$= \cos\{(\frac{\pi}{3} + \frac{\pi}{6}) - (\alpha + \beta)\}$

$= \cos\{\frac{\pi}{2} - (\alpha + \beta)\}$

$= \sin(\alpha + \beta) =$ R.H.S. (Proved)

(b) প্রমাণ কর যে, $B = \tan 20° + 2\tan 50°$

[চ.'০৫; ঢা'১০,'১৫]

প্রমাণ : $B = \tan 70° = \tan(50° + 20°)$

$\Rightarrow \tan 70° = \frac{\tan 50^0 + \tan 20^0}{1 - \tan 50^0 \tan 20^0}$

$\Rightarrow \tan 70° - \tan 70° \tan 50° \tan 20° = \tan 50° + \tan 20°$

$\Rightarrow \tan 70° - \tan(90° - 20°)\tan 50° \tan 20° = \tan 50° + \tan 20°$

$\Rightarrow \tan 70° - \cot 20° \tan 50° \tan 20° = \tan 50° + \tan 20°$

$\Rightarrow \tan 70° - \tan 50° = \tan 50° + \tan 20°$

$\Rightarrow \tan 70° = \tan 20° + 2\tan 50°$

$\therefore B = \tan 20° + 2\tan 50°$

(c) $A = -\frac{3}{2}$ হলে দেখাও যে, $\Sigma\cos\alpha = 0$ এবং $\Sigma\sin\alpha = 0$

প্রমাণ : দেওয়া আছে , $A = -\frac{3}{2}$

$\Rightarrow \cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$

$\Rightarrow 2(\cos\beta\cos\gamma + \sin\beta\sin\gamma + \cos\gamma\cos\alpha + \sin\gamma\sin\alpha + \cos\alpha\cos\beta + \sin\alpha\sin\beta) = -3$

$\Rightarrow 2(\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha) + 2(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha) + 1 + 1 + 1 = 0$

$\Rightarrow 2(\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha) + 2(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha) + (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + (\sin^2\gamma + \cos^2\gamma) = 0$

$\Rightarrow \{\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2(\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha)\} + \{\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha)\} = 0$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

$\therefore \cos\alpha + \cos\beta + \cos\gamma = 0$ এবং $\sin\alpha + \sin\beta + \sin\gamma = 0$

[∵ দুইটি সংখ্যার বর্গের সমষ্টি শূন্য হলে সংখ্যা দুইটি পৃথক পৃথক ভাবে শূন্য হয় ।]

$\therefore \sum\cos\alpha = 0$ এবং $\sum\sin\alpha = 0$

প্রশ্নমালা VII C

1. প্রমাণ কর যে,

(a) $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$

L.H.S.$= \sin 10° \sin 30° \sin 50° \sin 70°$

$= \sin 10° . \frac{1}{2} . \frac{1}{2}\{\cos(70° - 50°) - \cos(70° + 50°)\}$

$= \frac{1}{4}\sin 10°(\cos 20° - \cos 120°)$

$= \frac{1}{4}\sin 10° \cos 20° - \frac{1}{4}\cdot(-\frac{1}{2})\sin 10°$

$= \frac{1}{4}.\frac{1}{2}\{\sin(20° + 10°) - \sin(20° - 10°)\} + \frac{1}{8}\sin 10°$

$= \frac{1}{8}\sin 30° - \frac{1}{8}\sin 10° + \frac{1}{8}\sin 10°$

$= \frac{1}{8}.\frac{1}{2} = \frac{1}{16} =$ R.H.S. (Proved)

1(b) $\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$

L.H.S.$= \cos 20° \cos 40° \cos 60° \cos 80°$

$= \frac{1}{2}\{\cos(40° + 20°) + \cos(40° - 20°)\}\frac{1}{2}.\cos 80°$

$= \frac{1}{4}\{\cos 60° + \cos 20°\}\cos(90° - 10°)$

$= \frac{1}{4}(\frac{1}{2} + \cos 20°)\sin 10°$

$= \frac{1}{8}\sin 10° + \frac{1}{4}\cos 20° \sin 10°$

$= \frac{1}{8}\sin 10° + \frac{1}{8}\{\sin(20° + 10°)$

$- \sin(20° - 10°)\}$

$= \frac{1}{8}\sin 10° + \frac{1}{8}\sin 30° - \frac{1}{8}\sin 10°$

$= \frac{1}{8}.\frac{1}{2} = \frac{1}{16}$ = R.H.S. (Proved)

বিকল্প পদ্ধতি: $\cos 20° \cos 40° \cos 60° \cos 80°$

$= \cos 20° \cos(60° - 20^0) \frac{1}{2} \cos(60° - 20^0)$

$= \frac{1}{2} \cos 20° \{\cos^2 20^0 - \sin^2 60^0)$

$= \frac{1}{2} \cos 20° \{\cos^2 20^0 - \frac{3}{4})$

$= \frac{1}{2}.\frac{4\cos^3 20^0 - 3\cos 20^0}{4})$

$= \frac{1}{8} \cos(3 \times 20^0)$

$[\cos 3A = 4\cos^3 A - 3\cos A]$

$= \frac{1}{8} \cos 60^0 = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ = R.H.S.

