নিচের সীমাগুলির মান নির্ণয় কর :

1.(a) $\lim_{x \to 2} \frac{x^2-4}{x^2-5x+6}$

সমাধানঃ ধরি $x = 2 + h$. $\therefore$ $h \to 0$, যখন $x \to 2$

$\therefore \lim_{x \to 2} \frac{x^2-4}{x^2-5x+6}$

$= \lim_{h \to 0} \frac{(2+h)^2-4}{(2+h)^2-5(2+h)+6}$

$= \lim_{h \to 0} \frac{4+4h+h^2-4}{4+4h+h^2-10-5h+6}$

$= \lim_{h \to 0} \frac{h(h+4)}{h(h-1)} = \lim_{h \to 0} \frac{h+4}{h-1} = \frac{0+4}{0-1}$

$= -4$ (Ans.)

বিকল্প পদ্ধতিঃ $\lim_{x \to 2} \frac{x^2-4}{x^2-5x+6}$

$= \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x-3)} = \lim_{x \to 2} \frac{x+2}{x-3}$

$= \lim_{x \to 2} \frac{x+2}{x-3} = \frac{2+2}{2-3} = -4$ (Ans.)

1(b) $\lim_{x \to 0} \frac{(x+4)^3-(x-8)^2}{x(x-3)}$

$= \lim_{x\to 0} \frac{x^3+12x^2+48x+64-x^2+16x-64}{x(x-3)}$

$= \lim_{x \to 0} \frac{x^3+12x^2+48x+64-x^2+16x-64}{x(x-3)}$

$= \lim_{x\to 0} \frac{x^3+11x^2+64x}{x(x-3)} = \lim_{x\to 0} \frac{x(x^2+11x+64)}{x(x-3)}$

$= \lim_{x\to 0} \frac{x^2+11x+64}{x-3} = \frac{0^2+11.0+64}{0-3}$

$= \frac{64}{-3} = -21\frac{1}{3}$ (Ans.)

2(a) $\lim_{x \to 0} \frac{\sqrt{1+3x}-\sqrt{1-4x}}{x}$ [সি. '০৩]

$= \lim_{x\to 0} \frac{(\sqrt{1+3x}-\sqrt{1-4x})(\sqrt{1+3x}+\sqrt{1-4x})}{x(\sqrt{1+3x}+\sqrt{1-4x})}$

$= \lim_{x\to 0} \frac{(\sqrt{1+3x})^2-(\sqrt{1-4x})^2}{x(\sqrt{1+3x}+\sqrt{1-4x})}$

$= \lim_{x\to 0} \frac{1+3x-1+4x}{x(\sqrt{1+3x}+\sqrt{1-4x})}$

$= \lim_{x\to 0} \frac{7x}{x(\sqrt{1+3x}+\sqrt{1-4x})}$

$= \lim_{x\to 0} \frac{7}{\sqrt{1+3x}+\sqrt{1-4x}}$

$= \frac{7}{\sqrt{1+3.0}+\sqrt{1-4.0}} = \frac{7}{1+1} = \frac{7}{2}$

2(b) $\lim_{x \to 0} \frac{\sqrt{1+2x}-\sqrt{1-3x}}{x}$ [ব. '০৯,'১৩]

$= \lim_{x\to 0} \frac{(\sqrt{1+2x}-\sqrt{1-3x})(\sqrt{1+2x}+\sqrt{1-3x})}{x(\sqrt{1+2x}+\sqrt{1-3x})}$

$= \lim_{x\to 0} \frac{(\sqrt{1+2x})^2-(\sqrt{1-3x})^2}{x(\sqrt{1+2x}+\sqrt{1-3x})}$

$= \lim_{x\to 0} \frac{1+2x-1+3x}{x(\sqrt{1+2x}+\sqrt{1-3x})}$

$= \lim_{x\to 0} \frac{5x}{x(\sqrt{1+2x}+\sqrt{1-3x})} = \lim_{x\to 0} \frac{5}{\sqrt{1+2x}+\sqrt{1-3x}}$

$= \frac{5}{\sqrt{1+2.0}+\sqrt{1-3.0}} = \frac{5}{1+1} = \frac{5}{2}$ (Ans.)

2(c) $\lim_{x \to 0} \frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$

$= \lim_{x\to 0} \{\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}} \times \frac{\sqrt{1+x^2}+\sqrt{1+x}}{\sqrt{1+x^2}+\sqrt{1+x}}$

$\times \frac{\sqrt{1+x^3}+\sqrt{1+x}}{\sqrt{1+x^3}+\sqrt{1+x}}\}$

$= \lim_{x\to 0} \frac{(1+x^2-1-x)(\sqrt{1+x^3}+\sqrt{1+x})}{(1+x^3-1-x)(\sqrt{1+x^2}+\sqrt{1+x})}$

$$= \lim_{x\to 0} \frac{x(x-1)(\sqrt{1+x^3}+\sqrt{1+x})}{x(x^2-1)(\sqrt{1+x^2}+\sqrt{1+x})}$$

$$= \lim_{x\to 0} \frac{(x-1)(\sqrt{1+x^3}+\sqrt{1+x})}{(x^2-1)(\sqrt{1+x^2}+\sqrt{1+x})}$$

$$= \frac{(0-1)(\sqrt{1+0^3}+\sqrt{1+0})}{(0^2-1)(\sqrt{1+0^2}+\sqrt{1+0})} = \frac{2}{2} = 1$$

3(a) $\lim_{x\to\infty} \frac{2x^4-3x^2+1}{6x^4+x^3-3x}$

$$= \lim_{x\to\infty} \frac{x^4(2-\frac{3}{x^2}+\frac{1}{x^4})}{x^4(6+\frac{1}{x}-\frac{3}{x^3})}$$

$$= \lim_{x\to\infty} \frac{2-\frac{3}{x^2}+\frac{1}{x^4}}{6+\frac{1}{x}-\frac{3}{x^3}} = \frac{2-0+0}{6+0-0} = \frac{2}{6} = \frac{1}{3}$$

3(b) $\lim_{x\to\infty} \frac{3^x-3^{-x}}{3^x+3^{-x}}$ [চ.'০০]

$$= \lim_{x\to\infty} \frac{3^x(1-\frac{1}{3^{2x}})}{3^x(1+\frac{1}{3^{2x}})} = \lim_{x\to\infty} \frac{1-\frac{1}{3^{2x}}}{1+\frac{1}{3^{2x}}}$$

$$= \frac{1-0}{1+0} = \frac{1-0}{1+0} = 1$$

3(c) $\lim_{x\to\infty}\{\ln(2x-1)-\ln(x+5)\}$ [প্র.ভ.প.'০৪]

$$= \lim_{x\to\infty} \ln\frac{2x-1}{x+5} = \lim_{x\to\infty} \ln\frac{x(2-\frac{1}{x})}{x(1+\frac{5}{x})}$$

$$= \lim_{x\to\infty} \ln\frac{2-\frac{1}{x}}{1+\frac{5}{x}} = \ln\frac{2-0}{1+0}$$

$= \ln 2$ (Ans.)

