

$$\begin{aligned} 8(a) \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx \\ = \int \frac{2\cos^2 x - 1 - (2\cos^2 \theta - 1)}{\cos x - \cos \theta} dx \\ = 2 \int \frac{\cos^2 x - \cos^2 \theta}{\cos x - \cos \theta} dx \\ = 2 \int \frac{(\cos x + \cos \theta)(\cos x - \cos \theta)}{\cos x - \cos \theta} dx \end{aligned}$$

$$\begin{aligned} &= 2 \int (\cos x + \cos \theta) dx \\ &= 2(\int \cos x dx + \cos \theta \int dx) \\ &= 2(\sin x + \cos \theta \cdot x) + c \\ &= 2(\sin x + x \cos \theta) + c \end{aligned}$$

$$\begin{aligned} 8(b) \int (\sec x + \tan x)^2 dx \\ = \int (\sec^2 x + \tan^2 x + 2\sec x \tan x) dx \\ = \int (\sec^2 x + \sec^2 x - 1 + 2\sec x \tan x) dx \\ = \int (2\sec^2 x - 1 + 2\sec x \tan x) dx \\ = 2 \tan x - x + 2\sec x + c \end{aligned}$$

$$\begin{aligned} 9(a) \int \sqrt{1 \pm \sin x} dx \\ = \int \sqrt{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} \pm 2\sin \frac{x}{2} \cos \frac{x}{2}} dx \end{aligned}$$

$$\begin{aligned} &= \int \sqrt{(\sin \frac{x}{2} \pm \cos \frac{x}{2})^2} dx \\ &= \int (\sin \frac{x}{2} \pm \cos \frac{x}{2}) dx \text{ বা } \int (\cos \frac{x}{2} \pm \sin \frac{x}{2}) dx \\ &= 2(-\cos \frac{x}{2} \pm \sin \frac{x}{2}) + c \end{aligned}$$

$$\text{বা } 2(\sin \frac{x}{2} \mp \cos \frac{x}{2}) + c \quad (২)$$

$$\begin{aligned} 9(b) \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx \\ = \int \frac{\sin x + \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}} dx \\ = \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} dx \end{aligned}$$

$$= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int dx = x + c \quad (১)$$

$$9(c) \int \frac{\cos x + \sin x}{\cos x - \sin x} (1 - \sin 2x) dx \quad (১)$$

$$= \int \frac{\cos x + \sin x}{\cos x - \sin x} (\cos x - \sin x)^2 dx \quad (১)$$

$$= \int (\cos x + \sin x)(\cos x - \sin x) dx \quad (১)$$

$$= \int (\cos^2 x - \sin^2 x) dx = \int \cos 2x dx \quad (১)$$

$$= \frac{1}{2} \sin 2x + c \quad (২)$$

$$9(d) \int (\sin \frac{x}{2} + \cos \frac{x}{2})^2 dx$$

$$= \int (\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2\sin \frac{x}{2} \cos \frac{x}{2}) dx$$

$$= \int (1 + \sin x) dx = x - \cos x + c \quad (২)$$

$$10. \int \cos^3 x dx = \int \frac{1}{4} (3\cos x + \cos 3x) dx \quad (১)$$

$$= \frac{1}{4} (3\sin x + \frac{1}{3}\sin 3x) + c \quad (১)$$

প্রশ্নমালা X B

নিচের যোগজগুলি নির্ণয় কর :

$$1.(a) \int \frac{1}{\sqrt[3]{(1-4x)}} dx = \int \frac{1}{(1-4x)^{1/3}} dx$$

$$= \int (1-4x)^{-1/3} dx = \frac{(1-4x)^{-1/3+1}}{(-1/3+1)(-4)} + c$$

$$= \frac{(1-4x)^{2/3}}{\frac{2}{3}(-4)} + c = -\frac{3}{8}(1-4x)^{2/3} + c$$

$$1(b) \int \frac{e^{5x} + e^{3x}}{e^x + e^{-x}} dx \quad [\text{প্র.ভ.প. '৯২}]$$

$$= \int \frac{e^{4x}(e^x + e^{-x})}{e^x + e^{-x}} dx = \int e^{4x} dx = \frac{e^{4x}}{4} + c$$

$$1(c) \text{ ধরি, } I = \int \sin x^0 dx$$

এক $x^\circ = \frac{\pi x}{180} = z$.

তাহলে $\frac{\pi}{180} dx = dz \Rightarrow dx = \frac{180}{\pi} dz$ এবং

$I = \frac{180}{\pi} \int \sin z dz = \frac{180}{\pi} (-\cos z) + c$

$\therefore \int \sin x^\circ dx = -\frac{180}{\pi} \cos x^\circ + c$

2(a) ধরি, $I = \int \sin 5x dx$ [সি.'০৫]

এক $5x = z$. তাহলে $5dx = dz \Rightarrow dx = \frac{1}{5} dz$

এক $I = \frac{1}{5} \int \sin z dz = -\frac{1}{5} \cos z + c$

$\therefore \int \sin 5x dx = -\frac{1}{5} \cos 5x + c$

2(b) ধরি, $I = \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$ [কু.'০০]

এক $\sqrt{x} = z$. তাহলে $\frac{dx}{2\sqrt{x}} = dz \Rightarrow \frac{dx}{\sqrt{x}} = 2dz$

এক $I = 2 \int \cos z dz = 2 \sin z + c$

$\therefore \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = 2 \sin \sqrt{x} + c$

2(c) $\int \frac{1}{x^2} \sin \frac{1}{x} dx$ [সি.'০৪; য.'০৭]

ধরি, $\frac{1}{x} = z \therefore -x^{-2} dx = dz \Rightarrow \frac{1}{x^2} dx = -dz$

$\therefore \int \frac{1}{x^2} \sin \frac{1}{x} dx = \int \frac{\sin(1/x)}{x^2} dx$

$= -\int \sin z dz = -(-\cos z) + c = \cos \frac{1}{x} + c$

3. (a) ধরি, $I = \int x e^{x^2} dx$ [ব.'০৩]

এক $x^2 = z$. তাহলে, $2x dx = dz \Rightarrow x dx = \frac{dz}{2}$

এক $I = \frac{1}{2} \int e^z dz = \frac{1}{2} e^z + c = \frac{1}{2} e^{x^2} + c$

3(b) ধরি, $I = \int x^2 a^{x^3} dx$ [মা.'০৯]

এক $x^3 = z$. তাহলে, $3x^2 dx = dz \Rightarrow x^2 dx = \frac{dz}{3}$

এক $I = \frac{1}{3} \int a^z dz = \frac{a^z}{3 \ln a} + c = \frac{a^{x^3}}{3 \ln a} + c$

3(c) $\int e^x \tan e^x \sec e^x dx$

$= \int \sec e^x \tan e^x d(e^x)$ [$\because d(e^x) = e^x dx$]

$= \sec e^x + c$

3(d) ধরি, $I = \int e^{2x} \tan e^{2x} \sec e^{2x} dx$ [চ.'০৭]

এক $e^{2x} = z$. তাহলে, $2e^{2x} dx = dz$ এবং

$I = \frac{1}{2} \int \sec z \tan z dz = \frac{1}{2} \sec z + c$

$\therefore \int e^{2x} \tan e^{2x} \sec e^{2x} dx = \frac{1}{2} \sec e^{2x} + c$

4(a) ধরি, $I = \int \sin^2 x \cos x dx$ [সি.'০২]

এক $\sin x = z$. তাহলে, $\cos x dx = dz$ এবং

$I = \int z^2 dz = \frac{1}{3} z^3 + c = \frac{1}{3} \sin^3 x + c$

4(b) ধরি, $I = \int (1 + \cos x)^3 \sin x dx$ [কু.'০৩]

এক $1 + \cos x = z$. তাহলে, $-\sin x dx = dz$ এবং

$I = -\int z^3 dz = -\frac{z^4}{4} + c = -\frac{(1 + \cos x)^4}{4} + c$

4(c) ধরি, $I = \int \sin^2 \frac{x}{2} \cos \frac{x}{2} dx$ [চ.'০৩]

এক $\sin \frac{x}{2} = z$. তাহলে, $\frac{1}{2} \cos \frac{x}{2} dx = dz$ এবং

$I = 2 \int z^2 dz = 2 \cdot \frac{1}{3} z^3 + c = \frac{2}{3} \sin^3 \frac{x}{2} + c$

4(d) ধরি, $I = \int \sqrt{1 - \sin x} \cos x dx$ [সি.'০১]

এক $1 - \sin x = z$. তাহলে, $-\cos x dx = dz$ এবং

$$I = - \int z^{\frac{1}{2}} dz = - \frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = - \frac{2}{3} z^{\frac{3}{2}} + c$$

$$\therefore \int \sqrt{1-\sin x} \cos x dx = - \frac{2}{3} (1-\sin x)^{\frac{3}{2}} + c$$

$$4(e) \int \frac{\cos x dx}{(1-\sin x)^2} \quad [\text{রা. '০৪, কু. '০৬; ব. '১১}]$$

ধরি, $1-\sin x = z$. তাহলে, $-\cos x dx = dz$ এবং

$$\begin{aligned} \int \frac{\cos x dx}{(1-\sin x)^2} &= - \int \frac{dz}{z^2} = - \int z^{-2} dz \\ &= - \frac{z^{-2+1}}{-2+1} + c = z^{-1} + c = \frac{1}{1-\sin x} + c \end{aligned}$$

$$4(f) \int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx \quad [\text{চ. '০৭; কু. '০৮; রা. '১১}]$$

ধরি, $\tan^{-1} x^3 = z$

$$\therefore \frac{1}{1+(x^3)^2} \cdot 3x^2 dx = dz$$

$$\Rightarrow \frac{x^2}{1+x^6} dx = \frac{1}{3} dz$$

$$\begin{aligned} \therefore \int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx &= \frac{1}{3} \int z dz \\ &= \frac{1}{3} \cdot \frac{z^2}{2} + c = \frac{1}{6} (\tan^{-1} x^3)^2 + c \quad (\text{Ans.}) \end{aligned}$$

$$5(a) \text{ ধরি, } I = \int \frac{1}{x(1+\ln x)^3} dx \quad [\text{ঢা. '০১}]$$

এবং $1+\ln x = z$. তাহলে, $\frac{1}{x} dx = dz$ এবং

$$I = \int \frac{1}{z^3} dz = \int z^{-3} dz = \frac{z^{-3+1}}{-3+1} + c = - \frac{1}{2z^2} + c$$

$$\therefore \int \frac{1}{x(\ln x)^2} dx = - \frac{1}{2(1+\ln x)^2} + c$$

$$5(b) \text{ ধরি, } I = \int \frac{(\log_{10} x)^2}{x} dx \quad [\text{প্র.ভ.প. '৮৩}]$$

