches its ultimate strength, the value of the nominal moment $M_n$ can be written as,

$$M_u = T\left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{a}{2}\right)$$

And the usable flexural strength is

$$\varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2}\right)$$

Alternately,

$$\varphi M_n = \varphi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

### Balanced Steel Percentage

A beam that has a *balanced steel ratio* is one for which the tensile steel will theoretically just reach its yield point at the same time the extreme compression concrete fibers attain a strain equal to 0.003.

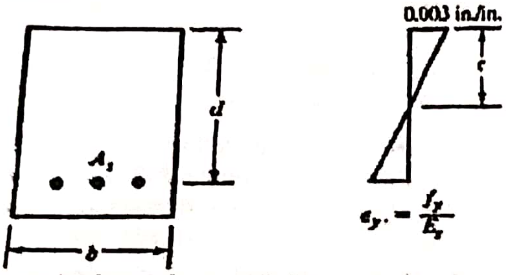



Figure: Balanced conditions

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{87000}{87000+f_y}$$

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{\varepsilon_c}{\varepsilon_c+\varepsilon_y}$$

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{0.003}{0.003+0.002}$$ (For 60 grade rebar, $\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29000} = 0.00207$)

Previous codes (1963–1999) limited flexural members to 75% of the balanced steel ratio $\rho_b$. However, this approach was changed in the 2002 code to the new philosophy, whereby the member capacity is penalized by reducing the $\varphi$ factor when the strain in the reinforcing steel at ultimate is less than 0.005.

### Minimum Percentage of Steel

Minimum reinforcement area, $A_{s,min} = \frac{3\sqrt{fc'}\,b_w\,d}{f_y} \geq \frac{200\,b_w\,d}{f_y}$

Statically determinate T section with the flange in tension or for cantilevers,

$$A_{s,min} = \frac{6\sqrt{fc'}\,b_w\,d}{f_y} \geq \frac{200\,b_w\,d}{f_y}$$

This ACI equation for the minimum amount of flexural reinforcement as a percentage as follows:

Minimum reinforcement ratio, $\rho_{min} = \frac{3\sqrt{fc'}}{f_y} \geq \frac{200}{f_y}$

Minimum reinforcement ratio, $\rho_{min} = \frac{200}{f_y}$ when $f_c' < 4500$ psi

Minimum reinforcement ratio, $\rho_{min} = \frac{3\sqrt{fc'}}{f_y}$ when $f_c' \geq 4500$ psi

## Maximum reinforcement ratio

Maximum reinforcement ratio allowed by the ACI codes for beams,

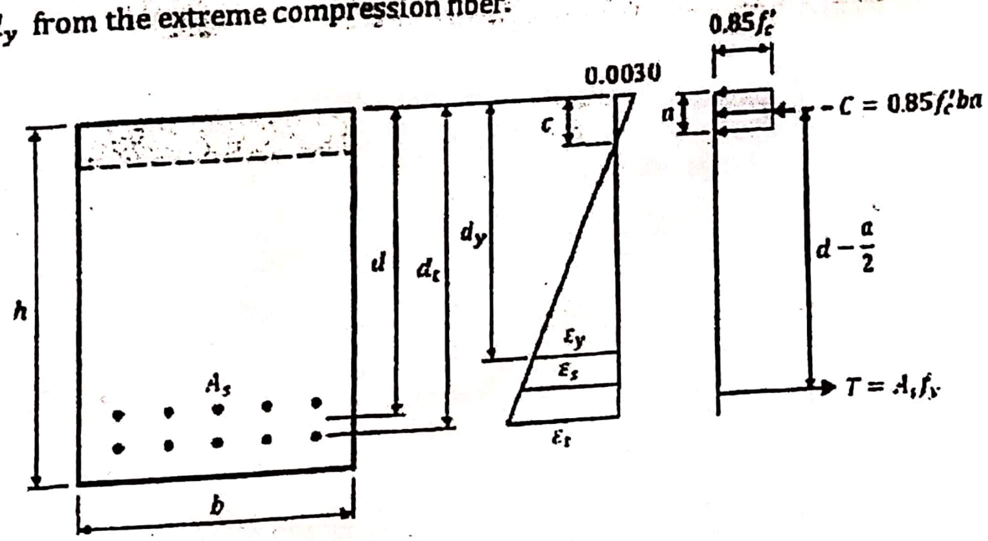
$$\rho_{max} = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{\varepsilon_c}{\varepsilon_c+\varepsilon_t} = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{\varepsilon_c}{\varepsilon_c+0.004}$$

Maximum reinforcement ratio for a tensioned controlled beams,

$$\rho_{max} = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{\varepsilon_c}{\varepsilon_c+\varepsilon_t} = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{\varepsilon_c}{\varepsilon_c+0.005}$$

For a given concrete cross section, using $\varepsilon_t = 0.004$ will result in a higher reinforcement ratio and thus a higher flexural strength than using $\varepsilon_t = 0.005$. This higher strength cannot be used to full advantage in design the increase in flexural strength is canceled by the drop in $\varphi$ as $\varepsilon_t$ decreases from 0.005 to 0.004. As a result the maximum practical reinforcement ratio for beams is attained at a net tensile strain of 0.005. Values of $\varepsilon_t$ below 0.005 are not recommended for the design of members with low axial loads.

**Multiple Layers of Tension Reinforcement** Under certain conditions, the required tension reinforcement cannot adequately fit within one layer in a section, in such cases the bars are provided in more than one layer. When determining the nominal flexural strength, it is important to check that all of the reinforcement yields. Assume that the yield strain $\varepsilon_y$ occurs at a distance of $d_y$ from the extreme compression fiber.




$$\frac{c}{0.003} = \frac{d_y - c}{\varepsilon_y}$$

Reinforcement located a distance equal to or greater than $d_y$ from the extreme compression fiber yields ( $\varepsilon_s \geq \varepsilon_y$ and $f_s = f_y$).

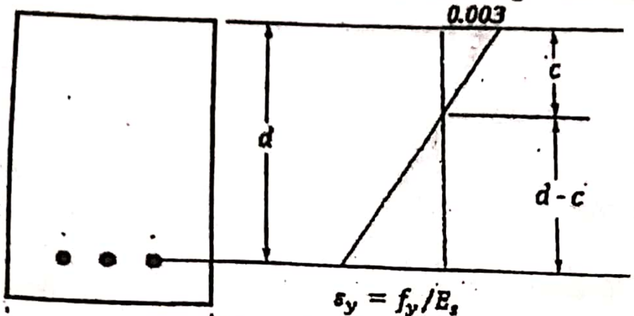
**Question:** What do you mean by "balanced steel ratio"? Prove that the balanced steel ratio of a singly rectangular beam is given by,

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y}\frac{87000}{87000+f_y} \qquad \text{(37th BCS)}$$

**Solution:**

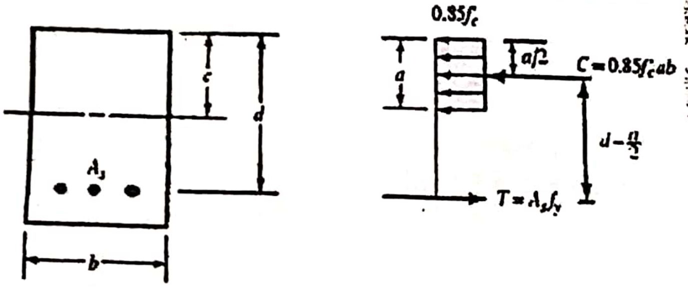
A balanced strain condition exists at a cross-section of a reinforced concrete member when the strain in the tension reinforcement reach its yield strain at the same time the extreme compression concrete fibers attain a failure strain equal to 0.003.

A reinforcement ratio $\rho_b$ producing balanced strain conditions can be established based on the condition that in a balanced failure the steel strain is exactly equal to $\varepsilon_y$ when the strain is the concrete simultaneously reaches the crushing strain of $\varepsilon_u = 0.003$.




Form triangular relationship, $\frac{\varepsilon_u}{c} = \frac{\varepsilon_y}{d-c}$

$$c = \frac{\varepsilon_u}{\varepsilon_u+\varepsilon_y}d = \frac{\varepsilon_u}{\varepsilon_u + f_y/E_s}d = \frac{0.003}{0.003+f_y/(29\times10^6)}d = \frac{87000}{87000+f_y}d$$




At equilibrium condition, $C = T$

$$0.85\,f_c'\,a\,b = A_s\,f_y$$

$$a = \frac{A_s\,f_y}{0.85\,f_c'\,b} = \frac{\rho_b\,b\,d\,f_y}{0.85\,f_c'\,b} = \frac{\rho_b\,d\,f_y}{0.85\,f_c'}$$

$$c = \frac{a}{\beta_1} = \frac{\rho_b\,d\,f_y}{0.85\,\beta_1\,f_c'}$$

Now, $\frac{\rho_b\,d\,f_y}{0.85\,\beta_1\,f_c'} = \frac{87000}{87000+f_y}d$

$$\rho_b = 0.85\,\beta_1\frac{f_c'}{f_y}\frac{87000}{87000+f_y}$$

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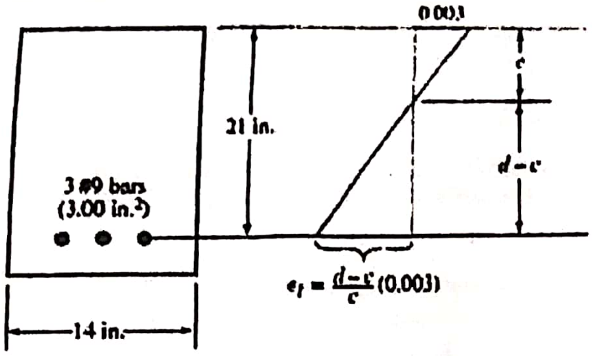
**Question:** Determine the values of a, c, and $\varepsilon_t$ for the beam shown in Figure $f_y$ = 60,000 psi and $f_c'$ = 3000 psi.

**Solution:**




$$a = \frac{A_s f_s}{0.85 f_c' b} = \frac{3 \times 60}{0.85 \times 3 \times 14} = 5.04 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{5.04}{0.85} = 5.93 \text{ in}$$

$$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{21-5.93}{5.93}(0.003) = 0.00762$$

This value of strain is much greater than the yield strain of 0.002. This is an indication of ductile behavior of the beam, because the steel is well into its yield plateau before concrete crushes.

**Question:** A rectangular beam of section 12" x 20" carries a moment 180 k-ft. Find out the reinforcment required to resist the moment and either failure occurs by cracking of concrete or yield of steel. Consider $f_c' = 3$ ksi, $f_y = 60$ ksi and $d' = 3$ in. (WASA – 2017)

**Solution:**

$$M_u = \varphi A_s f_y \left(d - \frac{a}{2}\right)$$

$$180 \times 12 = 0.9 \times A_s \times 60 \left(17 - \frac{5.5}{2}\right) \qquad \text{[Assume, } a = 5.5 \text{ in]}$$

$$A_s = 2.80 \text{ in}^2$$

$$\text{Check, } a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.80 \times 60}{0.85 \times 3 \times 12} = 5.5 \text{ in}$$

$$\rho = \frac{A_s}{b d} = \frac{2.8}{12 \times 17} = 0.0137$$

$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_t} = 0.85 \times 0.85 \times \frac{3000}{60000} \times \frac{0.003}{0.003 + 0.004} = 0.0154$$

$\rho_{max} > \rho$, Beam will fail by yield of steel.

**Question:** A simply supported beam has a span of 20 ft. If the cross [illegible] in figure, $f_c' = 3$ ksi and $f_y = 60$ ksi, determine the allowable uniformly distributed service load in the beam assuming the dead load is that due to beam weight. Given $b = 12$ in, $d = 17$ in, total depth $h = 20$ in and reinforced with three no. bars ($A_s = 2.37$ in$^2$)

**Solution:**

Reinforcement ratio, $\rho = \frac{A_s}{b\,d} = \frac{3 \times 0.79}{12 \times 17} = 0.0116$

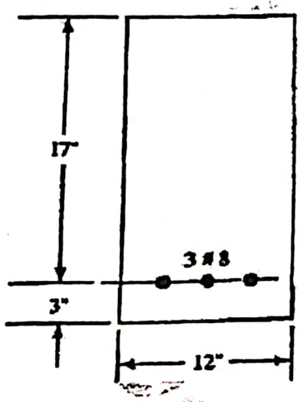
Maximum reinforcement ratio for a tensioned controlled beams,

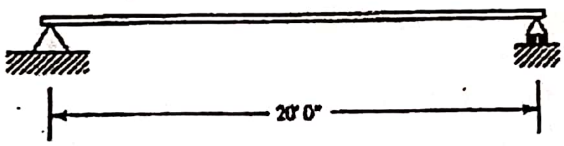
$$\rho_{max} = 0.85\,\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_t} = 0.85 \times 0.85 \times \frac{3000}{60000} \times \frac{0.003}{0.003 + 0.005} = 0.0135$$

$\rho < \rho_{max} \rightarrow$ Therefore it is a tension controlled section and $\varphi = 0.9$

$$a = \frac{A_s\,f_y}{0.85\,f_c'\,b} = \frac{2.37 \times 60}{0.85 \times 3 \times 12} = 4.64 \text{ in}$$

$$M_u = \varphi\,A_s\,f_y\left(d - \frac{a}{2}\right) = 0.9 \times 2.37 \times 60\left(17 - \frac{4.64}{2}\right) = 1875 \text{ k-in} = 156.5 \text{ k-ft}$$




The dead load acting on the beam is self-weight

$$W_D = \frac{12 \times 20}{144} \times 150 = 250 \text{ lb./ft} = 0.25 \text{ k/ft}$$

The external factored moment is,

$$M_u = 1.2\,M_D + 1.6\,M_L$$

$$156.5 = 1.2\left(\frac{0.25}{8} \times 20^2\right) + 1.6\left(\frac{W_L}{8} \times 20^2\right) = 15 + 80\,W_L$$

$$156.5 = 15 + 80\,W_L$$

$$W_L = 1.77 \text{ k/ft}$$

The allowable uniform service live load on the beam is 1.77 kip/ft.

**Question:** Determine the ACI design moment capacity, $\varphi M_n$ of the beam shown in Figure, if $f_c' = 4000$ psi and $f_y = 60{,}000$ psi.

**Solution:**

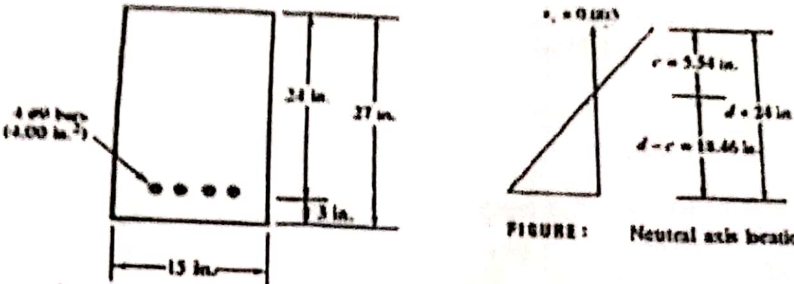



FIGURE: Neutral axis location

$$\rho = \frac{A_s}{b\,d} = \frac{4}{15 \times 24} = 0.0111 > \rho_{min} = 0.0033$$

$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} = \frac{4 \times 60}{0.85 \times 4 \times 15} = 4.71 \text{ in}$$

$\beta_1 = 0.85$ for 4000 psi concrete

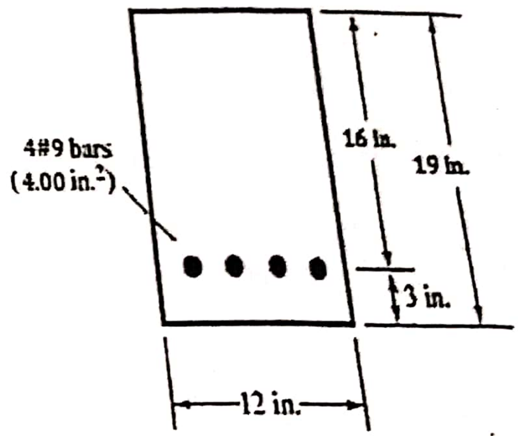
$$c = \frac{a}{\beta_1} = \frac{4.71}{0.85} = 5.54 \text{ in}$$

$$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{24-5.54}{5.54}(0.003) = 0.01 > 0.005$$

$$M_u = \varphi\, A_s\, f_y\left(d - \frac{a}{2}\right) = 0.9 \times 4 \times 60\left(24 - \frac{4.71}{2}\right) = 5194.8\ \text{k-in} = 432.9\ \text{k-ft}$$

**Question:** A rectangular beam of width 12" and effective depth 16" has four no. 9 bars in a row on tension side. Check whether the beam is tension controlled or not if $f_c' = 4$ ksi and $f_y = 60$ ksi. Determine design capacity of the beam. (BWDB – 2016)

**Solution:**




$$\rho = \frac{A_s}{b\,d} = \frac{4}{12 \times 16} = 0.0208 > \rho_{min} = 0.0033$$

$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} = \frac{4 \times 60}{0.85 \times 4 \times 12} = 5.88 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92 \text{ in}$$

$$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{16-6.92}{6.92}(0.003) = 0.00393 < 0.004$$

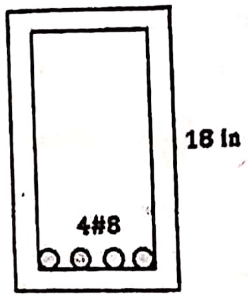
Transition region.

∴ Section is not ductile and may not be used as per ACI 10.5.3 code.

**DESIGN INTEGRITY, Call: 01633905761**

**Question: Determine the ultimate moment for the beam shown below, where $f_c' = 4$ ksi, $f_y = 60$ ksi, Clear cover = 1.5" (BWDB – 2018)**

**Solution:**




$$A_s = 4 \times 0.79 = 3.16 \text{ in}^2$$

$$\rho = \frac{A_s}{b\,d} = \frac{4}{12 \times 16.5} = 0.02 > \rho_{min} = 0.0033$$

$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} = \frac{3.16 \times 60}{0.85 \times 4 \times 12} = 4.65 \text{ in}$$

$\beta_1 = 0.85$ for 4000 psi concrete

$$c = \frac{a}{\beta_1} = \frac{4.65}{0.85} = 5.47 \text{ in}$$

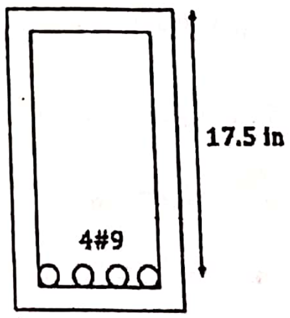
$$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{16.5-5.47}{5.47}(0.003) = 0.00605 > 0.005$$

$\varepsilon_t > 0.005$, So tension controlled section and $\varphi = 0.9$

$$M_u = \varphi\, A_s\, f_y\left(d - \frac{a}{2}\right) = 0.9 \times 3.16 \times 60\left(16.5 - \frac{4.65}{2}\right) = 2418.82 \text{ k} - \text{in}$$

**Question: A rectangular beam has a width of 12 in an effective depth to the centroid of the reinforcing steel of 17.5 in. It is reinforced with four no 9 bar in one row. If $f_y = 60$ ksi and $f_c' = 4$ ksi. What is the ultimate moment capacity of the beam? Also check what would be the φ value. (DESCO – 2015)**

**Solution:**




$$A_s = 4 \times 1 = 4 \text{ in}^2$$

$$\rho = \frac{A_s}{b\,d} = \frac{4}{12 \times 17.5} = 0.02 > \rho_{min} = 0.0033$$

$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} = \frac{4 \times 60}{0.85 \times 4 \times 12} = 5.88 \text{ in}$$

$\beta_1 = 0.85$ for 4000 psi concrete

$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92 \text{ in}$$

$$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{17.5-6.92}{6.92}(0.003) = 0.0046$$

$\varepsilon_t > 0.004$ and $\varepsilon_t < 0.005$

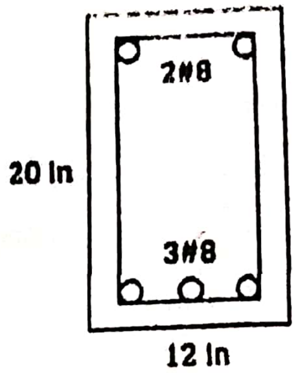
∴ Beam is in transition zone

$$\varphi = 0.65 + (\varepsilon_t - 0.002)\frac{250}{3} = 0.65 + (0.0046 - 0.002)\frac{250}{3} = 0.87$$

$$M_u = \varphi\, A_s\, f_y\left(d - \frac{a}{2}\right) = 0.9 \times 4 \times 60\left(17.5 - \frac{5.88}{2}\right) = 3040.12 \text{ k} - \text{in}$$

**Question:** Calculate the ultimate moment capacity of the following beam, given $h = 20"$, $b = 12"$, $d' = 2.5"$, $f_c' = 4$ ksi, $f_y = 60$ ksi. Assume singly reinforced beam (DMTCL – 2019)

**Solution:**




$A_s = 3 \times 0.79 = 2.37\ in^2$

$\rho = \frac{A_s}{b\,d} = \frac{2.37}{12 \times 17.5} = 0.0111 > \rho_{min} = 0.0033$

$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} = \frac{2.37 \times 60}{0.85 \times 4 \times 12} = 3.84$ in

$\beta_1 = 0.85$ for 4000 psi concrete

$c = \frac{a}{\beta_1} = \frac{3.84}{0.85} = 4.51$ in

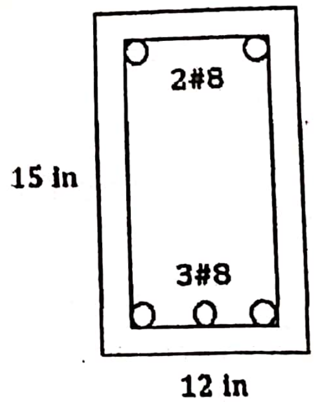
$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{17.5 - 4.51}{4.51}(0.003) = 0.008 > 0.005$

$\varepsilon_t > 0.005$, So tension controlled section and $\varphi = 0.9$

$M_u = \varphi\, A_s\, f_y \left(d - \frac{a}{2}\right) = 0.9 \times 2.37 \times 60 \left(17.5 - \frac{3.84}{2}\right) = 2017\ k-in$

**Question:** Determine the flexural moment capacity of the cantilever beam having 15" depth and concrete clear cover 2.5 inch. Given $f_c' = 4$ ksi & $f_y = 60$ ksi (Assume singly reinforced) (PGCB – 2019)

**Solution:**




$A_s = 2 \times \pi \times \left(\frac{8}{8}\right)^2 = 1.57\ in^2$, $d = 15 - 2.5 = 12.5$ in

$\rho = \frac{A_s}{b\,d} = \frac{1.57}{12 \times 12.5} = 0.01 > \rho_{min} = 0.0033$

$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} = \frac{1.57 \times 60}{0.85 \times 4 \times 12} = 2.31$ in

$\beta_1 = 0.85$ for 4000 psi concrete.

$c = \frac{a}{\beta_1} = \frac{2.31}{0.85} = 2.71$ in

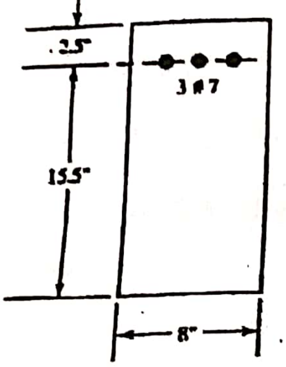
$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{12.5 - 2.71}{2.71}(0.003) = 0.01 > 0.005$

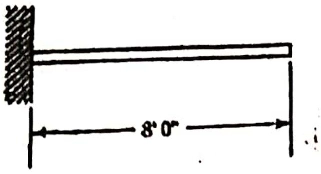
As $\varepsilon_t > 0.005 \rightarrow \varphi = 0.9$

Nominal moment, $M_n = A_s\, f_y (d - \frac{a}{2}) = 1.57 \times 60 \times (12.5 - \frac{2.31}{2}) = 961.83\ k-in$

Ultimate moment, $M_u = \varphi\, M_n = 0.9 \times 961.83 = 865.64\ k-in$

Question: An 8 feet span cantilever beam has a rectangular section and reinforcement as shown in figure. The beam carries a dead load including its own weight of 1.5 k/ft and a live load of 0.9 k/ft. Using $f_c' = 4$ ksi and $f_y = 60$ ksi, check if the beam is safe to carry the above loads. (BCIC – 2017, BUET M.Sc – 2019, EGCB – 2020)




**Solution:**

Factored load = $1.2\,DL + 1.6\,LL = 1.2 \times 1.5 + 1.6 \times 0.9 = 3.24$ k/ft

Factored moment, $M_u = \frac{W_u L^2}{2} = \frac{3.24 \times 8^2}{2} = 103.68$ k – ft = 1244 k – in

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.8 \times 60}{0.85 \times 4 \times 8} = 3.97 \text{ in}$$

$\beta_1 = 0.85$ for 4000 psi concrete

$$c = \frac{a}{\beta_1} = \frac{3.97}{0.85} = 4.67 \text{ in}$$

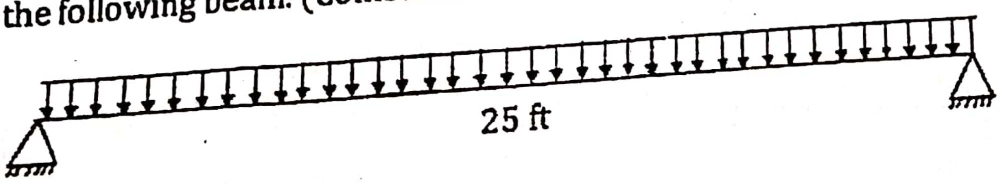
$$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{15.5 - 4.67}{4.67}(0.003) = 0.007 > 0.005$$

Therefore, it is a tension controlled section and $\varphi = 0.9$

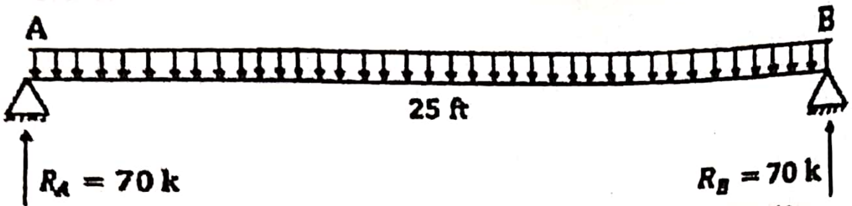
$$M_u = \varphi A_s f_y \left(d - \frac{a}{2}\right) = 0.9 \times 1.8 \times 60 \left(15.5 - \frac{3.97}{2}\right) = 1312 \text{ k} - \text{in} > 1244 \text{ k} - \text{in}$$

The beam is safe to carry the above loads

Question: A rectangular beam of section 12" x 24" carries $DL = 2$ k/ft and $LL = 2$ k/ft. If the value of $f_c' = 4$ ksi, $f_y = 60$ ksi & $d' = 1.5$", design the reinforcement at 10 ft from the support for the following beam. (Combined Bank – 2020)



**Solution:**




Factored load, $W = 1.2\,DL + 1.6\,LL = 1.2 \times 2 + 1.6 \times 2 = 5.6$ k/ft

Moment at 10 ft from support, $M_u = 70 \times 10 - 5.6 \times 10 \times 5 = 420$ k – ft

$M_u = \varphi\, A_s\, f_y \left(d - \frac{a}{2}\right)$

$420 \times 12 = A_s \times 0.9 \times 60 \times \left(22.5 - \frac{7.28}{2}\right)$ [Assume, $a = 7.28$ in & $\varphi = 0.9$]

$A_s = 4.94\ \text{in}^2$

Check, $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4.94 \times 60}{0.85 \times 4 \times 12} = 7.28$ in → as assumed (ok)

$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{22.5 - 7.28}{7.28}(0.003) = 0.0062 > 0.005 \rightarrow \varphi = 0.9$

Provide 5#9 bar as main reinforcement ($A_s = 5.00\ \text{in}^2$)

**Question: A rectangular beam that must carry a service live load of 2.47 kip/ft and a calculated dead load of 1.05 kip/ft on an 18 ft. Simple span is limited in cross section for architectural reason to 10 inch width and 20 inch total depth. If $f_y = 60000$ ksi and $f_c' = 4000$ psi. Estimate the steel requirement. (36th BCS)**

**Solution:**

Effective depth, d = 20 – 1.5 = 18.5 inch

Factored load = $1.2\,DL + 1.6\,LL = 1.2 \times 1.05 + 1.6 \times 2.47 = 5.212$ k/ft

Factored moment, $M_u = \frac{W_u L^2}{8} = \frac{5.212 \times 18^2}{8} = 211.08$ k – ft = 2532.96 k – in

$M_u = \varphi\, A_s\, f_y \left(d - \frac{a}{2}\right)$

$211.08 \times 12 = A_s \times 0.9 \times 60 \times \left(18.5 - \frac{5.20}{2}\right)$ [Assume, $a = 5.20$ in and $\varphi = 0.9$]

$A_s = 2.95\ \text{in}^2$

Check, $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.95 \times 60}{0.85 \times 4 \times 10} = 5.20$ in → as assumed (ok)

$c = \frac{a}{\beta_1} = \frac{5.20}{0.85} = 6.11$ in

$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{18.5 - 6.11}{6.11}(0.003) = 0.006 > 0.005$

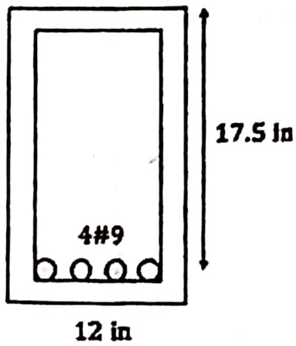
$\varepsilon_t > 0.005 \rightarrow \varphi = 0.9$, as assumed

Provide 5#6 bar as main reinforcement ($A_s = 2.95\ \text{in}^2$)

DESIGN INTEGRITY, Call: 01633905761

**Question:** A beam having b = 12" and d = 16" is reinforced with 4 nos. #8 bars in a row as main reinforcement and #3 bars 6 in c/c spacing as shear reinforcement. Considering $f_c'$ = 3 ksi and $f_y$ = 60 ksi. Check if the section is tension controlled or not and determine the allowable shear strength. (SGFCL – 2017)

**Solution:**




$A_s = 4 \times 0.79 = 3.16\ in^2$

$\rho = \frac{A_s}{b\,d} = \frac{4}{12 \times 13.5} = 0.0195 > \rho_{min} = 0.0033$

$a = \frac{A_s f_s}{0.85 f_c' b} = \frac{3.16 \times 60}{0.85 \times 3 \times 12} = 6.19$ in

$c = \frac{a}{\beta_1} = \frac{6.19}{0.85} = 7.28$ in

$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{13.5 - 7.28}{7.28}(0.003) = 0.00256 < 0.005$

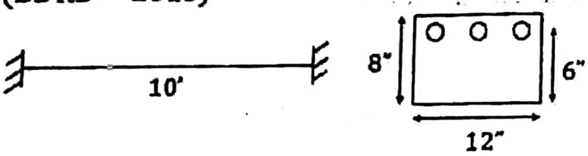
So, Section isn't tension controlled.

