$\therefore \quad \frac{d}{dx}\left(\frac{x\ln x}{\sqrt{1+x^2}}\right) = \frac{1+x^2+\ln x}{(\sqrt{1+x^2})^3}$ (Ans.)

প্রশ্নমালা IX E

1.(a) $(1+\sin 2x)^2$ [চ.'০৪]

ধরি, $y = (1+\sin 2x)^2$

$\therefore \frac{dy}{dx} = 2(1+\sin 2x)\frac{d}{dx}(1+\sin 2x)$

$= 2(1+\sin 2x)(0+\cos 2x)\frac{d}{dx}(2x)$

$= 2(1+\sin 2x)\cos 2x\,(2.1)$

$\therefore \frac{d}{dx}\{(1+\sin 2x)^2\} = 4\cos 2x(1+\sin 2x)$

1(b) a^{px+q} [চ'০১]

ধরি, $y = a^{px+q}$

$\therefore \frac{dy}{dx} = a^{px+q}.\ln a\,\frac{d}{dx}(px+q)$

$[\because \frac{d}{dx}(a^x) = a^x \ln a\,]$

$= a^{px+q}.\ln a\,(p.1+0)$

$\therefore \frac{d}{dx}(a^{px+q}) = p\,a^{px+q}.\ln a$ (Ans.)

1(c) $a^{\cos x}$ [চ.'০০]

$\frac{d}{dx}(a^{\cos x}) = a^{\cos x}.\ln a.\frac{d}{dx}(\cos x)$

$= a^{\cos x}.\ln a.(-\sin x)$

$= -a^{\cos x}\sin x.\ln a$

1(d) $10^{\ln(\sin x)}$ [সি.'০২ '০৫; চ.'০৭]

ধরি, $y = 10^{\ln(\sin x)}$

$\therefore \frac{dy}{dx} = 10^{\ln(\sin x)}.\ln 10\,\frac{d}{dx}\{\ln(\sin x)\}$

$= 10^{\ln(\sin x)}.\ln 10\,\frac{1}{\sin x}\frac{d}{dx}(\sin x)$

$= 10^{\ln(\sin x)}.\ln 10\,\frac{1}{\sin x}(\cos x)$

$\therefore \frac{d}{dx}\{10^{\ln(\sin x)}\} = 10^{\ln(\sin x)}.\ln 10.\cot x$

1(e) $10^{\ln(\tan x)}$

ধরি, $y = 10^{\ln(\tan x)}$

$\therefore \frac{dy}{dx} = 10^{\ln(\tan x)}.\ln 10\,\frac{d}{dx}\{\ln(\tan x)\}$

$= 10^{\ln(\tan x)}.\ln 10\,\frac{1}{\tan x}\frac{d}{dx}(\tan x)$

$= 10^{\ln(\sin x)}.\ln 10\,\frac{\cos x}{\sin x}(\sec^2 x)$

$= 10^{\ln(\sin x)}.\ln 10\,\frac{\cos x}{\sin x}.\frac{1}{\cos^2 x}$

$= 10^{\ln(\sin x)}.\ln 10\,\frac{2}{2\sin x\cos x}$

$= 10^{\ln(\sin x)}.\ln 10\,\frac{2}{\sin 2x}$

$= 2\,\mathrm{cosec}\,2x.10^{\ln(\sin x)}.\ln 10$

1(f) $a^{\ln(\cos x)}$ [রা.'০৫]

ধরি, $y = a^{\ln(\cos x)}$

$\therefore \frac{dy}{dx} = a^{\ln(\cos x)}.\ln a\,\frac{d}{dx}\{\ln(\cos x)\}$

$= a^{\ln(\cos x)}.\ln a\,\frac{1}{\cos x}\frac{d}{dx}(\cos x)$

$= a^{\ln(\cos x)}.\ln a\,\frac{1}{\cos x}(-\sin x)$

$\therefore \frac{d}{dx}\{a^{\ln(\cos x)}\} = -\tan x\,a^{\ln(\cos x)}\ln a$

1(g) $e^{2\ln(\tan 5x)}$ [ব.'০৬,'১১; কু.'০৭; সি.'১০,'১৩]

$e^{2\ln(\tan 5x)} = e^{\ln(\tan 5x)^2} = (\tan 5x)^2$

$\therefore \frac{d}{dx}\{e^{2\ln(\tan 5x)}\} = 2\tan 5x\frac{d}{dx}(\tan 5x)$

$= 2\tan 5x(\sec^2 5x)\frac{d}{dx}(5x)$

$= 2\tan 5x\sec^2 5x(5)$

$= 10\tan 5x\sec^2 5x$

1(h) $(\ln\sin x^2)^n$ [সি.'০৬; রা.'০৯]