1(c) $\tan 20° \tan 40° \tan 60° \tan 80° = 3$

L.H.S. $= \tan 20° \tan 40° \tan 60° \tan 80°$

$= \tan 20° \tan 40° . \sqrt{3} . \tan 80°$

$= \sqrt{3} \tan 20° \tan 40° \tan 60°$

$= \sqrt{3} . \frac{2\sin 20^0 \sin 40^0 \sin 80^0}{2\cos 20^0 \cos 40^0 \cos 80^0}$

$= \frac{\sqrt{3}\{\cos(40^0 - 20^0) - \cos(40^0 + 20^0)\}\sin(90^0 - 10^0)}{\{\cos(40^0 + 20^0) + \cos(40^0 - 20^0)\}\cos(90^0 - 10^0)}$

$= \sqrt{3} \frac{(\cos 20^0 - \cos 60^0)\cos 10^0}{(\cos 60^0 + \cos 20^0)\sin 10^0}$

$= \sqrt{3} \frac{\cos 20^0 \cos 10^0 - \frac{1}{2}\cos 10^0}{\frac{1}{2}\sin 10^0 + \cos 20^0 \sin 10^0}$

$= \sqrt{3} \frac{\frac{1}{2}\{\cos(20^0 + 10^0) + \cos(20^0 - 10^0)\} - \frac{1}{2}\cos 10^0}{\frac{1}{2}\sin 10^0 + \frac{1}{2}\{\sin(20^0 + 10^0) - \sin(20^0 - 10^0)\}}$

$= \sqrt{3}. \frac{\frac{1}{2}\cos 30^0 + \frac{1}{2}\cos 10^0 - \frac{1}{2}\cos 10^0}{\frac{1}{2}\sin 10^0 + \frac{1}{2}\sin 30^0 - \frac{1}{2}\sin 10^0}$

$= \sqrt{3}. \frac{\frac{1}{2}.\frac{\sqrt{3}}{2}}{\frac{1}{2}.\frac{1}{2}} = \sqrt{3}.\frac{\sqrt{3}}{4} \times 4$

$= \sqrt{3}.\sqrt{3} = 3$ = R.H.S.

বিকল্প পদ্ধতি:

L.H.S. $= \tan 20° \tan 40° \tan 60° \tan 80°$

$= \sqrt{3} \tan 20° \tan(60° - 20°) \tan(60° + 20°)$

$= \sqrt{3} \tan 20° \frac{\tan^2 60^0 - \tan^2 20^0}{1 - \tan^2 60^0 \tan^2 20^0}$

$= \sqrt{3} \tan 20° \frac{3 - \tan^2 20^0}{1 - 3\tan^2 20^0}$

$= \sqrt{3} \frac{3\tan 20^0 - \tan^3 20^0}{1 - 3\tan^2 20^0}$

$= \sqrt{3} \tan(3 \times 20^0)$

$[\because \tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}]$

$= \sqrt{3} \tan 60^0 = \sqrt{3} \times \sqrt{3} = 3$

2.(a) $\cos\theta \cos(60° - \theta) \cos(60° + \theta) = \frac{1}{4} \cos 3\theta$

L.H.S. $= \cos\theta \cos(60° - \theta) \cos(60° + \theta)$

$= \cos\theta . \frac{1}{2} \{\cos(60° + \theta + 60° - \theta)$

$+ \cos(60° + \theta - 60° + \theta)\}$

$= \frac{1}{2} \cos\theta(\cos 120° + \cos 2\theta)$

$= \frac{1}{2}\cos\theta(-\frac{1}{2}) + \frac{1}{2}\cos\theta\cos2\theta$

$= -\frac{1}{4}\cos\theta + \frac{1}{2}.\frac{1}{2}\{\cos(2\theta+\theta) + \cos(2\theta-\theta)\}$

$= -\frac{1}{4}\cos\theta + \frac{1}{4}\cos3\theta + \frac{1}{4}\cos\theta$

$= \frac{1}{4}\cos3\theta$ = R.H.S. (Proved)

বিকল্প পদ্ধতি:

L.H.S. $= \cos\theta\cos(60° - \theta)\cos(60° + \theta)$

$= \cos\theta\ (\cos^2\theta - \sin^2 60°)$

$= \cos\theta\ (\cos^2\theta - \frac{3}{4}) = \cos\theta.\frac{4\cos^2\theta - 3}{4}$

$= \frac{4\cos^3\theta - 3\cos\theta}{4} = \frac{1}{4}\cos3\theta$ = R.H.S.

[$\because \cos3\theta = 4\cos^3\theta - 3\cos\theta$]

2(b) $\cos(36° - \theta)\cos(36° + \theta) + \cos(54° + \theta)\cos(54° - \theta) = \cos2\theta$

L.H.S.$= \cos(36° - \theta)\cos(36° + \theta) + \cos(54° + \theta)\cos(54° - \theta)$

$= \frac{1}{2}(\cos72° + \cos2\theta) + \frac{1}{2}(\cos108° + \cos2\theta)$

$= \frac{1}{2}\{\cos(90° - 18°) + \cos2\theta\} + \frac{1}{2}\{\cos(90° + 18°) + \cos2\theta\}$

$= \frac{1}{2}(\cos2\theta + \cos18°) + \frac{1}{2}(\cos2\theta - \cos18°)$

$= \frac{1}{2}(\cos2\theta + \cos18° + \cos2\theta - \cos18°)$

$= \frac{1}{2}.2\cos2\theta = \cos2\theta$ = R.H.S. (Proved)

3. প্রমাণ কর যে,

(a) $\cos(60° - \theta) + \cos(60° + \theta) - \cos\theta = 0$

L.H.S. $= \cos(60° - \theta) + \cos(60° + \theta) - \cos\theta$

$= 2\cos60°\cos\theta - \cos\theta$

$= 2.\frac{1}{2}\cos\theta - \cos\theta$

$= \cos\theta - \cos\theta = 0$ = R.H.S. (Proved)

(b) $\sin\theta + \sin(120° + \theta) + \sin(240° + \theta) = 0$ [ঢা.'১২]