Allowable shear strength, $V_u = \varphi V_s + \varphi V_c$

$V_u = \varphi \frac{A_v f_y d}{S} + 2\varphi\sqrt{f_c'}\, b_w\, d$

$V_u = 0.75 \times \frac{0.22 \times 60 \times 13.5}{6} + 2 \times 0.75 \times \sqrt{3000} \times 12 \times 17.5 = 35.57$ kip

**Question:** The span of a beam 10' long and ends are fixed. The maximum load controlled by the capacity in the negative moment region. Based on the flexural requirements, what is the maximum uniform live load the beam can carry? ($f_y$ = 40ksi, $f_c'$ = 3.5 ksi, $M_n$ = 140.2 k-in, $A_s$ = 0.62 in²) (BB AD – 2018)




**Solution:**

Given, $M_n$ = 140.2 k-in.

$M_u = \varphi M_n = 0.9 \times 140.2 = 126.18$ k – in

For both end fixed beam, $M_u = \frac{W L^2}{12}$

$126.18 = \frac{W\,(10 \times 12)^2}{12}$

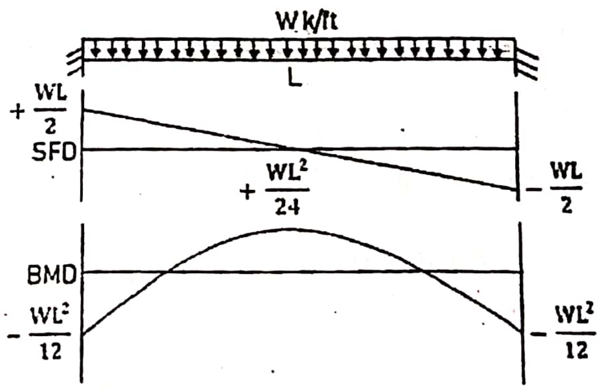
$W = 0.105$ kip/inch = 1.26 k/ft

Here, Dead load = $\frac{8 \times 12}{144} \times 150 = 100$ lb/ft

Total load = 1.4 $DL$ + 1.7 $LL$

$1.26 \times 1000 = 1.4 \times 100 + 1.7 \times LL$

$LL = 658.8$ lb/ft



**Doubly Reinforced Beam:** The beam that is reinforced with steel both in tension and compression zone of the section is known as doubly reinforced beam. This type of beam is mainly provided when the depth of the beam is restricted. If a beam with limited depth is reinforced on the tension side only it might not have sufficient resistance to oppose the bending moment. Reinforcement in the compression zone contributes to the total nominal flexural strength of a section. Compression reinforcement is also added to help reduce long-term deflections.

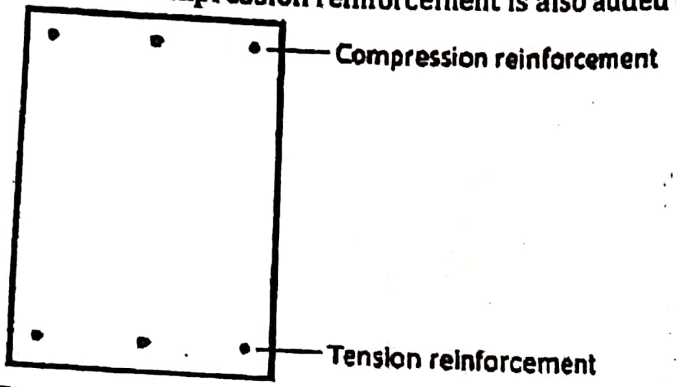



Figure: Doubly reinforced beam

***Nominal flexural strength when compression reinforcement yields***

The nominal flexural strength of a doubly reinforced section depends on whether the compression reinforcement yields or not. Assume that the yield strain $\varepsilon_y$ occurs at a distance of $d'_y$ from the extreme compression fiber.

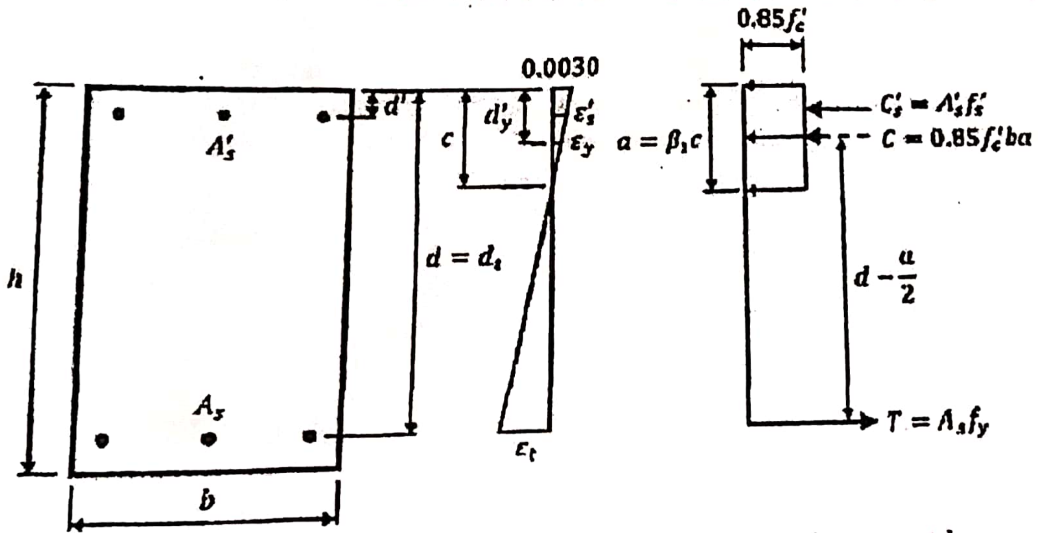



Figure: Strain and stress distributions in a doubly reinforced concrete beam.

From similar triangles,

$$\frac{c}{0.003} = \frac{c - d'_y}{\varepsilon_y}$$

For 60 grade reinforcement, $\varepsilon_y = 60/29000 = 0.00207$

$d'_y = 0.31\,c$

Compression reinforcement located a distance equal to or less than $d'_y$ from the extreme compression fiber yields (i.e. $\varepsilon'_s \geq \varepsilon_y$ and $f'_s = f_y$).

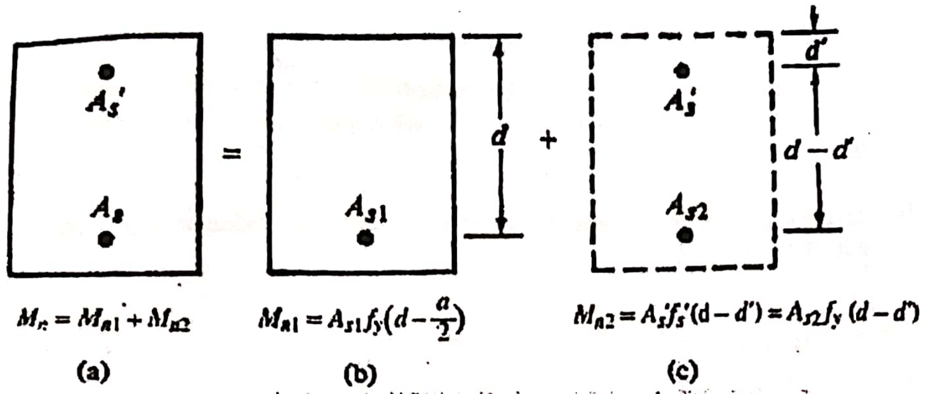



Figure: Doubly reinforced beam broken into parts.

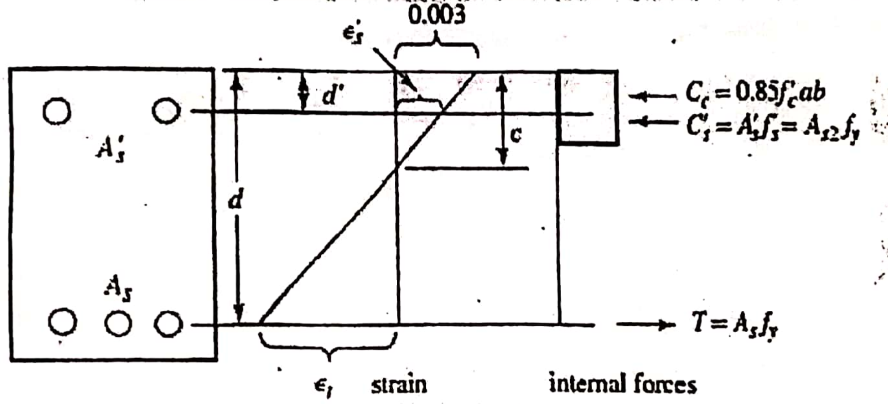
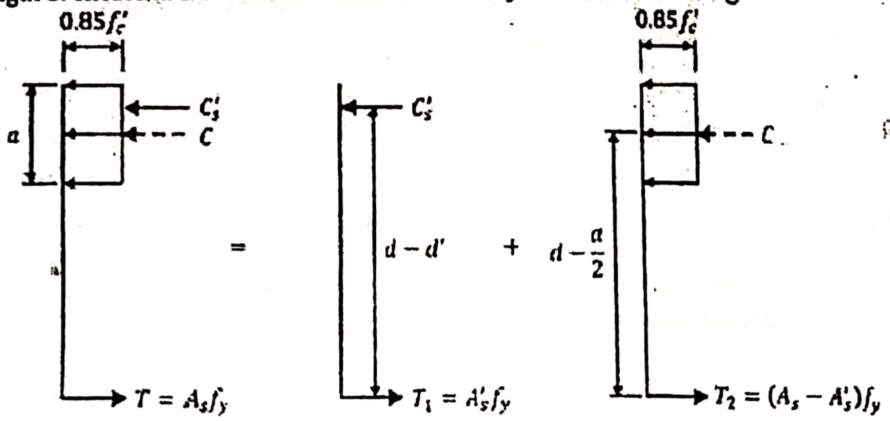



Figure: Internal strains and forces for doubly reinforced rectangular beam.




Force distribution in a doubly reinforced beam when the compression reinforcement yields.

When the compression steel yields, the depth of the equivalent stress block $a$ can be obtained by satisfying force equilibrium:

$T = C + C'_s$

$$A_s f_y = 0.85 f_c' a b + A_s' f_y$$

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b}$$

The total nominal flexural strength $M_n$ is considered to be the sum of two parts. The first part $M_{n1}$ is provided by the couple consisting of the force in the compression steel $A_s'$ and the force in an equal area of tension steel,

$$M_{n1} = A_s' f_y (d - d')$$

The second part $M_{n2}$ is provided by the couple consisting of the remaining tension steel $A_s - A_s'$ and the compression force in the concrete C:

$$M_{n2} = (A_s - A_s') f_y \left(d - \frac{a}{2}\right)$$

The total nominal flexural strength of a doubly reinforced section where $f_s' = f_y$ is,

$$M_n = (A_s - A_s') f_y \left(d - \frac{a}{2}\right) + A_s' f_y (d - d')$$

**_Nominal flexural strength when compression reinforcement does not yield_**

When the compression reinforcement does not yield ($f_s' < f_y$) the depth of the stress block '$a$' cannot be determined by the above Eq. because the magnitude of $f_s'$ is unknown. The strain in the compression reinforcement $\varepsilon_s'$ is related to $c$ as follows:

$$\frac{c}{0.003} = \frac{c - d'}{\varepsilon_s'}$$

Substituting, $\varepsilon_s' = \frac{f_s'}{E_s}$

$$f_s' = 0.003\, E_s \left(\frac{c - d'}{c}\right)$$

If $\varepsilon_s' < \varepsilon_y$, the compression steel is not yielding and the value of $c$ calculated above is not correct. A new equilibrium equation must be written that assumes $f_s' < f_y$

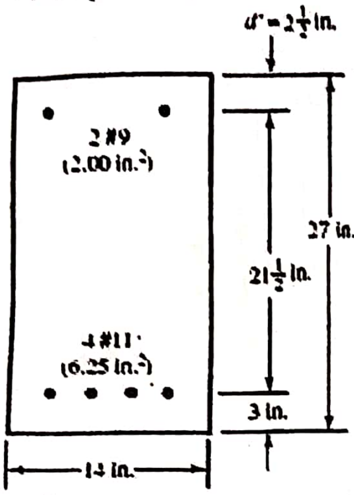
$$A_s f_y = 0.85 f_c' \beta_1 c b + A_s' \left(\frac{c - d'}{c}\right) (0.003) E_s$$

Where, $E_s = 29{,}000$ ksi

$$M_n = 0.85 f_c' a b \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$

Regardless of whether the compression steel yields or not, the strain $\varepsilon_t$ must be equal to or greater than 0.0040 in order to satisfy the provisions of ACI 10.3.5.

**Question:** Determine the design moment capacity of the beam shown in Figure. For which $f_y$ = 60,000 psi and $f_c'$ = 3000 psi.




**Solution:**

Assuming, $f_s' = f_y$

$$a = \frac{(A_s - A_s')\, f_y}{0.85\, f_c'\, b} = \frac{(6.25 - 2) \times 60}{0.85 \times 3 \times 14} = 7.14 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{8.40}{0.85} = 8.40 \text{ in}$$

Computing strains in compression steel to verify assumption that it is yielding,

$$\varepsilon_s' = 0.003 \left(\frac{c - d'}{c}\right) = 0.003 \left(\frac{8.40 - 2.5}{8.40}\right) = 0.00210$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60000}{29000000} = 0.00206 < \varepsilon_s' \rightarrow f_s' = f_y$$

$$A_{s2} = \frac{A_s' f_s'}{f_y} = \frac{2 \times 60000}{60000} = 2 \text{ in}^2$$

$$A_{s1} = A_s - A_{s2} = 6.25 - 2 = 4.25 \text{ in}^2$$

$$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{24 - 8.40}{8.40}(0.003) = 0.00557 > 0.005 \rightarrow \varphi = 0.90$$

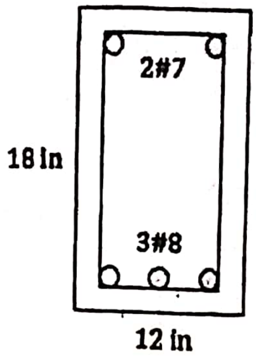
The design moment strength is,

$$\varphi M_n = \varphi \left[A_{s1} f_y \left(d - \frac{a}{2}\right) + A_s' f_y (d - d')\right]$$

$$\varphi M_n = 0.9 \left[4.25 \times 60 \left(24 - \frac{7.14}{2}\right) + 2 \times 60\ (24 - 2.5)\right]$$

$$= 7010 \text{ kip} - \text{in} = 584.2 \text{ kip} - \text{ft}$$

**Question: Determine the ultimate moment capacity of the beam which clear cover is 1.5 inch. Consider $f_c' = 4$ ksi and $f_y = 60$ ksi. (NPCBL – 2019, BIWTA – 2019)**




**Solution:**

$A_s' = 2 \times 0.6 = 1.2 \text{ in}^2$

$A_s = 3 \times 0.79 = 2.37 \text{ in}^2$

Assuming $f_s' = f_y$

$$a = \frac{(A_s - A_s')\, f_y}{0.85\, f_c'\, b} = \frac{(2.37 - 1.2) \times 60}{0.85 \times 4 \times 12} = 1.72 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{1.72}{0.85} = 2.02 \text{ in}$$

Computing strains in compression steel to verify assumpt

$$\varepsilon_s' = 0.003 \left(\frac{c - d'}{c}\right) = 0.003 \left(\frac{2.02 - 1.5}{2.02}\right) = 0.000772$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.0027 > \varepsilon_s' \rightarrow So\ f_s' \neq f_y$$

Since the assumption is not valid, we have to use the equilibrium equation that is based on $f_s'$ not yielding.

$$A_s f_y = 0.85\, f_c'.\beta_1\, c\, b + A_s' \left(\frac{c - d'}{c}\right)(0.003)\, E_s$$

$$2.37 \times 60 = 0.85 \times 4 \times 0.85 \times c \times 12 + 1.20 \left(\frac{c - 1.5}{c}\right)(0.003) \times 29{,}000$$

Solving the equation, $c = 2.74$ in

$$a = c\, \beta_1 = 2.74 \times 0.85 = 2.32 \text{ in}$$

$$\varepsilon_s' = 0.003 \left(\frac{c - d'}{c}\right) = 0.003 \left(\frac{2.74 - 1.5}{2.74}\right) = 0.00135 < \varepsilon_y$$

$$f_s' = \varepsilon_s' E_s = 0.00135 \times 29{,}000 = 39.15 \text{ ksi}$$

$$A_{s2} = \frac{A_s' f_s'}{f_y} = \frac{1.2 \times 39.15}{60{,}000} = 0.78 \text{ in}^2$$

$$A_{s1} = A_s - A_{s2} = 2.37 - 0.78 = 1.59 \text{ in}^2$$

$$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{16.5 - 2.74}{2.74}(0.003) = 0.015 > 0.005 \rightarrow \varphi = 0.90$$

The design moment strength is,

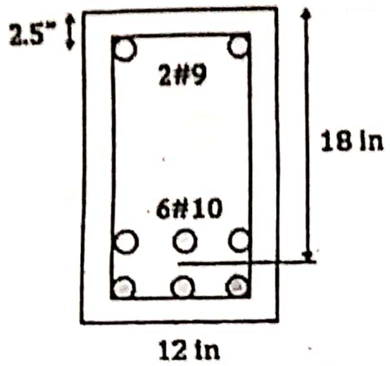
$$\varphi M_n = \varphi \left[A_{s1} f_y \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')\right]$$

$$\varphi M_n = 0.9 \left[1.59 \times 60 \left(16.5 - \frac{2.32}{2}\right) + 1.2 \times 39.15\,(16.5 - 1.5)\right]$$

$$= 1951.32 \text{ kip} - \text{in} = 162.61 \text{ kip} - \text{ft}$$

**Question:** A rectangular beam has a width 12 inch (305 mm) and an effective depth to the centroid of the tension reinforcement of 18 inch (457 mm). The tension reinforcement consists of six no. 10 bars (7.59 in²) in two rows. Compression reinforcement of two no. 9 bars (2.0 in²) is placed 2.5 inch from the compression face of the beam. If $f_y$ = 50 ksi (345 MPa) and $f_c'$ = 5 ksi (34.5 MPa), what is the design moment capacity of the beam? (40th BCS)

**Solution:**




Assuming, $f_s' = f_y$

$$a = \frac{(A_s - A_s')f_y}{0.85\, f_c'\, b} = \frac{(7.59 - 2) \times 50}{0.85 \times 5 \times 12} = 5.48 \text{ in}$$

$$\beta_1 = 0.85 - 0.005\left(\frac{f_c' - 4000}{1000}\right) = 0.85 - 0.005\left(\frac{5000 - 4000}{1000}\right) = 0.80$$

$$c = \frac{a}{\beta_1} = \frac{5.48}{0.80} = 6.85 \text{ in}$$

Computing strains in compression steel to verify assumption that it is yielding,

$$\varepsilon_s' = 0.003\left(\frac{c - d'}{c}\right) = 0.003\left(\frac{6.85 - 2.5}{6.85}\right) = 0.001905$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{50000}{29000000} = 0.0017 < \varepsilon_s' \rightarrow f_s' = f_y$$

$$A_{s2} = \frac{A_s' f_s'}{f_y} = \frac{2 \times 50000}{50000} = 2 \text{ in}^2$$

$$A_{s1} = A_s - A_{s2} = 7.59 - 2 = 5.59 \text{ in}^2$$

$$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{18 - 6.85}{6.85}(0.003) = 0.0048 < 0.005$$

$0.004 < \varepsilon_t < 0.005 \rightarrow$ Beam is in transition zone

$$\varphi = 0.65 + (\varepsilon_t - 0.002)\frac{250}{3} = 0.65 + (0.0048 - 0.002)\frac{250}{3} = 0.88$$

The design moment strength is,

$$\varphi' M_n = \varphi' \left[ A_{s1} f_y \left( d - \frac{a}{2} \right) + A'_s f_y (d - d') \right]$$

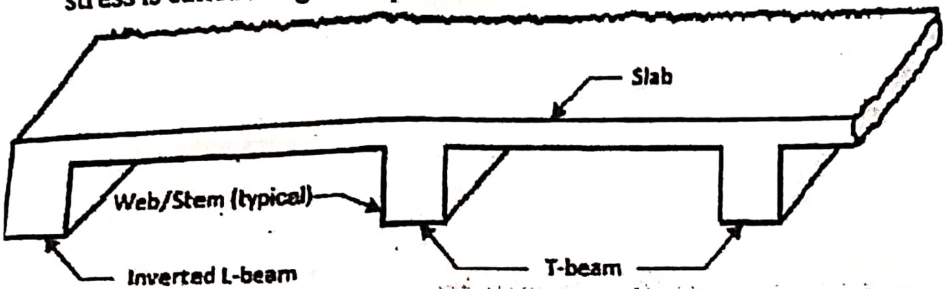
$$\varphi M_n = 0.88 \left[ 5.59 \times 50 \left( 18 - \frac{5.48}{2} \right) + 2 \times 50 \, (18 - 2.5) \right]$$

$$\varphi M_n = 5117.34 \text{ kip} - \text{in} = 426.44 \text{ kip} - \text{ft}$$

**Question: What is T beam and what are the advantages of a T beam over rectangular beam in case of RCC member? (34 & 40th BCS)**

**Solution:**

**The beam consists of a flange and a rib in the form of a T, generally made of RC concrete or metal is known as T-beam. The top part of the Slab which acts along the beam to resist the compressive stress is called flange. The part which lies below the slab and resists the shear stress is called rib.**




**Advantages:**

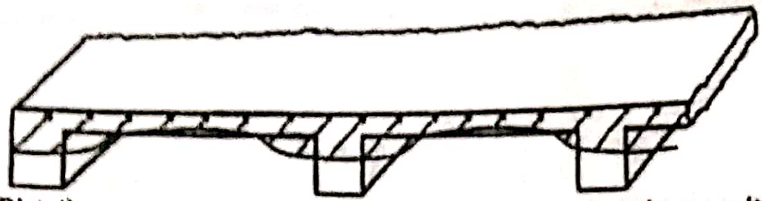
- Since the beam is casted monolithically with the slab, the flange also takes up the compressive stresses which mean it will be more effective in resisting the sagging moment acting on the beam.
- Better head room, this is direct outcome of the first point since the depth of the beam can be considerably reduced.
- For larger spans, t beams are usually preferred rather than rectangular beam as the deflection is reduced to a good extent.

**Disadvantages:**

- There is a considerable increase in the shear stress at the junction of the flange and the web of the beam due to the change in cross section. So casting should be done very carefully to ensure both are bonded well.
- Since the beam slab is monolithic (rigid), it becomes very weak in resisting lateral shear forces. (Cracks develop quickly). Hence usually in earthquake prone zones using t beams for high rise building is reinforced with mechanical stiffeners in the junction.
- There will be small savings in steel too (not a significant amount though)

**Effective Flange Width**

The beams depicted in figure are subjected to a positive moment that produces compressive stresses at the top of the section above the neutral axis. As seen in the figure, the compressive stresses are greatest over the web and decrease between the webs. In lieu of using a variable compressive stress, the maximum uniform compressive stress is assumed to act over an effective slab (flange) width. The effective width is determined such that the compressive force obtained by multiplying the maximum compressive stress by the effective width is equal to the resultant force of the actual compressive stress distribution.



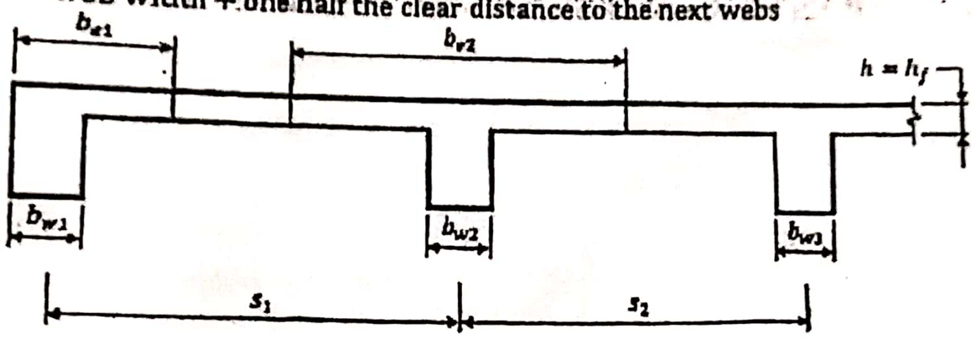
Distribution of compressive stress in a T-beam subjected to a positive moment

For beams with slabs on both sides of the web ($T-$beams)
Total effective flange width = the lesser of the following:

$$b_e = \begin{cases} \text{Span length}/4 \\ \text{web width} + 16\,(\text{slab thickness}) \\ \text{web width} + \text{one half the clear distance to the next webs} \end{cases}$$

For beams with a slab on only one side of the web (inverted $L-$beams)
Total effective flange width = the lesser of the following:

$$b_e = \begin{cases} \text{Span length}/4 \\ \text{web width} + 6\,(\text{slab thickness}) \\ \text{web width} + \text{one half the clear distance to the next webs} \end{cases}$$




$$b_{e1} \le \begin{cases} b_{w1} + \dfrac{\text{Span length}}{12} \\ b_{w1} + 6h \\ \dfrac{3b_{w1}}{4} - \dfrac{b_{w2}}{4} + \dfrac{s_1}{2} \end{cases} \qquad b_{e2} \le \begin{cases} \dfrac{\text{Span length}}{4} \\ b_{w2} + 16h \\ \dfrac{b_{w2}}{2} - \dfrac{b_{w1} + b_{w3}}{4} + \dfrac{s_1 + s_2}{2} \end{cases}$$

Effective flange widths for a T-beam and an inverted L-beam.

**Depth of stress block less than or equal to flange thickness ($a \le h_f$)**

When the depth of the compression zone $a$ falls within the flange of a T-beam or an inverted L-beam, the compressive zone is rectangular with a width equal to the effective flange width $b_e$. The nominal flexural strength of the section is determined by the equations developed earlier for rectangular sections.

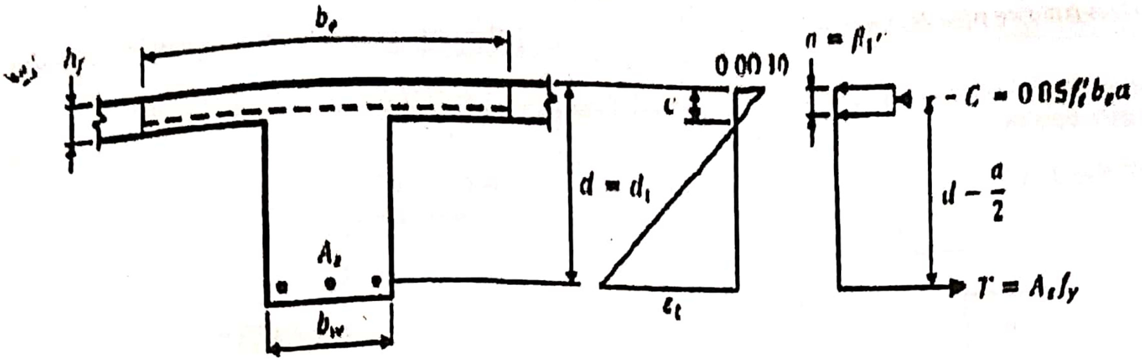



Figure: Strain and stress distributions in a T-beam with the flange in compression and $a \le h_f$.

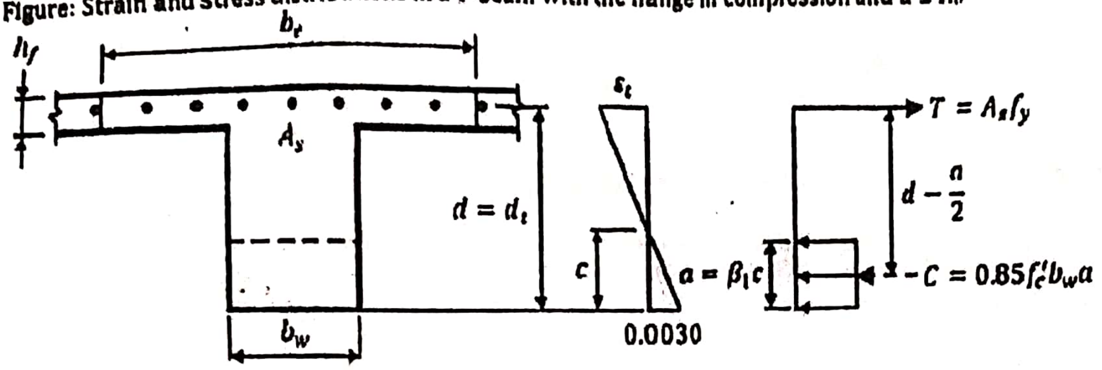



Figure: Strain and stress distributions in a T-beam with the flange in tension.