3.(d) $\lim_{x\to\infty} 2^x \sin\frac{b}{2^x}$ [সি.'০৫]

ধরি, $\frac{b}{2^x} = \theta$. এখানে $x\to\infty$ বলে $2^x\to\infty$

$\therefore\ \theta = \frac{b}{2^x} \to 0$

$$\therefore \lim_{x\to\infty} 2^x \sin\frac{b}{2^x} = \lim_{\theta\to 0} \frac{b}{\theta}\sin\theta$$

$$= b\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = b.1 = b$$

4.(a) $\lim_{x\to a} \frac{x^{7/2}-a^{7/2}}{\sqrt{x}-\sqrt{a}}$ [ঢা.'০৩]

$$= \frac{\lim_{x\to a}(x^{7/2}-a^{7/2})}{\lim_{x\to a}(x^{1/2}-a^{1/2})} = \frac{\lim_{x\to a}\frac{x^{7/2}-a^{7/2}}{x-a}}{\lim_{x\to a}\frac{x^{1/2}-a^{1/2}}{x-a}}$$

$$= \frac{\frac{7}{2}a^{\frac{7}{2}-1}}{\frac{1}{2}a^{\frac{1}{2}-1}} \quad [\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}]$$

$$= (\frac{7}{2}\times\frac{2}{1})a^{\frac{7}{2}-1-\frac{1}{2}+1} = 7a^{\frac{7}{2}-\frac{1}{2}} = 7a^3 \text{ (Ans.)}$$

4(b) $\lim_{x\to a} \frac{x^{5/2}-a^{5/2}}{x^{3/5}-a^{3/5}}$

$$= \frac{\lim_{x\to a}(x^{5/2}-a^{5/2})}{\lim_{x\to a}(x^{3/5}-a^{3/5})} = \frac{\lim_{x\to a}\frac{x^{5/2}-a^{5/2}}{x-a}}{\lim_{x\to a}\frac{x^{3/5}-a^{3/5}}{x-a}}$$

$$= \frac{\frac{5}{2}a^{\frac{5}{2}-1}}{\frac{3}{5}a^{\frac{3}{5}-1}} \quad [\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}]$$

$$= (\frac{5}{2}\times\frac{5}{3})a^{\frac{5}{2}-1-\frac{3}{5}+1} = \frac{25}{6}a^{\frac{5}{2}-\frac{3}{5}}$$

$$= \frac{25}{6}a^{\frac{25-6}{10}} = \frac{25}{6}a^{\frac{19}{10}} \text{ (Ans.)}$$

5(a) $\lim_{x\to 0} \frac{1-\cos 3x}{3x^2}$ [প্র.ভ.প.'৮৫]

$$= \lim_{x\to 0} \frac{2\sin^2\frac{3x}{2}}{3x^2} = \lim_{x\to 0} \frac{2\sin^2\frac{3x}{2}}{\frac{9x^2}{4}.\frac{4}{3}}$$

$$= \frac{2.3}{4} \lim_{x\to 0} \left\{\frac{\sin(3x/2)}{3x/2}\right\}^2 = \frac{3}{2}.1 = \frac{3}{2}$$

5.(b) $\lim_{x\to 0} \frac{1-\cos 7x}{3x^2}$ [সি.'০৮,'১২;কু.'১১; রা.'০৭,'১০; চ.'০৬; য.'০৮,'১২; ব.'০৮; ঢা.'১০; দি.'১১]

$$= \lim_{x\to 0} \frac{2\sin^2\frac{7x}{2}}{3.\frac{49x^2}{4}.\frac{4}{49}}$$

$$= (\frac{2}{3}\times\frac{49}{4}) \lim_{x\to 0} \left\{\frac{\sin(7x/2)}{7x/2}\right\}^2$$

$$= \frac{49}{6}.1 = \frac{49}{6} \text{ (Ans.)}$$

6. (a) $\lim_{x\to 0} \frac{\cos 2x - \cos 3x}{x^2}$

[ব.'০১; যা.'০৫ সি.'০৪]

$$= \lim_{x\to 0} \frac{2\sin\frac{1}{2}(2x+3x)\sin\frac{1}{2}(3x-2x)}{x^2}$$

$$= \lim_{x\to 0} \frac{2\sin\frac{5x}{2}\sin\frac{x}{2}}{x^2}$$

$$= 2\lim_{x\to 0} \frac{\sin\frac{5x}{2}}{\frac{5x}{2}} \times \lim_{x\to 0} \frac{\sin\frac{x}{2}}{\frac{x}{2}} \times \frac{5}{2}\times\frac{1}{2}$$

$$= 2\times 1\times\frac{5}{4} = \frac{5}{2} \text{ (Ans.)}$$

6(b) $\lim_{x\to 0} \frac{\cos 2x - \cos 4x}{x^2}$ [কু.'০৩]

$$= \lim_{x\to 0} \frac{2\sin\frac{1}{2}(2x+4x)\sin\frac{1}{2}(4x-2x)}{x^2}$$

$$= \lim_{x\to 0} \frac{2\sin 3x \sin x}{x^2}$$

$$= 2.\lim_{x\to 0} \frac{\sin 3x}{3x} \times \lim_{x\to 0} \frac{\sin x}{x} \times 3$$

$$= 2\times 1\times 1\times 3 = 6 \text{ (Ans.)}$$

6. (c) $\lim_{x\to 0} \frac{\cos ax - \cos bx}{x^2}$ [ব'১২; য.'১৩]

$$= \lim_{x\to 0} \frac{2\sin\frac{1}{2}(ax+bx)\sin\frac{1}{2}(bx-ax)}{x^2}$$

$$= 2\lim_{x\to 0} \frac{\sin\frac{(a+b)x}{2}}{\frac{(a+b)x}{2}} \times \frac{a+b}{2} \times \lim_{x\to 0} \frac{\sin\frac{(b-a)x}{2}}{\frac{(b-a)x}{2}} \times \frac{b-a}{2}$$

$$= 2\times 1\times\frac{a+b}{2}\times 1\times\frac{b-a}{2} = \frac{1}{2}(b^2-a^2)$$

6(d) $\lim_{x\to 0} \frac{1-2\cos x+\cos 2x}{x^2}$

[য.'০৫; কু.'১৪]

$$= \lim_{x\to 0} \frac{1-2\cos x+2\cos^2 x-1}{x^2}$$

$$= \lim_{x\to 0} \frac{2\cos x(\cos x-1)}{x^2}$$

$$= \lim_{x\to 0} \frac{2\cos x(-2\sin^2\frac{x}{2})}{x^2}$$

$$= -4\lim_{x\to 0} \left\{\frac{\sin(x/2)}{x/2}\right\}^2 \times \frac{1}{4}\times \lim_{x\to 0} \cos x$$

$$= -4\times 1\times\frac{1}{4}\times\cos 0 = -1\times 1 = -1$$

6(e) $\lim_{x\to 0} \frac{x(\cos x+\cos 2x)}{\sin x}$

[য.'০৯; রা.'১১; চ,'১৩]

$= \lim_{x\to 0} \frac{x}{\sin x} \times \lim_{x\to 0} (\cos x + \cos 2x)$

$= 1 \times (\cos 0 + \cos 0)$

$= 1 + 1 = 1$ (Ans.)

7.(a) $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$ [রা.'০৯; ব.'১১, '১৪; কু.'১০; সি.'০৯; মা.'১৩; ঢা.'১১,'১৫]

$= \lim_{x\to 0} \frac{\tan x(1-\cos x)}{x^3} = \lim_{x\to 0} \frac{\tan x . 2\sin^2 \frac{x}{2}}{x^3}$

$= 2 \lim_{x\to 0} \frac{\tan x}{x} \times \lim_{x\to 0} \left\{\frac{\sin(x/2)}{x/2}\right\}^2 \times \frac{1}{4}$

$= 2 \times 1 \times 1 \times \frac{1}{4} = \frac{1}{2}$ (Ans.)

7(b) $\lim_{x \to 0} \frac{\tan 2x - \sin 2x}{x^3}$ [মা.'০৪,'০৭]

$= \lim_{x\to 0} \frac{\tan 2x(1-\cos 2x)}{x^3} = \lim_{x\to 0} \frac{\tan 2x . 2\sin^2 x}{x^3}$

$= 2 \lim_{x\to 0} \frac{\tan 2x}{2x} \times 2 \times \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2$

$= 2 \times 1 \times 2 \times 1 = 4$ (Ans.)

7(c) $\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x}$ [ঢা.'০৯]

$= \lim_{x\to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x} = \lim_{x\to 0} \frac{1-\cos x}{x \sin x}$

$= \lim_{x\to 0} \frac{2\sin^2 \frac{x}{2}}{x . 2\sin\frac{x}{2}\cos\frac{x}{2}} = \lim_{x\to 0} \frac{\tan\frac{x}{2}}{\frac{x}{2}} \times \frac{1}{2}$

$= 1 \times \frac{1}{2} = \frac{1}{2}$ (Ans.)

7(d) $\lim_{x \to y} \frac{\sin x - \sin y}{x - y}$ [কু.'০৫]

$= \lim_{x\to y} \frac{2\sin\frac{x-y}{2}\cos\frac{x+y}{2}}{x-y}$

$= 2 . \lim_{x \to y} \frac{\sin\frac{x-y}{2}}{\frac{x-y}{2}} \times \frac{1}{2} \times \lim_{x\to y} \cos\frac{x+y}{2}$

$= 2 \times 1 \times \frac{1}{2} \cos\frac{y+y}{2} = \cos y$ (Ans.)

