1(a) $\sin(-1230°) - \cos\{(2n+1)\pi + \frac{\pi}{3}\}$

$= -\sin 1230° - \cos\{2n\pi + (\pi + \frac{\pi}{3})\}$

$= -\sin(3.360° + 150°) - \cos(\pi + \frac{\pi}{3})$

$= -\sin 150° - (-\cos\frac{\pi}{3})$

$= -\sin(180° - 30°) + \cos\frac{\pi}{3}$

$= -\sin 30° + \cos\frac{\pi}{3} = -\frac{1}{2} + \frac{1}{2} = 0$ (Ans.)

1(b) $\sin 780° \cos 390° + \sin(-330°)\cos(-300°)$ [চ.'০১]

$= \sin 780° \cos 390° - \sin 330° \cos 300°$

$= \sin(2.360° + 60°)\cos(360° + 30°) - \sin(360° - 30°)\cos(360° - 60°)$

$= \sin 60° \cos 30° - (-\sin 30°)\cos 60°$

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ (Ans.)

2. মান নির্ণয় কর ঃ

(a) $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$

[ঢা. '০২; সি. '০৯; মা.বো.'০৯; ব.'১০; য.'১১]

$= \sin^2\frac{\pi}{7} + \sin^2(\frac{\pi}{2} - \frac{\pi}{7}) + \sin^2(\pi + \frac{\pi}{7}) + \sin^2(\frac{\pi}{2} + \frac{\pi}{7})$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}$

$= 2(\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}) = 2.1 = 2$ (Ans.)

2.(b) $\sin^2\frac{17\pi}{18} + \sin^2\frac{5\pi}{8} + \cos^2\frac{37\pi}{18} + \cos^2\frac{3\pi}{8}$

$= \sin^2(\pi - \frac{\pi}{18}) + \sin^2(\pi - \frac{3\pi}{8}) + \cos^2(2\pi + \frac{\pi}{18}) + \cos^2\frac{3\pi}{8}$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{3\pi}{8} + \cos^2\frac{\pi}{18} + \cos^2\frac{3\pi}{8}$

$= (\sin^2\frac{\pi}{18} + \cos^2\frac{\pi}{18}) + (\sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8})$

$= 1 + 1 = 2$ (Ans.)

3.(a) $\sec^2\frac{14\pi}{17} - \sec^2\frac{39\pi}{17} + \cot^2\frac{41\pi}{34} - \cot^2\frac{23\pi}{34}$

$= \sec^2(\pi - \frac{3\pi}{17}) - \sec^2(2\pi + \frac{5\pi}{17}) + \cot^2(\pi + \frac{7\pi}{34}) - \cot^2(\pi - \frac{11\pi}{34})$

$= \sec^2\frac{3\pi}{17} - \sec^2\frac{5\pi}{17} + \cot^2\frac{7\pi}{34} - \cot^2\frac{11\pi}{34}$

$= \sec^2\frac{3\pi}{17} - \sec^2\frac{5\pi}{17} + \cot^2(\frac{\pi}{2} - \frac{5\pi}{17}) - \cot^2(\frac{\pi}{2} - \frac{3\pi}{17})$

$= \sec^2\frac{3\pi}{17} - \sec^2\frac{5\pi}{17} + \tan^2\frac{5\pi}{17} - \tan^2\frac{3\pi}{17}$

$= (\sec^2\frac{3\pi}{17} - \tan^2\frac{3\pi}{17}) - (\sec^2\frac{5\pi}{17} - \tan^2\frac{5\pi}{17})$

$= 1 - 1 = 0$ (Ans.)

3(b) $\tan 15° + \tan 45° + \tan 75° + \ldots + \tan 165°$

$= \tan 15° + \tan 45° + \tan 75° + \tan 105° + \tan 135° + \tan 165°$

$= \tan 15° + \tan 45° + \tan(90° - 15°) + \tan(90° + 15°) + \tan(180 - 45°) + \tan(180° - 15°)$

$= \tan 15° + \tan 45° + \cot 15° - \cot 15° - \tan 45° - \tan 15° = 0$ (Ans.)