### Depth of stress block greater than flange thickness ($a > h_f$)

When the depth of the stress block $a$ falls within the web of the beam, the compressive zone is T or L shaped as opposed to rectangular (see Figure). The resultant force $C$ is equal to 0.85fc' times the area of the compressive zone and is located at its centroid. In such cases, it is convenient to divide the tensile reinforcement into two parts. The first part $A_{sf}$ is defined as the area of steel that is required to balance the compressive force in the overhanging portions of the flange. The following equation for $A_{sf}$ is obtained from horizontal equilibrium, assuming that the tension reinforcement yields:

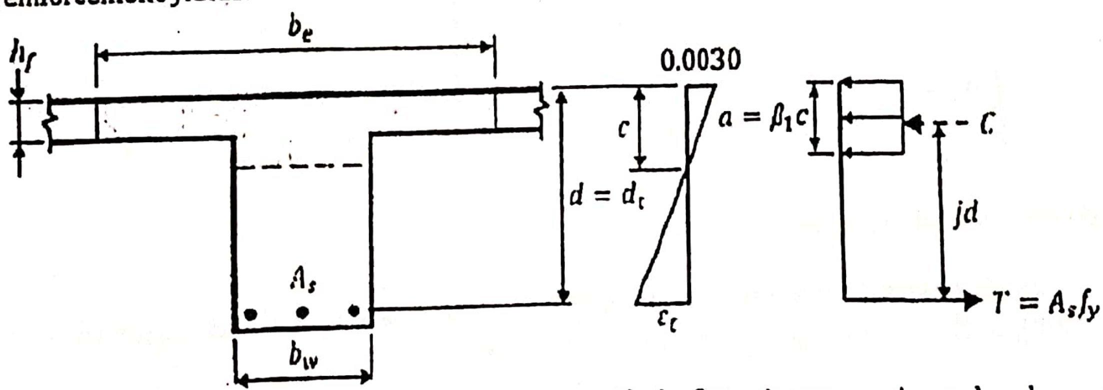



Figure: Strain and stress distributions in a T-beam with the flange in compression and $a > h_f$.

$$A_{sf} = \frac{0.85 f_c' (b_e - b_w) h_f}{f_y}$$

The nominal flexural resistance provided by the tensile reinforcement $A_{sf}$ is obtained from moment equilibrium:

$$M_{n1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

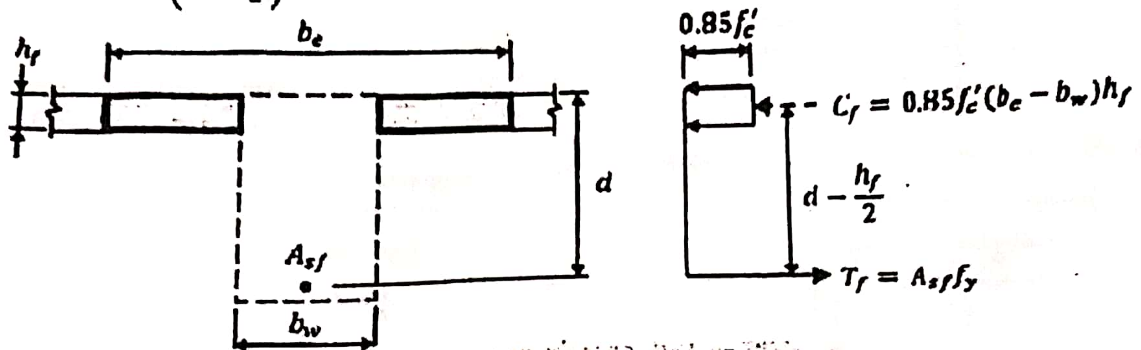



Figure: Stress distribution corresponding to overhanging flanges.

The remaining part of the tensile reinforcement $A_s - A_{sf}$ is balanced by the compression force in the rectangular part of the web (see Figure). As usual, the depth of the stress block 'a' is determined from horizontal equilibrium:

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w}$$

The nominal flexural resistance provided by the tensile reinforcement $(A_s - A_{sf})$ is obtained from moment equilibrium:

$$M_{n2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)$$

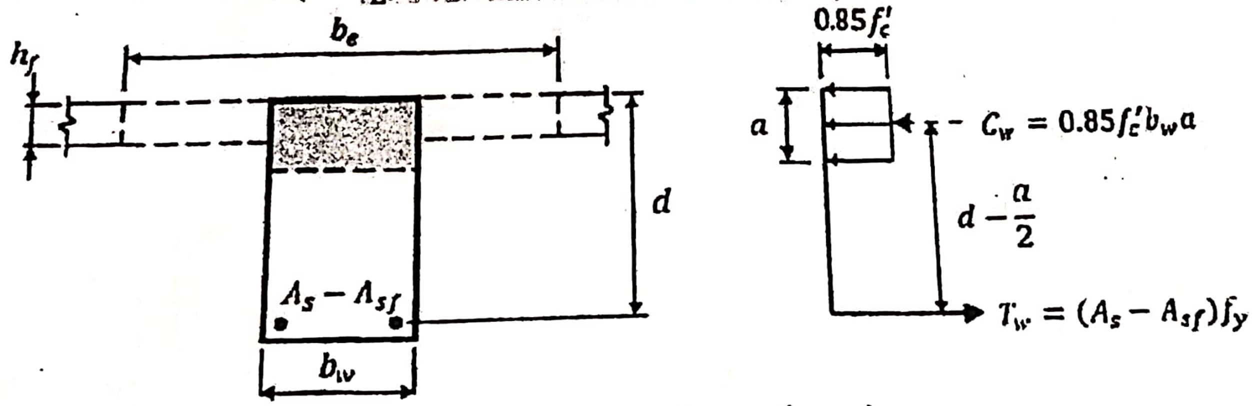



Figure: Stress distribution corresponding to the web.

Thus, the total nominal flexural strength of the section $M_n$ where $a > h_f$ is the addition of the two parts corresponding to the overhanging flanges and the web:

$$M_n = M_{n1} + M_{n2}$$

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)$$

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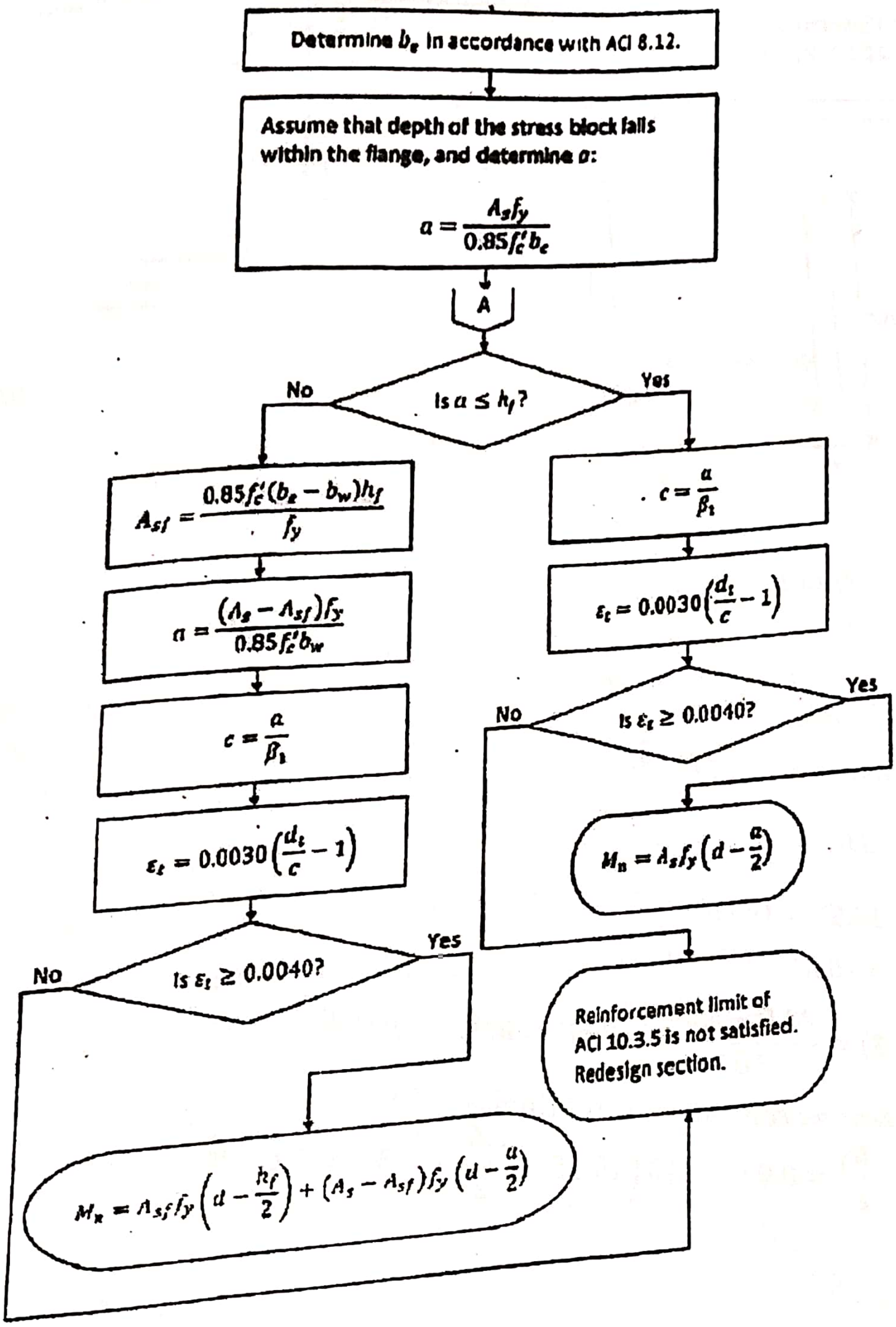




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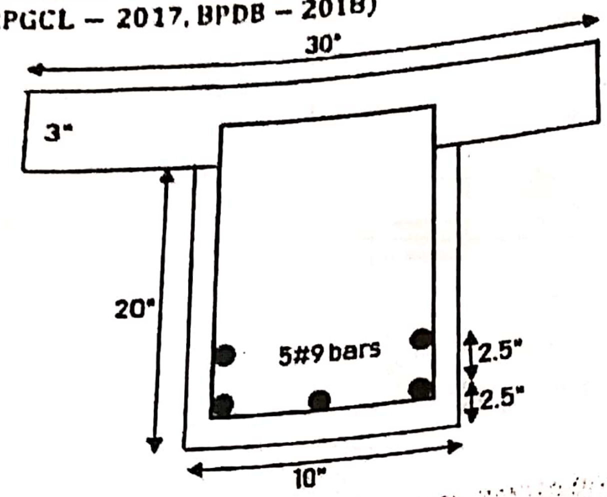
**Question: Determine the ultimate moment capacity of the T beam $f_c'$ = 3 ksi and $f_y$ = 40 ksi.**
**(RPGCL – 2017, BPDB – 2018)**




**Solution:**

Effective flange width, $b_e = 30"$

$A_s = 5 \times 1 = 5 \text{ in}^2$

$$a = \frac{A_s f_y}{0.85 f_c' b_e} = \frac{5 \times 40}{0.85 \times 3 \times 30} = 2.61"$$

$a < h_f$, so rectangular section analysis.

$$c = \frac{a}{\beta_1} = \frac{2.61}{0.85} = 3.07"$$

$d = 23 - (2.5 + 1.25) = 19.25"$

$d_t = 23" - 2.5" = 20.5"$

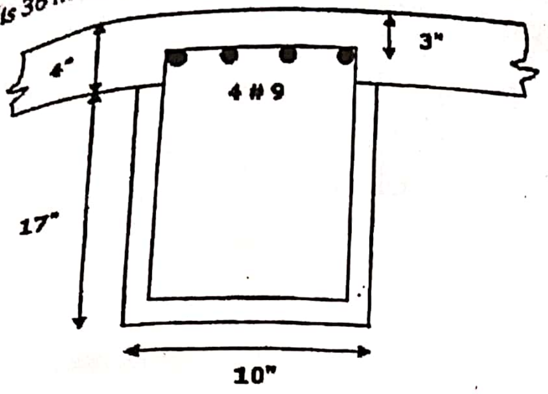
$$\varepsilon_t = \frac{d_t - c}{c}(0.003) = \frac{20.5 - 3.07}{3.07}(0.003) = 0.017 > 0.005$$

As $\varepsilon_t > 0.005$, so tension controlled section and $\varphi = 0.9$

$$M_u = \varphi A_s f_y \left(d - \frac{a}{2}\right) = 0.9 \times 5 \times 40\left(19.25 - \frac{2.61}{2}\right) = 3230.1 \text{ k-in}$$

$f_y = 40$ ksi

**Question:** Calculate the negative nominal moment of the beam. Clear distance between the webs is 36 in and span of the beam is 20 feet, $f_y = 60$ ksi, $f'_c = 5$ ksi, $\beta_1 = 0.8$ (PGCB – 2018)




**Solution:**

When T beams are resisting negative moments, their flanges will be in tension and the bottom of their stems will be in compression. For such situations the rectangular beam design formulas will be used.

$A_s = 4 \times 1 = 4\ in^2$

$a = \dfrac{A_s f_y}{0.85 f'_c b} = \dfrac{4 \times 60}{0.85 \times 5 \times 10} = 5.64''$

$\beta_1 = 0.80$ for 5000 psi concrete

$c = \dfrac{a}{\beta_1} = \dfrac{5.64}{0.80} = 7.05''$

$d = 21'' - 3'' = 18''$

$\varepsilon_t = \dfrac{d - c}{c}(0.003) = \dfrac{18 - 7.05}{7.05}(0.003) = 0.00465 < 0.005$

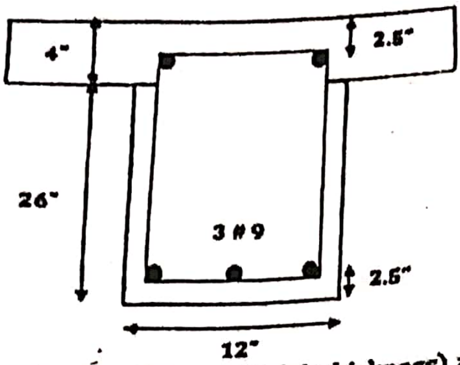
$0.004 < \varepsilon_t < 0.005 \rightarrow$ beam is in transition zone, so $\varphi$ will be less than 0.9

$\varphi = 0.65 + (\varepsilon_t - 0.002)\dfrac{250}{3} = 0.65 + (0.00465 - 0.002)\dfrac{250}{3} = 0.87$

$M_n = A_s f_y \left(d - \dfrac{a}{2}\right) = 4 \times 60\left(18 - \dfrac{5.64}{2}\right) = 3643.2\ k - in$

**Question:** A reinforced concrete T-beam has a cross section in the positive region of span as shown below. The beam has a 30-ft span and is cast integrally with a floor slab that is 4 in. thick. The clear distance between webs is 18 in. Determine the design positive moment capacity. Given $f_c'$ = 3000 psi and $f_y$ = 60000 psi. (GTCL – 2018)

**Solution:**




$b_e \leq$ Web width + 16 (slab thickness) = 12 + 16 x 4 = 76 in.

$b_e \leq$ Average clear distance to adjacent webs + Web width = 18 + 12 = 30 in.

$$b_e \leq \frac{\text{Span length}}{4} = \frac{30 \times 12}{4} = 90 \text{ in.}$$

Effective flange width, $b_e = 30$ in

$A_s = 3 \times 1 = 3 \text{ in}^2$

$$a = \frac{A_s f_y}{0.85 f_c' b_e} = \frac{3 \times 60}{0.85 \times 3 \times 30} = 2.35''$$

$a < h_f$, so rectangular section analysis.

$$c = \frac{a}{\beta_1} = \frac{2.35}{0.85} = 2.76''$$

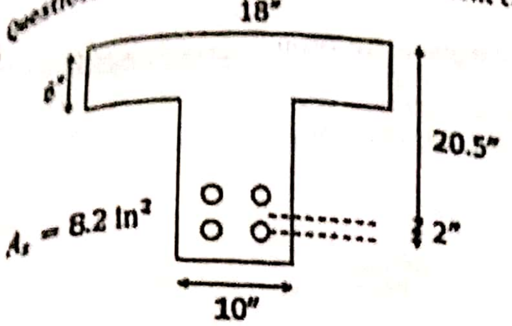
$d = 30'' - 2.5'' = 27.5''$

$$\varepsilon_t = \frac{d-c}{c}(0.003) = \frac{27.5 - 2.76}{2.76}(0.003) = 0.026 > 0.005$$

As $\varepsilon_t > 0.005$, so tension controlled section and $\varphi = 0.9$

$$M_u = \varphi A_s f_y \left(d - \frac{a}{2}\right) = 0.9 \times 3 \times 60 \left(27.5 - \frac{2.35}{2}\right) = 4264.65 \text{ k} - \text{in}$$

Question: Determine the ultimate moment capacity of the T beam in the figure (FED 2019)




Given,
$f'_c$ = 3000 psi
$f_y$ = 40,000 psi

**Solution:**

Effective flange width, $b_e = 18$ in

$$a = \frac{A_s f_y}{0.85 f'_c b_e} = \frac{8.2 \times 40}{0.85 \times 3 \times 18} = 7.14 \text{ in}$$

$a > h_f$, so T − beam analysis is required.

$$A_{sf} = \frac{0.85 f'_c (b_e - b_w) h_f}{f_y} = \frac{0.85 \times 3 (18 - 10) 6}{40} = 3.06 \text{ in}^2$$

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} = \frac{(8.2 - 3.06) 40}{0.85 \times 3 \times 10} = 8.06 \text{ in}$$

$$M_n = M_{n1} + M_{n2}$$

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)$$

$$M_n = 3.06 \times 40 \times \left(20.5 - \frac{6}{2}\right) + (8.2 - 3.06) \times 40 \times \left(20.5 - \frac{8.06}{2}\right) = 5065.63 \text{ k} - \text{in}$$

$\beta_1 = 0.85$ for 3000 psi concrete

$$c = \frac{a}{\beta_1} = \frac{8.06}{0.85} = 9.48 \text{ in}$$

$$\varepsilon_t = \frac{d_t - c}{c} (0.003) = \frac{22.5 - 9.48}{9.48} (0.003) = 0.00412 < 0.005$$

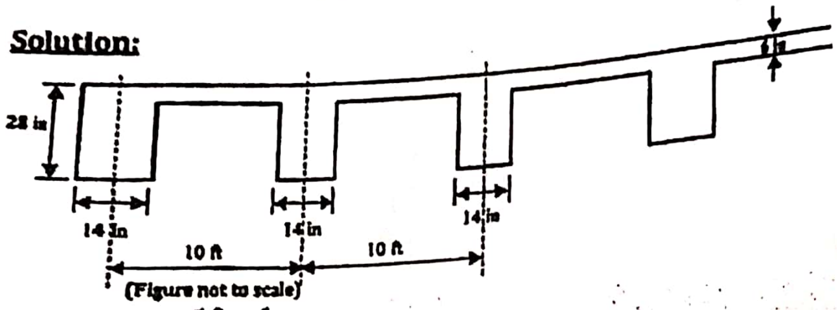
$0.005 > \varepsilon_t > 0.004$, So transition controlled section.

$$\varphi = 0.65 + (\varepsilon_t - 0.002) \frac{250}{3} = 0.65 + (0.0041 - 0.002) \frac{250}{3} = 0.825$$

$$M_u = \varphi M_n = 0.825 \times 5065.63 = 41793.14 \text{ k} - \text{in}$$

**Question:** A concrete floor system consist of parallel T-beam spaced 10 ft on centers and spanning 32 ft between supports. The 6 inch thick slab is poured monolithically with T-beam webs having width $b_w$ =14 inch and total depth measured from the top of the slab of h = 28 inch. In addition to its own weight, each T-beam must carry a superimposed dead load of 50 psf and service live load of 225 psf. Material strength $f_y$ = 60 ksi and $f_c'$ = 3 ksi. Determine the required tensile steel area and needed re-bars for typical member. (33th BCS)

**Solution:**



(Figure not to scale)

$$\text{Slab weight} = \frac{10 \times 6}{12} \times 0.15 = 0.75 \text{ kip/ft}$$

$$\text{Stem weight} = \frac{14 \times 22}{12 \times 12} \times 0.15 = 0.32 \text{ kip/ft}$$

Superimposed dead load = 10 x 0.050 = 0.5 kip/ft

Service live load = 10 x 0.225 = 2.55 kip/ft

Factored load, $W_u = 1.2\,DL + 1.6\,LL$

$W_u = 1.2\,(0.75 + 0.32 + 0.5) + 1.6\,(2.55) = 5.964$ kip/ft

$$\text{Moment, } M_u = \frac{W_u L^2}{8} = \frac{5.964 \times 32^2}{8} = 763.392 \text{ k-ft}$$

Assume an effective depth, d = h − 3 in. = 28 − 3 = 25 inch

$b_e \leq$ Web width + 16 (slab thickness) = 14 + 16 x 6 = 110"

$b_e \leq$ Average clear distance to adjacent webs + Web width = 10 x 12 = 120"

$b_e \leq$ (Span length)/4 = (32 x 12)/4 = 96"

Effective flange width, $b_e$ = 96"

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{763.392 \times 12}{0.9 \times 60 \left(25 - \frac{6}{2}\right)} = 7.71 \text{ in}^2$$ [Assume, $\varphi = 0.9$ & $a = t = 6"$]

$$\text{Check, } a = \frac{A_s f_y}{0.85 f_c' b_e} = \frac{7.71 \times 60}{0.85 \times 3 \times 96} = 1.88" \ll t\,(= 6")$$

Rectangular beam analysis with $b_e$ = 96" will be done.

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)} = \frac{763.392 \times 12}{0.9 \times 60 \left(25 - \frac{1.7}{2}\right)} = 7.02 \text{ in}^2$$ [Assume, $\varphi = 0.9$ & $a = 1.7"$]

Check, $a = \frac{A_s f_y}{0.85 f'_c b_e} = \frac{7.02 \times 60}{0.85 \times 3 \times 96} = 1.72" \rightarrow$ Ok

$c = \frac{a}{\beta_1} = \frac{1.7}{0.85} = 2"$

Check, $\varphi$

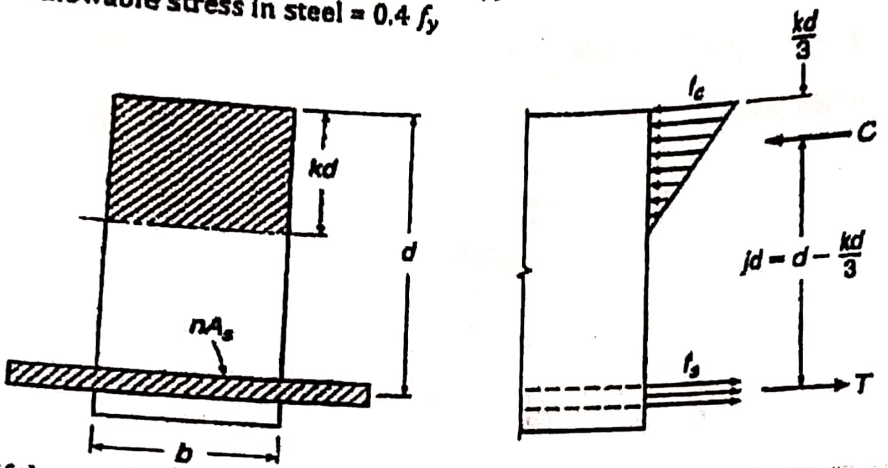
$\varepsilon_t = \frac{d_t - c}{c}(0.003) = \frac{25-2}{2}(0.003) = 0.0345 > 0.005 \rightarrow$ Ok

So, $A_s = 7.02$ in$^2$ **(Use 8#9 bars in two layer)**

**Working Stress Design (WSD) of Beam:**

**A method of proportioning structures or members for prescribed working loads at stresses well below the ultimate, and assuming linear distribution of flexural stresses.**

**Allowable stress in concrete = 0.45 $f'_c$**
**Allowable stress in steel = 0.4 $f_y$**




If the total compression force C and total tension force T then we can write,

C = Area x Average Compressive Stress

$$C = b \times k\,d \times \frac{0 + f_c}{2} = \frac{f_c}{2}\,b\,k\,d \text{ and } T = A_s\,f_s$$

Taking moments about C gives

$$M = T\,j\,d = A_s\,f_s\,j\,d$$

Where $jd$ is the internal lever arm between $C$ and $T$. From the above equation steel stress is·

$$f_s = \frac{M}{A_s\,j\,d}$$

Conversely, taking moment about T gives

$$M = C\,j\,d = \frac{f_c}{2}\,b\,k\,d\,j\,d = \frac{f_c}{2}\,k\,j\,b\,d^2$$

From which the concrete stress is

$$f_c = \frac{2\,M}{k\,j\,b\,d^2}$$

If one is to designing a cross section to resist a given moment, it is often convenient to express k in term of the desired stress

$$\frac{\varepsilon_c}{\varepsilon_s} = \frac{k\,d}{d - k\,d}$$

Substituting, $\varepsilon_c = \frac{f_c}{E_c}$ and $\varepsilon_s = \frac{f_s}{E_s}$

$$\frac{f_c}{E_c}\frac{E_s}{f_s} = \frac{k}{1 - k}$$

ntroducing the modular ratio, $n = \frac{E_s}{E_c}$ and the stress ratio, $r = \frac{f_s}{f_c}$ and $\frac{n}{r} = \frac{k}{1 - k}$

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And solving for k

$k = \frac{n}{n+r}$

For balance stress design

$\frac{f_c}{2} b\,k\,d = \rho\, f_s\, b\, d$

Dividing both sides by $bd$ and solving for $\rho$, which is then designated as $\rho_e$,

$\rho_e = \frac{k}{2}\frac{f_c}{f_s} = \frac{k}{2r}$

$\rho_e = \frac{n}{2r\,(n+r)}$

The ratio $\rho_e$ is called the balanced stress steel ratio. If $\rho < \rho_e$ , the steel will reach its allowable stresses at a lower load than the concrete. If $\rho > \rho_e$, then concrete will reach its allowable stress first.

It is convenient to have equations by which k and j may be found directly for analysis case. To determine the location of neutral axis, the moment of the tension is about the axis is set equal to the moment of the compression area which gives

$$b\frac{(kd)^2}{2} - nA_s(d - kd) = 0$$

$$\therefore k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

n = ratio of modulus of elasticity of steel to that of concrete $= \frac{E_s}{E_c}$

$E_s$ = modulus of elasticity of steel

$E_c$ = modulus of elasticity of concrete

$f_c$ = allowable compressive stress of concrete

$f_s$ = allowable tensile stress of steel reinforcement

$f_c'$ = specified compressive strength of concrete

$f_y$ = specified yield strength of steel reinforcement

b = the width of the rectangular beam

d = distance from extreme concrete fiber to centroid of steel reinforcement

kd = distance from the neutral axis to the extreme fiber of concrete

jd = distance between compressive force C and tensile force T

$\rho$ = ratio of the area of steel to the effective area of concrete, $\rho = \frac{A_{st}}{A_g} = \frac{A_{st}}{bd}$

k = ratio of distance of the neural axis of the cross section from extreme fibers in compression to the effective depth of the beam

j = ratio of the distance between the resultant of the compressive stresses and centre of the tensile stresses to d, the effective depth of the beam.

## Analysis versus Design

Two different types of problems arise in the study of reinforced concrete:

**Analysis:** Given cross section, concrete strength, reinforcement size, location, and yield strength, compute resistance or capacity.

**Design:** Given factored load effect such as $M_u$, select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. Although this general referred to as a "Design Problem," a better description might be proportioning.

Formulas to be used in solving problems

| *Analysis type problem* | *Design type problem* |
| --- | --- |
| allowable $f_c = 0.45 f_c'$ | allowable $f_c = 0.45 f_c'$ |
| allowable $f_s = 0.45 f_y$ | allowable $f_s = 0.45 f_y$ |
| Reinforcement ratio, $\rho = \frac{A_s}{b\,d}$ | Calculation of Moment |
| $k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$ | $k = \frac{n}{n+r}$ where, $n = \frac{E_s}{E_c}$ & $r = \frac{f_s}{f_c}$ |
| $j = 1 - \frac{k}{3}$ | $j = 1 - \frac{k}{3}$ |
| $M = A_s f_s j d$ | $M = \frac{f_c}{2} k j b d^2$ for finding out b |
| $M = \frac{f_c}{2} k j b d^2$ | $M = A_s f_s j d$ for finding out $A_s$ |

## Doubly Reinforced Rectangular Beams

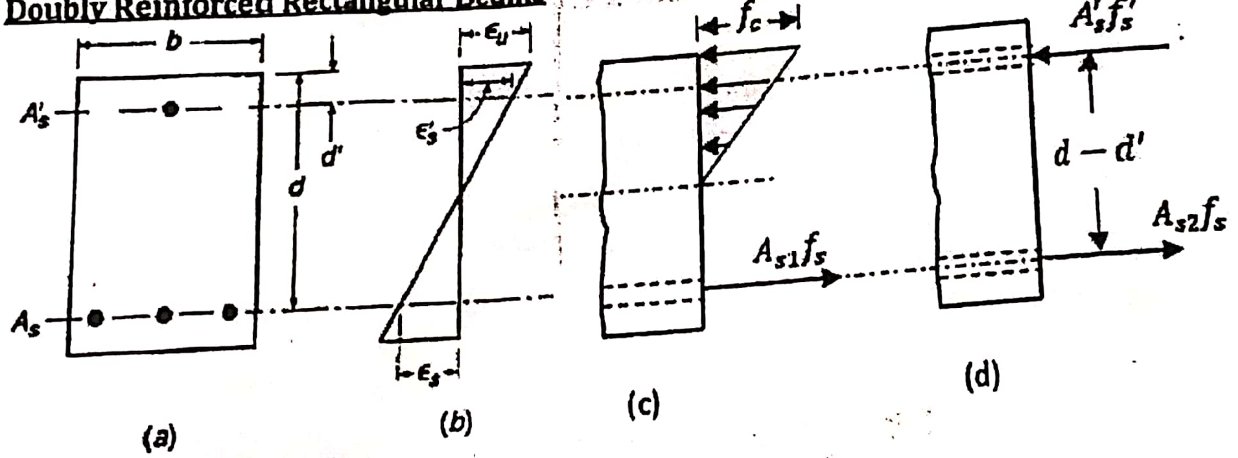



Fig: Doubly reinforced rectangular beam

A rectangular-beam cross section is shown with compression steel $A_s'$ placed at a distance $d'$ from the compression face and tensile steel $A_s$ at an effective depth $d$. The stress in the tensile steel is $f_s$, while that in compression is $f_s'$. It is a design convenience to divide the total resisting moment into two parts.

If $M$ is the total resisting moment and

$M_1$ = moment that can be developed by a given cross section of concrete without compression reinforcement working with a partial steel area $A_{s1}$ to balance concrete compression.
$M_2$ = moment in excess of $M_1$, developed by compression reinforcement, acting with additional tensile area $A_{s2}$

$M = M_1 + M_2$ and $A_s = A_{s1} + A_{s2}$

The moment $M_1$ depends on the concrete dimensions and allowable stresses is

$$M_1 = \frac{f_c}{2} kj\,b\,d^2$$

The area of tensile steel required for balanced-stress of a singly reinforced beam having a resisting moment $M_1$ is

$$A_{s1} = \frac{M_1}{f_s\,j\,d}$$

A moment $M_2$ is provided by compression steel and the remaining tensile steel $A_{s2} = A_s - A_{s1}$, acting with an internal lever arm $(d - d')$:

$$M_2 = A_{s2}\,f_s\,(d - d') = A_s'\,f_s'(d - d')$$

The additional moment $M_2$ may be govern by either the tensile or compressive steel, depending on the stresses in each and their relative areas. From the geometry

$$\frac{\varepsilon_s}{\varepsilon_s'} = \frac{d - kd}{kd - d'} = \frac{f_s}{f_s'}$$

$$f_s' = f_s \frac{k - d'/d}{1 - k}$$

However, the ACI code recommends that $f_s'$ be taken equal to twice this value, but not greater then $f_s$.