7(e) $\lim_{x\to\alpha} \frac{\tan x - \tan\alpha}{x - \alpha} = \lim_{x\to\alpha} \frac{\frac{\sin x}{\cos x} - \frac{\sin\alpha}{\cos\alpha}}{x-\alpha}$

$= \lim_{x\to\alpha} \frac{\sin x\cos\alpha - \cos x\sin\alpha}{(x-\alpha)\cos x\cos\alpha}$

$= \lim_{x\to\alpha} \frac{\sin(x-\alpha)}{(x-\alpha)\cos x\cos\alpha}$

$= \frac{1}{\cos\alpha} \lim_{(x-\alpha)\to 0} \frac{\sin(x-\alpha)}{x-\alpha} \times \lim_{x\to\alpha} \frac{1}{\cos x}$

$= \frac{1}{\cos\alpha} \times 1 \times \frac{1}{\cos\alpha} = \sec^2\alpha$ (Ans.)

8.(a) $\lim_{x \to 0} \frac{\tan ax}{\sin bx}$ [ঢা.'০৬]

$= \lim_{x \to 0} \frac{\tan ax}{\sin bx} = \frac{\lim_{ax\to 0} \frac{\tan ax}{ax} \times a}{\lim_{bx\to 0} \frac{\sin bx}{bx} \times b}$

$= \frac{1 \times a}{1 \times b} = \frac{a}{b}$ (Ans.)

8(b) $\lim_{x \to 0} \frac{1 - \cos ax}{1 - \cos bx}$ [চ.'০১]

$= \lim_{x \to 0} \frac{2\sin^2\frac{ax}{2}}{2\sin^2\frac{bx}{2}}$

$= \frac{\lim_{x\to 0}\left\{\frac{\sin(ax/2)}{ax/2}\right\}^2 \times \frac{a^2}{4}}{\lim_{x\to 0}\left\{\frac{\sin(bx/2)}{bx/2}\right\}^2 \times \frac{b^2}{4}} = \frac{1 \times \frac{a^2}{4}}{1 \times \frac{b^2}{4}} = \frac{a^2}{b^2}$

8(c) $\lim_{x \to 0} \frac{\cos 7x - \cos 9x}{\cos 3x - \cos 5x}$ [ঢা.'০৫; কু.'০৭]

$$= \lim_{x\to 0} \frac{2\sin\frac{1}{2}(7x+9x)\sin\frac{1}{2}(9x-7x)}{2\sin\frac{1}{2}(3x+5x)\sin\frac{1}{2}(5x-3x)}$$

$$= \lim_{x\to 0} \frac{\sin 8x \sin x}{\sin 4x \sin x} = \lim_{x\to 0} \frac{2\sin 4x\cos 4x}{\sin 4x}$$

$$= 2\lim_{x\to 0} \cos 4x = 2\cos 0 = 2\,.\,1 = 2$$

8(d) $\lim_{x\to 0} \frac{\sin 7x - \sin x}{\sin 6x}$ [চ.,মা.'০৩; দি.'১২]

$$= \lim_{x\to 0} \frac{2\sin\frac{1}{2}(7x-x)\cos\frac{1}{2}(7x+x)}{\sin 6x}$$

$$= \lim_{x\to 0} \frac{2\sin 3x\cos 4x}{2\sin 3x\cos 3x} = \lim_{x\to 0} \frac{\cos 4x}{\cos 3x}$$

$$= \frac{\cos 0}{\cos 0} = \frac{1}{1} = 1 \text{ (Ans.)}$$

8(e) $\lim_{x\to \frac{\pi}{2}} \{\sec x(\sec x - \tan x)\}$ [ঢা.'০৭]

$$= \lim_{x\to \frac{\pi}{2}} \frac{1}{\cos x}\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos^2 x} = \lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{1-\sin^2 x}$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{(1-\sin x)(1+\sin x)}$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{1}{1+\sin x} = \frac{1}{1+\sin\frac{\pi}{2}} = \frac{1}{1+1} = \frac{1}{2}$$

8. (f) $\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)$

[চ.'০৯; ব.'১০; সি.'১৪; প্র.ভ.প.'০৪]

$$= \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right) = \lim_{x\to 0} \frac{1-\cos x}{\sin x}$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \lim_{x\to 0} \tan\frac{x}{2}$$

$$= \tan\frac{0}{2} = \tan 0 = 0 \text{ (Ans.)}$$

8(g) $\lim_{\theta\to 0} \frac{1}{\theta}\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)$ [ঢা.'০১; রা.'১৩]

$$= \lim_{\theta\to 0} \frac{1}{\theta}\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) = \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta\sin\theta}$$

$$= \lim_{\theta\to 0} \frac{2\sin^2\frac{\theta}{2}}{\theta.2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \lim_{x\to 0} \frac{\tan\frac{\theta}{2}}{\theta}$$

$$= \lim_{\theta\to 0} \frac{\tan\frac{\theta}{2}}{\frac{\theta}{2}} \times \frac{1}{2} = 1 \times \frac{1}{2} = \frac{1}{2} \text{ (Ans.)}$$

8(h) $\lim_{x\to 0} \frac{1+\sin x}{\cos x}$ [রা.'০৪]

$$= \frac{1+\sin 0}{\cos 0} = \frac{1+0}{1} = 1 \text{ (Ans.)}$$

9(a) $\lim_{x\to 0} \frac{\sin 2x}{2x^2+x}$ [চ.'০২]

$$= \lim_{x\to 0} \frac{\sin 2x}{x(2x+1)} = \frac{\lim_{x\to 0} \frac{\sin 2x}{2x} \times 2}{\lim_{x\to 0} (2x+1)}$$

$$= \frac{1\times 2}{2\times 0+1} = 2 \text{ (Ans.)}$$

9(b) $\lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x^2\to 0} \frac{\sin x^2}{x^2} \times \lim_{x\to 0} x$

$$= 1\times 0 = 0 \text{ (Ans.)}$$

10.(a) $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$

[য. '০৪; ব. '০৬; ঢা.'১৩ রা.'১৪]

ধরি, $x = \frac{\pi}{2} + h$. $\because x \to \frac{\pi}{2} \quad \therefore h \to 0$

$$\therefore \lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} = \lim_{h\to 0} \frac{1-\sin(\frac{\pi}{2}+h)}{\cos(\frac{\pi}{2}+h)}$$

$$= \lim_{h \to 0} \frac{1-\cos h}{-\sin h} = \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{-2\sin\frac{h}{2}\cos\frac{h}{2}}$$

$$= -\lim_{h \to 0} \tan\frac{h}{2} = -\tan\frac{0}{2} = -\tan 0 = 0$$

10(b) $\lim_{x \to \frac{\pi}{2}} (\frac{\pi}{2} - x)\tan x$ [চ.'১০]

ধরি, $\frac{\pi}{2} - x = h$. $\because x \to \frac{\pi}{2}$ $\therefore h \to 0$

$$\therefore \lim_{x \to \frac{\pi}{2}} (\frac{\pi}{2} - x)\tan x$$

$$= \lim_{x \to \frac{\pi}{2}} h\tan(\frac{\pi}{2} - h) = \lim_{h\to 0} h\cot h$$

$$= \lim_{h\to 0} \frac{h}{\tan h} = 1$$

10(c) $\lim_{x \to \frac{\pi}{2}} \frac{\sec x - \tan x}{\frac{\pi}{2} - x}$ [ব.'০২]

ধরি, $\frac{\pi}{2} - x = h$. $\because x \to \frac{\pi}{2}$ $\therefore h \to 0$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\sec x - \tan x}{\frac{\pi}{2} - x}$$

$$= \lim_{h\to 0} \frac{\sec(\frac{\pi}{2} - h) - \tan(\frac{\pi}{2} - h)}{h}$$

$$= \lim_{h\to 0} \frac{\operatorname{cosec} h - \cot h}{h} = \lim_{h\to 0} \frac{\frac{1}{\sin h} - \frac{\cos h}{\sin h}}{h}$$

$$= \lim_{h\to 0} \frac{1-\cos h}{h\sin h} = \lim_{h\to 0} \frac{2\sin^2\frac{h}{2}}{h.2\sin\frac{h}{2}\cos\frac{h}{2}}$$

$$= \lim_{h\to 0} \frac{\tan\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} = 1 \times \frac{1}{2} = \frac{1}{2} \text{ (Ans.)}$$