L.H.S.$= \sin\theta + \sin(120° + \theta) + \sin(240° + \theta)$

$= \sin\theta + \sin\{180° - (60° - \theta)\} + \sin\{180° + (60° + \theta)\}$

$= \sin\theta + \sin(60° - \theta) - \sin(60° + \theta)$

$= \sin\theta - \{\sin(60° + \theta) - \sin(60° - \theta)\}$

$= \sin\theta - 2\cos60°\sin\theta = \sin\theta - 2.\frac{1}{2}\sin\theta$

$= \sin\theta - \sin\theta = 0$ = R.H.S. (Proved)

3(c) $\cos70° - \cos10° + \sin40° = 0$

L.H.S. $= \cos70° - \cos10° + \sin40°$

$= 2\sin\frac{1}{2}(70° + 10°)\sin\frac{1}{2}(10° - 70°) + \sin40°$

$= 2\sin40°\sin(-30°) + \sin40°$

$= -2\sin40°.(\frac{1}{2}) + \sin40°$

$= -\sin40° + \sin40° = 0$ = R.H.S.

4(a) $\sin18° + \cos18° = \sqrt{2}\cos27°$ [ব'১১]

L.H.S.$= \sin18° + \cos18°$

$= \sin(90° - 72°) + \cos18°$

$= \cos72° + \cos18°$

$= 2\cos\frac{1}{2}(72° + 18°)\cos\frac{1}{2}(72° - 18°)$

$= 2\cos45°\cos27° = 2.\frac{1}{\sqrt{2}}\cos27°$

$= \sqrt{2}\cos27°$

4.(b) $\frac{\cos10^0 - \sin10^0}{\cos10^0 + \sin10^0} = \tan35°$

L.H.S.$= \frac{\cos10^0 - \sin10^0}{\cos10^0 + \sin10^0}$

$= \frac{\cos10^0(1 - \tan10^0)}{\cos10^0(1 + \tan10^0)} = \frac{\tan45^0 - \tan10^0}{1 + \tan45^0\tan10^0}$

$= \tan(45° - 10°) = \tan35°$ = R.H.S. (Proved)

5.(a) $\cot(A + 15°) - \tan(A - 15°)$

$$=\frac{4\cos 2A}{2\sin 2A+1}$$

L.H.S. $=\cot(A+15°)-\tan(A-15°)$

$=\frac{\cos(A+15^0)}{\sin(A+15^0)}-\frac{\sin(A-15^0)}{\cos(A-15^0)}$

$=\frac{\cos(A+15^0)\cos(A-15^0)-\sin(A+15^0)\sin(A-15^0)}{\sin(A+15^0)\cos(A-15^0)}$

$=\frac{\cos(A+15^0+A-15^0)}{\frac{1}{2}(\sin 2A+\sin 30^0)}=\frac{2\cos 2A}{\sin 2A+\frac{1}{2}}$

$=\frac{4\cos 2A}{2\sin 2A+1}=$ R.H.S. (Proved)

5(b) $(\cos\alpha+\cos\beta)^2+(\sin\alpha-\sin\beta)^2$
$=4\cos^2\frac{1}{2}(\alpha+\beta)$ [য.'১২]

L.H.S. $=(\cos\alpha+\cos\beta)^2+(\sin\alpha-\sin\beta)^2$

$=\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta+$
$\sin^2\alpha+\sin^2\beta-2\sin^2\alpha\sin\beta$

$=1+1+2(\cos\alpha\cos\beta-\sin\alpha\sin\beta)$

$=2\{1+\cos(\alpha+\beta)\}$

$=2.2\cos^2\frac{1}{2}(\alpha+\beta)$

$=4\cos^2\frac{1}{2}(\alpha+\beta)=$ R.H.S. (Prived)

6. $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$

$=2\cot^n\frac{1}{2}(A-B)$ অথবা **0** যখন **n** যথাক্রমে জোড় অথবা বিজোড় সংখ্যা ।

$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$

$=\left(\frac{2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}{2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)}\right)^n+$

$\left(\frac{2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}{2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(B-A)}\right)^n$

$=\left(\cot\frac{1}{2}(A-B)\right)^n+\left(\frac{\cos\frac{1}{2}(A-B)}{-\sin\frac{1}{2}(A-B)}\right)^n$

$=\cot^n\frac{1}{2}(A-B)+\left(-\cot\frac{1}{2}(A-B)\right)^n$

$=\cot^n\frac{1}{2}(A-B)+(-1)^n\cot^n\frac{1}{2}(A-B)$

যখন n বিজোড় সংখ্যা,

$\cot^n\frac{1}{2}(A-B)+(-1)^n\cot^n\frac{1}{2}(A-B)$

$=\cot^n\frac{1}{2}(A-B)-\cot^n\frac{1}{2}(A-B)=0$,

যখন n জোড় সংখ্যা,

$\cot^n\frac{1}{2}(A-B)+(-1)^n\cot^n\frac{1}{2}(A-B)$

$=\cot^n\frac{1}{2}(A-B)+\cot^n\frac{1}{2}(A-B)$

$=2\cot^n\frac{1}{2}(A-B)$

$\therefore\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n=$

$2\cot^n\frac{1}{2}(A-B)$ অথবা o যখন যথাক্রমে জোড় অথবা বিজোড় সংখ্যা।

7. (a) $a\cos\alpha+b\sin\alpha=a\cos\beta+b\sin\beta$ হলে দেখাও যে, $\cos^2\frac{\alpha+\beta}{2}-\sin^2\frac{\alpha+\beta}{2}=\frac{a^2-b^2}{a^2+b^2}$