**Question:** A rectangular beam has a width of 12 in. and an effective depth to the centroid of the reinforcing steel of 17.5 in. It is reinforced with three No. 8 bars in one row. Find out the maximum working moment that can be resisted by the section if $f_c' = 4000$ psi, $f_y = 60{,}000$ psi.

**Solution:**

$f_c = 0.45 f_c' = 0.45 \times 4000 = 1800$ psi

$f_s = 0.45 f_y = 0.40 \times 60000 = 24000$ psi

Reinforcement ratio, $\rho = \frac{A_s}{b\,d} = \frac{3 \times 0.69}{12 \times 17.5} = 0.0112$

$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n = \sqrt{(0.0112 \times 8)^2 + 2 \times 0.0112 \times 8} - 0.0112 \times 8 = 0.343$.

$j = 1 - \frac{k}{3} = 1 - \frac{0.343}{3} = 0.886$

If the steel stress controls,

$M_s = A_s f_s j d = 2.35 \times 24 \times 0.886 \times 17.5 = 875$ kip-in

If the concrete stress controls,

$M_c = \frac{f_c}{2} k j b d^2 = \frac{1.8}{2} \times 0.343 \times 0.886 \times 12 \times 17.5^2 = 1010$ kip-in

The allowable moment capacity of the beam is controlled by allowable steel stress in this case.

$M = 875$ kip-in

Alternately, this problem could be solved by calculating the balanced-stress steel ratio and comparing the actual steel ratio with that value. For example,

$\rho_e = \frac{n}{2r(n+r)} = \frac{8}{2\,(24/1.80)\,(8 + 24/1.80)} = 0.0141$

Since $\rho = 0.0112 < \rho_e$ the steel stress control, and M = 875 in.-kips as before.

**Question:** A rectangular beam has a width of 12 in. and an effective depth to the centroid of the reinforcing steel of 17.5 in. It is reinforced with 3-20 mm bars in one row. Find out the allowable working moment that can be resisted by the section if $f_s = 24000$ psi, $f_c = 1200$ psi. (WASA – 2017, PGCL – 2017, MPA – 2019)

**Solution:**

Total steel area, $A_s = 3 \times \frac{\pi}{4}\left(\frac{20}{25.4}\right)^2 = 1.46\ \text{in}^2$

Reinforcement ratio, $\rho = \frac{A_s}{b\,d} = \frac{1.46}{12 \times 17.5} = 0.0069$

$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2 \times 0.0069 \times 9 + (0.0069 \times 9)^2} - (0.0069 \times 9) = 0.295$

$j = 1 - \frac{k}{3} = 0.90$

*If the steel stress controls,*

$M_s = A_s f_s j d = 1.46 \times 24 \times 0.9 \times 17.5 = 552.75$ kip-in

*If the concrete stress controls,*

$M_c = \frac{f_c}{2} k j b d^2 = \frac{1.2}{2} \times 0.295 \times 0.9 \times 12 \times 17.5^2 = 585.42$ kip-in

Allowable working moment = 552.75 k-in.

**Question: A simply supported beam of 10" x 18" section is a span length of 18' and distributed load is 1355 plf in additional of its self-weight. If $f_c' = 3000$ psi and $f_s = 20000$ psi, then calculate the required steel area and draw the long and cross section of the beam. d' = 2.5". (BHP – 2017)**

**Solution:**

$f_c = 0.45 f_c' = 0.45 \times 3000 = 1350$ psi

$$\text{Self-weight} = \frac{10}{12} \times \frac{18}{12} \times 150 = 187.5 \text{ plf}$$

Total load, $W = 1355 + 187.5 = 1542.5$ plf $= 1.5425$ klf

$$\text{Moment, } M_u = \frac{WL^2}{8} = \frac{1542.5 \times 18^2}{8} = 62.47 \text{ k-ft} = 749.64 \text{ k-in}$$

$$r = \frac{f_s}{f_c} = \frac{20}{1.35} = 14.81$$

$$k = \frac{n}{n+r} = \frac{9}{9 + 14.81} = 0.37$$

$$j = 1 - \frac{k}{3} = 0.87$$

Now, $M = A_s f_s j d$

$$A_s = \frac{M}{f_s j d} = \frac{749.64}{20 \times 0.87 \times 15.5} = 2.87 \text{ in}^2$$

Provided three No. 9 bars all in a row (Total $A_s = 3.00$ in²)

**Question:** Design a simply supported rectangular beam for a bending moment of 1440 k-in. Concrete strength, $f_c' = 3{,}500$ psi, steel strength, $f_y = 60{,}000$ psi, modular ratio, $n = 9$. Design the beam using WSD. (31th BCS)

**Solution:**

Maximum moment carried by the beam, $M_c = M_s = 1440$ kip – in

Allowable stress in concrete, $f_c = 0.45 f_c' = 0.45 \times 3.5 = 1.575$ ksi

Allowable tensile stress in steel, $f_s = 0.4 f_y = 0.40 \times 60 = 24$ ksi

Modular ratio, $n = 9$

$$\text{Stress ratio, } r = \frac{f_s}{f_c} = 15.24$$

$k = \frac{n}{n+r} = 0.37, j = 1 - \frac{k}{3} = 0.88$

Moment carried by concrete, $M_c = \frac{1}{2} f_c j\, k\, b\, d^2$

Or, $1440 = 0.5 \times 1.575 \times 0.88 \times 0.37 \times b \times (2b)^2$ [Assume, $d = 2b$]

$\therefore$ width of beam, $b = 11.2'' \cong 12''$

Depth of beam, $d = 2b = 24''$

Total depth, $h = d + 2'' = 24'' + 2'' = 26''$

Moment carried by steel, $M_s = A_s f_s j\, d$

Or, $1440 = A_s \times 24 \times 0.88 \times 24$

$\therefore A_s = 2.84 \text{ in}^2$

Use 4 $\neq$ 8 bar

**Question: A rectangular beam of 12" x 24" section must carry a uniformly distributed live load 680 plf and support the dead load of a wall weighing 380 plf in addition to its own self weight, on a simple span of 24 ft. Find out the area of steel required, using intermediate grade steel at a working stress of 20,000 psi and 3000 psi concrete at a working stress of 1350 psi. d' = 2.5". (BKB – 2018)**

**Solution:**

Self – weight of beam $= \frac{12 \times 24}{144} \times 150 = 300$ plf

Total working load, W = 300 + 380 + 680 = 1360 plf = 1.36 klf

Maximum moment, $M = \frac{W L^2}{8} = \frac{1.36 \times 24^2}{8} = 98.2$ k – ft

$r = \frac{f_s}{f_c} = \frac{20000}{1350} = 14.81$

$k = \frac{n}{n+r} = \frac{9}{9 + 14.81} = 0.378$

$j = 1 - \frac{k}{3} = 0.874$

Now, $M = A_s f_s j\, d$

$A_s = \frac{M}{f_s j\, d} = \frac{98.2 \times 12}{20 \times 0.874 \times 21.5} = 3.13 \text{ in}^2$

Provided four No. 8 bars all in a row (Total $A_s = 3.16 \text{ in}^2$)

**Question:** A rectangular beam must carry a distributed load of 680 plf and support the dead load of a wall weighing 380 plf, in addition to its own weight, on a simple span of 24 ft. Design the beam for flexure, using intermediate grade steel at a working stress of 20,000 psi and 3000 psi concrete at a working stress of 1350 psi.

**Solution:**

Given, $f_s = 20000$ psi and $f_c = 1350$ psi

The total load acting on the beam includes its own weight, which must be estimated. As a trial a total depth equal to 1 in. per ft of span and a width of one-half that amount are assumed.

The beam weight is estimated as $= \frac{12 \times 24 \times 150}{144} = 300$ plf

The total working load $= 680 + 380 + 300 = 1360$ plf

Maximum moment, $M = \frac{wl^2}{8} = \frac{1.36 \times 24^2}{8} = 98.2$ kip-ft $= 1178.4$ kip-in

We will choose $\rho = \rho_{b}$, such that steel and concrete reach their maximum allowable stresses for the same applied moment. For given materials, with $\rho = \rho_{b}$,

$$k = \frac{n}{n+r} = \frac{9}{9 + 20/1.35} = 0.378$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.378}{3} = 0.874$$

Equating the concrete resisting moment to the total working moment of 1178.4 kip-in.

$$M = \frac{f_c}{2} kjb\,d^2$$

$$bd^2 = \frac{2M}{f_c k j} = \frac{2 \times 1178.4}{1.35 \times 0.378 \times 0.874} = 5284.28 \text{ in}^2$$

Assuming $d = 20$ inch

$b = 5284/400 = 13.21$ in

And $M = A_s f_s j d$

$$A_s = \frac{M}{f_s j d} = \frac{1178.4}{20 \times 0.874 \times 20} = 3.38 \text{ in}^2$$

The area will be provided using two No. 10 bars and one No. 9 bars, all in a row. Concrete dimension will be rounded off to $b = 13.5$ in. and $d = 20$in. with a total depth of 22.5 in.

**Question:** A rectangular beam has a width of 12 in. and an effective depth to the centroid of the tensile reinforcement of 18 in. the tensile reinforcement consists of four No. 10 bars. Compressive reinforcement is provided by three No. 8 bars, the centroid of which is 2.5 in. from the compression face of the beam. If 60,000 psi yield steel is used at a working stress $f_s = 24{,}000$ psi and $f_c = 1800$ psi, what is the allowable working moment that the beam can resist?

**Solution:**

For the balance stress beam of 12 in. width and 18 in. effective depth.

$$k = \frac{n}{n+r} = \frac{8}{8+24/1.8} = 0.375$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.375}{3} = 0.875$$

The moment which could be resisted by the beam if singly reinforced with $\rho = \rho_e$, is

$$M_1 = \frac{f_c}{2} k\, j\, b\, d^2 = \frac{1.8}{2} 0.375 \times 0.875 \times 12 \times 18^2 = 1150 \text{ in.-kips}$$

And the corresponding steel area is

$$A_{s1} = \frac{M_1}{f_s\, j\, d} = \frac{1150}{24 \times 0.875 \times 18} = 3.05 \text{ in}^2$$

$$A_{s2} = A_s - A_{s1} = 5.06 - 3.05 = 2.01 \text{ in}^2$$

The stress in the compressive steel is to be taken as twice the modular value, but not to exceed the tensile steel stress. Thus,

$$f_s' = 2\, f_s \frac{k - d'/d}{1-k} = 2 \times 24 \frac{0.375 - 2.5/18}{1 - 0.375} = 18.1 \text{ ksi}$$

Since the value is less than $f_s = 24.0$, then compressive steel is assumed to be stressed to 18.1 ksi.

$$A_{s2}\, f_s = 2.01 \times 24.0 = 48.2 \text{ kips}$$

$$A_s'\, f_s' = 2.35 \times 18.1 = 42.5 \text{ kips}$$

The smaller of these two values controls, and

$$M_2 = A_s' f_s'(d - d') = 42.5 \times 15.5 = 660 \text{ in.-kips}$$

The total resisting moment is then the sum of $M_1$ and $M_2$:

$$M = M_1 + M_2 = 1150 + 660 = 1810 \text{ in.-kips}$$

**Question:** A rectangular beam is limited by architectural consideration to a width of 13 in. and a total depth of 24 in. It must resist a total working moment of 2530 in.-kips. If $f_s = 20{,}000$ psi and $f_c' = 4000$ psi, what reinforcement is required for flexural design?

**Solution:**

Under the assumption that two layer of tensile reinforcement will be required, 4 inch will be allowed from the bottom face to the tensile steel centroid. Compressive steel, if required will be located 3 inch from the compression beam face. For $\rho = \rho_e$, the maximum allowable resisting moment of the singly reinforced section is determined: with $f_c' = 4000$ psi, according to ACI Code, $f_c = 1800$ psi, and $n = 8$

$$k = \frac{n}{n+r} = \frac{8}{8+20/1.8} = 0.418$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.418}{3} = 0.861$$

$$M = \frac{f_c}{2} k\, j\, b\, d^2 = \frac{1.8}{2} 0.418 \times 0.861 \times 13 \times 20^2 = 1690 \text{ kip} - \text{in}$$

This is substantially less than the design moment of 2530 kip – in, indicating the necessity for compressive steel. With $M_1 = 1690$ kip – in

$$A_{s1} = \frac{M_1}{f_s\, j\, d} = \frac{1690}{20 \times 0.861 \times 20} = 4.90 \text{ in}^2$$

$$M_2 = M - M_1 = 2530 - 1690 = 840 \text{ kip} - \text{in}$$

$$A_{s2} = \frac{M_2}{f_s(d - d')} = \frac{840}{20 \times 17} = 2.47 \text{ in}^2$$

$$A_s = A_{s1} + A_{s2} = 4.90 + 2.47 = 7.37 \text{ in}^2$$

The stress in the compressive steel is then found:

$$f_s' = 2\, f_s \frac{k - d'/d}{1 - k} = 2 \times 20 \frac{0.418 - 3/20}{1 - 0.418} = 18.4 \text{ ksi}$$

But $f_s'$ is not to exceed the stress in the tensile steel. In this case, $f_s' = 18.4$ ksi, and

$$A_s' = \frac{M_2}{f_s'(d - d')} = \frac{840}{18.4 \times 17} = 2.68 \text{ in}^2$$

The required tensile steel area of 7.37 in² will be provided by six No. 10 bars placed in two layers. The compression steel requirement is met by one No. 8 and two No. 9 bars in one row.

## Shear Reinforcement

Shear failure of reinforced concrete, more properly called *diagonal tension failure*, is one of the causes of sudden collapse of beam. Shear failure is difficult to predict accurately. Furthermore, if a beam without properly designed shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly, with no advance warning of distress. This is in strong contrast with the nature of flexural failure.

The design for shear consists of calculating the maximum factored shear force $V_u$ and requiring that it is equal to or less than the design shear strength $\varphi V_n$:

$$V_u \leq \varphi V_n$$

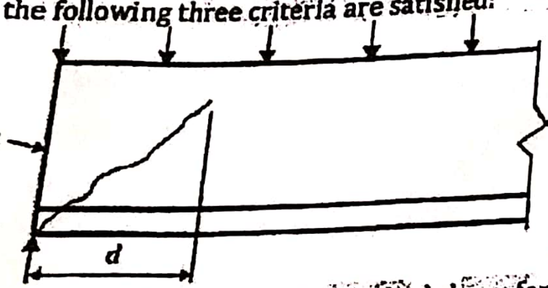
The factored shear force $V_u$ is determined by combining the nominal shear forces determined from analysis in accordance with the load combinations given in ACI 9.2. According to ACI, the strength reduction factor $\varphi$ is equal to 0.75 for shear.

The nominal shear strength $V_n$ is determined by ACI

$$V_n = V_c + V_s$$

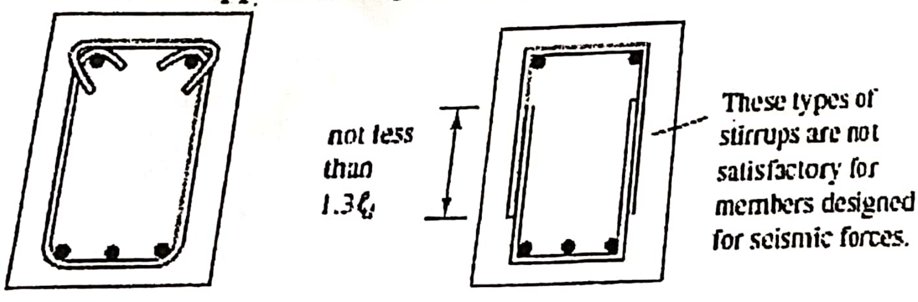
In this equation, $V_c$ is the nominal shear strength provided by concrete and $V_s$ is the nominal shear strength provided by shear reinforcement.

The locations of the critical sections for shear are given in ACI 11.1.3. It is permitted to design sections located less than a distance $d$ from the face of a support for $V_u$ computed at the distance $d$, provided the following three criteria are satisfied:

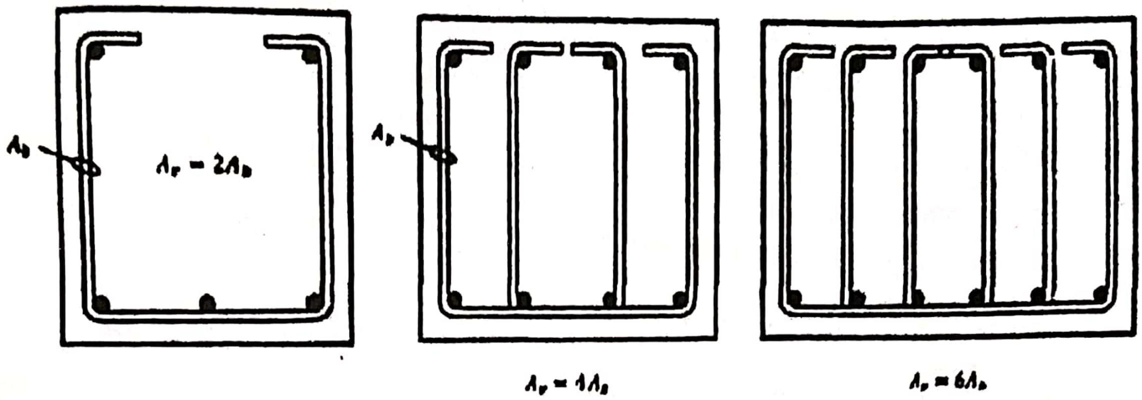



Support reactions in the direction of the applied shear force introduce compression into the end regions of the member. Loads are applied at or near the top of the member. No concentrated loads occur between the face of the support and a distance $d$ from the face of the support.

It has been shown from numerous experiments that the inclined shear crack that is closest to the support will extend upward from the face of the support, reaching the compression zone at a distance of approximately $d$ from the face of the support.




Types of stirrups.




Multiple-legged stirrup configurations.

**Question:** What do you mean by "stirrup"? Write down the purpose of shear/web Reinforcement? (BEPZA – 2019)

**Solution:**

**Stirrup:** A stirrup is a closed loop of reinforcement bar that is used to hold the main reinforcement bars together in an RCC structure. In a column, the stirrups provide the lateral support to the main bars against buckling.

**Purpose of shear/web Reinforcement:**

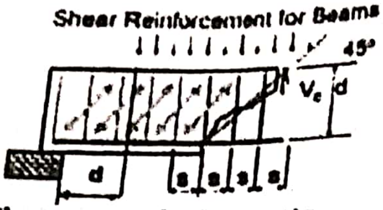
Shear reinforcement is provided to serve following purposes;

- to provide ductility to section i.e. to prevent sudden formation of diagonal crack in an unreinforced web, possibly leading to abrupt failures
- to resist any torsion
- to restrain the growth of inclined shear cracks
- to improve dowel action of longitudinal bars

**Question:** Why does ACI/BNBC code recommend a relatively smaller value of shear as design shear that occur at a distance d from the support face? (30th BCS)

**Solution:**

A diagonal crack that might result in shear failure, can form no closer to the face of the support than the distance d from the face of the support. Consequently, Section 11.1.3.1 of ACI 318-05 permits the maximum required value of shear Vu to be determined at a distance d from the face of such a support when the support provides compression resistance at the face of the beam opposite the loading face. If loads had been suspended from the bottom of the beam or if the support were no deeper than the beam itself, maximum required shear must be taken as the shear at the face of the support.




**Shear strength of concrete**

The shear strength provided by the concrete

$V_c = 2\lambda\sqrt{f_c'}\,b_w\,d$

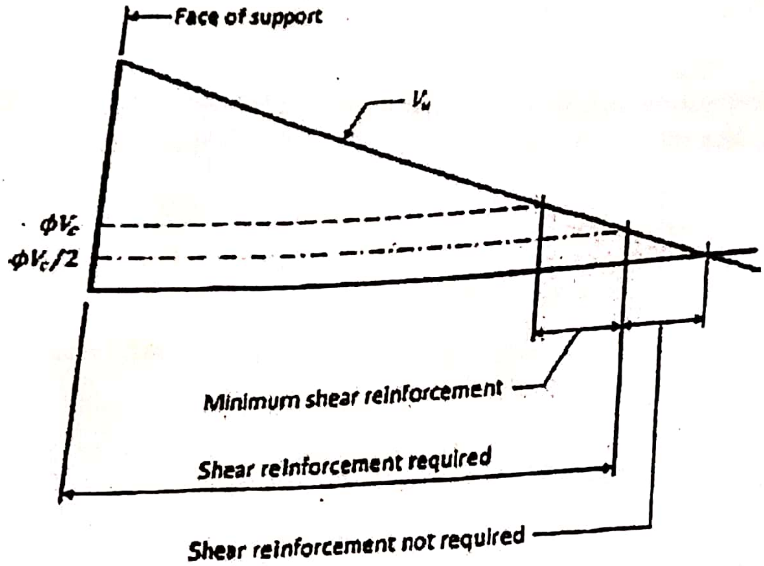
$\lambda = 0.75$ for all − lightweight concrete.

$\lambda = 1.0$ for normal − weight concrete.

Design shear strength of concrete

$\varphi V_c = \varphi\, 2\lambda\sqrt{f_c'}\,b_w\,d$

$\varphi = 0.75$ reduction factor for shear




***Case 1:*** $V_u \le \varphi V_c/2$

No shear reinforcement is required.

***Case 2:*** $\varphi V_c/2 \le V_u \le \varphi V_c$

Minimum area of shear reinforcement is required.

$$A_{v,min} = \frac{0.75\sqrt{f_c'}\,b_w\,s}{f_{yt}} \ge \frac{50\,b_w\,s}{f_{yt}}$$

Maximum spacing to provide minimum $Av$

$$s = \frac{A_v\,f_{yt}}{0.75\sqrt{f_c'}\,b_w} \le \frac{A_v\,f_{yt}}{50\,b_w}$$

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*Case* 3: $V_u \geq \varphi V_c$

$\varphi V_c + \varphi V_s = V_u$

Nominal shear strength of shear reinforcement, $V_s = \dfrac{V_u - \varphi V_c}{\varphi}$

Theoretical stirrup spacing, $s = \dfrac{A_v f_{yt} d}{V_s}$

If $V_s \leq 4\sqrt{f_c'}\, b_w d$, maximum spacing $= d/2 \leq 24$

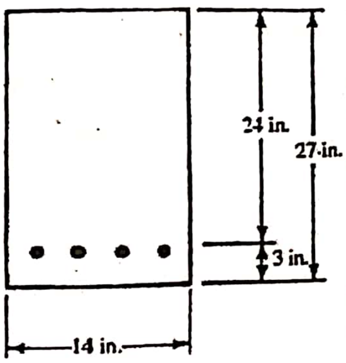
If $V_s > 4\sqrt{f_c'}\, b_w d$, maximum spacing $= d/4 \leq 12$

If $V_s > 8\sqrt{f_c'}\, b_w d$, need larger beam

**Question:** The beam shown in Figure, was selected using $f_y = 60{,}000$ psi and $f_c' = 3000$ psi, normal weight. Determine the theoretical spacing of #3 stirrups for each of the following shears:

(*a*) $V_u = 12{,}000\ lb.$
(*b*) $V_u = 40{,}000\ lb.$
(*c*) $V_u = 60{,}000\ lb.$
(*d*) $V_u = 150{,}000\ lb.$

**Solution:**




***When*** $V_u = 12000\ lb.$

Using $\lambda = 1.0$ for normal weight concrete
Shear strength provided by the concrete, $\varphi V_c = \varphi\, 2\, \lambda \sqrt{f_c'}\, b_w d$

$\varphi V_c = 0.75 \times 2 \times 1 \times \sqrt{3000} \times 14 \times 24 = 27605$ lb.

$\dfrac{\varphi V}{2} = 13803$ lb. $> 12000$ lb.

Stirrups not required.

***When*** $V_u = 40000\ lb.$

Stirrups needed because $V_u > \dfrac{\varphi V_c}{2}$

Theoretical spacing, $\varphi V_c + \varphi V_s = V_u$

$$V_s = \frac{V_u - \varphi V_c}{\varphi} = \frac{40000 - 27605}{0.75} = 16527 \text{ lb}$$

$$s = \frac{A_v f_{yt} d}{V_s} = \frac{2 \times 0.11 \times 60000 \times 24}{16527} = 19.17 \text{ in}$$

Maximum spacing to provide minimum $A_v$

$$s = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} = \frac{2 \times 0.11 \times 60000}{0.75\sqrt{3000} \times 14} = 22.95 \text{ in}$$

$$s = \frac{A_v f_{yt}}{50\, b_w} = \frac{2 \times 0.11 \times 60000}{50 \times 14} = 18.86 \text{ in}$$

$V_s = 16527 \text{ lb} < 4\sqrt{f_c'}\, b_w\, d = 4 \times \sqrt{3000} \times 14 \times 24 = 73614 \text{ lb}$

Maximum spacing $= \frac{d}{2} = 12$ in

Stirrups spacing, $s = 12$ in

***When*** $V_u = 60000$ ***lb.***

$$V_s = \frac{V_u - \varphi V_c}{\varphi} = \frac{60000 - 27605}{0.75} = 43193 \text{ lb}$$

$$s = \frac{A_v f_{yt} d}{V_s} = \frac{2 \times 0.11 \times 60000 \times 24}{43193} = 7.33 \text{ in}$$

Maximum spacing to provide minimum $A_v$

$$s = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} = \frac{2 \times 0.11 \times 60000}{0.75\sqrt{3000} \times 14} = 22.95 \text{ in}$$

$$s = \frac{A_v f_{yt}}{50\, b_w} = \frac{2 \times 0.11 \times 60000}{50 \times 14} = 18.86 \text{ in}$$

$V_s = 43193 \text{ lb} < 4\sqrt{f_c'}\, b_w\, d = 4 \times \sqrt{3000} \times 14 \times 24 = 73614 \text{ lb}$

Maximum spacing $= \frac{d}{2} = 12$ in

Stirrups spacing, $s = 7.33$ in

***When*** $V_u = 150000$ ***lb.***

$$V_s = \frac{V_u - \varphi V_c}{\varphi} = \frac{150000 - 27605}{0.75} = 163193 \text{ lb}$$

$V_s = 163193 \text{ lb} > 8\sqrt{f_c'}\, b_w\, d = 8 \times \sqrt{3000} \times 14 \times 24 = 147228 \text{ lb}$

$V_s$ may not be taken $> 8\sqrt{f_c'}\, b_w\, d$

Need larger beam or one with larger $f_c'$ value.



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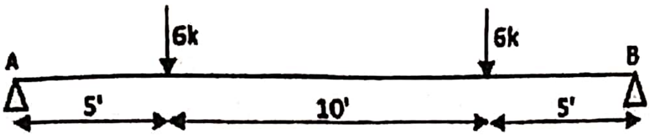
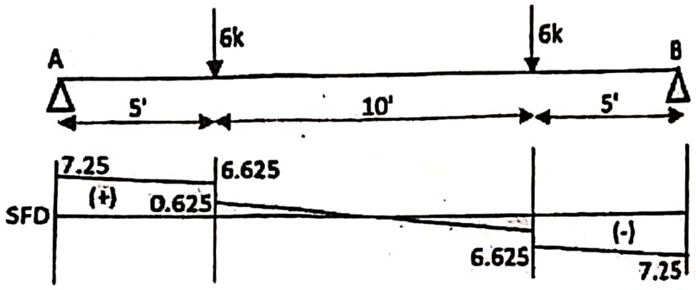
**Question:** A rectangular beam of size 10" x 12". Design the shear reinforcement of the below beam if $f_y = 60$ ksi and $f_c' = 3000$ psi. (BPDB – 2015)




**Solution:**




Beam self – weight $= \frac{12}{12} \times \frac{10}{12} \times 0.15 = 0.125$ k/ft

Taking moment at A, $M_A = 0$

$6 \times 5 + 6 \times 15 + 0.125 \times 20 \times 10 - R_B \times 20 = 0$

$R_A = R_B = 7.25$ k

$V_u$ at support $= 7.25$ k

$V_u$ at d distance, $V_u = 7.25 - \frac{10.5}{12} \times 0.125 = 7.14$ k

$\varphi V_c = 2 \varphi \lambda \sqrt{f_c'}\, b_w\, d = 2 \times 0.75 \times 1 \times \sqrt{3000} \times 10 \times (12 - 1.5) = 8.62$ kip

$\frac{\varphi V_c}{2} = 4.31$ k

$V_u > \frac{\varphi V_c}{2}$ and $V_u < \varphi V_c$, so minimum shear reinforcement is required.

$S_2 = \frac{d}{2} = \frac{10.5}{2} = 5.25$ in

$S_3 = \frac{A_v f_y}{50 b_w} = \frac{0.22 \times 60000}{50 \times 10} = 26.4$ in

$S_4 = 24$ in

Use 10 mm bar @ 5.25 inch c/c

**Question:** Design stirrup of a beam which size is 21" x 15" and 10#10 bar used in this beam. Given, $V_u$ = 40000 lb, $f_y$ = 60 ksi and $f_c'$ = 4 ksi and clear cover is 3 inch. (EGCB – 2015)

**Solution:**

Here, $V_u = 40000$ lb.

$\varphi V_c = \varphi\, 2\,\lambda \sqrt{f_c'}\, b_w\, d = 0.75 \times 2 \times 1 \times \sqrt{4000} \times 15 \times 18 = 25615$ lb

$\frac{\varphi V_c}{2} = 12807.5$ lb

$V_u > \frac{\varphi V_c}{2}$, so stirrup needed.

