নির্দিষ্ট যোগজ ও এর প্রয়োগ
প্রশ্নমালা X D

মান নির্ণয় কর :

1(a) $\int_0^3 (3-2x+x^2)dx$ [কু.'০৬,'০৭]

$= \left[3x - 2.\frac{x^2}{2} + \frac{x^3}{3}\right]_0^3 = \{(3.3 - 3^2 + \frac{3^3}{3}) - 0\}$

$= (9-9+9) = 9$

(b) $\int_0^{\pi/2} (\sin\theta + \cos\theta)\, d\theta$ [চ.'০৪]

$= [-\cos\theta + \sin\theta]_0^{\pi/2} = [-\cos\theta + \sin\theta]_0^{\pi/2}$

$= (\sin\frac{\pi}{2} - \cos\frac{\pi}{2}) - (\sin 0 - \cos 0)$

$= (1-0) - (0-1) = 2$

(c) $\int_0^{\pi} \frac{1-\cos 2x}{2} dx = \left[\frac{1}{2}(x - \frac{1}{2}\sin 2x)\right]_0^{\pi}$

$= \frac{1}{2}\{(\pi - \frac{1}{2}\sin 2\pi) - (0 - \frac{1}{2}\sin 2.0)\} = \frac{\pi}{2}$

(d) $\int_{-\pi/2}^{\pi/2} \frac{\sec x + 1}{\sec x} dx$ [য.'০৬; কু.,'০৯]

$= \int_{-\pi/2}^{\pi/2} (1 + \frac{1}{\sec x}) dx = \int_{-\pi/2}^{\pi/2} (1+\cos x) dx$

$= x[1 + \sin x)]_{-\pi/2}^{\pi/2}$

$= \frac{\pi}{2} + \sin\frac{\pi}{2} - \{-\frac{\pi}{2} + \sin(-\frac{\pi}{2})\}$

$= \frac{\pi}{2} + 1 - (-\frac{\pi}{2} - 1) = \frac{\pi}{2} + \frac{\pi}{2} + 2 = \pi + 2$

(e) $\int_{-1}^{1} |x|\, dx$ [প্র.ভ.প.'০৬]

$= \int_{-1}^{0} |x|\, dx + \int_0^1 |x|\, dx = \int_{-1}^{0} (-x) dx + \int_0^1 x dx$

$[\because |x| = x, x \ge 0; |x| = -x, x \le 0]$

$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = -0 + \frac{1}{2} + \frac{1}{2} - 0 = 1$

2(a) $\int_0^{\pi/3} \frac{1}{1-\sin x} dx$

[ঢা. '০৯,'১৩; য.'০৯; সি.'১০; রা.'১৩]

$= \int_0^{\pi/3} \frac{1+\sin x}{(1-\sin x)(1+\sin x)} dx$

$= \int_0^{\pi/3} \frac{1+\sin x}{1-\sin^2 x} dx = \int_0^{\pi/3} \frac{1+\sin x}{\cos^2 x} dx$

$= \int_0^{\pi/3} \{\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\} dx$

$= \int_0^{\pi/3} \{\sec^2 x + \sec x \tan x\} dx$

$= [\tan x + \sec x]_0^{\pi/3}$

$= \tan\frac{\pi}{3} + \sec\frac{\pi}{3} - (\tan 0 + \sec 0)$

$= \sqrt{3} + 2 - 0 - 1 = \sqrt{3} + 1$

2(b) $\int_0^{\pi/2} \frac{1}{1+\cos x} dx$ [ব.'০৮; ঢা.,সি.'১১]

$= \int_0^{\pi/2} \frac{1}{2\cos^2\frac{x}{2}} dx = \frac{1}{2}\int_0^{\pi/2} \sec^2\frac{x}{2} dx$

$= \frac{1}{2}\left[2\tan\frac{x}{2}\right]_0^{\pi/2} = \tan\frac{\pi}{4} - \tan 0 = 1$

3. $\int_0^{\pi/4} \frac{\cos 2\theta}{\cos^2\theta} d\theta$ [ব.'১১]

$= \int_0^{\pi/4} \frac{2\cos^2\theta - 1}{\cos^2\theta} d\theta$

$= \int_0^{\pi/4} (2 - \sec^2\theta) dx = [2\theta - \tan\theta]_0^{\pi/4}$

$= 2.\frac{\pi}{4} - \tan\frac{\pi}{4} - (2.0 - \tan 0) = \frac{\pi}{2} - 1$

4(a) $\int_0^{\pi/2} \cos^2 x dx$ [চ. '০৪; রা.'০৫ ,'০৯; সি.'১১]

$= \int_0^{\pi/2} \frac{1}{2}(1+\cos 2x) dx = \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/2}$

$= \frac{1}{2}\{(\frac{\pi}{2} + \frac{1}{2}\sin\pi) - (0 + \frac{1}{2}\sin 0)\} = \frac{\pi}{4}$

4 (b) $\int_0^{\pi/2} \cos^3 x dx$ [সি.'০৬,'০৭; য. '০৭,'০৯, '১৩; ব. '০৮; মা.'০৬; দি.'১৩]

$=\int_0^{\pi/2}\frac{1}{4}(3\cos x+\cos 3x)dx$

$=\frac{1}{4}\left[3\sin x+\frac{1}{3}\sin 3x\right]_0^{\pi/2}$

$=\frac{1}{4}(3\sin\frac{\pi}{2}+\frac{1}{3}\sin\frac{3\pi}{2}-3\sin 0-\frac{1}{3}\sin 0)$

$=\frac{1}{4}(3.1+\frac{1}{3}(-1)-0-0)=\frac{1}{4}\times\frac{8}{3}=\frac{2}{3}$

4(c) $\int_0^{\pi/2}\cos^4 x\,dx$ [য.'০৪]

$\cos^4 x=\frac{1}{4}(2\cos^2 x)^2=\frac{1}{4}(1+\cos 2x)^2$

$=\frac{1}{4}(1+2\cos 2x+\cos^2 2x)$

$=\frac{1}{4}\{1+2\cos 2x+\frac{1}{2}(1+\cos 4x)\}$

$=\frac{1}{4}(\frac{3}{2}+2\cos 2x+\frac{1}{2}\cos 4x)$

$\therefore\int_0^{\pi/2}\cos^4 x\,dx$

$=\frac{1}{4}\left[\frac{3}{2}x+\frac{2}{2}\sin 2x+\frac{1}{2}.\frac{1}{4}\sin 4x\right]_0^{\pi/2}$

$=\frac{1}{4}(\frac{3}{2}.\frac{\pi}{2}+\sin\pi+\frac{1}{8}\sin 2\pi-0)$

$=\frac{1}{4}(\frac{3\pi}{4}+0)=\frac{3\pi}{16}$

4(d) $\int_0^{\pi/4}\tan^2 x\,dx=\int_0^{\pi/4}(\sec^2 x-1)\,dx$

$=[\tan x-x]_0^{\pi/4}=\tan\frac{\pi}{4}-\frac{\pi}{4}-(\tan 0-0)$

$=1-\frac{\pi}{4}$

4(e) $\int_0^{\pi/2}\sin^2 2\theta\,d\theta$ [মা.বো.'০৯]

$=\int_0^{\pi/2}\frac{1}{2}(1-\cos 4\theta)\,d\theta=\frac{1}{2}\left[x-\frac{\sin 4x}{4}\right]_0^{\pi/2}$

$=\frac{1}{2}\{\frac{\pi}{2}-\frac{\sin 2\pi}{4}-(0-\frac{\sin 0}{4})\}$

$=\frac{1}{2}\{\frac{\pi}{2}-0-(0-0)\}=\frac{\pi}{4}$

5 (a) $\int_0^{\pi/2}\cos^5 x\sin x dx$ [ঢা.'০৩; দি.'১০; য.'১১]

$=-\int_0^{\pi/2}(\cos x)^5(-\sin x)dx$

$=-\left[\frac{1}{6}(\cos x)^6\right]_0^{\pi/2}$

$=-\frac{1}{6}\{(\cos\frac{\pi}{2})^6-(\cos 0)^6\}$

$=-\frac{1}{6}\{0-1\}=\frac{1}{6}$

5(b) ধরি $I=\int_0^{\pi/4}\sin^4 x\cos^4 x\,dx$ [প্র.ভ.প'৮৯]

$\sin^4 x\cos^4 x=\frac{1}{16}(2\sin x\cos x)=\frac{1}{16}\sin^4 2x$

$=\frac{1}{16}.\{\frac{1}{2}(1-\cos 4x)\}^2$

$=\frac{1}{64}(1-2\cos 4x+\cos^2 4x)$

$=\frac{1}{64}\{1-2\cos 4x+\frac{1}{2}(1+\cos 8x)\}$

$=\frac{1}{128}(3-4\cos 4x+\cos 8x)$

$\therefore I=\frac{1}{128}\left[3x-4.\frac{1}{4}\sin 4x+\frac{1}{8}\sin 8x\right]_0^{\pi/4}$

$=\frac{1}{128}(\frac{3\pi}{4}-\sin\pi+\frac{1}{8}\sin 2\pi-0)$

$=\frac{1}{128}\times\frac{3\pi}{4}=\frac{3\pi}{512}$

5(c) $\int_0^{\pi/2}\sin^2 x\sin 3x\,dx$

[ব.'০৫; মা.'০৪; য.'১৪]