দেওয়া আছে ,

$a\cos\alpha+b\sin\alpha=a\cos\beta+b\sin\beta$

$\Rightarrow a(\cos\alpha-\cos\beta)=b(\sin\beta-\sin\alpha)$

$\Rightarrow a.2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$

$=b.2\sin\frac{\beta-\alpha}{2}\cos\frac{\alpha+\beta}{2}$

$$\Rightarrow \frac{\cos\frac{\alpha+\beta}{2}}{\sin\frac{\alpha+\beta}{2}} = \frac{a}{b} \Rightarrow \frac{\cos^2\frac{\alpha+\beta}{2}}{\sin^2\frac{\alpha+\beta}{2}} = \frac{a^2}{b^2}$$

$$\Rightarrow \frac{\cos^2\frac{\alpha+\beta}{2}+\sin^2\frac{\alpha+\beta}{2}}{\cos^2\frac{\alpha+\beta}{2}-\sin^2\frac{\alpha+\beta}{2}} = \frac{a^2+b^2}{a^2-b^2}$$

[যোজন – বিয়োজন করে ।]

$$\Rightarrow \frac{1}{\cos^2\frac{\alpha+\beta}{2}-\sin^2\frac{\alpha+\beta}{2}} = \frac{a^2+b^2}{a^2-b^2}$$

$$\therefore \cos^2\left(\frac{\alpha+\beta}{2}\right) - \sin^2\left(\frac{\alpha+\beta}{2}\right) = \frac{a^2-b^2}{a^2+b^2}$$

7.(b) $\cos x = k\cos y$ হলে দেখাও যে, $\tan\frac{x+y}{2} = \frac{k-1}{k+1}\cot\frac{y-x}{2}$

প্রমাণ ঃ দেওয়া আছে , $\cos x = k\cos y$

$$\Rightarrow \frac{\cos x}{\cos y} = \frac{k}{1} \Rightarrow \frac{\cos x + \cos y}{\cos x - \cos y} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{2\cos\frac{x+y}{2}\cos\frac{y-x}{2}}{2\sin\frac{y-x}{2}\sin\frac{x+y}{2}} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\cot\frac{y-x}{2}}{\tan\frac{x+y}{2}} = \frac{k+1}{k-1}$$

$$\therefore \tan\frac{x+y}{2} = \frac{k-1}{k+1}\cot\frac{x+y}{2}$$

7(c) $\sin\theta = k\sin(\alpha-\theta)$ হলে দেখাও যে, $\tan\left(\theta-\frac{\alpha}{2}\right) = \frac{k-1}{k+1}\tan\frac{\alpha}{2}$

প্রমাণ ঃ দেওয়া আছে , $\sin\theta = k\sin(\alpha-\theta)$

$$\Rightarrow \frac{\sin\theta}{\sin(\alpha-\theta)} = \frac{k}{1}$$

$$\Rightarrow \frac{\sin\theta+\sin(\alpha-\theta)}{\sin\theta-\sin(\alpha-\theta)} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{2\sin\frac{\theta+\alpha-\theta}{2}\cos\frac{\theta-\alpha+\theta}{2}}{2\cos\frac{\theta+\alpha-\theta}{2}\sin\frac{\theta-\alpha+\theta}{2}} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\tan\frac{\alpha}{2}}{\tan(\theta-\frac{\alpha}{2})} = \frac{k+1}{k-1}$$

$$\therefore \tan\left(\theta-\frac{\alpha}{2}\right) = \frac{k-1}{k+1}\tan\frac{\alpha}{2}$$ (Showed).

7(d) $\frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)} = \frac{a}{b}$ হলে দেখাও যে, $\frac{a+b}{a-b}\sin^2(\alpha-\beta) = \sin^2(\theta+\alpha) - \sin^2(\theta+\beta)$

প্রমাণ ঃ দেওয়া আছে , $\frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)} = \frac{a}{b}$

$$\Rightarrow \frac{\tan(\theta+\alpha)+\tan(\theta+\beta)}{\tan(\theta+\alpha)-\tan(\theta+\beta)} = \frac{a+b}{a-b}$$

[যোজন – বিয়োজন করে ।]

$$\Rightarrow \frac{\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)}+\frac{\sin(\theta+\beta)}{\cos(\theta+\beta)}}{\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)}-\frac{\sin(\theta+\beta)}{\cos(\theta+\beta)}} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cos(\theta+\beta)+\sin(\theta+\beta)\cos(\theta+\alpha)}{\sin(\theta+\alpha)\cos(\theta+\beta)-\sin(\theta+\beta)\cos(\theta+\alpha)} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin\{(\theta+\alpha)+(\theta+\beta)\}}{\sin\{(\theta+\alpha)-(\theta+\beta)\}} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{a+b}{a-b}\sin(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}$$

$$\Rightarrow \frac{a+b}{a-b}\sin^2(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}\sin\{(\theta+\alpha)-(\theta+\beta)\}$$

$$\therefore \frac{a+b}{a-b}\sin^2(\alpha-\beta) = \sin^2(\theta+\alpha) - \sin^2(\theta+\beta)$$

[$\because \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$]

৪. $\dfrac{x}{\tan(\theta+\alpha)} = \dfrac{y}{\tan(\theta+\beta)} = \dfrac{z}{\tan(\theta+\gamma)}$ হলে দেখাও যে, $\dfrac{x+y}{x-y}\sin^2(\alpha-\beta) + \dfrac{y+z}{y-z}\sin^2(\beta-\gamma) + \dfrac{z+x}{z-x}\sin^2(\gamma-\alpha) = 0$

প্রমাণ ঃ দেওয়া আছে,

$$\frac{x}{\tan(\theta+\alpha)} = \frac{y}{\tan(\theta+\beta)} = \frac{z}{\tan(\theta+\gamma)}$$

১ম ও ২য় অনুপাত হতে পাই,

$$\frac{x}{\tan(\theta+\alpha)} = \frac{y}{\tan(\theta+\beta)}$$

$$\Rightarrow \frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)} = \frac{x}{y}$$

$$\Rightarrow \frac{\tan(\theta+\alpha)+\tan(\theta+\beta)}{\tan(\theta+\alpha)-\tan(\theta+\beta)} = \frac{x+y}{x-y}$$