Theoretical spacing, $\varphi V_c + \varphi V_s = V_u$

$$V_s = \frac{V_u - \varphi V_c}{\varphi} = \frac{40000 - 25615}{0.75} = 19180 \text{ lb} < 8\sqrt{f_c'}\, b_w\, d = 136610 \text{ lb} \rightarrow \text{(ok)}$$

$$s = \frac{A_v f_{yt} d}{V_s} = \frac{2 \times 0.11 \times 60000 \times 18}{19180} = 12.4 \text{ in}$$

Maximum spacing to provide minimum $A_v$

$$s = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} = \frac{2 \times 0.11 \times 60000}{0.75\sqrt{4000} \times 15} = 18.55 \text{ in}$$

$$s = \frac{A_v f_{yt}}{50\, b_w} = \frac{2 \times 0.11 \times 60000}{50 \times 15} = 17.6 \text{ in}$$

$V_s = 19180 \text{ lb} < 4\sqrt{f_c'}\, b_w\, d = 4 \times \sqrt{4000} \times 15 \times 18 = 68305$ lb

$$\text{Maximum spacing} = \frac{d}{2} = \frac{18}{2} = 9 \text{ in}$$

Provide 10 mm bar @ 9 in c/c

**Question:** Determine the minimum cross section required for a rectangular beam from a shear standpoint so that no web reinforcing is required by the ACI code if $V_u$ = 38 kip and $f_c'$ = 4000 psi. Use the conservative value of $V_c = 2\,\lambda\sqrt{f_c'}\, b_w\, d$

**Solution:**

Shear strength provided by the concrete, $\varphi V_c = \varphi\, 2\,\lambda\sqrt{f_c'}\, b_w\, d$

Web reinforcement is not required if $V_u < \frac{\varphi V_c}{2}$

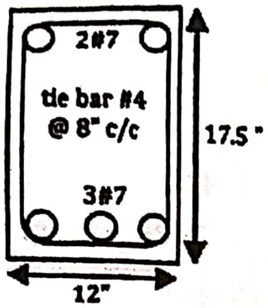
$$\text{Thus, } V_u = \frac{\varphi V_c}{2} = \frac{1}{2}\varphi\,(2\,\lambda\sqrt{f_c'}\, b_w\, d)$$

$$38{,}000 = \frac{0.75 \times 2 \times 1\sqrt{4000}\; b_w\, d}{2}$$

$b_w\, d = 801.1 \text{ in}^2$

Use 24" x 36" beam ($d$ = 33.5 in)

**Question:** Determine ultimate shear capacity of the beam if $f'_c = 3$ ksi and $f_y = 60$ ksi. (BCMCL – 2020)




**Solution:**

$\varphi V_c = 2\varphi\lambda\sqrt{f'_c}\, b_w d$

$\varphi V_c = 2 \times 0.75 \times 1 \times \sqrt{3000} \times 12 \times 17.5 = 17253.26 \text{ lb} = 17.25 \text{ kip}$

Spacing, $S = \dfrac{\varphi A_v f_y d}{V_u - \varphi V_c}$

$8 = \dfrac{0.75 \times (2 \times 0.2) \times 60 \times 17.5}{V_u - 17.25}$

$V_u = 56.625$ kip

**Question:** A RCC beam is to be designed to carry a shear force $V_u$ of 30k. No web reinforcement is to be used and $f'_c = 4$ ksi. What is the minimum cross section if controlled by shear? (BB AD – 2018)

**Solution:**

If no web reinforcement is to be used, the cross sectional dimensions must be selected so that the applied shear $V_u$ is no larger than one-half the design shear strength $\varphi V_c$

Web reinforcement is not required if $V_u < \dfrac{\varphi V_c}{2}$

Thus, $V_u = \dfrac{1}{2}\varphi\,(2\lambda\sqrt{f'_c}\, b_w d)$

$b_w d = \dfrac{30{,}000}{0.75 \times 1 \times \sqrt{4000}} = 632.45 \text{ in}^2$

Consider width of the beam, $b_w = 18$ in

depth of the beam $d = 35.13 \approx 35.5$ in and $h = 35.5 + 2.5 = 38$ in

Check: $\varphi V_c = 2\varphi\lambda\sqrt{f'_c}\, b_w d = 2 \times 0.75 \times 1 \times \sqrt{4000} \times 35.5 \times 18 = 60620 \text{ lb} = 60.62 \text{ kip}$

$\dfrac{\varphi V_c}{2} = 30.31 > V_u$ (Web reinforcement is not required)

Minimum cross section controlled by shear is 18" x 38"

**Question:** A rectangular beam having $b = 12''$ and $d = 22''$ spans 20′ face to face of simple supports. It has to carry service dead load 1.63 k/ft and service live load of 3.10 k/ft, both uniformly distributed along the span. Design the shear reinforcement using #3 bar as vertical U stirrups. Material strengths are $f_c' = 4000$ psi and $f_y = 60000$ psi. (32th BCS)

**Solution:**

Factored uniform load $= 1.2\,DL + 1.66\,LL = 1.2 \times 1.63 + 1.6 \times 3.10 = 6.916$ k/ft

$V_u$ (at face of support) $= \dfrac{6.916 \times 20}{2} = 69.16$ kip

Design, $V_u$ (at d distance from the face of support) $= 69.16 - \dfrac{6.916 \times 22}{12} = 56.48$ kip

$$\varphi V_c = \varphi\, 2\, \lambda \sqrt{f_c'}\, b_w\, d = \frac{0.75 \times 2 \times 1 \times \sqrt{4000} \times 12 \times 22}{1000} = 25.045 \text{ kip}$$

$V_u > \dfrac{\varphi V_c}{2}$, so stirrup needed.

Theoretical spacing, $\varphi V_c + \varphi V_s = V_u$

$$V_s = \frac{V_u - \varphi V_c}{\varphi} = \frac{56.48 - 25.045}{0.75} = 41.91 \text{ kip} < 8\sqrt{f_c'}\, b_w\, d = 133.57 \text{ kip} \rightarrow \text{(ok)}$$

$$s = \frac{A_v f_{yt}\, d}{V_s} = \frac{2 \times 0.11 \times 60 \times 22}{41.91} = 6.92 \text{ in}$$

Maximum spacing to provide minimum $A_v$

$$s = \frac{A_v f_{yt}}{0.75\sqrt{f_c'}\, b_w} = \frac{2 \times 0.11 \times 60000}{0.75\sqrt{4000} \times 12} = 23.19 \text{ in}$$

$$s = \frac{A_v f_{yt}}{50\, b_w} = \frac{2 \times 0.11 \times 60000}{50 \times 12} = 22 \text{ in}$$

$$V_s = 41.91 \text{ kip} < 4\sqrt{f_c'}\, b_w\, d = \frac{4 \times \sqrt{4000} \times 12 \times 22}{1000} = 66.78 \text{ kip}$$

Maximum spacing $= \dfrac{d}{2} = \dfrac{22}{2} = 11$ in

Provide #3 bar @ 6.5 in c/c

**Question:** What are simply supported beam and cantilever beam? (EED – 2015)

**Solution:**

Simply supported Beam: A simply supported beam is a type of beam that has pinned support at one end and roller support at the other end. Depending on the load applied, it undergoes shearing and bending. It is the one of the simplest structural elements in existence.
Cantilever Beam: A cantileverbeam is a rigid structural element, such as a beam or a plate, anchored at one end to a support from which it protrudes; this connection could also be perpendicular to a flat, vertical surface such as a wall.

**Question:** Where doubly reinforced beam should be provided? Give reasons. (40th BCS)

**Solution:**

The beam that is reinforced with steel both in tension and compression zone, it is known as doubly reinforced beam. This type of beam is mainly provided when the depth of the beam is restricted. If a beam with limited depth is reinforced on the tension side only it might not have sufficient resistance to oppose the bending moment. The resistance moment cannot be increased by increasing the amount of steel in the stress zone. Doubly reinforce beam is also provided when the loads are eccentric and when the beam is subjected to accidental or sudden lateral loads. When the external loads may occur on either face of the member i.e., the loads are alternating or reversing and may cause tension on both faces of the member.

**Question:** What is double reinforcement beam? Minimum clear cover slab, beam, column. (BSEC – 2014)

**Solution:**

The beam that is reinforced with steels both in tension and compression zone, it is known as doubly reinforced beam. This type of beam is mainly provided when the depth of the beam is restricted. If a beam with limited depth is reinforced on the tension side only it might not have sufficient resistance to oppose the bending moment. The moment of resistance cannot be increased by increasing the amount of steel in tension zone. Thus a doubly reinforced beam is provided to increase the moment of resistance of a beam having limited dimensions. In most of the general cases the minimum values of clear cover are 75 mm for Foundation, 40 mm for Column, 40 mm for Beam, 25 mm for shear wall, 15 mm for Slab and Stair.

**Question:** Write down ACI code provision for effective width of T-Beam. What is minimum slab thickness for one way slab? (BCPCL – 2016)

**Solution:**

Total effective flange width of a T-Beam is the lesser of the following:

$b_e \leq$ Web width + 16 (slab thickness)

$b_e \leq$ Average clear distance to adjacent webs + Web width

$b_e \leq$ Span length/4

| Member | Minimum Thickness, h | | | |
|---|---|---|---|---|
| | Simply supported | One end continuous | Both ends continuous | Cantilever |
| | Members not supporting or attached to partitions or other construction likely to be damaged by large deflections | | | |
| Solid one-way slabs | l/20 | l/24 | l/28 | l/10 |
| Beams or ribbed one-way slabs | l/16 | l/18.5 | l/21 | l/8 |

**Question: Why strength reduction factor is 0.75 for shear and 0.9 for flexure? Explain. (MES – 2015)**

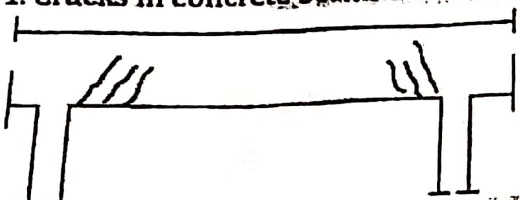
**Solution:**

If a beam without properly designed shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly with no advance warning (brittle failure). Therefore, concrete must be provided by "special shear reinforcement" to ensure flexural failure would occur before shear failure. In other words, we want to make sure that beam will fail in a ductile manner and in flexure not in shear. That's why strength reduction factor is 0.75 for shear and 0.9 for flexure.

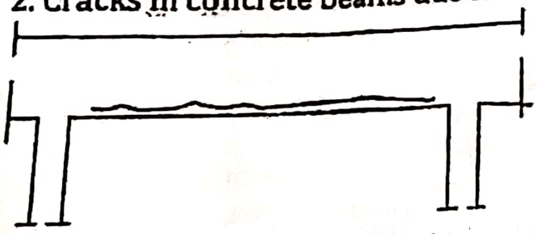
**Question: Draw the different type's cracks in a beam, Difference between normal stress, shear stress and flexural stress. (AB – 2017, BKB – 2018, CPGCL – 2018)**

**Solution:**

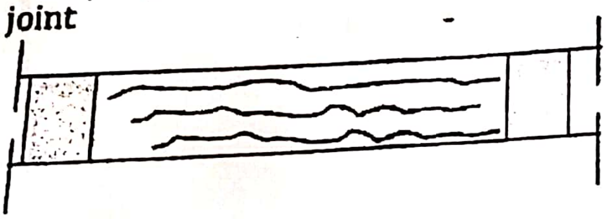
1. Cracks in concrete beams due to increased shear stress.



2. Cracks in concrete beams due to corrosion or insufficient concrete cover



3. Cracks parallel to main steel in case of corrosion in beams due to moisture infiltration at the joint



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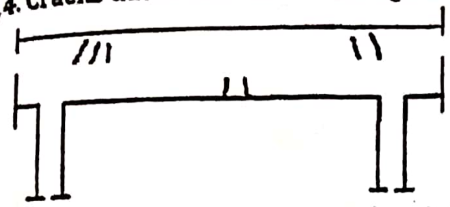
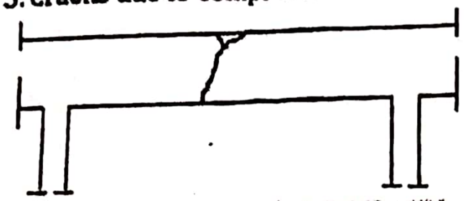
4. Cracks due to increased bending stress in beams



5. Cracks due to compression failure in beams



**Normal Stress:** A normal stress is a stress that occurs when a member is loaded by an axial force. A normal stress will occur when a member is placed in tension or compression.

$\sigma = P/A$

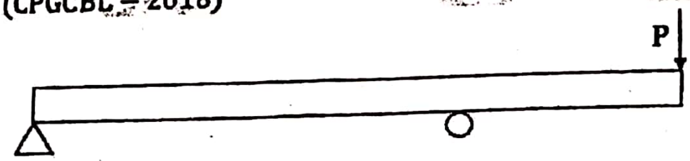
**Flexural stress:** When a member is being loaded similar to that in figure one bending stress (or flexure stress) will result. Stress produced by the bending force in the body is what we call flexural stress. Bending stress is maximum at top or bottom most fibre and it is zero at neutral axis.

Flexural stress, $\sigma = \frac{MY}{I}$

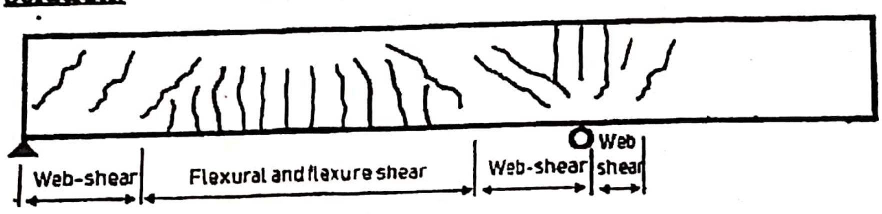
**Shear stress:** Normal stress is a result of load applied perpendicular to a member. Shear stress however results when a load is applied parallel to an area.

Shear stress, $\tau = \frac{VQ}{Ib}$

**Question:** Draw qualitative shear and flexural crack in a simply supported RC beam. (CPGCBL – 2018)

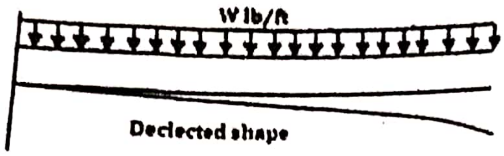


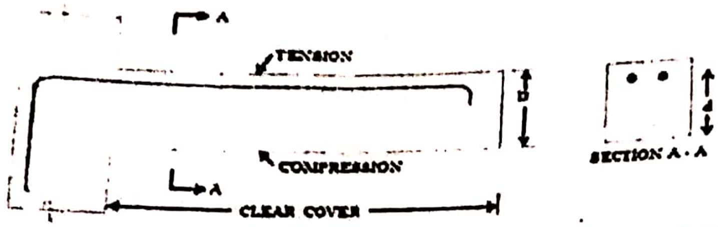

**Solution:**



**Question:** Draw the deflected shape and show the reinforcement of the following beam (WASA – 2014)

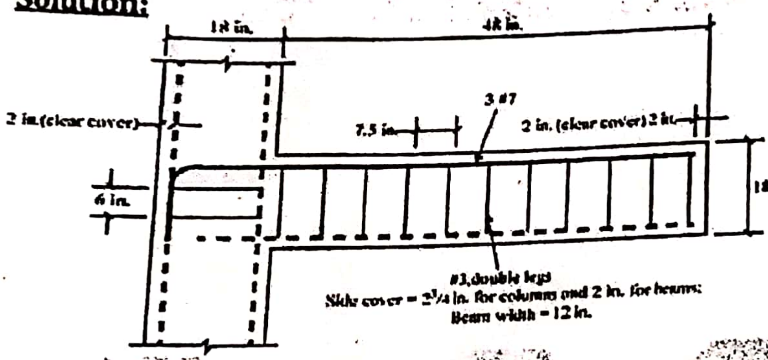
**Solution:**




**Question:** Show typical cantilever beam reinforcement detailing. (PGCB – 2017, RAJUK – 2016)

**Solution:**

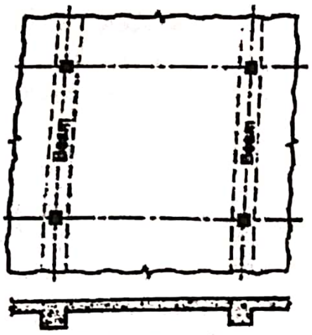


# Slab Design

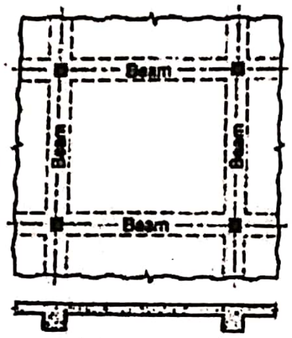
**Question: What is a Slab?**

**Slabs are constructed to provide flat surfaces, usually horizontal, in building floors, roofs, bridges, and other types of structures. The slab may be supported by walls, by reinforced concrete beams usually cast monolithically with the slab, by structural steel beams, by columns, or by the ground.**

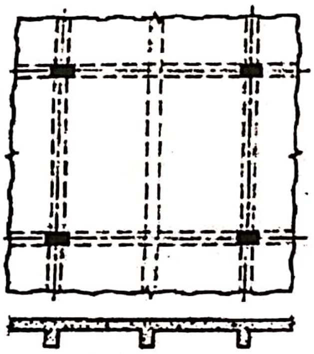
## Types of slab:

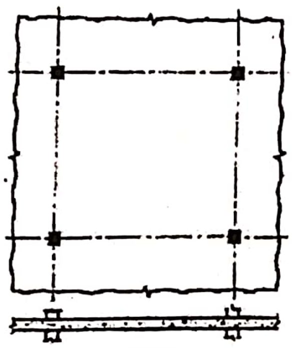


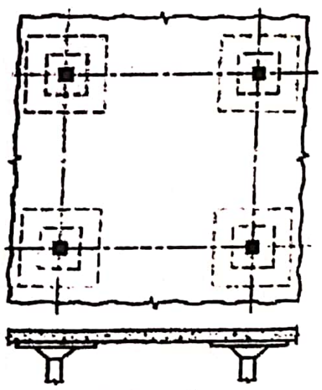
(a) One-way slab

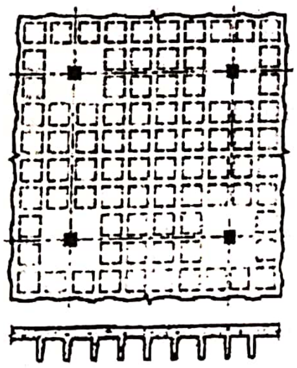


(b) Two-way slab


(c) One-way slab

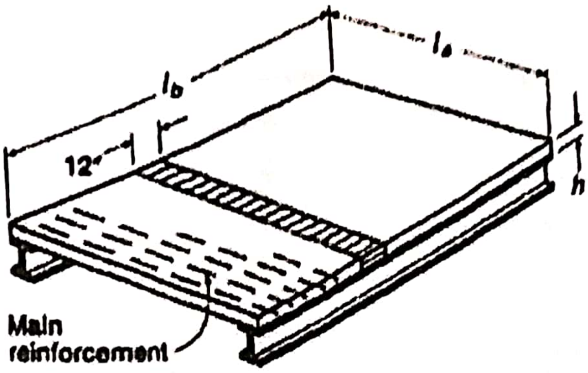

(d) Flat plate


(e) Flat slab


(f) Grid or waffle slab

## One-Way Slabs

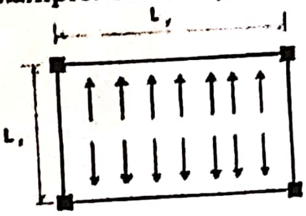
If the slab are supported on two opposite sides only, they are referred to as one-way slabs because the bending is in one direction only that is, perpendicular to the supported edges. Should the slab be supported by beams on all four edges, it is referred to as a two-way slab because the bending is in both directions. Actually, if a rectangular slab is supported on all four sides, but the long side is two or more times as long as the short side, the slab will, for all practical purposes, act as a one-way slab, with bending primarily occurring in the short direction. Such slabs are designed as one-way slabs. Normally, a 12-in.-wide piece of such a slab is designed as a beam.



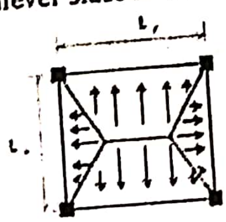

**Question: What is the difference between One Way Slab and Two Way Slab?**

**One way slab:** One way slab is a slab which is supported by beams on the two opposite sides to carry the load along one direction. The ratio of longer span (l) to shorter span (b) is equal or greater than 2, considered as one way slab because this slab will bend in one direction i.e in the direction along its shorter span. Due to the huge difference in lengths, load is not transferred to the shorter beams. Main reinforcement is provided in shorter span and distribution reinforcement in longer span.

**Example:** Generally all the Cantilever slabs are one Way slab.




One-way slab




Two-way slab

**Two way slab:** Two way slab is a slab supported by beams on all the four sides and the loads are carried by the supports along both directions, it is known as two way slab. In two way slab, the ratio of longer span (l) to shorter span (b) is less than 2. In two way slabs, load will be carried in both the directions. So, main reinforcement is provided in both direction for two way slabs.

**Example:** These types of slabs are used in constructing floors.

## Minimum slab thickness for one way slab

Minimum thickness of beams or one-way Slabs unless deflections are computed.

| Member | Minimum Thickness, $h$ | | | |
|---|---|---|---|---|
| | Simply supported | One end continuous | Both ends continuous | Cantilever |
| | Members not supporting or attached to partitions or other construction likely to be damaged by large deflections | | | |
| Solid one-way slabs | $\ell/20$ | $\ell/24$ | $\ell/28$ | $\ell/10$ |
| Beams or ribbed one-way slabs | $\ell/16$ | $\ell/18.5$ | $\ell/21$ | $\ell/8$ |

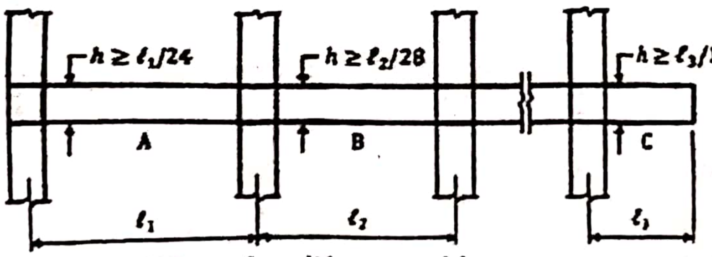



Figure: Slab thickness for solid one way slab

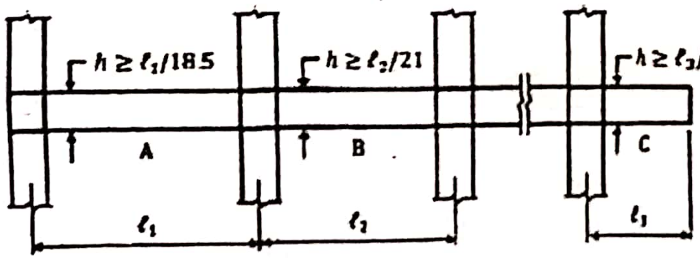



Figure: Slab thickness for beams or ribbed one way slab

### Minimum area of steel

The minimum amount of reinforcement for slabs and footings of uniform cross-sections is equal to the minimum amount required by ACI 7.12.2.1 for shrinkage and temperature reinforcement (ACI 10.5.4):

$A_{s,min} = 0.0020\, b\, h$ for 40 or 50 grade deformed bars.

$A_{s,min} = 0.0018\, b\, h$ for 60 grade deformed bars.

$$A_{s,min} = \frac{(0.0018\, b\, h)\, 60{,}000}{f_y}$$ where reinforcement with $f_y > 60{,}000$ psi

In these expressions, $b$ is the width of the member and $h$ is the overall thickness. A design width of 12 in is typically used in such cases, and reinforcement is specified in square inches per foot. The maximum spacing of this reinforcement is the smaller of three times the member thickness or 18 in. Maximum spacing of flexural reinforcement = lesser of $3h$ or 18 in.

**Moment coefficient method:**

If the slab is one span only and rests freely on its supports, the maximum positive moment M for a uniformly distributed load of w psf is $M = (w\,L^2)/8$ where L is the span length between the supports. If the same slab is built monolithically with the supporting beams or is continuous over several supports, the positive and negative moments are calculated either by structural analysis or by moment coefficients as for continuous beams. The ACI code section 8.3 permits the use of moment and shear coefficients in the case of two or more approximately equal spans. The conditions under which the moment coefficient for continuous beams and slabs should be used can be summarized as follows:

1. Spans are approximately equal: Longer span ≤ 1.2 (shorter span).
2. Loads are uniformly distributed.
3. The ratio (live load / dead load) is less than or equal to 3.
4. For slabs with spans less than or equal to 10 ft, negative bending moment at face of all supports is $\frac{1}{12}\,W_u\,l_n^2$
5. For an unrestrained discontinuous end at A, the coefficient is 0 at A and $+\frac{1}{11}$ at B.
6. Shearing force at C is 1.15 $W_u\,l_n/2$ and at the face of all other support $W_u\,l_n/2$
7. $M_u$ = (coefficient) $(W_u\,l_n^2)$; $l_n$ = clear span

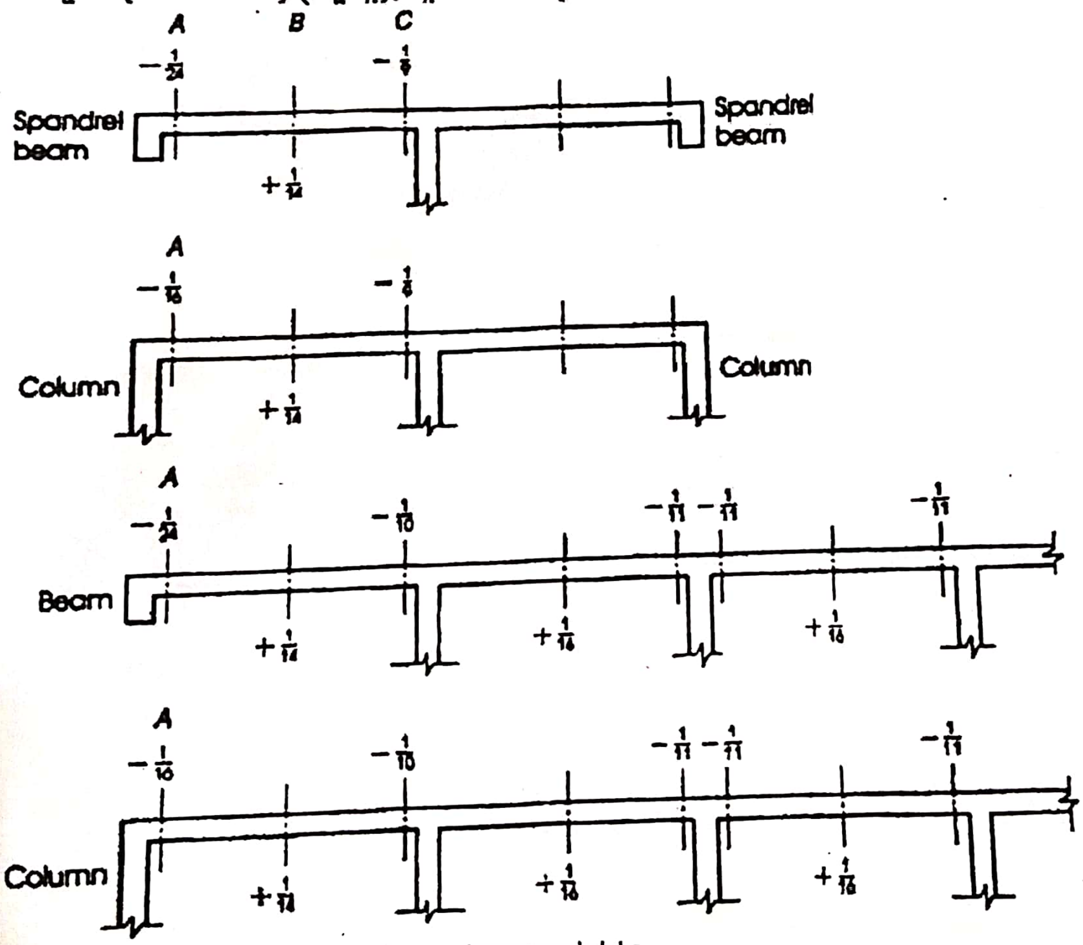



Figure: Moment coefficient for continuous beams and slabs

## Reinforcement detailing

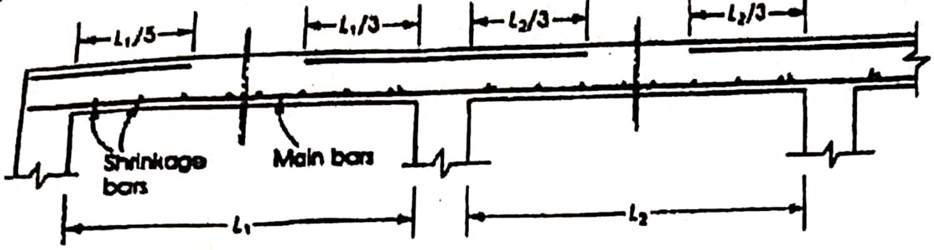



**Figure:** Reinforcement details in continuous one way slabs for straight bars

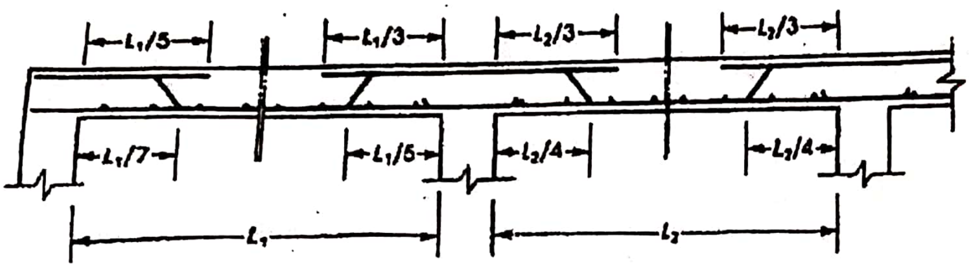



**Figure:** Reinforcement details in continuous one way slabs for bent bars

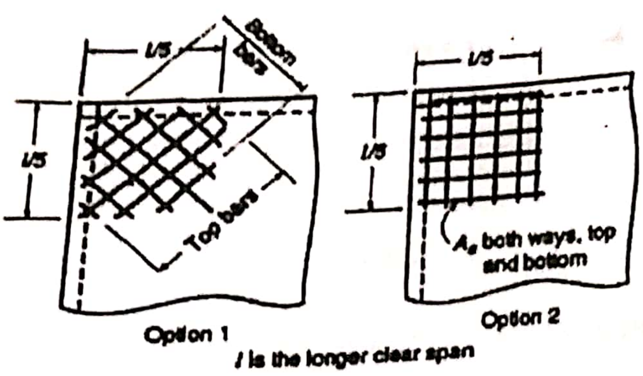
**Question:** What is the minimum slab thickness of one way slab? (MES-2015)

- Minimum Thickness of Non pre-stressed Beams or One-Way Slabs unless deflections are computed.

| | Minimum Thickness, *h* | | | |
|---|---|---|---|---|
| | Simply supported | One end continuous | Both ends continuous | Cantilever |
| Member | Members not supporting or attached to partitions or other construction likely to be damaged by large deflections | | | |
| Solid one-way slabs | *l*/20 | *l*/24 | *l*/28 | *l*/10 |
| Beams or ribbed one-way slabs | *l*/16 | *l*/18.5 | *l*/21 | *l*/8 |

**Question:** When and why does corner reinforcement used in a two way RC slab. (32th BCS)

Torsional moments are quite high nearer to the corner of the slab. In two-way slabs which are restrained along the edges & are not free to lift, cracks get developed at the corners because of the high torsional moment. Thus we need to prevent such types of the cracks formations in the two-way slab. Therefore we reinforce these sections by providing reinforcement to resist the crack formation at the corners.