3(c) $\cos^2 15° + \cos^2 25° + \cos^2 35° + \ldots\ldots + \cos^2 75°$

$= \cos^2 15° + \cos^2 25° + \cos^2 35° + \cos^2 45° + \cos^2 55° + \cos^2 65° + \cos^2 75°$

$= \cos^2 15° + \cos^2 25° + \cos^2 35° + (\frac{1}{\sqrt{2}})^2 + \cos^2(90° - 35°) +$

$\cos^2(90°-25°)+\cos^2(90°-15°)$

$=\cos^2 15°+\cos^2 25°+\cos^2 35°+\frac{1}{2}+$

$\sin^2 35°+\sin^2 25°+\sin^2 15°$

$=(\sin^2 5°+\cos^2 5°)+(\sin^2 25°+\cos^2 25°)$

$+(\sin^2 35°+\cos^2 35°)+\frac{1}{2}$

$=1+1+1+\frac{1}{2}=3+\frac{1}{2}=\frac{7}{2}$ (Ans.)

4. প্রমাণ ঃ দেওয়া আছে , [দি.'১৪;' য.'১২; চ.'০৯]

$\sin\theta=\frac{5}{13}$ এবং $\frac{\pi}{2}<\theta<\pi$

$\therefore \text{cosec}\theta=\frac{13}{5}$, $\cos\theta=-\sqrt{1-\sin^2\theta}$

$=-\sqrt{1-\frac{25}{169}}=-\sqrt{\frac{144}{169}}=-\frac{12}{13}$

$\therefore \sec\theta=-\frac{13}{12}$ এবং

$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{5}{13}\times(-\frac{13}{12})=-\frac{5}{12}$

$\Rightarrow \cot\theta=-\frac{12}{5}$

এখন , $\frac{\tan\theta+\sec(-\theta)}{\cot\theta+\text{cosec}(-\theta)}=\frac{\tan\theta+\sec\theta}{\cot\theta-\text{cosec}\theta}$

$=\frac{\frac{-5}{12}+\frac{-13}{12}}{\frac{-12}{5}-\frac{13}{5}}=\frac{\frac{-5-13}{12}}{\frac{-12-13}{5}}$

$=(-\frac{18}{12})\times(-\frac{5}{25})=\frac{3}{2}\times\frac{1}{5}=\frac{3}{10}$

$\therefore \frac{\tan\theta+\sec(-\theta)}{\cot\theta+\text{cosec}(-\theta)}=\frac{3}{10}$

5. সমাধান ঃ

(a) $\sin x+\sin(\pi+x)+\sin(2\pi+x)+\cdots\cdots$ (n +1)তম পদ পর্যন্ত

$=\sin x-\sin x+\sin x-\sin x+\cdots\cdots$ (n +1) তম পদ পর্যন্ত

n = 1 হলে , (1+ 1) বা ২য় পদ পর্যন্ত যোগফল

$=\sin x-\sin x=0$

n = 3 হলে, (3 + 1) বা ৪র্থ পদ পর্যন্ত

যোগফল $=\sin x-\sin x+\sin x-\sin x=0$

তদ্রুপ, n যেকোন বিজোড় সংখ্যা হলে নির্ণেয় যোগফল = 0

আবার, n = 2 হলে (2 + 1) বা ৩য় পদ পর্যন্ত

যোগফল $=\sin x-\sin x+\sin x=\sin x$

n = 4 হলে , (4 + 1) বা ৫ম পদ পর্যন্ত যোগফল

$=\sin x-\sin x+\sin x-\sin x+\sin x$

$=\sin x$

তদ্রুপ,nযেকোন জোড় সংখ্যা হলে নির্ণেয় যোগফল = sinx

5(b) $\tan\theta+\tan(\pi+\theta)+\tan(2\pi+\theta)+\cdots+\tan(n\pi+\theta)$

$=\tan\theta+\tan\theta+\tan\theta+\cdots$ n তম পদ পর্যন্ত

$=(n+1)\tan\theta$ (Ans.)

6(a) দেওয়া আছে, $\theta=\frac{\pi}{20}\Rightarrow\frac{\pi}{2}=10\theta$

L.H.S.= $\cot\theta\cot3\theta\cot5\theta\cot7\theta$ $\cot9\theta\cot11\theta\cot13\theta\cot15\theta\cot17\theta$ $\cot19\theta$

$=\cot\theta\cot3\theta\cot5\theta\cot7\theta\cot9\theta$

$\cot(10\theta+\theta)\cot(10\theta+3\theta)$

$\cot(10\theta+5\theta)\cot(10\theta+7\theta)$

$\cot(10\theta+9\theta)$

$=\cot\theta\cot3\theta\cot5\theta\cot7\theta\cot9\theta$

$\cot(\frac{\pi}{2}+\theta)\cot(\frac{\pi}{2}+3\theta)\cot(\frac{\pi}{2}+5\theta)$

$\cot(\frac{\pi}{2}+7\theta)\cot(\frac{\pi}{2}+9\theta)$

$=\frac{1}{\tan\theta\tan3\theta\tan5\theta\tan7\theta\tan9\theta}(-\tan\theta)$

$(-\tan3\theta)(-\tan5\theta)(-\tan7\theta)(-\tan9\theta)$

$=-1=$ R.H.S.