$=\int_0^{\pi/2}\frac{1}{2}(1-\cos 2x)\sin 3x\,dx$

$=\int_0^{\pi/2}(\frac{1}{2}\sin 3x-\frac{1}{2}\cos 2x\sin 3x)\,dx$

$=\int_0^{\pi/2}\{\frac{1}{2}\sin 3x-\frac{1}{4}(\sin 5x+\sin x)\}\,dx$

$= \left[-\frac{1}{2}.\frac{1}{3}\cos 3x - \frac{1}{4}(-\frac{1}{5}\cos 5x - \cos x)\right]_0^{\pi/2}$

$= -\frac{1}{6}(\cos\frac{3\pi}{2} - \cos 0) + \frac{1}{20}(\cos\frac{5\pi}{2} - \cos 0)$

$+\frac{1}{4}(\cos\frac{\pi}{2} - \cos 0)$

$= -\frac{1}{6}(0-1) + \frac{1}{20}(0-1) + \frac{1}{4}(0-1)$

$= \frac{1}{6} - \frac{1}{20} - \frac{1}{4} = \frac{10-3-15}{60} = \frac{-8}{60} = \frac{-2}{15}$

5(d) ধরি $I = \int_0^{\pi} 3\sqrt{1-\cos x}\sin x\, dx$ [কু.'০৪]

এবং $z = \cos x \ \therefore \ dz = -\sin x dx$

সীমা: $x = 0$ হলে $z = 1$, $x = \pi$ হলে $z = -1$

$\therefore I = -3\int_1^{-1}\sqrt{1-z}dz = -3\left[-\frac{2}{3}(1-z)^{\frac{3}{2}}\right]_1^{-1}$

$= 2\{(1+1)^{\frac{3}{2}} - (1-1)^{\frac{3}{2}}\} = 2\times 2\sqrt{2} = 4\sqrt{2}$

5(e) $\int_0^{\pi/2}(1+\cos\theta)^2\sin\theta\, d\theta$

[বুয়েট,'০৮-০৯;চ.'১১]

ধরি, $z = 1+\cos x \ \therefore \ dz = -\sin x dx$

সীমা: $x = 0$ হলে $z = 2$ এবং $x = \frac{\pi}{2}$ হলে $z = 1$

$\therefore \int_0^{\pi/2}(1+\cos\theta)^2\sin\theta\, d\theta = -\int_2^1 z^2 dz$

$= \left[-\frac{z^3}{3}\right]_2^1 = -(\frac{1^3}{3} - \frac{2^3}{3}) = -(\frac{1}{3} - \frac{8}{3}) = \frac{7}{3}$

6(a) $\int_0^{\pi/2}\sin x\sin 2x\, dx$

[য.০৮;ব.'০৪,'০৬; চ. '০৬;দি.'১৩]

$= \int_0^{\pi/2}\frac{1}{2}(\cos x - \cos 3x)\, dx$

$= \frac{1}{2}\left[\sin x - \frac{1}{3}\sin 3x\right]_0^{\pi/2}$

$= \frac{1}{2}(\sin\frac{\pi}{2} - \frac{1}{3}\sin\frac{3\pi}{2} - \sin 0 + \frac{1}{3}\sin 0)$

$= \frac{1}{2}\{1 - \frac{1}{3}(-1) - 0 + 0\} = \frac{1}{2}\times\frac{4}{3} = \frac{2}{3}$

6(b) $\int_0^{\pi/2}\cos 2x\cos 3x\, dx$ [কু. '০০; চ. '০৩]

$= \int_0^{\pi/2}\frac{1}{2}(\cos 5x + \cos x)\, dx$

$= \frac{1}{2}\left[\frac{1}{5}\sin 5x + \sin x\right]_0^{\pi/2}$

$= \frac{1}{2}(\frac{1}{5}\sin\frac{5\pi}{2} + \sin\frac{\pi}{2} - \frac{1}{5}\sin 0 - \sin 0)$

$= \frac{1}{2}(\frac{1}{5}.1 + 1) = \frac{1}{2}\times\frac{6}{5} = \frac{3}{5}$

6(c) $\int_0^{\pi/2}\sin 2x\cos x\, dx$ [য.'০৫; রা.'০৮; মা. '০৫]

$= \int_0^{\pi/2}\frac{1}{2}(\sin 3x + \sin x)\, dx$

$= \frac{1}{2}\left[-\frac{1}{3}\cos 3x - \cos x\right]_0^{\pi/2}$

$= \frac{1}{2}(-\frac{1}{3}\cos 3\frac{\pi}{2} - \cos\frac{\pi}{2} + \frac{1}{3}\cos 0 + \cos 0)$

$= \frac{1}{2}[-\frac{1}{3}(\cos 3\frac{\pi}{2} - \cos 0) - (\cos\frac{\pi}{2} - \cos 0)]$

$= \frac{1}{2}[-\frac{1}{3}(0-1) - (0-1)] = \frac{1}{2}(\frac{1}{3}+1) = \frac{2}{3}$

7(ক) ধরি, $I = \int_0^{\pi/2}\sqrt{\cos x}\sin^3 x dx$ [চ.'০১; য.১০]

$= \int_0^{\pi/2}\sqrt{\cos x}\sin^2 x\sin x dx$

$= \int_0^{\pi/2}\sqrt{\cos x}(1-\cos^2 x)\sin x dx$

এবং $z = \cos x \ \therefore \ dz = -\sin x dx$

সীমা: $x = 0$ হলে $z = 1$ এবং $x = \frac{\pi}{2}$ হলে $z = 0$

$\therefore I = -\int_1^0\sqrt{z}(1-z^2)\, dz$

$= -\int_1^0(\sqrt{z} - z^{5/2})\, dz = -\left[\frac{z^{3/2}}{3/2} - \frac{z^{7/2}}{7/2}\right]_1^0$

$= -\{\frac{2}{3}(0-1) - \frac{2}{7}(0-1)\} = -(-\frac{2}{3} + \frac{2}{7})$

$= \frac{-14+6}{21} = \frac{8}{21}$

7(b) ধরি, $I = \int_0^{\pi/2} \frac{\cos^3 x dx}{\sqrt{\sin x}}$ [ব.,চ.'১০; রা,'১২]

$= \int_0^{\pi/2} \frac{\cos^2 x \cos x dx}{\sqrt{\sin x}}$

$= \int_0^{\pi/2} \frac{(1-\sin^2 x)\cos x dx}{\sqrt{\sin x}}$

এবং $z = \sin x \therefore dz = \cos x\, dx$

সীমা: $x = 0$ হলে $z = 0$ এবং $x = \frac{\pi}{2}$ হলে $z = 1$

$\therefore I = \int_0^1 (\frac{1-z^2}{\sqrt{z}})\, dz = \int_0^1 (\frac{1}{\sqrt{z}} - z^{3/2})\, dz$

$= \left[2\sqrt{z} - \frac{z^{5/2}}{5/2}\right]_0^1 = 2(1-0) - \frac{2}{5}(1-0)$

$= 2 - \frac{2}{5} = \frac{8}{5}$

8(a) ধরি, $I = \int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} dx$ [চ.'০৪; রা.'১১]

এবং $z = \sin^{-1} x \therefore dz = \frac{1}{\sqrt{1-x^2}} dx$

সীমা: $x = 0$ হলে $z = 0$ এবং $x = 1$ হলে $z = \frac{\pi}{2}$

$\therefore I = \int_0^{\pi/2} z^2 dz = \left[\frac{z^3}{3}\right]_0^{\pi/2} = \frac{1}{3}\{(\frac{\pi}{2})^3 - 0\}$

$= \frac{\pi^3}{24}$

8(b) $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ [য.'০৪;সি.'০৭;ব.'০৮; দি.'০৯]

ধরি, $z = \sin^{-1} x \therefore dz = \frac{1}{\sqrt{1-x^2}} dx$

সীমা: $x = 0$ হলে $z = 0$ এবং $x = 1$ হলে $z = \frac{\pi}{2}$

$\therefore \int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int_0^{\pi/2} z dz = \left[\frac{z^2}{2}\right]_0^{\pi/2}$

$= \frac{1}{2}\{(\frac{\pi}{2})^2 - 0\} = \frac{\pi^2}{8}$

8(c) ধরি, $I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$ [বুয়েট'০৯; কু.'১১]

এবং $z = \tan^{-1} x \therefore dz = \frac{1}{1+x^2} dx$

সীমা: $x = 0$ হলে $z = 0$ এবং $x = 1$ হলে $z = \frac{\pi}{4}$

$\therefore I = \int_0^{\pi/4} z dz = \left[\frac{z^2}{2}\right]_0^{\pi/4} = \frac{1}{2} \cdot \frac{\pi^2}{16} = \frac{\pi^2}{32}$

9(a) $\int_0^1 \frac{x dx}{\sqrt{1-x^2}}$ [ঢা.'০৭; য.'০৭]

$= -\frac{1}{2}\int_0^1 \frac{(-2x)dx}{\sqrt{1-x^2}} = -\frac{1}{2}\left[2\sqrt{1-x^2}\right]_0^1$

$= -(\sqrt{1-1^2} - \sqrt{1-0^2}) = -(0-1) = 1$

9(b) $\int_4^8 \frac{x dx}{\sqrt{x^2-15}} = \frac{1}{2}\int_4^8 \frac{d(x^2-15)}{\sqrt{x^2-15}}$

