$\therefore \cot(\theta+\varphi) = \frac{x-y}{x+y}\cot\theta$ (Showed) (১)

সৃজনশীল প্রশ্ন:

17. $x\cos\alpha + y\sin\alpha = k = x\cos\beta + y\sin\beta$ ……(i)

$A = \sin\frac{\pi}{16}.\sin\frac{3\pi}{16}.\sin\frac{5\pi}{16}.\sin\frac{7\pi}{16}$ ……(ii)

(a) প্রমাণ কর যে,

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$$

প্রমাণ : L.H.S.

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{1}{2}(\frac{5\pi}{13}+\frac{3\pi}{13})\cos\frac{1}{2}(\frac{5\pi}{13}-\frac{3\pi}{13})$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos(\pi-\frac{9\pi}{13})\cos\frac{\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}-2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}$

$= 0 =$ R.H.S. (Proved)

(b) (ii) এর সাহায্যে A এর মান নির্ণয় কর।

সমাধান: $A = \sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$

$= \frac{1}{4}(2\sin\frac{7\pi}{16}\sin\frac{\pi}{16})(2\sin\frac{5\pi}{16}\sin\frac{3\pi}{16})$

$= \frac{1}{4}\{\cos(\frac{7\pi}{16}-\frac{\pi}{16})-\cos(\frac{7\pi}{16}+\frac{\pi}{16})\}\{\cos(\frac{5\pi}{16}-\frac{3\pi}{16})-\cos(\frac{5\pi}{16}+\frac{3\pi}{16})\}$

$= \frac{1}{4}(\cos\frac{3\pi}{8}-\cos\frac{\pi}{2})(\cos\frac{\pi}{8}-\cos\frac{\pi}{2})$

$= \frac{1}{4}\{\cos(\frac{\pi}{2}-\frac{\pi}{8})-0\}(\cos\frac{\pi}{8}-0)$

$= \frac{1}{4}\sin\frac{\pi}{8}\cos\frac{\pi}{8}=\frac{1}{8}\sin 2.\frac{\pi}{8}$

$= \frac{1}{8}\sin\frac{\pi}{4}=\frac{1}{8}.\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{16}$ (Ans.)

(c) (i) এর সাহায্যে দেখাও যে,

$$\frac{x}{\cos\frac{1}{2}(\alpha+\beta)}=\frac{y}{\sin\frac{1}{2}(\alpha+\beta)}=\frac{k}{\cos\frac{1}{2}(\alpha-\beta)}$$

প্রমাণ : দেওয়া আছে ,

$x\cos\alpha + y\sin\alpha - k = 0$………(1)

$x\cos\beta + y\sin\beta - k = 0$………(2)

বজ্রগুণন প্রক্রিয়ায় সাহায্যে (1) ও (2) হতে আমরা পাই ,

$\frac{x}{-\sin\alpha+\sin\beta}=\frac{y}{-\cos\beta+\cos\alpha}$

$=\frac{k}{\cos\alpha\sin\beta-\sin\alpha\cos\beta}$

$\Rightarrow \frac{x}{2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)}$

$=\frac{y}{2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)}=\frac{k}{\sin(\beta-\alpha)}$

$\Rightarrow \frac{x}{2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)}$

$=\frac{y}{2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)}$ উ: $\sqrt{2}/16$

$=\frac{k}{2\sin\frac{1}{2}(\beta-\alpha)\cos\frac{1}{2}(\beta-\alpha)}$

$\therefore \frac{x}{\cos\frac{1}{2}(\alpha+\beta)}=\frac{y}{\sin\frac{1}{2}(\alpha+\beta)}=\frac{k}{\cos\frac{1}{2}(\alpha-\beta)}$

প্রশ্নমালা –VII D

প্রমাণ কর যে,

1. (a) $\frac{1+\cos 2\theta}{\sin 2\theta}=\cot\theta$

L.H.S.$=\frac{1+\cos 2\theta}{\sin 2\theta}=\frac{2\cos^2\theta}{2\sin\theta\cos\theta}=\frac{\cos\theta}{\sin\theta}$

$= \cot\theta =$ R.H.S. (proved)

1(b) $\sin 2x \tan 2x = \dfrac{4\tan^2 x}{1-\tan^4 x}$

L.H.S. $= \sin 2x \tan 2x$

$= \dfrac{2\tan x}{1+\tan^2 x} \times \dfrac{2\tan x}{1-\tan^2 x}$

$= \dfrac{4\tan^2 x}{1-\tan^4 x} =$ R.H.S. (proved)

2.(a) $4(\sin^3 10° + \cos^3 20°) = 3(\sin 10° + \cos 20°)$

L.H.S. $= 4(\sin^3 10° + \cos^3 20°)$

$= 4\sin^3 10° + 4\cos^3 20°$

$= 3\sin 10° - \sin(3.10°) + \cos(3.20°) + 3\cos 20°$

$= 3(\sin 10° + \cos 20°) - \sin 30° + \cos 60°$

$= 3(\sin 10° + \sin 20°) - \dfrac{1}{2} + \dfrac{1}{2}$

$= 3(\sin 10° + \cos 20°) =$ R.H.S. (Proved)

(b) $\sin^2(60° + A) + \sin^2 A + \sin^2(60° - A) = \dfrac{3}{2}$

L.H.S. $= \sin^2(60° + A) + \sin^2 A + \sin^2(60 - A)$

$= \dfrac{1}{2}\{1 - \cos 2(60° + A) + 1 - \cos 2A + 1 - \cos 2(60° - A)\}$

$= \dfrac{1}{2}\{3 - \cos(120° + 2A) - \cos(120° - 2A) - \cos 2A\}$

$= \dfrac{1}{2}[3 - \{\cos(120° + 2A) + \cos(120° - 2A)\} - \cos 2A]$

$= \dfrac{1}{2}\{3 - 2.\cos 120° \cos 2A - \cos 2A\}$

$= \dfrac{1}{2}\{3 - 2(-\dfrac{1}{2})\cos 2A - \cos 2A\}$

$= \dfrac{1}{2}\{3 + \cos 2A - \cos 2A\} = \dfrac{3}{2} =$ R.H.S.

