$\sqrt{x+h} = z + k \cdots \cdots$ (iii)

(iii) – (ii) $\Rightarrow \sqrt{x+h} - \sqrt{x} = k$

$h \to 0$ হলে $k \to 0$

$$\therefore \frac{d}{dx}(e^{\sqrt{x}}) = \lim_{h\to 0} \frac{e^{z+k} - e^z}{k} \cdot \frac{k}{h}$$

$$= \lim_{k\to 0} \frac{e^{z+k} - e^z}{k} \cdot \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{k\to 0} \frac{e^z . e^k - e^z}{k} \cdot \lim_{h\to 0} \frac{\sqrt{x}\sqrt{1+\frac{h}{x}} - \sqrt{x}}{h}$$

$$= \lim_{k\to 0} \frac{e^z(e^k - 1)}{k} \cdot \sqrt{x} \lim_{h\to 0} \frac{\sqrt{1+\frac{h}{x}} - 1}{h}$$

$$= e^z \lim_{k\to 0} \frac{(e^k - 1)}{k} \cdot \sqrt{x} \lim_{h\to 0} \frac{(1+\frac{h}{x})^{1/2} - 1}{h}$$

$$= e^{\sqrt{x}} . 1 . \sqrt{x} \lim_{h\to 0} \frac{\{1 + \frac{1}{2}.\frac{h}{x} + \frac{1}{2}(\frac{1}{2}-1)\frac{h^2}{x^2} + \cdots\} - 1}{h}$$

$$= e^{\sqrt{x}} . \sqrt{x} \lim_{h\to 0} \{\frac{1}{2}.\frac{1}{x} + \frac{1}{2}(\frac{1}{2}-1)\frac{h}{x^2} + \cdots\}$$

$$= e^{\sqrt{x}} . \sqrt{x} . \frac{1}{2} . \frac{1}{x} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

3. (a) মূল নিয়মে $x = 2$ -তে x^5 এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = x^5$. $\therefore f(2) = 2^5$

$$\therefore f'(2) = \lim_{x\to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2} \frac{x^5 - 2^5}{x - 2}$$

$$= 5 \times (2)^4 \quad [\because \lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}]$$

$$= 5 \times 16 = 80$$

3 (b) মূল নিয়মে $x = a$ -তে e^{mx} এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = e^{mx}$ $\therefore f(a) = e^{ma}$

$$\therefore f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x\to a} \frac{e^{mx} - e^{ma}}{x - a} = \lim_{x\to a} \frac{e^{ma}(e^{mx-ma} - 1)}{x - a}$$

$$= e^{ma} \lim_{x-a\to 0} \frac{e^{m(x-a)} - 1}{m(x-a)} \times m$$

$$= me^{ma} . 1 \qquad [\because \lim_{x\to 0} \frac{e^x - 1}{x} = 1]$$

$$= me^{ma}$$

3(c) মূল নিয়মে $x = \frac{\pi}{4}$ -তে $\tan x$ এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = \tan x$. $\therefore f(\frac{\pi}{4}) = \tan\frac{\pi}{4}$

$$\therefore f'(\frac{\pi}{4}) = \lim_{x\to \frac{\pi}{4}} \frac{f(x) - f(\frac{\pi}{4})}{x - \frac{\pi}{4}}$$

$$= \lim_{x\to \frac{\pi}{4}} \frac{\tan x - \tan\frac{\pi}{4}}{x - \frac{\pi}{4}}$$

$$= \lim_{x\to \frac{\pi}{4}} \frac{\sin x \cos\frac{\pi}{4} - \cos x \sin\frac{\pi}{4}}{(x - \frac{\pi}{4})\cos x \cos\frac{\pi}{4}}$$

$$= \lim_{x\to \frac{\pi}{4}} \frac{\sin(x - \frac{\pi}{4})}{(x - \frac{\pi}{4})\cos x \cos\frac{\pi}{4}}$$

$$= \lim_{x-\frac{\pi}{4}\to 0} \frac{\sin(x - \frac{\pi}{4})}{x - \frac{\pi}{4}} \times \lim_{x\to \frac{\pi}{4}} \frac{1}{\cos x \cos\frac{\pi}{4}}$$

$$= 1 . \frac{1}{\cos\frac{\pi}{4}\cos\frac{\pi}{4}} = \frac{1}{(1/\sqrt{2})^2} = 2$$

প্রশ্নমালা IX D

x এর সাপেক্ষে অন্তরক সহগ নির্ণয় কর ঃ

1(a) $\frac{d}{dx}\{x^2 \ln(x)\}$

$= x^2 \frac{d}{dx}\{ln(x)\} + ln(x) \frac{d}{dx}(x^2)$

$= x^2 \frac{1}{x} + ln(x).(2x) = x + 2x\, ln(x)$

1(b) $5e^x \log_a x$ [ব.'০৮;দি.'১৩]