$= \frac{1}{2}\left[2\sqrt{x^2-15}\right]_4^8 = \sqrt{64-15} - \sqrt{16-15}$

$= \sqrt{64-15} - \sqrt{16-15} = \sqrt{49} - \sqrt{1} = 6$

9(c) $\int_0^2 \frac{x dx}{\sqrt{9-2x^2}}$

[বুয়েট'০৯; ব.'১০; রা.'১২; দি.'১৩; সি.,চ.'১৪]

$= -\frac{1}{4}\int_0^2 \frac{d(9-2x^2)}{\sqrt{9-2x^2}} = -\frac{1}{4}\left[2\sqrt{9-2x^2}\right]_0^2$

$= -\frac{1}{2}(\sqrt{9-8} - \sqrt{9-0}) = -\frac{1}{2}(1-3) = 1$

9(d) ধরি, $I = \int_0^1 \frac{x dx}{\sqrt{4-x^2}}$

[সি.'০৯; ঢা., রা., কু.'১০; দি.'১৩]

এবং $z = 4 - x^2 \therefore dz = -2x dx$

সীমা: $x = 0$ হলে $z = 4$ এবং $x = 1$ হলে $z = 3$

$\therefore I = -\frac{1}{2}\int_4^3 \frac{dz}{\sqrt{z}} = -\frac{1}{2}\left[2\sqrt{z}\right]_4^3$

$= -(\sqrt{3}-\sqrt{4}) = 2-\sqrt{3}$

9(e) ধরি, $I=\int_{-2}^{5} \frac{7x}{\sqrt{x^2+3}}dx$ [মা.বো.'০৪]

এবং $z = x^2+3 \therefore dz = 2xdx$

সীমা: $x=-2$ হলে $z=7$ এবং $x=5$ হলে $z=28$

$\therefore I = \frac{7}{2}\int_7^{28}\frac{dz}{\sqrt{z}} = \frac{7}{2}\left[2\sqrt{z}\right]_7^{28}$

$= 7(\sqrt{28}-\sqrt{7}) = 7(2\sqrt{7}-\sqrt{7}) = 7\sqrt{7}$

9(f) ধরি, $I=\int_0^1 x^3\sqrt{1+3x^4}\,dx$ [ব. '০৯; চ. '০৮ ; রা. ০৭,'০৯; কু. '০৭,'১০; সি. '০৮;মা.'০৫, '০৯]

এবং $z=1+3x^4 \therefore dz = 12x^3dx$

সীমা: $x=0$ হলে $z=1$ এবং $x=1$ হলে $z=4$

$\therefore I = \frac{1}{12}\int_1^4 \sqrt{z}dz = \frac{1}{12}\left[\frac{z^{3/2}}{3/2}\right]_1^4$

$= \frac{1}{12}\times\frac{2}{3}(4^{3/2}-1) = \frac{1}{18}(8-1) = \frac{7}{18}$

10. (a) $\int_1^2 x^2e^{x^3}dx$ [ঢা.'০৯; রা.'০৪,'০৬; ব.'১০]

ধরি, $z=x^3 \therefore dz = 3x^2dx \Rightarrow x^2dx = \frac{1}{3}dz$

সীমা: $x=1$ হলে $z=1$ এবং $x=2$ হলে $z=8$

$\therefore \int_1^2 x^2e^{x^3}dx = \frac{1}{3}\int_1^8 e^z dz = \frac{1}{3}\left[e^z\right]_1^8$

$= \frac{1}{3}(e^8-e^1) = \frac{1}{3}(e^8-e)$

10(b) $\int_0^1 xe^{x^2}dx$ [ঢা.,ব.'০৫; চ., য. '০৬; সি. '০৭,'১০; চ.,কু.'দি.'১২]

ধরি, $z=x^2 \therefore dz=2xdx \Rightarrow xdx=\frac{1}{2}dz$

সীমা: $x=0$ হলে $z=0$ এবং $x=1$ হলে $z=1$

$\therefore \int_0^1 xe^{x^2}dx = \frac{1}{2}\int_0^1 e^z dz = \frac{1}{2}\left[e^z\right]_0^1$

$= \frac{1}{2}(e^1-e^0) = \frac{1}{2}(e-1)$

10(c) $\int_0^{\ln 2}\frac{e^x}{1+e^x}dx$ [য. '০৯,'১১; রা.'১০; ঢা. '১০, '১৩; কু.'১৩; চ.'০৯,'১১; দি.'০৯; সি. '০৮,'১২; মা.'১৪]

ধরি, $z=1+e^x \therefore dz = e^xdx$

সীমা: $x=0$ হলে $z=1+e^0=1+1=2$ এবং $x=\ln 2$ হলে $z=1+e^{\ln 2}=1+2=3$

$\therefore \int_0^{\ln 2}\frac{e^x}{1+e^x}dx = \int_2^3\frac{dz}{z} = \left[\ln z\right]_2^3$

$= \ln 3 - \ln 2 = \ln\frac{3}{2}$

10(d) $\int_1^3 \frac{1}{x}\cos(\ln x)\,dx$ [ঢা.'০৮; কু.'০৮,'১৪; ব.'১২]

ধরি, $z=\ln x \therefore dz = \frac{dx}{x}$

সীমা: $x=1$ হলে $z=\ln 1 = 0$ এবং $x=3$ হলে $z=\ln 3$

$\therefore \int_1^3 \frac{1}{x}\cos(\ln x)\,dx = \int_0^{\ln 3}\cos z\,dz$

$= \left[\sin z\right]_0^{\ln 3} = \sin(\ln 3) - \sin 0 = \sin(\ln 3)$

11. (a) $\int_{\pi/3}^{\pi/2}\frac{\cos^5 x}{\sin^7 x}dx$ [য.'০৫; রা.'০৭; চ.'০৮; দি.'১১; ঢা.'১২; বুয়েট'০৯]

$= \int_{\pi/3}^{\pi/2}\cot^5 x\,\text{cosec}^2 x\,dx$

ধরি, $\cot x = z \therefore -\text{cosec}^2 x\,dx = dz$

সীমা: $x=\frac{\pi}{3}$ হলে $z=\cot\frac{\pi}{3}=\frac{1}{\sqrt{3}}$ এবং $x=\frac{\pi}{2}$ হলে $z=\cot\frac{\pi}{2}=0$

$\therefore \int_{\pi/3}^{\pi/2}\frac{\cos^5 x}{\sin^7 x}dx = \int_{1/\sqrt{3}}^0 z^5(-dz)$

$= -\left[\frac{1}{6}z^6\right]_{1/\sqrt{3}}^0 = -\frac{1}{6}\{0-(\frac{1}{\sqrt{3}})^6\} = \frac{1}{162}$

11(b) ধরি, $I=\int_0^{\pi/4} \tan^3 x \sec^2 x\, dx$ [য.'০৬; রা.'০৬,'০৮; কু., সি , দি.'০৯; ঢা.,ব.'১১; সি.'১৩]

এবং $\tan x = z \therefore \sec^2 x\, dx = dz$

সীমা: $x=0$ হলে $z=\tan 0 = 0$ এবং

$x=\frac{\pi}{4}$ হলে $z=\tan\frac{\pi}{4}=1$

$\therefore I=\int_0^1 z^3 dz = \left[\frac{1}{4}z^4\right]_0^1 = \frac{1}{4}(1^4-0^4)=\frac{1}{4}$

11(c) $\int_0^{\pi/4}(\tan^3 x+\tan x)dx$ [কু.'০৮]

$=\int_0^{\pi/4}(\tan^2 x+1)\tan x dx$

$=\int_0^{\pi/4}\sec^2 x\tan x dx$

$=\int_0^{\pi/4}(\tan x)d(\tan x)=\left[\frac{1}{2}(\tan x)^2\right]_0^{\pi/4}$

$=\frac{1}{2}\{(\tan\frac{\pi}{4})^2-(\tan 0)^2\}=\frac{1}{2}\{(1)^2-0\}=\frac{1}{2}$

11(d) $\int_0^{\pi/4}\tan^2 x\sec^2 x\, dx$ [ঢা.'০৩,'১৩; কু.'০৪,'০৬; য.'০৪; ঢা.'০৫; রা.'০৫; চ.'১১]

ধরি, $\tan x = z \therefore \sec^2 x\, dx = dz$

সীমা: $x=0$ হলে $z=\tan 0=0$ এবং

$x=\frac{\pi}{4}$ হলে $z=\tan\frac{\pi}{4}=1$

$\therefore \int_0^{\pi/4}\tan^2 x\sec^2 x\, dx = \int_0^1 z^2\, dz$

$=\left[\frac{1}{3}z^3\right]_0^1=\frac{1}{3}(1^3-0^3)=\frac{1}{3}$

12. (a) $\int xe^{-3x}dx$ [দি.'১০]

$=x\int e^{-3x}dx-\int\{\frac{d}{dx}(x)\int e^{-3x}dx\}dx$

$=x(-\frac{1}{3}e^{-3x})-\int 1.(-\frac{1}{3}e^{-3x})dx$

$=-x\frac{1}{3}e^{-3x}+\frac{1}{3}(-\frac{1}{3}e^{-3x})$

$=-\frac{1}{3}xe^{-3x}-\frac{1}{9}e^{-3x}=-\frac{1}{9}(3x+1)e^{-3x}$

$\therefore \int_0^1 xe^{-3x}dx=\left[-\frac{1}{9}(3x+1)e^{-3x}\right]_0^1$

$=-\frac{1}{9}\{(3+1)e^{-3}-(0+1)e^{-0}\}$

$=-\frac{1}{9}(4e^{-3}-1)=\frac{1}{9}(1-4e^{-3})$

12(b) $\int \ln(2x)dx$ [য. '০১;ব.'০৯]