এবং $\log_{10} x = z$. তাহলে, $\frac{1}{x \ln 10} dx = dz$ এবং

$$I = \ln 10 \int z^2 dz = \ln 10 \cdot \frac{1}{3} z^3 + c$$

$$\therefore \int \frac{(\log_{10} x)^2}{x} dx = \frac{\ln 10}{3} (\log_{10} x)^3$$

$$6(a) \text{ ধরি, } I = \int e^{\tan^{-1} x} \cdot \frac{1}{1+x^2} dx$$

[ঢা. '০৯; মা. '১২, '১৪]

এবং $\tan^{-1} x = z$. তাহলে, $\frac{1}{1+x^2} dx = dz$ এবং

$$I = \int e^z dz = e^z + c = e^{\tan^{-1} x} + c$$

$$6(b) \int e^{\sin^{-1} x} \cdot \frac{dx}{\sqrt{1-x^2}} \quad [\text{চ. '০১; প্র.ভ.প. '০৬}]$$

ধরি, $\sin^{-1} x = z$. তাহলে, $\frac{1}{\sqrt{1-x^2}} dx = dz$ এবং

$$\begin{aligned} \int e^{\sin^{-1} x} \cdot \frac{dx}{\sqrt{1-x^2}} &= \int e^z dz = e^z + c \\ &= e^{\sin^{-1} x} + c \end{aligned}$$

$$6(c) \text{ ধরি, } I = \int \frac{x}{\sqrt{1-x^2}} dx \quad [\text{য. '০৬; দি. '১১; ঢা. '১৪}]$$

এবং $1-x^2 = z$. তাহলে, $-2x dx = dz$ এবং

$$I = - \frac{1}{2} \int \frac{dz}{\sqrt{z}} = - \frac{1}{2} \cdot 2\sqrt{z} + c$$

$$\therefore \int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} + c$$

$$6(d) \text{ ধরি, } I = \int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}} dx \text{ এবং}$$

$\sin^{-1} x = z$ [কু. '০৭; ব. '১১, '১৪; য. '০৯, '১৩; ঢা. '১৩]

তাহলে, $\frac{1}{\sqrt{1-x^2}} dx = dz$ এবং

$$\begin{aligned} \therefore I &= \int \tan z dz = \ln |\sec z| + c \\ &= \ln |\sec(\sin^{-1} x)| + c \end{aligned}$$

7(a) ধরি, $I = \int \frac{\sin x}{3 + 4 \cos x} dx$ [ঢা.'০৭, ব.'১৩]

এবং $3 + 4 \cos x = z$. তাহলে, $-4 \sin x dx = dz$

এবং $I = -\frac{1}{4} \int \frac{dz}{z} = -\frac{1}{4} \ln |3 + 4 \cos x| + c$

7(b) ধরি, $I = \int \frac{\sin x}{1 + 2 \cos x} dx$ [রা.'০৩]

এবং $1 + 2 \cos x = z$. তাহলে, $-2 \sin x dx = dz$

এবং $I = -\frac{1}{2} \int \frac{dz}{z} = -\frac{1}{2} \ln |1 + 2 \cos x| + c$

7(c) $\int \frac{\sec^2 x}{3 - 4 \tan x} dx = -\frac{1}{4} \int \frac{-4 \sec^2 x dx}{3 - 4 \tan x}$
 $= -\frac{1}{4} \ln |3 - 4 \tan x| + c$

7(d) ধরি, $I = \int \frac{dx}{(1 + x^2) \tan^{-1} x}$

[ব.'০৪; ঢা.'১০; সি.'১১; কু.'১৩]

এবং $\tan^{-1} x = z$. তাহলে, $\frac{1}{1 + x^2} dx = dz$ এবং

$I = \int \frac{dz}{z} = \ln |z| + c = \ln |\tan^{-1} x| + c$

8 $\int \frac{1}{x(1 + \ln x)} dx$ [ব.'০৯; কু.'১২]

ধরি, $1 + \ln x = z$. তাহলে, $\frac{1}{x} dx = dz$ এবং

$\int \frac{1}{x(1 + \ln x)} dx = \int \frac{dz}{z} = \ln |z| + c$
 $= \ln (1 + \ln x) + c$

9(a) $\int \frac{e^{3x}}{e^{3x} - 1} dx = \frac{1}{3} \int \frac{(3e^{3x} - 0) dx}{e^{3x} - 1}$

$= \frac{1}{3} \ln |e^{3x} - 1| + c$

9(b) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \int \frac{d(e^x + e^{-x})}{e^x + e^{-x}}$ [দি.'১০]

$= \ln |e^x + e^{-x}| + c$

9(c) $\int \frac{1}{e^x + 1} dx = \int \frac{e^{-x}}{e^{-x}(e^x + 1)} dx$ [য.'১০]

$= \int \frac{e^{-x}}{1 + e^{-x}} dx = -\int \frac{(0 - e^{-x}) dx}{1 + e^{-x}}$
 $= -\ln |1 + e^{-x}| + c$

10. (a) ধরি, $I = \int \frac{1}{\sqrt[3]{1 - 6x}} dx$ [প্র.ভ.প.'০৫]

এবং $1 - 6x = z$. তাহলে, $-6 dx = dz$

$I = -\frac{1}{6} \int \frac{1}{\sqrt[3]{z}} dz = -\frac{1}{6} \int \frac{dz}{z^{1/3}} = -\frac{1}{6} \int z^{-1/3} dz$

$= -\frac{1}{6} \frac{z^{-1/3+1}}{-1/3+1} + c = -\frac{1}{6} \frac{z^{2/3}}{2/3} + c$

$= -\frac{1}{4} (1 - 6x)^{2/3} + c$

10(b) ধরি, $I = \int \frac{x^3 dx}{\sqrt{(1 - 2x^4)}}$ [চ.'০১]

এবং $1 - 2x^4 = z$. তাহলে, $-8x^3 dx = dz$ এবং

$I = -\frac{1}{8} \int \frac{dz}{\sqrt{z}} = -\frac{1}{8} \cdot 2\sqrt{z} + c = -\frac{1}{4} \sqrt{z} + c$

$\therefore \int \frac{x^3 dx}{\sqrt{(1 - 2x^4)}} = -\frac{1}{4} \sqrt{1 - 2x^4} + c$

10(c) $\int \frac{dx}{\cos^2 x \sqrt{\tan x - 1}}$ [সি.'০২]

$= \int \frac{\sec^2 x dx}{\sqrt{\tan x - 1}} = \int \frac{(\sec^2 x - 0) dx}{\sqrt{\tan x - 1}}$

$= 2\sqrt{\tan x - 1} + c$ [$\because \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$]

10(d) ধরি, $I = \int \frac{\cos x}{\sqrt{\sin x}} dx$ [কু.'০৫; রা.'১০]

এবং $\sin x = z$. তাহলে, $\cos x dx = dz$ এবং

$I = \int \frac{dz}{\sqrt{z}} = 2\sqrt{z} + c = 2\sqrt{\sin x} + c$

10(e) ধরি, $I = \int \frac{dx}{x\sqrt{1+\ln x}}$

[কু.'০৩]

এবং $1 + \ln x = z$. তাহলে, $\frac{1}{x} dx$ এবং

$I = \int \frac{dz}{\sqrt{z}} = 2\sqrt{z} + c = 2\sqrt{1+\ln x} + c$

11(a) $\int \frac{dx}{4x^2+9} = \frac{1}{2} \int \frac{2xdx}{(2x)^2+3^2}$
 $= \frac{1}{2} \cdot \frac{1}{3} \tan^{-1} \frac{2x}{3} + c = \frac{1}{6} \tan^{-1} \frac{2x}{3} + c$

11(b) $\int \frac{xdx}{x^4+1}$ [রা.'০৮; ব.'১১]
 $= \frac{1}{2} \int \frac{2xdx}{1+(x^2)^2} = \frac{1}{2} \cdot \tan^{-1}(x^2) + c$

11(c) ধরি, $I = \int \frac{3x^2}{1+x^6} dx$ [রা.'০১, চ.'০৮]

এবং $x^3 = z$. তাহলে, $3x^2 dx = dz$ এবং

$I = \int \frac{dz}{1+z^2} = \tan^{-1} z + c$

$\therefore \int \frac{3x^2}{1+x^6} dx = \tan^{-1}(x^3) + c$

11(d) ধরি, $I = \int \frac{e^x}{1+e^{2x}} dx$ [সি.'০৪]

এবং $e^x = z$. তাহলে, $e^x dx = dz$ এবং

$I = \int \frac{dz}{1+z^2} = \tan^{-1} z + c = \tan^{-1}(e^x) + c$

11(e) $\int \frac{5e^{2x}}{1+e^{4x}} dx = \frac{5}{2} \int \frac{2e^{2x} dx}{1+(e^{2x})^2}$ [চ.'০১, '১১]
 $= \frac{5}{2} \tan^{-1}(e^{2x}) + c$

11(f) $\int \frac{1}{e^x + e^{-x}} dx$ [জা.'০৬; য.'০৫, '১২; রা.'০৭, '১৪; ব.'০৫, '০৭, '০৯; চ.'০৮; কু.'১২, '১৪; দি.'১৩; মা.'১৪]
 $= \int \frac{e^x}{e^x(e^x + e^{-x})} dx = \int \frac{e^x}{(e^x)^2 + 1} dx$
 ধরি, $e^x = z$. তাহলে, $e^x dx = dz$ এবং

$\int \frac{1}{e^x + e^{-x}} dx = \int \frac{dz}{1+z^2} = \tan^{-1} z + c$
 $= \tan^{-1}(e^x) + c$

12. (a) $\int \frac{dx}{x^2 - x + 1}$ [চ.'০৩]

$= \int \frac{dx}{(x - \frac{1}{2})^2 + 1 - \frac{1}{4}} = \int \frac{dx}{(x - \frac{1}{2})^2 + \frac{3}{4}}$
 $= \int \frac{d(x - \frac{1}{2})}{(\frac{\sqrt{3}}{2})^2 + (x - \frac{1}{2})^2}$ [$\because d(x - \frac{1}{2}) = dx$]

$= \frac{1}{\sqrt{3}/2} \tan^{-1} \frac{x - \frac{1}{2}}{\sqrt{3}/2} + c$
 $= \frac{2}{\sqrt{3}} \tan^{-1} \frac{2x - 1}{\sqrt{3}} + c$