ধরি, $y = (\ln\sin x^2)^n$

$\frac{dy}{dx} = n(\ln\sin x^2)^{n-1}\frac{d}{dx}(\ln\sin x^2)$

$= n(\ln\sin x^2)^{n-1}.\frac{1}{\sin x^2}\frac{d}{dx}(\sin x^2)$

$= n\,(ln \sin x^2)^{n-1} \cdot \frac{1}{\sin x^2}(\cos x^2)\,(2x)$

$\therefore \frac{d}{dx}\{(ln \sin x^2)^n\} = nx\cot x^2 (ln \sin x^2)^{n-1}$

1(i) $\cos(e^{\tan^2 2x})$

$\frac{d}{dx}\{\cos(e^{\tan^2 2x})\} = \frac{d\{\cos(e^{\tan^2 2x})\}}{d(e^{\tan^2 2x})}$

$\frac{d(e^{\tan^2 2x})}{d(\tan^2 2x)} \cdot \frac{d(\tan^2 2x)}{d(\tan 2x)} \frac{d(\tan 2x)}{d(2x)} \frac{d(2x)}{dx}$

$= -\sin(e^{\tan^2 2x}).\, e^{\tan^2 2x}.2\tan 2x.\sec^2 2x.2$

$= -4\tan 2x \sec^2 2x \sin(e^{\tan^2 2x})\; e^{\tan^2 2x}$

1(j) $\frac{d}{dx}(\sin^3 x^2)$ [চ.'০৯]

$= \frac{d(\sin x^2)^3}{d(\sin x^2)} \frac{d(\sin x^2)}{d(x^2)} \frac{d(x^2)}{dx}$

$= 3(\sin x^2)^2.\cos x^2.2x$

$= 6x \sin^2 x^2 \cos x^2$ (Ans.)

1(k) $e^{5\ln(\tan x)}$ [চ.'১২]

$= e^{\ln(\tan x)^5} = (\tan x)^5$

$\therefore \frac{d}{dx}\{e^{5\ln(\tan x)}\} = 5\tan^4 x \frac{d}{dx}(\tan x)$

$= 5\tan^4 x \sec^2 x$

1(*l*) $x^n \ln(2x)$. [চ.'০৭]

মনে করি, $y = x^n \ln(2x)$

$\therefore \frac{dy}{dx} = x^n \frac{d}{dx}\{ln(2x)\} + ln(2x)\frac{d}{dx}(x^n)$

$= x^n \frac{1}{2x}\frac{d}{dx}(2x) + ln(2x).nx^{n-1}$

$= x^{n-1}\frac{1}{2}.(2) + nx^{n-1} ln(2x)$

$\therefore \frac{d}{dx}\{x^n \ln(2x)\} = x^{n-1}\{1 + n\, ln(2x)\}$

1(m) $x\sqrt{\sin x}$ [ঢা.'০৮]

মনে করি, $y = x\sqrt{\sin x} = x(\sin x)^{\frac{1}{2}}$

$\therefore \frac{dy}{dx} = x\frac{d}{dx}\{(\sin x)^{\frac{1}{2}}\} + (\sin x)^{\frac{1}{2}}\frac{d}{dx}(x)$

$= x.\frac{1}{2}(\sin x)^{-\frac{1}{2}}\frac{d}{dx}(\sin x) + \sqrt{\sin x}.1$

$= \frac{1}{2}x\frac{1}{\sqrt{\sin x}}(\cos x) + \sqrt{\sin x}$

$\therefore \frac{d}{dx}(x\sqrt{\sin x}) = \frac{x\cos x + 2\sin x}{2\sqrt{\sin x}}$

1(n) $e^{ax}\tan^2 x$ [ঢা.'০৯]

মনে করি, $y = e^{ax}\tan^2 x$

$\therefore \frac{dy}{dx} = e^{ax}\frac{d}{dx}(\tan^2 x) + \tan^2 x\frac{d}{dx}(e^{ax})$

$= e^{ax}(2\tan x)\frac{d}{dx}(\tan x) + \tan^2 x.e^{ax}(a)$

$= e^{ax}\tan x(2\sec^2 x + a\tan x)$ (Ans.)