মনে করি, $y = 5e^x \log_a x$

$\therefore \frac{dy}{dx} = 5\{e^x \frac{d}{dx}(\log_a x) + \log_a x \frac{d}{dx}(e^x)\}$

$= 5\{e^x \frac{1}{x \ln a} + \log_a x . e^x\}$

$\therefore \frac{d}{dx}\{5e^x \log_a x\} = 5e^x \{\frac{1}{x \ln a} + \log_a x\}$

1(c) $\log_{10} x$ [দি.'১১,'১৩]

মনে করি, $y = \log_{10} x = \log_{10} e \times \log_e x$

$\Rightarrow y = \frac{1}{\log_e 10} \times \ln x = \frac{1}{\ln 10} \times \ln x$

$\therefore \frac{dy}{dx} = \frac{1}{\ln 10} \frac{d}{dx}(\ln x) = \frac{1}{\ln 10} \times \frac{1}{x}$

$\therefore \frac{d}{dx}(\log_{10} x) = \frac{1}{x \ln 10}$ (Ans.)

1(d) $\log_a x$ [ঢা.'১৩]

মনে করি, $y = \log_a x = \log_a e \times \log_e x$

$\Rightarrow y = \frac{1}{\log_e a} \times \ln x = \frac{1}{\ln a} \times \ln x$

$\therefore \frac{dy}{dx} = \frac{1}{\ln a} \frac{d}{dx}(\ln x) = \frac{1}{\ln a} \times \frac{1}{x}$

$\therefore \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$ (Ans.)

2. (a) $a^x \ln(x) + be^x \sin x$

$\frac{d}{dx}\{a^x \ln(x) + be^x \sin x\} = a^x \frac{d}{dx}\{\ln(x)\}$

$+ \ln(x) \frac{d}{dx}(a^x) + b\{e^x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(e^x)\}$

$= a^x \frac{1}{x} + \ln(x)(a^x \ln a) + b\{e^x(\cos x) + \sin x (e^x)\}$

$= a^x \{\frac{1}{x} + \ln a \ln(x)\} + b e^x (\cos x + \sin x)$

2(b) $x^2 \log_a x - x^3 \ln a^x + 6x e^x \ln x$

ধরি, $y = x^2 \log_a x - x^3 \ln a^x + 6x e^x \ln x$

$= x^2 \log_a x - x^4 \ln a + 6x e^x \ln x$

$\therefore \frac{dy}{dx} = x^2 \frac{d}{dx}(\log_a x) + \log_a x \frac{d}{dx}(x^2) - \ln a \frac{d}{dx}(x^4) + 6\{x e^x \frac{d}{dx}(\ln x) + x \ln x \frac{d}{dx}(e^x) + e^x \ln x \frac{d}{dx}(x)\}$

$= x^2 \frac{1}{x \ln a} + \log_a x .(2x) - \ln a.(4x^3) + 6\{x e^x . \frac{1}{x} + x \ln x . e^x + e^x \ln x . 1\}$

$= x(\frac{1}{\ln a} + 2\log_a x - 4x^2 \ln a) + 6e^x(1 + x \ln x + \ln x)$

3. (a) মনে করি, $y = \frac{x}{x^2 + a^2}$

$\therefore \frac{dy}{dx} = \frac{(x^2 + a^2)\frac{d}{dx}(x) - x\frac{d}{dx}(x^2 + a^2)}{(x^2 + a^2)^2}$

$= \frac{(x^2 + a^2).1 - x(2x + 0)}{(x^2 + a^2)^2} = \frac{x^2 + a^2 - 2x^2}{(x^2 + a^2)^2}$

$\therefore \frac{d}{dx}(\frac{x}{x^2 + a^2}) = \frac{a^2 - x^2}{(x^2 + a^2)^2}$

3(b) $\frac{d}{dx}(\frac{1 - \tan x}{1 + \tan x})$ [দি.'১০; ব.'১৩]