6. (b) দেওয়া আছে, $\theta=\frac{\pi}{28}\Rightarrow\frac{\pi}{2}=14\theta$

L.H.S = $\tan\theta\tan3\theta\tan5\theta\tan7\theta$

$\tan9\theta\tan11\theta\tan13\theta$

$=\tan\theta\tan3\theta\tan5\theta\tan7\theta$

$\tan(14\theta-5\theta)\tan(14\theta-3\theta)$

$\tan(14\theta-\theta)$

$= \dfrac{1}{\tan\theta\tan 3\theta\tan 5\theta}\tan\dfrac{\pi}{4}$

$\tan(\dfrac{\pi}{2}-5\theta)\ \tan(\dfrac{\pi}{2}-3\theta)\tan(\dfrac{\pi}{2}-\theta)$

$= \dfrac{1}{\tan\theta\tan 3\theta\tan 5\theta}.1.\tan 5\theta.\ \tan 3\theta.\ \tan\theta$

$= 1 = $ R.H.S.

(c) $\cos\dfrac{\pi}{11}.\cos\dfrac{2\pi}{11}.\cos\dfrac{3\pi}{11}\cdots\cos\dfrac{10\pi}{11} = -2^n$ হলে n এর মান নির্ণয় কর।

ধরি, $\dfrac{\pi}{11}=\theta \Rightarrow 11\theta=\pi \Rightarrow 6\theta+5\theta=\pi$

এখন, $\cos\dfrac{\pi}{11}.\cos\dfrac{2\pi}{11}.\cos\dfrac{3\pi}{11}\cdots\cos\dfrac{10\pi}{11}$

$= (\cos\theta.\cos 2\theta.\cos 3\theta.\cos 4\theta.\cos 5\theta)$
$(\cos 6\theta.\cos 7\theta.\cos 8\theta.\cos 9\theta.\cos 10\theta)$

$= (\cos\theta.\cos 2\theta.\cos 3\theta.\cos 4\theta.\cos 5\theta)\cdot$
$\{\cos(\pi-5\theta).\cos(\pi-4\theta).\cos(\pi-3\theta).$
$\cos(\pi-2\theta).\ \cos(\pi-\theta)\}$

$= -\cos^2\theta\cos^2 2\theta\cos^2 3\theta\cos^2 4\theta\cos^2 5\theta$

$= -\cos^2\theta\cos^2 2\theta\cos^2 3\theta\cos^2 4\theta\cos^2 5\theta$

$= -\cos^2\theta\cos^2 2\theta\cos^2 3\theta\cos^2 4\theta\cos^2 5\theta$

$= -(\cos\theta\cos 2\theta\cos 3\theta\cos 4\theta\cos 5\theta)^2$

$= -\dfrac{1}{2^2\sin^2\theta}(2\sin\theta\cos\theta)^2$
$(\cos 2\theta\cos 3\theta\cos 4\theta\cos 5\theta)^2$

$= -\dfrac{1}{2^2\sin^2\theta}(\sin 2\theta)^2$
$(\cos 2\theta\cos 3\theta\cos 4\theta\cos 5\theta)^2$

$= -\dfrac{1}{2^2.2^2\sin^2\theta}(2\sin 2\theta\cos 2\theta)^2$
$(\cos 3\theta\cos 4\theta\cos 5\theta)^2$

$= -\dfrac{1}{2^4.2^2\sin^2\theta}(2\sin 4\theta\cos 4\theta)^2$
$(\cos 3\theta\ \cos 5\theta)^2$