[যোজন – বিয়োজন করে ।]

$$\Rightarrow \frac{\dfrac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} + \dfrac{\sin(\theta+\beta)}{\cos(\theta+\beta)}}{\dfrac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} - \dfrac{\sin(\theta+\beta)}{\cos(\theta+\beta)}} = \frac{x+y}{x-y}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cos(\theta+\beta)+\sin(\theta+\beta)\cos(\theta+\alpha)}{\sin(\theta+\alpha)\cos(\theta+\beta)-\sin(\theta+\beta)\cos(\theta+\alpha)} = \frac{x+y}{x-y}$$

$$\Rightarrow \frac{\sin\{(\theta+\alpha)+(\theta+\beta)\}}{\sin\{(\theta+\alpha)-(\theta+\beta)\}} = \frac{x+y}{x-y}$$

$$\Rightarrow \frac{x+y}{x-y}\sin(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}$$

$$\Rightarrow \frac{x+y}{x-y}\sin^2(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}\ \sin\{(\theta+\alpha)-(\theta+\beta)\}$$

$$\therefore \frac{x+y}{x-y}\sin^2(\alpha-\beta) = \sin^2(\theta+\alpha)-\sin^2(\theta+\beta)$$

অনুরূপভাবে, $\dfrac{y}{\tan(\theta+\beta)} = \dfrac{z}{\tan(\theta+\gamma)}$

$$\Rightarrow \frac{y+z}{y-z}\sin^2(\beta-\gamma) = \sin^2(\theta+\beta)-\sin^2(\theta+\gamma)$$

এবং $\dfrac{z}{\tan(\theta+\gamma)} = \dfrac{x}{\tan(\theta+\alpha)}$

$$\Rightarrow \frac{z+x}{z-x}\sin^2(\gamma-\alpha) = \sin^2(\theta+\gamma)-\sin^2(\theta+\alpha)$$

$$\therefore \frac{x+y}{x-y}\sin^2(\alpha-\beta) + \frac{y+z}{y-z}\sin^2(\beta-\gamma) + \frac{z+x}{z-x}\sin^2(\gamma-\alpha) = \sin^2(\theta+\alpha)-\sin^2(\theta+\beta)+\sin^2(\theta+\beta)-\sin^2(\theta+\gamma)+\sin^2(\theta+\gamma)-\sin^2(\theta+\alpha)=0$$

9. (a) $\sin A + \cos A = \sin B + \cos B$ হলে দেখাও যে, $A + B = \dfrac{\pi}{2}$ [সি.'০৯; চ.,দি.'১০; কু.'১২]

প্রমাণঃ দেওয়া আছে, $\sin A + \cos A = \sin B + \cos B$

$$\Rightarrow \sin A - \sin B = \cos B - \cos A$$

$$\Rightarrow 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B) = 2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\Rightarrow \frac{\sin\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A+B)} = 1$$

$$\Rightarrow \tan\frac{1}{2}(A+B) = \tan\frac{\pi}{4} \Rightarrow \frac{1}{2}(A+B) = \frac{\pi}{4}$$

$$\therefore A+B = \frac{\pi}{2}$$

9(b) $\sin\theta+\sin\varphi = a$ এবং $\cos\theta + \cos\varphi = b$ হলে দেখাও যে, $\tan\dfrac{\theta-\varphi}{2} = \pm\sqrt{\dfrac{4-a^2-b^2}{a^2+b^2}}$

প্রমাণ ঃ দেওয়া আছে, $\sin\theta + \sin\varphi = a$

$$\Rightarrow 2\sin\frac{1}{2}(\theta+\varphi)\cos\frac{1}{2}(\theta-\varphi) = a$$

উভয় পক্ষকে বর্গ করে আমরা পাই,

$$4\sin^2\frac{1}{2}(\theta+\varphi)\cos^2\frac{1}{2}(\theta-\varphi) = a^2 \cdots (1)$$

এবং $\cos\theta + \cos\varphi = b$

$$\Rightarrow 2\cos\frac{1}{2}(\theta+\varphi)\cos\frac{1}{2}(\theta-\varphi) = b$$

উভয় পক্ষকে বর্গ করে আমরা পাই ,

$$4\cos^2\frac{1}{2}(\theta+\varphi)\cos^2\frac{1}{2}(\theta-\varphi)=b^2\cdots(2)$$

(1) ও (2) যোগ করে আমরা পাই ,

$$4\cos^2\frac{1}{2}(\theta-\varphi)\{\sin^2\frac{1}{2}(\theta+\varphi)+\cos^2\frac{1}{2}(\theta+\varphi)\}=a^2+b^2$$

$$\Rightarrow \cos^2\frac{1}{2}(\theta-\varphi)=\frac{a^2+b^2}{4}$$

$$\Rightarrow \sec^2\frac{1}{2}(\theta-\varphi)=\frac{4}{a^2+b^2}$$

$$\Rightarrow 1+\tan^2\frac{1}{2}(\theta-\varphi)=\frac{4}{a^2+b^2}$$

$$\Rightarrow \tan^2\frac{1}{2}(\theta-\varphi)=\frac{4}{a^2+b^2}-1$$

$$=\frac{4-a^2-b^2}{a^2+b^2}$$

$$\therefore \tan\frac{1}{2}(\theta-\varphi)=\pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

9.(c) cosec A + sec A = cosec B + sec B হলে দেখাও যে, tan A tan B = cot $\frac{A+B}{2}$

