প্রশ্নমালা X C

[সূত্র (MCQ এর ক্ষেত্রে) ঃ $\int x^m e^{nx} dx = \{\frac{1}{n}x^m - \frac{1}{n^2}\frac{d}{dx}(x^m) + \frac{1}{n^3}\frac{d^2}{dx^2}(x^m) - \frac{1}{n^4}\frac{d^3}{dx^3}(x^m) + \dots\dots\} e^{nx}$]

1.(a) $\int xe^x dx$

$= x\int e^x dx - \int\{\frac{d}{dx}(x)\int e^x dx\}dx$

$= xe^x - \int 1.e^x dx = xe^x - e^x + c$

(b) $\int x^2 e^x dx$ [কু.'০৪; সি.'০৯]

$= x^2\int e^x dx - \int\{\frac{d}{dx}(x^2)\int e^x dx\}dx$

$= x^2 e^x - \int(2x)e^x dx$

$= x^2 e^x - 2[x\int e^x - \int\{\frac{d}{dx}(x)\int e^x dx\}dx]$

$= x^2 e^x - 2[xe^x - \int 1.e^x dx]$

$= x^2 e^x - 2xe^x + 2e^x + c$

$= (x^2 - 2x + 2)e^x + c$

(c) $\int x^2 e^{-3x} dx$

$= x^2\int e^{-3x} dx - \int\{\frac{d}{dx}(x^2)\int e^{-3x} dx\}dx$

$= x^2(-\frac{1}{3})e^{-3x} - \int(2x)(-\frac{1}{3})e^{-3x} dx$

$= -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}[x\int e^{-3x} - \int\{\frac{d}{dx}(x)\int e^{-3x} dx\}dx]$

$= -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}[x(-\frac{e^{-3x}}{3}) - \int(-\frac{e^{-3x}}{3})dx]$

$= -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}[\frac{xe^{-3x}}{3} + \frac{1}{3}(-\frac{e^{-3x}}{3})] + c$

$= -\frac{1}{3}(x^2 + \frac{2}{3}x + \frac{2}{9})e^{-3x} + c$

(d) ধরি, $I = \int x^3 e^{x^2} dx$ এবং $x^2 = z$. তাহলে $2xdx = dz \Rightarrow xdx = \frac{1}{2}dz$ এবং

$I = \int x^2 e^{x^2}(xdx) = \frac{1}{2}\int ze^z dz$

$= \frac{1}{2}[z\int e^z dz - \int\{\frac{d}{dz}(z)\int e^z dz\}dz]$

$= \frac{1}{2}[ze^z - \int 1.e^z dz] = \frac{1}{2}(ze^z - e^z) + c$

$= \frac{1}{2}(x^2 - 1)e^{x^2} + c$

2. সূত্র (MCQ এর জন্য)ঃ $\int x^n \sin xdx = x^n(-\cos x) - (nx^{n-1})(-\sin x) + \cdots$

(a) $\int x\sin 3xdx$

$= x\int \sin 3xdx - \int\{\frac{d}{dx}(x)\int \sin 3x\, dx\}dx$

$= x(-\frac{1}{3}\cos 3x) - \int 1.(-\frac{1}{3}\cos 3x)dx$

$= -\frac{1}{3}x\cos 3x + \frac{1}{3}(\frac{1}{3}\sin 3x) + c$

$= \frac{1}{9}\sin 3x - \frac{1}{3}x\cos 3x + c$

(b) $\int x^3 \sin x\, dx$

$= x^3\int \sin xdx - \int\{\frac{d}{dx}(x^3)\int \sin x\, dx\}dx$

$= x^3(-\cos x) - \int 3x^2(-\cos x)dx$

$= -x^3\cos x + 3[x^2\int \cos x - \int\{\frac{d}{dx}(x^2)\int \cos x\, dx\}dx]$

$= -x^3\cos x + 3[x^2\sin x - \int 2x\sin xdx]$

$= -x^3\cos x + 3[x^2\sin x - 2\{x(-\cos x) - \int 1(-\cos x)dx\}]$

$= -x^3\cos x + 3[x^2\sin x -$

$2(-x\cos x+\sin x)]+c$

$=-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+c$

[MCQ এর ক্ষেত্রে, $\int x^3\sin x\,dx=x^3(-\cos x)$

$-(3x^2)(-\sin x)+(6x)(\cos x)-6\sin x$

$=-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+c]$.

(c) ধরি, $I=\int e^{2x}\cos e^x dx$ এবং $e^x=z$.

তাহলে $e^x dx=dz$ এবং

$I=\int e^x\cos e^x\,(e^x dx)=\int z\cos z\,dz$

$=z\int\cos z\,dz-\int\{\frac{d}{dz}(z)\int\cos z\,dz\}dz$

$=z\sin z-\int 1.\sin z\,dz$

$=z\sin z-(-\cos z)+c$

$=e^x\sin e^x+\cos e^x+c$

(d) ধরি, $I=\int\sin\sqrt{x}\,dx$ এবং $\sqrt{x}=z$

তাহলে $\frac{1}{2\sqrt{x}}dx=dz\Rightarrow dx=2z\,dz$ এবং

$I=\int 2z\sin z\,dz$

$=2[z\int\sin z\,dz-\int\{\frac{d}{dz}(z)\int\sin z\,dz\}\,dz]$

$=2[z(-\cos z)-\int 1.(-\cos z)dz]$

$=-2z\cos z+2\sin z+c$

$=-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}+c$

3 (a) $\int x\sin^2\frac{x}{2}dx$ [য.বো.'০২]

$=\int x\frac{1}{2}(1-\cos x)dx$

$=\frac{1}{2}\int x dx-\frac{1}{2}\int x\cos x\,dx$

$=\frac{1}{2}.\frac{x^2}{2}-\frac{1}{2}[x\int\cos x\,dx-\int\{\frac{d}{dx}(x)\int\cos x dx\}dx]$

$=\frac{x^2}{4}-\frac{1}{2}[x\sin x-\int 1.\sin x\,dx]$

$=\frac{x^2}{4}-\frac{1}{2}[x\sin x-(-\cos x)]+c$

$=\frac{x^2}{4}-\frac{1}{2}x\sin x-\frac{1}{2}\cos x+c$

(b) $\int x^2\cos^2\frac{x}{2}dx=\int x^2\frac{1}{2}(1+\cos x)dx$

$=\frac{1}{2}[\int x^2 dx+\int x^2\cos x dx]$

$=\frac{1}{2}[\frac{x^3}{3}+x^2(\sin x)-(2x)(-\cos x)+$
$(2)(-\sin x)]+c$

$=\frac{1}{2}[\frac{x^3}{3}+x^2\sin x+2x\cos x-2\sin x]+c$

(c) $\int x\cos 2x\cos 3x dx$

$=\int x\frac{1}{2}(\cos 5x-\cos x)dx$

$=\frac{1}{2}[x\int\cos 5x dx-\int\{\frac{d}{dx}(x)\int\cos 5x dx\}dx$
$+x\int\cos x dx-\int\{\frac{d}{dx}(x)\int\cos x dx\}dx]$

$=\frac{1}{2}[x(\frac{\sin 5x}{5})-\int 1.(\frac{\sin 5x}{5})dx$
$+x\sin x-\int 1.\sin x dx]$

$=\frac{1}{2}[\frac{1}{5}x\sin 5x+\frac{\cos 5x}{25}+x\sin x+\cos x]+c$

4. (a) $\int x\sec^2 x dx$ [ঢা.'০১,'১৪]

$=x\int\sec^2 x dx-\int\{\frac{d}{dx}(x)\int\sec^2 x dx\}dx$

$=x\tan x-\int 1.\tan x dx$

$=x\tan x+\ln|\cos x|+c$

4.(b) $\int x\sec^2 3x\,dx$ [ঢা.'০১]

$=x\int\sec^2 3x dx-\int\{\frac{d}{dx}(x)\int\sec^2 3x dx\}dx$

$=x\frac{\tan 3x}{3}-\int 1.\frac{\tan 3x}{3}dx$

$=\frac{x}{3}\tan 3x+\frac{1}{9}\ln|\cos 3x|+c$

(c) $\int x\tan^2 xdx$ [রা.'০৫; সি.'০৫]

$= \int x(\sec^2 x - 1)dx = \int x\sec^2 xdx - \int xdx$

$= x\int \sec^2 xdx - \int \{\frac{d}{dx}(x)\int \sec^2 xdx\}dx - \frac{x^2}{2}$

$= = x\tan x - \int 1.\tan xdx - \frac{x^2}{2}$

$= x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$

(d) ধরি, $I = \int \cos ec^3 x\, dx$

$= \int \cos ec^2 x \cos ecx\, dx$

$= \cos ecx \int \cos ec^2 x\ \ dx -$

$\int \{\frac{d}{dx}(\cos ecx)\int \cos ec^2 xdx\}dx$

$= -\cos ecx \cot x - \int (-\cos ecx \cot x).(-\cot x)dx =$

$-\cos ecx \cot x - \int \cos ecx(\cos ec^2 x - 1)dx$

$= -\cos ecx \cot x - \int \cos ec^3 xdx + \int \cos ecxdx$

$\Rightarrow I = -\cos ecx \cot x - I + \ln|\tan\frac{x}{2}| + c_1$

$\Rightarrow 2I = -\cos ecx \cot x + \ln|\tan\frac{x}{2}| + c_1$

$\Rightarrow I = -\frac{1}{2}\cos ecx \cot x + \frac{1}{2}\ln|\tan\frac{\pi}{2}| + \frac{1}{2}c_1$