2(c) $\cos^2(A - 120°) + \cos^2 A + \cos^2(A + 120°) = 3/2$ [ঢা.'০৩; কু.'০৭; য.'০৮]

L.H.S. $= \cos^2(A - 120°) + \cos^2 A + \cos^2(A + 120°)$

$= \dfrac{1}{2}\{1 + \cos 2(A - 120°) + 1 + \cos 2A + 1 + \cos 2(A + 120°)\}$

$= \dfrac{1}{2}\{3 + \cos(2A - 240°) + \cos(2A + 240°) + \cos 2A\}$

$= \dfrac{1}{2}\{3 + 2\cos 2A \cos 240° + \cos 2A\}$

$= \dfrac{1}{2}\{3 + 2\cos 2A \cos(180° + 60°) + \cos 2A\}$

$= \dfrac{1}{2}\{3 + 2\cos 2A(-\cos 60°) + \cos 2A\}$

$= \dfrac{1}{2}\{3 + 2.\cos 2A(-\dfrac{1}{2}) + \cos 2A\}$

$= \dfrac{1}{2}(3 - \cos 2A + \cos 2A) = \dfrac{3}{2} =$ R.H.S.

3.(a) $\cos^3 x + \cos^3(60° - x) + \cos^3(60° + x) = \dfrac{1}{4}(6\cos x - \cos 3x)$

L.H.S. $= \cos^3 x + \cos^3(60° - x) + \cos^3(60° + x)$

$= \dfrac{1}{4}\{3\cos x + \cos 3x + 3\cos(60° - x) + \cos 3(60° - x) + 3\cos(60° + x) + \cos 3(60° + x)\}$

$= \dfrac{1}{4}[3\{\cos x + \cos(60° + x) + \cos(60° - x)\} + \cos 3x + \cos(180° + 3x) + \cos(180° - 3x)]$

$= \dfrac{1}{4}[3(\cos x + 2\cos 60° \cos x) + \cos 3x - \cos 3x - \cos 3x]$

$= \dfrac{1}{4}[3(\cos x + 2.\dfrac{1}{2}\cos x) - \cos 3x]$

$= \dfrac{1}{4}(3.2\cos x - \cos 3x)$

$= \dfrac{1}{4}(6\cos x - \cos 3x) =$ R.H.S. (Proved)

(b) $\cos^3 x \cos 3x + \sin^3 x \sin 3x = \cos^3 2x$ [য.'০৩]

L.H.S.= $\cos^3 x \cos 3x + \sin^3 x \sin 3x$

$= \frac{1}{4}(\cos 3x + 3\cos x)\cos 3x +$

$\frac{1}{4}(3\sin x - \sin 3x)\sin 3x$

$= \frac{1}{4}(\cos^2 3x + 3\cos x\cos 3x +$

$3\sin x\sin 3x - \sin^2 3x)$

$= \frac{1}{4}\{\cos 2.3x + 3\cos(3x - x)\}$

$= \frac{1}{4}\{\cos 3.2x + 3\cos 2x\} = \cos^3 2x =$ R.H.S.

3. (c) $\cos^4 x = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

L.H.S. = $\cos^4 x = (\cos^2 x)^2$

$= \{\frac{1}{2}(1 + \cos 2x)\}^2$

$= \frac{1}{4}\{1 + 2\cos 2x + \cos^2 2x\}$

$= \frac{1}{4}\{1 + 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\}$

$= \frac{1}{4}\{1 + 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x)\}$

$= \frac{1}{4}\{\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x)\}$

$= \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$ = R.H.S.

3(d) $\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\sin^2 2x$

L.H.S.= $\sin^4 x + \cos^4 x$

$= (\sin^2 x)^2 + (\cos^2 x)^2$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$= 1^2 - \frac{1}{2}(2\sin x\cos x)^2 = 1 - \frac{1}{2}(\sin 2x)^2$

$= 1 - \frac{1}{2}\sin^2 2x$ = R.H.S. (Proved)

3(e) $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$= \frac{3}{2}$ [ঢা'১৫]

L.H.S.

$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$= \cos^4\frac{\pi}{8} + \cos^4(\frac{\pi}{2} - \frac{\pi}{8}) + \cos^4(\frac{\pi}{2} + \frac{\pi}{8}) +$

$\cos^4(\pi - \frac{\pi}{8})$

$= \cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}$

$= 2\{(\cos^2\frac{\pi}{8})^2 + (\sin^2\frac{\pi}{8})^2\}$

$= 2\{(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8})^2 - 2\cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\}$

$= 2\{(1)^2 - \frac{1}{2}(2\cos\frac{\pi}{8}\sin\frac{\pi}{8})^2\}$

$= 2\{1 - \frac{1}{2}(\sin\frac{\pi}{4})^2\} = 2\{1 - \frac{1}{2}(\frac{1}{\sqrt{2}})^2\}$

$= 2\{1 - \frac{1}{4} = 2 \times \frac{3}{4} = \frac{3}{2}$ = R.H.S (Proved)

4.(a) $\sec\theta = \frac{2}{\sqrt{2 + \sqrt{2 + 2\cos 4\theta}}}$ [দি.'০৯; ঢা.'১৪]

L.H.S. = $\sec\theta = \frac{1}{\cos\theta} = \frac{2}{2\cos\theta}$

$= \frac{2}{\sqrt{4\cos^2\theta}} = \frac{2}{\sqrt{2(1 + \cos 2\theta)}}$

$= \frac{2}{\sqrt{2 + 2\cos 2\theta}} = \frac{2}{\sqrt{2 + \sqrt{4\cos^2 2\theta}}}$

$= \frac{2}{\sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}} = \frac{2}{\sqrt{2 + \sqrt{2 + 2\cos 4\theta)}}}$

= R.H.S.

4.(b) $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0} = 4$ [কু.'০৬; রা.'০৭; ঢা.'০৭; চ., ব.'০৮; দি.'১১; সি.'১২; য.'১৩]

$$\text{L.H.S.} = \frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0}$$

$$= \frac{\cos 10^0 - \sqrt{3}\sin 10^0}{\sin 10^0 \cos 10^0}$$

$$= \frac{\frac{1}{2}\cos 10^0 - \frac{\sqrt{3}}{2}\sin 10^0}{\frac{1}{2}\sin 10^0 \cos 10^0}$$

$$= \frac{\cos 60^0 \cos 10^0 - \sin 60^0 \sin 10^0}{\frac{1}{4}\sin 20^0}$$

$$= \frac{4\cos(60^0 + 10^0)}{\sin(90^0 - 70^0)} = \frac{4\cos 70^0}{\cos 70^0} = 4 = \text{R.H.S.}$$

5. (a) $\tan\theta = \frac{1}{7}$ এবং $\tan\varphi = \frac{1}{3}$ হলে দেখাও যে, $\cos 2\theta = \sin 4\varphi$.