2.(a) $ln(\cos x)$ [রা.'০৩,'০৫,'১০]

$\frac{d}{dx}\{ln(\cos x)\} = \frac{1}{\cos x}\frac{d}{dx}(\cos x)$

$= \frac{1}{\cos x}(-\sin x) = -\tan x$ (Ans.)

2(b) $ln(e^x + e^{-x})$ [কু.'০৮]

$\frac{d}{dx}\{ln(e^x + e^{-x})\} = \frac{1}{e^x + e^{-x}}\frac{d}{dx}(e^x + e^{-x})$

$= \frac{1}{e^x + e^{-x}}(e^x - e^{-x}) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

2(c) $\log_x a$ [রা.'০১; চ.'০৬;'০৮]

$\log_x a = \log_x e \times \log_e a = ln\, a\frac{1}{\log_e x}$

$= ln\, a\frac{1}{\ln x} = ln\, a\,(ln\, x)^{-1}$

$\therefore \frac{d}{dx}(\log_x a) = ln\, a\{-1(ln\, x)^{-2}\frac{d}{dx}(ln\, x)\}$

$= -ln\, a\frac{1}{(\ln x)^2}.\frac{1}{x} = -\frac{\ln a}{x(\ln x)^2}$

2(d) $\log_{10} 3x$ [য.'০৬,'১৩]

$\log_{10} 3x = \log_{10} e \times \log_e 3x = \frac{1}{\log_e 10}\ln(3x)$

$\therefore \frac{d}{dx}(\log_{10} 3x) = \frac{1}{\ln 10}\frac{1}{3x}\frac{d}{dx}(3x)$

$= \frac{1}{\ln 10}\frac{1}{3x}(3.1) = \frac{1}{x\ln 10}$ (Ans.)

2(e) $\log_a x + \log_x a$

$= \log_a e \times \log_e x + \log_x e \times \log_e a$

$= \frac{1}{\log_e a}\times \ln x + \frac{1}{\log_e x}\times \ln a$

$= \frac{1}{\ln a}\times \ln x + \ln a \times (\ln x)^{-1}$

$\therefore \frac{d}{dx}(\log_a x + \log_x a)$

$= \frac{1}{\ln a}\frac{1}{x} + \ln a \times \{-1(\ln x)^{-2}\frac{1}{x}\}$

$= \frac{1}{x\ln a} - \frac{\ln a}{x(\ln x)^2}$

2(f) ধরি, $y = \log_x \tan x = \log_x e \times \log_e \tan x$

$= \frac{1}{\log_e x}\times \ln(\tan x) = \frac{\ln(\tan x)}{\ln x}$

$\therefore \frac{dy}{dx} = \frac{\ln x\frac{d}{dx}\{\ln(\tan x)\} - \ln(\tan x)\frac{d}{dx}(\ln x)}{(\ln x)^2}$

$= \frac{\ln x\frac{1}{\tan x}\sec^2 x - \ln(\tan x).\frac{1}{x}}{(\ln x)^2}$

$= \frac{\ln x\frac{\cos x}{\sin x}\frac{1}{\cos^2 x} - \frac{1}{x}\ln(\tan x)}{(\ln x)^2}$

$= \frac{\ln x\frac{2}{\sin 2x} - \frac{1}{x}\ln(\tan x)}{(\ln x)^2}$

$= \frac{2x\ln x \operatorname{cosec} 2x - \ln(\tan x)}{x(\ln x)^2}$ (Ans.)

2(g) $ln(\sin 2x)$ [ঢা.'১১; সি.'১৩]

$\frac{d}{dx}\{ln(\sin 2x)\} = \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x)$

$= \frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x) = 2\cot 2x$

(h) $\ln(\sin x^2)$ [রা.'১১]

$\frac{d}{dx}\{\ln(\sin x^2)\} = \frac{1}{\sin x^2}\frac{d}{dx}(\sin x^2)$

$= \frac{1}{\sin x^2}(\cos x^2)\frac{d}{dx}(x^2) = 2x\cot x^2$

(i) $\ln(\ln x)$ [মা.'১৪]

$\frac{d}{dx}\{\ln(\ln x)\} = \frac{1}{\ln x}\frac{d}{dx}(\ln x) = \frac{1}{\ln x}.\frac{1}{x}$

$= \frac{1}{x\ln x}$ (Ans.)