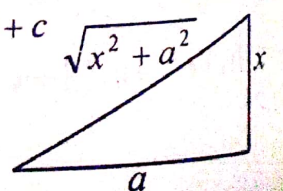
12(b) $\int \frac{dx}{\sqrt{x^2 + 4x + 13}}$ [রা.'০২]

$= \int \frac{dx}{\sqrt{(x+2)^2 + 13 - 4}}$
 $= \int \frac{d(x+2)}{\sqrt{(x+2)^2 + 3^2}}$
 $= \ln |\sqrt{(x+2)^2 + 3^2} + x + 2| + c$
 $= \ln |\sqrt{x^2 + 4x + 13} + x + 2| + c$

12(c) $\int \frac{dx}{(a^2 + x^2)^{3/2}}$ [য.'০২; প্র.ভ.প.'০৬]

ধরি, $x = a \tan \theta$. তাহলে $dx = a \sec^2 \theta d\theta$

$\therefore \int \frac{dx}{(a^2 + x^2)^{3/2}} = \int \frac{a \sec^2 \theta d\theta}{(a^2 + a^2 \tan^2 \theta)^{3/2}}$
 $= \int \frac{a \sec^2 \theta d\theta}{a^3 (1 + \tan^2 \theta)^{3/2}} = \int \frac{\sec^2 \theta d\theta}{a^2 \sec^3 \theta}$
 $= \frac{1}{a^2} \int \cos \theta d\theta = \frac{1}{a^2} \sin \theta + c$



$$= \frac{x}{a^2 \sqrt{x^2 + a^2}} + c$$

$$[\text{চিত্র হতে } \tan \theta = \frac{x}{a} \text{ এবং } \sin \theta = \frac{x}{\sqrt{x^2 + a^2}}]$$

$$12(d) \int x^2 \sqrt{1-x^2} dx$$

ধরি, $x = \sin \theta$. তাহলে $dx = \cos \theta d\theta$

$$\therefore \int x^2 \sqrt{1-x^2} dx$$

$$= \int \sin^2 \theta \sqrt{1-\sin^2 \theta} \cos \theta d\theta$$

$$= \int \sin^2 \theta \cos^2 \theta d\theta = \int \frac{1}{4} (2 \sin \theta \cos \theta)^2 d\theta$$

$$= \int \frac{1}{4} \sin^2 2\theta d\theta = \int \frac{1}{8} (1 - \cos 4\theta) d\theta$$

$$= \frac{1}{8} \left(\theta - \frac{\sin 4\theta}{4} \right) + c = \frac{1}{8} \left(\theta - \frac{2 \sin 2\theta \cos 2\theta}{4} \right) + c$$

$$= \frac{1}{8} \left(\theta - \frac{2 \sin \theta \cos \theta \cos 2\theta}{2} \right) + c$$

$$= \frac{1}{8} \left\{ \theta - \frac{2 \sin \theta \sqrt{1-\sin^2 \theta} (1-2\sin^2 \theta)}{2} \right\} + c$$

$$= \frac{1}{8} \{ \sin^{-1} x - x \sqrt{1-x^2} (1-2x^2) \} + c$$

$$13(a) \int \frac{dx}{1-x^2} = \int \frac{dx}{1^2-x^2} \quad [\text{ব. '০৩}]$$

$$= \frac{1}{2.1} \ln \left| \frac{1+x}{1-x} \right| + c \quad [\text{সূত্র প্রয়োগ করে।}]$$

$$= \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + c$$

$$13(b) \int \frac{dx}{9-4x^2} = \int \frac{dx}{3^2-(2x)^2} \quad [\text{সি. '১১}]$$

$$= \frac{1}{2} \int \frac{2dx}{3^2-(2x)^2} = \frac{1}{2} \cdot \frac{1}{2.3} \ln \left| \frac{3+2x}{3-2x} \right| + c$$

$$= \frac{1}{12} \ln \left| \frac{3+2x}{3-2x} \right| + c$$

$$13(c) \text{ ধরি, } I = \int \frac{dx}{9x^2-16} \quad [\text{ঢা. '০৩}]$$

$$= \int \frac{dx}{(3x)^2-4^2} \text{ এবং } 3x=z. \text{ তাহলে, } 3dx=dz \text{ এবং}$$

$$I = \frac{1}{3} \int \frac{dz}{z^2-4^2} = \frac{1}{3} \cdot \frac{1}{2.4} \ln \left| \frac{z-4}{z+4} \right| + c$$

$$\therefore \int \frac{dx}{9x^2-16} = \frac{1}{24} \ln \left| \frac{3x-4}{3x+4} \right| + c$$

$$13(d) \int \frac{dx}{16-4x^2} \quad [\text{ক. '০০; সি. '০১}]$$

$$= \frac{1}{4} \int \frac{dx}{4-x^2} = \frac{1}{4} \int \frac{dx}{2^2-x^2}$$

$$= \frac{1}{4} \cdot \frac{1}{2.2} \ln \left| \frac{2+x}{2-x} \right| + c = \frac{1}{16} \ln \left| \frac{2+x}{2-x} \right| + c$$

$$13(e) \int \frac{\cos x dx}{3+\cos^2 x} \quad [\text{প্র.ভ.প. '০৫}]$$

$$= \int \frac{\cos x dx}{3+1-\sin^2 x} = \int \frac{d(\sin x)}{2^2-(\sin x)^2}$$

$$= \frac{1}{2.2} \ln \left| \frac{2+\sin x}{2-\sin x} \right| + c = \frac{1}{4} \ln \left| \frac{2+\sin x}{2-\sin x} \right| + c$$

$$13(f) \int \frac{1}{e^x-e^{-x}} dx \quad [\text{রা. '০১; য. '০২}]$$

$$= \int \frac{1}{e^x-e^{-x}} dx = \int \frac{e^x}{e^x(e^x-e^{-x})} dx$$

$$= \int \frac{e^x}{(e^x)^2-1} dx = \int \frac{d(e^x)}{(e^x)^2-1^2}$$

$$= \frac{1}{2.1} \ln \left| \frac{e^x-1}{e^x+1} \right| + c = \frac{1}{2} \ln \left| \frac{e^x-1}{e^x+1} \right| + c$$

$$14(a) \int \frac{dx}{\sqrt{25-x^2}} = \int \frac{dx}{\sqrt{5^2-x^2}} \quad [\text{দি. '১০; চ. '১৩}]$$

$$= \sin^{-1} \frac{x}{5} + c$$

$$14(b) \int \frac{dx}{\sqrt{2-3x^2}}$$

$$[\text{য. '০৫; কু. '০৭, '১০, '১৪; জা. ব. '১২; সি. '১৩}]$$

$$= \frac{1}{\sqrt{3}} \int \frac{\sqrt{3} dx}{\sqrt{(\sqrt{2})^2-(\sqrt{3}x)^2}} = \frac{1}{\sqrt{3}} \sin^{-1} \frac{\sqrt{3}x}{\sqrt{2}} + c$$

14(c) $\int \frac{dx}{\sqrt{5-4x^2}}$

[ব.'০৬, '০৯; রা.'০৮; টা.'০৯; চ. য.'১১]

$= \int \frac{dx}{\sqrt{(\sqrt{5})^2 - (2x)^2}}$

ধরি, $2x = z$. তাহলে $2dx = dz$

$\therefore \int \frac{dx}{\sqrt{5-4x^2}} = \frac{1}{2} \int \frac{dz}{\sqrt{(\sqrt{5})^2 - z^2}}$

$= \frac{1}{2} \sin^{-1} \frac{z}{\sqrt{5}} + c = \frac{1}{2} \sin^{-1} \frac{2x}{\sqrt{5}} + c$

14(d) $\int \frac{dx}{\sqrt{25-16x^2}}$

[সি.'০৪]

$= \frac{1}{4} \int \frac{d(4x)}{\sqrt{5^2 - (4x)^2}}$

[$\because d(4x) = 4dx$]

$= \frac{1}{4} \sin^{-1} \frac{4x}{5} + c.$

14(e) $\int \frac{\sin x}{\sqrt{5-\cos^2 x}} dx$

[কু.'০৪]

$= - \int \frac{-\sin x dx}{\sqrt{(\sqrt{5})^2 - (\cos x)^2}} = \cos^{-1} \left(\frac{\cos x}{\sqrt{5}} \right) + c$

14(f) ধরি, $I = \int \frac{x^2}{\sqrt{1-x^6}} dx$

[ব.'০৮; য.'১১; দি.'১২]

এবং $x^3 = z$. তাহলে, $3x^2 dx = dz$

$I = \int \frac{x^2 dx}{\sqrt{1-(x^3)^2}} = \frac{1}{3} \int \frac{dz}{\sqrt{1-z^2}} = \frac{1}{3} \sin^{-1} z + c$

$= \frac{1}{3} \sin^{-1} x^3 + c$

14(g) $\int \frac{dx}{\sqrt{2ax-x^2}}$

[য.'০৯]

$= \int \frac{dx}{\sqrt{a^2 - (x^2 - 2ax + a^2)}}$

$= \int \frac{(1-0)dx}{\sqrt{a^2 - (x-a)^2}} = \sin^{-1} \left(\frac{x-a}{a} \right) + c$

14(h) ধরি, $I = \int \sqrt{1-9x^2} dx$

[ব.'০১]

এবং $3x = z$ তাহলে, $3dx = dz$ এবং

$I = \int \sqrt{1-(3x)^2} dx = \frac{1}{3} \int \sqrt{1-z^2} dz$

$= \frac{1}{3} \left[\frac{z\sqrt{1-z^2}}{2} + \frac{1}{2} \sin^{-1} z \right] + c$

$= \frac{1}{3} \left[\frac{3x\sqrt{1-(3x)^2}}{2} + \frac{1}{2} \sin^{-1}(3x) \right] + c$

$= \frac{1}{6} [3x\sqrt{1-9x^2} + \sin^{-1}(3x)] + c$

15. $\int \frac{3x-2}{\sqrt{3+2x-4x^2}} dx$

$= \int \frac{-\frac{3}{8}(-8x+2) + \frac{3}{4} - 2}{\sqrt{3+2x-4x^2}} dx$

$= -\frac{3}{8} \int \frac{(-8x+2)dx}{\sqrt{3+2x-4x^2}}$

$= -\frac{5}{4} \int \frac{dx}{\sqrt{-\{(2x)^2 - 2.2x \cdot \frac{1}{2} + (\frac{1}{2})^2\} + 3 + \frac{1}{4}}}$

$= -\frac{3}{8} \int \frac{d(3+2x-4x^2)}{\sqrt{3+2x-4x^2}}$

$= -\frac{5}{4} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(2x - \frac{1}{2}\right)^2}}$