$=\ln(2x)\int dx-\int[\frac{d}{dx}\{\ln(2x)\}\int dx]dx$

$=x\ln(2x)-\int\frac{2}{2x}.xdx$

$=x\ln(2x)-\int dx = x\ln(2x)-x+c$

$\therefore \int_2^4 \ln(2x)\,dx = [x\ln(2x)-x]_2^4$

$=4\ln 8-4-(2\ln 4-2)$

$=4\ln 2^3-4-2\ln 2^2+2$

$=12\ln 2-2-4\ln 2 = 8\ln 2-2$

12(c) $\int\frac{\ln x}{\sqrt{x}}dx$ [প্র.ভ.প'১৬]

$=\ln x\int\frac{1}{\sqrt{x}}dx-\int[\frac{d}{dx}(\ln x)\int\frac{1}{\sqrt{x}}dx]dx$

$=2\sqrt{x}\ln x-\int\frac{1}{x}.2\sqrt{x}dx$

$=2\sqrt{x}\ln x-2\int\frac{1}{\sqrt{x}}dx$

$=2\sqrt{x}\ln x-2.2\sqrt{x}+c$

$=2\sqrt{x}(\ln x-2)+c$

$\int_1^4\frac{\ln x}{\sqrt{x}}dx=[2\sqrt{x}(\ln x-2)]_1^4$

$=2\sqrt{4}(\ln 4-2)-2\sqrt{1}(\ln 1-2)$

$=4\ln 2^2-8-2(0-2)$

$=8\ln 2-8+4=8\ln 2-4$

12(d) $\int x^2\cos x\, dx$ [কু.'০৪]

$= x^2\int \cos x\,dx - \int\{\frac{d}{dx}(x^2)\int \cos x\,dx\}dx$

$= x^2 \sin x - \int 2x\sin x\,dx$

$= x^2 \sin x - 2[x\int \sin x\,dx - \int 1.(-\cos x)dx]$

$= x^2 \sin x - 2[x(-\cos x) + \sin x] + c$

$= x^2 \sin x + 2x\cos x - 2\sin x + c$

$\int_0^{\pi/2} x^2 \cos x\,dx$ =

$[x^2 \sin x + 2x\cos x - 2\sin x]_0^{\pi/2}$

$= (\frac{\pi}{2})^2 \sin\frac{\pi}{2} + 2.\frac{\pi}{2}\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2} - 0$

$= \frac{\pi^2}{4}.1 + 2.\frac{\pi}{2}.0 - 2.1 = \frac{\pi^2}{4} - 2$

12(e) $\int x\tan^{-1} x\,dx$

[রা. '০৮,'১২; চ. '০৮,'১২; য.'১১; দি.'১২; কু.'১৪]

$= \tan^{-1} x\int xdx - \int\{\frac{d}{dx}(\tan^{-1} x)\int xdx\}dx$

$= \frac{x^2}{2}\tan^{-1} x - \int \frac{1}{1+x^2}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}dx$

$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int (1 - \frac{1}{1+x^2})dx$

$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}(x - \tan^{-1} x) + c$

$= \frac{1}{2}\{(x^2+1)\tan^{-1} x - x\} + c$

$\int_1^{\sqrt{3}} x\tan^{-1} x\,dx = \left[\frac{(x^2+1)\tan^{-1} x - x}{2}\right]_1^{\sqrt{3}}$

$= \frac{(3+1)\tan^{-1}\sqrt{3} - \sqrt{3} - (1+1)\tan^{-1} 1 + 1}{2}$

$= \frac{1}{2}(4.\frac{\pi}{3} - \sqrt{3} - 2.\frac{\pi}{4} + 1)$

$= \frac{1}{2}(\frac{4\pi}{3} - \frac{\pi}{2} - \sqrt{3} + 1)$

$= \frac{1}{2}(\frac{8\pi - 3\pi}{6} - \sqrt{3} + 1) = \frac{1}{12}(5\pi - 6\sqrt{3} + 6)$

12(f) ধরি, $I = \int_0^{\pi/2} e^x(\sin x + \cos x)\,dx$

[কু.'০৫,'১১; রা.'১০]

এবং $f(x) = \sin x \;\therefore f'(x) = \cos x$

$\therefore I = \int_0^{\pi/2} e^x\{f(x) + f'(x)\}\,dx$

$= [e^x f(x)]_0^{\pi/2} = [e^x \sin x]_0^{\pi/2}$

$= e^{\pi/2}\sin\frac{\pi}{2} - e^0 \sin 0 = e^{\pi/2} - 0 = e^{\pi/2}$

12(g) $\int \ln x\,dx$ [প্র.ভ.প.'০৫]

$= \ln x\int dx - \int\{\frac{d}{dx}(\ln x)\int dx\}dx$

$= x\ln x - \int \frac{1}{x}.xdx = x\ln x - \int dx$

$= x\ln x - x + c = x(\ln x - 1) + c$

$\therefore \int_1^0 \ln x\,dx = [x(\ln x - 1)]_1^0$

$= 0 - 1(\ln 1 - 1) = -1(0 - 1) = 1$

12(h) $\int x\sin^2 x\,dx$ [প্র.ভ.প.'০৫]

$= \int \frac{x}{2}(1 - \cos 2x)\,dx = \frac{1}{2}\int (x - x\cos 2x)\,dx$

$= \frac{1}{2}.\frac{x^2}{2} - \frac{1}{2}[x\int \cos 2xdx - \int\{1.\frac{1}{2}\sin 2x\,dx\}]$

$= \frac{1}{4}x^2 - \frac{1}{2}[x.\frac{1}{2}\sin 2x - \frac{1}{2}\int \sin 2x\,dx]$

$= \frac{1}{4}x^2 - \frac{1}{4}[x\sin 2x - (-\frac{1}{2}\cos 2x)] + c$

$= \frac{1}{4}(x^2 - x\sin 2x - \frac{1}{2}\cos 2x) + c$

$\therefore \int_0^{\pi} x\sin^2 x\,dx = \frac{1}{4}\left[x^2 - x\sin 2x - \frac{1}{2}\cos 2x\right]_0^{\pi}$

$= \frac{1}{4}\{(\pi^2 - \pi\sin 2\pi - \frac{1}{2}\cos 2\pi) + \frac{1}{2}\cos 0\}$

$= \frac{1}{4}\{\pi^2 - 0 - \frac{1}{2}.1) + \frac{1}{2}.1\} = \frac{1}{4}\pi^2$

12(i) $\int x\cot^{-1}x\,dx$ [বুয়েট'০৯]

$= \cot^{-1}x\int xdx - \int\{\frac{d}{dx}(\cot^{-1}x)\int xdx\}dx$

$= \frac{x^2}{2}\cot^{-1}x + \int\frac{1}{1+x^2}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\cot^{-1}x + \frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$

$= \frac{x^2}{2}\cot^{-1}x + \frac{1}{2}\int(1-\frac{1}{1+x^2})dx$

$= \frac{x^2}{2}\cot^{-1}x + \frac{1}{2}(x+\cot^{-1}x)+c$

$= \frac{1}{2}\{(x^2+1)\cot^{-1}x + x\}+c$

$\int_1^{\sqrt{3}} x\cot^{-1}x\,dx = \left[\frac{(x^2+1)\cot^{-1}x+x}{2}\right]_1^{\sqrt{3}}$

$= \frac{(3+1)\cot^{-1}\sqrt{3}+\sqrt{3}-(1+1)\cot^{-1}1-1}{2}$

$= \frac{1}{2}(4.\frac{\pi}{6}+\sqrt{3}-2.\frac{\pi}{4}-1)$

$= \frac{1}{2}(\frac{2\pi}{3}-\frac{\pi}{2}+\sqrt{3}-1)$

$= \frac{1}{2}(\frac{4\pi-3\pi}{6}+\sqrt{3}-1) = \frac{1}{12}(\pi+6\sqrt{3}-6)$

(j) $\int x\ln x\,dx$ [য.'০৫; রা.'১৪]

$= \ln x\int x\,dx - \int\{\frac{d}{dx}(\ln x)\int xdx\}dx$

$= \ln x.\frac{x^2}{2} - \int(\frac{1}{x}\times\frac{x^2}{2})dx$

$= \ln x.\frac{x^2}{2} - \frac{1}{2}\int xdx = \frac{x^2}{2}\ln x - \frac{1}{2}\times\frac{x^2}{2}+c$

$\therefore \int_1^{\sqrt{e}} x\ln x\,dx = \left[\frac{x^2}{2}\ln x - \frac{1}{4}x^2\right]_1^{\sqrt{e}}$

$= \frac{(\sqrt{e})^2}{2}\ln\sqrt{e} - \frac{1}{4}(\sqrt{e})^2 - \frac{1}{2}\ln 1 + \frac{1}{4}$

$= \frac{e}{2}.\frac{1}{2}\ln e - \frac{1}{4}e - \frac{1}{2}\times 0 + \frac{1}{4}$

$= \frac{e}{4}.1 - \frac{1}{4}e - \frac{1}{2}\times 0 + \frac{1}{4} = \frac{1}{4}$

13(a) $\int_0^1 \frac{x\,dx}{1+x^4}$ [প্র.ভ.প.'০৬]

