$= \frac{1}{2}\sqrt{2+\sqrt{2(1+\cos 30^0)}}$

$= \frac{1}{2}\sqrt{2+\sqrt{2.2\cos^2 15^0)}} = \frac{1}{2}\sqrt{2+2\cos 15^0}$

$= \frac{1}{2}\sqrt{2(1+\cos 15^0)} = \frac{1}{2}\sqrt{2.2\cos^2(7\frac{1}{2})^0}$

$= \frac{1}{2}.2\cos(7\frac{1}{2})^0 = \cos(7\frac{1}{2})^0$

$\therefore \angle B = (7\frac{1}{2})^0$; যেহেতু $\angle B$ সূক্ষ্মকোণ

(c) দৃশ্যকল্প -২ হতে প্রমাণ কর যে,

$\tan(\frac{\pi}{4}-\frac{\theta}{2}) = \sqrt{\frac{b}{a}}$.

প্রমাণ : দৃশ্যকল্প-১ হতে পাই, $\sin\theta = \frac{QR}{PQ}$

$\Rightarrow \sin\theta = \frac{a-b}{a+b}$

$\therefore$ L.H.S. $= \tan(\frac{\pi}{4}-\frac{\theta}{2}) = \frac{\sin(\frac{\pi}{4}-\frac{\theta}{2})}{\cos(\frac{\pi}{4}-\frac{\theta}{2})}$

$= \frac{2\sin(\frac{\pi}{4}-\frac{\theta}{2})\cos(\frac{\pi}{4}-\frac{\theta}{2})}{2\cos^2(\frac{\pi}{4}-\frac{\theta}{2})}$

$= \frac{\sin 2(\frac{\pi}{4}-\frac{\theta}{2})}{1+\cos 2(\frac{\pi}{4}-\frac{\theta}{2})} = \frac{\sin(\frac{\pi}{2}-\theta)}{1+\cos(\frac{\pi}{2}-\theta)}$

$= \frac{\cos\theta}{1+\sin\theta} = \frac{\sqrt{1-\sin^2\theta}}{1+\sin\theta} = \frac{\sqrt{1-(\frac{a-b}{a+b})^2}}{1+\frac{a-b}{a+b}}$

$= \frac{\sqrt{(a+b)^2-(a-b)^2}}{\frac{a+b+a-b}{a+b}} = \frac{\sqrt{4ab}}{2a}$

$= \frac{2\sqrt{a}\sqrt{b}}{2a} = \sqrt{\frac{b}{a}}$ = R.H.S.

বিকল্প পদ্ধতি : দৃশ্যকল্প-১ হতে পাই, $\sin\theta = \frac{QR}{PQ}$

$\Rightarrow \sin\theta = \frac{a-b}{a+b}$

$\Rightarrow \frac{1}{\sin\theta} = \frac{a+b}{a-b} \Rightarrow \frac{1-\sin\theta}{1+\sin\theta} = \frac{a+b-a+b}{a+b+a-b}$

[বিয়োজন-যোজন করে।]

$\Rightarrow \frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\cos\frac{\theta}{2}\sin\frac{\theta}{2}} = \frac{2b}{2a}$

$\Rightarrow \frac{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2}{(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})^2} = \frac{b}{a} \Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}} = \sqrt{\frac{b}{a}}$

$\Rightarrow \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}} = \sqrt{\frac{b}{a}} \Rightarrow \frac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\theta}{2}} = \sqrt{\frac{b}{a}}$

$\therefore \tan(\frac{\pi}{4}-\frac{\theta}{2}) = \sqrt{\frac{b}{a}}$

প্রশ্নমালা VII F

$A+B+C=\pi$ হলে প্রমাণ কর যে,

1. (a) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ **[য.'০২]**

প্রমাণ : L.H.S. $= \sin A + \sin B + \sin C$

$= 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B) + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\sin(\frac{\pi}{2}-\frac{C}{2})\cos\frac{1}{2}(A-B) + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\cos\frac{1}{2}(A-B) + 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\{\cos\frac{1}{2}(A-B) + \sin\frac{C}{2}\}$

$= 2\cos\frac{C}{2}\{\cos\frac{1}{2}(A-B) + \sin(\frac{\pi}{2}-\frac{A+B}{2})\}$

$= 2\cos\frac{C}{2}\{\cos(\frac{A}{2}-\frac{B}{2}) + \cos(\frac{A}{2}+\frac{B}{2})\}$

$= 2\cos\frac{C}{2}(2\cos\frac{A}{2}\cos\frac{B}{2})$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ = R.H.S. (Proved)

1.(b) sin A + sin B – sin C = $4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$ [য.'০৮]

প্রমাণ ঃ L.H.S.= sinA + sinB – sinC

$=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$=2\sin(\frac{\pi}{2}-\frac{C}{2})\cos\frac{1}{2}(A-B) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\cos\frac{1}{2}(A-B) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\{\cos\frac{1}{2}(A-B) - \sin\frac{C}{2}\}$

$=2\cos\frac{C}{2}\{\cos\frac{1}{2}(A-B) - \sin(\frac{\pi}{2}-\frac{A+B}{2})\}$

$= 2\cos\frac{C}{2}\{\cos(\frac{A}{2}-\frac{B}{2}) - \cos(\frac{A}{2}+\frac{B}{2})\}$

$= 2\cos\frac{C}{2}(2\sin\frac{A}{2}\sin\frac{B}{2})$

$= 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$ = R.H.S. (Proved)

1. (c) sin2A – sin2B + sin2C = 4cosA sinB cosC [কু.'০১]