$= -\dfrac{1}{2^6\sin^2\theta}(\sin 8\theta\ \cos 3\theta\ \cos 5\theta)^2$

$= -\dfrac{1}{2^6\sin^2\theta}\{\sin(\pi-3\theta)\cos 3\theta\cos 5\theta\}^2$

$= -\dfrac{1}{2^6\sin^2\theta}\{\sin 3\theta\cos 3\theta\cos 5\theta\}^2$

$= -\dfrac{1}{2^6.2^2\sin^2\theta}\{2\sin 3\theta\cos 3\theta\cos 5\theta\}^2$

$= -\dfrac{1}{2^8\sin^2\theta}\{\sin 6\theta\cos 5\theta\}^2$

$= -\dfrac{1}{2^{10}\sin^2\theta}\{2\sin 6\theta\cos 5\theta\}^2$

উ: n = -8

$= -\dfrac{1}{2^{10}\sin^2\theta}\{\sin 11\theta+\cos\theta\}^2$

$= -\dfrac{1}{2^{10}\sin^2\theta}\{\sin\pi+\sin\theta\}^2$

$= -\dfrac{1}{2^{10}\sin^2\theta}\{0+\sin\theta\}^2$

$= -\dfrac{1}{2^{10}\sin^2\theta}\sin^2\theta = -\dfrac{1}{2^{10}} = -2^{-10}$

$\therefore\ -2^n = -2^{-10} \Rightarrow 2^n = 2^{-10} \Rightarrow n = -10$

সম্ভাব্য ধাপসহ সমস্যা

7. মান নির্ণয় কর :

(a) $\tan(-1590°) = -\tan(1590°)$ (১)

$= -\tan(4.360°+150°) = -\tan 150°$ (১)

$= -\tan(180°-30°) = +\tan 30° = \dfrac{1}{\sqrt{3}}$ (১)

(b) $\cos 420°\sin(-300°) - \sin 870°\cos 570°$

$= \cos 420°(-\sin 300°) - \sin 870°\cos 570°$ (১)

$= -\cos(360°+60°)\sin(360°-60°)$
$-\sin(2.360°+150°)\cos(2.360°-150°)$

$= -\cos 60°(-\sin 60°) - \sin 150°\cos 150°$ (১)

$= \cos 60°\sin 60° - \sin(180°-30°)$
$\cos(180°-30°)$

$= \cos 60°\sin 60° - \sin 30°(-\cos 30°)$

$= \frac{1}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{\sqrt{3}}{2} = 2.\frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (Ans.) (১)

8. $\cos^2\frac{\pi}{24} + \cos^2\frac{19\pi}{24} + \cos^2\frac{31\pi}{24} + \cos^2\frac{37\pi}{24}$

$= \cos^2\frac{\pi}{24} + \cos^2\frac{19\pi}{24} + \cos^2(\frac{\pi}{2} + \frac{19\pi}{24})$

$+ \cos^2(3.\frac{\pi}{2} + \frac{\pi}{24})$

$= \cos^2\frac{\pi}{24} + \cos^2\frac{19\pi}{24} + \sin^2\frac{\pi}{24} + \sin^2\frac{19\pi}{24}$ (১)

$= (\sin^2\frac{\pi}{24} + \cos^2\frac{\pi}{24}) + (\sin^2\frac{19\pi}{24} + \cos^2\frac{19\pi}{24})$

$= 1 + 1 = 2$ (Ans.) (১)

9(a) $\cos^2 25° + \cos^2 35° + \cos^2 45° + \cos^2 55° + \cos^2 65°$

$= \cos^2 25° + \cos^2 35° + \left(\frac{1}{\sqrt{2}}\right)^2 +$

$\cos^2(90° - 35°) + \cos^2(90° - 25°)$

$= \cos^2 25° + \cos^2 35° + \frac{1}{2} + \sin^2 35°$

$+ \sin^2 25°$ (১)

$= (\sin^2 25° + \cos^2 25°) + \frac{1}{2} +$

$(\sin^2 25° + \cos^2 25°)$

$= 1 + \frac{1}{2} + 1 = \frac{5}{2}$ (Ans.) (১)

9(b) $\sin^2 10° + \sin^2 20° + \sin^2 30° + \cdots + \sin^2 80°$

$= \sin^2 10° + \sin^2 20° + \sin^2 30°$

$+ \sin^2 40° + \sin^2 50° + \sin^2 60°$

$+ \sin^2 70° + \sin^2 80°$

$= \sin^2 10° + \sin^2 20° + \sin^2 30° +$

$\sin^2 40° + \sin^2(90° - 40°) +$

$\sin^2(90° - 30°) + \sin^2(90° - 20°)$

$+ \sin^2(90° - 10°)$

$= \sin^2 10° + \sin^2 20° + \sin^2 30°$

$+ \sin^2 40° + \cos^2 40° + \cos^2 30°$

$+ \cos^2 20° + \cos^2 10°$ (১)