$= -\frac{3}{8} \cdot 2\sqrt{3+2x-4x^2}$

$= -\frac{5}{4} \int \frac{\frac{1}{2} d\left(2x - \frac{1}{2}\right)}{\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(2x - \frac{1}{2}\right)^2}}$

$= -\frac{3}{4} \sqrt{3+2x-4x^2} - \frac{5}{8} \sin^{-1} \frac{2x - \frac{1}{2}}{\frac{\sqrt{13}}{2}} + c$

$$= -\frac{3}{4}\sqrt{3+2x-4x^2} - \frac{5}{8}\sin^{-1}\frac{4x-1}{\sqrt{13}} + c$$

16(a) $\int \frac{x+25}{x-25} dx$ [সি.'০৭]

$$= \int \frac{x-25+50}{x-25} dx = \int \left(\frac{x-25}{x-25} + \frac{50}{x-25}\right) dx$$

$$= \int \left(1 + \frac{50}{x-25}\right) dx = \int dx + 50 \int \frac{1}{x-25} dx$$

$$= x + 50 \ln|x-25| + c$$

16(b) $\int \frac{x^2 dx}{x^2-4}$ [সি.'০৮; ব.'০৮; রা.'০৮, '১১]

$$= \int \frac{x^2-4+4}{x^2-4} dx = \int \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4}\right) dx$$

$$= \int \left(1 + \frac{4}{x^2-2^2}\right) dx$$

$$= x + \frac{4}{2 \cdot 2} \ln \left| \frac{x-2}{x+2} \right| + c = x + \ln \left| \frac{x-2}{x+2} \right| + c$$

16(c) $\int \frac{x^2-1}{x^2-4} dx$ [কু.'০৯; সি.'০৫, '১২; য.'০৯; ঢা.'১১; ব.'১৩]

$$= \int \frac{x^2-4+3}{x^2-4} dx = \int \left(\frac{x^2-4}{x^2-4} + \frac{3}{x^2-4}\right) dx$$

$$= \int \left(1 + \frac{3}{x^2-2^2}\right) dx = x + \frac{3}{2 \cdot 2} \ln \left| \frac{x-2}{x+2} \right| + c$$

$$= x + \frac{3}{4} \ln \left| \frac{x-2}{x+2} \right| + c$$

16(d) $\int \frac{x dx}{(1-x)^2} = -\int \frac{1-x-1}{(1-x)^2} dx$

$$= -\int \left\{ \frac{1-x}{(1-x)^2} - \frac{1}{(1-x)^2} \right\} dx$$

$$= -\int \frac{1}{1-x} dx + \int \frac{1}{(1-x)^2} dx$$

$$= -\int \frac{-d(1-x)}{1-x} - \int \frac{d(1-x)}{(1-x)^2}$$

$$= \ln|1-x| - \left(-\frac{1}{1-x}\right) + c$$

$$= \ln|1-x| + \frac{1}{1-x} + c$$

17(a) $\int \sqrt{\frac{5-x}{5+x}} dx = \int \frac{5-x}{\sqrt{5^2-x^2}} dx$

$$= \int \frac{5}{\sqrt{5^2-x^2}} dx - \int \frac{x}{\sqrt{25-x^2}} dx$$

$$= \int \frac{5}{\sqrt{5^2-x^2}} dx + \frac{1}{2} \int \frac{d(25-x^2)}{\sqrt{25-x^2}}$$

$$= 5 \sin^{-1} \frac{x}{5} + \frac{1}{2} \cdot 2 \sqrt{25-x^2} + c$$

$$= 5 \sin^{-1} \frac{x}{5} + \sqrt{25-x^2} + c$$

17(b) $\int x \sqrt{\frac{1-x}{1+x}} dx = \int x \frac{\sqrt{1-x} \times \sqrt{1-x}}{\sqrt{1+x} \times \sqrt{1-x}} dx$

$$= \int x \frac{1-x}{\sqrt{1-x^2}} dx = \int \frac{x-x^2}{\sqrt{1-x^2}} dx$$

$$= \int \frac{(1-x^2) - \frac{1}{2}(-2x) - 1}{\sqrt{1-x^2}} dx$$

$$= \int \frac{1-x^2}{\sqrt{1-x^2}} dx - \frac{1}{2} \int \frac{(-2x)}{\sqrt{1-x^2}} dx - \int \frac{1}{\sqrt{1-x^2}} dx$$

$$= \int \sqrt{1-x^2} dx - \frac{1}{2} \cdot 2 \sqrt{1-x^2} - \sin^{-1} x$$

$$= \frac{x\sqrt{1-x^2}}{2} + \frac{1}{2} \sin^{-1} x - \sqrt{1-x^2} - \sin^{-1} x + c$$

$$= \frac{x\sqrt{1-x^2}}{2} - \frac{1}{2} \sin^{-1} x - \sqrt{1-x^2} + c \text{ (Ans.)}$$

নিয়ম : $\int \frac{1}{g(x)\sqrt{\phi(x)}} dx$ আকারের জন্য,

(a) $g(x)$ ও $\phi(x)$ উভয়ে একঘাত হলে, $\phi(x) = z^2$ ধরতে হয়।

(b) $g(x)$ একঘাত ও $\phi(x)$ দ্বিঘাত হলে, $g(x) = \frac{1}{z}$ ধরতে হয়।

(c) $g(x)$ দ্বিঘাত ও $\phi(x)$ একঘাত হলে, $\phi(x) = z^2$ ধরতে হয়।

(d) $g(x)$ ও $\varphi(x)$ উভয়ে দ্বিঘাত হলে, $x = \frac{1}{z}$ ধরতে হয়।

(e) $\int \frac{x}{g(x)\sqrt{\varphi(x)}} dx$ এবং $g(x)$ ও $\varphi(x)$ উভয়ে দ্বিঘাত হলে, $\varphi(x) = z^2$ ধরতে হয়।

18(a) ধরি, $I = \int \frac{dx}{(x-3)\sqrt{x+1}}$ এবং
[ঢা.'১০; ব.'১৩]

$x+1 = z^2$. তাহলে $dx = 2zdz$ এবং

$$I = \int \frac{2zdz}{(z^2 - 1 - 3)\sqrt{z^2}}$$

$$\Rightarrow I = \int \frac{2zdz}{(z^2 - 4)z} = 2 \int \frac{dz}{z^2 - 2^2}$$

$$= 2 \cdot \frac{1}{2 \cdot 2} \ln \left| \frac{z-2}{z+2} \right| + c = \frac{1}{2} \ln \left| \frac{\sqrt{x+1}-2}{\sqrt{x+1}+2} \right| + c$$

$$18(b) \int \frac{dx}{(x-1)\sqrt{x^2-2x}} = \int \frac{d(x-1)}{(x-1)\sqrt{(x-1)^2-1}}$$

$$= \sec^{-1}(x-1) + c$$

নিয়ম : (a) যদি কোন যোগজ $\int \frac{a+bx^l}{p+qx^m} dx$ আকারে থাকে, যেখানে l ও m উভয়ে ভগ্নাংশ এবং তাদের হরের ল.সা.গু n হয়, তবে $x = z^n$ ধরতে হয়।

(b) $\int \frac{dx}{x(a+bx^n)}$ আকারের যোগজের জন্য, $x^n = \frac{1}{z}$ ধরতে হয়।

(c) $\int \frac{dx}{x\sqrt{a+bx^n}}$ আকারের যোগজের জন্য, $x^n = \frac{1}{z^2}$ ধরতে হয়।

(d) $\int \frac{dx}{x^m(a+bx^n)}$ আকারের যোগজের জন্য, $a+bx = zx$ ধরতে হয়।

(e) $\int \frac{dx}{(x-a)^m(x-b)^n}$ আকারের যোগজের জন্য, $z = \frac{x-b}{x-a}$ ধরতে হয়।

$$19(a) \int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx = \int \frac{x^{1/2}}{1+x^{1/3}} dx$$

ধরি, $x = z^6$. তাহলে, $dx = 6z^5 dz$

$$\therefore \int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx = \int \frac{\sqrt{z^6} \cdot 6z^5 dz}{1+\sqrt[3]{z^6}}$$

$$= \int \frac{z^3 \cdot 6z^5 dz}{1+z^2} = 6 \int \frac{z^8 dz}{1+z^2}$$

$$= 6 \int \frac{1}{z^2+1} \{z^6(z^2+1) - z^4(z^2+1) + z^2(z^2+1) - (z^2+1) + 1\} dz$$

$$= 6 \int (z^6 - z^4 + z^2 - 1 + \frac{1}{1+z^2}) dz$$

$$= 6 \left(\frac{z^7}{7} - \frac{z^5}{5} + \frac{z^3}{3} - z + \tan^{-1} z \right) + c$$

$$= \frac{6}{7} x^{7/6} - \frac{6}{5} x^{5/6} + \frac{6}{3} x^{3/6} - 6x^{1/6} + \tan^{-1} x^{1/6} + c$$

$$19(b) \text{ ধরি, } I = \int \frac{dx}{x(4+5x^{20})} \text{ এবং } x^{20} = \frac{1}{z}$$

$$\text{তাহলে, } 20x^{19} dx = -\frac{dz}{z^2} \Rightarrow x^{19} dx = -\frac{dz}{20z^2}$$

$$\text{এবং } I = \int \frac{x^{19} dx}{x^{20}(4+5x^{20})} = \int \frac{-\frac{dz}{20z^2}}{\frac{1}{z}(4+5\frac{1}{z})}$$

$$= -\frac{1}{20} \int \frac{dz}{4z+5} = -\frac{1}{20} \cdot \frac{1}{4} \int \frac{d(4z+5)}{4z+5}$$

$$= -\frac{1}{80} \ln |4z+5| + c = -\frac{1}{80} \ln \left| \frac{4}{x^{20}} + 5 \right| + c$$

$$19(c) \text{ ধরি, } I = \int \frac{dx}{x\sqrt{x^4-1}} \quad [\text{ক.'০১; রা.'১১}]$$

$$\text{এবং } x^4 = \frac{1}{z^2}. \text{ তাহলে, } 4x^3 dx = -\frac{2dz}{z^3} \text{ এবং}$$

$$I = \int \frac{x^3 dx}{x^4 \sqrt{x^4-1}} = \int \frac{-\frac{2dz}{2z^3}}{\frac{1}{z^2} \sqrt{\frac{1}{z^2}-1}}$$

$$= -\frac{1}{2} \int \frac{dz}{\sqrt{1-z^2}} = \frac{1}{2} \cos^{-1} z + c$$

$$= \frac{1}{2} \cos^{-1} \left(\frac{1}{x^2} \right) + c = \frac{1}{2} \sec^{-1} (x^2) + c$$