$= \frac{(1 + \tan x)\frac{d}{dx}(1 - \tan x) - (1 - \tan x)\frac{d}{dx}(1 + \tan x)}{(1 + \tan x)^2}$

$$= \frac{(1+\tan x)(-\sec^2 x)-(1-\tan x)(\sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{(-1-\tan x-1+\tan x)\sec^2 x}{(1+\tan x)^2}$$

$$= \frac{-2\sec^2 x}{(1+\tan x)^2} \quad \text{(Ans.)}$$

3(c) $\frac{d}{dx}\left(\frac{1+\sin x}{1+\cos x}\right) =$ **[কু.'০৪]**

$$\frac{(1+\cos x)\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2} =$$

$$\frac{(1+\cos x)(\cos x)-(1+\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\cos x+\cos^2 x+\sin x+\sin^2 x}{(1+\cos x)^2}$$

$$= \frac{\cos x+\sin x+1}{(1+\cos x)^2} \quad \text{(Ans.)}$$

3(d) $\frac{1+\sin x}{1-\sin x}$

[ঢা.'১৩; ব. '০৭; রা.'০৯; চ.'১২; দি.'১৪]

$$\frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right) =$$

$$\frac{(1-\sin x)\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$$

$$= \frac{(1-\sin x)(\cos x)-(1+\sin x)\frac{d}{dx}(-\cos x)}{(1-\sin x)^2}$$

$$= \frac{(1-\sin x+1+\sin x)\cos x}{(1-\sin x)^2}$$

$$= \frac{2\cos x}{(1-\sin x)^2} \quad \text{(Ans.)}$$

3(e) $\frac{\cos x-\cos 2x}{1-\cos x}$

[ব.'১০; রা., কু.'০৮; য.'১৩; ঢা.'১৪]

$$\frac{\cos x-\cos 2x}{1-\cos x} = \frac{\cos x-(2\cos^2 x-1)}{1-\cos x}$$

$$= \frac{1+\cos x-2\cos^2 x}{1-\cos x}$$

$$= \frac{(1-\cos x)(1+2\cos x)}{1-\cos x} = 1+2\cos x$$

$$\therefore \frac{d}{dx}\left(\frac{\cos x-\cos 2x}{1-\cos x}\right) = -2\sin x$$

3(f) $\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}$ **[ঢা.'০৯; ব.'০৯,'১১; য.'১৪]**

$$\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}} = \frac{\sin x+\cos x}{\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}}$$

$$= \frac{\sin x+\cos x}{\sqrt{(\sin x+\cos x)^2}} = \frac{\sin x+\cos x}{\sin x+\cos x} = 1$$

$$\therefore \frac{d}{dx}\left(\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\right) = 0 \text{ (Ans.)}$$

3(g) ধরি, $y = \frac{x\ln x}{\sqrt{1+x^2}}$ **[প্র.ভ.প.'০৫]**

$$\therefore \frac{dy}{dx} = \frac{\sqrt{1+x^2}\frac{d}{dx}(x\ln x)-x\ln x\frac{d}{dx}(\sqrt{1+x^2})}{(\sqrt{1+x^2})^2}$$

$$= \frac{1}{1+x^2}\left[\sqrt{1+x^2}\left(x.\frac{1}{x}+\ln x\right)-x\ln x\frac{2x}{2\sqrt{1+x^2}}\right]$$

$$= \frac{1}{1+x^2}\left[\frac{(1+x^2)(1+\ln x)-x^2\ln x}{\sqrt{1+x^2}}\right]$$

$$\therefore \frac{d}{dx}\left(\frac{x\ln x}{\sqrt{1+x^2}}\right) = \frac{1+x^2+\ln x}{(\sqrt{1+x^2})^3}$$

বিকল্প পদ্ধতি ঃ ধরি, $y = \frac{x\ln x}{\sqrt{1+x^2}}$

$$\frac{dy}{dx} = \frac{x\ln x}{\sqrt{1+x^2}}\left[\frac{1}{x}\frac{d}{dx}(x)+\frac{1}{\ln x}\frac{d}{dx}(\ln x)-\right.$$

$$\left.\frac{1}{\sqrt{1+x^2}}\frac{d}{dx}(\sqrt{1+x^2})\right]$$

$$= \frac{x\ln x}{\sqrt{1+x^2}}\left[\frac{1}{x}+\frac{1}{\ln x}.\frac{1}{x}-\frac{1}{\sqrt{1+x^2}}.\frac{2x}{2\sqrt{1+x^2}}\right]$$

$$= \frac{x\ln x}{\sqrt{1+x^2}}\,\frac{\ln x(1+x^2)+1+x^2-x^2\ln x}{x(1+x^2)\ln x}$$

$$\therefore \frac{d}{dx}\left(\frac{x\ln x}{\sqrt{1+x^2}}\right)=\frac{1+x^2+\ln x}{(\sqrt{1+x^2})^3} \quad \text{(Ans.)}$$

প্রশ্নমালা IX E

1.(a) $(1+\sin 2x)^2$ [চ.'০৪]