**Question: Write down the rules of corner reinforcement according to BNBC code. (RAJUK – 2014)**

**Solution:**

a. Special reinforcement shall be provided at exterior corner in both top and bottom of the slab for a distance in each direction from the corner equal to one-fifth of the longer span of the corner panel.

b. Corner reinforcement shall be parallel to the slab at the top to a line bisecting the angle at the relevant corner.

c. Corner reinforcement shall be perpendicular to the slab at the bottom to a line bisecting the angle at the relevant corner.

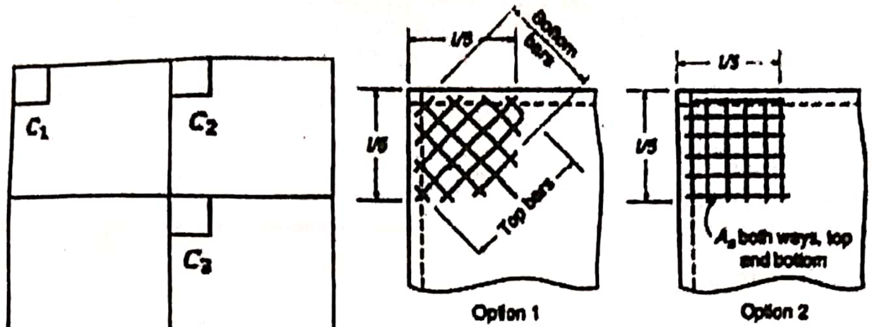
d. The top and bottom corner reinforcement shall be of size and spacing equivalent to that required for the maximum positive moment in the panel.

**Question: What is corner reinforcement in RC slab? Why and where are they used? State ACI/BNBC specification for such reinforcement. (30th BCS)**

**Solution:**

Torsional moments are quite high nearer to the corner of the slab. In two-slabs which are restrained along the edges & are not free to lift, cracks get developed at the corners because of the high torsional moment. Thus we need to prevent such types of the cracks formations in the two-way slab. Therefore we reinforce these sections by providing reinforcement to resist the crack formation at the corners.

However, torsional reinforcing bar have three different requirements for three different types of corner C1, C2 and C3 as shown in fig given below:



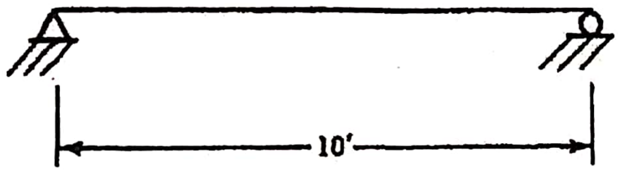

*l* is the longer clear span

- At corner C1 - The slab is discontinuous on both the sides, the torsion reinforcement shall consist of top and bottom bars each with layers of bar placed parallel to the sides of the slab and extending a minimum distance of one fifth of the shorter span from the edges. The amount of reinforcement in each of the four layers shall be 75 per cent of the area required for the maximum mid span moment in the slab.

- At corner C2 - Contained by edges over one of which is continuous, the torsional reinforcement shall be half of the amount required for corner C1.

- At Corner C3 - Contained by edges over both of which the slab is continuous, torsional reinforcement need not to be provided.

**Question:** Design a one-way slab for the inside of a building using the span, loads, and other data given in Figure. Normal-weight aggregate concrete is specified with a density of 150 pcf.




LL = 200 psf
$f'_c$ = 4000 psi
$f_y$ = 60,000 psi

**Solution:**

Minimum Total Slab Thickness,

$$h = \frac{l}{20} = \frac{(20\ \text{in/ft})\ (10\ \text{ft})}{20} = 6\ \text{in}$$

Assume 6-in. slab (with d = approximately 6 in. – 3/4 in. Cover –1/4 in. for estimated half diameter of bar size = 5.0 in.) Design a 12-in.-wide strip of the slab. Thus, b = 12 in.

$$DL = \text{slab weight} = \frac{6}{12} \times 150\ \text{pcf} = 75\ \text{psf.}$$

$LL = 200$ psf.

$W_u = (1.2)\ (75\ \text{psf.}) + (1.6)\ (200\ \text{psf.}) = 410\ \text{psf.}$

$$M_u = \frac{(0.410 \text{ klf})\,(10 \text{ ft})^2}{8} = 5.125 \text{ ft} - \text{k}$$

$$M_u = \varphi\, A_s\, f_y\, (d - \frac{a}{2})$$

$$A_s = \frac{5.125 \times 12}{0.9 \times 60\,(5 - 0.14)} = 0.236 \text{ in}^2/\text{ft (assume, } a = 0.34 \text{ in.)}$$

$$\text{Check, } a = \frac{As\,fy}{0.85\,f_c\,b} = \frac{0.236 \times 60}{0.85 \times 4 \times 12} = 0.34 \text{ inch.}$$

$\rho_{min} = 0.0033$

$A_{s,min} = \rho\, b\, d = (0.0033)(12 \text{ in.})(5 \text{ in.}) = 0.198 \text{ in}^2/\text{ft}$

So, $A_s = 0.236 \text{ in}^2/\text{ft}$

Use #4 @ 10 in. c/c ($A_s = 0.24 \text{ in}^2/\text{ft}$)

Spacing < maximum of 18 in. as per ACI 7.6.5

**Transverse Direction—Shrinkage and Temperature Steel**

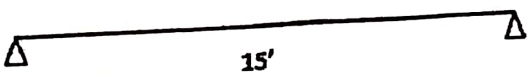
$A_s = 0.0018\, \text{b t} = (0.0018)\,(12 \text{ in.})\,(6 \text{ in.}) = 0.1296 \text{ in}^2/\text{ft}$

Use #3 @ 10 in. c/c ($0.13 \text{ in}^2/\text{ft}$)

Spacing < maximum of 18 in. as per ACI 7.12.2.2 OK

The #4 bars are placed below the #3 bars in this case. The #4 bars are the primary flexural reinforcing, and the value of $d$ is based on this assumption. The #3 bars are for temperature and shrinkage control, and their depth within the slab is not as critical.

**Question: Determine the slab thickness and spacing required for the temperature and shrinkage reinforcement of one way slab, considering $f_y$ = 60 ksi. (LGD – 2018, 51 BMA)**




**Solution:**

$$\text{Minimum slab thickness, } h = \frac{L}{20} = \frac{15 \times 12}{20} = 9 \text{ in}$$

Transverse direction – Shrinkage and Temperature steel

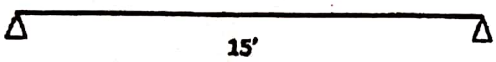
$A_s = 0.0018\, \text{b t} = 0.0018 \times 12 \times 9 = 0.1944 \text{ in}^2/\text{ft}$

$$\text{Using 10 mm bar} = \frac{0.11 \times 12}{0.1944} = 6.79 \text{ in}$$

Provide 10 mm bar @ 6.5 in c/c

**Question:** A 15' long and 10' wide uniformly loaded slab is simply supported along two short sides; the long sides are unsupported. The slab thickness is 8" with #5 bar spaced 6" c/c as main flexural steel. Calculate the maximum uniformly load that the slab can carry. $f_c' = 4$ ksi and $f_y = 60$ ksi. (33th BCS)

**Solution:**




The slab is simply supported along two short sides; the long sides are unsupported. So the slab is one way slab.

Effective depth, $d = 8 - 0.75\text{ (cover)} - 0.3125\text{ (half bar diameter)} = 6.9375$ in

Area of #5 bar $= \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.306\ \text{in}^2$

Reinforcement provided, $A_{st} = \frac{0.306 \times 12}{6} = 0.612\ \text{in}^2/\text{ft}$

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{0.612 \times 60}{0.85 \times 4 \times 12} = 0.9\ \text{in}$$

$$M_u = \varphi A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_u = 0.9 \times 0.612 \times 60\left(6.9375 - \frac{0.9}{2}\right) = 214.39\ k-in = 17.86\ k-ft$$

For simply supported one way slab, $M_u = \frac{W_u L^2}{8}$

$$\frac{W_u \times 15^2}{8} = 17.86$$

$$W_u = 0.635\ \text{ksf} = 635\ \text{psf}$$

$$DL = \text{slab self weight} = \frac{8}{12} \times 150\ \text{pcf} = 100\ \text{psf}.$$

Now, $W_u = 1.2\,DL + 1.6\,LL$

$1.2 \times 100 + 1.6 \times LL = 635.$

$LL = 321.875$ psf

# Column Design

**Question:** What is Column? Describe types of column.

Column may be defined as a vertical structural member that carry loads chiefly in compression. It can carry moment as well, about one or both axes of the cross section.

**Types of Column:**

A. Based on reinforcement column can be divided into the following was:

- Column reinforced with longitudinal bar and lateral ties.
- Column reinforced with longitudinal bar and continuous spirals.
- Composite compression members reinforced with structural steel shapes, pipe or tubing.

B. Based on length:

Short column: Short columns those whose ratio of effective length to the minimum thickness is less than 12 ($\leq 12$). Material strength is the main factor for its failure.

Long Column: Long columns those whose ratio of effective length to the minimum thickness is greater than 12 ($\geq 12$). Material strength and buckling are the main factors for its failure.

C. Depending on position at a building or loading pattern:
1. Axially loaded column
2. Uni-axial column
3. Bi-axial column.

**Question:** Why ties are used in column?

1. To hold longitudinal bars in position in forms while the concrete is being placed. For this purpose both of them are wired together to form a cage of reinforcement.
2. To prevent the highly stressed slender longitudinal bars from buckling outward bursting the thin concrete cover.

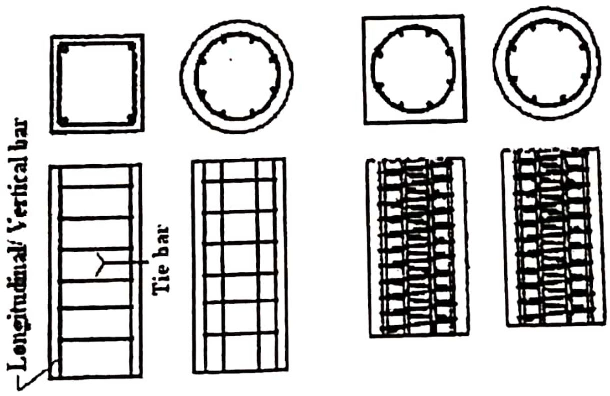



Fig: Tied column Fig: Spiral column

| Serial No. | Contents | Tied | Spiral |
|---|---|---|---|
| a) | Least Dimension | Min$^m$ 10", in EQ regions 12" | Min$^m$ 12" |
| b) | Least area | -- | -------- |
| c) | Steel ratio, $\rho_{g,min}$ | 1% | 1% |
| d) | Steel ratio, $\rho_{g,max}$ | 8% | 8% |
| e) | Min$^m$ No. of longitudinal bar | 4 | 6 |
| f) | Min$^m$ longitudinal bar size | #5 bar | #5 bar |
| g) | Min$^m$ No. of tie bar | #3 (for longitudinal bar size > #10, use #4 bar as tie) | #3 |
| h) | Min$^m$ Clear cover | 1.5 in | 1.5 in |

**Note: Generally $\rho_{g,max}$ is preferable between 1%- 4% to avoid steel congestion.**

**Question: Write down the guideline for Tie/ spiral reinforcement**

**Solution:**

1) All bars of tied columns shall be enclosed by lateral ties, at least No. 3 in size for longitudinal bars up to No. 10, and at least No. 4 in size for Nos. 11, 14, and 18 and bundled longitudinal bars.
2) The spacing of the ties shall not exceed 16 diameters of longitudinal bars (16$D$), 48 diameters of tie bars (48$d$), nor the least dimension ($t_{min}$) of the column.
3) No bar shall be farther than 6 in. clear on either side from such a laterally supported bar.
4) The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135°.
5) Spirals shall be continuous bar of not less than #3. The clear spacing between turns of the spiral must not exceed 3 inch (but not less than 1 inch).

**Question: Why strength reduction factor in column is less than beam?**

**Solution:**

The strength-reduction factor for columns is much lower than those for flexure ($\Phi = 0.9$) and shear ($\Phi = 0.75$). This is because in axially loaded columns, the strength depends mainly on the concrete compression strength, whereas the strength of members in bending is less affected by the variation of concrete strength, especially in the case of an under-reinforced section. Furthermore, the concrete in columns is subjected to more segregation than in the case of beams. Columns are cast vertically in long, narrow forms, but the concrete in beams is cast in shallow, horizontal forms. The failure of a column is generally a more severe matter than is the failure of a beam, because a column generally supports a larger part of a structure than does a beam. In other words, if a column fails in a building, a larger part of the building will fall down than if a beam fails.

**Question: What is column? Write ACI code of column & ties (BSEC – 2014)**

**Solution:**

**A column or pillar in architecture and structural engineering is a structural element that transmits, through compression, the weight of the structure above to other structural elements below. In other words, a column is a compression member.**

- **For axially as well as eccentrically loaded columns, the ACI Code sets the strength-reduction factors at $\Phi = 0.65$ for tied columns and $\Phi = 0.75$ for spirally reinforced columns.**
- **The minimum longitudinal steel percentage is 1%, and the maximum percentage is 8% of the gross area of the section.**
- ***At least four bars are required for tied circular and rectangular members and six bars are needed for circular members enclosed by spirals.***
- ***The minimum diameter of spirals is 3/8 in., and their clear spacing should not be more than 3 in. nor less than 1 in.***
- ***Ties for columns must have a minimum diameter of 3/8 in. to enclose longitudinal bars of no. 10 size or smaller and a minimum diameter of ½ in. for larger bar diameters.***
- ***Spacing of ties shall not exceed the smallest of 48 times the tie diameter, 16 times the longitudinal bar diameter, or the least dimension of the column.***

***Question: What are meant by (i) Short column; (ii) Long column; (iii) Tied column; (iv) Spiral column. (31th BCS)***

**Solution:**

***Short column: Short column is basically defined as the column in which the ratio of effective length of the column to the least lateral dimension of the column is less than 12. Short columns usually fail by crushing***

***Long column: Long column is basically defined as the column in which the ratio of effective length of the column to the least lateral dimension of the column is more than 12. Slender column is liable to fail by buckling.***

***Tied Column: When the main longitudinal bars of the column are confined within closely spaced lateral ties, it is called a tied column.***

***Spiral Column: When the main longitudinal bars of the column are enclosed with in closely spaced and continuously wound spiral reinforcement, it is called as a spiral column.***

**Question:** What are the values of maximum and minimum reinforcement of steel in column? Why does the code suggest those values? (36th BCS, MES 2015)

**Solution:**

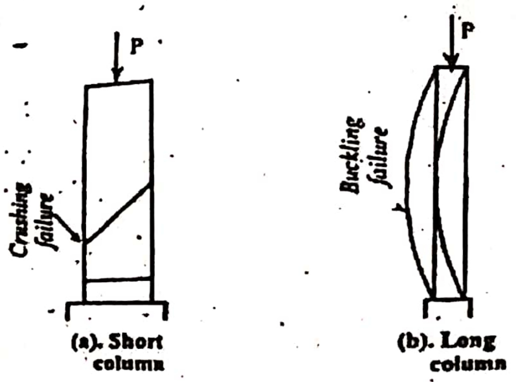
As per the ACI Code, the minimum reinforcing steel in a column is 1% and the maximum is 8% of the gross area.

The minimum reinforcement ratio is used to make sure you provide at least enough reinforcement to ensure your column will have some ductility, confinement, resistance to thermal loading, and ability to limit the width of any cracks that will form. The primary motive for the minimum limit is to ensure that the moment capacity is higher than the cracking moment.

The maximum reinforcement ratio is used to prevent you from overcrowding the interior of the column with reinforcement that would, in turn, restrict the flow of concrete/aggregate and reduce the availability of concrete for reinforcement to be embedded in. Too much reinforcement would also make the column impossible to construct.

**Question:** Short column and long column failure pattern. (PGCB – 2017)

**Solution:**

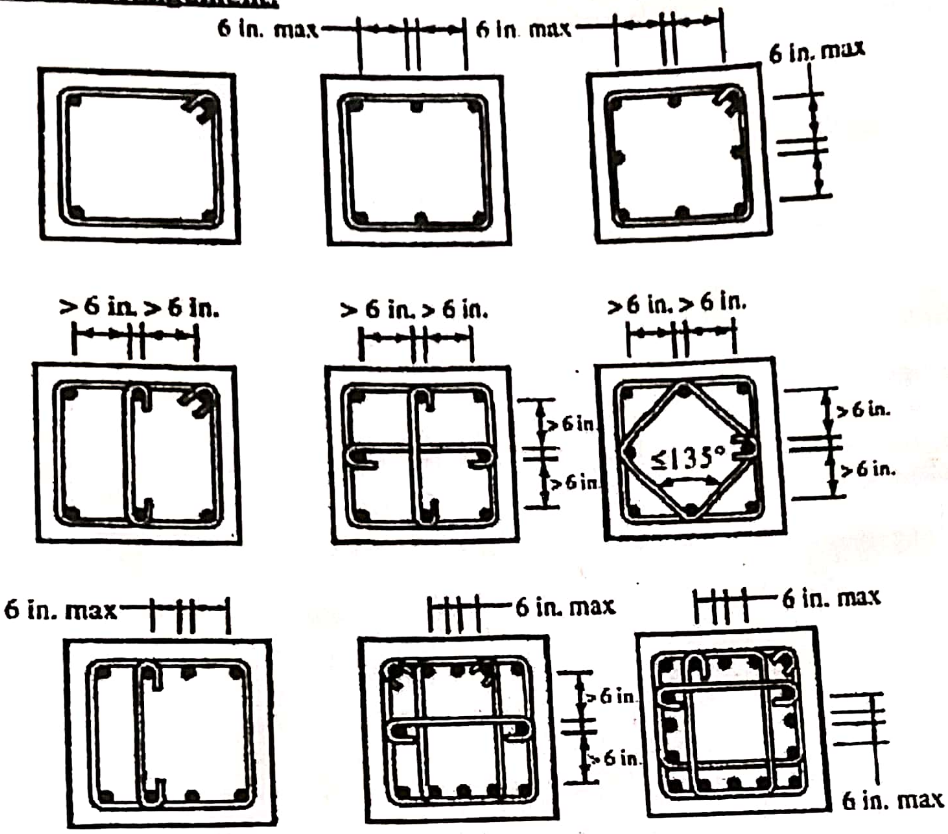



**Question:** Why load capacity in tied column is less than spiral column?

**Solution:**

Concrete columns reinforced with spiral reinforcement can withstand more loads than tied column. This phenomenon happens because when load eccentricities are small, spirally reinforced columns shows greater toughness, greater ductility than the columns with ties. Therefore, it is expected from civil engineering professionals to use spiral reinforcement instead of ties whenever possible in reinforced concrete columns. In loaded condition, tied column concrete fails by crushing and shearing but in spiral column concrete prevented by spiral core.

**Tie bar arrangement:**




**Design of axially loaded column:**

**WSD Method:**

For concentrically loaded spirally reinforced column Code provides the following formula for maximum allowable load:

$$P = 0.25\, f_c'\, A_g + f_s\, A_s = A_g(0.25\, f_c' + f_s\, \rho_g)$$

For concentrically loaded tied reinforced column Code provides the following formula for maximum allowable load:

$$P = 0.85\,(0.25\, f_c'\, A_g + f_s\, A_s) = 0.85\, A_g(0.25\, f_c' + f_s\, \rho_g)$$

**Short column or slender column?**

If, $\frac{L}{b} < 12$ → The column is short column

If, $\frac{L}{b} > 12$ → The column is slender column

For slender column, the permissible stresses in concrete and steel are reduced by multiplication with a strength reduction coefficient $C_r$ given by:

$$C_r = 1.25 - \frac{l_e}{48\, b}$$

Where $b$ is the least lateral dimension of the column (or diameter of the core in a spiral column).
Alternatively, for more exact calculations,

$$C_r = 1.25 - \frac{l_e}{160\, r_{min}}$$

Where, $r_{min}$ is the least radius of gyration of the column.

If reduction factor is used, $P = C_r\, 0.85\, (0.25\, f_c'\, A_g + f_s\, A_s)$

**USD Method:**

The maximum compressive axial load that can be resisted by the concrete $P_c$

$P_c = 0.85\, f_c'(A_g - A_{st})$

Here, full strength of $f_c'$ is not used rather $0.85\, f_c'$

The maximum axial load that can be carried by the longitudinal reinforcement $P_s$

$P_s = f_y\, A_{st}$

The maximum concentric axial load $Po$ that can be carried by a short column

$P_o = 0.85\, f_c'\, (A_g - A_{st}) + f_y\, A_{st}$

The maximum nominal axial strength $P_{n,max}$ is equal to a constant times the concentric axial load strength $P_o$. The constant depends on the type of transverse reinforcement utilized in the section and accounts for any accidental eccentricities or any accidental bending moments that may exist in a compression member and were not considered in the analysis. These eccentricities can arise from unbalanced moments in the beams framing into the column, misalignment of columns from floor to floor or misalignment of the longitudinal reinforcement in the column.

The nominal load strength of an axially loaded column was given in the previous equation. Because a perfect axially loaded column does not exist, some eccentricity occurs on the column section, thus reducing its load capacity. To take into consideration, the ACI code specifies that the maximum nominal load should be multiplied by a factor equal to 0.8 for tied columns and 0.85 for spirally reinforced columns.

For members with spiral reinforcement, the constant $\alpha$ is equal to 0.85.

$P_{n,max} = 0.85\,[0.85\, f_c'\, (A_g - A_{st}) + f_y\, A_{st}]$

For members with tie reinforcement, the constant $\alpha$ is equal to 0.80

$P_{n,max} = 0.80\,[0.85\, f_c'\, (A_g - A_{st}) + f_y\, A_{st}]$

Design axial strength $\varphi\, P_n$ formed by multiplying the maximum nominal axial strength $P_{n,max}$ by the corresponding strength reduction factor $\varphi$.

For members with spiral reinforcement

$\varphi\, P_{n,max} = 0.85\, \varphi\, [0.85\, f_c'\, (A_g - A_{st}) + f_y\, A_{st}]$

For members with tie reinforcement

$\varphi\, P_{n,max} = 0.80\, \varphi\, [0.85\, f_c'\, (A_g - A_{st}) + f_y\, A_{st}]$

The strength reduction factor $\varphi$ is equal to 0.75 for compression-controlled sections with spiral reinforcement and 0.65 for other reinforced members including tied reinforcement.

**For spiral reinforcement**

minimum percentage of steel, $\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_y}$

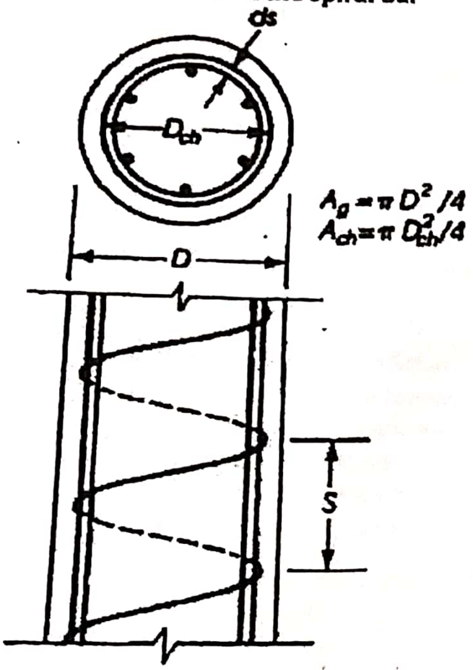
Spacing of the spiral, $s = \frac{4a_s(D_c - d_b)}{\rho_s D_c^2}$

$A_s =$ Area of the core

$a_s =$ Cross – sectional area of the spiral bar

$D_c =$ Diameter of the core out to out of the spiral

$d_b =$ Diameter of the spiral bar




**Question:** Design tie size and spacing of a 24" x 16" rectangular column which is reinforced with 12#9 bar. (BGFCL – 2017)

**Solution:**

As longitudinal bar is less than #10 bar, use #3 bar as tie bar.

1. $48\,d = 48 \times \frac{3}{8} = 18$ in
2. $16\,D = 16 \times \frac{9}{8} = 18$ in
3. Least dimension $= 16$ in

Use #3 bar @16" c/c

**Question:** Determine the maximum allowable axial compressive force for a 15" x 20" tied column having 2.5% steel reinforcement. $f_c' = 4$ ksi and $f_y = 60$ ksi. (WRGCL – 2014)

**Solution:**

$A_g = 15 \times 20 = 300 \text{ in}^2$

Given, $\rho = 0.025$

$P_u = \varphi\, \alpha\, A_g\, [0.85 f_c'(1-\rho) + \rho f_y]$

$P_u = 0.8 \times 0.65 \times 300 \times [0.85 \times 4 \times (1-0.025) + 0.025 \times 60] = 751.14 \text{ k}$

**Question:** Design a circular spiral column to support an axial dead load 475 k and a live load 250 k using fc' = 4 ksi, fy = 60 ksi. And a steel ratio of about 3%. Also, design the necessary spirals.

**Solution:**

$P_u = 1.2\, DL + 1.6\, LL = 1.2 \times 475 + 1.6 \times 250 = 970 \text{ k}$

$\varphi P_n = \varphi\, 0.85\, [0.85 f_c' (A_g - A_{st}) + f_y A_{st}]$

$970 = 0.75 \times 0.85\, [0.85 \times 4\, (A_g - 0.03\, A_g) + 60 \times 0.03\, A_g]$

$A_g = 299 \text{ in}^2$

Use 20 in diameter column ($314.2 \text{ in}^2$)

$\varphi P_n = \varphi\, 0.85\, [0.85 f_c' (A_g - A_{st}) + f_y A_{st}]$

$970 = 0.75 \times 0.85\, [0.85 \times 4\, (314.2 - A_{st}) + 4 \times A_{st}]$

$A_{st} = 8 \text{ in}^2$

Use 8#10 bars ($A_{st} = 10.16 \text{ in}^2$)

**Design of spirals:**

diameter of core, $D_c = 20 - 2 \times 1.5 = 17$ in.

Area of the core, $A_c = \dfrac{\pi D_c^2}{4} = \dfrac{\pi \times 17^2}{4} = 226.98 \text{ in}^2$

Area of the column, $A_g = \dfrac{\pi D^2}{4} = \dfrac{\pi \times 20^2}{4} = 314.16 \text{ in}^2$

minimum percentage of steel, $\rho_s = 0.45 \left(\dfrac{A_g}{A_c} - 1\right) \dfrac{f_c'}{f_y}$

$\rho_s = 0.45 \left(\dfrac{314.16}{226.98} - 1\right) \dfrac{4}{60} = 0.01152$

Using #3 bar as spiral reinforcement, $a_s = 0.11 \text{ in}^2$ and $d_b = 0.375$ in

Spacing, $s = \dfrac{4a_s(D_c - d_b)}{\rho_s D_c^2} = \dfrac{4 \times 0.11\,(17 - 0.375)}{0.01152 \times 17^2} = 2.2$ in

Use #3 bar spiral at $S = 2$ in.

**Question: Determine the allowable compressive load in the following column having cross section 20" x 20" with 10-3/4" bar used given fc' = 3 ksi and $f_y$ = 60 ksi. (BPDB – 2016)**

**Solution:**

Here, $A_g = 20 \times 20 = 400\ in^2, A_{st} = 10 \times \frac{\pi}{4} \times \left(\frac{3}{4}\right)^2 = 3.53\ in^2$

$P_u = \varphi\ \alpha\ [0.85\ f_c'\ (A_g - A_{st}) + A_{st}\ f_y]$

$P_u = 0.8 \times 0.65\ [0.85 \times 4\ (400 - 3.53) + 3.53 \times 60\ ] = 811.23\ k$

**Question: Design a square tied column to support dead load of 500 kips and live load of 400 kips. Reinforcement ratio must be within 1 to 2%. Assume, fc' = 4 ksi and $f_y$ = 60 ksi. (NPCBL – 2017)**

**Solution:**

$P_u = 1.4\ DL + 1.7\ LL = 1.4 \times 500 + 1.7 \times 400 = 1380$ kips

Given, $\rho = 0.02$

$P_u = \varphi\ \alpha\ A_g\ [0.85\ fc'(1 - \rho) + \rho\ f_y]$

Or, $1380 = 0.65 \times 0.80 \times A_g \times [0.85 \times 4 \times (1 - 0.02) + 0.02 \times 60]$

$A_g = 585.58\ in^2$

Square column, $h = \sqrt{585.58} = 24.19 \approx 25$ in

So, 25" x 25" square tied column. $A_g = 625\ in^2$

Again, $P_u = \varphi\ \alpha\ [0.85\ f_c'\ (A_g - A_{st}) + A_{st}\ f_y]$

Or, $1380 = 0.65 \times 0.80\ [0.85 \times 4 \times (625 - A_{st}) + A_{st} \times 60]$

$\therefore A_{st} = 9.34\ in^2$

Area of #10 bar, $A_{st} = 1.23\ in^2$

Use 8 Nos. #10 bar. (Ans.)