প্রমাণ ঃ দেওয়া আছে , $\tan\theta = \frac{1}{7}$, $\tan\varphi = \frac{1}{3}$.

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-(1/7)^2}{1+(1/7)^2}$$

$$= \frac{1-1/49}{1+1/49} = \frac{49-1}{49+1} = \frac{48}{50} = \frac{24}{25}$$

$$\sin 4\varphi = 2\sin 2\varphi \cos 2\varphi$$

$$= 2\frac{2\tan\varphi}{1+\tan^2\varphi}\frac{1-\tan^2\varphi}{1+\tan^2\varphi}$$

$$= \frac{4.\frac{1}{3}(1-\frac{1}{9})}{(1+\frac{1}{9})^2} = \frac{4.\frac{1}{3}.\frac{8}{9}}{(\frac{10}{9})^2} = \frac{32}{27}\times\frac{81}{100} = \frac{24}{25}$$

$\therefore\ \cos 2\theta = \sin 4\varphi$ (Showed)

5.(b) $2\tan\alpha = 3\tan\beta$ হলে প্রমাণ কর যে, $\tan(\alpha-\beta) = \frac{\sin 2\beta}{5-\cos 2\beta}$

প্রমাণ ঃ দেওয়া আছে , $2\tan\alpha = 3\tan\beta$

$$\Rightarrow \tan\alpha = \frac{3}{2}\tan\beta$$

$$\text{L.H.S.} = \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$= \frac{(\frac{3}{2}-1)\tan\beta}{1+\frac{3}{2}\tan^2\beta} = \frac{\tan\beta}{2+3\tan^2\beta}$$

$$= \frac{\frac{\sin\beta}{\cos\beta}}{2+3\frac{\sin^2\beta}{\cos^2\beta}} = \frac{\sin\beta\cos\beta}{2\cos^2\beta+3\sin^2\beta}$$

$$= \frac{2\sin\beta\cos\beta}{2.2\cos^2\beta+3.2\sin^2\beta}$$

$$= \frac{\sin 2\beta}{2(1+\cos 2\beta)+3(1-\cos 2\beta)}$$

$$= \frac{\sin 2\beta}{2+2\cos 2\beta+3-3\cos 2\beta} = \frac{\sin 2\beta}{5-\cos 2\beta}$$

= R.H.S. (Proved)

6.(a) $x = \sin\frac{\pi}{18}$ হলে দেখাও যে,

$$8x^4 + 4x^3 - 6x^2 - 2x + \frac{1}{2} = 0$$

প্রমাণ ঃ আমরা জানি, $4\sin^3\theta = 3\sin\theta - \sin 3\theta$

$$\therefore\ 4\sin^3\frac{\pi}{18} = 3\sin\frac{\pi}{18} - \sin 3\frac{\pi}{18}$$

$$\Rightarrow 4x^3 = 3x - \sin\frac{\pi}{6} \quad [\because x = \sin\frac{\pi}{18}]$$

$$\Rightarrow 4x^3 - 3x + \frac{1}{2} = 0$$

এখন, $8x^4 + 4x^3 - 6x^2 - 2x + \frac{1}{2}$

$$= 2x(4x^3 - 3x + \frac{1}{2}) + 1(4x^3 - 3x + \frac{1}{2})$$

$= 2x \times 0 + 1 \times 0 = 0$ (Showed)

6(b) প্রমাণ কর ঃ $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ [রা.'১১]

প্রমাণ : $\cos 5\theta = \cos(3\theta + 2\theta)$

$= \cos 3\theta\cos 2\theta - \sin 3\theta\sin 2\theta$

$= (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1) - (3\sin\theta - 4\sin^3\theta).\ 2\sin\theta\cos\theta$

$= 8\cos^5\theta - 6\cos^3\theta - 4\cos^3\theta + 3\cos\theta - 2\cos\theta(3\sin^2\theta - 4\sin^4\theta)$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta -$

$2\cos\theta\{3(1-\cos^2\theta) - 4(1-\cos^2\theta)^2\}$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2\cos\theta\{3 - 3\cos^2\theta - 4(1 - 2\cos^2\theta + \cos^4\theta)\}$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - (6\cos\theta - 6\cos^3\theta - 8\cos\theta + 16\cos^3\theta - 8\cos^5\theta)$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 6\cos\theta + 6\cos^3\theta + 8\cos\theta - 16\cos^3\theta + 8\cos^5\theta$

$\therefore \cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

7.(a) $\tan\alpha \tan\beta = \sqrt{\frac{a-b}{a+b}}$ হলে প্রমাণ কর যে, $(a - b\cos 2\alpha)(a - b\cos 2\beta) = a^2 - b^2$

প্রমাণ ঃ দেওয়া আছে, $\tan\alpha \tan\beta = \sqrt{\frac{a-b}{a+b}}$

$\Rightarrow \tan^2\alpha \tan^2\beta = \frac{a-b}{a+b}$

$\Rightarrow (a-b) = (a+b)\tan^2\alpha \tan^2\beta \cdots\cdots(1)$

L.H.S $= (a - b\cos 2\alpha)(a - b\cos 2\beta)$

$= \left\{a - b\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right\}\left\{a - b\frac{1-\tan^2\beta}{1+\tan^2\beta}\right\}$

$= \frac{a + a\tan^2\alpha - b + b\tan^2\alpha}{1+\tan^2\alpha} \times \frac{a + a\tan^2\beta - b + b\tan^2\beta}{1+\tan^2\beta}$

$= \frac{(a-b) + (a+b)\tan^2\alpha}{1+\tan^2\alpha} \times \frac{(a-b) + (a+b)\tan^2\beta}{1+\tan^2\beta}$

$= \frac{(a+b)\tan^2\alpha\tan^2\beta + (a+b)\tan^2\alpha}{1+\tan^2\alpha} \times \frac{(a+b)\tan^2\alpha\tan^2\beta + (a+b)\tan^2\beta}{1+\tan^2\beta}$

$= \frac{(a+b)\tan^2\alpha(\tan^2\beta+1)}{1+\tan^2\alpha} \times \frac{(a+b)\tan^2\alpha(\tan^2\beta+1)}{1+\tan^2\beta}$