3(a) $ln[x - \sqrt{x^2-1}]$ [রা. '০২; কু. '০৩; চ.'০৫]

$\frac{d}{dx}\{ln(x-\sqrt{x^2-1})\}$

$= \frac{1}{x-\sqrt{x^2-1}}\frac{d}{dx}(x-\sqrt{x^2-1})$

$= \frac{1}{x-\sqrt{x^2-1}}\{1 - \frac{1}{2\sqrt{x^2-1}}(2x)\}$

$= \frac{1}{x-\sqrt{x^2-1}}\{\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}}\}$

$= -\frac{1}{\sqrt{x^2-1}}$ (Ans.)

3(b) $ln[x - \sqrt{x^2+1}]$ [রা. '০২; কু. '০৩,'১০]

$\frac{d}{dx}\{ln(x-\sqrt{x^2+1})\}$

$= \frac{1}{x-\sqrt{x^2+1}}\frac{d}{dx}(x-\sqrt{x^2+1})$

$= \frac{1}{x-\sqrt{x^2+1}}\{1 - \frac{1}{2\sqrt{x^2+1}}(2x)\}$

$= \frac{1}{x-\sqrt{x^2+1}}\{\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\}$

$= -\frac{1}{\sqrt{x^2+1}}$ (Ans.)

3(c) $ln(\sqrt{x-a}+\sqrt{x-b})$ [কু.'০১]

$\frac{d}{dx}\{ln(\sqrt{x-a}+\sqrt{x-b})\}$

$=\frac{1}{\sqrt{x-a}+\sqrt{x-b}}\frac{d}{dx}(\sqrt{x-a}+\sqrt{x-b})$

$=\frac{1}{\sqrt{x-a}+\sqrt{x-b}}\{\frac{1}{2\sqrt{x-a}}+\frac{1}{2\sqrt{x-b}}\}$

$=\frac{1}{\sqrt{x-a}+\sqrt{x-b}}\{\frac{\sqrt{x-b}+\sqrt{x-a}}{2\sqrt{x-a}\sqrt{x-b}}\}$

$=\frac{1}{2\sqrt{(x-a)(x-b)}}$ (Ans.)

3(d) $ln\left\{e^x\left(\frac{x-1}{x+1}\right)^{3/2}\right\}$ [চ.'০০]

ধরি, $y=ln\left\{e^x\left(\frac{x-1}{x+1}\right)^{3/2}\right\}$

$=ln\,e^x+\frac{3}{2}\{ln(x-1)-ln(x+1)\}$

$=x+\frac{3}{2}\{ln(x-1)-ln(x+1)\}$

$\therefore \frac{dy}{dx}=1+\frac{3}{2}\{\frac{1}{x-1}-\frac{1}{x+1}\}$

$=1+\frac{3}{2}\{\frac{x+1-x+1}{(x-1)(x+1)}\}$

$=1+\frac{3}{2}\{\frac{2}{x^2-1}\}=\frac{x^2-1+3}{x^2-1}$

$=\frac{x^2+2}{x^2-1}$ (Ans.)

4. (a) $\frac{\tan x-\cot x}{\tan x+\cot x}$ [চ.'০৭; য.'০৬]

$\frac{\tan x-\cot x}{\tan x+\cot x}=\frac{\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$

$=\frac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x}=\frac{-\cos 2x}{1}=-\cos 2x$

$\therefore \frac{d}{dx}(\frac{\tan x-\cot x}{\tan x+\cot x})=\sin 2x.2=2\sin 2x$

4(b) $\left(\frac{\sin 2x}{1+\cos 2x}\right)^2$ [কু.'০৩]

$=\left(\frac{2\sin x\cos x}{2\cos^2 x}\right)^2=\left(\frac{\sin x}{\cos x}\right)^2=\tan^2 x$

$\therefore \frac{d}{dx}\left(\frac{\sin 2x}{1+\cos 2x}\right)^2=2\tan x\frac{d}{dx}(\tan x)$

$=2\tan x\sec^2 x$

4(c) $ln\sqrt{\frac{1-\cos x}{1+\cos x}}$ [ঢা.'০৭,'১৩; রা.'১১; কু.'১৪]

$=ln\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}=ln\sqrt{\tan^2\frac{x}{2}}=ln\tan\frac{x}{2}$

$\therefore \frac{d}{dx}\{ln\sqrt{\frac{1-\cos x}{1+\cos x}}\}=\frac{1}{\tan\frac{x}{2}}\sec^2\frac{x}{2}.\frac{1}{2}$

$=\frac{1}{2}\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\frac{1}{\cos^2\frac{x}{2}}=\frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$=\frac{1}{\sin x}=\text{cosec}\,x$ (Ans.)