প্রমাণ ঃ দেওয়া আছে ,

$$\text{cosec } A+\sec A=\text{cosec } B+\sec B$$

$$\Rightarrow \text{cosec } A-\text{cosec } B=\sec B-\sec A$$

$$\Rightarrow \frac{1}{\sin A}-\frac{1}{\sin B}=\frac{1}{\cos B}-\frac{1}{\cos A}$$

$$\Rightarrow \frac{\sin B-\sin A}{\sin A\sin B}=\frac{\cos A-\cos B}{\cos A\cos B}$$

$$\Rightarrow \frac{\sin B-\sin A}{\cos A-\cos B}=\frac{\sin A\sin B}{\cos A\cos B}$$

$$\Rightarrow \frac{2\cos\frac{A+B}{2}\sin\frac{B-A}{2}}{2\sin\frac{A+B}{2}\sin\frac{B-A}{2}}=\tan A\tan B$$

$$\therefore \tan A\tan B=\cot\left(\frac{A+B}{2}\right)$$

সম্ভাব্য ধাপসহ সমস্যা :

প্রমাণ কর যে,

10(a) cos 10° cos 50° cos70° = $\frac{\sqrt{3}}{8}$

$$\text{L.H.S}=\cos10^\circ\cos50^\circ\cos70^\circ$$

$$=\cos10^\circ\{\cos(60^\circ-10^0)\cos(60^\circ+10^0)\}$$

$$=\cos10^\circ(\cos^2 10^0-\sin^2 60^\circ)$$ (১)

$$=\cos10^\circ(\cos^2 10^0-\frac{3}{4})$$

$$=\frac{1}{4}(4\cos^3 10^0-3\cos10^\circ)$$

$$=\frac{1}{4}\cos(3\times10^0)$$ (১)

$$[\because \cos3A=4\cos^3A-3\cos A]$$

$$=\frac{1}{4}\cos30^0=\frac{1}{4}\times\frac{\sqrt{3}}{4}$$

$$=\frac{\sqrt{3}}{8}=\text{R.H.S. (Proved)}$$ (১)

10.(b) sin20° sin40° sin60° sin80° = $\frac{3}{16}$

$$\text{L.H.S}=\sin20^\circ\sin40^\circ\sin60^\circ\sin80^\circ$$

$$=\frac{1}{2}\{\cos(40^\circ-20^\circ)-\cos(40^\circ+20^\circ)\}.\frac{\sqrt{3}}{2}.\sin80^\circ$$ (১)

$$=\frac{\sqrt{3}}{4}(\cos20^\circ-\cos60^\circ)\sin(90^\circ-10^\circ)$$

$$=\frac{\sqrt{3}}{4}(\cos20^\circ-\frac{1}{2})\cos10^\circ$$ (১)

$$=\frac{\sqrt{3}}{4}\cos20^\circ\cos10^\circ-\frac{\sqrt{3}}{8}\cos10^\circ$$

$$=\frac{\sqrt{3}}{4}\frac{1}{2}\{\cos(20^\circ-10^\circ)+\cos(20^\circ+10^\circ)\}-\frac{\sqrt{3}}{8}\cos10^\circ$$ (১)

$$=\frac{\sqrt{3}}{8}\cos10^\circ+\frac{\sqrt{3}}{8}\cos30^\circ-\frac{\sqrt{3}}{8}\cos10^\circ$$

$$=\frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2}=\frac{3}{16}=\text{R.H.S. (Proved)}$$ (১)

10(c) $\cos 10° \cos 30° \cos 50° \cos 70° = \frac{3}{16}$

L.H.S. $= \cos10°\cos30°\cos50°\cos70°$

$= \cos10°.\frac{\sqrt{3}}{2}\frac{1}{2}\{\cos(70° + 50°) + \cos(70° - 50°)\}$ (১)

$= \frac{\sqrt{3}}{4}.\cos10°\cos120° + \frac{\sqrt{3}}{4}\cos20°\cos10°$

$= \frac{\sqrt{3}}{4}\cos10°.(-\frac{1}{2}) + \frac{\sqrt{3}}{4}.\frac{1}{2}\{\cos(20°+10°) + \cos(20°-10°)\}$

$= -\frac{\sqrt{3}}{8}\cos10° + \frac{\sqrt{3}}{8}\cos30° + \frac{\sqrt{3}}{8}\cos10°$

$= \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2} = \frac{3}{16}$ = R.H.S. (Proved) (১)

11(a) $4\cos\theta\cos(\frac{2\pi}{3}+\theta)\cos(\frac{4\pi}{3}+\theta) = \cos 3\theta$

L.H.S. $= 4\cos\theta\cos(\frac{2\pi}{3}+\theta)\cos(\frac{4\pi}{3}+\theta)$

$= 4\cos\theta.\frac{1}{2}\{\cos(\frac{4\pi}{3}+\frac{2\pi}{3}+2\theta) + \cos(\frac{4\pi}{3}-\frac{2\pi}{3})\}$ (১)

$= 2\cos\theta\{\cos(2\pi + 2\theta) + \cos\frac{2\pi}{3}\}$

$= 2\cos\theta\cos2\theta + 2\cos\theta(-\frac{1}{2})$ (১)

$= \cos(2\theta+\theta) + \cos(2\theta-\theta) - \cos\theta$

$= \cos3\theta + \cos\theta - \cos\theta$

$= \cos3\theta =$ R.H.S. (Proved) (১)

11(b) $\sin(45°+A)\sin(45°-A) = \frac{1}{2}\cos2A$

L.H.S. $= \sin(45° + A)\sin(45° - A)$

$= \frac{1}{2}\{\cos(45°+A-45°+A) - \cos(45°+A+45°-A)\}$ (১)

$= \frac{1}{2}(\cos2A - \cos90°) = \frac{1}{2}(\cos2A - 0)$

$= \frac{1}{2}\cos2A$ = R.H.S. (Proved) (১)