$= (\sin^2 10° + \cos^2 10°) + (\sin^2 20° + \cos^2 20°)$

$+ (\sin^2 30° + \cos^2 30°) + (\sin^2 40° + \cos^2 40°)$

$= 1 + 1 + 1 + 1 = 4$ (Ans.) (১)

10. $\tan\theta = \frac{3}{4}$ এবং $\cos\theta$ ঋণাত্মক হলে,

$\frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta}$ এর মান নির্ণয় কর।

সমাধান ঃ দেওয়া আছে,

$\tan\theta = \frac{3}{4}$ এবং $\cos\theta$ ঋণাত্মক

$\therefore \sec\theta = -\sqrt{1 + \tan^2\theta} = -\sqrt{1 + \frac{9}{16}}$ (১)

$= -\sqrt{\frac{25}{16}} = -\frac{5}{4}$ $\therefore \cos\theta = -\frac{4}{5}$ এবং

$\sin\theta = \tan\theta\cos\theta = \frac{3}{4}(-\frac{4}{5}) = -\frac{3}{5}$ (১)

এখন, $\frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} = \frac{-\frac{3}{5} - \frac{4}{5}}{-\frac{5}{4} + \frac{3}{4}}$

$= -\frac{3+4}{5} \times \frac{4}{-5+3} = -\frac{7}{5} \times \frac{4}{-2} = \frac{14}{5}$ (Ans.) (১)

11. $\sin\theta = \frac{12}{13}$ এবং $90° < \theta < 180°$ হলে

দেখাও যে, $\frac{\tan\theta + \sec(-\theta)}{\cot\theta + \text{cosec}(-\theta)} = \frac{10}{3}$

প্রমাণ ঃ যেহেতু $\sin\theta = \frac{12}{13} \Rightarrow \text{cosec}\theta = \frac{13}{12}$ এবং

$90° < \theta < 180°$,

$\therefore \cos\theta = -\sqrt{1 - \sin^2\theta}$ (১)

$= -\sqrt{1 - \frac{144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$

$\therefore \sec\theta = -\frac{13}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{13} \times (-\frac{13}{5}) = -\frac{12}{5}$

$\Rightarrow \cot\theta = -\dfrac{5}{12}$ (১)

এখন , $\dfrac{\tan\theta + \sec(-\theta)}{\cot\theta + \operatorname{cosec}(-\theta)} = \dfrac{\tan\theta + \sec\theta}{\cot\theta - \operatorname{cosec}\theta}$ (১)

$= \dfrac{-\dfrac{12}{5} - \dfrac{13}{5}}{-\dfrac{5}{12} - \dfrac{13}{12}} = \dfrac{-25}{5} \times \dfrac{12}{-5-13}$

$= 5 \times \dfrac{12}{18} = \dfrac{10}{3}$ (১)

12. যোগফল নির্ণয় কর: $\cos\theta + \cos(\pi + \theta) + \cos(2\pi + \theta) + \cdots + \cos(n\pi + \theta)$

সমাধান: $\cos\theta + \cos(\pi + \theta) + \cos(2\pi + \theta) + \cdots + \cos(n\pi + \theta)$

$= \cos\theta + \{-\cos\theta + \cos\theta - \cos\theta + \cdots + (-1)^n \cos\theta\}$ (১)

$n = 2$ হলে যোগফল $= \cos\theta + \{-\cos\theta + \cos\theta\}$

$= \cos\theta$ (১)

$n = 4$ হলে যোগফল $= \cos\theta + \{-\cos\theta + \cos\theta - \cos\theta + \cos\theta\} = \cos\theta$

তদ্রূপ, n যেকোনো জোড় হলে নির্ণেয় যোগফল $= \cos x$

$n = 1$ হলে যোগফল $= \cos\theta + (-\cos\theta) = 0$ (১)

$n = 3$ হলে যোগফল $= \cos\theta + \{-\cos\theta + \cos\theta -$

তদ্রূপ, n যেকোনো বিজোড় হলে নির্ণেয় যোগফল $= 0$ (১)

13. $n \in \mathbb{Z}$ হলে , $\sin\{n\pi + (-1)^n \dfrac{\pi}{4}\}$ এর মান নির্ণয় কর ।

সমাধান ঃ (a) $\sin\{n\pi + (-1)^n \dfrac{\pi}{4}\}$

n জোড় সংখ্যা হলে মনে করি, $n = 2m$, যেখানে $m \in \mathbb{N}$.