10(d) $\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{(\frac{\pi}{2} - x)^2}$ [য.'০৬,'১০; কু.'০৮]

ধরি, $\frac{\pi}{2} - x = h$. $\because x \to \frac{\pi}{2}$ $\therefore h \to 0$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{(\frac{\pi}{2} - x)^2} = \lim_{h \to 0} \frac{1-\sin(\frac{\pi}{2} - h)}{h^2}$$

$$= \lim_{h \to 0} \frac{1-\cos h}{h^2} = \lim_{h \to 0} \frac{2\sin^2(h/2)}{(h/2)^2 \times 4}$$

$$= \frac{1}{2}\lim_{h \to 0} \left\{\frac{\sin(h/2)}{h/2}\right\}^2 = \frac{1}{2} \times 1 = \frac{1}{2} \text{ (Ans.)}$$

10(e) $\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{1-\tan x} = \lim_{x \to \frac{\pi}{4}} \frac{\sin(\frac{\pi}{2} - 2x)}{\tan\frac{\pi}{4} - \tan x}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sin(\frac{\pi}{2} - 2x)}{\frac{\sin(\pi/4)}{\sin(\pi/4)} - \frac{\sin x}{\cos x}}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x.\cos\frac{\pi}{4}.\sin(\frac{\pi}{2} - 2x)}{\sin(\pi/4)\cos x - \sin x\cos(\pi/4)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x.\sin 2(\frac{\pi}{4} - x)}{\sin(\frac{\pi}{4} - x)}.\frac{1}{\sqrt{2}}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x.2\sin(\frac{\pi}{4} - x)\cos(\frac{\pi}{4} - x)}{\sin(\frac{\pi}{4} - x)}.\frac{1}{\sqrt{2}}$$

$$= \lim_{x \to \frac{\pi}{4}} 2\cos(\frac{\pi}{4} - x)\cos x.\frac{1}{\sqrt{2}}$$

$$= 2\cos(\frac{\pi}{4} - \frac{\pi}{4})\cos\frac{\pi}{4}.\frac{1}{\sqrt{2}} = 2.1.\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}$$

$$= 1 \text{ (Ans.)}$$

11.(a) $\lim\limits_{x\to 0} \dfrac{\sin^{-1}x}{x}$

ধরি, $\sin^{-1}x = \theta \Rightarrow \sin\theta = x$

$\because x\to 0 \quad \therefore \theta\to 0$

$\therefore \lim\limits_{x\to 0} \dfrac{\sin^{-1}x}{x} = \lim\limits_{\theta\to 0} \dfrac{\theta}{\sin\theta} = 1$

11(b) $\lim\limits_{x\to 0} \dfrac{\sin^{-1}(3x)}{4x}$

ধরি, $\sin^{-1}(3x) = \theta \Rightarrow \sin\theta = 3x$

$\because x\to 0 \quad \therefore \theta\to 0$

$\therefore \lim\limits_{x\to 0} \dfrac{\sin^{-1}(3x)}{4x} = \lim\limits_{\theta\to 0} \dfrac{\theta}{\frac{4}{3}\sin\theta}$

$= \dfrac{3}{4}\lim\limits_{\theta\to 0} \dfrac{\theta}{\sin\theta} = \dfrac{3}{4}\times 1 = \dfrac{3}{4}$ (Ans.)

12. (a) $\lim\limits_{x\to 0} \dfrac{e^{2x}-(1+x)^7}{\ln(1+x)}$

$= \lim\limits_{x\to 0} \dfrac{\{1+2x+\frac{(2x)^2}{2!}\cdots\}-(1+7x+21x^2+\cdots)}{x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots}$

$= \lim\limits_{x\to 0} \dfrac{(2-7)x+(2-21)x^2+\cdots}{x(1-\frac{x}{2}+\frac{x^2}{3}-\cdots)}$

$= \lim\limits_{x\to 0} \dfrac{-5-19x+\cdots}{1-\frac{x}{2}+\frac{x^2}{3}-\cdots} = \dfrac{-5-19\times 0+0+\cdots}{1-\frac{0}{2}+\frac{0^2}{3}-0+\cdots}$

$= \dfrac{-5}{1} = -5$ (Ans.)

12(b) $\lim\limits_{x\to 0} \dfrac{a^x-1}{x}$

$= \lim\limits_{x\to 0} \dfrac{\{1+x\ln a+\frac{(x\ln a)^2}{2!}+\cdots\}-1}{x}$

$= \lim\limits_{x\to 0} \dfrac{x\{\ln a+\frac{x(\ln a)^2}{2!}+\frac{x^2(\ln a)^3}{3!}+\cdots\}}{x}$

$= \lim\limits_{x\to 0}\{\ln a+\frac{x(\ln a)^2}{2!}+\frac{x^2(\ln a)^3}{3!}+\cdots\}$

$= \ln a+\frac{0\times(\ln a)^2}{2!}+\frac{0^2(\ln a)^3}{3!}+\cdots\cdots\}$

$= \ln a$

12(c) $\lim\limits_{x\to 0} \dfrac{e^{\sin x}-1}{\sin x}$ [কু.'০১; মা.বো.'০৯; রা.'১২]

$= \lim\limits_{x\to 0} \dfrac{\{1+\sin x+\frac{\sin^2 x}{2!}+\frac{\sin^3 x}{3!}+\cdots)-1}{\sin x}$

$= \lim\limits_{x\to 0} \dfrac{\sin x+\frac{\sin^2 x}{2!}+\frac{\sin^3 x}{3!}+\cdots}{\sin x}$

$= \lim\limits_{x\to 0}(1+\frac{\sin x}{2!}+\frac{\sin^2 x}{3!}+\cdots)$

$= 1+\frac{\sin 0}{2!}+\frac{\sin^2 0}{2!}+\cdots = 1+0+0\cdots$

$= 1$

12(d) $\lim\limits_{x\to 0} \dfrac{a^x-a^{-x}}{x}$ [প্র.ভ.প.'০৬]

$= \lim\limits_{x\to 0}\frac{1}{x}[\{1+x\ln a+\frac{(x\ln a)^2}{2!}+\frac{(x\ln a)^3}{3!}+\cdots\}$

$-\{1-x\ln a+\frac{(x\ln a)^2}{2!}-\frac{(x\ln a)^3}{3!}+\cdots\}]$

$= \lim\limits_{x\to 0}\frac{1}{x}\{2x\ln a+2\frac{(x\ln a)^3}{3!}+\cdots\cdots\}$

$= 2\lim\limits_{x\to 0}\{\ln a+\frac{x^2(\ln a)^3}{3!}+\frac{x^4(\ln a)^5}{5!}+\cdots\}$

$= 2\lim\limits_{x\to 0}\{\ln a+\frac{0^2(\ln a)^3}{3!}+\frac{0^4(\ln a)^5}{5!}+\cdots\}$

$= 2\ln a$ (Ans.)