**Question: The value of $f_c'$ is 4000 psi, $f_y$ is 60000 psi and steel area is 2% and the column section is 12" x 12". Find the design ultimate axial stress of the column with zero eccentricity. (GTCL – 2016)**

**Solution:**

$A_g = 12 \times 12 = 144\ in^2$

Given, $\rho = 0.02$

$P_u = \varphi\ \alpha\ A_g\ [0.85\ f_c'\ (1 - \rho) + \rho\ f_y]$ {For zero eccentricity, $\alpha = 1$}

$P_u = 1 \times 0.65 \times 144 \times [0.85 \times 4 \times (1 - 0.02) + 0.02 \times 60] = 424.19\ k$

Design ultimate stress, $\sigma = \frac{P_u}{A_g} = \frac{424.19}{144} = 2.94\ kip/in^2$

**Question:** A rectangular column of 15" x 15" section has reinforcement of 8 no. 9 bars. If the value of fc' is 3000 psi and $f_y$ is 60000 psi, find the ultimate design axial load of the column with zero eccentricity. (TGTDCL – 2018, BEPZA – 2019, NHA – 2020)

**Solution:**

$A_g = 15 \times 15 = 225 \text{ in}^2$

$A_{st} = 8 \times \frac{\pi}{4} \times \left(\frac{9}{8}\right)^2 \approx 8 \text{ in}^2$

$P_u = \varphi\, \alpha\, [0.85\, f_c'\, (A_g - A_{st}) + A_{st}\, f_y]$ {For zero eccentricity, $\alpha = 1$}

$P_u = 0.65 \times 1 \times [0.85 \times 4 \times (225 - 8) + 8 \times 60] = 791.57 \text{ k}$

**Question:** Determine the nominal axial compression force for RCC column of 12" x 22" section having 2.5% mild steel reinforcement. Given $f_c' = 4$ ksi and $f_y = 60$ ksi (PGCL – 2014, DNCC – 2016, ERL – 2017, BGFCL – 2017)

**Solution:**

$Ag = 12 \times 22 = 264 \text{ in}^2$

$A_{st} = \rho\, b\, d = 0.025 \times 12 \times 22 = 6.6 \text{ in}^2$

Nominal axial compression load, $P_n = \alpha\, [0.85\, f_c'\, (A_g - A_{st}) + A_{st}\, f_y]$

$P_n = 0.80\, [0.85 \times 4 \times (264 - 6.6) + 6.6 \times 60] = 1016.928 \text{ k}$

**Question:** Find the axial load of column 30" x 12" which is reinforced with 12 Nos. 20 mm diameter bar, consider $f_c' = 3.5$ ksi and $f_y = 60$ ksi.
(DPDC – 2014, PGCB – 2015, CPGCBL – 2018, CUET M.Sc – 2020)

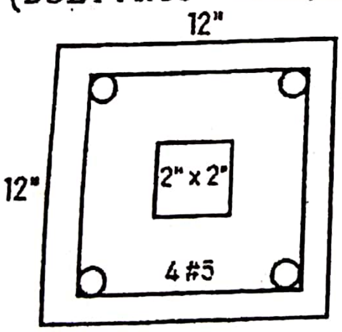
**Solution:**

Here, $A_g = 30 \times 12 = 360 \text{ in}^2$, $A_{st} = 12 \times 0.48 = 5.76 \text{ in}^2$

We know, $P_u = \varphi\, \alpha\, [0.85\, f_c'\, (A_g - A_{st}) + A_{st}\, f_y]$

$P_u = 0.80 \times 0.65\, [0.85 \times 3.5 \times (360 - 5.76) + (5.76 \times 60)] = 727.72 \text{ k}$

**Question:** A hollow rectangular column has shown in figure, the column is reinforced with four #5 bar. Determine the capacity of the column and also design the tie reinforcement.
(BUET M.Sc – 2018, DPDC – 2019)



**Solution:**

$A_g = (12 \times 12) - (2 \times 2) = 140 \text{ in}^2$

$A_{st} = 4 \times \frac{\pi}{4} \times \left(\frac{5}{8}\right)^2 = 1.23 \text{ in}^2$

$P_u = \varphi \alpha [0.85 f_c' (A_g - A_{st}) + A_{st} f_y]$

$P_u = 0.8 \times 0.65 \times [0.85 \times 4 \times (140 - 1.23) + 1.23 \times 60] = 283.72$ k

As main bar is less than #10 bar, Use #3 bar as tie bar.

**Spacing:**

a. $48d = 48 \times \frac{3}{8} = 18"$

b. $16D = 16 \times \frac{5}{8} = 10"$

c. Least dimension = 12"

Use #3 bar @ 12" c/c

**Question:** Design a square tied column to support dead load of 130 kips and live load of 180 kips. Reinforcement ratio must be within 1 to 2%. Assume; $f_c' = 4$ ksi and $f_y = 60$ ksi. (JB – 2017)

**Solution:**

$P_u = 1.4\,DL + 1.7\,LL = 1.4 \times 130 + 1.7 \times 180 = 488$ kips

Given, $\rho = 0.02$

$P_u = \varphi \alpha A_g [0.85 f_c' (1 - \rho) + \rho f_y]$

Or, $488 = 0.65 \times 0.80 \times A_g \times [0.85 \times 4 \times (1 - 0.02) + 0.02 \times 60]$

$\therefore A_g = 207.07 \text{ in}^2$

Square column, $h = \sqrt{207.07} = 14.39 \approx 15$ in

So, 15" x 15" square tied column. $A_g = 225 \text{ in}^2$

Again, $P_u = \varphi \alpha [0.85 f_c' (A_g - A_{st}) + A_{st} f_y]$

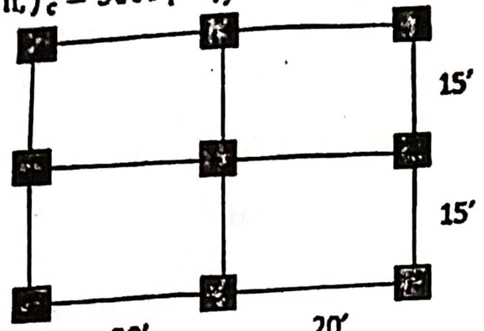
Or, $488 = 0.65 \times 0.80\ [0.85 \times 4 \times (225 - A_{st}) + A_{st} \times 60]$

$\therefore A_{st} = 3.06 \text{ in}^2$

Area of #6 bar, $A_{st} = 0.44 \text{ in}^2$

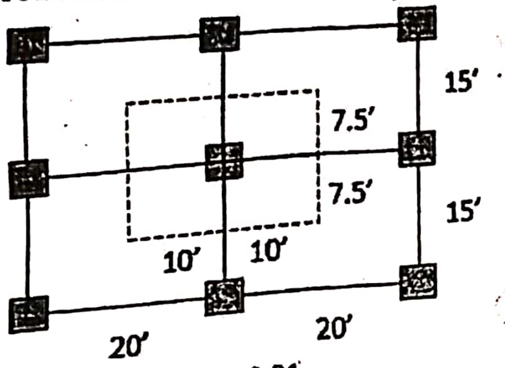
Use 8 Nos. #6 bar.

**Question:** A load of 250 psf per floor, the column is at the ground floor of a five storied building. If, $f'_c = 3000$ psi, $f_y = 60000$ psi design the central column. (SGFL - 2021)




**Solution:**

Total load on the column, $P_u = \left(\frac{15}{2}+\frac{15}{2}\right) \times \left(\frac{20}{2}+\frac{20}{2}\right) \times 250 \times 5 = 375000 \text{ lb} = 375 \text{ kip}$




Assume, $\rho = 1\% = 0.01$

$P_u = \varphi \, \alpha \, A_g \, [0.85 \, fc'(1-\rho) + \rho \, f_y]$

Or, $375 = 0.65 \times 0.80 \times A_g \times [0.85 \times 3 \times (1-0.01) + 0.01 \times 60]$

$A_g = 230.80 \text{ in}^2$

Considering square column, $h = \sqrt{230.80} = 15.19 \approx 16$ in

So, 16" x 16" square tied column. $A_g = 256 \text{ in}^2$

$\therefore \; A_{st} = \rho \, A_g = 0.01 \times 256 = 2.56 \text{ in}^2$

Area of #5 bar, $A_{st} = 0.31 \text{ in}^2$

Use 10 Nos. #5 bar.

**Question:** Design a circular spiral column to support an axial dead load 500 k and a live load 25 k using $f_c' = 4.5$ ksi, $f_y = 60$ ksi and a steel ratio of about 3%. Also, design the necessary spirals. (36th BCS)

**Solution:**

$P_u = 1.2\,DL + 1.6\,LL = 1.2 \times 500 + 1.6 \times 25 = 640$ k

$P_u = \varphi\, 0.85\, [0.85\, f_c'\, (A_g - A_{st}) + f_y\, A_{st}]$

$640 = 0.75 \times 0.85\, [0.85 \times 4.5\, (A_g - 0.03\, A_g) + 60 \times 0.03\, A_g]$

$A_g = 182.19\ in^2$

Use 16 in diameter column (201.06 in²)

$P_u = \varphi\, 0.85\, [0.85\, f_c'\, (A_g - A_{st}) + f_y\, A_{st}]$

$640 = 0.75 \times 0.85\, [0.85 \times 4.5\, (201.06 - A_{st}) + 60 \times A_{st}]$

$A_{st} = 4.18\ in^2$

Use 6#8 bars ($A_{st} = 4.74\ in^2$)

**Design of spirals:**

diameter of core, $D_c = 16 - 2 \times 1.5 = 13$ in.

Area of the core, $A_c = \dfrac{\pi\, D_c^{\,2}}{4} = \dfrac{\pi \times 13^2}{4} = 132.73\ in^2$

Area of the column, $A_g = \dfrac{\pi\, D^2}{4} = \dfrac{\pi \times 16^2}{4} = 201.06\ in^2$

minimum percentage of steel, $\rho_s = 0.45 \left(\dfrac{A_g}{A_c} - 1\right) \dfrac{f_c'}{f_y}$

$\rho_s = 0.45 \left(\dfrac{201.06}{132.73} - 1\right) \dfrac{4.5}{60} = 0.0173$

Using #3 bar as spiral reinforcement, $a_s = 0.11\ in^2$ and $d_b = 0.375$ in

Spacing, $S = \dfrac{4\, a_s (D_c - d_b)}{\rho_s D_c^2} = \dfrac{4 \times 0.11\,(13 - 0.375)}{0.0173 \times 13^2} = 1.89$ in

Use #3 bar spiral at $S = 1.9$ in.

**Question:** (a) A reinforced concrete column 4 m long (effective) and 400 mm in diameter is reinforced with 8 bars of 20 mm diameter. Find safe load of columns can carry using WSD method. Concrete mix used is M15 and $f_y$ = 415Mpa. The column carries lateral ties. (b) If the effective length is increased to 8 m, what will be the safe load of column can carry?

**Solution:**

$A_g = \pi\, r^2 = \pi \times 200^2 = 125664\ mm^2$

$A_s = 8 \times 314.16 = 2513.28\ mm^2$

$f_c' = 15\ MPa = 15\ N/mm^2$

$f_y = 415\ MPa = 415\ N/mm^2$

$f_s = 0.4 f_y = 0.4 \times 415 = 166\ N/mm^2$

(a) $\frac{L_e}{b} = \frac{4 \times 1000}{400} = 10 < 12 \rightarrow$ Thus the column is short column.

$P = (0.25\ f_c'\ A_g + f_s\ A_s)$

$= 0.25 \times 15 \times 125664 + 166 \times 2513.28$

$= 888444.48\ N = 888.444\ KN$

(b) $\frac{L_e}{b} = \frac{8 \times 1000}{400} = 20 > 12$

Hence the column is a long column, Reduction factor will be used

$C_r = 1.25 - \frac{l_e}{48\,b} = 1.25 - \frac{8 \times 1000}{48 \times 400} = 0.83$

$P = C_r\ (0.25\ f_c'\ A_g + f_s\ A_s)$

$= 0.83\ (0.25 \times 15 \times 125664 + 166 \times 2513.28)$

$= 737408.91\ N = 737.408\ KN$

**Question:** A reinforced concrete column of effective length 7 m and column diameter is 300 mm with 8 – 16 mm rebar. Concrete mix used is M15 and $f_y$ = 415Mpa. Using working stress design (WSD) method to determine the column strength. (AB – 2017)

**Solution:**

$A_g = \pi\, r^2 = \pi \times 150^2 = 70686\ mm^2$

$A_s = 8 \times 201.06 = 1608.5\ mm^2$

$f_c' = 15\ MPa = 15\ N/mm^2$

$f_y = 415\ MPa = 415\ N/mm^2$

$f_s = 0.4 f_y = 0.4 \times 415 = 166\ N/mm^2$

$\frac{L_e}{b} = \frac{7 \times 1000}{300} = 23.33 > 12$

So it's a long column, Reduction factor will be used

$C_r = 1.25 - \frac{l_e}{48\,b} = 1.25 - \frac{7 \times 1000}{48 \times 300} = 0.76$

$P = C_r\ (0.25\ f_c'\ A_g + f_s\ A_s)$

$= 0.76 \times [0.25 \times 15 \times 70686 + 166 \times 1608.5]$

$= 404383.46\ N = 404.383\ KN$

**Question:** A reinforced concrete column of effective length 4.8 m and overall dimension 250mm x 400mm. Design axial load on column is 60 tones. Concrete mix used is M20 and mild steel used. Using working stress design (WSD) method to determine the reinforcement required for column also design ties spacing. (HBFC – 2018, Combined Bank – 2019)

**Solution:**

$A_g = 250 \times 400 = 10^5 \text{ mm}^2$

$f'_c = 20 \text{ MPa} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ MPa} = 415 \text{ N/mm}^2$

$f_s = 0.4 f_y = 0.4 \times 415 = 166 \text{ N/mm}^2$

$P = 60 \text{ tones} = 60 \times 9806 = 5.88 \times 10^5 \text{ N}$

$\frac{L_e}{b} = \frac{4.8 \times 1000}{250} = 19.2 > 12$

so it's a long column, Reduction factor will be used

$C_r = 1.25 - \frac{l_e}{48\,b} = 1.25 - \frac{4.8 \times 1000}{48 \times 250} = 0.85$

Design load, $P = \frac{5.88 \times 10^5}{0.85} = 6.91 \times 10^5 \text{ N}$

$P = 0.85\, A_g\, [0.25 f'_c + f_s \rho_g]$

$6.91 \times 10^5 = 0.85 \times 10^5\, [0.25 \times 20 + 166 \times \rho_g]$

$\rho_g = 0.01855$

$A_{st} = \rho_g \times A_g = 0.01855 \times 10^5 = 1885.18 \text{ mm}^2$

Area of 16mm bar, $A_{st} = 201.06 \text{ mm}^2$

No. of bars required = 1885.18 / 201.06 = 9.37 = 10 bar

**Tie bar design:** As main bar is 16 mm bar, use 10 mm bar as tie bar

1. $48d = 48 \times 10 = 480$ mm
2. $16D = 16 \times 16 = 256$ mm
3. Least dimension = 250 mm

Use 10 mm bar @ 250 mm c/c

# Footing Design

**What is footing?**

Footings are structural members used to support columns and walls and transmit their loads to the underlying soils.

**Question:** Draw and discuss different types of footing. (33th BCS)

**Solution:**

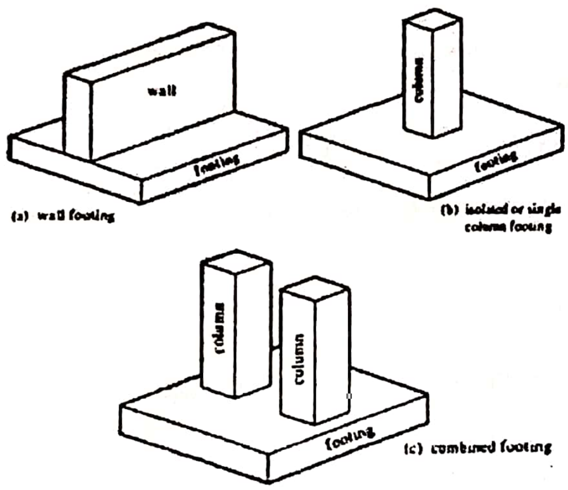
**Wall footing:** A wall footing, is simply an enlargement of the bottom of a wall that will sufficiently distribute the load to the foundation soil. Wall footihgs are normally used around the perimeter of a building and perhaps for some of the interior walls.

**Isolated footing:** An isolated or single-column footing, is used to support the load of a single column. These are the most commonly used footings, particularly where the loads are relatively light and the columns are not closely spaced.

**Combined footing:** Combined footings are used to support two or more column loads, a combined footing might be economical where two or more heavily loaded columns are so spaced that normally designed single-column footings would run into each other.

**Mat footing:** A mat or raft or floating foundation, is a continuous reinforced concrete slab over a large area used to support many columns and walls. This kind of foundation is used where soil strength is low or where column loads are large but where piles or caissons are not used.

**Pile cap:** Pile caps, are slabs of reinforced concrete used to distribute column loads to groups of piles.

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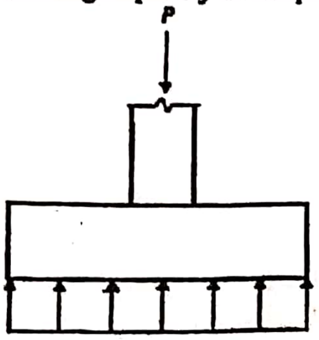
## Allowable Bearing Capacity

It's the load per unit area of the foundation can be resisted by the underlying soil without any unsafe movement occurs (shear failure) and if this load is exceeded, the shear failure will not occur in the underlying soil till reaching the ultimate load. The bearing capacity of soil or rock can be obtained from soil borings and tests performed by a geotechnical engineer. Allowable bearing pressure $q_a$ under service loads are usually based on a factor of 2.5 to 3.0 against exceeding the bearing capacity of the particular soil and to keep settlements within tolerable limits.




$q = \frac{P}{A_f}$

## Required Footing Area

For a concentrically loaded isolated spread footing, the required area of the footing $A$ is determined by dividing the total service load $P$ by the permissible soil pressure $q_e$:

$$\text{Footing area required, } A = \frac{\text{Service load}}{\text{Effective soil pressure}} = \frac{DL + LL}{q_e}$$

## One-Way Shear or Beam Shear

For footings with bending action in one direction, the critical section is located at a distance $d$ from the face of the column.

The allowable shear strength, $\varphi V_c = 2\,\varphi\,\lambda\sqrt{fc'}\,b\,d$

$\varphi = 0.75$ strength reduction factor for shear

Factored shear force $V_u$ at the critical section, $V_u = q_u\,B\,(c - d)$

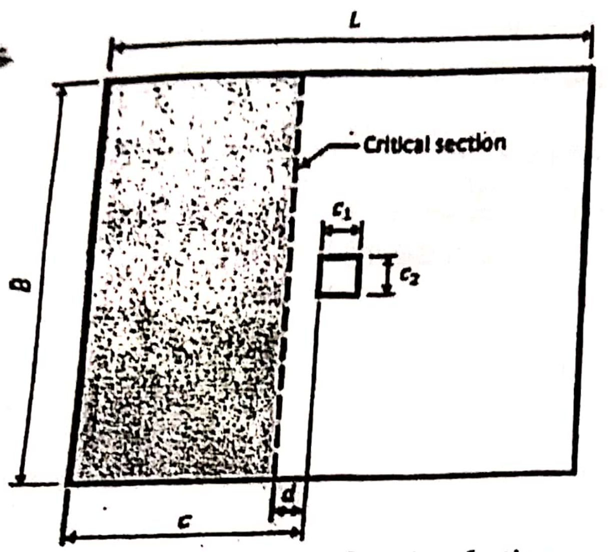
The following equation must be satisfied at the critical section, which in this case is located a distance $d$ from the face of the column

$$V_u = q_u\,B\,(c - d) \le \varphi V_c = 2\,\varphi\,\lambda\sqrt{fc'}\,b\,d$$

If no shear reinforcement is to be used then $d$ can be determined, assuming

$$V_u = \varphi V_c$$

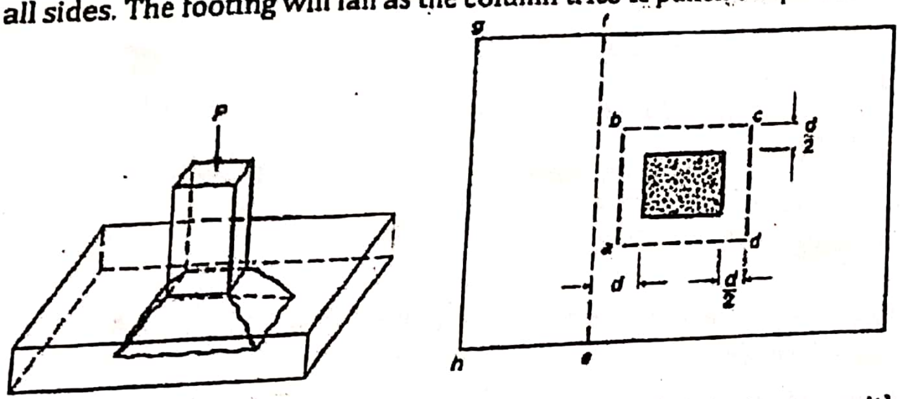
$$d = \frac{Vu}{2\,\phi\,\lambda\sqrt{fc'}\,b}$$




Critical section for one-way shear in a footing.

**Two-Way Shear or Punching Shear**

Two-way shear is a measure of the diagonal tension caused by the effect of the column load on the footing. Inclined cracks may occur in the footing at a distance $d/2$ from the face of the column on all sides. The footing will fail as the column tries to punch out part of the footing.




The AC1 Code. Section 11.11.2 allows a shear strength, $V_c$ in footings without shear reinforcement for two-way shear action the smallest of,

$$\varphi V_{c1} = \varphi\, 4\lambda \sqrt{f_c'}\, b_0\, d$$

$$\varphi V_{c2} = \varphi \left(2 + \frac{4}{\beta}\right) \lambda \sqrt{f_c'}\, b_0\, d$$

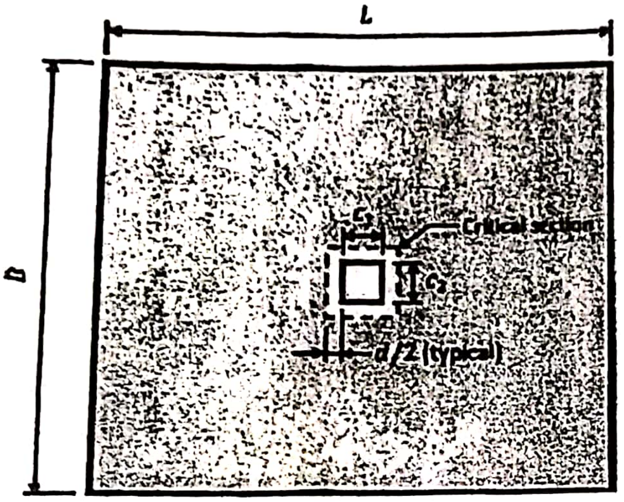
$$\varphi V_{c2} = \varphi \left(2 + \frac{\alpha\, d}{b_0}\right) \lambda \sqrt{f_c'}\, b_0\, d$$

Where,

$\beta$ = Ratio of long side to short side of the rectangular column

$b_o$ = perimeter of the critical section taken at d/2 from the loaded area

d = effective depth of footing
λ = is a modification factor for type of concrete (ACI 8.6.1)
λ = 1.0 Normal – weight concrete
λ = 0.85 for sand light – weight concrete
λ = 0.75 for all light – weight concrete
φ = 0.75 strength reduction factor for shear
α = 40 for interior columns, 30 for edge columns and 20 for corner columns.




The following equation must be satisfied at the critical section, which in this case is located a distance $d/2$ from the face of the column

$$V_u = q_u\,[B\,L - (C_1 + d)(C_2 + d)] \le \varphi\, V_c$$

$$\text{Or, } V_u = P_u - q_u\,[(C_1 + d)(C_2 + d)] \le \varphi\, V_c$$

$P_u$ = vertical downward factored load
$q_u$ = vertical upward pressure
Based on the preceding three values of $V_c$, the effective depth $d$ required for two way shear is the largest obtained from the following formulas.

$$d_1 = \frac{V_{u1}}{4\,\varphi\,\lambda\sqrt{f_c'}\,b_o} \quad \text{(Where } \beta \le 2\text{)}$$

$$d_1 = \frac{V_{u2}}{\left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'}\,b_o} \quad \text{(Where } \beta \ge 2\text{)}$$

$$d_2 = \frac{V_{u2}}{\left(2 + \frac{\alpha\,d}{b_o}\right)\lambda\sqrt{f_c'}\,b_o}$$

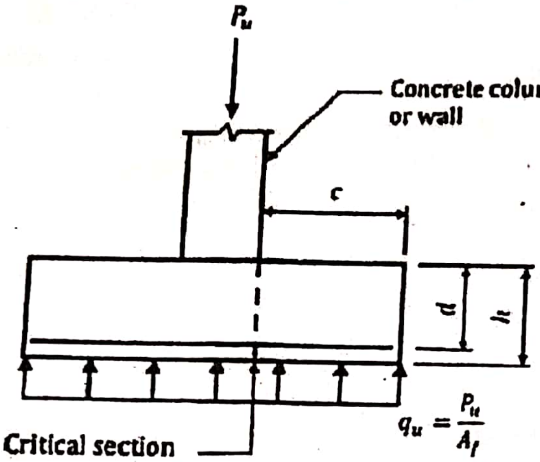
$b_o = 4\,(C + d)$ for square columns, where one side = $C$
$b_o = 2\,(C_1 + d) + 2\,(C_2 + d)$ for rectangular columns of sides $C_1$ and $C_2$

### Critical section for flexure

A spread footing must be designed for the bending moments that are induced because of the pressure developed at the base of the footing from the factored loads. The critical section for flexure for an isolated footing supporting a concrete column, pedestal, or wall is located at the face of the supported member. The maximum factored bending moment *Mu* at this critical section in this direction can be determined by the following equation, which is applicable to cantilevered members:

$$M_u = \frac{q_u\, B\, c^2}{2}$$

*c* is the distance from the critical section to the edge of the footing




Critical section for an isolated footing supporting a column, pedestal, or wall.

#### Determining the Required Reinforcement

Once the maximum factored moment $M_u$ at the critical section has been determined, the required area of reinforcing steel $A_s$ can be calculated using the strength design requirements

$$M_u = \frac{q_u\, B\, c^2}{2} \le \varphi\, M_n = \varphi\, A_s\, f_y \left(d - \frac{a}{2}\right)$$

An efficient design for footings would be one where the section is tension-controlled. Thus, the strength reduction factor $\varphi$ is equal to 0.9 in accordance with ACI 9.3.2.1

#### Minimum area of steel

$A_{s,min} = 0.0020\, b\, h$ for 40 or 50 grade deformed bars.

$A_{s,min} = 0.0018\, b\, h$ for 60 grade deformed bars.

$$A_{s,min} = \frac{(0.0018\, b\, h)\ 60{,}000}{f_y}$$ where reinforcement with $f_y > 60{,}000$ psi

Maximum spacing of flexural reinforcement = lesser of 3h or 18 in.

**Question:** A 12" square column reinforced with a 10 ft square footing. The footing thickness is 20" and clear cover is 3.5". Calculate the ultimate punching shear capacity if $f_c' = 3000$ psi, $f_y = 60000$ psi. (BWDB - 2014)

**Solution:**

Given, $t = 20"$

Assume, Clear cover = 3.5"

Effective depth, $d = 20" - 3.5" = 16.5"$

Punching perimeter, $b_0 = 4\,(a + d) = 4\,(12 + 16.5) = 114"$

Punching shear capacity $= 4\,\varphi\,\lambda\sqrt{f_c'}\,b_o d$

$\varphi\,V_c = 4 \times 0.75 \times 1 \times \sqrt{3000} \times 114 \times 16.5 = 309.08$ kip

**Question:** A RCC footing is to be constructed 6 feet below the grade. If allowable soil pressure is 5 ksf and self-weight of soil is 100 pcf, what will be the effective soil pressure at the bottom of footing if depth of footing is 20"? (WRGCL - 2014)

**Solution:**

Allowable soil pressure = 5 ksf

Weight of footing $= \frac{20}{12} \times \frac{150}{1000} = 0.25$ ksf

Weight of soil above footing $= (6 - \frac{20}{12}) \times \frac{100}{1000} = 0.433$ ksf

So, effective soil pressure $= 5 - 0.25 - 0.433 = 4.317$ ksf

**Question:** A footing size 10.5' x 10.5'. Factored load on footing is 330 kip and column size is 20" x 20". Calculate the depth of footing considering punching shear check only. $f_c' = 4$ ksi. (BCIC – 2017)

**Solution:**

Assume, depth of footing, $d = 14$ in

Factored load = 330 kip

Net upward pressure, $q_u = \frac{330}{10.5 \times 10.5} = 2.99$ ksf.

Punching perimeter, $b_0 = 4\,(c + d) = 4\,(20 + 14) = 136$ in

Developed shear strength, $V_u = P_u - q_u\,(c + d)^2 = 330 - 2.99 \times \left(\frac{20}{12} + \frac{14}{12}\right)^2 = 306$ kip

Now, $V_u = \varphi\,V_c = \varphi\,4\,\lambda\sqrt{f_c'}\,b_o\,d$

$$d = \frac{V_u}{4\,\varphi\,\lambda\sqrt{f_c'}\,b_o} = \frac{306 \times 1000}{4 \times 0.75 \times 1\sqrt{4000} \times 136} = 13.69 \text{ inch.}$$

**Question: If DL is 225 kips & LL is 185 kips on column, determine the footing size when bearing capacity of soil is 2 tsf. (BSEC – 2014)**

**Solution:**

Total service load $= DL + LL = 225 + 185 = 410$ kip

Bearing capacity = 2 tsf = 4 ksf

Footing Area $= \frac{\text{Service load}}{\text{Soil pressure}} = \frac{410}{4} = 102.5\ ft^2$

Considering square footing $= \sqrt{102.5} = 10.124\ ft \approx 10.5\ ft$

Footing size = 10.5 ft x 10.5 ft

**Question: A 16" square column reinforced with 8 nos. 20 mm bar up to footing and the column carries total 550 kip load. If the footing size is 9' x 9' and effective depth is 24", find the punching shear stress developed in the footing. (BWDB – 2019)**

**Solution:**

Here, Total load, $P_u = 550$ kip

Effective depth, $d = 24" = 2$ ft.

Punching perimeter, $b_0 = 4\,(c + d) = 4\left(\frac{16}{12} + \frac{24}{12}\right) = 13.33$ ft

Net upward pressure, $q_u = \frac{\text{Total load}}{\text{Area}} = \frac{550}{9 \times 9} = 6.79\ k/ft^2$

Developed shear strength, $V_u = P_u - q_u\,(c + d)^2$

$V_u = 550 - 13.33\left(\frac{16}{12} + \frac{24}{12}\right)^2 = 401.88$ kip

Punching shear stress developed $= \frac{V_u}{b_o\, d} = \frac{401.88}{13.33 \times 2} = 15.08\ k/ft^2$

**Question: A 20 inch square tied column 8 #9 bar carries load of 380 kip, Design a square footing by working stress method by following information, Allowable soil pressure 7000 lb./ft², $f_c' = 3000$ psi, $f_s = 20000$ psi (EED – 2019)**

**Solution:**

Assume, footing depth, $d = 19$ in and total depth, $h = 19 + 3 = 22$ in

Assume, the unit weight of soil = 110 pcf & the depth of foundation from GL = 6 ft

Effective soil pressure, $q_e = 7000 - 1.58 \times 150 - 4.42 \times 110 = 6276.8$ psf = 6.276 ksf

Footing area required, $A = \frac{\text{Load}}{\text{Effective soil pressure}} = \frac{380}{6.276} = 60.54\ ft^2$

Cosider 8'x 8' footing size, $A = 64\ ft^2 > 60.54\ ft^2$

Net upward pressure, $q_u = \frac{\text{Load}}{\text{Footing area}} = \frac{380}{8 \times 8} = 5.9375$ ksf.