$\therefore \sin\{n\pi + (-1)^n \dfrac{\pi}{4}\}$

$= \sin\{2m\pi + (-1)^{2m} \dfrac{\pi}{4}\}$

$= \sin(2m\pi + \dfrac{\pi}{4}) = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ (১)

n বিজোড় সংখ্যা হলে মনে করি , $n = 2m + 1;\ m \in \mathbb{N}$.

$\therefore \sin\{n\pi + (-1)^n \dfrac{\pi}{4}\}$

$= \sin\{(2m+1)\pi + (-1)^{2m+1} \dfrac{\pi}{4}\}$

$= \sin\{2m\pi + (\pi - \dfrac{\pi}{4})\}$

$= \sin(\pi - \dfrac{\pi}{4}) = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ (Ans.) (১)

14. দেখাও যে , $\tan\dfrac{\pi}{12}\tan\dfrac{5\pi}{12}\tan\dfrac{7\pi}{12}\tan\dfrac{11\pi}{12} = 1$

প্রমাণ: $\tan\dfrac{\pi}{12}\ \tan\dfrac{5\pi}{12}\ \tan\dfrac{7\pi}{12}\ \tan\dfrac{11\pi}{12}$

$= \tan\dfrac{\pi}{12}\ \tan\dfrac{5\pi}{12}\ \tan(\dfrac{\pi}{2} + \dfrac{\pi}{12})\ \tan(\dfrac{\pi}{2} + \dfrac{5\pi}{12})$

$= \tan\dfrac{\pi}{12}\ \tan\dfrac{5\pi}{12}\ \cot\dfrac{\pi}{12}\ \cot\dfrac{5\pi}{12}$ (১)

$= (\tan\dfrac{\pi}{12} . \cot\dfrac{\pi}{12})(\tan\dfrac{5\pi}{12} . \cot\dfrac{5\pi}{12})$

$= 1.1 = 1 \quad [\because \tan\theta.\cot\theta = 1]$ (১)

সৃজনশীল প্রশ্ন :

15. $A = \dfrac{\cot(-\theta) + \operatorname{cosec}\theta}{\cos\theta + \sin(-\theta)}$ এবং $f(\theta) = \tan\theta$

(a) $\sin^2\dfrac{\pi}{12} + \sin^2\dfrac{3\pi}{12} + \sin^2\dfrac{5\pi}{12} + \sin^2\dfrac{7\pi}{12} + \sin^2\dfrac{9\pi}{12} + \sin^2\dfrac{11\pi}{12}$ এর মান নির্ণয় কর।

প্রমাণ: $\sin^2\dfrac{\pi}{12} + \sin^2\dfrac{3\pi}{12} + \sin^2\dfrac{5\pi}{12} + \sin^2\dfrac{7\pi}{12} + \sin^2\dfrac{9\pi}{12} + \sin^2\dfrac{11\pi}{12}$

$= \sin^2\dfrac{\pi}{12} + \sin^2\dfrac{3\pi}{12} + \sin^2\dfrac{5\pi}{12} + \sin^2(\dfrac{\pi}{2} + \dfrac{\pi}{12}) + \sin^2(\dfrac{\pi}{2} + \dfrac{3\pi}{12}) + \sin^2(\dfrac{\pi}{2} + \dfrac{5\pi}{12})$

$= \sin^2\dfrac{\pi}{12} + \sin^2\dfrac{3\pi}{12} + \sin^2\dfrac{5\pi}{12} + \cos^2\dfrac{\pi}{12}$

$+\cos^2\frac{3\pi}{12}+\cos^2\frac{5\pi}{12}$

$=(\sin^2\frac{\pi}{12}+\cos^2\frac{\pi}{12})+(\sin^2\frac{3\pi}{12}+\cos^2\frac{3\pi}{12})$

$+(\sin^2\frac{5\pi}{12}+\cos^2\frac{5\pi}{12})$

$=1+1+1=3$ (Ans.)