(d) ধরি, $I = \int \frac{dx}{(x-1)^2(x-2)^3}$ এবং $z = \frac{x-1}{x-2}$

$$\Rightarrow zx - 2z = x - 1 \Rightarrow x(1-z) = 1 - 2z$$

$$\Rightarrow x = \frac{1-2z}{1-z} \Rightarrow x-2 = \frac{1-2z}{1-z} - 2$$

$$\Rightarrow x-2 = \frac{1-2z-2+2z}{1-z} = -\frac{1}{1-z}$$

$$\Rightarrow dx = -\frac{dz}{(1-z)^2}$$

$$\therefore I = \int \frac{dx}{\left(\frac{x-1}{x-2}\right)^2(x-2)^5} = \int \frac{-\frac{dz}{(1-z)^2}}{z^2 \cdot \frac{-1}{(1-z)^5}}$$

$$= \int \frac{(1-z)^3 dz}{z^2} = \int \frac{(1-3z+3z^2-z^3) dz}{z^2}$$

$$= \int \left(\frac{1}{z^2} - 3\frac{1}{z} + 3 - z \right) dz$$

$$= -\frac{1}{z} - 3 \ln |z| + 3z - \frac{z^2}{2} + c$$

$$= -\frac{x-2}{x-2} - 3 \ln \left| \frac{x-1}{x-2} \right| + 3 \left(\frac{x-1}{x-2} \right)$$

$$-\frac{1}{2} \left(\frac{x-1}{x-2} \right)^2 + c$$

20(a) $\int \frac{x^2+1}{x^4+1} dx = \int \frac{x^2(1+\frac{1}{x^2})}{x^2(x^2+\frac{1}{x^2})} dx$

$$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2} dx = \int \frac{d\left(x-\frac{1}{x}\right)}{\left(x-\frac{1}{x}\right)^2+(\sqrt{2})^2}$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{x-\frac{1}{x}}{\sqrt{2}} + c = \frac{1}{\sqrt{2}} \tan^{-1} \frac{x^2-1}{\sqrt{2}x} + c$$

20(b) $\int \frac{x^2-1}{x^4+1} dx = \int \frac{x^2(1-\frac{1}{x^2})}{x^2(x^2+\frac{1}{x^2})} dx$

$$= \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2} dx = \int \frac{d\left(x+\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)^2-(\sqrt{2})^2}$$

$$= \frac{1}{2\sqrt{2}} \ln \left| \frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}} \right| + c$$

$$= \frac{1}{2\sqrt{2}} \ln \left| \frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x} \right| + c$$

(c) $\int \frac{x^2 dx}{x^4+a^4} = \frac{1}{2} \int \frac{(x^2+a^2)+(x^2-a^2)}{x^4+a^4} dx$

$$= \frac{1}{2} \left[\int \frac{x^2+a^2}{x^4+a^4} dx + \int \frac{x^2-a^2}{x^4+a^4} dx \right]$$

$$= \frac{1}{2} \left[\int \frac{x^2(1+\frac{a^2}{x^2})}{x^2(x^2+\frac{a^4}{x^2})} dx + \int \frac{x^2(1-\frac{a^2}{x^2})}{x^2(x^2+\frac{a^4}{x^2})} dx \right]$$

$$= \frac{1}{2} \left[\int \frac{d\left(x-\frac{a^2}{x}\right)}{\left(x-\frac{a^2}{x}\right)^2+(\sqrt{2}a)^2} + \int \frac{d\left(x+\frac{a^2}{x}\right)}{\left(x+\frac{a^2}{x}\right)^2-(\sqrt{2}a)^2} \right]$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{2}a} \tan^{-1} \frac{x-\frac{a^2}{x}}{\sqrt{2}a} + \frac{1}{2\sqrt{2}a} \ln \left| \frac{x+\frac{a^2}{x}-\sqrt{2}a}{x+\frac{a^2}{x}+\sqrt{2}a} \right| + c \right]$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{2}a} \tan^{-1} \frac{x-\frac{a^2}{x}}{\sqrt{2}a} + \frac{1}{2\sqrt{2}a} \ln \left| \frac{x+\frac{a^2}{x}-\sqrt{2}a}{x+\frac{a^2}{x}+\sqrt{2}a} \right| + c \right]$$

$$= \frac{1}{2\sqrt{2}a} \left[\tan^{-1} \frac{x^2-a^2}{\sqrt{2}ax} + \frac{1}{2\sqrt{2}a} \ln \left| \frac{x+\frac{a^2}{x}-\sqrt{2}a}{x+\frac{a^2}{x}+\sqrt{2}a} \right| + c \right]$$

$$\frac{1}{2} \ln \left| \frac{x^2 + a^2 - \sqrt{2} ax}{x^2 + a^2 + \sqrt{2} ax} \right| + c$$

21(a) $\int \sin^2 x \cos^2 x dx$ [য. '০৮; রা., জ. '১৩]

$$\begin{aligned} &= \int \frac{1}{4} (2 \sin x \cos x) dx = \frac{1}{4} \int \sin^2 2x dx \\ &= \frac{1}{8} \int (1 - \cos 4x) dx = \frac{1}{8} \left(x - \frac{1}{4} \sin 4x \right) + c \end{aligned}$$

21(b) ধরি, $I = \int \sin^3 x \cos^3 x dx$ [য. '০৬]

$$= \int \sin^3 x (1 - \sin^2 x) \cos x dx \text{ এবং } \sin x = z.$$

তাহলে, $\cos x dx = dz$ এবং

$$\begin{aligned} I &= \int z^3 (1 - z^2) dz = \int (z^3 - z^5) dz \\ &= \frac{1}{4} z^4 - \frac{1}{6} z^6 + c = \frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + c \end{aligned}$$

21(c) ধরি, $I = \int \sin^3 x \cos^4 x dx$ [রা. '০১]

$$= \int (1 - \cos^2 x) \cos^4 x \sin x dx \text{ এবং } \cos x = z$$

তাহলে, $-\sin x dx = dz$ এবং

$$\begin{aligned} I &= - \int (1 - z^2) z^4 dz = \int (z^6 - z^4) dz \\ &= \frac{1}{7} z^7 - \frac{1}{5} z^5 + c = \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + c \end{aligned}$$

21(d) ধরি, $I = \int \sin^4 x \cos^4 x dx$

$$\begin{aligned} \sin^4 x \cos^4 x &= \frac{1}{16} (2 \sin x \cos x)^4 \\ &= \frac{1}{16} \sin^4 2x = \frac{1}{16} \cdot \left\{ \frac{1}{2} (1 - \cos 4x) \right\}^2 \\ &= \frac{1}{64} (1 - 2 \cos 4x + \cos^2 4x) \\ &= \frac{1}{64} \left\{ 1 - 2 \cos 4x + \frac{1}{2} (1 + \cos 8x) \right\} \\ &= \frac{1}{128} (3 - 4 \cos 4x + \cos 8x) \end{aligned}$$

$$\begin{aligned} \therefore I &= \int \frac{1}{128} (3 - 4 \cos 4x + \cos 8x) dx \\ &= \frac{1}{128} \left(3x - 4 \cdot \frac{1}{4} \sin 4x + \frac{1}{8} \sin 8x \right) + c \end{aligned}$$

$$= \frac{1}{128} \left(3x - \sin 4x + \frac{1}{8} \sin 8x \right) + c$$

21(e) $\int \sin^2 x \cos 2x dx$

[চ. '০৯; য. '০৫; কু. '০৭; সি. '১১]

$$\begin{aligned} &= \int \frac{1}{2} (1 - \cos 2x) \cos 2x dx \\ &= \frac{1}{2} \int (\cos 2x - \cos^2 2x) dx \\ &= \frac{1}{2} \int \left\{ \cos 2x - \frac{1}{2} (1 + \cos 4x) \right\} dx \\ &= \frac{1}{2} \left\{ \frac{1}{2} \sin 2x - \frac{1}{2} \left(x + \frac{1}{4} \sin 4x \right) \right\} + c \\ &= \frac{1}{4} \left(\sin 2x - x - \frac{1}{4} \sin 4x \right) + c \end{aligned}$$

22(a) $\int \tan^2 x dx$ [জ. '০৫, '০৭]

$$= \int (\sec^2 x - 1) dx = \tan x - x + c$$

22(b) ধরি, $I = \int \frac{\tan^2(\ln x)}{x} dx$ [ব. '০২]

এবং $\ln x = z$. তাহলে, $\frac{1}{x} dx = dz$ এবং

$$\begin{aligned} I &= \int \tan^2 z dz = \int (\sec^2 z - 1) dz \\ &= \tan z - z + c = \tan(\ln x) - \ln x + c \end{aligned}$$

22(c) $\int \frac{dx}{\sin x \cos^3 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} dx$

$$= \int \left(\tan x \sec^2 x + \frac{2}{2 \sin x \cos x} \right) dx$$

$$= \int \tan x \sec^2 x dx + 2 \int \frac{dx}{\sin 2x}$$

$$= \int \tan x d(\tan x) + \int \sec 2x d(2x)$$

$$= \frac{1}{2} \tan^2 x + \ln \left| \tan \frac{2x}{2} \right| + c$$

$$= \frac{1}{2} \tan^2 x + \ln |\tan x| + c$$

23. $\int \frac{1 - \tan x}{1 + \tan x} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

$$\int \frac{d(\sin x + \cos x)}{\sin x + \cos x} = \ln |\sin x + \cos x| + c$$

24. (a) ধরি, $I = \int \frac{\sin 4x}{\sin^4 x + \cos^4 x} dx$ এবং

$z = \sin^4 x + \cos^4 x$ তাহলে,

$$dz = (4 \sin^3 x \cos x - 4 \cos^3 x \sin x) dx$$

$$= 4 \sin x \cos x (\sin^2 - \cos^2) dx$$

$$= -2 \sin 2x \cos 2x dx = -\sin 4x dx \text{ এবং}$$

$$I = \int \frac{-dz}{z} = -\ln |z| + c$$

$$= -\ln |\sin^4 x + \cos^4 x| + c$$

24(b) ধরি, $I = \int \frac{dx}{1 + \cos^2 x}$ [রা.'০৬]

$$= \int \frac{\sec^2 x dx}{\sec^2 x (1 + \cos^2 x)} = \int \frac{\sec^2 x dx}{\sec^2 x + 1}$$

$$= \int \frac{\sec^2 x dx}{1 + \tan^2 x + 1} \text{ এবং } z = \tan x \Rightarrow dz = \sec^2 x dx$$

$$\therefore I = \int \frac{dz}{(\sqrt{2})^2 + z^2} = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{z}{\sqrt{2}} \right) + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right) + c$$