12(e) $\lim\limits_{x\to\infty}(1+\frac{b}{x})^{\frac{x}{a}}, a>0, b>0$

$= \lim\limits_{x\to\infty}(1+\frac{b}{x})^{\frac{x}{a}}$

$$= \lim_{x\to\infty}\{1+\frac{\frac{x}{a}}{1!}.\frac{b}{x}+\frac{\frac{x}{a}(\frac{x}{a}-1)}{2!}(\frac{b}{x})^2+$$

$$+\frac{\frac{x}{a}(\frac{x}{a}-1)(\frac{x}{a}-2)}{3!}(\frac{x}{a})^3+\cdots\cdots\}$$

$$= \lim_{x\to\infty}\{1+\frac{b}{a}+\frac{\frac{x^2}{a^2}(1-\frac{a}{x})}{2!}\frac{b^2}{x^2}+$$

$$\frac{\frac{x^3}{a^3}(1-\frac{a}{x})(1-\frac{2a}{x})}{3!}\frac{b^3}{x^3}+\cdots\}$$

$$= \lim_{x\to\infty}\{1+\frac{b}{a}+\frac{1-\frac{a}{x}}{2!}\frac{b^2}{a^2}+$$

$$\frac{(1-\frac{a}{x})(1-\frac{2a}{x})}{3!}\frac{b^3}{a^3}+\cdots\}$$

$$=1+\frac{b}{a}+\frac{1-0}{2!}\frac{b^2}{a^2}+\frac{(1-0)(1-0)}{3!}\frac{b^3}{a^3}+\cdots$$

$$=1+\frac{b}{a}+\frac{1}{2!}(\frac{b}{a})^2+\frac{1}{3!}(\frac{b}{a})^3+\cdots\cdots=e^{\frac{b}{a}}$$

12(f) $f(x)=\sin x$ হলে, $\lim\limits_{h\to 0}\frac{f(x+nh)-f(x)}{h}$ এর মান নির্ণয় কর। [প্র.ভ.প.'০০]

$$\lim_{h\to 0}\frac{f(x+nh)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sin(x+nh)-\sin x}{h}$$

$$=\lim_{h\to 0}\frac{2\sin\frac{nh}{2}\cos\frac{1}{2}(2x+nh)}{h}$$

$$=2\lim_{h\to 0}\frac{\sin\frac{nh}{2}}{\frac{nh}{2}}\times\frac{n}{2}\lim_{h\to 0}\cos\frac{1}{2}(2x+nh)$$

$$=2\times 1\times\frac{n}{2}\times\cos\frac{1}{2}(2x+n\times 0)$$

$$=n\cos x \text{ (Ans.)}$$

(g) $$\lim_{x\to 0}\frac{e^{ax}-\cos ax}{e^{bx}-\cos bx}$$

$$=\lim_{x\to 0}\frac{\{1+ax+\frac{(ax)^2}{2!}+\cdots\}-\{1-\frac{(ax)^2}{2!}+\frac{(ax)^4}{4!}-\cdots\}}{\{1+bx+\frac{(bx)^2}{2!}+\cdots\}-\{1-\frac{(bx)^2}{2!}+\frac{(bx)^4}{4!}-\cdots\}}$$

$$=\lim_{x\to 0}\frac{ax+2\frac{(ax)^2}{2!}+\frac{(ax)^3}{3!}+\cdots}{bx+2\frac{(bx)^2}{2!}+\frac{(bx)^3}{3!}+\cdots}$$

$$=\lim_{x\to 0}\frac{a+2\frac{a^2x}{2!}+\frac{a^3x^2}{3!}+\cdots}{b+2\frac{b^2x}{2!}+\frac{b^3x^2}{3!}+\cdots}$$

$$=\frac{a+2.0+0+\cdots}{b+2.0+0+\cdots}=\frac{a}{b} \text{ (Ans.)}$$

(h) $$\lim_{x\to 0}\frac{12-6x^2-12\cos x}{x^4}$$

ধরি, $x=2y \quad \because x\to 0 \quad \therefore y\to 0$

$$\therefore \lim_{x\to 0}\frac{12-6x^2-12\cos x}{x^4}$$

$$=\lim_{y\to 0}\frac{12-6(2y)^2-12\cos 2y}{(2y)^4}$$

$$=\lim_{y\to 0}\frac{12-24y^2-12\cos 2y}{16y^4}$$

$$=\lim_{y\to 0}\frac{3(1-\cos 2y)-6y^2}{4y^4}$$

$$=\lim_{y\to 0}\frac{3.2\sin^2 y-6y^2}{4y^4}$$

$$=\lim_{y\to 0}\frac{3\{y-\frac{y^3}{3!}+\frac{y^5}{5!}-\cdots\infty\}^2-3y^2}{2y^4}$$

$$=\lim_{y\to 0}\frac{3[y^2-\frac{2y^4}{3!}+\{\frac{1}{(3!)^2}+\frac{2}{5!}\}y^6+\cdots\infty\}^2-3y^2}{2y^4}$$

$$=\lim_{y\to 0}\frac{3[y^2-\frac{2y^4}{3!}+\{\frac{1}{(3!)^2}+\frac{2}{5!}\}y^6+\cdots\infty]-3y^2}{2y^4}$$

$$= \lim_{y\to 0}\left[\frac{-\frac{6}{3!}+3\{\frac{1}{(3!)^2}+\frac{2}{5!}\}y^2+\cdots\infty}{2}\right]$$

$$= \lim_{y\to 0}\left[\frac{-1+3\{\frac{1}{(3!)^2}+\frac{2}{5!}\}y^2+\cdots\infty}{2}\right]$$

$$= -\frac{1}{2} \text{ (Ans.)}$$

13. (a) $\lim_{n\to\infty}\frac{1^2+2^2+\cdots\cdots+n^2}{n^3}$

$$=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=\lim_{n\to\infty}\frac{n^3(1+\frac{1}{n})(2+\frac{1}{n})}{6n^3}$$

$$=\lim_{n\to\infty}\frac{(1+\frac{1}{n})(2+\frac{1}{n})}{6}=\frac{(1+0)(2+0)}{6}$$

$$=\frac{2}{6}=\frac{1}{3} \text{ (Ans.)}$$

13(b) $\lim_{n\to\infty}\frac{1}{n^4}\sum_{r=1}^{n}r^3$

$$=\lim_{n\to\infty}\frac{1}{n^4}(1^3+2^3+3^3+\cdots+n^3)$$

$$=\lim_{n\to\infty}\frac{n^2(n+1)^2}{4n^4}=\lim_{n\to\infty}\frac{n^4(1+\frac{1}{n})^2}{4n^4}$$

$$=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^2}{4}=\frac{(1+0)^2}{4}=\frac{1}{4} \text{ (Ans.)}$$

13(c) $\lim_{n\to\infty}\frac{1.3+2.4+\cdots\cdots+n(n+2)}{n^3}$

সমাধান :মনে করি, $1.3+2.4+\cdots+n(n+2)$ ধারার nতম পদ u_n.

$\therefore u_n = n(n+2) = n^2+2n$

$$\therefore 1.3+2.4+\cdots+n(n+2)=\sum_{n=1}^{n}n^2+2\sum_{n=1}^{n}n$$

$$=\frac{n(n+1)(2n+1)}{6}+2\frac{n(n+1)}{2}$$

$$=n(n+1)(\frac{2n+1}{6}+1)$$

$$=n(n+1)\frac{2n+1+6}{6}=\frac{n(n+1)(2n+7)}{3}$$

$$\lim_{n\to\infty}\frac{1.3+2.4+\cdots\cdots+n(n+2)}{n^3}$$

$$=\lim_{n\to\infty}\frac{n(n+1)(2n+7)}{6n^3}=\lim_{n\to\infty}\frac{n^3(1+\frac{1}{n})(2+\frac{7}{n})}{6n^3}$$

$$=\lim_{n\to\infty}\frac{(1+\frac{1}{n})(2+\frac{7}{n})}{6}=\frac{(1+0)(2+0)}{6}=\frac{1}{3}$$

14. যদি $f(x)=\frac{2x}{1-x}$ হয় , তবে (a) $\lim_{x\to 1+}f(x)$ এবং $\lim_{x\to 1-}f(x)$ এর মান নির্ণয় কর।