প্রমাণ ঃ L.H.S. = sin2A – sin2B + sin2C

$= 2\sin\frac{2A-2B}{2}\cos\frac{2A+2B}{2} + \cos 2C$

$= 2\sin(A-B)\cos(A+B) + 2\sin C\cos C$

$= 2\sin(A-B)\cos(\pi - C) + 2\sin C\cos C$

$= -2\cos C\sin(A-B) + 2\sin C\cos C$

$= 2\cos C\{\sin C - \sin(A-B)\}$

$= 2\cos C[\sin\{\pi-(A+B)\} - \sin(A-B)]$

$= 2\cos C\{\sin(A+B) - \sin(A-B)\}$

$= 2\cos C.2\sin B\cos A = 4\cos A\sin B\cos C$

= R.H.S. (Proved)

1.(d) cos2A – cos 2B + cos 2C = 1– 4sin A cos B sin C

প্রমাণ ঃ L.H.S. = cos 2A – cos 2B + cos 2C

= cos 2A + cos 2C – cos 2B

$= 2\cos(A+C)\cos(A-C) - (2\cos^2 B - 1)$

$= 2\cos(\pi - B)\cos(A-C) - 2\cos^2 B + 1$

$= -2\cos B\cos(A-C) - 2\cos^2 B + 1$

$= 1 - 2\cos B\{\cos(A-C) + \cos B\}$

$= 1 - 2\cos B[\cos(A-C) + \cos\{\pi-(A+C)\}]$

$= 1 - 2\cos B\{\cos(A-C) - \cos(A+C)\}$

$= 1 - 2\cos B.2\sin A\sin C$

$= 1 - 4\sin A\cos B\sin C$ = R.H.S. (Proved)

(e) $\frac{\cos A}{\sin B\sin C} + \frac{\cos B}{\sin C\sin A} + \frac{\cos C}{\sin A\sin B} = 2$ [চ.'১১]

প্রমাণ ঃ

L.H.S. $= \frac{\cos A}{\sin B\sin C} + \frac{\cos B}{\sin C\sin A} + \frac{\cos C}{\sin A\sin B}$

$= \frac{\cos A\sin A + \cos B\sin B + \cos C\sin C}{\sin A\sin B\sin C}$

$= \frac{\sin 2A + \sin 2B + \sin 2C}{2\sin A\sin B\sin C}$

এখন, sin 2A + sin 2B + sin 2C

$= 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$

$= 2\sin(\pi - C)\cos(A-B) + 2\sin C\cos C$

$= 2\sin C\cos(A-B) + 2\sin C\cos C$

$= 2\sin C\{\cos(A-B) + \cos C\}$

$= 2\sin C\{\cos(A-B) + \cos(\pi - \overline{A+B})\}$

$= 2\sin C\{\cos(A-B) - \cos(A+B)\}$

$= 2\sin C.2\sin A\sin B = 4\sin A\sin B\sin C$

$\therefore$ L.H.S. $= \frac{4\sin A\sin B\sin C}{2\sin A\sin B\sin C} = 2$ = R.H.S.

2.(a) sin (B + 2C) + sin (C+2A) + sin (A+2B) $= 4\sin\frac{B-C}{2}\sin\frac{C-A}{2}\sin\frac{A-B}{2}$

প্রমাণ ঃ L.H.S.= sin(B + 2C) + sin(C + 2A) + sin(A + 2B)

$= \sin\{A+B+C+(C-A)\} + \sin\{A+B+C+(A-B)\} + \sin\{A+B+C+(B-C)\}$

$= \sin\{\pi-(A-C)\} + \sin\{\pi-(B-A)\} + \sin\{\pi-(C-B)\}$

$= \sin(A-C) + \sin(B-A) + \sin(C-B)$

$= 2\sin\frac{1}{2}(A-C+B-A)\cos\frac{1}{2}(A-C-B+A)$

$- \sin(B-C)$

$= 2\sin\frac{1}{2}(B-C)\cos\frac{1}{2}(2A-B-C) -$

$2\sin\frac{1}{2}(B-C)\cos(B-C)$

$= 2\sin\frac{1}{2}(B-C)\{\cos\frac{1}{2}(2A-B-C) -$

$\cos(B-C)$

$= 2\sin\frac{B-C}{2}\{2\sin\frac{1}{2}(\frac{2A-B-C+B-C}{2})$

$\sin\frac{1}{2}(\frac{B-C-2A+B+C}{2})\}$

$= 2\sin\frac{B-C}{2}.2\sin\frac{A-C}{2}\sin\frac{B-A}{2}$

$= 4\sin\frac{B-C}{2}\sin\frac{C-A}{2}\sin\frac{A-B}{2} = \text{R.H.S.}$

2.(b) $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} =$

$4\cos\frac{\pi-A}{4}\cos\frac{\pi-B}{4}\cos\frac{\pi-C}{4}$

R.H.S.$= 4\cos\frac{\pi-A}{4}\cos\frac{\pi-B}{4}\cos\frac{\pi-C}{4}$

$= 2.2\cos\frac{B+C}{4}\cos\frac{C+A}{4}\cos\frac{A+B}{4}$

$= 2[\cos(\frac{B+C}{4}+\frac{C+A}{4})$

$+\cos(\frac{B+C}{4}-\frac{C+A}{4})]\cos\frac{A+B}{4}$

$= 2[\cos\frac{A+B+2C}{4}+\cos\frac{B-A}{4}]\cos\frac{A+B}{4}$

$= 2\cos\frac{A+B+2C}{4}\cos\frac{A+B}{4} +$

$2\cos\frac{B-A}{4}\cos\frac{A+B}{4}$

$= \cos\frac{A+B+C}{2} + \cos\frac{C}{2} + \cos\frac{B}{2} + \cos(-\frac{A}{2})$

$= \cos\frac{\pi}{2} + \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$

$= 0 + \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$

$= \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = \text{R.H.S.(Proved)}$

3.(a) $\tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} + \tan\frac{A}{2}\tan\frac{B}{2} = 1$

প্রমাণ ঃ দেওয়া আছে , $A + B + C = \pi$

$\Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2} \Rightarrow \frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\therefore \tan(\frac{A}{2}+\frac{B}{2}) = \tan(\frac{\pi}{2}-\frac{C}{2})$