$= (a+b)^2 \tan^2\alpha \tan^2\beta = (a+b)^2 . \frac{a-b}{a+b}$

$= a^2 - b^2 =$ R.H.S. (Proved)

7. (b) যদি α ও β কোণদ্বয় ধনাত্মক ও সূক্ষ্ম এবং $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$ হয়, তবে দেখাও যে, $\tan\alpha = \sqrt{2}\tan\beta$

প্রমাণ ঃ দেওয়া আছে, $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$

$\Rightarrow \frac{1}{\cos 2\alpha} = \frac{3 - \cos 2\beta}{3\cos 2\beta - 1}$

$\Rightarrow \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{3 - \cos 2\beta - 3\cos 2\beta + 1}{3 - \cos 2\beta + 3\cos 2\beta - 1}$

$\Rightarrow \frac{2\sin^2\alpha}{2\cos^2\alpha} = \frac{4(1 - \cos 2\beta)}{2(1 + \cos 2\beta)}$

$\Rightarrow \tan^2\alpha = \frac{2.2\sin^2\beta}{2\cos^2\beta} = 2\tan^2\beta$

$\therefore \tan\alpha = \sqrt{2}\tan\beta$ (Showed)

7(c) $\cos A \sin(A - \frac{\pi}{6})$ এর মান বৃহত্তম হলে A এর মান নির্ণয় কর।

সমাধান : $\cos A \sin(A - \frac{\pi}{6})$

$= \frac{1}{2}.2\cos A \cos(A - \frac{\pi}{6})$

$= \frac{1}{2}\{\sin(A + A - \frac{\pi}{6}) - \sin(A - A + \frac{\pi}{6})\}$

$= \frac{1}{2}\{\sin(2A - \frac{\pi}{6}) - \sin\frac{\pi}{6}\}$

$= \frac{1}{2}\{\sin(2A - \frac{\pi}{6}) - \frac{1}{2}\}$

ইহা বৃহত্তম হলে, $\sin(2A - \frac{\pi}{6}) = 1$

$\Rightarrow \sin(2A - \frac{\pi}{6}) = \sin\frac{\pi}{2}$

$\therefore 2A - \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow 2A = \frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi + \pi}{6}$

$\Rightarrow 2A = \frac{4\pi}{6} \quad \therefore A = \frac{\pi}{3}$ (Ans.)

সম্ভাব্য ধাপসহ সমস্যা :

প্রমাণ কর যে,

8. (a) $\tan\theta\,(1+\sec 2\theta) = \tan 2\theta$

L.H.S.= $\tan\theta\,(1+\sec 2\theta)$

$= \tan\theta\left(1+\frac{1}{\cos 2\theta}\right)$

$= \tan\theta\left(1+\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$ (১)

$= \tan\theta\left(\frac{1-\tan^2\theta+1+\tan^2\theta}{1-\tan^2\theta}\right)$

$= \frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$ = R.H.S. (proved) (১)

8.(b) $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A} = \tan A$

L.H.S.= $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A}$

$= \frac{\sin A+2\sin A\cos A}{1+\cos A+2\cos^2 A-1}$ (১)

$= \frac{\sin A(1+2\cos A)}{\cos A(1+2\cos A)} = \tan A$ = R.H.S. (১)

8. (c) $\frac{\cos^3 x+\sin^3 x}{\cos x+\sin x} = 1-\frac{1}{2}\sin 2x$

L.H.S.= $\frac{\cos^3 x+\sin^3 x}{\cos x+\sin x}$

$= \frac{(\cos x+\sin x)(\cos^2 x+\sin^2 x-\cos x\sin x)}{\cos x+\sin x}$

$= 1-\cos x\sin x = 1-\frac{1}{2}\sin 2x$ = R.H.S. (১)

9. $\frac{\tan^2(\theta+\frac{\pi}{4})-1}{\tan^2(\theta+\frac{\pi}{4})+1} = \sin 2\theta$

L.H.S.= $\frac{\tan^2(\theta+\frac{\pi}{4})-1}{\tan^2(\theta+\frac{\pi}{4})+1}$

$= -\frac{1-\tan^2(\theta+\frac{\pi}{4})}{1+\tan^2(\theta+\frac{\pi}{4})} = -\cos 2\left(\theta+\frac{\pi}{4}\right)$ (১)

$= -\cos\left(\frac{\pi}{2}+2\theta\right) = -(-\sin 2\theta)$

$= \sin 2\theta$ = R.H.S (Proved) (১)

10. $\tan\left(\alpha+\frac{\pi}{3}\right)+\tan\left(\alpha-\frac{\pi}{3}\right) = \frac{4\sin 2\alpha}{1-4\sin^2\alpha}$

L.H.S.= $\tan\left(\alpha+\frac{\pi}{3}\right)+\tan\left(\alpha-\frac{\pi}{3}\right)$

$= \frac{\sin(\alpha+\frac{\pi}{3})}{\cos(\alpha+\frac{\pi}{3})}+\frac{\sin(\alpha-\frac{\pi}{3})}{\cos(\alpha-\frac{\pi}{3})}$

$= \frac{\sin(\alpha+\frac{\pi}{3})\cos(\alpha-\frac{\pi}{3})+\cos(\alpha+\frac{\pi}{3})\sin(\alpha-\frac{\pi}{3})}{\cos(\alpha+\frac{\pi}{3})\cos(\alpha-\frac{\pi}{3})}$

$= \frac{\sin(\alpha+\frac{\pi}{3}+\alpha-\frac{\pi}{3})}{\frac{1}{2}(\cos 2\alpha+\cos 2\frac{\pi}{3})}$ (১) + (১)

$= \frac{2\sin 2\alpha}{\cos 2\alpha+(-\frac{1}{2})}$

$= \frac{4\sin 2\alpha}{2\cos 2\alpha-1} = \frac{4\sin 2\alpha}{2(1-2\sin^2\alpha)-1}$ (১)

$= \frac{4\sin 2\alpha}{1-4\sin^2\alpha}$ = R.H.S. (Proved) (১)

11. $4\cos^3 x\sin 3x+4\sin^3 x\cos 3x = 3\sin 4x$

L.H.S. = $4\cos^3\sin 3x+4\sin^3 x\cos 3x$

$= (\cos 3x+3\cos x)\sin 3x + (3\sin x-\sin 3x)\cos 3x$ (১)