সমাধানঃ ধরি $x=1+h$

$$\therefore \lim_{x\to 1+}f(x)=\lim_{h\to 0+}\frac{2(1+h)}{1-(1+h)}=\lim_{h\to 0+}\frac{2+2h}{1-1-h}$$

$$=\lim_{h\to 0+}\frac{2+2h}{-h}=\lim_{h\to 0+}(-\frac{2}{h}-2)$$

$$=-\infty-2=-\infty \text{ (Ans.)}$$

$$\lim_{x\to 1-}f(x)=\lim_{h\to 0-}\frac{2(1+h)}{1-(1+h)}=\lim_{h\to 0-}\frac{2+2h}{1-1-h}$$

$$=\lim_{h\to 0-}\frac{2+2h}{-h}=\lim_{h\to 0-}(-\frac{2}{h}-2)$$

$$=+\infty-2=+\infty \text{ (Ans.)}$$

(b) $\lim_{x\to\infty}f(x)$ এবং $\lim_{x\to-\infty}f(x)$ এর মান নির্ণয় কর।

সমাধান ঃ $\lim_{x\to\infty}f(x)=\lim_{x\to\infty}\frac{2x}{1-x}$

$$=\lim_{x\to\infty}\frac{2x}{x(\frac{1}{x}-1)}=\lim_{x\to\infty}\frac{2}{\frac{1}{x}-1}$$

$$=\frac{2}{0-1}=-2 \text{ (Ans.)}$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{2x}{1-x}$$

$$=\lim_{x\to-\infty}\frac{2x}{x(\frac{1}{x}-1)}=\lim_{x\to-\infty}\frac{2}{\frac{1}{x}-1}$$

$$=\frac{2}{-0-1}=-2 \text{ (Ans.)}$$

সম্ভাব্য ধাপসহ প্রশ্ন:

15. $\lim\limits_{x\to1}\frac{1}{x-1}(\frac{1}{x+3}-\frac{2}{3x+5})$

$=\lim\limits_{x\to1}\frac{3x+5-2x-6}{(x-1)(x+3)(3x+5)}$ (১)

$=\lim\limits_{x\to1}\frac{x-1}{(x-1)(x+3)(3x+5)}$

$=\lim\limits_{x\to1}\frac{1}{(x+3)(3x+5)}=\frac{1}{(1+3)(3.1+5)}$ (১)

$=\frac{1}{4.8}=\frac{1}{32}$ (Ans.)

16.(a) $\lim\limits_{x\to2}\frac{4-x^2}{3-\sqrt{x^2+5}}$

$=\lim\limits_{x\to2}\frac{(4-x^2)(3+\sqrt{x^2+5})}{(3-\sqrt{x^2+5})(3+\sqrt{x^2+5})}$ (১)

$=\lim\limits_{x\to2}\frac{(4-x^2)(3+\sqrt{x^2+5})}{3^2-(x^2+5)}$

$=\lim\limits_{x\to2}\frac{(4-x^2)(3+\sqrt{x^2+5})}{9-x^2-5}$

$=\lim\limits_{x\to2}\frac{(4-x^2)(3+\sqrt{x^2+5})}{4-x^2}$

$=\lim\limits_{x\to2}(3+\sqrt{x^2+5})=3+\sqrt{2^2+5}$

$=3+3=6$ (Ans.) (১)

16(b) $\lim\limits_{x\to1}\frac{x-1}{\sqrt{x^2-1}+\sqrt{x-1}}$ [প্র.ভ.প. ৮৩]

$=\lim\limits_{x\to1}\frac{(x-1)(\sqrt{x^2-1}-\sqrt{x-1})}{(\sqrt{x^2-1}+\sqrt{x-1})(\sqrt{x^2-1}-\sqrt{x-1})}$ (১)

$=\lim\limits_{x\to1}\frac{(x-1)(\sqrt{x^2-1}-\sqrt{x-1})}{(x^2-1)-(x-1)}$

$=\lim\limits_{x\to1}\frac{(x-1)(\sqrt{x^2-1}-\sqrt{x-1})}{x^2-1-x+1}$

$=\lim\limits_{x\to1}\frac{(x-1)(\sqrt{x^2-1}-\sqrt{x-1})}{x(x-1)}$

$=\lim\limits_{x\to1}\frac{\sqrt{x^2-1}-\sqrt{x-1}}{x}$

$=\frac{\sqrt{1^2-1}-\sqrt{1-1}}{1}=\frac{0}{1}=0$ (Ans.) (১)

16(c) $\lim\limits_{h\to0}\frac{(x+h)^{1/2}-x^{1/2}}{h}$ [সি.'০১]

$=\lim\limits_{h\to0}\frac{\{(x+h)^{1/2}-x^{1/2}\}\{(x+h)^{1/2}+x^{1/2}\}}{h\{(x+h)^{1/2}+x^{1/2}\}}$ (১)

$=\lim\limits_{h\to0}\frac{\{(x+h)^{1/2}\}^2-\{x^{1/2}\}^2}{h\{(x+h)^{1/2}+x^{1/2}\}}$

$=\lim\limits_{h\to0}\frac{x+h-x}{h\{(x+h)^{1/2}+x^{1/2}\}}$

$=\lim\limits_{h\to0}\frac{h}{h\{(x+h)^{1/2}+x^{1/2}\}}$

$=\lim\limits_{h\to0}\frac{1}{(x+h)^{1/2}+x^{1/2}}$

$=\frac{1}{(x+0)^{1/2}+x^{1/2}}=\frac{1}{x^{1/2}+x^{1/2}}=\frac{1}{2\sqrt{x}}$ (১)

16.(d) $\lim\limits_{x\to0}\frac{a-\sqrt{a^2-x^2}}{x^2}$

$=\lim\limits_{x\to0}\frac{a-\sqrt{a^2-x^2}}{x^2}$

$=\lim\limits_{x\to0}\frac{(a-\sqrt{a^2-x^2})(a+\sqrt{a^2+x^2})}{x^2(a+\sqrt{a^2-x^2})}$ (১)

$$=\lim_{x\to0}\frac{a^2-(\sqrt{a^2-x^2})^2}{x^2(a+\sqrt{a^2-x^2})}=\lim_{x\to0}\frac{a^2-a^2+x^2}{x^2(a+\sqrt{a^2-x^2})}$$

$$=\lim_{x\to0}\frac{x^2}{x^2(a+\sqrt{a^2-x^2})}=\lim_{x\to0}\frac{1}{a+\sqrt{a^2-x^2}}$$

$$=\lim_{x\to0}\frac{1}{a+\sqrt{a^2-0^2}}=\frac{1}{a+a}=\frac{1}{2a}$$ (১)

17. $\lim\limits_{x\to\infty}\dfrac{2x^2+1}{6+x-3x^2}$

$$=\lim_{x\to\infty}\frac{x^2(2+\frac{1}{x^2})}{x^2(\frac{6}{x^2}+\frac{1}{x}-3)}=\lim_{x\to\infty}\frac{2+\frac{1}{x^2}}{\frac{6}{x^2}+\frac{1}{x}-3}$$ (১)

$$=\frac{2+0}{0+0-3}=-\frac{2}{3}$$ (১)

18.(a) $\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2}$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}}{x^2}=\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{2.\frac{x^2}{4}}$$ (১)

$$=\frac{1}{2}\lim_{x\to0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2=\frac{1}{2}.1=\frac{1}{2}\text{ (Ans.)}$$ (১)

18(b) $\lim\limits_{x\to0}\dfrac{1-\cos x}{x}=\lim\limits_{x\to0}\dfrac{2\sin^2\frac{x}{2}}{x}$ (১)

$$=\lim_{x\to0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\lim_{x\to0}\sin\frac{x}{2}=1.\sin\frac{0}{2}$$

$$=1.0=0\text{ (Ans.)}$$ (১)

19. $\lim\limits_{x\to0}\dfrac{3\sin\pi x-\sin3\pi x}{x^3}$

$$=\lim_{x\to0}\frac{4\sin^3\pi x}{x^3}=4\lim_{x\to0}\left(\frac{\sin\pi x}{\pi x}\right)^3.\pi^3$$ (১)

$$=4\times1\times\pi^3=4\pi^3$$ (১)

20.(a) $\lim\limits_{x\to0}\dfrac{\sin5x}{\sin3x}=\dfrac{\lim\limits_{x\to0}\frac{\sin5x}{5x}\times5}{\lim\limits_{x\to0}\frac{\sin3x}{3x}\times3}$ (১)

$$=\frac{1\times5}{1\times3}=\frac{5}{3}\text{ (Ans.)}$$ (১)

20. (b) $\lim\limits_{x\to0}\dfrac{6x-\sin2x}{2x+3\sin4x}$

$$=\lim_{x\to0}\frac{x(6-\frac{\sin2x}{x})}{x(2+3\frac{\sin4x}{x})}=\frac{6-\lim\limits_{x\to0}\frac{\sin2x}{2x}\times2}{2+3\lim\limits_{x\to0}\frac{\sin4x}{4x}\times4}$$ (১)

$$=\frac{6-1\times2}{2+3\times1\times4}=\frac{6-2}{2+12}=\frac{4}{14}=\frac{2}{7}\text{ (Ans.)}$$ (১)