24(c) $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$ [য.'০৩]

$$= \int \frac{2 \sin^2 x}{2 \cos^2 x} dx = \int \tan^2 x dx$$

$$= \int (\sec^2 x - 1) dx = \tan x - x + c$$

24(d) $\int \frac{1 - \cos 5x}{1 + \cos 5x} dx$ [য.'০১; সি.'০২]

$$= \int \frac{2 \sin^2 \frac{5x}{2}}{2 \cos^2 \frac{5x}{2}} dx = \int \tan^2 \frac{5x}{2} dx$$

$$= \int (\sec^2 \frac{5x}{2} - 1) dx = \frac{2}{5} \tan \frac{5x}{2} - x + c$$

25(a) ধরি, $I = \int \frac{dx}{(e^x - 1)^2} = \int \frac{dx}{\{e^x(1 - e^{-x})\}^2}$

$$= \int \frac{dx}{e^{2x}(1 - e^{-x})^2} = \int \frac{e^{-x} \cdot e^{-x} dx}{(1 - e^{-x})^2} \text{ এবং}$$

$e^{-x} = z$. তাহলে $-e^{-x} dx = dz$ এবং

$$I = -\int \frac{z dz}{(1 - z)^2} = \int \frac{(1 - z) - 1}{(1 - z)^2} dz$$

$$= \int \left\{ \frac{1}{1 - z} - \frac{1}{(1 - z)^2} \right\} dz$$

$$= -\int \left\{ \frac{1}{1 - z} - \frac{1}{(1 - z)^2} \right\} d(1 - z)$$

$$= -\left\{ \ln |1 - z| + \frac{1}{1 - z} \right\} + c$$

$$= -\ln |1 - e^{-x}| - \frac{1}{1 - e^{-x}} + c$$

25(b) $\int \frac{\sin x dx}{\sin(x + a)} = \int \frac{\sin x dx}{\sin x \cos a + \cos x \sin a}$

ধরি, $\sin x = l(\sin x \cos a + \cos x \sin a) +$

$m(\cos x \cos a - \sin x \sin a) + n$

$$\Rightarrow \sin x = (l \cos a - m \sin a) \sin x + (l \sin a + m \cos a) \cos x + n$$

উভয়পক্ষে $\sin x$, $\cos x$ ও ধ্রুবপদ সমীকৃত করে পাই,

$$n = 0, l \sin a + m \cos a = 0 \Rightarrow m = -\frac{l \sin a}{\cos a}$$

এবং $l \cos a - m \sin a = 1$

$$\Rightarrow l \cos a + \frac{l \sin a}{\cos a} \sin a = 1$$

$$\Rightarrow l(\sin^2 a + \cos^2 a) = \cos a \Rightarrow l = \cos a$$

$$\therefore m = -\frac{\cos a \sin a}{\cos a} = -\sin a$$

$$\therefore \int \frac{\sin x dx}{\sin(x + a)} = \int \frac{\cos a \sin(x + a) dx}{\sin(x + a)}$$

$$\int \frac{\sin a (\cos x \cos a - \sin x \sin a) dx}{\sin x \cos a + \sin a \cos x}$$

$$= \cos a \int dx - \sin a \ln |\sin(x + a)|$$

$$= x \cos a - \sin a \ln |\sin(x + a)| + c$$

25(c) $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$

$$\begin{aligned}
 &= \int \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} \right) dx \\
 &= \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx = \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}} dx \\
 &= \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx \\
 &= \sqrt{2} \int \frac{d(\sin x - \cos x)}{\sqrt{1 - (\sin x - \cos x)^2}} \\
 &= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c
 \end{aligned}$$

সম্ভাব্য ধাপসহ প্রশ্ন

নিচের যোগজগুলি নির্ণয় কর:

$$26(a) \int (e^{\frac{x}{2}} + e^{-\frac{x}{2}}) dx = \frac{e^{\frac{x}{2}}}{\frac{1}{2}} + \frac{e^{-\frac{x}{2}}}{-\frac{1}{2}} + c \quad (২)$$

$$= 2(e^{\frac{x}{2}} - e^{-\frac{x}{2}}) + c$$

$$(b) \int a^{4x} dx = \frac{a^{4x}}{\ln a \cdot 4} + c = \frac{a^{4x}}{4 \ln a} + c \quad (২)$$

$$\begin{aligned}
 27(a) \int x^2 \cos x^3 dx &= \frac{1}{3} \int \cos(x^3)(3x^2 dx) \quad (১) \\
 &= \frac{1}{3} \sin x^3 + c \quad (১)
 \end{aligned}$$

$$(b) \int \frac{(1 + \tan \frac{3x}{2})^2 dx}{1 + \sin 3x} \quad [প্র.ভ.প. ৮৬]$$

$$= \int \frac{(1 + \tan \frac{3x}{2})^2 dx}{1 + \frac{2 \tan(3x/2)}{1 + \tan^2(3x/2)}} \quad (১)$$

$$= \int \frac{\{1 + \tan(3x/2)\}^2 \{1 + \tan^2(3x/2)\} dx}{1 + \tan^2(3x/2) + 2 \tan(3x/2)}$$

$$= \int \frac{\{1 + \tan(3x/2)\}^2 \{1 + \tan^2(3x/2)\} dx}{\{1 + \tan(3x/2)\}^2}$$

$$= \int \{1 + \tan^2(3x/2)\} dx = \int \sec^2(3x/2) dx$$

$$= \frac{2}{3} \tan \frac{3x}{2} + c \quad (২)$$

$$28. \int \frac{2x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

ধরি, $\sin^{-1} x^2 = z$

$$\therefore \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x dx = dz \quad (১)$$

$$\Rightarrow \frac{2x dx}{\sqrt{1-x^4}} = dz$$

$$\therefore \int \frac{2x \sin^{-1} x^2}{\sqrt{1-x^4}} dx = \int z dz$$

$$= \frac{z^2}{2} + c = \frac{1}{2} (\sin^{-1} x^2)^2 + c \quad (\text{Ans.}) \quad (১)$$

$$29. \int \frac{1}{x(\ln x)^2} dx = \int (\ln x)^{-2} d(\ln x) \quad (১)$$

$$= \frac{(\ln x)^{-2+1}}{-2+1} + c = -\frac{1}{\ln x} + c \quad (১)$$

$$30(a) \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int \sin^{-1} x d(\sin^{-1} x) \quad (১)$$

$$= \frac{(\sin^{-1} x)^2}{2} + c \quad (১)$$

$$(b) \int \frac{1 + \tan^2 x}{(1 + \tan x)^2} dx \quad [প্র.ভ.প. ৯৩]$$

$$= \int \frac{\sec^2 x}{(1 + \tan x)^2} dx$$

$$= \int (1 + \tan x)^{-2} d(1 + \tan x) \quad (১)$$

$$= \frac{(1 + \tan x)^{-2+1}}{-2+1} + c = -\frac{1}{1 + \tan x} + c \quad (১)$$

$$(c) \text{ ধরি, } I = \int \frac{\cos 2x}{(\sqrt{\sin 2x+3})^3} dx \quad [প্র.ভ.প. ৯৫]$$

এবং $\sin 2x + 3 = z$. তাহলে, $2 \cos 2x dx = dz$ এবং

$$I = \frac{1}{2} \int \frac{dz}{z^{3/2}} = \frac{1}{2} \int z^{-3/2} dz \quad (১)$$

$$\frac{1}{2} z^{\frac{3}{2}+1} + c = \frac{1}{2} z^{\frac{5}{2}} + c = \frac{1}{2} \cdot \frac{1}{z^{\frac{1}{2}}} + c = -\frac{1}{\sqrt{z}} + c \quad (১)$$

$$= -\frac{1}{\sqrt{\sin 2x + 3}} + c$$

$$31. (a) \int \operatorname{cosec} \frac{x}{2} dx = \frac{1}{1/2} \ln \left| \tan \left(\frac{x/2}{2} \right) \right| + c \quad (১)$$

$$= 2 \ln \left| \tan \frac{x}{4} \right| + c$$

$$31(b) \int \sec \sqrt{x} \frac{dx}{\sqrt{x}} = 2 \int \sec(\sqrt{x}) \left(\frac{1}{2\sqrt{x}} dx \right) \quad (১)$$

$$= 2 \ln \left| \sec \sqrt{x} + \tan \sqrt{x} \right| + c \quad (১)$$

$$31(c) \int \left(\frac{3}{x-1} - \frac{4}{x-2} \right) dx$$

$$= 3 \ln |x-1| - 4 \ln |x-2| + c \quad (১)$$

$$31(d) \int \frac{\sin x}{1+\cos x} dx = - \int \frac{(-\sin x dx)}{1+\cos x} \quad (১)$$

$$= - \ln |1+\cos x| + c \quad (১)$$

$$32. \int \frac{1}{x \ln x} dx$$

$$= \int \frac{d(\ln x)}{\ln x} \quad [\because d(\ln x) = \frac{1}{x} dx] \quad (১)$$

$$= \ln(\ln x) + c \quad (১)$$

$$33. (a) \int \frac{dx}{16+x^2} = \int \frac{dx}{4^2+x^2} = \frac{1}{4} \tan^{-1} \frac{x}{4} + c \quad (১)$$

$$33(b) \int \frac{4}{16a^2+x^2} dx = 4 \int \frac{dx}{(4a)^2+x^2} \quad (১)$$

$$= 4 \cdot \frac{1}{4a} \tan^{-1} \frac{x}{4a} + c = \frac{1}{a} \tan^{-1} \frac{x}{4a} + c \quad (১)$$

$$33(c) \int \frac{x^2 dx}{e^{x^3} + e^{-x^3}} \quad [\text{প্র.ভ.প. '৮৯, '০১}]$$

$$= \int \frac{x^2 e^{x^3} dx}{e^{x^3} (e^{x^3} + e^{-x^3})} = \int \frac{x^2 e^{x^3} dx}{(e^{x^3})^2 + 1}$$

$$= \int \frac{d(e^{x^3})}{1+(e^{x^3})^2} \cdot \frac{1}{3} \quad [\because d(e^{x^3}) = e^{x^3} 3x^2 dx] \quad (১)$$

$$= \frac{1}{3} \tan^{-1}(e^{x^3}) + c \quad (১)$$

$$34 (a) \int \frac{dx}{x^2+6x+25} = \int \frac{dx}{(x+3)^2+25-9}$$

$$= \int \frac{d(x+3)}{(x+3)^2+4^2} = \frac{1}{4} \tan^{-1} \frac{x+3}{4} + c \quad (১)$$