$\Rightarrow I = -\frac{1}{2}\cos ecx \cot x + \frac{1}{2}\ln|\tan\frac{\pi}{2}| + c$

5. সূত্র (MCQ এর জন্য)ঃ

$\int x^n \ln xdx = \frac{x^{n+1}}{n+1}(\ln x - \frac{1}{n+1})$

(a) $\int x\ln xdx$ [য.'০৩; ঢা.'০৬; ব.'০৮]

$= \ln x\int xdx - \int \{\frac{d}{dx}(\ln x)\int xdx\}dx$

$= \ln x.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int xdx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}.\frac{x^2}{2} + c = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$

(b) $\int x^n \ln xdx$ [প্র.ভ.প.'১৩]

$= \ln x\int x^n dx - \int \{\frac{d}{dx}(\ln x)\int x^n dx\}dx$

$= \ln x.\frac{x^{n+1}}{n+1} - \int \frac{1}{x}.\frac{x^{n+1}}{n+1}dx$

$= \frac{x^{n+1}}{n+1}\ln x - \frac{1}{n+1}\int x^n dx$

$= \frac{x^{n+1}}{n+1}\ln x - \frac{1}{n+1}.\frac{x^{n+1}}{n+1} + c$

$= \frac{x^{n+1}}{n+1}\ln x - \frac{x^{n+1}}{(n+1)^2} + c$

(c) $\int x^2(\ln x)^2 dx$ [প্র.ভ.প.'০৫]

$= (\ln x)^2\int x^2 dx - \int \{\frac{d}{dx}(\ln x)^2\int x^2 dx\}dx$

$= (\ln x)^2\frac{x^3}{3} - \int 2\ln x.\frac{1}{x}.\frac{x^3}{3}dx$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}\int x^2 \ln xdx$

$= \frac{x^3}{3}(\ln x)^2 -$

$\frac{2}{3}[\ln x\int x^2 dx - \int \{\frac{d}{dx}(\ln x)\int x^2 dx\}dx]$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\ln x.\frac{x^3}{3} - \int \frac{1}{x}.\frac{x^3}{3}dx]$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx]$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\frac{x^3}{3}\ln x - \frac{1}{3}.\frac{x^3}{3}] + c$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\frac{x^3}{3}\ln x - \frac{x^3}{9}] + c$

$= \frac{x^3}{27}[9(\ln x)^2 - 6\ln x + 2] + c$

(d) $\int(\ln x)^2 dx$ [য.'০৫; চ.'০৭; প্র.ভ.প. '১০]

$= (\ln x)^2\int dx - \int \{\frac{d}{dx}(\ln x)^2\int dx\}dx$

$= (\ln x)^2.x - \int 2\ln x.\frac{1}{x}.xdx$

$= x(\ln x)^2 - 2\int \ln xdx$

$= x(\ln x)^2 - 2[\ln x\int dx - \int\{\frac{d}{dx}(\ln x)\int dx\}dx] =$

$x(\ln x)^2 - 2[\ln x.x - \int \frac{1}{x}.xdx]$

$= x(\ln x)^2 - 2[x\ln x - \int dx]$

$= x(\ln x)^2 - 2[x\ln x - x] + c$

$= x\{(\ln x)^2 - 2\ln x + 2\} + c$

(e) ধরি, $I = \int \frac{\ln(\ln x)dx}{x}$ এবং $\ln x = z$.

তাহলে $\frac{1}{x}dx = dz$ এবং $I = \int \ln z\, dz$

$\Rightarrow I = \ln z\int dz - \int\{\frac{d}{dz}(\ln z)\int dz\}dz$

$= \ln z.z - \int \frac{1}{z}.zdz = z\ln z - \int dz$

$= z\ln z - z + c = \ln x\{\ln(\ln x) - 1\} + c$

(f) ধরি, $I = \int \frac{\ln \sec^{-1} x}{x\sqrt{x^2-1}}dx$ [ঢা.'০৮; সি.'১৪]

এবং $\sec^{-1} x = z \Rightarrow \frac{dx}{x\sqrt{x^2-1}} = dz$

$\therefore I = \int \ln z\, dz$

$= \ln z\int dz - \int\{\frac{d}{dz}(\ln z)\int dz\}dz$

$= \ln z.z - \int \frac{1}{z}.zdz = z\ln z - \int dz$

$= z\ln z - z + c$

$= \{\ln(\sec^{-1} x) - 1\}\sec^{-1} x + c$

6(a) $\int \tan^{-1} xdx$ [কু.'০২; ঢা.'০৪; ব.'১০]

$= \tan^{-1} x\int dx - \int\{\frac{d}{dx}(\tan^{-1} x)\int dx\}dx$

$= x\tan^{-1} x - \int \frac{x}{1+x^2}dx$

$= x\tan^{-1} x - \frac{1}{2}\int \frac{(0+2x)dx}{1+x^2}$

$= x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + c$

(b) $\int x\sin^{-1} xdx$ [ঢা.'০৭]

$= \sin^{-1} x\int xdx - \int\{\frac{d}{dx}(\sin^{-1} x)\int xdx\}dx$

$= \sin^{-1} x.\frac{x^2}{2} - \int \frac{1}{\sqrt{1-x^2}}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}dx$

$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}[\int \sqrt{1-x^2}dx - \int \frac{1}{\sqrt{1-x^2}}dx]$

$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1} x -$

$-\sin^{-1} x] + c$

$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}[\frac{x\sqrt{1-x^2}}{2} - \frac{1}{2}\sin^{-1} x] + c$

(c) $\int \sin^{-1} x\, dx$ [সি.'০৩; য.'১০; ঢা.'১৪]

$= \sin^{-1} x\int dx - \int\{\frac{d}{dx}(\sin^{-1} x)\int dx\}dx$

$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1} x - (-\frac{1}{2})\int \frac{(0-2x)dx}{\sqrt{1-x^2}}$

$= x\sin^{-1} x + \frac{1}{2}.2\sqrt{1-x^2} + c$

$= x\sin^{-1} x + \sqrt{1-x^2} + c$

(d) $\int \cos^{-1} xdx$ [কু.'০৫,'১৪; চ.'০৬; য.'০৮; রা.'১০]

$= \cos^{-1} x\int dx - \int\{\frac{d}{dx}(\cos^{-1} x)\int dx\}dx$

$= x\cos^{-1} x + \int \frac{x}{\sqrt{1-x^2}}dx$

$= x\cos^{-1}x + (-\frac{1}{2})\int \frac{(0-2x)dx}{\sqrt{1-x^2}}$

$= x\cos^{-1}x - \frac{1}{2}.2\sqrt{1-x^2} + c$

$= x\cos^{-1}x - \sqrt{1-x^2} + c$

(e) $\int x\sin^{-1}x^2 dx$

[ঢা.'০৫; রা.'০৬; প্র.ভ.প. '০৪,'০৬]

$= \sin^{-1}x^2\int xdx - \int\{\frac{d}{dx}(\sin^{-1}x^2)\int xdx\}dx$

$= \sin^{-1}x^2.\frac{x^2}{2} - \int\frac{2x}{\sqrt{1-x^4}}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\sin^{-1}x^2 - \int\frac{x^3}{\sqrt{1-x^4}}dx$

$= \frac{x^2}{2}\sin^{-1}x^2 - (-\frac{1}{4})\int\frac{d(1-x^4)}{\sqrt{1-x^4}}$

$= \frac{x^2}{2}\sin^{-1}x^2 + \frac{1}{4}.2\sqrt{1-x^4} + c$

$= \frac{x^2}{2}\sin^{-1}x^2 + \frac{1}{2}\sqrt{1-x^4} + c$

6.(f) $\int x\tan^{-1}xdx$

[য.'০৬;সি. '০৪,'০৮; রা.'০৬ ; কু.'১০ ; ব.'১১]

$= \tan^{-1}x\int x\,dx - \int\{\frac{d}{dx}(\tan^{-1}x)\int x\,dx\}\,dx$

$= \tan^{-1}x.\frac{x^2}{2} - \int\frac{1}{1+x^2}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int(1-\frac{1}{1+x^2})\,dx$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}(x-\tan^{-1}x) + c$

$= \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c$ (Ans.)