ধরি, $y=(1+\sin 2x)^2$

$$\therefore \frac{dy}{dx}=2(1+\sin 2x)\frac{d}{dx}(1+\sin 2x)$$

$$=2(1+\sin 2x)(0+\cos 2x)\frac{d}{dx}(2x)$$

$$=2(1+\sin 2x)\cos 2x\,(2.1)$$

$$\therefore \frac{d}{dx}\{(1+\sin 2x)^2\}=4\cos 2x(1+\sin 2x)$$

1(b) a^{px+q} [চ'০১]

ধরি, $y=a^{px+q}$

$$\therefore \frac{dy}{dx}=a^{px+q}.\ln a\frac{d}{dx}(px+q)$$

$$\left[\because \frac{d}{dx}(a^x)=a^x\ln a\right]$$

$$=a^{px+q}.\ln a\,(p.1+0)$$

$$\therefore \frac{d}{dx}(a^{px+q})=p\,a^{px+q}.\ln a \text{ (Ans.)}$$

1(c) $a^{\cos x}$ [চ.'০০]

$$\frac{d}{dx}(a^{\cos x})=a^{\cos x}.\ln a.\frac{d}{dx}(\cos x)$$

$$=a^{\cos x}.\ln a.(-\sin x)$$

$$=-a^{\cos x}\sin x.\ln a$$

1(d) $10^{\ln(\sin x)}$ [সি.'০২ '০৫; চ.'০৭]

ধরি, $y=10^{\ln(\sin x)}$

$$\therefore \frac{dy}{dx}=10^{\ln(\sin x)}.\ln 10\frac{d}{dx}\{\ln(\sin x)\}$$

$$=10^{\ln(\sin x)}.\ln 10\frac{1}{\sin x}\frac{d}{dx}(\sin x)$$

$$=10^{\ln(\sin x)}.\ln 10\frac{1}{\sin x}(\cos x)$$

$$\therefore \frac{d}{dx}\{10^{\ln(\sin x)}\}=10^{\ln(\sin x)}.\ln 10.\cot x$$

1(e) $10^{\ln(\tan x)}$

ধরি, $y=10^{\ln(\tan x)}$

$$\therefore \frac{dy}{dx}=10^{\ln(\tan x)}.\ln 10\frac{d}{dx}\{\ln(\tan x)\}$$

$$=10^{\ln(\tan x)}.\ln 10\frac{1}{\tan x}\frac{d}{dx}(\tan x)$$

$$=10^{\ln(\sin x)}.\ln 10\frac{\cos x}{\sin x}(\sec^2 x)$$

$$=10^{\ln(\sin x)}.\ln 10\frac{\cos x}{\sin x}.\frac{1}{\cos^2 x}$$

$$=10^{\ln(\sin x)}.\ln 10\frac{2}{2\sin x\cos x}$$

$$=10^{\ln(\sin x)}.\ln 10\frac{2}{\sin 2x}$$

$$=2\operatorname{cosec}2x\,.10^{\ln(\sin x)}.\ln 10$$

1(f) $a^{\ln(\cos x)}$ [রা.'০৫]

ধরি, $y=a^{\ln(\cos x)}$

$$\therefore \frac{dy}{dx}=a^{\ln(\cos x)}.\ln a\frac{d}{dx}\{\ln(\cos x)\}$$

$$=a^{\ln(\cos x)}.\ln a\frac{1}{\cos x}\frac{d}{dx}(\cos x)$$

$$=a^{\ln(\cos x)}.\ln a\frac{1}{\cos x}(-\sin x)$$

$$\therefore \frac{d}{dx}\{a^{\ln(\cos x)}\}=-\tan x\,a^{\ln(\cos x)}\ln a$$

1(g) $e^{2\ln(\tan 5x)}$ [ব.'০৬,'১১; কু.'০৭; সি.'১০,'১৩]

$$e^{2\ln(\tan 5x)}=e^{\ln(\tan 5x)^2}=(\tan 5x)^2$$

$$\therefore \frac{d}{dx}\{e^{2\ln(\tan 5x)}\}=2\tan 5x\frac{d}{dx}(\tan 5x)$$

$$=2\tan 5x(\sec^2 5x)\frac{d}{dx}(5x)$$

$$=2\tan 5x\sec^2 5x\,(5)$$

$$=10\tan 5x\sec^2 5x$$

1(h) $(ln\sin x^2)^n$ [সি.'০৬; রা.'০৯]

ধরি, $y=(ln\sin x^2)^n$

$$\frac{dy}{dx}=n(ln\sin x^2)^{n-1}\frac{d}{dx}(ln\sin x^2)$$

$$=n(ln\sin x^2)^{n-1}.\frac{1}{\sin x^2}\frac{d}{dx}(\sin x^2)$$