$= \frac{1}{2}\int_0^1\frac{2x\,dx}{1+(x^2)^2} = \left[\frac{1}{2}\tan^{-1}(x^2)\right]_0^1$

$= \frac{1}{2}(\tan^{-1}1 - \tan^{-1}0) = \frac{1}{2}(\frac{\pi}{4}-0) = \frac{\pi}{8}$

13(b) $\int_0^1\frac{1+x}{1+x^2}dx$

[রা. '০৬, '০৯; ব. '০৭ ; ঢা. '০৯; কু.,সি.'১২,'১৪]

$= \int_0^1(\frac{1}{1+x^2}+\frac{x}{1+x^2})dx =$

$\int_0^1(\frac{1}{1+x^2}+\frac{1}{2}\frac{2x}{1+x^2})dx$

$= \left[\tan^{-1}x + \frac{1}{2}\ln(1+x^2)\right]_0^1$

$= \tan^{-1}1 + \frac{1}{2}\ln 2 - \tan^{-1}0 - \frac{1}{2}\ln 1$

$= \frac{\pi}{4}+\frac{1}{2}\ln 2 - 0 + 0 = \frac{\pi}{4}+\frac{1}{2}\ln 2$

13(c) $\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$ [ঢা.'০৭]

$= -\int_0^{\pi}\frac{(-\sin x)}{1^2+\cos^2 x}dx = -[\tan^{-1}(\cos x)]_0^{\pi}$

$= -\{\tan^{-1}(\cos\pi) - \tan^{-1}(\cos 0)\}$

$= -\{\tan^{-1}(-1) - \tan^{-1}(1)\}$

$= -(-\frac{\pi}{4}-\frac{\pi}{4}) = \frac{\pi}{2}$

13(d) ধরি, $I = \int_0^{\pi/4}\frac{\sin 2x}{\cos^4 x+\sin^4 x}dx$ [প্র.ভ.প.'০৭]

$\cos^4 x + \sin^4 x = (\cos^2 x)^2 + (\sin^2 x)^2$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$= 1-\frac{1}{2}(2\sin x\cos x)^2 = 1-\frac{1}{2}\sin^2 2x$

$= 1-\frac{1}{2}(1-\cos^2 2x) = \frac{1}{2}(1+\cos^2 2x)$

$\therefore I = 2\int_0^{\pi/4}\frac{\sin 2x}{1+\cos^2 2x}dx$

$= 2(-\frac{1}{2})\int_0^{\pi/4}\frac{(-2\sin 2x)}{1^2+(\cos 2x)^2}dx$

$= -\left[\tan^{-1}(\cos 2x)\right]_0^{\pi/4}$

$= -\{\tan^{-1}(\cos\frac{\pi}{2}) - \tan^{-1}(\cos 0)\}$

$= -\{\tan^{-1}0 - \tan^{-1}1\} = -\{0-\frac{\pi}{4}\} = \frac{\pi}{4}$

13(e) $\int_0^1 \frac{dx}{e^x+e^{-x}}$

[রা.'১২; সি. '০৭; কু.'০৮; ব.'১৩; ঢা.'১৪]

$= \int_0^1\frac{e^x dx}{e^x(e^x+e^{-x})} = \int_0^1\frac{e^x dx}{(e^x)^2+1}$

ধরি, $e^x = z \quad \therefore e^x dx = dz$

সীমা : $x=0$ হলে, $z=e^0=1$

$x=1$ হলে, $z=e^1=e$

$\therefore \int_0^1\frac{dx}{e^x+e^{-x}} = \int_1^e\frac{dz}{z^2+1} = \left[\tan^{-1}z\right]_1^e =$

$\tan^{-1}e - \tan^{-1}(1) = \tan^{-1}e - \frac{\pi}{4}$

14(a) $\int_3^4\frac{dx}{25-x^2}$ [ব.'১৩]

$= \int_3^4\frac{dx}{5^2-x^2} = \left[\frac{1}{2.5}\ln\left|\frac{5+x}{5-x}\right|\right]_3^4$

$= \frac{1}{10}(\ln\left|\frac{5+4}{5-4}\right| - \ln\left|\frac{5+3}{5-3}\right|)$

$= \frac{1}{10}(\ln 9 - \ln 4) = \frac{1}{10}\ln\frac{9}{4} = \frac{1}{10}\ln(\frac{3}{2})^2$

$= \frac{1}{10}\times 2\ln(\frac{3}{2}) = \frac{1}{5}\ln(\frac{3}{2})$

(b) $\int_0^{\pi/2}\frac{\cos x dx}{9-\sin^2 x}dx$ [ঢা.'০৫; মা.'০৮; চ., সি.'০৯]

ধরি, $\sin x = z$. $\therefore \cos x dx = dz$

সীমা : $x=0$ হলে $z=0$ এবং $x=\frac{\pi}{2}$ হলে $z=1$

$\therefore \int_0^{\pi/2}\frac{\cos x dx}{9-\sin^2 x}dx = \int_0^1\frac{dz}{3^2-z^2}$

$= \left[\frac{1}{2.3}\ln\left|\frac{3+z}{3-z}\right|\right]_0^1 = \frac{1}{6}(\ln\left|\frac{3+1}{3-1}\right| - \ln\left|\frac{3+0}{3-0}\right|)$

$= \frac{1}{6}(\ln 2 - \ln 1) = \frac{1}{6}\ln 2$

15 (a) $\int_0^1\frac{dx}{\sqrt{2x-x^2}} = \int_0^1\frac{dx}{\sqrt{1-(x^2-2x+1)}}$

$= \int_0^1\frac{d(x-1)}{\sqrt{1-(x-1)^2}} = \left[\sin^{-1}(x-1)\right]_0^1$

$= \sin^{-1}(1-1) - \sin^{-1}(0-1) = \sin^{-1}0 + \sin^{-1}1$

$= \frac{\pi}{2}$

15 (b) $\int_{1/2}^1\frac{dx}{x\sqrt{4x^2-1}}$ [প্র.ভ.প. '০৪]

$= \int_{1/2}^1\frac{2dx}{2x\sqrt{(2x)^2-1}} = \left[\sec^{-1}(2x)\right]_{1/2}^1$

$= \sec^{-1}2 - \sec^{-1}1 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$

15(c) ধরি $I = \int_1^2\frac{dx}{x^2\sqrt{4-x^2}}$ [প্র.ভ.প.'০৪]

এবং $x = 2\cos\theta$. তাহলে $dx = -2\sin\theta\, d\theta$

সীমা : $x=1$ হলে $\theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$ এবং

$x=2$ হলে $\theta = \cos^{-1}1 = 0$

$\therefore I = \int_{\pi/3}^0\frac{-2\sin\theta\, d\theta}{4\cos^2\theta\sqrt{4(1-\cos^2\theta)}}$

$= \int_{\pi/3}^0\frac{-2\sin\theta\, d\theta}{4\cos^2\theta . 2\sin\theta} = -\frac{1}{4}\int_{\pi/3}^0\sec^2\theta\, d\theta$

$= -\frac{1}{4}\left[\tan\theta\right]_{\pi/3}^0 = -\frac{1}{4}(\tan 0 - \tan\frac{\pi}{3})$

$= -\frac{1}{4}(0-\sqrt{3}) = \frac{\sqrt{3}}{4}$

15 (d) $\int_0^{\pi/6}\frac{dx}{1-\tan^2 x}$ [বুয়েট ০৭–০৮]

$=\int_0^{\pi/6}\frac{\cos^2 x dx}{\cos^2 x-\sin^2 x}$

$=\int_0^{\pi/6}\frac{\frac{1}{2}(1+\cos 2x)dx}{\cos 2x}=\frac{1}{2}\int_0^{\pi/6}(\sec 2x+1)dx$

$=\frac{1}{2}\left[\frac{1}{2}\ln|\tan 2x+\sec 2x|+x\right]_0^{\pi/6}$

$=\frac{1}{2}\{\frac{1}{2}\ln|\tan\frac{\pi}{3}+\sec\frac{\pi}{3}|+\frac{\pi}{6}-0\}$

$=\frac{1}{4}\ln|\sqrt{3}+2|+\frac{\pi}{12}=\frac{1}{4}\ln(\sqrt{3}+2)+\frac{\pi}{12}$

16. (a) ধরি $I=\int_0^a\sqrt{a^2-x^2}\,dx$ [সি. '০৭; রা. '০৫; কু.'০৯,'১৩; চ. '০৯; য.,ব. '১২, দি.'১২,'১৪]

এবং $x=a\sin\theta$. তাহলে $dx=a\cos\theta\,d\theta$

সীমা : $x=0$ হলে $\theta=\sin^{-1}0=0$ এবং

$x=a$ হলে $\theta=\sin^{-1}1=\frac{\pi}{2}$

$\therefore I=\int_0^{\pi/2}\sqrt{a^2(1-\sin^2\theta)}\,a\cos\theta\,d\theta$

$=a^2\int_0^{\pi/2}\cos^2\theta\,d\theta=\frac{a^2}{2}\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta$

$=\frac{a^2}{2}\left[\theta+\frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$=\frac{a^2}{2}\{(\frac{\pi}{2}+\frac{1}{2}\sin\pi)-(0+\frac{1}{2}\sin 0)\}$

$=\frac{a^2}{2}.\frac{\pi}{2}=\frac{1}{4}\pi a^2$

16(b) ধরি $I=\int_0^{\sqrt{2}}\frac{x^2}{(4-x^2)^{3/2}}dx$ [প্র.ভ.প.'৮৫]