$= \cos 3x\sin 3x-\sin 3x\cos 3x + 3(\sin 3x\cos x+\sin x\cos 3x)$

$= 3\sin(3x+x)$ (১)

$= 3\sin 4x$ = R.H. S **(Proved)**

12. $\tan^2\theta = 1 + 2\tan^2\varphi$ হলে দেখাও যে, $\cos 2\varphi = 1 + 2\cos 2\theta$

প্রমাণ ঃ দেওয়া আছে, $\tan^2\theta = 1 + 2\tan^2\varphi$

এখন, $1 + 2\cos 2\theta = 1 + 2\frac{1-\tan^2\theta}{1-\tan^2\theta}$ (১)

$= \frac{1+\tan^2\theta + 2 - 2\tan^2\theta}{1+\tan^2\theta} = \frac{3-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{3-1-2\tan^2\varphi}{1+1+2\tan^2\varphi} = \frac{2(1-\tan^2\varphi)}{2(1+\tan^2\varphi)}$

$= \frac{1-\tan^2\varphi}{1+\tan^2\varphi} = \cos 2\varphi$

$\therefore \cos 2\varphi = 1 + \cos 2\theta$ (Showed) (১)

বিকল্প পদ্ধতি: দেওয়া আছে, $\tan^2\theta = 1 + 2\tan^2\varphi$

$\Rightarrow \tan^2\theta - 1 = 2\tan^2\varphi$

$\Rightarrow \frac{1}{\tan^2\varphi} = \frac{2}{\tan^2\theta - 1}$

$\Rightarrow \frac{1-\tan^2\varphi}{1+\tan^2\varphi} = \frac{2-\tan^2\theta+1}{2+\tan^2\theta-1}$

$\Rightarrow \cos 2\varphi = \frac{3-\tan^2\theta}{1+\tan^2\theta}$ (১)

$= \frac{1+\tan^2\theta + 2(1-\tan^2\theta)}{1+\tan^2\theta}$

$= \frac{1+\tan^2\theta}{1+\tan^2\theta} + 2\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\therefore \cos 2\varphi = 1 + 2\cos 2\theta$ (১)

13. $\cos\alpha = \frac{1}{2}(x + \frac{1}{x})$ হলে প্রমাণ কর যে,

$\cos 2\alpha = \frac{1}{2}(x^2 + \frac{1}{x^2})$, $\cos 3\alpha = \frac{1}{2}(x^3 + \frac{1}{x^3})$, $\cos 4\alpha = \frac{1}{2}(x^4 + \frac{1}{x^4})$

প্রমাণ ঃ দেওয়া আছে, $\cos\alpha = \frac{1}{2}(x + \frac{1}{x})$

$\cos 2\alpha = 2\cos^2\alpha - 1$ (১)

$= 2.\left(\frac{1}{2}(x + \frac{1}{x})\right)^2 - 1$

$= 2.\frac{1}{4}(x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2}) - 1$

$= \frac{1}{2}(x^2 + 2 + \frac{1}{x^2} - 2) = \frac{1}{2}(x^2 + \frac{1}{x^2})$

$\therefore \cos 2\alpha = \frac{1}{2}(x^2 + \frac{1}{x^2})$ (১)

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ (১)

$= 4\left(\frac{1}{2}(x + \frac{1}{x})\right)^3 - 3.\frac{1}{2}(x + \frac{1}{x})$

$= 4.\frac{1}{8}(x^3 + 3x^2.\frac{1}{x} + 3x\frac{1}{x^2} + \frac{1}{x^3})$

$- 3.\frac{1}{2}(x + \frac{1}{x})$

$= \frac{1}{2}(x^3 + 3x + 3.\frac{1}{x} + \frac{1}{x^3} - 3x - 3.\frac{1}{x})$

$= \frac{1}{2}(x^3 + \frac{1}{x^3})$

$\therefore \cos 3\alpha = \frac{1}{2}(x^3 + \frac{1}{x^3})$ [যোজন-বিয়োজন করে] (১)

$\cos 4\alpha = \cos 2.2\alpha = 2\cos^2 2\alpha - 1$ (১)

$= 2.\{\frac{1}{2}(x^2 + \frac{1}{x^2})\}^2 - 1$

$= \frac{1}{2}(x^4 + 2.x^2.\frac{1}{x^2} + \frac{1}{x^4}) - 1$

$= \frac{1}{2}(x^4 + 2 + \frac{1}{x^4} - 2)$

$\therefore \cos 4\alpha = (x^4 + \frac{1}{x^4})$ (১)

14. $\tan\theta = \frac{\tan x + \tan y}{1 + \tan x \tan y}$ হলে দেখাও যে, $\sin 2\theta = \frac{\sin 2x + \sin 2y}{1 + \sin 2x.\sin 2y}$

প্রমাণঃ দেওয়া আছে, $\tan\theta = \frac{\tan x + \tan y}{1 + \tan x \tan y}$

$$=\frac{\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}}{1+\frac{\sin x}{\cos x}\frac{\sin y}{\cos y}}=\frac{\sin x\cos y+\sin y\cos x}{\cos x\cos y+\sin x\sin y}$$

$$\therefore \quad \tan\theta=\frac{\sin(x+y)}{\cos(x-y)}$$

$$\sin 2\theta=\frac{2\tan\theta}{1+\tan^2\theta}=\frac{2\frac{\sin(x+y)}{\cos(x-y)}}{1+\{\frac{\sin(x+y)}{\cos(x-y)}\}^2} \quad (১)$$

$$=\frac{2\sin(x+y)}{\cos(x-y)}\times\frac{\cos^2(x-y)}{\cos^2(x-y)+\sin^2(x+y)}$$

$$=\frac{2\sin(x+y)\cos(x-y)}{\frac{1}{2}\{1+\cos 2(x-y)\}+\frac{1}{2}\{1-\cos 2(x+y)\}} \quad (১)$$

$$=\frac{\sin(x+y+x-y)+\sin(x+y-x+y)}{\frac{1}{2}\{2+\cos 2(x-y)-\cos 2(x+y)\}} \quad (১)$$

$$=\frac{\sin 2x+\sin 2y}{1+\frac{1}{2}.2\sin\frac{2(x-y)+2(x+y)}{2}\sin\frac{2(x+y)-2(x-y)}{2}}$$

$$\therefore \quad \sin 2\theta=\frac{\sin 2x+\sin 2y}{1+\sin 2x+\sin 2y} \quad \text{(Showed)} \quad (১)$$