**Punching shear check**

Punching perimeter, $b_0 = 2\,(C_1 + d) + 2\,(C_2 + d) = 2\,(20 + 19) + 2\,(20 + 19) = 156$ in

Shear developed, $V = P - q_u\,[(C_1 + d)(C_2 + d)]$

$$V = 380 - 5.9375\left[\frac{(20 + 19)(20 + 19)}{12 \times 12}\right] = 317.285 \text{ kip}$$

Developed shear stress, $v = \frac{V}{b_0\,d} = \frac{317.285 \times 1000}{156 \times 19} = 107.09$ psi

Allowable shear stress, $v_a = 2\sqrt{f_c'} = 2\sqrt{3000} = 109.54$ psi

Here, $v_a > v \rightarrow$ ok

**Beam shear check**

Length of footing outer face to column face, $c = \frac{8}{2} - \frac{20}{2 \times 12} = 3.17$ ft

Shear force at the critical section, $V = q_u\,B\,(c - d)$

$V = 5.9375 \times 8\,(3.17 - 1.58) = 84.96$ kip

Developed shear stress, $v = \frac{V}{b\,d} = \frac{84.96 \times 1000}{8 \times 12 \times 19} = 46.57$ psi

Allowable shear stress, $v_a = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.24$ psi

Here, $v_a > v \rightarrow$ ok

**Reinforcement calculation**

Moment, $M = \frac{q_u\,B\,c^2}{2} = \frac{5.9375 \times 8 \times 3.17^2}{2} = 238.71$ k – ft

$f_c = 0.45\,fc' = 0.45 \times 3000 = 1350$ psi

$$r = \frac{f_s}{f_c} = \frac{20000}{1350} = 14.81$$

$$k = \frac{n}{n + r} = \frac{9}{9 + 14.81} = 0.37$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.37}{3} = 0.87$$

Reinforcement required, $A_s = \frac{M}{f_s\,j\,d} = \frac{238.71 \times 1000 \times 12}{20000 \times 0.87 \times 19} = 8.66$ in$^2$

Using 22 mm dia (0.59 in$^2$) bar, Spacing $= \frac{0.59 \times 8 \times 12}{8.66} = 6.54$ in

Provide 22 mm diameter bar @ 6.5" c/c in both direction.

**Question:** A 15 in brick wall supports a DL = 12 k/ft and LL = 8 k/ft. The wall footing is to be placed at 5'-0" below the G.L and $q_a$ = 4 ksf is reported at that level. Design a RC footing using $fc'$ = 3.5 ksi and $f_y$ = 60 ksi. (34th BCS)

**Solution:**

Assume, total depth, $h = 12$ in and effective depth, $d = 12 - 3 = 9$ in

Assume, the unit weight of soil = 100 pcf

**Footing area calculation:**

The depth of foundation from GL = 5 ft

Effective soil pressure, $q_e = 4000 - 1 \times 150 - 4 \times 100 = 3450$ psf = 3.450 ksf

Calculate the width of the footing for a 1 ft length of the wall:

$$\text{Width of footing} = \frac{\text{Service load}}{\text{Effective soil pressure}} = \frac{12 + 8}{3.450} = 5.79 \text{ ft} \rightarrow \text{Use } 6' \text{ft}$$

$$\text{Net upward pressure}, q_u = \frac{\text{Factored load}}{\text{Footing area}} = \frac{1.2 \times 12 + 1.6 \times 8}{6 \times 1} = 4.5333 \text{ ksf}$$

**One way shear check:**

$$\text{Length of footing outer face to column face}, c = \frac{6}{2} - \frac{15}{2 \times 12} = 2.375$$

Factored shear force $V_u$ at the critical section, $V_u = q_u (c - d)$

$V_u = 4.5333 (2.375 - 0.75) = 7.366$ kip

Allowable one way shear strength, $\varphi V_c = 2 \varphi \lambda \sqrt{fc'}\, b\, d$

$\varphi V_c = 2 \times 0.75 \times 1 \sqrt{3500} \times 12 \times 9 = 9584.04$ lb = 9.584 kip

Here, $\varphi V_c > V_u \rightarrow$ ok

**Moment and Reinforcement calculation:**

$$\text{Moment}, M = \frac{q_u c^2}{2} = \frac{4.5333 \times 2.375^2}{2} = 12.78 \text{ k} - \text{ft/ft}$$

$$M_u = \varphi A_s f_y \left(d - \frac{a}{2}\right)$$

$$A_s = \frac{12.78 \times 12}{0.9 \times 60 \times (9 - 0.27)} = 0.32 \text{ in}^2/\text{ft} \qquad (\text{Assume, a} = 0.54 \text{ in})$$

$$\text{Check } a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.32 \times 60}{0.85 \times 3.5 \times 12} = 0.54 \text{ in} \rightarrow \text{Ok}$$

$$\rho_{min} = \frac{200}{f_y} = 0.0033$$

$$A_{s,min} = \rho\, b\, d = 0.0033 \times 12 \times 9 = 0.36 \text{ in}^2/\text{ft}$$

Using #5 bar $= \frac{0.31 \times 12}{0.36} = 10.33$ in

Provide #5 bar @ 10 in c/c as main bar

Transverse direction – Shrinkage and Temperature steel

$A_s = 0.0018\, b\, t = 0.0018 \times 12 \times 12 = 0.25\ \text{in}^2/\text{ft}$

Use #4 bar @ 9 in c/c

**Question: Design a suitable spread footing for 15" x 15" RC column reinforced with W 7/8" φ under the following condition: $P_{DL} = 100$ k, $P_{LL} = 70$ k, $f_c' = 3$ ksi, $f_y = 60$ ksi, $q_a = 1.5$ tsf at 5' below the G.L. Make a suitable sketch of footing mentioning the reinforcement details. (30th BCS)**

**Solution:**

Assume, total depth, $h = 16$ in and footing effective depth, $d = 16 - 3.5 = 12.5$ in

Assume, the unit weight of soil = 100 pcf

Allowable soil pressure, $q_a = 1.5$ tsf = 3000 ksf

Factored laod, $P_u = 1.2\, DL + 1.6\, LL = 1.2 \times 100 + 1.6 \times 70 = 232$ kip

**Footing area calculation:**

The depth of foundation from GL = 5 ft

Effective soil pressure, $q_e = 3000 - \frac{16}{12} \times 150 - \frac{44}{12} \times 100 = 2433.33$ psf = 2.433 ksf

Width of footing $= \frac{\text{Service load}}{\text{Effective soil pressure}} = \frac{100 + 70}{2.433} = 69.87\ \text{ft}^2$

Use 8.5' x 8.5' square footing = 72.25 ft²

Net upward pressure, $q_u = \frac{\text{Factored load}}{\text{Footing area}} = \frac{1.2 \times 100 + 1.6 \times 70}{72.25} = 3.21$ ksf.

**Punching shear check**

Punching perimeter, $b_0 = 4\,(c + d) = 4\,(15 + 12.5) = 110$ in = 9.17 ft

Shear developed, $V_u = P_u - q_u\,(c + d)^2$

$V_u = 232 - 3.21 \times \left(\frac{15}{12} + \frac{12.5}{12}\right)^2 = 215.14$ kip

Allowable shear, $\varphi V_c = \varphi\, 4\, \lambda \sqrt{f_c'}\, b_0\, d$

$\varphi V_c = 0.75 \times 4 \times 1 \times \sqrt{3000} \times 110 \times 12.5 = 225935.55$ lb = 225.93 kip

Here, $\varphi V_c > V_u \rightarrow$ ok

**One way shear check:**

Length of footing outer face to column face, $c = \frac{8.5}{2} - \frac{15}{2 \times 12} = 3.625$ ft

Shear force $V_u$ at the critical section, $V_u = q_u B (c - d)$

$V_u = 3.21 \times 8.5 (3.625 - 1.04) = 70.53$ kip

Allowable shear strength, $\varphi V_c = 2 \varphi \lambda \sqrt{fc'}\, b\, d$

$\varphi V_c = 2 \times 0.75 \times 1 \sqrt{3000} \times 8.5 \times 12 \times 12.5 = 104751.93$ lb $= 104.75$ kip

Here, $\varphi V_c > V_u \rightarrow$ ok

**Moment and Reinforcement calculation:**

Moment, $M = \frac{q_u B c^2}{2} = \frac{3.21 \times 8.5 \times 3.625^2}{2} = 179.27$ k−ft

$M_u = \varphi A_s f_y (d - \frac{a}{2})$

$A_s = \frac{179.27 \times 12}{0.9 \times 60 \times (12.5 - 0.37)} = 3.28$ in$^2$ (Assume, a = 0.74 in)

Check $a = \frac{A_s f_y}{0.85\, f_c'\, b} = \frac{3.28 \times 60}{0.85 \times 3 \times 8.5 \times 12} = 0.75$ in $\rightarrow$ 0k

$\rho_{min} = \frac{200}{f_y} = 0.0033$

$A_{s,min} = \rho\, b\, d = 0.0033 \times 8.5 \times 12 \times 12.5 = 4.20$ in$^2$

Use 14 nos # 5 bar with, $A_s = 4.34$ in$^2$

Spacing, $S = \frac{102 - 6}{13} = 7.38$ inch $c/c$ in both direction.

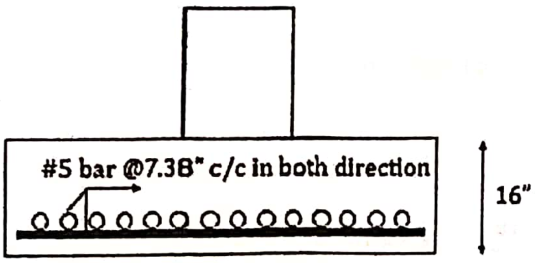



Figure: Typical reinforcement details

**Question:** An 18" x 18" column with $fc' = 4$ ksi, reinforced with eight no. 8 bars of $f_y = 50$ ksi, supports a dead load of 225 kips and a live load of 175 kips. The allowable soil pressure $q_a$ is 5 ksf. Design a square footing 5′ below grade, using $fc' = 4$ ksi and $f_y = 50$ ksi. (40th BCS)

**Solution:**

Assume, total depth, $h = 22$ in and footing effective depth, $d = 22 - 3.5 = 18.5$ in

Assume, the unit weight of soil = 100 pcf

Allowable soil pressure, $q_a = 5000$ ksf

Factored laod, $P_u = 1.2\,DL + 1.6\,LL = 1.2 \times 225 + 1.6 \times 175 = 550$ kip

**Footing area calculation:**

The depth of foundation from GL = 5 ft

Effective soil pressure, $q_e = 5000 - \frac{22}{12} \times 150 - \frac{38}{12} \times 100 = 4408.33$ psf $= 4.408$ ksf

Width of footing $= \frac{\text{Service load}}{\text{Effective soil pressure}} = \frac{225 + 175}{4.408} = 90.74$ ft$^2$

Use 10′ x 10′ square footing = 100 ft$^2$

Net upward pressure, $q_u = \frac{\text{Factored load}}{\text{Footing area}} = \frac{1.2 \times 225 + 1.6 \times 175}{100} = 5.5$ ksf.

**Punching shear check**

Punching perimeter, $b_0 = 4\,(c + d) = 4\,(18 + 18.5) = 146$ in $= 12.17$ ft

Shear developed, $V_u = P_u - q_u\,(c + d)^2$

$$V_u = 550 - 5.5 \times \left(\frac{18}{12} + \frac{18.5}{12}\right)^2 = 499.11 \text{ kip}$$

Allowable shear, $\varphi V_c = \varphi\, 4\, \lambda \sqrt{f_c'}\, b_0\, d$

$\varphi V_c = 0.75 \times 4 \times 1 \times \sqrt{4000} \times 146 \times 18.5 = 512478.7$ lb $= 512.47$ kip

Here, $\varphi V_c > V_u \rightarrow$ ok

**One way shear check:**

Length of footing outer face to column face, $c = \frac{10}{2} - \frac{18}{2 \times 12} = 4.25$ ft

Shear force $V_u$ at the critical section, $V_u = q_u\, B\, (c - d)$

$V_u = 5.5 \times 10\,(4.25 - 1.54) = 149.05$ kip

Allowable shear strength, $\varphi V_c = 2\, \varphi\, \lambda \sqrt{fc'}\, b\, d$

$\varphi V_c = 2 \times 0.75 \times 1\, \sqrt{4000} \times 10 \times 12 \times 18.5 = 210607.69$ lb $= 210.60$ kip

Here, $\varphi V_c > V_u \rightarrow$ ok

**Moment and Reinforcement calculation:**

Moment, $M = \frac{q_u B c^2}{2} = \frac{5.5 \times 10 \times 4.25^2}{2} = 496.71\ k-ft$

$M_u = \varphi A_s f_y (d - \frac{a}{2})$

$A_s = \frac{496.71 \times 12}{0.9 \times 50 \times (18.5 - 0.45)} = 7.33\ in^2$ (Assume, a = 0.9 in)

Check $a = \frac{A_s f_y}{0.85\ f_c'\ b} = \frac{7.33 \times 50}{0.85 \times 4 \times 10 \times 12} = 0.89$ in → Ok

$\rho_{min} = \frac{200}{f_y} = 0.0033$

$A_{s,min} = \rho\ b\ d = 0.0033 \times 10 \times 12 \times 18.5 = 7.32\ in^2$

Use 24 nos # 5 bar with, $A_s = 7.44\ in^2$

Spacing, $S = \frac{120 - 6}{23} = 4.95$ inch $c/c$ in both direction.

**Question: Differentiate between pre-stressed beam and reinforced beam? (MES – 2015)**

**Solution:**

Pre-stressed concrete is a concrete block which is pre-tensioned during its construction. Here the concrete is pre-stressed in tension so when the service load is applied the entire section remains in compression. Stress distribution diagram will always show compressive stress in the precast pre-stressed concrete block. Whereas reinforced cement concrete uses reinforcing bars to resist the tension stresses which develops below the neutral axis during sagging moment. The stress distribution diagram for the same would show compression above the neutral axis and tensile stress below the neutral axis hence the tensile stresses developed are counteracted by reinforcing bars.

**Question: Write down the advantages and disadvantages of pre-stressing member? (MES – 2015)**

**Solution:**

**Advantages of pre-stressed concrete**

- Pre-stressed concrete is more durable since there are no tensile cracks, whereas in reinforced concrete tensile cracks are unavoidable.
- As high strength concrete is used and also since dead load moments can be neutralized pre-stressed concrete, dead weight of pre-stressed concrete member is much less compared to what is required for reinforced concrete member.
- The material cost in pre-stressed concrete is much less than that in equivalent reinforced concrete members.
- The deformation of pre-stressed members is much less compared to that for an equivalent reinforced concrete member.
- Higher moment of inertia (high stiffness)
- Increase in shear capacity.
- Reduction of formwork
- Rapid construction
- The fatigue strength of pre-stressed is very good.
- Reduces consumption of reinforcement steel.
- High span-to-depth ratios larger spans possible with pre-stressing.

**Disadvantages of pre-stressed concrete**

- The main disadvantage of pre-stressing is that it requires some special equipment like jacks, anchorage etc.
- High tensile steel is required for pre-stressing that is very difficult to procure.
- It requires highly skilled workers and should be prepared under expert supervision.
- It needs special technique to apply pre-stressing forces and Anchorage the wires.

**Question:** Why high strength concrete is used in pre-stressed concrete? (DMTCL – 2019, 31th BCS)

**Solution:**

High strength concrete is needed in pre-stressing over the low strength concrete due to following reasons,

- Due to large pre-stressing force to be applied by tendon, large bearing stress gets developed in concrete at the ends by anchoring devices.
- Bursting stresses at the ends cannot be effectively resisted by low strength concrete.
- Stresses transferred from tendons to concrete takes due to bond between concrete and steel. High strength bond can be achieved by the use of high strength concrete.
- With the use of high strength concrete in pre-stress, Shrinkage cracks are low as compared to low strength concrete.
- Use of high strength concrete reduces the size of concrete member thereby resulting in reduction of concrete and the self-weight of member.

**Question:** Distinguish between Pre-stress concrete and traditional RCC (BHP – 2017)

**Solution:**

- In RCC beam the concrete in the compression side of the neutral side of the axis alone is effective. The concrete in the tension side of the neutral axis is ineffective. But in the pre-stressed concrete beam, the entire section is effective.
- Reinforced concrete beams are generally heavy. They always need shear reinforcements besides the longitudinal reinforcement for flexure. Pre-stressed concrete beams are lighter. By providing the curved tendons and the pre-compression, a considerable part of the shear is resisted.
- In reinforced concrete beams, high strength concrete is not needed. But in pre-stressed concrete beams, high strength concrete and high strength steel are necessary. High strength concrete is needed to resist high stresses at the anchorages. High strength steel is needed to transfer large pre-stressing force.
- Reinforced concrete beams being massive and heavy are more suitable in situations where the weight is more desired than strength. Pre-stressed concrete beams are very suitable for heavy loads and longer spans. They are slender and artistic treatments can be easily provided. Cracks do not occur under working loads. Even if a minute crack occurs when overloaded, such crack gets closed when the overload is removed. The deflections of the pre-stressed concrete beams are small.
- In reinforced concrete beams, there is no way testing the steel and the concrete. In pre-stressed concrete beams, testing of steel and concrete can be made while pre-stressing.

**Question:** Describe the precautions to be taken during construction for the workers and pedestrians for a construction site. (EED – 2019)

**Solution:**

**Safety training:** All workers must complete safety training, to point out any high risk areas and provide instructions for emergency management.

**Minimize and manage risk:** Conducting regular safety audits and having procedures in place to report, assess and address potential risks.

**Site security:** Security in and outside of work hours is important to protect potential construction hazards. This includes supervision or authorized site visitors.

**Safe work method assessment:** A safe work method statement (SWMS) must be prepared for all high risk construction projects, before work commences. The SWMS should outline the scope of work involved, any potential safety issues, and how risks will be prevented and managed.

**Use clear signage:** Should use clear signage throughout the construction site, for both workers and pedestrians. Including a 24 hour emergency contact number and a map or directions to the site office.

**Chemical storage:** Chemicals need to be stored very carefully to minimize fires, explosions, chemical injury and pollution on worksites.

**Environmental conditions:** Extreme weather conditions can cause serious safety hazards. That's why environmental safety should follow

**Provide personal protective (PPS) equipment:** In many situations an employer is obligated to provide PPS such as high vis vests, safety goggles and safety harnesses to construction site workers.

**Dropped objects:** It is your responsibility to secure objects onsite and minimize the risk of them falling. This type of risks can be avoided by putting preventative safety measures in place.

**Question:** Write down the causes why building collapse? (EED – 2019)

**Solution:**

**Artificial reasons:**

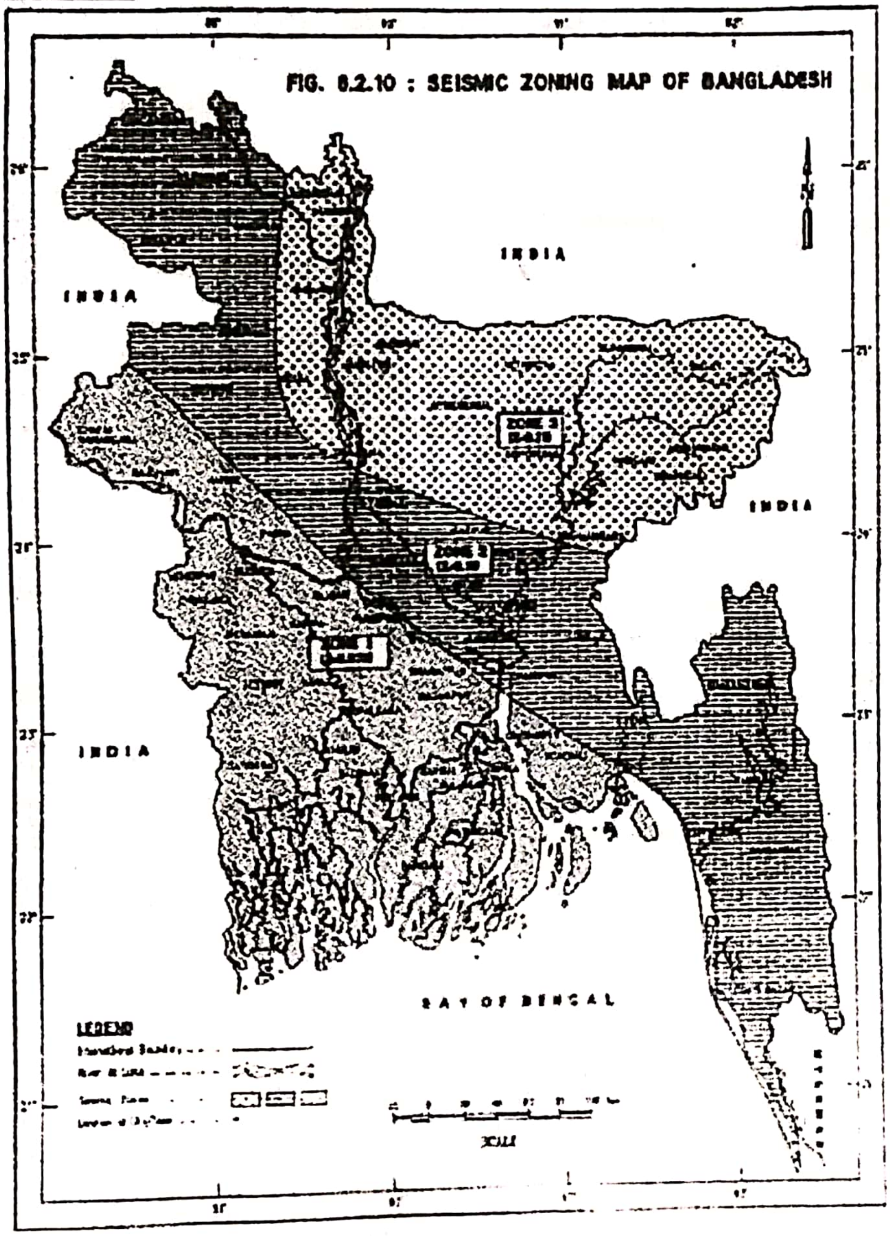
1. Inadequate foundation design
2. Errors in structural design.
3. Quality building materials aren't used.
4. Inexperienced & unskilled labor
5. The load is heavier than calculated
6. Use buildings for multipurpose (Ex: Using residential building as commercial building)
7. Use after the expiry of life of buildings.
8. Maintenance not done properly.

**Natural reasons:**

1. Earthquake
2. Landslides or settlement of Sub-base soil due to over drainage.
3. Tsunami
4. Typhoon and winds with high speed.

**Question: 1993 সন অনুযায়ী বাংলাদেশের সিসমিক জোন ম্যাপ টি অংকন করো এবং ম্যাপে বিভিন্ন জোন এবং zone-coefficient গুলো লিখুন। (LGED – 2019)**

**Solution:**



**Question:** What is meant by PPE? Name the PPEs for supervision Engineer (EED – 2019)

**Solution:**

**PPE:** Personal protective equipment (PPE) is protective clothing, helmets, goggles, or other safety equipment at a construction site.

**PPEs for supervision Engineer:** Depending on the circumstances and potential hazards present, additional PPE may be required. This determination will be made by your supervisor based on the preliminary Job Hazard Analysis; EHS may also be consulted.

PPE may include:

1. Protective gloves
2. Hearing protection
3. Full face shields when cutting, grinding, or chipping
4. Chemical splash goggles
5. Respiratory protection
6. Fall protection equipment when working above 6 feet
7. Specific protective clothing such as welding leathers when welding

**Question:** Name some non-destructive tests for checking quality and quantity of infrastructure. (EED – 2019)

**Solution:**

Non-destructive tests performed to determine the mechanical characteristics of materials & build a diagnostic picture of the structures in reference to their safety , expire of structure and relevant.

1. Visual inspection
2. Ultrasonic pulse velocity
3. Acoustic emission
4. Electromagnetic cover measurement
5. Rebound hammer test
6. Corrosion test
7. Ground Penetrating Radar (GPR)
8. Load and monitoring tests core extraction for compressive strength test
9. Structural monitoring of crack patterns
10. Thermography.

**Question:** Difference between bridge and culvert in detail. (EED – 2015)

**Solution:**

| Bridge | Culvert |
|---|---|
| A bridge is a passage of transportation over a large body of water or physical obstruction. | A culvert is generally a tunnel-like structure that allows water to pass under a roadway or railway. |
| Bridges are constructed at a height more than 20 feet. | Culverts are built at less than 20 feet high over the obstruction. |
| A bridge spans from 6 meters (minor bridges) to more than 120 meters. | The length of culverts is typically not more than 6 meters. |
| Piers and abutments are the supporting structures of a bridge. | Culverts are usually embedded in the soil which bears the major portion of the culvert load. |
| A bridge is usually a linear and straight passage. | Culverts are totally enclosed structures that can be semi-circular, rectangular, elliptical or pear-shaped. |
| Bridges are usually constructed at the site or pre-constructed in smaller parts. | Culverts can be pre-constructed or built at the site (in situ culverts). |
| Bridges provide an easier route of transportation that saves time and reduces distance. | Culverts prevent water logging, flood, and erosion, and allows water to flow its natural course under a roadway or railway. |

**Question:** Write down the percentage losses of pretension and post tension materials. (31th BCS)

**Solution:**

| TYPES OF LOSSES | PRE-TENSIONING | POST-TENSIONING |
|---|---|---|
| Elastic deformation of concrete | Yes | No loss due to elastic deformation if all the wires are simultaneously tensioned. If the wires are successively tensioned, there will be loss of pre-stress due to elastic deformation of concrete. |
| Relaxation of stress in steel | Yes | Yes |
| Shrinkage of concrete | Yes | Yes |
| Creep of concrete | Yes | Yes |
| Friction | No | Yes |
| Anchorage grip | No | Yes |

**Question: write down the factors influencing development length of bar. (32th BCS)**

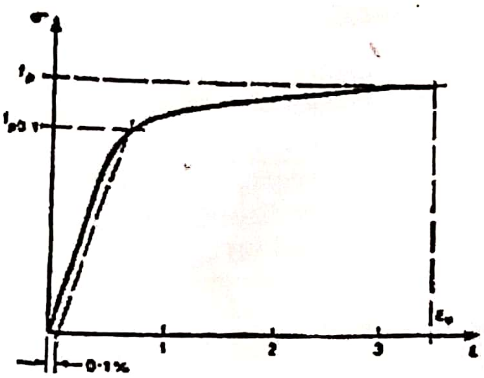
**Solution:**

**Factors influencing development length of bar are given below:**

1. **Compressive Strength of the**
2. **Density of the concrete (Light weight or normal weight)**
3. **Rebar Clear**
4. **Rebar Center to Center Spacing**
5. **Transverse Reinforcement**
6. **Vertical location of the horizontal re-bars**
7. **Coating of rebar**
8. **Rebar**

**Question: Draw typical stress strain curve for pre-stressing steels. (40th BCS)**

**Solution:**




**Question: Discuss various ways of failure of reinforced concrete structure. (40th BCS)**

**Solution:**

Any structure may experience a phenomenon known as structural failure or collapse. Structural failure is the total or partial loss of the structure's integrity and its load bearing capacity. Such an incident occurs when the material of the construction exceeds its strength limits and breaks or experiences excessive deformations. When it comes to concrete buildings, the usual types of structural failure are the following:

**Crushing failure of columns:** When reinforced concrete columns are subjected to intense loading conditions, both steel and concrete yield and the column fails. Earthquake loads that apply lateral stresses highly affect the solidity of the column which may get crushed and lose its bearing capacity.

**Shear cracking of columns:** Reinforced concrete columns may undergo shear failure mostly caused by seismic forces. The cracks usually appear diagonally and may take a spiral shape when the structure experience twisting.

**Cracking of column-beam junction:** The joint of a column and a beam usually experiences high bending and axial stresses which may lead to severe cracking reducing the strength of the junction

**Short Column Effect:** This phenomenon is related to the construction of infills walls attached to the columns. The interaction of the column with the wall leads to high stress concentration. The column is restricted by the wall and therefore, its lateral deformability (compared to its height) is highly increased. Consequently, a short column is much stiffer than other columns and accumulates higher shear stress that leads to diagonal cracking and failure.

**Infill Walls collapse:** Infill walls that are connected with the building's framework may collapse. To prevent such failures, walls should be tightly jointed with the concrete beams and columns.

**Gable Frames failure:** Gable frames are commonly used for large structures such as schools and cinema halls. They are prone to collapse and they do not present any residual strength.

**Reinforcing bars pull out:** Reinforcing bars can be pulled out due to tensile stresses caused in the column when their anchor length inside the column is inadequate and they cannot fully reach their tensile strength.

**Articulated beams:** Articulated beams are prone to sliding when large lateral stresses capable of exceeding frictional resistance, are applied.

**Sinking and tilting of the foundation:** The foundation of a building may experience sinking or tilting during an earthquake. This is caused due to ground's response and is amplified in soft soils that experiences large deformations. In both cases, the superstructure will suffer critical damage which may lead to total collapse.

**Question:** Difference between Pre cast and cast in situ pile. (*HED – 2017*)

**Solution:**

**Cast in situ piles**

Advantages

1. Piles of any size and length may be constructed at the site.
2. Damage due to driving and handling that is common in precast piles is eliminated in this case.
3. These piles are ideally suited in places where vibrations of any type are required to be avoided to preserve the safety of the adjoining structure.

Disadvantages

1. Installation of cast-in-situ piles requires careful supervision and quality control of all the materials used in the construction.
2. The method is quite cumbersome. It needs sufficient storage space for all the materials used in the construction.
3. Construction of piles in holes where there is heavy current of ground water flow or artesian pressure is very difficult.

**Pre-cast Piles**

**Advantages**

1. Precast piles can be cast well before the initiation of the work bringing about fast execution work.
2. Construction of precast piles can be inspected easily and if any deformity distinguished can be redressed before utilize.
3. The reinforcement stays at the right position and doesn't get dislocated.
4. They can be driven under water.
5. The driving of neighboring pile does not create unfriendly impact on the piles already driven.

**Disadvantages**

1. Due to their heavy weight handling and transportation of precast piles is very difficult.
2. The correct length of a pile can once in a while be pre-decided and hence it must be extended sometimes, rendering the piles becomes weaker at the joint.
3. Piles are subjected to the driving shocks after the concrete has completely set. This may lead to defective construction.