4(d) $\sqrt{\frac{1+x}{1-x}}$ [প্র.ভ.প.'৮৩; রা.'১১]

ধরি, $y=\sqrt{\frac{1+x}{1-x}}=\frac{\sqrt{1+x}}{\sqrt{1-x}}$

$\therefore \frac{dy}{dx}=\frac{\sqrt{1-x}\frac{d}{dx}(\sqrt{1+x})-\sqrt{1+x}\frac{d}{dx}(\sqrt{1-x})}{(\sqrt{1-x})^2}$

$=\frac{\sqrt{1-x}\frac{1}{2\sqrt{1+x}}.1-\sqrt{1+x}\frac{1}{2\sqrt{1-x}}(-1)}{1-x}$

$=\frac{\sqrt{1-x}\frac{1}{2\sqrt{1+x}}.1-\sqrt{1+x}\frac{1}{2\sqrt{1-x}}(-1)}{1-x}$

$$= \frac{1-x+1+x}{2(1-x)\sqrt{(1+x)(1-x)}} = \frac{2}{2(1-x)\sqrt{1-x^2}}$$

$$\therefore \frac{d}{dx}\left(\sqrt{\frac{1+x}{1-x}}\right) = \frac{1}{(1-x)\sqrt{1-x^2}}$$

4.(e) $\ln \sqrt[3]{\frac{1-\cos x}{1+\cos x}}$ [দি.'১২; প্র.ভ.প.'০৫]

$$= \ln\left(\frac{2\sin^2(x/2)}{2\cos^2(x/2)}\right)^{1/3} = \frac{1}{3}\ln\tan^2\frac{x}{2}$$

$$= \frac{2}{3}\ln\tan\frac{x}{2}$$

$$\therefore \frac{d}{dx}\left(\ln\sqrt[3]{\frac{1-\cos x}{1+\cos x}}\right) = \frac{2}{3}\frac{\sec^2(x/2)}{\tan(x/2)}.\frac{1}{2}$$

$$= \frac{1}{3}\frac{\cos\frac{x}{2}}{\cos^2\frac{x}{2}\sin\frac{x}{2}} = \frac{2}{3}\frac{1}{2\cos\frac{x}{2}\sin\frac{x}{2}}$$

$$= \frac{2}{3}\frac{1}{\sin x} = \frac{2}{3}\cos ec x$$

5. (a) $\sin^2[ln(\sec x)]$ [রা.'০৭,'১৩; কু.,সি., মা.বো.'০৯; চ.'১১; ঢা.'১২; য.; দি.'১৩]

ধরি, $y = \sin^2[ln(\sec x)]$

$$\therefore \frac{dy}{dx} = \frac{d\{\sin[\ln(\sec x)]\}^2}{d\{\sin[\ln(\sec x)]\}}\frac{d\{\sin[\ln(\sec x)]\}}{d\{\ln(\sec x)\}}\frac{d\{\ln(\sec x)\}}{d(\sec x)}\frac{d(\sec x)}{dx}$$

$$= 2\sin[\ln(\sec x)]\cos[\ln(\sec x)]\frac{1}{\sec x}\sec x\tan x$$

$$= \tan x \sin[2\ln(\sec x)]$$

5(b) $\sin^2\{ln(x^2)\}$

[য.'০৭,'০৮; চ.'০৬,'১৩; ঢা.,সি,'১৪]

$$\frac{d}{dx}[\sin^2\{ln(x^2)\}] = \frac{d[\sin\{\ln(x^2)\}]^2}{d[\sin\{\ln(x^2)\}]}\frac{d[\sin\{\ln(x^2)\}]}{d[\ln(x^2)]}\frac{d[\ln(x^2)]}{d(x^2)}\frac{d(x^2)}{dx}$$

$$= 2\sin\{\ln(x^2)\}\cos\{\ln(x^2)\}\frac{1}{x^2}.2x$$

$$= \frac{2}{x}\sin\{2\ln(x^2)\} = \frac{2}{x}\sin\{4\ln(x)\}$$

5(c) $\sqrt{\sin\sqrt{x}}$ [চ.'০১; ঢা.'০৫,'০৭]

$$\frac{d}{dx}(\sqrt{\sin\sqrt{x}})$$

$$= \frac{d(\sqrt{\sin\sqrt{x}})}{d(\sin\sqrt{x})}\frac{d(\sin\sqrt{x})}{d(\sqrt{x})}\frac{d(\sqrt{x})}{dx}$$