15. $\tan\theta=\frac{y}{x}$ **হলে দেখাও যে,**

$$x\cos 2\theta+y\sin 2\theta=x.$$

প্রমাণ ঃ দেওয়া আছে , $\tan\theta=\frac{y}{x}$

$$x\cos 2\theta+y\sin 2\theta$$

$$=x\frac{1-\tan^2\theta}{1+\tan^2\theta}+y\frac{2\tan\theta}{1+\tan^2\theta} \quad (১)$$

$$=x\frac{1-\frac{y^2}{x^2}}{1+\frac{y^2}{x^2}}+y\frac{2\frac{y}{x}}{1+\frac{y^2}{x^2}}$$

$$=x\frac{x^2-y^2}{x^2+y^2}+y(\frac{2y}{x}\times\frac{x^2}{x^2+y^2})$$

$$=\frac{x^3-xy^2}{x^2+y^2}+\frac{2xy^2}{x^2+y^2}$$

$$=\frac{x^3-xy^2+2xy^2}{x^2+y^2}=\frac{x(x^2+y^2)}{x^2+y^2}$$

$$\therefore \quad x\cos 2\theta+y\sin 2\theta=x \quad \text{(Showed)} \quad (১)$$

16. $\sqrt{2}\cos A=\cos B+\cos^3 B$ **এবং** $\sqrt{2}\sin A=\sin B-\sin^3 B$ **হলে দেখাও যে,** $\sin(A-B)=\pm\frac{1}{3}$.

প্রমাণ ঃ দেওয়া আছে, $\sqrt{2}\cos A=\cos B+\cos^3 B$

$$\sqrt{2}\sin A=\sin B-\sin^3 B$$

$$\sin(A-B)=\sin A\cos B-\sin B\cos A \quad (১)$$

$$=\frac{1}{\sqrt{2}}(\sin B-\sin^3 B)\cos B-\frac{1}{\sqrt{2}}\sin B(\cos B+\cos^3 B)$$

$$\Rightarrow \sqrt{2}\sin(A-B)=\sin B\cos B-\sin^3 B\cos B-\sin B\cos B-\sin B\cos^3 B$$

$$\Rightarrow \sqrt{2}\sin(A-B=-\sin B\cos B(\sin^2 B+\cos^2 B)$$

$$\Rightarrow \sqrt{2}\sin(A-B)=-\frac{1}{2}\sin 2B \quad (১)$$

$$\Rightarrow 2\sqrt{2}\sin(A-B)=-\sin 2B \quad \cdots\cdots (1)$$

$$\sqrt{2}\cos(A-B)=\sqrt{2}\cos A\cos B-\sqrt{2}\sin A\sin B$$

$$=(\cos B+\cos^3 B)\cos B-\sin B(\sin B-\sin^3 B)$$

$$=\cos^2 B+\sin^2 B+\cos^4 B-\sin^4 B$$

$$=1+(\cos^2 B+\sin^2 B)(\cos^2 B-\sin^2 B)$$

$$\therefore \sqrt{2}\cos(A-B)=1+\cos 2B \quad (১)$$

$$\Rightarrow \sqrt{2}\cos(A-B)-1=\cos 2B \quad \cdots\cdots\cdots\cdots (2)$$

(1) ও (2) বর্গ করে যোগ করলে আমরা পাই ,

$$(2\sqrt{2})^2\sin^2(A-B)+(\sqrt{2})^2\cos^2(A-B)+1-2\sqrt{2}\cos(A-B)=\sin^2 2B+\cos^2 2B$$

$$\Rightarrow 8\{1-\cos^2(A-B)\}+2\cos^2(A-B)+1-2\sqrt{2}\cos(A-B)=1$$

$$\Rightarrow 8-8\cos^2(A-B)+2\cos^2(A-B)-2\sqrt{2}\cos(A-B)=0$$

$$\Rightarrow 6\cos^2(A-B)+2\sqrt{2}\cos(A-B)-8=0$$

$$\Rightarrow 3\cos^2(A-B)+\sqrt{2}\cos(A-B)-4=0$$

$$\Rightarrow 3\cos^2(A-B)+3\sqrt{2}\cos(A-B)-2\sqrt{2}\cos(A-B)-4=0$$

$\Rightarrow 3\cos(A-B)\{\cos(A-B)+\sqrt{2}\}-2\sqrt{2}\{\cos(A-B)+\sqrt{2}\}=0$

$\Rightarrow\{\cos(A-B)+\sqrt{2}\}\{3\cos(A-B)-2\sqrt{2}\}=0$

$\therefore \cos(A-B)=-\sqrt{2}$ অথবা, $\cos(A-B)=\frac{2\sqrt{2}}{3}$ (১)

কিন্তু $-1\le\cos\theta\le1$ বলে $\cos(A-B)\ne-\sqrt{2}$ (১)

$\therefore \cos(A-B)=\frac{2\sqrt{2}}{3}$

$\therefore \sin(A-B)=\pm\sqrt{1-\cos^2(A-B)}$

$=\pm\sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2}=\pm\sqrt{1-\frac{8}{9}}$

$\therefore \sin(A-B)=\pm\sqrt{\frac{1}{9}}=\pm\frac{1}{3}$ (১)

17. দেখাও যে, $\frac{\tan 2^n\theta}{\tan\theta}=(1+\sec 2\theta)(1+\sec 2^2\theta)(1+\sec 2^3\theta)\ldots\ldots(1+\sec 2^n\theta)$

প্রমাণঃ $\tan\theta(1+\sec 2\theta)=\tan\theta\left(1+\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$ (১)

$=\tan\theta\left(\frac{1-\tan^2\theta+1+\tan^2\theta}{1-\tan^2\theta}\right)$

$=\tan\theta\cdot\frac{2}{1-\tan^2\theta}=\frac{2\tan\theta}{1-\tan^2\theta}=\tan 2\theta$ (১)

$\therefore \frac{\tan 2\theta}{\tan\theta}=1+\sec 2\theta$

অনুরূপভাবে, $\frac{\tan 2^2\theta}{\tan 2\theta}=1+\sec 2^2\theta$,

$\frac{\tan 2^3\theta}{\tan 2^2\theta}=1+\sec 2^3\theta,\ldots,$

$\frac{\tan 2^n\theta}{\tan 2^{n-1}\theta}=1+\sec 2^n\theta$ (১)