(b) $\cot\theta=\frac{3}{4}$ এবং $\cos\theta$ ঋণাত্মক হলে, A এর মান নির্ণয় কর।

প্রমাণ: যেহেতু $\cot\theta=\frac{3}{4}\Rightarrow\tan\theta=\frac{4}{3}$ এবং $\cos\theta$ ঋণাত্মক

$\therefore\sec\theta=-\sqrt{1+\tan^2\theta}=-\sqrt{1+\frac{16}{9}}$

$=-\sqrt{\frac{25}{9}}=-\frac{5}{3}$

$\therefore\cos\theta=-\frac{3}{5}$ এবং

$\sin\theta=\tan\theta\cos\theta=\frac{4}{3}\times(-\frac{3}{5})=-\frac{4}{5}$

$\therefore\operatorname{cosec}\theta=-\frac{5}{4}$

এখন, $\frac{\cot(-\theta)+\operatorname{cosec}\theta}{\cos\theta+\sin(-\theta)}=\frac{-\cot\theta+\operatorname{cosec}\theta}{\cos\theta-\sin\theta}$

$=\frac{-\frac{3}{4}+(-\frac{5}{4})}{-\frac{3}{5}-\frac{-4}{5}}=\frac{-3-5}{4}\times\frac{5}{-3+4}$

$=-\frac{40}{4}=-10$ (Ans.)

(c) $4n\theta=\pi$ হলে দেখাও যে, $f(\theta)f(2\theta)f(3\theta)\cdots\cdots\cdots\cdots f\{(2n-1)\theta\}=1$

প্রমাণ: $f(\theta)f(2\theta)f(3\theta)\cdots\cdots f\{(2n-1)\theta\}$

$=\tan\theta.\tan2\theta.\tan3\theta.\cdots\tan(2n-1)\theta$

এখানে, পদসংখ্যা $=2n-1$, যা বিজোড় সংখ্যা।

$\frac{2n-1+1}{2}$ অর্থাৎ n তম পদ মধ্যপদ।

$\therefore$ মধ্যপদ $=\tan n\theta=\tan\frac{\pi}{4}=1$ [$\because 4n\theta=\pi$]

$\tan\theta.\tan(2n-1)\theta=\tan\theta.\tan(2n\theta-\theta)$

$=\tan\theta.\tan(\frac{\pi}{2}-\theta)$ [$\because 4n\theta=\pi$]

$=\tan\theta.\cot\theta=1$

$\tan2\theta.\tan(2n-2)\theta=\tan2\theta.\tan(2n\theta-2\theta)$

$=\tan2\theta.\tan(\frac{\pi}{2}-2\theta)$ উ: −10

$=\tan2\theta.\cot2\theta=1$

অনুরূপভাবে, $\tan3\theta.\tan(2n-3)\theta=1$

$\tan4\theta.\tan(2n-4)\theta=1,\cdots$ ইত্যাদি।

অর্থাৎ,মধ্যপদ হতে সমদূরবর্তী পদ দুইটির গুণফল = 1

$\therefore f(\theta)f(2\theta)f(3\theta)\cdots\cdots f\{(2n-1)\theta\}=1$

প্রশ্নমালা VII B

1. মান নির্ণয় কর ঃ (a) tan 105° (b) cosec 165°

(a) $\tan105^\circ=\tan(60^\circ+45^\circ)$

$=\frac{\tan60^0+\tan45^0}{1-\tan60^0\tan45^0}=\frac{\sqrt{3}+1}{1-\sqrt{3}.1}$

$=\frac{(1+\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})}=\frac{1+2\sqrt{3}+3}{1-3}$

$=\frac{2(\sqrt{3}+2)}{-2}=-(\sqrt{3}+2)$

1(b) $\operatorname{cosec}165^\circ=\operatorname{cosec}(90^\circ+75^\circ)$

$=\sec75^\circ=\frac{1}{\cos75^0}=\frac{1}{\cos(45^0+30^0)}$

$=\frac{1}{\cos45^0\cos30^0-\sin45^0\sin30^0}$

$=\frac{1}{\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}.\frac{1}{2}}=\frac{2\sqrt{2}}{\sqrt{3}-1}$

$=\frac{2\sqrt{2}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{2(\sqrt{6}+\sqrt{3})}{3-1}$

$=\frac{2(\sqrt{6}+\sqrt{3})}{2}=\sqrt{6}+\sqrt{3}$