এবং $x=2\sin\theta$. তাহলে $dx=2\cos\theta\,d\theta$

সীমা : $x=0$ হলে $\theta=\sin^{-1}0=0$ এবং

$x=\sqrt{2}$ হলে $\theta=\sin^{-1}\frac{1}{\sqrt{2}}=\frac{\pi}{4}$

$\therefore I=\int_0^{\pi/4}\frac{4\sin^2\theta.2\cos\theta\,d\theta}{\{4(1-\sin^2\theta)\}^{3/2}}$

$=\int_0^{\pi/4}\frac{8\sin^2\theta\cos\theta\,d\theta}{8\cos^3\theta}=\int_0^{\pi/4}\tan^2\theta\,d\theta$

$=\int_0^{\pi/4}(\sec^2\theta-1)\,d\theta=[\tan\theta-\theta]_0^{\pi/4}$

$=\tan\frac{\pi}{4}-\frac{\pi}{4}-(\tan 0-0)=1-\frac{\pi}{4}$

17. ধরি, $I=\int_0^4 y\sqrt{4-y}\,dy$

[ব.'০৫; রা.'০৭;ঢা.'০৯,'১২; রা.'১৩; চ.'১০,'১৪]

এবং $4-y=z^2$. $\therefore -dy=2z\,dz$

সীমা : $y=0$ হলে $z=2$ এবং $y=4$ হলে $z=0$

$\therefore I=\int_2^0(4-z^2)\sqrt{z^2}.(-2z\,dz)$

$=2\int_2^0(z^4-4z^2)\,dz=2\left[\frac{1}{5}z^5-\frac{4}{3}z^3\right]_2^0$

$=2(-\frac{1}{5}\times 2^5+\frac{4}{3}\times 2^3)=2^6(-\frac{1}{5}+\frac{1}{3})=\frac{128}{15}$

18. $\int_1^{15}\frac{x+2}{(x+1)(x+3)}dx$ [প্র.ভ.প.'৯৫]

$=\int_1^{15}\{\frac{-1+2}{(x+1)(-1+3)}+\frac{-3+2}{(-3+1)(x+3)}\}dx$

$=\int_1^{15}\{\frac{1}{2(x+1)}+\frac{1}{2(x+3)}\}dx$

$=\frac{1}{2}[\ln|x+1|+\ln|x+3|]_1^{15}$

$=\frac{1}{2}[\ln|(x+1)(x+3)|]_1^{15}$

$=\frac{1}{2}\{\ln|(15+1)(15+3)|-\ln|(1+1)(1+3)|\}$

$=\frac{1}{2}\{\ln(16\times 18)-\ln(2\times 4)\}$

$=\frac{1}{2}\ln\frac{16\times 18}{2\times 4}=\frac{1}{2}\ln 6^2=\frac{2}{2}\ln 6=\ln 6$

সম্ভাব্য ধাপসহ প্রশ্ন :

19. $\int_0^{\pi/2}\sqrt{1+\sin\theta}\,d\theta$

$= \int_0^{\pi/2} \sqrt{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\, d\theta$ (১)

$= \int_0^{\pi/2} \sqrt{(\sin\frac{\theta}{2} + \cos\frac{\theta}{2})^2}\, d\theta$

$= \int_0^{\pi/2} (\sin\frac{\theta}{2} + \cos\frac{\theta}{2})\, d\theta$

$= \left[-2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2}\right]_0^{\pi/2}$ (২)

$= 2\{-\cos\frac{\pi}{4} + \sin\frac{\pi}{4} - (-\cos 0 + \sin 0)\}$ (১)

$= 2\{-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (-1+0)\} = 2$

20. $\int_{\pi/2}^{\pi/4} \frac{dx}{\sin x} = \int_{\pi/2}^{\pi/4} \text{cosec}\, x dx$

$= \left[\ln|\tan\frac{x}{2}|\right]_{\pi/2}^{\pi/4}$ (১)

$= \ln|\tan\frac{\pi}{8}| - \ln|\tan\frac{\pi}{4}| = \ln(\tan\frac{\pi}{8}) - \ln 1$

$= \ln(\tan\frac{\pi}{8}) - 0 = \ln(\tan\frac{\pi}{8})$ (১)

21. $\int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} \frac{1}{4}(3\sin x - \sin 3x) dx$ (১)

$= \frac{1}{4}\left[-3\cos x + \frac{1}{3}\cos 3x\right]_0^{\pi/2}$ (২)

$= \frac{1}{4}\{-3\cos\frac{\pi}{2} + \frac{1}{3}\cos\frac{3\pi}{2} - (-3\cos 0 + \frac{1}{3}\cos 0)\}$

$= \frac{1}{4}\{(-0+0) - (-3.1 + \frac{1}{3})\} = \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$ (১)

22(a) $\int_0^{\pi/2} \sin^5 x \cos x dx$

$= \int_0^{\pi/2} (\sin x)^5 d(\sin x)$ (১)

$= \left[\frac{1}{6}(\sin x)^6\right]_0^{\pi/2} = \frac{1}{6}\{(\sin\frac{\pi}{2})^6 - (\sin 0)^6\}$ (২)

$= \frac{1}{6}\{1-0\} = \frac{1}{6}$

22(b) $\int_0^{\pi/4} \cos x \sin^3 x dx$

$= \int_0^{\pi/4} (\sin x)^3 d(\sin x)$ (১)

$= \left[\frac{1}{4}(\sin x)^4\right]_0^{\pi/4} = \frac{1}{4}\{(\sin\frac{\pi}{4})^4 - (\sin 0)^4\}$ (২)

$= \frac{1}{4}\{(\frac{1}{\sqrt{2}})^4 - 0\} = \frac{1}{4}.\frac{1}{4} = \frac{1}{16}$

23. $\int_0^{\pi/6} \sin 3x \cos 3x\, dx$

$= \int_0^{\pi/6} \frac{1}{2}\sin 6x\, dx = \frac{1}{2}\left[-\frac{\cos 6x}{6}\right]_0^{\pi/6}$ (২)

$= -\frac{1}{12}(\cos\pi - \cos 0) = -\frac{1}{12}(-1-1) = \frac{1}{6}$

24(a) $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \frac{1}{2}\int_0^1 e^{\sqrt{x}} d(\sqrt{x})$ (১)

$= 2\left[e^{\sqrt{x}}\right]_0^1 = 2(e^{\sqrt{1}} - e^{\sqrt{0}}) = 2(e-1)$ (১)

24(b) $\int_0^2 2x\cos(1+x^2)\, dx$

$= \int_0^2 \cos(1+x^2)\, d(1+x^2)$ (১)

$= \left[\sin(1+x^2)\right]_0^2 = \sin(1+2^2) - \sin(1+0^2)$ (২)

$= \sin(5) - \sin(1)$

25(a) ধরি, $I = \int 2x^3 e^{-x^2} dx$ এবং $x^2 = z$.

তাহলে $2x\,dx = dz$ এবং (১)

$I = \int x^2 e^{-x^2}(2xdx) = \int ze^{-z} dz$

$= z\int e^{-z}dz - \int\{\frac{d}{dz}(z)\int e^{-z}dz\}dz$ (১)

$= z(-e^{-z}) - \int 1.(-e^{-z})dz$ (১)

$= -ze^{-z} + (-e^{-z}) = -(x^2+1)e^{-x^2}$

$\therefore \int_0^1 2x^3 e^{-x^2} dx = \left[-(x^2+1)e^{-x^2}\right]_0^1$

$= -(1+1)e^{-1} + (0+1)e^0 = 1 - 2e^{-1}$ (১)

25(b) $\int \ln(1+x)dx$

$= \ln(1+x)\int dx - \int[\frac{d}{dx}\{\ln(1+x)\}\int dx]dx$ (১)

$= x\ln(1+x) - \int \frac{1}{1+x}.xdx$ (১)

$= x\ln(1+x) - \int \frac{1+x-1}{1+x}dx$

$= x\ln(1+x) - \int (1-\frac{1}{1+x})dx$

$= x\ln(1+x) - \{x - \ln(1+x)\} + c$ (১)

$= (x+1)\ln(1+x) - x + c$

$\therefore \int_0^1 \ln(1+x)dx = [(x+1)\ln(1+x) - x]_0^1$

$= 2\ln 2 - 1 - \ln 1 = 2\ln 2 - 1 - 0 = 2\ln 2 - 1$ (১)

26(a) $\int_1^{\sqrt{3}} \frac{3\,dx}{1+x^2} = 3[\tan^{-1} x]_1^{\sqrt{3}}$ (১)

$= 3(\tan^{-1}\sqrt{3} - \tan^{-1} 1) = 3(\frac{\pi}{3} - \frac{\pi}{4})$ (১)

$= 3\times\frac{\pi}{12} = \frac{\pi}{4}$

26(b) $\int_{-2}^{2} \frac{dx}{x^2+4} = \int_{-2}^{2} \frac{dx}{x^2+2^2} = \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_{-2}^{2}$ (১)

$= \frac{1}{2}\{\tan^{-1}1 - \tan^{-1}(-1)\} = \frac{1}{2}\{\frac{\pi}{4} + \frac{\pi}{4}\} = \frac{\pi}{4}$ (১)

26(c) $\int_0^a \frac{dx}{a^2+x^2} = \left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_0^a$