7(a) $\int e^x\cos x\,dx$ [ঢা.'০২;প্র.ভ.প.'০৪,'০৬]

ধরি, $I = \int e^x\cos x\,dx$

$= e^x\int\cos x\,dx - \int\{\frac{d}{dx}(e^x)\int\cos x\,dx\}\,dx$

$= e^x\sin x - \int e^x\sin dx$

$= e^x\sin x - e^x\int\sin x\,dx + \int\{\frac{d}{dx}(e^x)\int\sin x\,dx\}\,dx$

$= e^x\sin x - e^x(-\cos x) + \int e^x(-\cos x)\,dx$

$= e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$

$= e^x\sin x + e^x\cos x - I + c_1$

$\Rightarrow 2I = e^x\sin x + e^x\cos x + c_1$

$\Rightarrow I = \frac{1}{2}e^x(\sin x + \cos x) + \frac{1}{2}c_1$

$\therefore \int e^x\cos x\,dx = \frac{1}{2}e^x(\sin x + \cos x) + c$

7(b) $\int e^x\sin xdx$ [কু.'০৮,'১৩; মা.'০৯; রা.,দি.'১৪]

ধরি, $I = \int e^x\sin xdx$

$= e^x\int\sin x\,dx - \int\{\frac{d}{dx}(e^x)\int\sin x\,dx\}\,dx$

$= e^x(-\cos x) - \int e^x(-\cos x)\,dx$

$= -e^x\cos x + e^x\int\cos x\,dx - \int\{\frac{d}{dx}(e^x)\int\cos x\,dx\}\,dx$

$= -e^x\cos x + e^x\sin x - \int e^x\sin xdx$

$= e^x(\sin x - \cos x) - I + c_1$

$\Rightarrow 2I = e^x(\sin x - \cos x) + c_1$

$\Rightarrow I = \frac{1}{2}e^x(\sin x - \cos x) + \frac{1}{2}c_1$

$\therefore \int e^x\sin xdx = \frac{1}{2}e^x(\sin x - \cos x) + c$

7(c) $\int e^{2x}\sin xdx$ [সি.'০২]

ধরি, $I = \int e^{2x}\sin xdx$

$= e^{2x}\int\sin x\,dx - \int\{\frac{d}{dx}(e^{2x})\int\sin x\,dx\}\,dx$

$= e^{2x}(-\cos x) - \int 2e^{2x}(-\cos x)\,dx$

$= -e^{2x}\cos x + 2e^{2x}\int \cos x\,dx -$

$2\int\{\frac{d}{dx}(e^{2x})\int \cos x\,dx\}\,dx$

$= -e^{2x}\cos x + 2e^{2x}\sin x - 2\int 2e^{2x}\sin x dx$

$= -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x dx$

$= e^{2x}(2\sin x - \cos x) - 4\text{I} + c_1$

$\Rightarrow 5\,\text{I} = e^{2x}(2\sin x - \cos x) + c_1$

$\Rightarrow \text{I} = \frac{e^{2x}}{5}(2\sin x - \cos x) + \frac{1}{5}c_1$

$\therefore \text{I} = \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(2\sin x - \cos x) + c$

7(d) $\int e^{2x}\cos^2 x\,dx = \int e^{2x}\frac{1}{2}(1+\cos 2x)\,dx$

$= \frac{1}{2}[\int e^{2x}dx + \int e^{2x}\cos 2x\,dx\,]$

$= \frac{1}{2}[\frac{1}{2}e^{2x} + \frac{e^{2x}}{2^2+2^2}(2\cos 2x + 2\sin 2x)] + c$

$= \frac{1}{2}[\frac{1}{2}e^{2x} + \frac{e^{2x}}{8}(2\cos 2x + 2\sin 2x)] + c$

$= \frac{1}{8}(2 + \cos 2x + \sin 2x)e^{2x} + c$

8(a) $\int e^x(\sin x + \cos x)dx$

[সি.'০৫,'১১; ঢা.'১০; কু.'১১]

$= \int e^x \sin x dx + \int e^x \cos x dx$

$= \int e^x \sin x dx + e^x \int \cos x dx -$

$\int\{\frac{d}{dx}(e^x)\int \cos x dx\}dx$

$= \int e^x \sin x dx + e^x \sin x - \int e^x \sin x dx$

$= e^x \sin x + c$

বিকল্প পদ্ধতি ঃ

ধরি, $f(x) = \sin x$. $\therefore f'(x) = \cos x$ এবং

$\int e^x(\sin x + \cos x)dx = \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + c = e^x \sin x + c$

8(b) ধরি, $\text{I} = \int e^x \sec x(1+\tan x)dx$

[রা.'০৩; য.'১১; চ.'১৩; প্র.ভ.প.'০৪]

এবং $f(x) = \sec x$. $\therefore f'(x) = \sec x \tan x$ এবং

$\text{I} = \int e^x(\sec x + \sec x \tan x)dx$

$= \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

$\therefore \int e^x \sec x(1+\tan x)dx = e^x \sec x + c$

8.(c) ধরি, $\text{I} = \int e^x(\tan^{-1}x + \frac{1}{1+x^2})$ এবং

$f(x) = \tan^{-1}x \;\; \therefore f'(x) = \frac{1}{1+x^2}$ এবং

$\text{I} = \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

$\therefore \int e^x(\tan^{-1}x + \frac{1}{1+x^2}) = e^x \tan^{-1}x + c$

8(d) $\int e^x\{\tan x - \ln(\cos x)\}\,dx$ [প্র.ভ.প.'১২]

ধরি, $\text{I} = \int e^x\{\tan x - \ln(\cos x)\}\,dx$ [সূত্র প্রয়োগ করে] এবং

$f(x) = -\ln(\cos x)$

$\therefore f'(x) = -\frac{-\sin x}{\cos x} = \tan x$ এবং

$\text{I} = \int e^x\{-\ln(\cos x) + \tan x\}dx$

$= \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

$\therefore \int e^x\{\tan x + \ln(\sec x)\}dx = -e^x \ln(\cos x) + c$

9(a) $\int \frac{e^x}{x}(1 + x\ln x)dx$ [ব.'০১; য.'০৭; দি.'১৩]

ধরি, $\text{I} = \int \frac{e^x}{x}(1 + x\ln x)dx = \int e^x(\frac{1}{x} + \ln x)dx$

এবং $f(x) = \ln x$. তাহলে $f'(x) = \frac{1}{x}$ এবং

$\text{I} = \int e^x(\ln x + \frac{1}{x})dx = \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + c = e^x \ln x + c$

$\therefore \int \frac{e^x}{x}(1 + x\ln x)dx = e^x \ln x + c$

9(b) $\int e^{-2x}(\frac{1}{x}-2\ln x)\,dx$ [কু.'০২]

$= \int e^{-2x}\cdot\frac{1}{x}dx - 2\int e^{-2x}\ln x\,dx$

$= e^{-2x}\int\frac{1}{x}dx - \int\{\frac{d}{dx}(e^{-2x})\int\frac{1}{x}dx\}dx - 2\int e^{-2x}\ln x\,dx$

$= e^{-2x}\ln x - \int(-2e^{-2x})\ln x dx - 2\int e^{-2x}\ln x\,dx$

$= e^{-2x}\ln x + 2\int e^{-2x}\ln x dx - 2\int e^{-2x}\ln x dx$

$\therefore \int e^{-2x}(\frac{1}{x}-2\ln x)\,dx = e^{-2x}\ln x + c$

9(c) $\int e^{5x}\left\{5\ln x+\frac{1}{x}\right\}dx$ [চ.'০৯; প্র.ভ.প.'১৯]

$= \int 5e^{5x}\ln x dx + \int e^{5x}\frac{1}{x}dx$

$= \int 5e^{5x}\ln x dx + e^{5x}\int\frac{1}{x}dx - \int\{\frac{d}{dx}(e^{5x})\int\frac{1}{x}dx\}dx$

$= \int 5e^{5x}\ln x dx + e^{5x}\ln x - \int 5e^{5x}\ln x dx$

$\therefore \int e^{5x}\left\{5\ln x+\frac{1}{x}\right\}dx = e^{5x}\ln x + c$

10(a) $\int\frac{dx}{x^2+x}$ [ব.'০৩]

$= \int\frac{dx}{x(x+1)} = \int\{\frac{1}{x(0+1)}+\frac{1}{(x+1)(-1)}\}dx$

$= \int(\frac{1}{x}-\frac{1}{x+1})dx = \ln|x| - \ln|x+1| + c$

10(b) $\int\frac{x+35}{x^2-25}dx$ [চ.'০৪]

$= \int\frac{x+35}{(x-5)(x+5)}dx$

$= \int\{\frac{5+35}{(x-5)(5+5)}+\frac{-5+35}{(-5-5)(x+5)}\}dx$

$= \int\{\frac{40}{10(x-5)}-\frac{30}{10(x+5)}\}dx$

$= \int\{\frac{4}{x-5}-\frac{3}{x+5}\}dx$

$= 4\ln|x-5| - 3\ln|x+5| + c$

10(c) $\int\frac{2x-1}{x(x-1)(x-2)}dx$ [চা.'০৯]

$= \int\{\frac{2.0-1}{x(0-1)(0-2)}+\frac{2.1-1}{1(x-1)(1-2)}+\frac{2.2-1}{2(2-1)(x-2)}\}dx$

$= \int\{-\frac{1}{2}\frac{1}{x}-\frac{1}{x-1}+\frac{3}{2(x-2)}\}dx$

$= -\frac{1}{2}\ln|x| - \ln|x-1| + \frac{3}{2}\ln|x-2| + c$

10(d) $\int\frac{x^2dx}{x^4-1}$ [রা.'১১; প্র.ভ.প.'১১]

$= \int\frac{x^2dx}{(x^2-1)(x^2+1)}$

$= \int\{\frac{1}{(x^2-1)(1+1)}+\frac{-1}{(-1-1)(x^2+1)}\}dx$

$= \frac{1}{2}\int\frac{1}{x^2-1^2}dx + \frac{1}{2}\int\frac{1}{1+x^2}dx$

$= \frac{1}{2}\cdot\frac{1}{2.1}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\tan^{-1}x + c$

$= \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\tan^{-1}x + c$

10(e) ধরি, $I = \int\frac{dx}{e^{2x}-3e^x}$ [প্র.ভ.প.'০৪]