$$34(b) \int \frac{dx}{(x^2+9)^2} \quad [\text{প্র.ভ.প. '০০}]$$

$$= \frac{1}{18} \int \frac{(x^2+9)-(x^2-9)}{(x^2+9)^2} dx$$

$$= \frac{1}{18} \left\{ \int \frac{x^2+9}{(x^2+9)^2} dx - \int \frac{x^2-9}{(x^2+9)^2} dx \right\} \quad (১)$$

$$= \frac{1}{18} \left\{ \int \frac{dx}{x^2+9} - \int \frac{x^2(1-\frac{9}{x^2})}{x^2(x+\frac{9}{x})^2} dx \right\} \quad (১)$$

$$= \frac{1}{18} \left\{ \int \frac{dx}{x^2+3^2} - \int \frac{d(x+\frac{9}{x})}{(x+\frac{9}{x})^2} \right\} \quad (১)$$

$$= \frac{1}{18} \left\{ \frac{1}{3} \tan^{-1} \frac{x}{3} - \left(-\frac{1}{x+\frac{9}{x}} \right) \right\} + c \quad (১)$$

$$= \frac{1}{18} \left(\frac{1}{3} \tan^{-1} \frac{x}{3} + \frac{x}{x^2+9} \right) + c$$

$$\text{বিকল্প পদ্ধতি : ধরি, } x = 3 \tan \theta \text{ . তাহলে}$$

$$\theta = \tan^{-1} \frac{x}{3} \text{ এবং } dx = 3 \sec^2 \theta d\theta \quad (১)$$

$$\therefore \int \frac{dx}{(x^2+9)^2} = \int \frac{3 \sec^2 \theta d\theta}{(9 \tan^2 \theta + 9)^2}$$

$$= \int \frac{3 \sec^2 \theta d\theta}{81(\tan^2 \theta + 1)^2} = \int \frac{\sec^2 \theta d\theta}{27 \sec^4 \theta}$$

$$= \frac{1}{27} \int \cos^2 \theta d\theta = \frac{1}{27} \int \frac{1}{2} (1 + \cos 2\theta) d\theta \quad (১)$$

$$\begin{aligned}
 &= \frac{1}{54} \left(\theta + \frac{1}{2} \sin 2\theta \right) + c \\
 &= \frac{1}{54} \left(\theta + \frac{1}{2} \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) + c \\
 &= \frac{1}{54} \left(\tan^{-1} \frac{x}{3} + \frac{x/3}{1 + x^2/9} \right) + c \\
 &= \frac{1}{54} \left(\tan^{-1} \frac{x}{3} + \frac{3x}{9 + x^2} \right) + c
 \end{aligned}$$

$$35. \int \frac{dx}{x^2 - 3x + 2} \quad [\text{প্র.ভ.প.'০৪}]$$

$$\begin{aligned}
 &= \int \frac{dx}{(x - \frac{3}{2})^2 + 2 - \frac{9}{4}} = \int \frac{dx}{(x - \frac{3}{2})^2 - (\frac{1}{2})^2} \quad (১) \\
 &= \frac{1}{2 \cdot \frac{1}{2}} \ln \left| \frac{x - \frac{3}{2} - \frac{1}{2}}{x - \frac{3}{2} + \frac{1}{2}} \right| + c = \ln \left| \frac{x - 2}{x - 1} \right| + c \quad (২)
 \end{aligned}$$

$$36(a) \int \frac{dx}{\sqrt{x+4}\sqrt{x+3}} = \int \frac{dx}{\sqrt{x^2 + 7x + 12}}$$

$$= \int \frac{dx}{\sqrt{(x + \frac{7}{2})^2 + 12 - \frac{49}{4}}} = \int \frac{dx}{\sqrt{(x + \frac{7}{2})^2 - (\frac{1}{2})^2}} \quad (১)$$

$$= \ln \left| \sqrt{(x + \frac{7}{2})^2 - (\frac{1}{2})^2} + x + \frac{7}{2} \right| + c$$

$$= \ln \left| \sqrt{x^2 + 7x + 12} + x + \frac{7}{2} \right| + c \quad (২)$$

$$36(b) \int \sqrt{16 - 9x^2} dx$$

$$= \frac{1}{3} \sqrt{(4)^2 - (3x)^2} d(3x) \quad (১)$$

$$= \frac{1}{3} \left[\frac{3x \sqrt{4^2 - (3x)^2}}{2} + \frac{4^2}{2} \sin^{-1} \frac{3x}{4} \right] + c \quad (২)$$

$$= \frac{x \sqrt{16 - 9x^2}}{2} + \frac{8}{3} \sin^{-1} \frac{3x}{4} + c \quad (\text{Ans.})$$

$$37(a) \int \frac{x dx}{\sqrt{4+x}} = \int \frac{4+x-4}{\sqrt{4+x}} dx \quad (১)$$

$$\begin{aligned}
 (১) &= \int \left(\frac{4+x}{\sqrt{4+x}} - \frac{4}{\sqrt{4+x}} \right) dx \\
 &= \int \sqrt{4+x} dx - 4 \int \frac{1}{\sqrt{4+x}} dx \\
 &= \frac{(4+x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 4 \cdot 2 \sqrt{4+x} + c \quad (২) \\
 &= \frac{2}{3} (4+x)^{3/2} - 8 \sqrt{4+x} + c
 \end{aligned}$$

$$37(b) \int \frac{6x-10}{(2x+1)^2} dx = \int \frac{3(2x+1)-13}{(2x+1)^2} dx \quad (১)$$

$$\begin{aligned}
 &= \int \frac{3}{2x+1} dx - \int \frac{13}{(2x+1)^2} dx \\
 &= \frac{3}{2} \int \frac{d(2x+1)}{2x+1} - \frac{13}{2} \int (2x+1)^{-2} d(2x+1) \quad (১)
 \end{aligned}$$

$$= \frac{3}{2} \ln |2x+1| - \frac{13}{2} \frac{(2x+1)^{-2+1}}{-2+1} + c \quad (২)$$

$$= \frac{3}{2} \ln |2x+1| + \frac{13}{2(2x+1)} + c \quad (\text{Ans.})$$

$$(c) \int \frac{x dx}{4-x} = \int \frac{-(4-x-4)}{4-x} dx \quad [\text{প্র.ভ.প.'৮৮}] \quad (১)$$

$$= - \int \frac{4-x}{4-x} dx + 4 \int \frac{dx}{4-x}$$

$$= - \int dx - 4 \int \frac{d(4-x)}{4-x} = -x - 4 \ln |4-x| + c \quad (২)$$

$$38. \text{ ধরি, } I = \int \frac{\sqrt{x+3}}{x+2} dx \text{ এবং } x+3 = z^2 \text{ তাহলে,}$$

$$dx = 2z dz \text{ এবং } I = \int \frac{\sqrt{z^2} \cdot 2z dz}{z^2 - 3 + 2} \quad (১)$$

$$\Rightarrow I = \int \frac{2z^2 dz}{z^2 - 1} = 2 \int \frac{z^2 - 1 + 1}{z^2 - 1} dz \quad (১)$$

$$= 2 \int dz + 2 \int \frac{1}{z^2 - 1} dz$$

$$= 2z + 2 \cdot \frac{1}{2 \cdot 1} \ln \left| \frac{z-1}{z+1} \right| + c \quad (২)$$

$$= 2\sqrt{x+3} + \ln \left| \frac{\sqrt{x+3}-1}{\sqrt{x+3}+1} \right| + c$$

39. ধরি, $I = \int \frac{dx}{(2x+3)\sqrt{x^2+3x+2}}$ এবং

$2x+3 = \frac{1}{z}$ তাহলে $z = \frac{1}{2x+3}$ এবং

$2dx = -\frac{1}{z^2} dz \Rightarrow dx = -\frac{dz}{2z^2}$

$\therefore I = \int \frac{-dz/2z^2}{\frac{1}{z}\sqrt{\left(\frac{1-3z}{2z}\right)^2 + 3 \cdot \frac{1-3z}{2z} + 2}}$

$= -\int \frac{dz}{2z\sqrt{\frac{1-6z+9z^2}{4z^2} + \frac{3-9z}{2z} + 2}}$

$= -\int \frac{dz}{2z\sqrt{\frac{1-6z+9z^2+6z-18z^2+8z^2}{4z^2}}}$

$= -\int \frac{dz}{\sqrt{1-z^2}} = \cos^{-1} z + c$

$= \cos^{-1}\left(\frac{1}{2x+3}\right) + c = \sec^{-1}(2x+3) + c$

বিকল্প পদ্ধতি : $\int \frac{dx}{(2x+3)\sqrt{x^2+3x+2}}$

$= \int \frac{dx}{(2x+3)\sqrt{\frac{1}{4}(4x^2+12x+8)}}$

$= \int \frac{dx}{(2x+3)\frac{1}{2}\sqrt{(2x+3)^2-1}}$

$= \int \frac{d(2x+3)}{(2x+3)\sqrt{(2x+3)^2-1}}$

$= \sec^{-1}(2x+3) + c$

40(a) ধরি, $I = \int \frac{1+x^{1/4}}{1+x^{1/2}} dx$ এবং $x = z^4$. তাহলে,

$dx = 4z^3 dz$ এবং

$I = \int \frac{(1+z)4z^3 dz}{1+z^2} = 4 \int \frac{z^4+z^3}{1+z^2} dz$

$= 4 \int \frac{z^2(z^2+1)-(z^2+1)+z(z^2+1)-z-1}{1+z^2} dz$ (১)

$= 4 \left\{ \int (z^2-1+z) dz - \int \frac{z dz}{z^2+1} - \int \frac{dz}{z^2+1} \right\}$

$= 4 \left\{ \frac{z^3}{3} - z + \frac{z^2}{2} - \frac{1}{2} \ln(z^2+1) - \tan^{-1} z \right\} + c$ (২)

$= 4 \left\{ \frac{x^{3/4}}{3} - x^{1/4} + \frac{x^{1/2}}{2} - \frac{1}{2} \ln(x^{1/2}+1) - \tan^{-1} x^{1/4} \right\} + c$

40(b) ধরি, $I = \int \frac{dx}{x(x^3+2)}$ এবং $x^3 = \frac{1}{z}$ (১)

তাহলে, $3x^2 dx = -\frac{1}{z^2} dz \Rightarrow x^2 dx = -\frac{dz}{3z^2}$ (১)

এবং $I = \int \frac{x^2 dx}{x^3(x^3+2)} = \int \frac{-\frac{dz}{3z^2}}{\frac{1}{z}\left(\frac{1}{z}+2\right)}$