$\Rightarrow \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}} = \cot\frac{C}{2} = \frac{1}{\tan\frac{C}{2}}$

$\Rightarrow \tan\frac{A}{2}\tan\frac{C}{2} + \tan\frac{B}{2}\tan\frac{C}{2} = 1 - \tan\frac{A}{2}\tan\frac{B}{2}$

$\therefore \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} + \tan\frac{A}{2}\tan\frac{B}{2} = 1$

3(b) cot B cot C + cot C cot A + cot A cot B = 1 [প্র.ভ.প.'০৫]

প্রমাণ ঃ দেওয়া আছে , $A + B + C = \pi$

$\Rightarrow A+B = \pi - C \Rightarrow \cot(A+B) = \cot(\pi - C)$

$\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = -\cot C$

$\Rightarrow \cot A \cot B - 1 = -\cot B\cot C - \cot C\cot A$

$\therefore \cot B \cot C + \cot C \cot A + \cot A \cot B = 1$

4. (a) $\sin^2 A - \sin^2 B + \sin^2 C = 2 \sin A \cos B \sin C$ [ঢা.'০২ ; চ.'০২,'১৩; সি.'০৭; রা.'১১]

প্রমাণ ঃ **L.H.S.** $= \sin^2 A - \sin^2 B + \sin^2 C$

$= \frac{1}{2}(1 - \cos 2A + 1 - \cos 2C) - \sin^2 B$

$= 1 - \sin^2 B - \frac{1}{2}.2\cos(A+C)\cos(A-C)$

$= \cos^2 B - \cos(\pi - B)\cos(A - C)$
$= \cos^2 B + \cos B \cos(A - C)$
$= \cos B \{\cos B + \cos(A - C)\}$
$= \cos B [\cos\{\pi - (A + C)\} + \cos(A - C)]$
$= \cos B [-\cos(A + C) + \cos(A - C)]$
$= \cos B .2 \sin A \sin C$
$= 2 \sin A \cos B \sin C$ = R.H.S. (Proved)

(b) $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \cos C$ [ঢা.'০৩,'০৭,'০৯; য.'০৭]

প্রমাণ ঃ **L.H.S.** $= \cos^2 A + \cos^2 B - \cos^2 C$

$= \frac{1}{2}(1 + \cos 2A + 1 + \cos 2B) - \cos^2 C$

$= 1 + \frac{1}{2}.2 \cos(A + B)\cos(A - B) - \cos^2 C$

$= 1 + \cos(\pi - C)\cos(A - B) - \cos^2 C$
$= 1 - \cos C \cos(A - B) - \cos^2 C$
$= 1 - \cos C \{\cos(A - B) + \cos C\}$
$= 1 - \cos C [\cos(A - B) + \cos\{\pi - (A + B)\}]$
$= 1 - \cos C.[\cos(A - B) - \cos(A + B)]$
$= 1 - 2 \cos C \sin A \sin B$ = R.H.S

(c) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$ [সি.'০২, '০৭; দি.'০৯; ঢা.'১১; চ.'১৩]

প্রমাণ ঃ **L.H.S.** $= \cos^2 A + \cos^2 B + \cos^2 C$

$= \frac{1}{2}(1 + \cos 2A + 1 + \cos 2B) + \cos^2 C$

$= 1 + \frac{1}{2}.2\cos(A + B)\cos(A - B) + \cos^2 C$

$= 1 + \cos(\pi - C)\cos(A - B) + \cos^2 C$
$= 1 - \cos C \cos(A - B) + \cos^2 C$
$= 1 - \cos C [\cos(A - B) - \cos C]$
$= 1 - \cos C [\cos(A - B) - \cos\{\pi - (A + B)\}]$
$= 1 - \cos C[\cos(A - B) + \cos(A + B)]$
$= 1 - \cos C. 2 \cos A \cos B$
$= 1 - 2 \cos A \cos B \cos C$ = R.H.S.

4(d) $\cos^2 2A + \cos^2 2B + \cos^2 2C = 1 + 2 \cos 2A \cos 2B \cos 2C$

প্রমাণ ঃ **L.H.S.** $= \cos^2 2A + \cos^2 2B + \cos^2 2C$

$= \frac{1}{2}[1 + \cos 4A + 1 + \cos 4B] + \cos^2 2C$

$= 1 + \frac{1}{2}.2 \cos 2(A + B)\cos 2(A - B) + \cos^2 2C$
$= 1 + \cos(2\pi - 2C)\cos 2(A - B) + \cos^2 2C$
$= 1 + \cos 2C\{\cos 2(A - B) + \cos 2C\}$
$= 1 + \cos 2C [\cos 2(A - B) + \cos\{2\pi - 2(A + B)\}]$
$= 1 + \cos 2C[\cos 2(A - B) + \cos 2(A + B)]$
$= 1 + \cos 2C.2\cos 2A \cos 2B$
$= 1 + 2\cos 2A \cos 2B \cos 2C$ = R.H.C.
(Proved)

4(e) $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 1 - 2 \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ [কু.'০৯]

প্রমাণ ঃ **L.H.S.** $= \sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$= \frac{1}{2}(1 - \cos A + 1 - \cos B) + \sin^2\frac{C}{2}$

$= 1 - \frac{1}{2}.2 \cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B) + \sin^2\frac{C}{2}$

$= 1 - \cos(\frac{\pi}{2} - \frac{C}{2})\cos\frac{1}{2}(A - B) + \sin^2\frac{C}{2}$

$= 1 - \sin\frac{C}{2}\cos\frac{1}{2}(A - B) + \sin^2\frac{C}{2}$

$= 1 - \sin\frac{C}{2}\{\cos\frac{1}{2}(A - B) - \sin\frac{C}{2}\}$

$= 1 - \sin\frac{C}{2}[\cos\frac{1}{2}(A - B) - \sin\{\frac{\pi}{2} - \frac{1}{2}(A + B)\}]$