এবং $e^x = z$. তাহলে $e^x dx = dz \Rightarrow dx = \frac{dz}{z}$ এবং

$I = \int\frac{1}{z^2-3z}\frac{dz}{z} = \int\frac{dz}{z^2(z-3)}$

এখন ধরি, $\frac{1}{z^2(z-3)} \equiv \frac{A}{z}+\frac{B}{z^2}+\frac{C}{z-3}$

$\therefore\ 1 \equiv Az(z-3) + B(z-3) + Cz^2 \cdots (1)$

(1) এ $z=3$ বসিয়ে পাই, $1 = 9C \Rightarrow C = \frac{1}{9}$

(1) এ $z=0$ বসিয়ে পাই, $1=-3B \Rightarrow B=-\frac{1}{3}$

(1) এর উভয়পক্ষ থেকে z^2 এর সহগ সমীকৃত করে পাই,

$0=A+C \Rightarrow A=-C=-\frac{1}{9}$

$\therefore I=\int\{-\frac{1}{9}\frac{1}{z}-\frac{1}{3}\frac{1}{z^2}+\frac{1}{9(z-3)}\}dz$

$=-\frac{1}{9}\ln|z|-\frac{1}{3}(-\frac{1}{z})+\frac{1}{9}\ln|z-3|+c$

$=\frac{1}{9}\ln|\frac{z-3}{z}|+\frac{1}{3z}+c$

$\therefore \int\frac{dx}{e^{2x}-3e^x}=\frac{1}{9}\ln|\frac{e^x-3}{e^x}|+\frac{1}{3e^x}+c$

11. $\int\frac{1}{x^2(x-1)}dx$ [কু.,রা. '০২; ব.'০৫,'১০]

ধরি, $\frac{1}{x^2(x-1)}\equiv\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$\Rightarrow 1=Ax(x-1)+B(x-1)+Cx^2\cdots(1)$

(1) এ $x=0$ বসিয়ে পাই, $1=-B \Rightarrow B=-1$

(1) এ $x=1$ বসিয়ে পাই, $1=C \Rightarrow C=1$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0=A+C \Rightarrow A=-C=-1$

$\therefore \int\frac{1}{x^2(x-1)}dx=\int\{-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x-1}\}dz$

$=-\ln|x|-(-\frac{1}{x})+\ln|x-1|+c$

$=\ln|\frac{x-1}{x}|+\frac{1}{x}+c$

12 ধরি, $I=\int\frac{x+2}{(1-x)(x^2+4)}dx$ এবং

$\frac{x+2}{(1-x)(x^2+4)}\equiv\frac{A}{1-x}+\frac{Bx+C}{x^2+4}$

$\Rightarrow x+2=A(x^2+4)+(Bx+C)(1-x)\cdots(1)$

(1) এ $x=1$ বসিয়ে পাই, $1+2=5A \Rightarrow A=\frac{3}{5}$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0=A-B \Rightarrow B=A=\frac{3}{5}$

(1) এর উভয়পক্ষ থেকে ধ্রুবপদ সমীকৃত করে পাই,

$2=4A+C \Rightarrow C=2-\frac{12}{5}=-\frac{2}{5}$

$\therefore I=\frac{3}{5}\int\frac{1}{1-x}dx+\int\frac{\frac{3}{5}x-\frac{2}{5}}{x^2+4}dx$

$=-\frac{3}{5}\ln|1-x|+\frac{3}{10}\int\frac{2xdx}{x^2+4}-\frac{2}{5}\int\frac{dx}{x^2+2^2}$

$=-\frac{3}{5}\ln|1-x|+\frac{3}{10}\ln(x^2+4)-\frac{2}{5.2}\tan^{-1}\frac{x}{2}+c$

$=-\frac{3}{5}\ln|1-x|+\frac{3}{10}\ln(x^2+4)-\frac{1}{5}\tan^{-1}\frac{x}{2}+c$

13(a) $\int\frac{x^7}{(1-x^4)^2}dx=\int\frac{-x^3(1-x^4)+x^3}{(1-x^4)^2}dx$

$=\int\{\frac{-x^3}{1-x^4}+\frac{x^3}{(1-x^4)^2}\}dx$

$=\frac{1}{4}\int\frac{d(1-x^4)}{1-x^4}-\frac{1}{4}\int\frac{d(1-x^4)}{(1-x^4)^2}$

$=\frac{1}{4}\ln|1-x^4|-\frac{1}{4}(-\frac{1}{1-x^4})+c$

$=\frac{1}{4}(\ln|1-x^4|+\frac{1}{1-x^4})+c$

13(b) ধরি, $I=\int\frac{(x-2)^2}{(x+1)^2}dx=\int\frac{x^2-4x+4}{x^2+2x+1}dx$

$=\int\frac{(x^2+2x+1)-6x+3}{x^2+2x+2}dx$

$=\int\{1-\frac{6x-3}{(x+1)^2}\}dx$ এবং

$\frac{6x-3}{(x+1)^2}\equiv\frac{A}{x+1}+\frac{B}{(x+1)^2}$

$\Rightarrow 6x-3=A(x+1)+B\cdots(1)$

(1) এ $x=-1$ বসিয়ে পাই, $B=-6-3=-9$

(1) এর উভয়পক্ষ থেকে x এর সহগ সমীকৃত করে পাই,

$6=A \Rightarrow A=6$

$\therefore I=\int\{1-\frac{6}{x+1}+\frac{9}{(x+1)^2}\}dx$

$= x - 6\ln|x+1| - \frac{9}{x+1} + c$

13(c) ধরি, $I = \int \frac{\sin 2x\, dx}{3+5\cos x} = \int \frac{2\sin x \cos x\, dx}{3+5\cos x}$

এবং $\cos x = z$. তাহলে $-\sin x dx = dz$ এবং

$I = \int \frac{-2z\, dz}{3+5z} = -\frac{2}{5}\int \frac{3+5z-3}{3+5z} dz$

$= -\frac{2}{5}\int (1 - \frac{3}{3+5z}) dz$

$= -\frac{2}{5}(z - \frac{3}{5}\ln|3+5z|) + c$

$= \frac{2}{25}(3\ln|3+5z| - 5z) + c$

$= \frac{2}{25}(3\ln|3+5\cos x| - 5\cos x) + c$

সম্ভাব্য ধাপসহ প্রশ্ন

নিচের যোগজগুলি মান নির্ণয় কর ঃ

14. $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$

$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})dx}{(\sqrt{x+a}+\sqrt{x+b})(\sqrt{x+a}-\sqrt{x+b})}$ (১)

$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})dx}{(x+a)-(x+b)}$

$= \int \frac{(x+a)^{1/2} - (x+b)^{1/2}}{a-b} dx$

$= \frac{1}{a-b}\left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(x+b)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right] + c$

$= \frac{2}{3(a-b)}[(x+a)^{3/2} - (x+b)^{3/2}] + c$ (১)

15. $\int 3\sin x \cos x dx$

$= \int \frac{3}{2}(2\sin x\cos x)dx = \frac{3}{2}\int \sin 2x dx$ (১)

$= \frac{3}{2}(-\frac{1}{2}\cos 2x) + c = -\frac{3}{4}\cos 2x + c$ (১)

16. (a) $\int 3\cos 3x \cos x\, dx$

$= \int \frac{3}{2}\{\cos(3x+x) + \cos(3x-x)\}\, dx$ (১)

$= \int \frac{3}{2}(\cos 4x + \cos 2x)\, dx$

$= \frac{3}{2}(\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x) + c$

$= \frac{3}{8}(\sin 4x + 2\sin 2x) + c$ (১)

16(b) $\int \cos^2 \frac{x}{2} dx = \int \frac{1}{2}(1+\cos x)dx$ (১)

$= \frac{1}{2}(x + \sin x) + c$ (১)

17(a) $\int \cos x \cos(\sin x) dx$

$= \int \cos(\sin x)\, d(\sin x) = \cos(\sin x) + c$ (২)

(b) ধরি, $I = \int (e^x + \frac{1}{x})(e^x + \ln x)\, dx$ [রা.'০১]

এবং $e^x + \ln x = z$.

তাহলে $(e^x + \frac{1}{x})dx = dz$ এবং (১)

$I = \int z\, dz = \frac{1}{2}z^2 + c = \frac{1}{2}(e^x + \ln x)^2 + c$ (১)

18. $\int e^{3\cos 2x} \sin 2x\, dx$

$= -\frac{1}{6}\int e^{3\cos 2x}(-6\sin 3x dx)$ (১)

$= -\frac{1}{6}e^{3\cos 2x} + c$ (১)

19(a) ধরি, $I = \int \sin^3 x \cos x dx$

এবং $\sin x = z$. তাহলে, $\cos x\, dx = dz$ এবং (১)

$I = \int z^3\, dz = \frac{1}{4}z^4 + c = \frac{1}{4}\sin^4 x + c$ (১)

19(b) ধরি, $I = \int \tan^3 x \sec^2 x dx$ এবং $\tan x = z$ (১)

তাহলে, $\sec^2 x\, dx = dz$ এবং

$I=\int z^3dz=\frac{z^{3+1}}{3+1}+c=\frac{1}{4}\tan^4 x+c$ (১)

19(c) $\int \sin^2(3x+2)dx$

$=\int\frac{1}{2}\{1-\cos 2(3x+2)\}dx$ (১)

$=\frac{1}{2}\{\int dx-\int\cos(6x+4)dx\}$

$=\frac{1}{2}\{x-\frac{\sin(6x+4)}{6}\}+c$ (১)

$=\frac{1}{2}x-\frac{1}{12}\sin(6x+4)+c$

20.(a) $\int\frac{(\ln x)^2}{x}dx=\int(\ln x)^2 d(\ln x)$ (১)