11(c) $4\cos\frac{B+C}{2}\cos\frac{C+A}{2}\cos\frac{A+B}{2}$
$= \cos A + \cos B + \cos C + \cos(A+B+C)$

L.H.S. $= 4\cos\frac{B+C}{2}\cos\frac{C+A}{2}\cos\frac{A+B}{2}$

$= 2\{\cos\frac{1}{2}(B+C+C+A) + \cos\frac{1}{2}(B+C-C-A)\}\cos\frac{A+B}{2}$ (১)

$= 2\cos\frac{1}{2}(B+2C+A)\cos\frac{A+B}{2} + 2\cos\frac{1}{2}(B-A)\cos\frac{A+B}{2}$

$= \cos\frac{1}{2}(A+B+2C+A+B) + \cos\frac{1}{2}(A+B+2C-A-B) + \cos\frac{1}{2}(B-A+A+B) + \cos\frac{1}{2}(B-A-A-B)$

$= \cos(A+B+C) + \cos C + \cos B + \cos(-A)$

$= \cos A + \cos B + \cos C + \cos(A+B+C)$

= R.H.S. (Proved) (১)

12(a) $\sin\theta + \sin(60°-\theta) - \sin(60°+\theta) = 0$

L.H.S. $= \sin\theta + \sin(60°-\theta) - \sin(60°+\theta)$

$= \sin\theta - \{\sin(60°+\theta) - \sin(60°-\theta)\}$

$= \sin\theta - 2\sin\theta\cos60°$ (১)

$= \sin\theta - 2(\frac{1}{2})\sin\theta$

$= \sin\theta - \sin\theta = 0 =$ R.H.S. (Proved) (১)

(b) $\cos 40° + \cos 80° + \cos 160° = 0$

L.H.S. $= \cos40° + \cos80° + \cos160°$

$= \cos40° + 2\cos\frac{1}{2}(160° + 80°)$

$\cos\frac{1}{2}(160° - 80°)$ (১)

$= \cos 40° + 2\cos 120° \cos 40°$

$= \cos 40° + 2(-\frac{1}{2})\cos 40°$

$= \cos 40° - \cos 40° = 0 =$ R.H.S. (১)

13. $\sin 65° + \cos 65° = \sqrt{2}\cos 20°$

L.H.S. $= \sin 65° + \cos 65°$

$= \sin 65° + \cos(90° - 25°)$

$= \sin 65° + \sin 25°$ (১)

$= 2\sin\frac{1}{2}(65° + 25°)\cos(65° - 25°)$ (১)

$= 2\sin 45° \cos 20° = 2.\frac{1}{\sqrt{2}}\cos 20°$

$= \sqrt{2}\cos 20° =$ R.H.S. (Proved) (১)

14.(a) $\tan(\frac{\pi}{6} + \theta)\tan(\frac{\pi}{6} - \theta) = \frac{2\cos 2\theta - 1}{2\cos 2\theta + 1}$

L.H.S. $= \tan(\frac{\pi}{6} + \theta)\tan(\frac{\pi}{6} - \theta)$

$$= \frac{\sin(\frac{\pi}{6} + \theta)\sin(\frac{\pi}{6} - \theta)}{\cos(\frac{\pi}{6} + \theta)\cos(\frac{\pi}{6} - \theta)}$$

$$= \frac{2\sin(\frac{\pi}{6} + \theta)\sin(\frac{\pi}{6} - \theta)}{2\cos(\frac{\pi}{6} + \theta)\cos(\frac{\pi}{6} - \theta)}$$

$$= \frac{\cos(\frac{\pi}{6} + \theta - \frac{\pi}{6} + \theta) - \cos(\frac{\pi}{6} + \theta + \frac{\pi}{6} - \theta)}{\cos(\frac{\pi}{6} + \theta - \frac{\pi}{6} + \theta) + \cos(\frac{\pi}{6} + \theta + \frac{\pi}{6} - \theta)}$$ (১)+(১)

$$= \frac{\cos 2\theta - \cos\frac{\pi}{3}}{\cos 2\theta + \cos\frac{\pi}{3}} = \frac{\cos 2\theta - \frac{1}{2}}{\cos 2\theta + \frac{1}{2}}$$

$= \frac{2\cos 2\theta - 1}{2\cos 2\theta + 1} =$ R.H.S. (Proved) (১)

14.(b) $\sin(\alpha + \beta + \gamma) + \sin(\alpha - \beta - \gamma) + \sin(\alpha + \beta - \gamma) + \sin(\alpha - \beta + \gamma) = 4\sin\alpha\cos\beta\cos\gamma$

L.H.S. $= \sin(\alpha + \beta + \gamma) + \sin(\alpha - \beta - \gamma) + \sin(\alpha + \beta - \gamma) + \sin(\alpha - \beta + \gamma)$

$= \sin\{\alpha + (\beta + \gamma)\} + \sin\{\alpha - (\beta + \gamma)\} + \sin\{\alpha + (\beta - \gamma)\} + \sin\{\alpha - (\beta - \gamma)\}$

$= 2\sin\alpha\cos(\beta + \gamma) + 2\sin\alpha\cos(\beta - \gamma)$ (১)

$= 2\sin\alpha\{\cos(\beta + \gamma) + \cos(\beta - \gamma)\}$

$= 2\sin\alpha .2\cos\beta\cos\gamma$

$= 4\sin\alpha\cos\beta\cos\gamma =$ R.H.S. (Prived) (১)

15. $\sin x = k\sin y$ হলে দেখাও যে,

$\tan\frac{x - y}{2} = \frac{k - 1}{k + 1}\tan\frac{x + y}{2}$ [প্র.ভ.প.'১৭]