$= 1 - \sin\frac{C}{2}[\cos\frac{1}{2}(A - B) - \cos\frac{1}{2}(A + B)]$

$= 1 - 2 \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ = R.H.S. (Proved)

5. $A + B + C = \frac{\pi}{2}$ হলে প্রমাণ কর যে,

(a) $\sin^2 A + \sin^2 B + \sin^2 C + 2\sin A \sin B \sin C = 1$ [ঢা.,বা.'০১;মা., দি.'১২; কু.'১৪]

প্রমাণ ঃ L.H.S. $= \sin^2 A + \sin^2 B + \sin^2 C + 2\sin A \cos B \sin C$

$= \frac{1}{2}(1 - \cos 2A + 1 - \cos 2B) + \sin^2 C + 2\sin A \cos B \sin C$

$= 1 - \frac{1}{2}.2\cos(A+B)\cos(A-B) + \sin^2 C + 2\sin A \cos B \sin C$

$= 1 - \cos(\frac{\pi}{2} - C)\cos(A-B) + \sin^2 C + 2\sin A \cos B \sin C$

$= 1 - \sin C \cos(A-B) + \sin^2 C + 2\sin A \sin B \sin C$

$= 1 - \sin C\{\cos(A-B) - \sin C\} + 2\sin A \cos B \sin C$

$= 1 - \sin C[\cos(A-B) - \sin\{\frac{\pi}{2} - (A+B)\}] + 2\sin A \cos B \sin C$

$= 1 - \sin C[\cos(A-B) - \cos(A+B)] + 2\sin A \cos B \sin C$

$= 1 - \sin C .2\sin A \sin B + 2\sin A \cos B \sin C$

$= 1 - 2\sin A \sin B \sin C + 2\sin A \sin B \sin C$

$= 1 =$ R.H.S.

5(b) $\cot A + \cot B + \cot C = \cot A \cot B \cot C$

প্রমাণ ঃ দেওয়া আছে , $A + B + C = \frac{\pi}{2}$

$\Rightarrow A + B = \frac{\pi}{2} - C$

$\Rightarrow \cot(A+B) = \cot(\frac{\pi}{2} - C)$

$\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = \tan C$

$\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = \frac{1}{\cot C}$

$\Rightarrow \cot A + \cot B = \cot A \cot B \cot C + \cot C$

$\therefore \cot A + \cot B + \cot C = \cot A \cot B \cot C$

6. (a) $A + B + C = 2\pi$ হলে প্রমাণ কর যে, $\cos^2 A + \cos^2 B + \cos^2 C - 2\cos A \cos B \cos C = 1$ [সি.'০১]

প্রমাণ ঃ L.H.S. $= \cos^2 A + \cos^2 B + \cos^2 C - 2\cos A \cos B \cos C$

$= \frac{1}{2}(1 + \cos 2A + 1 + \cos 2B) + \cos^2 C - 2\cos A \cos B \cos C$

$= 1 + \frac{1}{2}.2\cos(A+B)\cos(A-B) + \cos^2 C - 2\cos A \cos B \cos C$

$= 1 + \cos(2\pi - C)\cos(A-B) + \cos^2 C - 2\cos A \cos B \cos C$

$= 1 + \cos C\{\cos(A-B) + \cos C\} - 2\cos A \cos B \cos C$

$= 1 + \cos C[\cos(A-B) + \cos\{2\pi - (A+B)\}] - 2\cos A \cos B \cos C$

$= 1 + \cos C[\cos(A-B) + \cos(A+B)] - 2\cos A \cos B \cos C$

$= 1 + \cos C.2\cos A.\cos B - 2\cos A \cos B \cos C$

$= 1 - 2\cos A \cos B \cos C + 2\cos A \cos B \cos C$

$= 1$

6(b) $A + B + C = 0$ হলে প্রমাণ কর যে, $\cos A + \cos B + \cos C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} - 1$

প্রমাণ ঃ L.H.S. $= \cos A + \cos B + \cos C$

$= 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B) + 2\cos^2\frac{1}{2}C - 1$

$= 2\cos\frac{1}{2}(-C)\cos\frac{1}{2}(A-B) + 2\cos^2\frac{1}{2}C - 1$

$= 2\cos\frac{1}{2}C[\cos\frac{1}{2}(A-B) + \cos\frac{1}{2}\{-(A+B)\}] - 1$

$= 2\cos\frac{1}{2}C[\cos\frac{1}{2}(A-B) + \cos\frac{1}{2}(A+B)] - 1$

$= 2\cos\frac{1}{2}C.2\cos\frac{1}{2}A.\cos\frac{1}{2}B - 1$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} - 1 =$ R.H.S. (Proved)

6(c) $A+B+C=(2n+1)\frac{\pi}{2}$ হলে দেখাও যে, $\tan A\tan C+\tan C\tan A+\tan A\tan B=1$

প্রমাণ ঃ দেওয়া আছে , $A+B+C=(2n+1)\frac{\pi}{2}$

$\Rightarrow A+B=(n\pi+\frac{\pi}{2})-C$

$\Rightarrow \tan(A+B)=\tan\{(n\pi+\frac{\pi}{2})-C\}$

$=\tan\{n\pi+(\frac{\pi}{2}-C)\}$

$=\tan(\frac{\pi}{2}-C)=\cot C$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{1}{\tan C}$

$\Rightarrow \tan A\tan C+\tan B\tan C=1-\tan A\tan B$

$\therefore \tan A\tan C+\tan C\tan A+\tan A\tan B=1$

7(a) $A+B+C=\pi$ এবং $\cot A+\cot B+\cot C=\sqrt{3}$ হলে দেখাও যে, $A=B=C$. [ব.'০৭]