$= \frac{1}{a}(\tan^{-1}1 - \tan^{-1}0) = \frac{1}{a}(\frac{\pi}{4} - 0) = \frac{\pi}{4a}$ (১)

27. $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1} x]_0^1$ (১)

$= \sin^{-1}1 - \sin^{-1}0 = \frac{\pi}{2}$ (১)

28(a) $\int_0^1 x(1-\sqrt{x})^2\,dx = \int_0^1 x(1-2\sqrt{x}+x)dx$

$= \int_0^1 (x - 2x^{\frac{3}{2}} + x^2)dx = \left[\frac{x^2}{2} - 2\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^3}{3}\right]_0^1$ (১)

$= (\frac{1}{2} - 2\times\frac{2}{5} + \frac{1}{3}) - 0 = \frac{15-24+10}{30} = \frac{1}{30}$ (১)

(b) $\int_1^2 \frac{(x^2-1)^2}{x^2}dx = \int_1^2 \frac{x^4-2x^2+1}{x^2}dx.$

$= \int_1^2 (x^2 - 2 + \frac{1}{x^2})dx = \left[\frac{x^3}{3} - 2x - \frac{1}{x}\right]_1^2$ (১)

$= (\frac{8}{3} - 4 - \frac{1}{2}) - (\frac{1}{3} - 2 - 1)$

$= \frac{8}{3} - 1 - \frac{1}{2} - \frac{1}{3} = \frac{16-6-3-2}{6} = \frac{5}{6}$ (১)

(c) $\int_{\pi/2}^{\pi} (1+\sin 2\theta)d\theta = \left[\theta - \frac{1}{2}\cos 2\theta\right]_{\pi/2}^{\pi}$ (১)

$= (\pi - \frac{1}{2}\cos 2\pi) - (\frac{\pi}{2} - \frac{1}{2}\cos 2.\frac{\pi}{2})$

$= \pi - \frac{1}{2}.1 - \frac{\pi}{2} + \frac{1}{2}(-1) = \frac{\pi}{2} - 1$ (১)

29. $\int_{-\pi/4}^{0} \tan(\frac{\pi}{4} + x)dx$

$= \left[-\ln|\cos(\frac{\pi}{4} + x)|\right]_{-\pi/4}^{0}$ (১)

$= -\ln|\cos\frac{\pi}{4}| + \ln|\cos(\frac{\pi}{4} - \frac{\pi}{4})|$ (১)

$= -\ln|\frac{1}{\sqrt{2}}| + \ln|\cos 0| = -\ln 2^{-\frac{1}{2}} + \ln 1$

$= \frac{1}{2}\ln 2 + 0 = \frac{1}{2}\ln 2$ (১)

30(a) $\int_0^{\pi/2} \sin^2 x dx$ [য.'০১; কু.'০২]

$= \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x)dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2}$ (২)

$= \frac{1}{2}\{(\frac{\pi}{2} - \frac{1}{2}\sin\pi) - (0 - \frac{1}{2}\sin 0)\} = \frac{\pi}{4}$ (১)

30(b) $\int_0^{\pi/2}\sin^5 x\cos^4 xdx$

$=\int_0^{\pi/2}\sin^4 x\cos^4 x\sin xdx$

$=\int_0^{\pi/2}(1-\cos^2 x)^2\cos^4 x\sin xdx$

মনে করি, $\cos x = z \quad \therefore -\sin x\, dx = dz$.

$x=0$ হলে, $z=\cos 0=1$; (১)

$x=\frac{\pi}{2}$ হলে, $z=\cos\frac{\pi}{2}=0$

$\therefore \int_0^{\pi/2}\sin^5 x\cos^4 xdx = -\int_1^0(1-z^2)^2 z^4 dz$

$=-\int_1^0(1-2z^2+z^4)z^4dz$

$=-\int_1^0(z^4-2z^6+z^8)dz$

$=-\left[\frac{1}{5}z^5-2.\frac{1}{7}z^7+\frac{1}{9}z^9\right]_1^0$ (১)

$=-\{0-(\frac{1}{5}-\frac{2}{7}+\frac{1}{9})\}=\frac{63-90+35}{315}$

$=\frac{98-90}{315}=\frac{8}{315}$ (১)

30(c) ধরি, $I=\int_0^{\pi/2}\frac{\cos x}{(1+\sin x)^3}dx$

এবং $z=1+\sin x \;\therefore\; dz=\cos xdx$ (১)

সীমা: $x=0$ হলে $z=1$ এবং $x=\frac{\pi}{2}$ হলে $z=2$

$\therefore I=\int_1^2\frac{dz}{z^3}=\int_1^2 z^{-3}dz=\left[\frac{z^{-2}}{-2}\right]_1^2=\left[-\frac{1}{2z^2}\right]_1^2$ (১)

$=-\frac{1}{2}(\frac{1}{2^2}-\frac{1}{1^2})=-\frac{1}{2}(\frac{1}{4}-1)=\frac{3}{8}$ (১)

31. ধরি, $I=\int_0^1\frac{\cos^{-1}x}{\sqrt{1-x^2}}dx$ [প্র.ভ.প.'০৪]

এবং $z=\cos^{-1}x \;\therefore\; dz=-\frac{1}{\sqrt{1-x^2}}dx$ (১)

সীমা: $x=0$ হলে $z=\frac{\pi}{2}$ এবং $x=1$ হলে $z=0$

$\therefore I=-\int_{\pi/2}^0 zdz=-\left[\frac{z^2}{2}\right]_{\pi/2}^0$ (১)

$=-\frac{1}{2}\{0-(\frac{\pi}{2})^2\}=\frac{\pi^2}{8}$ (১)

32(a) $\int_1^3\frac{2xdx}{1+x^2}=\int_1^3\frac{d(1+x^2)}{1+x^2}$ (১)

$=\left[\ln(1+x^2)\right]_1^3=\ln(1+9)-\ln(1+1)$ (১)

$=\ln\frac{10}{2}=\ln 5$ (১)

32(b) $\int_0^4\frac{dx}{\sqrt{(2x+1)}}=\frac{1}{2}\int_0^4\frac{d(2x+1)}{\sqrt{(2x+1)}}$ (১)

$=\frac{1}{2}\left[2\sqrt{2x+1}\right]_0^4=\sqrt{8+1}-\sqrt{0+1}=3-1=2$ (২)

33(a) $\int\ln(x^2+1)dx$

$=\ln(x^2+1)\int dx-\int[\frac{d}{dx}\{\ln(x^2+1)\}\int dx]dx$ (১)

$=x\ln(x^2+1)-\int\frac{2x}{x^2+1}.xdx$ (১)

$=x\ln(x^2+1)-2\int\frac{x^2+1-1}{x^2+1}dx$

$=x\ln(x^2+1)-2\int(1-\frac{1}{x^2+1})dx$

$=x\ln(x^2+1)-2(x-\tan^{-1}x)+c$ (১)

$=x\ln(x^2+1)-2x+2\tan^{-1}x)+c$

$\int_0^1\ln(x^2+1)dx=\left[x\ln(x^2+1)-2x+2\tan^{-1}x\right]_0^1$

$=\ln 2-2+2\tan^{-1}1-0$

$=\ln 2-2+2.\frac{\pi}{4}=\ln 2-2+\frac{\pi}{2}$ (১)

33(b) ধরি, $I=\int_2^e\{\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\}dx$ [প্র.ভ.প'১৪,'০২]

এবং $\ln x=y\Rightarrow x=e^y \;\therefore\; dx=e^ydy$ (১)

$\therefore \int\{\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\}dx=\int\{\frac{1}{y}-\frac{1}{y^2}\}e^ydy$

$=\int e^y\{\frac{1}{y}+D(\frac{1}{y})\}dy=\frac{e^y}{y}+c=\frac{x}{\ln x}$ (২)

$\therefore I=\left[\frac{x}{\ln x}\right]_2^e=\frac{e}{\ln e}-\frac{2}{\ln 2}=e-\frac{2}{\ln 2}$ (১)

34(a) $\int_0^1\frac{3\,dx}{1+x^2}=3\left[\tan^{-1}x\right]_0^1$ (১)

$=3(\tan^{-1}1-\tan^{-1}0)=\frac{3\pi}{4}$ (১)

34(b) $\int_0^{\pi/2}\frac{\cos x}{1+\sin^2 x}dx=\int_0^{\pi/2}\frac{d(\sin x)}{1^2+(\sin x)^2}$ (১)

$=\left[\tan^{-1}(\sin x)\right]_0^{\pi/2}=\tan^{-1}(\sin\frac{\pi}{2})-\tan^{-1}(\sin 0)$

$=\tan^{-1}1-\tan^{-1}0=\frac{\pi}{4}-0=\frac{\pi}{4}$ (১)

35(a) $\int_{-1}^{2}\frac{dx}{x^2-9}=\int_{-1}^{2}\frac{dx}{x^2-3^2}$

$=\left[\frac{1}{2.3}\ln\left|\frac{x-3}{x+3}\right|\right]_{-1}^{2}$ (১)

$=\frac{1}{6}\{\ln\left|\frac{2-3}{2+3}\right|-\ln\left|\frac{-1-3}{-1+3}\right|\}$

$=\frac{1}{6}(\ln\frac{1}{5}-\ln 2)=\frac{1}{6}\ln\frac{1}{5\times 2}=\frac{1}{6}\ln(0\cdot 1)$ (১)