$=\frac{(\ln x)^{2+1}}{2+1}+c=\frac{1}{3}(\ln x)^3+c$ (১)

20(b) $\int\frac{\sqrt{1+\ln x}}{x}dx$

$=\int(1+\ln x)^{\frac{1}{2}}d(1+\ln x)$ (১)

$=\frac{(1+\ln x)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c=\frac{2}{3}(1+\ln x)^{3/2}+c$ (১)

20(c) $\int\frac{\cos(\ln x)}{x}dx=\int\cos(\ln x)\,d(\ln x)$ (১)

$=\sin(\ln x)+c$ (১)

21. $\int\frac{e^{-x}dx}{(5+e^{-x})^2}$

$=\int(5+e^{-x})^{-2}\,d(5+e^{-x}).(-1)$ (১)

$=-\frac{(5+e^{-x})^{-2+1}}{-2+1}+c=\frac{1}{5+e^{-x}}+c$ (১)

22. $\int\frac{e^x(1+x)dx}{\cos^2(xe^x)}$

ধরি, $xe^x=z\ \therefore\ e^x(x+1)dx=dz$ (১)

$\therefore\ \int\frac{e^x(1+x)dx}{\cos^2(xe^x)}=\int\frac{dz}{\cos^2 z}=\int\sec^2 zdz$

$=\tan z+c=\tan(xe^x)+c$ (১)

23(a) ধরি, $I=\int\frac{\sin(2+5\ln x)}{x}dx$ এবং

$2+5\ln x=z$. তাহলে, $\frac{5}{x}dx=dz$ এবং (১)

$I=\frac{1}{5}\int\sin z\,dz=\frac{1}{5}(-\cos z)+c$

$=-\frac{1}{5}\cos(2+5\ln x)+c$ (১)

23(b) $\int\frac{dx}{\sin(x-a)\sin(x-b)}$

$=\int\frac{\sin\{(x-b)-(x-a)\}dx}{\sin(a-b)\sin(x-a)\sin(x-b)}$

$=\int\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)\}dx}{\sin(a-b)\sin(x-a)\sin(x-b)}$ (১)

$=\frac{1}{\sin(a-b)}\int\{\cot(x-a)-\cot(x-b)\}dx$

$=\frac{\ln|\sin(x-a)|-\ln|\sin(x-b)|}{\sin(a-b)}+c$

$=\frac{1}{\sin(a-b)}\ln\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$ (১)

24(a) $\int\frac{\sec^2 xdx}{\sqrt{1+\tan x}}=\int\frac{d(1+\tan x)}{\sqrt{1+\tan x}}$ (১)

$=2\sqrt{1+\tan x}+c$ (১)

24(b) $\int\frac{dx}{\sqrt{(\sin^{-1}x)}\sqrt{1-x^2}}=\int\frac{d(\sin^{-1}x)}{\sqrt{(\sin^{-1}x)}}$ (১)

$[\because d(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}dx]$

$=2\sqrt{\sin^{-1}x}+c$, $[\because\int\frac{dx}{\sqrt{x}}=2\sqrt{x}]$ (১)

24(c) ধরি, $I=\int\frac{dx}{(1+x^2)\sqrt{\tan^{-1}x+3}}$

এবং $\tan^{-1}x+3=z$. তাহলে, $\frac{dx}{1+x^2}=dz$ এবং (১)

$$I=\int\frac{dz}{\sqrt{z}}=2\sqrt{z}+c \quad [\because \int\frac{1}{\sqrt{x}}dx=2\sqrt{x}]$$

$$\therefore \int\frac{dx}{(1+x^2)\sqrt{\tan^{-1}x+3}}=2\sqrt{\tan^{-1}x+3}+c \quad (১)$$

24(d) $\int\frac{\tan(\ln|x|)}{x}dx=\int\tan(\ln|x|)\,d(\ln|x|)$ (১)

$=\ln\{\sec(\ln|x|)\}+c$ (১)

25(a) $\int\frac{\sec^2 x dx}{\sqrt{1-\tan^2 x}}=\int\frac{d(\tan x)}{\sqrt{1-\tan^2 x}}$ (১)

$=\sin^{-1}(\tan x)+c$ (১)

25(b) $\int\frac{dx}{\sqrt{15-4x-4x^2}}$

$=\int\frac{dx}{\sqrt{16-\{(2x)^2+2.2x.1+1^2\}}}$

$=\frac{1}{2}\int\frac{d(2x+1)}{\sqrt{4^2-(2x+1)^2}}=\frac{1}{2}\sin^{-1}(\frac{2x+1}{4})+c$ (২)

25(c) $\int\frac{dx}{\sqrt{x(4-x)}}=\int\frac{dx}{\sqrt{4x-x^2}}$

$=\int\frac{dx}{\sqrt{2^2-(x^2-4x+2^2)}}$

$=\int\frac{d(x-2)}{\sqrt{2^2-(x-2)^2}}=\sin^{-1}(\frac{x-2}{2})+c$ (২)

25(d) $\int\frac{dx}{\sqrt{a^2-b^2(1-x)^2}}$

$=-\frac{1}{b}\int\frac{d(b-bx)}{\sqrt{a^2-(b-bx)^2}}$ (১)

$=-\frac{1}{b}\sin^{-1}(\frac{b-bx}{a})+c$ (১)

25(e) ধরি, $I=\int\sqrt{\tan x}dx$ এবং $\tan x=z^2$

তাহলে, $\sec^2 x\,dx=2z\,dz$ (১)

$\Rightarrow dx=\frac{2zdz}{1+\tan^2 x}=\frac{2z}{1+z^4}$ এবং

$$I=\int\frac{2z^2dz}{1+z^4}=\int\frac{(z^2+1)-(z^2-1)}{1+z^4}dz$$

$=\int[\frac{z^2+1}{z^4+1}+\frac{z^2-1}{z^4+1}]dz$

$=\int[\frac{1+\frac{1}{z^2}}{z^2+\frac{1}{z^2}}+\frac{1-\frac{1}{z^2}}{z^2+\frac{1}{z^2}}]dz$

$=\int[\frac{1+\frac{1}{z^2}}{(z-\frac{1}{z})^2+2}+\frac{1-\frac{1}{z^2}}{(z+\frac{1}{z})^2-2}]dz$

$=\int\frac{d(z-\frac{1}{z})}{(z-\frac{1}{z})^2+(\sqrt{2})^2}+\int\frac{d(z+\frac{1}{z})}{(z+\frac{1}{z})^2-(\sqrt{2})^2}$ (১)

$=\frac{1}{\sqrt{2}}\tan^{-1}\frac{z-\frac{1}{z}}{\sqrt{2}}+\frac{1}{2\sqrt{2}}\ln|\frac{z-\frac{1}{z}-\sqrt{2}}{z-\frac{1}{z}+\sqrt{2}}|+c$ (১)

$=\frac{1}{\sqrt{2}}\tan^{-1}\frac{z^2-1}{\sqrt{2}z}+\frac{1}{2\sqrt{2}}\ln|\frac{z^2-1-\sqrt{2}z}{z^2-1+\sqrt{2}z}|+c$

$=\frac{1}{\sqrt{2}}\tan^{-1}\frac{\tan x-1}{\sqrt{2\tan x}}+$

$\frac{1}{2\sqrt{2}}\ln|\frac{\tan x-\sqrt{2\tan x}-1}{\tan x+\sqrt{2\tan x}-1}|+c$ (১)

26. ধরি, $I=\int 3\cos^3 x\cos 2x\,dx$

$\cos^3 x\cos 2x=\frac{1}{4}(3\cos x+\cos 3x)\cos 2x$ (১)

$=\frac{1}{4}[3\cos x\cos 2x+\cos 3x\cos 2x]$

$=\frac{1}{4}[3.\frac{1}{2}(\cos 3x+\cos x)+\frac{1}{2}(\cos 5x+\cos x)]$ (১)

$=\frac{1}{8}(3\cos 3x+4\cos x+\cos 5x)$

$\therefore I=\frac{3}{8}\int(3\cos 3x+4\cos x+\cos 5x)\,dx$

$=\frac{3}{8}(3.\frac{1}{3}\sin 3x+4\sin x+\frac{1}{5}\sin 5x)+c$ (২)

27(a) ধরি, $I = \int e^{2x}\cos x dx$

$= e^{2x}\int \cos x\, dx - \int \{\frac{d}{dx}(e^{2x})\int \cos x\, dx\}\, dx$ (১)

$= e^{2x}\sin x - \int 2e^{2x}\sin x\, dx$ (২)

$= e^{2x}\sin x - 2e^{2x}\int \sin x dx +$

$2\int \{\frac{d}{dx}(e^{2x})\int \sin x\, dx\}\, dx$

$= e^{2x}\sin x - 2e^{2x}(-\cos x) + 2\int 2e^{2x}(-\cos x)dx$

$= e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x dx$

$= e^{2x}(\sin x + 2\cos x) - 4I + c_1$

$\Rightarrow 5I = e^{2x}(\sin x + 2\cos x) + c_1$

$\Rightarrow I = \frac{e^{2x}}{5}(\sin x + 2\cos x) + \frac{1}{5}c_1$

$\therefore I = \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(\sin x + 2\cos x) + c$ (১)

27.(b) $\int e^{-3x}\cos 4x\, dx$

$= \frac{e^{-3x}}{3^2+4^2}(-3\cos 4x + 4\sin 4x) + c$

[সূত্র প্রয়োগ করে।]