প্রমাণ ঃ দেওয়া আছে, $A+B+C=\pi$

$\Rightarrow A+B=\pi-C$

$\Rightarrow \cot(A+B)=\cot(\pi-C)$

$\Rightarrow \frac{\cot A\cot B-1}{\cot B+\cot A}=-\cot C$

$\Rightarrow \cot A\cot B-1=\cot B\cot C-\cot C\cot A$

$\Rightarrow \cot A\cot B+\cot B\cot C+\cot C\cot A=1$

এখন, $\cot A+\cot B+\cot C=\sqrt{3}$

$\Rightarrow \cot^2 A+\cot^2 B+\cot^2 C+2(\cot A\cot B+\cot B\cot C+\cot C\cot A)=3(\cot A\cot B+\cot B\cot C+\cot C\cot A)$

$\Rightarrow \cot^2 A+\cot^2 B+\cot^2 C-(\cot A\cot B+\cot B\cot C+\cot C\cot A)=0$

$\Rightarrow \frac{1}{2}\{(\cot A-\cot B)^2+(\cot B-\cot C)^2+(\cot C-\cot A)^2\}=0$

প্রত্যেকটি শূন্য না হলে তিনটি বর্গের সমষ্টি শূন্য হতে পারে না।

$\therefore \cot A-\cot B=0\Rightarrow \cot A=\cot B$,

$\cot B-\cot C=0\Rightarrow \cot B=\cot C$

$\therefore \cot A=\cot B=\cot C$

$\Rightarrow A=B=C$

7(b) $A+B+C=\pi$ এবং $\sin^2 A+\sin^2 B+\sin^2 C=\sin B\sin C+\sin C\sin A+\sin A\sin B$ হলে দেখাও যে, $A=B=C$

প্রমাণ ঃ দেওয়া আছে , $\sin^2 A+\sin^2 B+\sin^2 C=\sin A\sin B+\sin B\sin C+\sin C\sin A$

$\Rightarrow \sin^2 A+\sin^2 B+\sin^2 C-(\sin A\sin B+\sin B\sin C+\sin C\sin A)=0$

$\Rightarrow \frac{1}{2}\{(\sin A-\sin B)^2+(\sin B-\sin C)^2+(\sin C-\sin A)^2\}=0$

প্রত্যেকটি শূন্য না হলে তিনটি বর্গের সমষ্টি শূন্য হতে পারে না।

$\therefore \sin A-\sin B=0\Rightarrow \sin A=\sin B$

$\Rightarrow \sin A=\sin B=\sin(\pi-B)$

$\therefore \sin A=\sin B$ অথবা, $\sin A=\sin(\pi-B$

$\therefore A=B$ অথবা, $A=\pi-B\Rightarrow A+B=\pi$

কিন্তু $A+B+C=\pi$ বলে, $A+B=\pi$ হতে পারে না।

$\therefore A=B$. অনুরূপভাবে, $B=C$

$\therefore A=B=C$ (Showed)

7(c) $\tan A+\tan B+\tan C=\tan A\tan B\tan C$ হলে দেখাও যে, $A+B+C=n\pi$, যখন $n\in\mathbb{Z}$.

প্রমাণ ঃ দেওয়া আছে , $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

$\Rightarrow \tan A+\tan B=-\tan C(1-\tan A\tan B)$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$

$\Rightarrow \tan(A+B)=-\tan C=\tan(\pi-C)=\tan(2\pi-C)=\tan(3\pi-C)=\cdots\cdots=\tan(n\pi-C)$, যেখানে $n\in\mathbb{Z}$.

$\therefore A+B=n\pi-C\Rightarrow A+B+C=n\pi$ (Showed)

সম্ভাব্য ধাপসহ প্রশ্ন:

$A+B+C=\pi$ হলে প্রমাণ কর যে,

8. $\cos A + \cos B - \cos C = 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} - 1$

প্রমাণ : L.H.S. $= \cos A + \cos B - \cos C$

$= 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B) - (1-2\sin^2\frac{C}{2})$ (১)+(১)

$= 2\cos(\frac{\pi}{2}-\frac{C}{2})\cos\frac{1}{2}(A-B) + 2\sin^2\frac{C}{2} - 1$

$= 2\sin\frac{C}{2}\cos\frac{1}{2}(A-B) + 2\sin^2\frac{C}{2} - 1$ (১)

$= 2\sin\frac{C}{2}\{\cos\frac{1}{2}(A-B) + \sin\frac{C}{2}\} - 1$

$= 2\sin\frac{C}{2}\{\cos\frac{1}{2}(A-B) + \sin(\frac{\pi}{2}-\frac{A+B}{2})\} - 1$

$= 2\sin\frac{C}{2}\{\cos(\frac{A}{2}-\frac{B}{2}) + \cos(\frac{A}{2}+\frac{B}{2})\} - 1$

$= 2\sin\frac{C}{2}(2\cos\frac{A}{2}\cos\frac{B}{2}) - 1$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} - 1 =$ R.H.S. (Proved) (১)

9.(a) $\sin(B+C-A) + \sin(C+A-B) + \sin(A+B-C) = 4\sin A \sin B \sin C$

প্রমাণ : L.H.S. $= \sin(B+C-A) + \sin(C+A-B) + \sin(A+B-C)$

$= \sin(A+B+C-2A) + \sin(A+B+C-2B) + \sin(A+B+C-2C)$

$= \sin(\pi - 2A) + \sin(\pi - 2B) + \sin(\pi - 2C)$

$= \sin 2A + \sin 2B + \sin 2C$ (১)

$= 2\sin\frac{1}{2}(2A+2B)\cos\frac{1}{2}(2A-2B) + \cos 2C$ (১)

$= 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$ (১)