21(a) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\sin2x}{\cos2x}$ [প্র.ভ.প.'৮৬]

ধরি, $x=\frac{\pi}{4}+h$. $\because x\to\frac{\pi}{4}\ \therefore h\to0$ (১)

$$\therefore\lim_{x\to\frac{\pi}{4}}\frac{1-\sin2x}{\cos2x}=\lim_{h\to0}\frac{1-\sin2(\frac{\pi}{4}+h)}{\cos2(\frac{\pi}{4}+h)}$$

$$=\lim_{h\to0}\frac{1-\sin(\frac{\pi}{2}+2h)}{\cos(\frac{\pi}{2}+2h)}=\lim_{h\to0}\frac{1-\cos2h}{-\sin2h}$$ (১)

$$=\lim_{h\to0}\frac{2\sin^2h}{-2\sin h\cos h}=-\lim_{h\to0}\tan h$$ (১)

$$=-\lim_{h\to0}\frac{\tan h}{h}\times h=-1\times0=0\text{ (Ans.)}$$ (১)

21(b) $\lim\limits_{x\to\pi}\dfrac{\sin x}{\pi-x}$

ধরি, $\pi-x=h$. $\because x\to\pi\ \therefore h\to0$ (১)

$$\therefore\lim_{x\to\pi}\frac{\sin x}{\pi-x}=\lim_{h\to0}\frac{\sin(\pi-h)}{h}$$

$$=\lim_{h\to0}\frac{\sin h}{h}=1\text{ (Ans.)}$$ (১) + (১)

22(a) $\lim\limits_{x\to0}\dfrac{x-\ln(1+x)}{1+x-e^x}$

$$= \lim_{x\to 0}\frac{x-(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots)}{1+x-(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots)} \quad (১)+(১)$$

$$= \lim_{x\to 0}\frac{x-x+\frac{x^2}{2}-\frac{x^3}{3}+\frac{x^4}{4}-\cdots}{1+x-1-x-\frac{x^2}{2!}-\frac{x^3}{3!}-\cdots}$$

$$= \lim_{x\to 0}\frac{x^2(\frac{1}{2}-\frac{x}{3}+\frac{x^2}{4}-\cdots\cdots)}{x^2(-\frac{1}{2!}-\frac{x}{3!}-\cdots\cdots)} \quad (১)$$

$$= \lim_{x\to 0}\frac{\frac{1}{2}-\frac{x}{3}+\frac{x^2}{4}-\cdots\cdots}{-\frac{1}{2!}-\frac{x}{3!}-\cdots\cdots} = \frac{\frac{1}{2}-\frac{0}{3}+\frac{0^2}{4}-\cdots}{-\frac{1}{2!}-\frac{0}{3!}-\cdots}$$

$$= \frac{\frac{1}{2}}{-\frac{1}{2}} = -1 \quad (১)$$

22.(b) $\lim\limits_{x\to 0}\frac{\ln(1+5x)}{\ln(1-5x)}$

$$= \lim_{x\to 0}\frac{5x-\frac{(5x)^2}{2}+\frac{(5x)^3}{3}-\frac{(5x)^4}{4}+\cdots}{-5x-\frac{(5x)^2}{2}-\frac{(5x)^3}{3}-\frac{(5x)^4}{4}-\cdots} \quad (২)$$

$$= \lim_{x\to 0}\frac{5-\frac{5^2x}{2}+\frac{5^3x^2}{3}-\frac{5^4x^3}{4}+\cdots}{-5-\frac{5^2x}{2}-\frac{5^3x^2}{3}-\frac{5^4x^3}{4}-\cdots} \quad (১)$$

$$= \frac{5-\frac{5^2.0}{2}+\frac{5^30^2}{3}-\frac{5^40^3}{4}+\cdots}{-5-\frac{5^2.0}{2}-\frac{5^30^2}{3}-\frac{5^40^3}{4}-\cdots}$$

$$= \frac{5}{-5} = -1 \text{ (Ans.)} \quad (১)$$

22.(c) $\lim\limits_{x\to 0}(1+2x)^{(2x+5)/x} = \lim\limits_{x\to 0}(1+2x)^{2+5/x}$

$$= \lim_{x\to 0}(1+2x)^2 \lim_{x\to 0}(1+2x)^{5/x} \quad (১)$$

$$= (1+2.0)^2 \times \{\lim_{x\to 0}(1+2x)^{\frac{1}{2x}}\}^{10} \quad (১)$$

$$= e^{10} \text{ (Ans.)} \quad (১)$$

23. $f(x) = \begin{cases} e^{-|x|/2}, & \text{যখন } -1 < x < 0 \\ x^2, & \text{যখন } 0 < x < 2 \end{cases}$ হলে

$\lim\limits_{x\to 0} f(x)$ এর মান কি বিদ্যমান আছে?

সমাধানঃ $x = 0$ বিন্দুতে

ডানদিকবর্তী লিমিট $= \lim\limits_{x\to 0_+} f(x) = \lim\limits_{x\to 0_+} x^2 = 0^2 = 0$ (১)

বামদিকবর্তী লিমিট $= \lim\limits_{x\to 0_-} f(x)$

$$= \lim_{x\to 0_-} e^{-|x|/2} = e^{-|0|/2} = e^0 = 1 \quad (১)$$

বামদিকবর্তী লিমিট ও ডানদিকবর্তী লিমিট বিদ্যমান আছে কিন্তু সমান নয়।

$\therefore \lim\limits_{x\to 0} f(x)$ বিদ্যমান নাই। (১)

24. $\lim\limits_{x\to 0}\frac{\sqrt{(x^2+2x-8)(x-2)}}{x^2-4}$ এর মান কি বিদ্যমান আছে?

সমাধানঃ $x = 2$ বিন্দুতে

ডানদিকবর্তী লিমিট $= \lim\limits_{x\to 2^+}\frac{\sqrt{(x^2+2x-8)(x-2)}}{x^2-4}$

$$= \lim_{x\to 2^+}\frac{\sqrt{(x+4)(x-2)(x-2)}}{(x-2)(x+2)}$$

$$= \lim_{x\to 2^+}\frac{\sqrt{(x-2)^2(x+4)}}{(x-2)(x+2)}$$

$$= \lim_{x\to 2^+}\frac{|x-2|\sqrt{(x+4)}}{(x-2)(x+2)} = \lim_{x\to 2^+}\frac{(2-x)\sqrt{(x+4)}}{(x-2)(x+2)}$$

$$= \lim_{x\to 2^+}\frac{-\sqrt{x+4}}{x+2} = -\frac{\sqrt{2+4}}{2+2} = -\frac{\sqrt{6}}{4}$$

বামদিকবর্তী লিমিট $= \lim\limits_{x\to 2^-}\frac{|x-2|\sqrt{(x+4)}}{(x-2)(x+2)}$

$$= \lim_{x \to 2^-} \frac{(x-2)\sqrt{(x+4)}}{(x-2)(x+2)}$$

$$= \lim_{x \to 2^-} \frac{\sqrt{x+4}}{x+2} = \frac{\sqrt{2+4}}{2+2} = \frac{\sqrt{6}}{4}$$

বামদিকবর্তী লিমিট ও ডানদিকবর্তী লিমিট বিদ্যমান কিন্তু সমান নয়।

$\therefore \lim_{x \to 0} \frac{\sqrt{(x^2+2x-8)(x-2)}}{x^2-4}$ এর মান বিদ্যমান নাই।

ভর্তি পরীক্ষার MCQ :

MCQ এর জন্য বিশেষ সূত্র :

L'Hospital's rule : কার্যপ্রণালী ঃ যদি x = a এর জন্য $\frac{f(x)}{g(x)}$ ভগ্নাংশটি অনির্ণেয় আকার যেমন $\frac{0}{0}$ বা $\frac{\infty}{\infty}$ হয়, তবে অনির্ণেয় আকার শেষ না হওয়া পর্যন্ত ভগ্নাংশের লব এবং হরকে পৃথকভাবে অন্তরীকরণ (differentiation) করতে হবে। অতঃপর নতুন ভগ্নাংশে পদত্ত x = a স্থাপন করে ফাংশনের সীমায়িত মান নির্ণয় করতে হয়।