$= \frac{e^{-3x}}{25}(-3\cos 4x + 4\sin 4x) + c$ (২)

28(a) ধরি, $I = \int e^x \frac{1+\sin x}{1+\cos x}dx$

$= \int e^x\{\frac{1}{1+\cos x} + \frac{\sin x}{1+\cos x}\}dx$

এবং $f(x) = \frac{\sin x}{1+\cos x}$

$\therefore f'(x) = \frac{(1+\cos x)\cos x - \sin x(0-\sin x)}{(1+\cos x)^2}$ (২)

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$ এবং

$I = \int e^x\{\frac{\sin x}{1+\cos x} + \frac{1}{1+\cos x}\}dx$

$= \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$ (১)

$\therefore I = \int e^x \frac{1+\sin x}{1+\cos x}dx = e^x\frac{\sin x}{1+\cos x} + c$ (১)

28(b) $\int e^{ax}(a\sin bx + b\cos bx)dx$

$= \int ae^{ax}\sin bx dx + \int be^{ax}\cos bx dx$

$= a\sin bx\int e^{ax}dx - \int\{\frac{d}{dx}(a\sin bx)\int e^{ax}dx\}dx$

$+ \int be^{ax}\cos bx dx$ (১)

$= a\sin bx.(\frac{e^{ax}}{a}) - \int (ab\cos bx)(\frac{e^{ax}}{a})dx$

$+ \int be^{ax}\cos bx dx$ (২)

$= e^{ax}\sin bx - \int be^{ax}\cos bx dx + \int be^{ax}\cos bx dx$

$\therefore \int e^{ax}(a\sin bx + b\cos bx)dx = e^{ax}\sin bx + c$ (১)

29(a) $\int \frac{x-3}{(1-2x)(1+x)}dx$

$= \int [\frac{\frac{1}{2}-3}{(1-2x)(1+\frac{1}{2})} + \frac{-1-3}{\{1-2(-1)\}(1+x)}]dx$ (১)

$= \int [\frac{-\frac{5}{2}}{\frac{3}{2}(1-2x)} + \frac{-4}{3(1+x)}]dx$

$= -\frac{5}{3}(-\frac{1}{2})\int \frac{d(1-2x)}{(1-2x)} - \frac{4}{3}\int \frac{1}{1+x}dx$ (১)

$= \frac{5}{6}\ln|1-2x| - \frac{4}{3}\ln|1+x| + c$ (১)

29(b) $\int \frac{dx}{x^4-1} = \int \frac{dx}{(x^2-1)(x^2+1)}$

$= \int \{\frac{1}{(x^2-1)(1+1)} + \frac{1}{(-1-1)(x^2+1)}\}dx$ (১)

$= \frac{1}{2}\int \frac{dx}{x^2-1^2} - \frac{1}{2}\int \frac{1}{1+x^2}dx$

$= \frac{1}{2}.\frac{1}{2.1}\ln|\frac{x-1}{x+1}| - \frac{1}{2}\tan^{-1}x + c$ (২)

$= \frac{1}{4}\ln|\frac{x-1}{x+1}| - \frac{1}{2}\tan^{-1}x + c$

30(a) $\int \frac{1}{x(x+1)^2}dx$

ধরি, $\frac{1}{x(x+1)^2} \equiv \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\Rightarrow 1 = A(x+1)^2 + Bx(x+1) + Cx \cdots(1)$

(1) এ $x=0$ বসিয়ে পাই, $1 = A \Rightarrow A = 1$

(1) এ $x=-1$ বসিয়ে পাই, $1 = -C \Rightarrow C = -1$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0 = A + B \Rightarrow B = -A = -1$

$\therefore \int \frac{1}{x(x+1)^2}dx = \int\{\frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2}\}dz$ (১)

$= \ln|x| - \ln|x+1| - (-\frac{1}{x+1}) + c$ (২)

$= \ln|\frac{x}{x+1}| + \frac{1}{x+1} + c$

30(b) $\int \frac{3x+1}{(x+1)^2}dx = \int \frac{3(x+1)-2}{(x+1)^2}dx$

$= \int\{\frac{3(x+1)}{(x+1)^2} - \frac{2}{(x+1)^2}\}dx$

$= \int\{\frac{3}{x+1} - \frac{2}{(x+1)^2}\}dx$

$= 3\ln|x+1| - 2(-\frac{1}{x+1}) + c$

$= 3\ln|x+1| + \frac{2}{x+1} + c$ (২)

31. (a) $\int \frac{dx}{x(x^2+1)} = \int \frac{(x^2+1) - x^2 dx}{x(x^2+1)}$

$= \int\{\frac{1}{x} - \frac{x}{x^2+1}\}dx$

$= \int \frac{1}{x}dx - \frac{1}{2}\int \frac{(2x+0)dx}{x^2+1}$ (১)

$= \ln|x| - \frac{1}{2}\ln(x^2+1) + c$ (১)

31(b) ধরি, $I = \int \frac{xdx}{(x-1)(x^2+4)}$ এবং

$\frac{x}{(x-1)(x^2+4)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$

$\Rightarrow x = A(x^2+4) + (Bx+C)(x-1) \cdots(1)$

(1) এ $x=1$ বসিয়ে পাই, $1 = 5A \Rightarrow A = \frac{1}{5}$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0 = A + B \Rightarrow B = -A = -\frac{1}{5}$

(1) এর উভয়পক্ষ থেকে ধ্রুবপদ সমীকৃত করে পাই,

$0 = 4A - C \Rightarrow C = 4A = \frac{4}{5}$

$I = \frac{1}{5}\int \frac{1}{x-1}dx + \int \frac{-\frac{1}{5}x + \frac{4}{5}}{x^2+4}dx$ (১)

$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\int \frac{2xdx}{x^2+4} + \frac{4}{5}\int \frac{dx}{x^2+2^2}$ (১)

$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\ln(x^2+4) + \frac{4}{5}.\frac{1}{2}\tan^{-1}\frac{x}{2} + c$

$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\ln(x^2+4) + \frac{2}{5}\tan^{-1}\frac{x}{2} + c$ (২)

32.(a) $\int xe^{-x}dx$

$= x\int e^{-x}dx - \int\{\frac{d}{dx}(x)\int e^{-x}dx\}dx$ (১)

$= -xe^{-x} - \int 1.(-e^{-x})dx = -xe^{x} - e^{-x} + c$ (১)

32(b) $\int xe^{ax}dx$

$= x\int e^{ax}dx - \int\{\frac{d}{dx}(x)\int e^{ax}dx\}dx$ (১)

$= x.\frac{1}{a}e^{ax} - \int 1.(\frac{1}{a}e^{ax})dx = \frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax} + c$

$= \frac{1}{a^2}(ax-1)e^{ax} + c$ (১)

32(c) $\int x^3e^{2x}dx$

$=x^3\int e^{2x}dx-\int\{\frac{d}{dx}(x^3)\int e^{2x}dx\}dx$ (১)

$=x^3(\frac{1}{2}e^{2x})-\int(3x^2)(\frac{1}{2}e^{2x})dx$ (২)

$=\frac{1}{2}x^3e^{2x}-\frac{3}{2}[x^2\int e^{2x}-\int\{\frac{d}{dx}(x^2)\int e^{2x}dx\}dx]$

$=\frac{1}{2}x^3e^{2x}-\frac{3}{2}[x^2.\frac{1}{2}e^{2x}-\int(2x).\frac{1}{2}e^{2x}dx]$

$=\frac{1}{2}x^3e^{2x}-\frac{3}{2}[\frac{x^2e^{2x}}{2}-\{x\int e^{2x}-\int 1.\frac{e^{2x}}{2}dx\}]$

$=\frac{1}{2}x^3e^{2x}-\frac{3}{2}[\frac{x^2e^{2x}}{2}-\{x\frac{e^{2x}}{2}-\frac{e^{2x}}{4}\}]+c$

$=(\frac{1}{2}x^3-\frac{3}{4}x^2+\frac{3}{4}x-\frac{3}{8})e^{2x}+c$ (১)

[MCQ এর ক্ষেত্রেঃ

$\int x^3e^{2x}dx=\{\frac{1}{2}x^3-\frac{1}{2^2}(3x^2)+\frac{1}{2^3}(6x)-\frac{1}{2^4}.6\}e^{2x}=\{\frac{1}{2}x^3-\frac{3}{4}x^2+\frac{3}{4}x-\frac{3}{8}\}e^{2x}$]

33. (a) $\int x\sin x dx$

$=x\int\sin xdx-\int\{\frac{d}{dx}(x)\int\sin x\,dx\}\,dx$ (১)

$=x(-\cos x)-\int 1.(-\cos x)dx$

$=-x\cos x+\sin x+c$ (১)

33. (b) $\int x\cos xdx$

$=x\int\cos xdx-\int\{\frac{d}{dx}(x)\int\cos x\,dx\}\,dx$ (১)

$=x\sin x-\int 1.\sin xdx$

$=x\sin x+\cos x+c$ (১)

33(c) $\int x^2\sin xdx$

$=x^2\int\sin xdx-\int\{\frac{d}{dx}(x^2)\int\sin x\,dx\}\,dx$ (১)

$=x^2(-\cos x)-\int 2x(-\cos x)dx$ (১)

$=-x^2\cos x+2[x\int\cos x-\int\{\frac{d}{dx}(x)\int\cos x\,dx\}\,dx]$

$=-x^2\cos x+2[x\sin x-\int 1.\sin x\,dx]$ (১)