$= 2\sin(\pi - C)\cos(A-B) + 2\sin C\cos C$

$= 2\sin C\cos(A-B) + 2\sin C\cos C$

$= 2\sin C\{\cos(A-B) + \cos C\}$

$= 2\sin C\{\cos(A-B) + \cos(\pi - \overline{A+B})\}$

$= 2\sin C\{\cos(A-B) - \cos(A+B)\}$

$= 2\sin C.2\sin A\sin B = 4\sin A\sin B\sin C$

$=$ R.H.S. (Proved) (১)

9. (b) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\frac{B+C}{4}\sin\frac{C+A}{4}\sin\frac{A+B}{4}$

$= 1 + 4\sin\frac{\pi-A}{4}\sin\frac{\pi-B}{4}\sin\frac{\pi-C}{4}$

M.H.S. $= 1 + 4\sin\frac{B+C}{4}\sin\frac{C+A}{4}\sin\frac{A+B}{4}$

$= 1 + 2.2\sin\frac{B+C}{4}\sin\frac{C+A}{4}\sin\frac{A+B}{4}$

$= 1 + 2[\cos\frac{B+C-C-A}{4} - \cos\frac{B+C+C+A}{4}]\sin\frac{A+B}{4}$ (১)

$= 1 + 2\cos\frac{B-A}{4}\sin\frac{A+B}{4} - 2\cos\frac{A+B+2C}{4}\sin\frac{A+B}{4}$

$= 1 + \sin(\frac{A+B}{4}+\frac{B-A}{4}) + \sin(\frac{A+B}{4}-\frac{B-A}{4}) - \{\sin(\frac{A+B}{4}+\frac{A+B+2C}{4}) + \sin(\frac{A+B}{4}-\frac{A+B+2C}{4})\}$ (১)

$= 1 + \sin\frac{B}{2} + \sin\frac{A}{2} - \sin\frac{A+B+C}{2} - \sin(-\frac{C}{2})$

$= 1 + \sin\frac{A}{2} + \sin\frac{B}{2} - \sin\frac{\pi}{2} + \sin\frac{C}{2}$ (১)

$= 1 + \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1$

$= \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} =$ L.H.S.

Again , $1 + 4\sin\frac{B+C}{4}\sin\frac{C+A}{4}\sin\frac{A+B}{4}$

$=1 + 4\sin\frac{\pi-A}{4}\sin\frac{\pi-B}{4}\sin\frac{\pi-C}{4}$ =R.H.S. (১)

(c) sin A cos B cos C + sin B cos C cos A + sin C cos A cos B = sin A sin B sin C

প্রমাণ ঃ L.H.S. = sinA cos B cos C + sin B cos C cos A + sin C cos A cos B

= (sin A cos B + sin B cos A) cos C + sin C cos A cos B

= sin(A + B) cos C + sinC cosA cos B (১)

= sin (π − C) cos{π − (A + B)} + sin C cos A cos B

= sin C { − cos (A + B) + cos A cos B} (১)

= sin C (− cos A cos B + sinA sinB + cos A cos B) (১)

= sin A sin B sin C = R.H.S. (Proved) (১)

10. tan 2A + tan 2B + tan 2C = tan 2A tan 2B tan 2C

প্রমাণ ঃ দেওয়া আছে , A + B + C = π

$\Rightarrow$ 2A + 2B = 2π − 2C

$\Rightarrow$ tan(2A + 2B) = tan(2π − 2C)

$\Rightarrow \frac{\tan 2A + \tan 2B}{1 - \tan 2A + \tan 2B} = -\tan 2C$ (১)

$\Rightarrow$ tan2A + tan2B = − tan2C + tan2Atan2Btan2C

$\therefore$ tan2A + tan2B + tan2C = tan2A tan2Btan2C (১)

11. $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

প্রমাণ ঃ L.H.S. = $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2}$

$= \frac{1}{2}(1 + \cos A + 1 + \cos B) + \cos^2\frac{C}{2}$ (১)

$= 1 + \frac{1}{2}.2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B) + \cos^2\frac{C}{2}$ (১)

$= 1 + \cos(\frac{\pi}{2} - \frac{C}{2})\cos\frac{1}{2}(A - B) + \cos^2\frac{C}{2}$

$= 1 + \sin\frac{C}{2}\cos\frac{1}{2}(A - B) + 1 - \sin^2\frac{C}{2}$ (১)

$= 2 + \sin\frac{C}{2}\{\cos\frac{1}{2}(A - B) - \sin\frac{C}{2}\}$

$= 2 + \sin\frac{C}{2}[\cos\frac{1}{2}(A - B) - \sin\{\frac{\pi}{2} - \frac{1}{2}(A + B)\}]$

$= 2 + \sin\frac{C}{2}[\cos\frac{1}{2}(A-B) - \cos\frac{1}{2}(A+B)]$

$= 2 + \sin\frac{C}{2}.2\sin\frac{A}{2}\sin\frac{B}{2}$

$= 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ =R.H.S (Proved)(১)

12. A + B + C = (2n + 1)$\frac{\pi}{2}$ হলে দেখাও যে, sin2A+ sin2B + sin2C=± 4cosA cosB cosC

প্রমাণ ঃ $\sin\{(2n+1)\frac{\pi}{2} - \theta\} = \sin\{n\pi + (\frac{\pi}{2} - \theta)\}$

$= \pm\sin(\frac{\pi}{2} - \theta) = \pm\cos\theta$ (১)

এখন, sin 2A + sin 2B + sin 2C

= 2sin(A + B) cos (A−B) + 2 sinC cosC (১)

$= 2\sin\{(2n+1)\frac{\pi}{2} - C\}\cos(A - B) + 2\sin\{(2n+1)\frac{\pi}{2} - (A + B)\}\cos C$

= 2 (± cos C) cos (A −B) + 2{± cos (A+ B)} cos C

= ± 2 cos C{ cos (A − B) + cos (A + B)} (১)

= ± 2 cos C (2 cos A cos B)

± 4 cos A cos B cos C

$\therefore$ sin 2A + sin 2B + sin 2C = ± 4 cos A cos B cos C (১)