35(b) $\int_0^{a/2}\frac{1}{a^2-x^2}dx=\left[\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|\right]_0^{a/2}$ (১)

$=\frac{1}{2a}\ln\left|\frac{a+\frac{a}{2}}{a-\frac{a}{2}}\right|=\frac{1}{2a}\ln\left|\frac{3a}{a}\right|=\frac{1}{2a}\ln 3$ (১)

36(a) $\int_0^a\frac{dx}{\sqrt{a^2-x^2}}=\left[\sin^{-1}\frac{x}{a}\right]_0^a$ (১)

$=\sin^{-1}\frac{a}{a}-\sin^{-1}\frac{0}{a}=\sin^{-1}1-\sin^{-1}0=\frac{\pi}{2}$ (১)

36(b) $\int_0^1\frac{dx}{\sqrt{4-3x^2}}$ [কু.বো.'০১; প্র.ভ.প. ৮৩]

$=\frac{1}{\sqrt{3}}\int_0^1\frac{\sqrt{3}dx}{\sqrt{2^2-(\sqrt{3}x)^2}}=\left[\frac{1}{\sqrt{3}}\sin^{-1}\frac{\sqrt{3}x}{2}\right]_0^1$ (১)

$=\frac{1}{\sqrt{3}}(\sin^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}0)=\frac{1}{\sqrt{3}}(\frac{\pi}{3}-0)=\frac{\pi}{3\sqrt{3}}$ (১)

36(c) ধরি, $I=\int_0^{\pi/2}\frac{\cos x dx}{\sqrt{4-\sin^2 x}}$ এবং

$\sin x=z$. তাহলে $\cos x dx=dz$ (১)

সীমা : $x=0$ হলে $z=0$ এবং $x=\frac{\pi}{2}$ হলে $z=1$

$\therefore I=\int_0^1\frac{dz}{\sqrt{2^2-z^2}}=\left[\sin^{-1}\frac{x}{2}\right]_0^1$ (১)

$=\sin^{-1}\frac{1}{2}-\sin^{-1}0=\frac{\pi}{2}-0=\frac{\pi}{2}$ (১)

36(d) $\int_2^3\frac{dx}{(x-1)\sqrt{x^2-2x}}$ [প্র.ভ.প.'০১,'০৩]

$=\int_2^3\frac{dx}{(x-1)\sqrt{(x^2-2x+1)-1}}$

$=\int_2^3\frac{d(x-1)}{(x-1)\sqrt{(x-1)^2-1}}$ (১)

$=\left[\sec^{-1}(x-1)\right]_2^3=\sec^{-1}(3-1)-\sec^{-1}(2-1)$ (২)

$=\sec^{-1}2-\sec^{-1}1=\frac{\pi}{3}-0=\frac{\pi}{3}$.

37. $\int_0^a\frac{a^2-x^2}{(a^2+x^2)^2}dx$ [প্র.ভ.প. '০০]

$=\int_0^a\frac{x^2(\frac{a^2}{x^2}-1)}{\{x(\frac{a^2}{x}+x)\}^2}dx=\int_0^a\frac{(\frac{a^2}{x^2}-1)}{(\frac{a^2}{x}+x)^2}dx$ (১)

$=\int_0^a\frac{-(-\frac{a^2}{x^2}+1)}{(\frac{a^2}{x}+x)^2}dx=-\left[-\frac{1}{\frac{a^2}{x}+x}\right]_0^a$ (২)

$=\left[\frac{x}{a^2+x^2}\right]_0^a=\frac{a}{a^2+a^2}-0=\frac{1}{2a}$ (১)

38. $\int_8^{27}\frac{dx}{x-x^{1/3}}=\int_8^{27}\frac{dx}{x(1-x^{-2/3})}$

ধরি $x^{-\frac{2}{3}} = z$. তাহলে $-\frac{2}{3}x^{-\frac{5}{3}}dx = dz$

$$\Rightarrow -\frac{2}{3}x^{-\frac{2}{3}}\frac{dx}{x} = dz \Rightarrow -\frac{2}{3}z\frac{dx}{x} = dz$$

$$\Rightarrow \frac{dx}{x} = -\frac{3}{2}\frac{dz}{z}$$

সীমা : $x = 8$ হলে $z = 2^{-2} = \frac{1}{4}$ এবং

$x = 27$ হলে $z = 3^{-2} = \frac{1}{9}$

$$\therefore \int_8^{27}\frac{dx}{x - x^{1/3}} = -\frac{3}{2}\int_{1/4}^{1/9}\frac{dz}{z(1-z)}$$

$$= \frac{3}{2}\int_{1/4}^{1/9}\left\{\frac{1}{z-1} - \frac{1}{z}\right\}dz$$

$$= \frac{3}{2}\left[\ln|z-1| - \ln|z|\right]_{1/4}^{1/9} = \frac{3}{2}\left[\ln\left|\frac{z-1}{z}\right|\right]_{1/4}^{1/9}$$

$$= \frac{3}{2}\left\{\ln\left|\frac{\frac{1}{9}-1}{\frac{1}{9}}\right| - \ln\left|\frac{\frac{1}{4}-1}{\frac{1}{4}}\right|\right\}$$

$$= \frac{3}{2}\{\ln|-8| - \ln|-3|\} = \frac{3}{2}(\ln 8 - \ln 3)$$

$$= \frac{3}{2}\ln\frac{8}{3}$$

39. $\int_{-1}^{1}\frac{1-x}{1+x}dx$ [প্র.ভ.প.'৮৪]

$$= \int_{-1}^{1}\frac{-(1+x)+2}{1+x}dx = \int_{-1}^{1}(-1+\frac{2}{1+x})dx$$

$$= \left[-x + 2\ln|1+x|\right]_{-1}^{1}$$

$$= -1 + 2\ln|1+1| - (1 + 2\ln|1-1|)$$

$$= -1 + 2\ln 2 - 1 - 2\ln 0$$

$$= 2(\ln 2 - 1)$$

40. ধরি, $I = \int_{-\pi/4}^{\pi/4}\frac{\sec^6 x - \tan^6 x}{(a^x+1)\cos^2 x}dx \quad (a>0)$

$$= \int_{-\pi/4}^{0}\frac{\sec^6 x - \tan^6 x}{(a^x+1)\cos^2 x}dx + \int_{0}^{\pi/4}\frac{\sec^6 x - \tan^6 x}{(a^x+1)\cos^2 x}dx$$

$= I_1 + I_2$, যেখানে

$$I_1 = \int_{-\pi/4}^{0}\frac{\sec^6 x - \tan^6 x}{(a^x+1)\cos^2 x}dx = -\int_{0}^{-\pi/4}\frac{\sec^6 x - \tan^6 x}{(a^x+1)\cos^2 x}dx$$

ধরি, $x = -y$ তাহলে $dx = -dy$

সীমা : $x=0$ হলে $y=0$ এবং $x=-\frac{\pi}{4}$ হলে $y=\frac{\pi}{4}$

$$\therefore I_1 = \int_0^{\pi/4}\frac{\sec^6 y - \tan^6 y}{(a^{-y}+1)\cos^2 y}dy = \int_0^{\pi/4}\frac{\sec^6 x - \tan^6 x}{(a^{-x}+1)\cos^2 x}dx$$

$$\therefore I = \int_0^{\pi/4}\frac{\sec^6 x - \tan^6 x}{(a^{-x}+1)\cos^2 x}dx + \int_0^{\pi/4}\frac{\sec^6 x - \tan^6 x}{(a^x+1)\cos^2 x}dx$$

$$= \int_0^{\pi/4}\left(\frac{1}{a^{-x}+1} + \frac{1}{a^x+1}\right)\frac{\sec^6 x - \tan^6 x}{\cos^2 x}dx$$

$$= \int_0^{\pi/4}\left(\frac{a^x}{1+a^x} + \frac{1}{a^x+1}\right)\frac{(1+\tan^2 x)^3 - \tan^6 x}{\cos^2 x}dx$$

$$= \int_0^{\pi/4}\frac{1+3\tan^2 x + 3\tan^4 x + \tan^6 x - \tan^6 x}{\cos^2 x}dx$$

$$= \int_0^{\pi/4}(1+3\tan^2 x + 3\tan^4 x)\sec^2 x\,dx$$

$$= \left[\tan x + 3\frac{\tan^3 x}{3} + 3\frac{\tan^5 x}{5}\right]_0^{\pi/4}$$

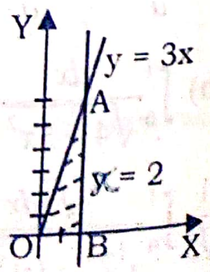
$$= (\tan\frac{\pi}{4} + \tan^3\frac{\pi}{4} + \frac{3}{5}\tan^5\frac{\pi}{4}) = 1 + 1 + \frac{3}{5}$$

$$= \frac{13}{5} \text{ (Ans.)}$$

প্রশ্নমালা X E

1(a) $y = 3x$ সরলরেখা , x-অক্ষ এবং কোটি $x = 2$ দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

সমাধান : নির্ণেয় ক্ষেত্রফল = $y = 3x$ সরলরেখা, x-অক্ষ এবং $x = 0$ ও $x = 2$ রেখাদ্বয় দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল = $\int_0^2 y\,dx = \int_0^2 3x\,dx$



$$= 3\left[\frac{x^2}{2}\right]_0^2 = \frac{3}{2}(2^2 - 0) = 6 \text{ বর্গ একক।}$$