$=-x^2\cos x+2[x\sin x-(-\cos x)]+c$

$=-x^2\cos x+2x\sin x+2\cos x+c$ (১)

33.(d) $\int x^2\cos^2 x\,dx$ [প্র.ভ.প.'৮৫,'৯৬]

$=\int x^2\frac{1}{2}(1+\cos 2x)\,dx$ (১)

$=\frac{1}{2}[\int x^2\,dx+\int x^2\cos 2x\,dx]$ (১)

$=\frac{1}{2}[\frac{x^3}{3}+x^2(\frac{1}{2}\sin 2x)-(2x)(-\frac{1}{2^2}\cos 2x)+2(-\frac{1}{2^3}\sin 2x)]+c$ (২)

$=\frac{1}{2}[\frac{x^3}{3}+\frac{x^2}{2}\sin 2x+\frac{1}{2}x\cos 2x-\frac{1}{4}\sin 2x]+c$

33(e) ধরি, $I=\int\cos\sqrt{x}\,dx$ এবং $\sqrt{x}=z$

তাহলে $\frac{1}{2\sqrt{x}}dx=dz\Rightarrow dx=2z\,dz$ এবং (১)

$I=\int 2z\cos z\,dz$

$=2[z\int\cos z\,dz-\int\{\frac{d}{dz}(z)\int\cos z\,dz\}\,dz]$ (১)

$=2[z\sin z)-\int 1.\sin zdz]$ (১)

$=2z\sin z-2(-\cos z)+c$

$=2\sqrt{x}\sin\sqrt{x}+2\cos\sqrt{x}+c$ (১)

34(a) $\int x\sin x\cos xdx=\frac{1}{2}\int x\sin 2xdx$ (১)

$=\frac{1}{2}[x\int\sin 2xdx-\int\{\frac{d}{dx}(x)\int\sin 2xdx\}dx]$ (১)

$=\frac{1}{2}[x(-\frac{\cos 2x}{2})-\int 1.(-\frac{\cos 2x}{2})dx]$ (১)

$=\frac{1}{4}[-x\cos 2x+\frac{\sin 2x}{2}]+c$ (১)

34(b) $\int x\sin x\sin 2x\,dx$

$= \int x \frac{1}{2}(\cos x - \cos 3x)\, dx$ (১)

$= \frac{1}{2}[x\int \cos x dx - \int\{\frac{d}{dx}(x)\int \cos x dx\}dx$ (১)

$- x\int \cos 3x dx + \int\{\frac{d}{dx}(x)\int \cos 3x dx\}dx]$

$= \frac{1}{2}[x\sin x - \int 1.\sin x dx$

$- x\frac{\sin 3x}{3} + \int 1.\frac{\sin 3x}{3}dx]$ (১)

$= \frac{1}{2}[x\sin x + \cos x - \frac{x\sin 3x}{3} - \frac{\cos 3x}{9}] + c$ (১)

34. (c) $\int \frac{x}{\sin^2 x}dx = \int x\cos ec^2 x\, dx$

$= x\int \cos ec^2 x dx - \int\{\frac{d}{dx}(x)\int \cos ec^2 x dx\}dx$ (১)

$= x(-\cot x) - \int 1.(-\cot x)dx$ (১)

$= -x\cot x + \ln|\sin x| + c$ (১)

34(d) ধরি, $I = \int \sec^3 x\, dx = \int \sec^2 x \sec x\, dx$

$= \sec x\int \sec^2 x\ dx - \int\{\frac{d}{dx}(\sec x)\int \sec^2 x dx\}dx$ (১)

$= \sec x \tan x - \int \sec x \tan x.\tan x dx$ (১)

$= \sec x \tan x - \int \sec x(\sec^2 x - 1)dx$

$= \sec x \tan x - \int \sec^3 x dx + \int \sec x dx$

$\Rightarrow I = \sec x \tan x - I + \ln|\tan(\frac{\pi}{4} + \frac{x}{2}) + c_1$ (১)

$\Rightarrow 2I = \sec x \tan x + \ln|\tan(\frac{\pi}{4} + \frac{x}{2}) + c_1$

$\Rightarrow I = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\tan(\frac{\pi}{4} + \frac{x}{2}) + \frac{1}{2}c_1$

$\Rightarrow I = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\tan(\frac{\pi}{4} + \frac{x}{2})| + c$ (১)

35(a) $\int x^2 \ln x dx$

$= \ln x\int x^2 dx - \int\{\frac{d}{dx}(\ln x)\int x^2 dx\}dx$ (১)

$= \ln x.\frac{x^3}{3} - \int \frac{1}{x}.\frac{x^3}{3}dx$ (২)

$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$

$= \frac{x^3}{3}\ln x - \frac{1}{3}.\frac{x^3}{3} + c = \frac{x^3}{3}\ln x - \frac{x^3}{9} + c$ (১)

35(b) $\int x^3 \ln x dx$

$= \ln x\int x^3 dx - \int\{\frac{d}{dx}(\ln x)\int x^3 dx\}dx$ (১)

$= \ln x.\frac{x^4}{4} - \int \frac{1}{x}.\frac{x^4}{4}dx$ (২)

$= \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx$

$= \frac{x^4}{4}\ln x - \frac{1}{4}.\frac{x^4}{4} + c = \frac{x^4}{4}\ln x - \frac{x^4}{16} + c$ (১)

35(c) $\int \frac{\ln x}{x^2}dx$

$= \ln x\int \frac{1}{x^2}dx - \int\{\frac{d}{dx}(\ln x)\int \frac{1}{x^2}dx\}dx$ (১)

$= \ln x.(-\frac{1}{x}) - \int \frac{1}{x}.(-\frac{1}{x})dx$ (২)

$= -\frac{1}{x}\ln x + \int \frac{1}{x^2}dx = -\frac{1}{x}\ln x + (-\frac{1}{x}) + c$

$= -\frac{1}{x}\ln x - \frac{1}{x} + c$ (১)

36(a) $\int 2^x \sin x\, dx = \int e^{x\ln 2}\sin x\, dx$

$= \frac{e^{x\ln 2}}{(\ln 2)^2 + 1^2}[\ln 2.\sin x - 1.\cos x] + c$ (২)

[সূত্র প্রয়োগ করে।]

$= \frac{2^x}{(\ln 2)^2 + 1}[\ln 2.\sin x - \cos x] + c$

36(b) $\int (3^x e^x + \ln x)dx$ [প্র.ভ.প.'৮৪]

$= \int (3e)^x dx + \int \ln x dx$

$=\frac{(3e)^x}{\ln(3e)}+\frac{1}{x}+c=\frac{3^x e^x}{\ln 3+\ln e}+\frac{1}{x}+c$ (২)

$=\frac{3^x e^x}{\ln 3+1}+\frac{1}{x}+c$

36(c) ধরি, $I=\int e^x\left\{\frac{1}{1-x}+\frac{1}{(x-1)^2}\right\}dx$ এবং

$f(x)=\frac{1}{1-x}=(1-x)^{-1}$

$\therefore f'(x)=-(1-x)^{-1-1}(-1)=\frac{1}{(1-x)^2}$ এবং (১)

$I=\int e^x\left\{\frac{1}{1-x}+\frac{1}{(1-x)^2}\right\}dx$

$=\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+c$ (১)

$\therefore \int e^x\left\{\frac{1}{1-x}+\frac{1}{(x-1)^2}\right\}dx=\frac{e^x}{1-x}+c$ (১)

36(d) ধরি, $I=\int\left[\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\right]dx$ এবং $\ln x=y.$

তাহলে, $x=e^y \Rightarrow dx=e^y\,dy$ এবং (১)

$I=\int e^y\left[\frac{1}{y}-\frac{1}{y^2}\right]dy=\int e^y\left[\frac{1}{y}+D(\frac{1}{y})\right]dy$ (১)

$\left[\because D(\frac{1}{y})=\frac{d}{dx}(\frac{1}{y})=-\frac{1}{y^2}\right]$

$=\frac{e^y}{y}+c=\frac{x}{\ln x}+c$ (১)

37(a) $\int e^{-x}\left\{\frac{1}{x}+\frac{1}{x^2}\right\}dx=\int\frac{e^{-x}}{x}dx+\int\frac{e^{-x}}{x^2}dx$

$=\frac{1}{x}\int e^{-x}dx-\int\{\frac{d}{dx}(\frac{1}{x})\int e^{-x}dx\}dx+\int\frac{e^{-x}}{x^2}dx$ (১)

$=\frac{1}{x}(-e^{-x})-\int(-\frac{1}{x^2})(-e^{-x})dx+\int\frac{e^{-x}}{x^2}dx$ (২)

$=-\frac{e^{-x}}{x}-\int\frac{e^{-x}}{x^2}dx+\int\frac{e^{-x}}{x^2}dx$

$\therefore \int e^{-x}\left\{\frac{1}{x}+\frac{1}{x^2}\right\}dx=-\frac{e^{-x}}{x}+c$ (১)

37(b) $\int e^x\{\tan x+\ln(\sec x)\}dx$ [প্র.ভ.প. '৯১]

ধরি, $I=\int e^x\{\tan x+\ln(\sec x)\}dx$ এবং

$f(x)=\ln(\sec x)$

$\therefore f'(x)=\frac{\sec x\tan x}{\sec x}=\tan x$ এবং (১)