$\therefore \frac{\tan 2\theta}{\tan\theta}\cdot\frac{\tan 2^2\theta}{\tan 2\theta}\cdot\frac{\tan 2^3\theta}{\tan 2^2\theta}\ldots\ldots\frac{\tan 2^n\theta}{\tan 2^{n-1}\theta}$
$=(1+\sec 2\theta)(1+\sec 2^2\theta)(1+\sec 2^3\theta)\ldots\ldots(1+\sec 2^n\theta)$

$\Rightarrow \frac{\tan 2^n\theta}{\tan\theta}=(1+\sec 2\theta)(1+\sec 2^2\theta)(1+\sec 2^3\theta)\ldots\ldots(1+\sec 2^n\theta)$ (১)

18.(a) দেখাও যে, $\frac{2\cos 2^n\theta+1}{2\cos\theta+1}=(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 2^2\theta-1)\ldots\ldots(2\cos 2^{n-1}\theta-1)$

প্রমাণ : আমরা পাই,

$(2\cos\theta+1)(2\cos\theta-1)=4\cos^2\theta-1$

$=4\cdot\frac{1}{2}(1+\cos 2\theta)-1$ (১)

$=2+2\cos 2\theta-1$

$\therefore 2\cos\theta-1=\frac{2\cos 2\theta+1}{2\cos\theta+1}$

অনুরূপভাবে,

$2\cos 2\theta-1=\frac{2\cos 2^2\theta+1}{2\cos 2\theta+1}$

$2\cos 2^2\theta-1=\frac{2\cos 2^3\theta+1}{2\cos 2^2\theta+1}$

$2\cos 2^{n-1}\theta-1=\frac{2\cos 2^n\theta+1}{2\cos 2^{n-1}\theta+1}$ (১)

গুণ করে আমরা পাই,

$(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 2^2\theta-1)\ldots\ldots(2\cos 2^{n-1}\theta-1)$

$\frac{2\cos 2\theta+1}{2\cos\theta+1}\cdot\frac{2\cos 2^2\theta+1}{2\cos 2\theta+1}\cdot\frac{2\cos 2^3\theta+1}{2\cos 2^2\theta+1}\ldots\ldots\frac{2\cos 2^n\theta+1}{2\cos 2^{n-1}\theta+1}=\frac{2\cos 2^n\theta+1}{2\cos\theta+1}$

$\therefore \frac{2\cos 2^n\theta+1}{2\cos\theta+1}=(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 2^2\theta-1)\cdots(2\cos 2^{n-1}\theta-1)$ (১)

18.(b) $13\theta=\pi$ হলে দেখাও যে, $\cos\theta.\cos 2\theta.\cos 3\theta.\cos 4\theta.\cos 5\theta.\cos 6\theta=\frac{1}{2^6}$

প্রমাণ : $\cos\theta \cos2\theta \cos3\theta \cos4\theta \cos5\theta \cos6\theta$

আমরা জানি, $2 \sin\theta \cos\theta = \sin 2\theta$ (১)

$\Rightarrow \sin\theta \cos\theta = \frac{1}{2}\sin 2\theta$

$\therefore \sin\theta \cos\theta \cos 2\theta = \frac{1}{2}\sin 2\theta \cos 2\theta$

$= \frac{1}{2^2}\sin 4\theta$

অনুরূপভাবে, $\sin\theta \cos\theta \cos 2\theta.\cos4\theta = \frac{1}{2^3}\sin 8\theta$

$\sin\theta \cos\theta \cos 2\theta.\cos4\theta \cos 8\theta = \frac{1}{2^3}\sin 16\theta$

… … … … … … … … … … … … …

… … … …… … … … …… … … …

$\sin\theta \cos\theta \cos 2\theta.\cos4\theta \cos 8\theta \cos 16\theta$

$\cos 32\theta = \frac{1}{2^6}\sin 64\theta$ (১)

$\Rightarrow \sin\theta \cos\theta \cos 2\theta.\cos4\theta \cos(13\theta - 5\theta)$
$\cos(13\theta + 3\theta) \cos(26\theta + 6\theta)$

$= \frac{1}{2^6}\sin(65\theta - \theta)$

$\Rightarrow \sin\theta \cos\theta \cos 2\theta.\cos 4\theta \cos(\pi - 5\theta)$
$\cos(\pi + 3\theta) \cos(2\pi + 6\theta)$

$= \frac{1}{2^6}\sin(5\pi - \theta)$

$\Rightarrow \sin\theta \cos\theta \cos 2\theta.\cos 4\theta (-\cos 5\theta)$

$(-\cos 3\theta).\cos 6\theta = \frac{1}{2^6}(\sin\theta)$ (১)

$\therefore \cos\theta \cos 2\theta \cos3\theta \cos 4\theta$

$\cos 5\theta \cos 6\theta = \frac{1}{2^6}$ (Showed) (১)

18.(c) $\theta = \frac{\pi}{2^n + 1}$ **হলে প্রমাণ কর যে, $2^n \cos\theta$ $\cos 2\theta \cos 2^2\theta \cdots\cdots \cos 2^{n-1}\theta = 1$.**

প্রমাণ : দেওয়া আছে, $\theta = \frac{\pi}{2^n + 1} \Rightarrow 2^n\theta + \theta = \pi$

$\Rightarrow 2^n\theta = \pi - \theta \Rightarrow \sin 2^n\theta = \sin(\pi - \theta)$

$\Rightarrow 2\sin 2^{n-1}\theta \cos 2^{n-1}\theta = \sin\theta$ (১) + (১)

$\Rightarrow 2\cos 2^{n-1}\theta(2\sin 2^{n-2}\theta \cos 2^{n-2}\theta) = \sin\theta$

$\Rightarrow 2^2 \cos 2^{n-1}\theta \cos 2^{n-2}\theta \sin 2^{n-2}\theta = \sin\theta$

$\Rightarrow 2^n \cos 2^{n-1}\theta \cos 2^{n-2}\theta \cos 2^{n-3}\theta \cdots\cdots$
$\cdots \sin 2^{n-n}\theta \cos 2^{n-n}\theta = \sin\theta$ (১)

$\Rightarrow 2^n \cos 2^{n-1}\theta \cos 2^{n-2}\theta \cos 2^{n-3}\theta \cdots\cdots$
$\cdots \sin 2^0\theta \cos 2^0\theta = \sin\theta$