13. cos ² A + cos ² B + cos ² C + 2cosA cos B cos C = 1 হলে দেখাও যে, A ± B ± C = (2n + 1) π, যেখানে n যে কোন অখন্ড সংখ্যা ।

প্রমাণ ঃ দেওয়া আছে ,

$\cos^2 A + \cos^2 B + \cos^2 C + 2\cos A \cos B \cos C = 1$

$\Rightarrow \frac{1}{2}(1+\cos 2A + 1 + \cos 2B) + \cos^2 C + \cos C \cdot 2\cos A \cos B = 1$ (১)

$\Rightarrow 1 + \frac{1}{2} \cdot 2\cos(A+B)\cos(A-B) + \cos^2 C + \cos C\{\cos(A+B) + \cos(A-B)\} = 1$ (১)

$\Rightarrow \cos(A+B)\cos(A-B) + \cos^2 C + \cos C \cos(A+B) + \cos(A-B)\cos C = 0$

$\Rightarrow \cos(A-B)\{\cos(A+B) + \cos C\} + \cos C\{\cos(A+B) + \cos C\} = 0$

$\Rightarrow \{\cos(A+B) + \cos C\}\{\cos(A-B) + \cos C\} = 0$

$\therefore \cos(A \pm B) + \cos C = 0$ (১)

$\Rightarrow \cos(A \pm B) = -\cos C = \cos(\pi \pm C)$
$= \cos(3\pi \pm C) = \ldots\ldots$
$= \cos\{(2n+1)\pi \pm C\}$, যেখানে $n \in \mathbb{Z}$.

$\Rightarrow A \pm B = (2n+1)\pi \pm C$

$\Rightarrow A \pm B \pm C = (2n+1)\pi$ (১)

14. $x + y + z = xyz$ হলে প্রমাণ কর যে,

$$\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2x}{1-x^2} \cdot \frac{2y}{1-y^2} \cdot \frac{2z}{1-z^2}$$

মনে করি, $x = \tan A \Rightarrow A = \tan^{-1} x$

$y = \tan B \Rightarrow A = \tan^{-1} y$

$z = \tan C \Rightarrow C = \tan^{-1} z$ (১)

$\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow \tan A + \tan B = -\tan C(1 - \tan A \tan B)$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\Rightarrow \tan(A+B) = \tan(\pi - C)$ (১)+(১)

$\Rightarrow A + B = \pi - C$

$\Rightarrow 2A + 2B = 2\pi - 2C$

$\Rightarrow \tan(2A+2B) = \tan(2\pi - 2C)$

$\Rightarrow \frac{\tan 2A + \tan 2B}{1 - \tan 2A \tan 2B} = -\tan 2C$

$\Rightarrow \tan 2A + \tan 2B = \tan 2C + \tan A \tan B \tan C$

$\Rightarrow \tan 2A + \tan 2B + \tan 2C = \tan A \tan B \tan C$

$\Rightarrow \frac{2\tan A}{1-\tan^2 A} + \frac{2\tan B}{1-\tan^2 B} + \frac{2\tan C}{1-\tan^2 C} = \frac{2\tan A}{1-\tan^2 A} \frac{2\tan B}{1-\tan^2 B} \frac{2\tan C}{1-\tan^2 C}$ (১)

$\Rightarrow \frac{2x}{1-x^2} + \frac{2x}{1-x^2} + \frac{2x}{1-x^2} = \frac{2x}{1-x^2} \frac{2x}{1-x^2} \frac{2x}{1-x^2}$ (১)

15. $x + y + z = xyz$ হলে প্রমাণ কর যে,

$$\frac{3x-x^3}{1-3x^2} + \frac{3y-y^3}{1-3y^2} + \frac{3z-z^3}{1-3z^2} = \frac{3x-x^3}{1-3x^2} \cdot \frac{3y-y^3}{1-3y^2} \cdot \frac{3z-z^3}{1-3z^2}$$

প্রমাণঃ মনে করি, $x = \tan A$, $y = \tan B$, $z = \tan C$

$\therefore \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$ $[\because x + y + z = xyz]$

$\Rightarrow \tan A + \tan B = \tan C(\tan A \cdot \tan B - 1)$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\Rightarrow \tan(A+B) = \tan(\pi - C)$ (১)+(১)

$\therefore A + B = \pi - C$

$\Rightarrow 3A + 3B + 3C = 3\pi$

$\therefore \tan(3A + 3B + 3C) = \tan 3\pi$

$\Rightarrow \frac{\tan 3A + \tan 3B + \tan 3C - \tan 3A \cdot \tan 3B \cdot \tan 3C}{1 - \tan 3A \cdot \tan 3B - \tan 3B \cdot \tan 3C - \tan 3C \cdot \tan 3A} = 0$ (১)

$\Rightarrow \tan 3A + \tan 3B + \tan 3C - \tan 3A \cdot \tan 3B \cdot \tan 3C = 0$

$$\Rightarrow \frac{3\tan A-\tan^3 A}{1-3\tan^2 A}+\frac{3\tan B-\tan^3 B}{1-3\tan^2 B}+\frac{3\tan C-\tan^3 C}{1-3\tan^2 C}$$

$$=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}\cdot\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}\cdot\frac{3\tan A-\tan^3 A}{1-3\tan^2 A} \quad (১)$$

$$\therefore \frac{3x-x^3}{1-3x^2}+\frac{3y-y^3}{1-3y^2}+\frac{3z-z^3}{1-3z^2}$$

$$=\frac{3x-x^3}{1-3x^2}\cdot\frac{3y-y^3}{1-3y^2}\cdot\frac{3z-z^3}{1-3z^2} \text{ (Proved)} \quad (১)$$

16. $yz+zx+xy=1$ হলে প্রমাণ কর যে,

$$\frac{(x^2-1)(y^2-1)}{xy}+\frac{(y^2-1)(z^2-1)}{yz}+\frac{(z^2-1)(x^2-1)}{zx}=4$$