$= -\frac{1}{3} \int \frac{dz}{1+2z} = -\frac{1}{3} \cdot \frac{1}{2} \int \frac{d(1+2z)}{1+2z}$ (১)

$= -\frac{1}{6} \ln|1+2z| + c = -\frac{1}{6} \ln\left|1+\frac{2}{x^3}\right| + c$ (১)

40(c) ধরি, $I = \int \frac{dx}{x\sqrt{2+3\sqrt{x}}}$ এবং $\sqrt{x} = \frac{1}{z^2}$

তাহলে, $\frac{1}{2\sqrt{x}} dx = -\frac{2}{z^3} dz \Rightarrow \frac{z^2}{2} dx = -\frac{2}{z^3} dz$ (১)

$\Rightarrow dx = -\frac{4dz}{z^5}$ এবং $I = \int \frac{-\frac{4dz}{z^5}}{\frac{1}{z^4}\sqrt{2+\frac{3}{z^2}}}$

(১)

(১)

$= -4 \int \frac{dz}{\sqrt{2z^2+3}} = -4 \int \frac{dz}{\sqrt{2}\sqrt{z^2+(\sqrt{3}/2)^2}}$

(১)

$= -2\sqrt{2} \ln\left|z + \sqrt{z^2 + \frac{3}{2}}\right| + c$ (১)

$= -2\sqrt{2} \ln\left|\frac{1}{x^{1/4}} + \sqrt{\frac{1}{x^{1/2}} + \frac{3}{2}}\right| + c$

40(d) ধরি, $I = \int \frac{dx}{x+x^n}, n \neq 1$ এবং $x^{n-1} = \frac{1}{z}$

তাহলে, $(n-1)x^{n-2} dx = -\frac{dz}{z^2}$

$\Rightarrow x^{n-2} dx = \frac{-dz}{(n-1)z^2}$

এবং $I = \int \frac{dx}{x(1+x^{n-1})} = \int \frac{x^{n-2} dx}{x^{n-1}(1+x^{n-1})}$

$= \int \frac{\frac{-dz}{(n-1)z^2}}{\frac{1}{z}(1+\frac{1}{z})} = -\frac{1}{n-1} \int \frac{dz}{1+z}$

$= -\frac{1}{n-1} \ln|1+z| + c$

$= -\frac{1}{n-1} \ln|1+\frac{1}{x^{n-1}}| + c$

41(a) ধরি, $I = \int \frac{dx}{x\sqrt{x^3+4}}$ এবং $x^3 = \frac{1}{z^2}$

তাহলে, $3x^2 dx = -\frac{2dz}{z^3} \Rightarrow x^2 dx = -\frac{2dz}{3z^3}$ এবং (১)

$I = \int \frac{x^2 dx}{x^3 \sqrt{x^3+4}} = \int \frac{-\frac{2dz}{3z^3}}{\frac{1}{z^2} \sqrt{\frac{1}{z^2}+4}}$

$= -\frac{2}{3} \int \frac{dz}{\sqrt{1+4z^2}} = -\frac{2}{3} \cdot \frac{1}{2} \int \frac{dz}{\sqrt{(\frac{1}{2})^2+z^2}}$

$= -\frac{1}{3} \ln|z + \sqrt{\frac{1}{4}+z^2}| + c$

$= -\frac{1}{3} \ln|\frac{1}{x^{3/2}} + \sqrt{\frac{1}{4} + \frac{1}{x^3}}| + c$ (১)

41(b) $\int \frac{dx}{x^3(3+5x)^2}$

ধরি, $3+5x = zx \Rightarrow (z-5)x = 3$

$\Rightarrow x = \frac{3}{z-5}$ তাহলে, $dx = -\frac{3dz}{(z-5)^2}$ এবং (১)

(১) $\int \frac{dx}{x^3(3+5x)^2} = \int \frac{\frac{-3dz}{(z-5)^2}}{\frac{27}{(z-5)^3} (3+5\frac{3}{z-5})^2}$

$= \int \frac{-3(z-5)^3 dz}{27(3z-15+15)^2}$

$= -\frac{1}{81} \int \frac{z^3 - 15z^2 + 75z - 125}{z^2} dz$

$= -\frac{1}{81} \int (z - 15 + \frac{75}{z} - 125\frac{1}{z^2}) dz$

$= -\frac{1}{81} \{ \frac{z^2}{2} - 15z + 75 \ln|z| - 125(-\frac{1}{z}) \} + c$ (২)

(১) $= -\frac{1}{81} \{ \frac{1}{2} (\frac{3+5x}{x})^2 - 15(\frac{3+5x}{x}) +$

$75 \ln|\frac{3+5x}{x}| + 125(\frac{x}{3+5x}) \} + c$ (১)

42(a) $\int \frac{a^2+x^2}{(x^2-a^2)^2} dx = \int \frac{x^2(1+\frac{a^2}{x^2})}{x^2(x-\frac{a^2}{x})^2} dx$

$= \int \frac{d(x-\frac{a^2}{x})}{(x-\frac{a^2}{x})^2} = -\frac{1}{x-\frac{a^2}{x}} + c = -\frac{x}{x^2-a^2} + c$ (২)

42(b) $\int \frac{(x^2-1)dx}{x^4+6x^3+7x^2+6x+1}$

$= \int \frac{(1-\frac{1}{x^2})dx}{x^2 + \frac{1}{x^2} + 6(x+\frac{1}{x}) + 7}$ (১)

$= \int \frac{(1-\frac{1}{x^2})dx}{(x+\frac{1}{x})^2 + 6(x+\frac{1}{x}) + 5}$

$= \int \frac{(1-\frac{1}{x^2})dx}{(x+\frac{1}{x}+3)^2 + 5-9} = \int \frac{d(x+\frac{1}{x}+3)}{(x+\frac{1}{x}+3)^2 - 2^2}$ (১)

$$= \frac{1}{2.2} \ln \left| \frac{x + \frac{1}{x} + 3 - 2}{x + \frac{1}{x} + 3 - 2} \right| + c$$

$$= \frac{1}{4} \ln \left| \frac{x^2 + 1 + x}{x^2 + 1 + 5x} \right| + c$$

$$43(a) \int \cot^2 x dx = \int (\operatorname{cosec}^2 x - 1) dx$$

$$= -\cot x - x + c$$

$$43(b) \int \tan^2 \frac{x}{2} dx = \int (\sec^2 \frac{x}{2} - 1) dx$$

$$= 2 \int \sec^2 \frac{x}{2} d(\frac{x}{2}) - \int dx = 2 \tan \frac{x}{2} - x + c \quad (২)$$

$$43(c) \int \frac{dx}{\sin x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x} dx$$

$$= \int \tan x \sec x dx + \int \operatorname{cosec} x dx$$

$$= \sec x + \ln(\operatorname{cosec} x - \cot x) + c \quad (২)$$

$$44(a) \int \frac{dx}{4 - 5 \sin^2 x} = \int \frac{\sec^2 x dx}{\sec^2 x (4 - 5 \sin^2 x)}$$

$$= \int \frac{\sec^2 dx}{4 \sec^2 x - 5 \tan^2 x}$$

$$= \int \frac{\sec^2 dx}{4(1 + \tan^2 x) - 5 \tan^2 x} = \int \frac{\sec^2 dx}{4 - \tan^2 x}$$

$$= \int \frac{d(\tan x)}{2^2 - (\tan x)^2} = \frac{1}{2.2} \ln \left| \frac{2 + \tan x}{2 - \tan x} \right| + c \quad (২)$$

$$= \frac{1}{4} \ln \left| \frac{2 + \tan x}{2 - \tan x} \right| + c$$

$$44(b) \int \frac{\sin 2x}{\sin x + \cos x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x + 2 \sin x \cos x - 1}{\sin x + \cos x} dx \quad (১)$$

$$= \int \frac{(\sin x + \cos x)^2 - 1}{\sin x + \cos x} dx$$

$$= \int (\sin x + \cos x - \frac{1}{\sin x + \cos x}) dx$$

$$= \cos x - \sin x - \int \frac{dx}{\sqrt{2}(\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4})} \quad (১)$$

$$= \cos x - \sin x - \frac{1}{\sqrt{2}} \int \frac{dx}{\sin(x + \frac{\pi}{4})} \quad (১)$$

$$= \cos x - \sin x - \frac{1}{\sqrt{2}} \int \operatorname{cosec}(x + \frac{\pi}{4}) dx$$

$$= \cos x - \sin x - \frac{1}{\sqrt{2}} \ln \left| \tan \frac{1}{2} (x + \frac{\pi}{4}) \right| + c \quad (১)$$

$$= \cos x - \sin x - \frac{1}{\sqrt{2}} \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{8} \right) \right| + c$$

$$45. \text{ ধরি, } I = \int \frac{dx}{\sqrt{x + \sqrt{1-x}}} \text{ এবং}$$

$$x = \sin^2 \theta. \text{ তাহলে } dx = 2 \sin \theta \cos \theta d\theta,$$

$$\sin \theta = \sqrt{x} \Rightarrow \theta = \sin^{-1} \sqrt{x} \text{ এবং} \quad (১)$$

$$I = \int \frac{2 \sin \theta \cos \theta d\theta}{\sqrt{\sin^2 \theta + \sqrt{1 - \sin^2 \theta}}}$$

$$= \int \frac{2 \sin \theta \cos \theta d\theta}{\sin \theta + \cos \theta}$$

$$= \int \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta + \cos \theta} d\theta$$

$$= \int \frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta + \cos \theta} d\theta$$

$$= \int (\sin \theta + \cos \theta - \frac{1}{\sin \theta + \cos \theta}) d\theta$$

$$= \cos \theta - \sin \theta - \int \frac{d\theta}{\sqrt{2}(\sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4})} \quad (১)$$

$$= \cos \theta - \sin \theta - \frac{1}{\sqrt{2}} \int \frac{d\theta}{\sin(\theta + \frac{\pi}{4})} \quad (১)$$

$$= \cos \theta - \sin \theta - \frac{1}{\sqrt{2}} \int \operatorname{cosec}(\theta + \frac{\pi}{4}) d\theta$$

$$= \sqrt{1 - \sin^2 \theta} - \sin \theta - \frac{1}{\sqrt{2}} \ln \left| \tan \frac{1}{2} (\theta + \frac{\pi}{4}) \right| + c \quad (১)$$

$$= \sqrt{1-x} - \sqrt{x} - \frac{1}{\sqrt{2}} \ln \left| \tan \left(\frac{1}{2} \sin^{-1} \sqrt{x} + \frac{\pi}{8} \right) \right| + c$$