$\Rightarrow 2^n \cos 2^{n-1}\theta \cos 2^{n-2}\theta \cos 2^{n-3}\theta \cdots\cdots$
$\cdots \sin\theta \cos\theta = 1$

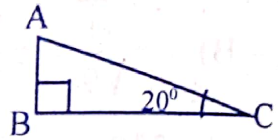
$\therefore 2^n \cos\theta. \cos 2\theta. \cos 2^2\theta \cdots \cos 2^{n-1}\theta = 1$ (১)

(Showed)

সৃজনশীল প্রশ্ন:

19. দৃশ্যকল্প -১: $A = \tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$;

দৃশ্যকল্প -২:



(a) প্রমাণ কর যে, $\sin^2\left(\frac{\pi}{8} + \frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\sin\theta$ [রা. '১১]

L.H.S. $= \sin^2\left(\frac{\pi}{8} + \frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{\theta}{2}\right)$

$= \frac{1}{2}\{1 - \cos2(\frac{\pi}{8} + \frac{\theta}{2})\} - \frac{1}{2}\{1 - \cos2(\frac{\pi}{8} - \frac{\theta}{2})\}$

$= \frac{1}{2}\{1 - \cos(\frac{\pi}{4} + \theta) - 1 + \cos(\frac{\pi}{4} - \theta)\}$

$= \frac{1}{2}\{\cos(\frac{\pi}{4} - \theta) - \cos(\frac{\pi}{4} + \theta)\}$

$= \frac{1}{2}.2\sin\frac{\pi}{4}\sin\theta = \frac{1}{\sqrt{2}}\sin\theta =$ R.H.S.

(b) দৃশ্যকল্প -১ হতে প্রমাণ কর যে, $A = \cot\theta$ [য. '০২; সি. '০৮]

প্রমাণ ঃ $4\tan 4\theta + 8\cot 8\theta$

$= 4\left(\frac{\sin 4\theta}{\cos 4\theta} + 2\frac{\cos 8\theta}{\sin 8\theta}\right)$

$= 4\left(\frac{\sin 4\theta}{\cos 4\theta} + \frac{2\cos 8\theta}{2\sin 4\theta\cos 4\theta}\right)$

$= 4(\frac{\sin^2 4\theta + 1 - 2\sin^2 4\theta}{\sin 4\theta \cos 4\theta})$

$= 4(\frac{1-\sin^2 4\theta}{\sin 4\theta \cos 4\theta}) = 4(\frac{\cos^2 4\theta}{\sin 4\theta \cos 4\theta})$

$= 4 \cot 4\theta$

অনুরূপভাবে প্রমাণ করা যায় ,

$2\tan 2\theta + 4\cot 4\theta = 2\cot 2\theta$ এবং

$\tan\theta + 2\cot 2\theta = \cot\theta$

L.H.S.= $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$

$= \tan\theta + 2\tan 2\theta + 4\cot 4\theta$

$= \tan\theta + 2\cot 2\theta = \cot\theta$ = R.H.S. (Proved)

(c) দৃশ্যকল্প -২ হতে দেখাও যে, $\sqrt{3}\frac{AC}{AB} - \frac{AC}{BC} = 4$

[ঢা.'১০;চ.'১৪,'১৫]

দৃশ্যকল্প -২ হতে পাই, $\frac{AC}{AB} = \text{cosec } 20^0$ ও

$\frac{AC}{BC} = \sec 20^0$.

$\sqrt{3}\frac{AC}{AB} - \frac{AC}{BC} = \sqrt{3}\,\text{cosec}\, 20^0 - \sec 20^0$

$= \frac{\sqrt{3}}{\sin 20^0} - \frac{1}{\cos 20^0}$

$= \frac{\sqrt{3}\cos 20^0 - \sin 20^0}{\sin 20^0 \cos 20^0}$

$= \frac{\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0}{\frac{1}{2}\sin 20^0 \cos 20^0}$

$= \frac{\cos 30^0 \cos 20^0 - \sin 30^0 \sin 10^0}{\frac{1}{4}\sin 40^0}$

$= \frac{4\cos(30^0 + 20^0)}{\sin(90^0 - 50^0)} = \frac{4\cos 50^0}{\cos 50^0} = 4$

$\therefore \sqrt{3}\frac{AC}{AB} - \frac{AC}{BC} = 4$

প্রশ্নমালা –VII E

প্রমাণ কর যে,

1. (a) $\frac{1-\sin x}{1+\sin x} = \tan^2(\frac{\pi}{4} - \frac{x}{2})$

L.H.S.= $\frac{1-\sin x}{1+\sin x}$

$= \frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{(\cos\frac{x}{2} - \sin\frac{x}{2})^2}{(\cos\frac{x}{2} + \sin\frac{x}{2})^2} = \left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)^2$

$= \left(\frac{\tan\frac{\pi}{4} - \tan\frac{x}{2}}{1+\tan\frac{\pi}{4}\tan\frac{x}{2}}\right)^2 = \tan^2(\frac{\pi}{4} - \frac{x}{2})$ = R.H.S

1. (b) $\cos^2\frac{\alpha}{2} + \cos^2(\frac{\alpha}{2} - 60°) + \cos^2(\frac{\alpha}{2} + 60°) = \frac{3}{2}$

L.H.S. $= \cos^2\frac{\alpha}{2} + \cos^2(\frac{\alpha}{2} - 60°) + \cos^2(\frac{\alpha}{2} + 60°)$

$= \frac{1}{2}\{1 + \cos 2.\frac{\alpha}{2} + 1 + \cos 2.(\frac{\alpha}{2} - 60°) + 1 + \cos 2(\frac{\alpha}{2} + 60°)\}$

$= \frac{1}{2}\{3 + \cos\alpha + \cos(\alpha - 120°) + \cos(\alpha + 120°)\}$

$= \frac{1}{2}\{3 + \cos\alpha + 2\cos\alpha\cos 120°\}$

$= \frac{1}{2}\{3 + \cos\alpha + 2\cos\alpha.(-\frac{1}{2})\}$

$= \frac{1}{2}\{3 + \cos\alpha - \cos\alpha\} = \frac{3}{2}$

1(c) $\sin^2(\frac{\alpha}{2} - 36°) + \sin^2(\frac{\alpha}{2} + 36°)$

$= \frac{1}{4}\{4 - (\sqrt{5} - 1)\cos\alpha\}$