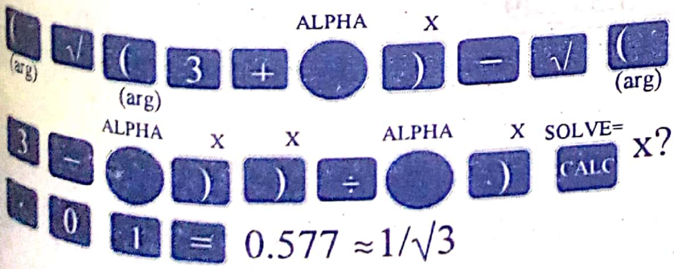
1. যখন $x \to 0$, লিমিট $\frac{\sqrt{3+x}-\sqrt{3-x}}{x}$ কত? [DU 04-05, NU 08-09, 05-06]

$Sol^n: \lim_{x \to 0} \frac{\sqrt{3+x}-\sqrt{3-x}}{x}$

$$= \lim_{x \to 0} \frac{\frac{1}{2\sqrt{3+x}} - \frac{1}{2\sqrt{3-x}}(-1)}{1}$$

$$= \frac{1}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

By Calculator : (Mode Radian এ নিতে হবে)



x?

0.577 ≈1/√3

2. যখন $x \to 0$, লিমিট $\frac{x(\cos x + \cos 2x)}{\sin x}$ কত?

[DU 03-04, RU 06-07, 04-05; KU 03-04]

$Sol^n: \lim_{x \to 0} \frac{x(\cos x + \cos 2x)}{\sin x}$

$$= \lim_{x \to 0} \frac{(\cos x + \cos 2x).1 + x(-\sin x - 2\sin x)}{\cos x}$$

$$= \frac{(\cos.0 + \cos 2.0).1 + 0.(-\sin 0 - 2\sin 0)}{\cos 0} = 2$$

3. যখন $x \to 0$, লিমিট $\frac{\sin 3x}{x}$ কত?

[DU 99-00, RU 06-07]

$Sol^n: \lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} \frac{3\cos 3x}{1}$

$= 3\cos 0 = 3$

4. যখন $x \to 0$, লিমিট $\frac{\tan x - \sin x}{x^3}$ কত?

[KU 03-04]

$Sol^n: \lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \to 0} \frac{\sec^2 x - \cos x}{3x^2}$

$$= \lim_{x \to 0} \frac{2\sec^2 x \tan x + \sin x}{6x}$$

$$= \frac{1}{6} \lim_{x \to 0} \{2(\sec^2 x.\sec^2 x + \tan x.2\sec^2 x \tan x) + \cos x\}$$

$$= \frac{1}{6}\{2(1+0)+1\} = \frac{1}{2}$$

5. $\lim_{x \to 0} \frac{\sin(2x)^2}{x} = ?$ **[DU 08-09]**

$Sol^n: \lim_{x \to 0} \frac{\sin(4x^2)}{x} = \lim_{x \to 0} \frac{\cos(4x^2).8x}{1}$

$= \cos(4.0).8.0 = 0$

6. যখন $x \to \frac{\pi}{2}$, লিমিট $\frac{1-\sin x}{\cos x}$ কত?

[DU 00-01 , RU 06-07]

$$Sol^n: \lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}$$

$= \frac{0}{1} = 0$

7. যখন $x \to 2$, লিমিট $\frac{\sin(x-2)}{x-2}$ কত? **[CU 07-08]**

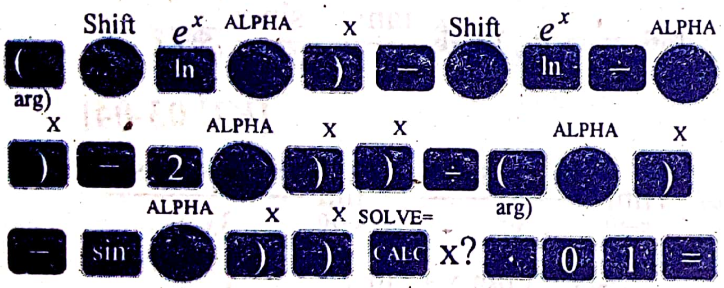
Sol^n: $\lim_{x\to 2}\frac{\sin(x-2)}{x-2} = \lim_{x\to 2}\frac{\cos(x-2)}{1}$

$= \cos(2-2) = \cos 0 = 1$

8. যখন $x \to 0$, লিমিট $\frac{e^x - e^{-x} - 2x}{x - \sin x}$ কত? **[SU 04-05]**

Sol^n: $\lim_{x\to 0}\frac{e^x - e^{-x} - 2x}{x-\sin x} = \lim_{x\to 0}\frac{e^x + e^{-x} - 2}{1-\cos x}$

$= \lim_{x\to 0}\frac{e^x - e^{-x}}{\sin x} = \lim_{x\to 0}\frac{e^x + e^{-x}}{\cos x} = \frac{1+1}{1} = 2$



1.99 ≈2

9. যখন $x \to 0$, লিমিট $\frac{a^x - 1}{x}$ কত? **[CU 08-09; RU 02-03]**

Sol^n: $\lim_{x\to 0}\frac{a^x - 1}{x} = \lim_{x\to 0}\frac{a^x \ln a}{1} = a^0 \log_e a$

$= \log_e a$

10. $\lim_{x\to 0}\frac{1-e^{-2x}}{\ln(1+x)}$, $0 < x < 1$ এর মান কত? **[SU 04-05, KU 03-04]**

Sol^n: $\lim_{x\to 0}\frac{1-e^{-2x}}{\ln(1+x)} = \lim_{x\to 0}\frac{2e^{-2x}}{\frac{1}{1+x}} = \lim_{x\to 0}\frac{2e^{-2x}}{\frac{1}{1+x}} = 2$

11. যখন $x \to 0$, লিমিট $\frac{\sin^{-1}x}{x}$ কত? **[CU 08-09; RU 07-08; IU 04-05]**

Sol^n: $\lim_{x\to 0}\frac{\sin^{-1}x}{x} = \lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$

12. যখন $x \to 0$, লিমিট $\frac{\tan^{-1}x}{x}$ কত? **[DU 06-07]**

Sol^n: $\lim_{x\to 0}\frac{\tan^{-1}x}{x} = \lim_{x\to 0}\frac{\frac{1}{1+x^2}}{1} = \frac{1}{1+0^2} = 1$

13. যখন $x \to 0$, লিমিট $\frac{\tan^{-1}(2x)}{x}$ কত? **[DU 07-08; CU 07-08; NU 06-07]**

Sol^n: $\lim_{x\to 0}\frac{\tan^{-1}2x}{x} = \lim_{x\to 0}\frac{\frac{2}{1+4x^2}}{1} = \frac{2}{1+4.0^2} = 2$

14. $\lim_{x\to 0}\frac{e^x + e^{-x} - 2}{x^2} = ?$ **[BUET 03-04]**

Sol^n: $\lim_{x\to 0}\frac{e^x + e^{-x} - 2}{x^2} = \lim_{x\to 0}\frac{e^x - e^{-x}}{2x}$

$= \lim_{x\to 0}\frac{e^x + e^{-x}}{2} = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1$

15. $\lim_{x\to 0}\frac{1-\cos 7x}{3x^2} = ?$ **[BUET 07-08]**

Sol^n: $\lim_{x\to 0}\frac{1-\cos 7x}{3x^2} = \lim_{x\to 0}\frac{0 + 7\sin 7x}{6x}$

$= \lim_{x\to 0}\frac{0 + 49\cos 7x}{6} = \frac{49}{6}$

16. $\lim_{x\to 3}\frac{x^3 - 27}{x^2 - 9} = ?$ **[KUET 05-06]**

Sol^n: $\lim_{x\to 3}\frac{x^3-27}{x^2-9} = \lim_{x\to 3}\frac{3x^2}{2x} = \lim_{x\to 3}\frac{3x}{2}$

$= \frac{3.3}{2} = \frac{9}{2}$