$I=\int e^x\{\ln(\sec x)+\tan x\}dx$

$=\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+c$

$\therefore \int e^x\{\tan x+\ln(\sec x)\}dx=e^x\ln(\sec x)+c$ (১)

38. ধরি, $I=\int e^x\frac{x^2+1}{(x+1)^2}dx$ [প্র.ভ.প.'০২]

$=\int e^x\frac{x^2-1+2}{(x+1)^2}dx$

$=\int e^x\{\frac{(x-1)(x+1)}{(x+1)^2}+\frac{2}{(x+1)^2}\}dx$

$=\int e^x\{\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\}dx$ এবং $f(x)=\frac{x-1}{x+1}$

$\therefore f'(x)=\frac{(x+1).1-(x-1).1}{(x+1)^2}$ (২)

$=\frac{x+1-x+1}{(x+1)^2}=\frac{2}{(x+1)^2}$ এবং

$I=\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+c$ (১)

$\therefore \int e^x\frac{x^2+1}{(x+1)^2}dx=e^x(\frac{x-1}{x+1})+c$ (১)

39. $\int\frac{x}{(x-1)^2(x+2)}dx$

ধরি, $\frac{x}{(x-1)^2(x+2)}\equiv\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}$

$\Rightarrow x=A(x-1)(x+2)+B(x+2)+C(x-1)^2\cdots(1)$

(1) এ $x=1$ বসিয়ে পাই, $1=3B\Rightarrow B=1/3$

(1) এ $x=-2$ বসিয়ে পাই, $-2=9C\Rightarrow C=-2/9$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0=A+C\Rightarrow A=-C=2/9$

$\therefore \int\frac{x}{(x-1)^2(x+2)}dx$

$= \int \{\frac{2/9}{x-1} + \frac{1/3}{(x-1)^2} + \frac{-2/9}{x+2}\}dz$ (১)

$= \frac{2}{9}\ln|x-1| + \frac{1}{3}(-\frac{1}{x-1}) - \frac{2}{9}\ln|x+2| + c$ (২)

$= \frac{2}{9}\ln|\frac{x-1}{x+2}| - \frac{1}{3(x-1)} + c$ (১)

40. ধরি, $I = \int \frac{x^2+1}{(x+2)^2}dx$

$= \int \frac{x^2+4x+4-(4x+3)}{(x+2)^2}dx$

$= \int \{1 - \frac{4x+3}{(x+2)^2}\}dx$ এবং (১)

$\frac{4x+3}{(x+2)^2} \equiv \frac{A}{x+2} + \frac{B}{(x+2)^2}$

$\Rightarrow 4x+3 = A(x+2) + B \cdots(1)$

(1) এ $x = -2$ বসিয়ে পাই, $B = -8+3 = -5$

(1) এর উভয়পক্ষ থেকে x এর সহগ সমীকৃত করে পাই, $4 = A \Rightarrow A = 4$

$\therefore I = \int \{1 - \frac{4}{x+2} + \frac{5}{(x+2)^2}\}dx$ (১)

$= x - 4\ln|x+2| - \frac{5}{x+2} + c$ (২)

ভর্তি পরীক্ষার MCQ

1. $\int \frac{dx}{\cos^2 x\sqrt{\tan x}} = ?$ **[DU 07-08; NU06-07]**

$Sol^n: I = \int \frac{\sec^2 x dx}{\sqrt{\tan x}} = \int \frac{d(\tan x)}{\sqrt{\tan x}} = 2\sqrt{\tan x}$

2. $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx = ?$ **[DU 07-08; NU07-08; KU 03-04]**

$Sol^n: I = \int \sec^2(xe^x)\, d(xe^x) = \tan(xe^x)$

3. $\int \frac{dx}{x+\sqrt{x}} = ?$ **[DU 02-03]**

$Sol^n: I = \int \frac{dx}{\sqrt{x}(\sqrt{x}+1)} = 2\int \frac{d(\sqrt{x}+1)}{\sqrt{x}+1})$

$= 2\ln(\sqrt{x}+1) + c$

4. $\int \sin^5 x \cos x dx = ?$ **[DU 98-99]**

$Sol^n: I = \int \sin^5 x d(\sin x) = \frac{1}{6}\sin^6 x + c$

5. $\int \frac{dx}{e^x + e^{-x}} = ?$ **[JU 06-07; CU 04-05]**

$Sol^n: I = \int \frac{e^x dx}{e^{2x}+1} = \int \frac{d(e^x)}{1+(e^x)^2}$

$= \tan^{-1}(e^x) + c$

6. $\int \sqrt{\frac{1+x}{1-x}}dx = ?$ **[DU 95-96; JU 07 08]**

$Sol^n: I = \int \frac{1+x}{\sqrt{1-x^2}}dx$

$= \int \frac{1}{\sqrt{1-x^2}}dx + (-\frac{1}{2})\int \frac{d(1-x^2)}{\sqrt{1-x^2}}$

$= \sin^{-1}x - \frac{1}{2}.2\sqrt{1-x^2} = \sin^{-1}x - \sqrt{1-x^2}$

7. $\int xe^x dx = ?$ **[JU 07-08]**

$Sol^n: I = (x-1)e^x + c$

8. $\int \frac{dx}{ay-bx} = ?$ **[SU 06-07]**

$Sol^n: I = -\frac{1}{b}\int \frac{d(ay-bx)}{ay-bx}$

$= -\frac{1}{6}\ln|ay-bx| + c$

9. $\int e^x \sec x(1+\tan x)dx = ?$ **[RU 06-07]**

$Sol^n: I = \int e^x(\sec x + \sec x \tan x)dx$

$= \int e^x\{\sec x + D(\sec x)\}dx = e^x \sec x$

10. $\int -\sin\phi\, dt = ?$ **[CU 04-05]**

$Sol^n: I = -\sin\phi \int dt = -t\sin\phi + c$

11. $\int \frac{dx}{\sqrt{9-16x^2}} = ?$ [KU 03-04]

$Sol^n: I = \frac{1}{4}\int \frac{d(4x)}{\sqrt{3^2-(4x)^2}} = \frac{1}{4}\sin^{-1}\frac{4x}{3}$

12. $\int \frac{xe^x}{(x+1)^2}dx = ?$ [DU 01-02; CU 02-03; RU 04-05, 05-06; JU 06-07;BUET 06-07]

$Sol^n: I = \int \frac{(x+1-1)e^x}{(x+1)^2}dx$

$= \int e^x\{\frac{1}{x+1} - \frac{1}{(x+1)^2}\}dx$

$= \int e^x\{\frac{1}{x+1} + D(\frac{1}{x+1})\}dx = \frac{e^x}{x+1} + c$

13. $\int x\cos x dx = ?$ [DU 96 - 97]

$= x\sin x - (1)(-\cos x) = x\sin x + \cos x + c$

14. $\int x\ln(1+2x)\,dx = ?$ [SU 96-97]

$Sol^n: I = \ln(1+2x).\frac{x^2}{2} - \int \frac{2.}{1+2x}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\ln(1+2x) - \int \frac{\frac{1}{2}x(2x+1) - \frac{1}{4}(2x+1) + \frac{1}{4}}{2x+1}dx$

$= \frac{x^2}{2}\ln(1+2x) - \int (\frac{1}{2}x - \frac{1}{4} + \frac{1}{4}\frac{1}{2x+1})dx$

$= \frac{x^2}{2}\ln(1+2x) - \{\frac{x^2}{4} - \frac{1}{4}x + \frac{1}{8}\ln(2x+1|\} + c$

$= \frac{x^2}{2}\ln(1+2x) - \frac{x^2}{4} + \frac{1}{4}x - \frac{1}{8}\ln(2x+1| + c$

15. $\int \log_3 x\,dx = ?$ [CU 06-07]

$Sol^n: I = x\log_3 x - \int \frac{1}{x\ln 3}.x\,dx$

$= x\log_3 x - \frac{x}{\ln 3} + c$

অন্তরক ও যোগজের মিশ্রিত সমস্যা

16. $y = x^2$ হলে $\int (\frac{dy}{dx})dx$ এর মান কত? [CU 02-03; IU 05-06]

$Sol^n: \int (\frac{dy}{dx})dx = y + c = x^2 + c$

17. যদি $\frac{dy}{dx} = 2a$ হয় তাহলে y এর মান কত? [CU 02-03]

$Sol^n: \frac{dy}{dx} = 2a \Rightarrow y = \int 2a dx = 2ax + c$

18. $\int f(x)dx = \cos x + k$ হলে $f(x)$ এর মান কত? [CU 02-03]

$Sol^n: f(x) = \frac{d}{dx}(\cos x + k) = -\sin x$

19. $\frac{d}{dx}(\int y dx)$ এর মান কত যখন $y = \sin x$ [CU 02-03]

$Sol^n: \frac{d}{dx}(\int y dx) = y = \sin x$

আংশিক ভগ্নাংশ

20. $\frac{x+17}{(x-3)(x+2)} = \frac{a}{x-3} + \frac{b}{x+2}$ হলে a ও b এর মান কত? [DU 08-09; JU, CU 07-08]

$Sol^n: a = \frac{3+17}{3+2} = 4;\ b = \frac{-2+17}{-2-3} = -3$

21. $\frac{x+A}{(x+1)(x-3)} \equiv \frac{B}{x+1} + \frac{1}{x-3}$

$Sol^n: \frac{3+A}{3+1} = 1 \Rightarrow A = 1;$

$B = \frac{-1+A}{-1-3} = \frac{-1+1}{-4} = 0$