প্রমাণ ঃ দেওয়া আছে , $\sin x = k\sin y$

$\Rightarrow \frac{\sin x}{\sin y} = \frac{k}{1} \Rightarrow \frac{\sin x + \sin y}{\sin x - \sin y} = \frac{k + 1}{k - 1}$

$$\Rightarrow \frac{2\sin\frac{x + y}{2}\cos\frac{x - y}{2}}{2\sin\frac{x - y}{2}\cos\frac{x + y}{2}} = \frac{k + 1}{k - 1}$$ (১)

$$\Rightarrow \frac{\tan\frac{x + y}{2}}{\tan\frac{x - y}{2}} = \frac{k + 1}{k - 1}$$

$\therefore \tan\frac{x - y}{2} = \frac{k - 1}{k + 1}\tan\frac{x + y}{2}$ (১)

16. $x\sin\varphi = y\sin(2\theta + \varphi)$ হলে দেখাও যে,

$\cot(\theta + \varphi) = \frac{x - y}{x + y}\cot\theta$

প্রমাণ ঃ দেওয়া আছে , $x\sin\varphi = y\sin(2\theta + \varphi)$

$\Rightarrow \frac{\sin(2\theta + \varphi)}{\sin\varphi} = \frac{x}{y}$

$\Rightarrow \frac{\sin(2\theta + \varphi) - \sin\varphi}{\sin(2\theta + \varphi) + \sin\varphi} = \frac{x - y}{x + y}$

$$\Rightarrow \frac{2\cos\frac{2\theta + \varphi + \varphi}{2}\sin\frac{2\theta + \varphi - \varphi}{2}}{2\sin\frac{2\theta + \varphi + \varphi}{2}\cos\frac{2\theta + \varphi - \varphi}{2}} = \frac{x - y}{x + y}$$ (১)

$\Rightarrow \frac{\cot(\theta + \varphi)}{\cot\theta} = \frac{x - y}{x + y}$

$\therefore \cot(\theta + \varphi) = \frac{x - y}{x + y} \cot\theta$ (Showed) (১)

সৃজনশীল প্রশ্ন:

17. $x\cos\alpha + y\sin\alpha = k = x\cos\beta + y\sin\beta$ …… (i)

$A = \sin\frac{\pi}{16}.\sin\frac{3\pi}{16}.\sin\frac{5\pi}{16}.\sin\frac{7\pi}{16}$ ……(ii)

(a) প্রমাণ কর যে,

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

প্রমাণ : L.H.S.

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{1}{2}(\frac{5\pi}{13} + \frac{3\pi}{13}) \cos\frac{1}{2}(\frac{5\pi}{13} - \frac{3\pi}{13})$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos(\pi - \frac{9\pi}{13})\cos\frac{\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} - 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}$

$= 0 =$ R.H.S. (Proved)

(b) (ii) এর সাহায্যে A এর মান নির্ণয় কর।

সমাধান: $A = \sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$

$= \frac{1}{4}(2\sin\frac{7\pi}{16}\sin\frac{\pi}{16})(2\sin\frac{5\pi}{16}\sin\frac{3\pi}{16})$

$= \frac{1}{4}\{\cos(\frac{7\pi}{16} - \frac{\pi}{16}) - \cos(\frac{7\pi}{16} + \frac{\pi}{16})\} \{\cos(\frac{5\pi}{16} - \frac{3\pi}{16}) - \cos(\frac{5\pi}{16} + \frac{3\pi}{16})\}$

$= \frac{1}{4}(\cos\frac{3\pi}{8} - \cos\frac{\pi}{2})(\cos\frac{\pi}{8} - \cos\frac{\pi}{2})$

$= \frac{1}{4}\{\cos(\frac{\pi}{2} - \frac{\pi}{8}) - 0\}(\cos\frac{\pi}{8} - 0)$

$= \frac{1}{4}\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \frac{1}{8}\sin 2.\frac{\pi}{8}$

$= \frac{1}{8}\sin\frac{\pi}{4} = \frac{1}{8}.\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{16}$ (Ans.)

(c) (i) এর সাহায্যে দেখাও যে,

$$\frac{x}{\cos\frac{1}{2}(\alpha+\beta)} = \frac{y}{\sin\frac{1}{2}(\alpha+\beta)} = \frac{k}{\cos\frac{1}{2}(\alpha-\beta)}$$

প্রমাণ : দেওয়া আছে ,

$x\cos\alpha + y\sin\alpha - k = 0$………(1)

$x\cos\beta + y\sin\beta - k = 0$………(2)

বজ্রগুণন প্রক্রিয়ায় সাহায্যে (1) ও (2) হতে আমরা পাই ,

$$\frac{x}{-\sin\alpha + \sin\beta} = \frac{y}{-\cos\beta + \cos\alpha} = \frac{k}{\cos\alpha\sin\beta - \sin\alpha\cos\beta}$$

$$\Rightarrow \frac{x}{2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)} = \frac{y}{2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)} = \frac{k}{\sin(\beta-\alpha)}$$

$$\Rightarrow \frac{x}{2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)} = \frac{y}{2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)}$$ উ: $\sqrt{2}/16$

$$= \frac{k}{2\sin\frac{1}{2}(\beta-\alpha)\cos\frac{1}{2}(\beta-\alpha)}$$

$$\therefore \frac{x}{\cos\frac{1}{2}(\alpha+\beta)} = \frac{y}{\sin\frac{1}{2}(\alpha+\beta)} = \frac{k}{\cos\frac{1}{2}(\alpha-\beta)}$$

প্রশ্নমালা –VII D

প্রমাণ কর যে,

1. (a) $\frac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$

L.H.S.$= \frac{1+\cos 2\theta}{\sin 2\theta} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta}$