প্রমাণ ঃ মনে করি, $x=\cot A \Rightarrow A=\cot^{-1}x$

$y=\cot B \Rightarrow A=\cot^{-1}y$

$z=\cot C \Rightarrow C=\cot^{-1}z$ (১)

$\therefore \cot A\cot B+\cot B\cot C+\cot C\cot A=1$

$\Rightarrow \cot A\cot B-1=-(\cot B+\cot A)\cot C$

$\Rightarrow \frac{\cot A\cot B-1}{\cot A+\cot B}=-\cot C$

$\Rightarrow \cot(A+B)=\cot(\pi-C)$ (১)+(১)

$\Rightarrow A+B=\pi-C$

$\Rightarrow 2A+2B=2\pi-2C$

$\Rightarrow \cot(2A+2B)=\cot(2\pi-2C)$

$\Rightarrow \frac{\cot 2A\cot 2B-1}{\cot 2B+\cot 2A}=-\cot 2C$

$\Rightarrow \cot 2A\cot 2B+\cot 2B\cot 2C+\cot 2C\cot 2A=1$

$\Rightarrow \frac{\cot^2 A-1}{2\cot A}\cdot\frac{\cot^2 B-1}{2\cot B}+\frac{\cot^2 B-1}{2\cot B}\cdot\frac{\cot^2 C-1}{2\cot C}+\frac{\cot^2 C-1}{2\cot C}\cdot\frac{\cot^2 A-1}{2\cot A}=1$ (১)

$\frac{x^2-1}{2x}\cdot\frac{y^2-1}{2y}+\frac{y^2-1}{2y}\cdot\frac{z^2-1}{2z}+\frac{z^2-1}{2z}\cdot\frac{x^2-1}{2x}=1$

$\therefore \frac{(x^2-1)(y^2-1)}{xy}+\frac{(y^2-1)(z^2-1)}{yz}+\frac{(z^2-1)(x^2-1)}{zx}=4$ (Showed) (১)

সৃজনশীল প্রশ্ন :

17. দৃশ্যকল্প - ১: $A+B+C=\pi$, $\sin 2A\cos 2B+\cos 2A\sin 2B+\sin 2C=0$

দৃশ্যকল্প - ২: $A+B+C=\frac{\pi}{2}$.

(a) $\cos A\cos B-\sin A\sin B+\cos C=0$ হলে প্রমাণ কর যে, $\cos^2 A+\cos^2 B+\cos^2 C+2\cos A\cos B\cos C=1$

প্রমাণ : দেওয়া আছে,

$\cos A\cos B-\sin A\sin B+\cos C=0$

$\Rightarrow \cos A\cos B+\cos C=\sin A\sin B$

$\Rightarrow (\cos A\cos B+\cos C)^2=\sin^2 A\sin^2 B$

$\Rightarrow \cos^2 A\cos^2 B+2\cos A\cos B\cos C+\cos^2 C=(1-\cos^2 A)(1-\cos^2 B)$

$\Rightarrow \cos^2 A\cos^2 B+2\cos A\cos B\cos C+\cos^2 C=1-\cos^2 A-\cos^2 B+\cos^2 A\cos^2 B$

$\therefore \cos^2 A+\cos^2 B+\cos^2 C+2\cos A\cos B\cos C=1$

(b) দৃশ্যকল্প - ১ এর সাহায্যে প্রমাণ কর যে, $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

প্রমাণ : দৃশ্যকল্প – ১ হতে পাই, $A+B+C=\pi$ এবং $\sin 2A\cos 2B+\cos 2A\sin 2B+\sin 2C=0$

$\Rightarrow \sin 2A(1-2\sin^2 B)+(1-2\sin^2 A)\sin 2B+\sin 2C=0$

$\Rightarrow \sin 2A - 2\sin 2A \sin^2 B + \sin 2B - 2\sin^2 A \sin 2B + \sin 2C = 0$

$\Rightarrow \sin 2A + \sin 2B + \sin 2C$
$= 2\sin 2A \sin^2 B + 2\sin^2 A \sin 2B$
$= 4\sin A \cos A \sin^2 B + 4\sin^2 A \sin B \cos B$
$= 4\sin A \sin B (\cos A \sin B + \sin A \cos B)$
$= 4\sin A \sin B \sin (A + B)$
$= 4\sin A \sin B \sin (\pi - C)$

$\therefore \sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$

(c) দৃশ্যকল্প - ২ এর সাহায্যে প্রমাণ কর যে, $\cos^2 A + \cos^2 B - \cos^2 C = 2\cos A \cos B \sin C$

প্রমাণ : দৃশ্যকল্প – ২ হতে পাই, $A + B + C = \frac{\pi}{2}$

$\Rightarrow A + B = \frac{\pi}{2} - C$

$\Rightarrow \cos (A + B) = \cos (\frac{\pi}{2} - C)$
$\Rightarrow \cos A \cos B - \sin A \sin B = \sin C$
$\Rightarrow \cos A \cos B - \sin C = \sin A \sin B$
$\Rightarrow (\cos A \cos B - \sin C)^2 = \sin^2 A \sin^2 B$
$\Rightarrow \cos^2 A \cos^2 B - 2\cos A \cos B \sin C + \sin^2 C = (1 - \cos^2 A)(1 - \cos^2 B)$
$\Rightarrow \cos^2 A \cos^2 B - 2\cos A \cos B \sin C + 1 - \cos^2 C = (1 - \cos^2 A)(1 - \cos^2 B)$
$\Rightarrow \cos^2 A \cos^2 B - 2\cos A \cos B \sin C + 1 - \cos^2 C = 1 - \cos^2 A - \cos^2 B + \cos^2 A \cos^2 B$

$\therefore \cos^2 A + \cos^2 B - \cos^2 C = 2\cos A \cos B \sin C$