$$= \frac{1}{2\sqrt{\sin\sqrt{x}}}\cos\sqrt{x}.\frac{1}{2\sqrt{x}}$$

$$= \frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}} \text{ (Ans.)}$$

5(d) $\cos(lnx) + ln(\tan x)$

[ব.'০৩; সি.'০৬]

$$\frac{d}{dx}\{\cos(lnx) + ln(\tan x)\}$$

$$= \frac{d}{dx}\{\cos(lnx)\} + \frac{d}{dx}\{ln(\tan x)\}$$

$$= -\sin(lnx).\frac{1}{x} + \frac{1}{\tan x}.\sec^2 x$$

$$= -\frac{1}{x}\sin(lnx) + \frac{\cos x}{\sin x}\frac{1}{\cos^2 x}$$

$$= \frac{2}{2\sin x\cos x} - \frac{1}{x}\sin(lnx)$$

$$= 2\text{ cesec } 2x - \frac{1}{x}\sin(lnx)$$

5(e) $2\text{cosec}2x\cos(ln\tan x)$ [রা.'০৬]

$$\frac{d}{dx}\{2\text{ cosec } 2x\cos(ln\tan x)\}$$

$$= 2\left[\text{cosec } 2x\frac{d}{dx}\{\cos(ln\tan x)\} + \cos(ln\tan x)\frac{d}{dx}(\text{cosec } 2x)\right]$$

$= 2\ [\text{cosec}\ 2x\ \{-\ \sin\ (ln\ \tan x)\}.\dfrac{1}{\tan x}.$

$\sec^2 x + \cos(ln \tan x)\ (-\text{cosec}2x\ \cot 2x.\ .2)]$

$= 2\ [-\ \text{cosec}\ 2x\ \ \sin\ (ln\ \tan x)\}.\dfrac{\cos x}{\sin x}.$

$\dfrac{1}{\cos^2 x} - 2\text{cosec}2x\ \cot 2x\ \cos(ln \tan x)]$

$= 2[-\text{cosec}\ 2x\ \ \sin\ (ln\ \tan x)\}\ \dfrac{2}{2\sin x \cos x}$

$-2\text{cosec}2x\ \cot 2x\ \cos(ln \tan x)]$

$= -4[\text{cosec}^2\ 2x\ \ \sin\ (ln\ \tan x)\}$

$\quad + \text{cosec}\ 2x\ \cot\ 2x\ \cos(\ ln\ \tan x)]$

5(f) $\dfrac{d}{dx}\{1+\tan(1+\sqrt{x})\}^{1/3}$

$= \dfrac{1}{3}\{1+\tan(1+\sqrt{x})\}^{\frac{1}{3}-1}\ \{0+\sec^2(1+\sqrt{x})\}$

$(0+\dfrac{1}{2\sqrt{x}})$

$= \dfrac{1}{6\sqrt{x}}\{1+\tan(1+\sqrt{x})\}^{-\frac{2}{3}}\ \sec^2(1+\sqrt{x})$

5(g) $\dfrac{d}{dx}(\sqrt{\tan e^{x^2}}\ )$ [য.'০৯]

$= \dfrac{d(\sqrt{\tan e^{x^2}})}{d(\tan e^{x^2})}\dfrac{d(\tan e^{x^2})}{d(e^{x^2})}\dfrac{d(e^{x^2})}{d(x^2)}\dfrac{d(x^2)}{dx}$

$= \dfrac{1}{2\sqrt{\tan e^{x^2}}}\sec^2 e^{x^2}\ .e^{x^2}\ .2x = \dfrac{xe^{x^2}\ \sec^2 e^{x^2}}{\sqrt{\tan e^{x^2}}}$

5(h) $\dfrac{d}{dx}\{\ \sin^2 \log(\sec x)\ \}$ [সি.'১২]

$= 2\sin\{\log(\sec x)\}.\cos\{\log(\sec x)\} \times$

$\dfrac{d}{dx}\{\log(\sec x)\}$

$= \sin\{2\log(\sec x)\} \times \dfrac{1}{\sec x \ln 10}\dfrac{d}{dx}(\sec x)$

$= \dfrac{\sin\{2\log(\sec x)\}}{\sec x \ln 10}\sec x\ .\ \tan x$

$= \dfrac{\sin\{2\log(\sec x)\}.\tan x}{\ln 10}$

5(i) $\dfrac{d}{dx}(\sin\sqrt{x}\ )$ [সি.'১২; কু.'১৩]

$= \cos\sqrt{x}\ \dfrac{d}{dx}(\sqrt{x}\ )$

$= \cos\sqrt{x}\ .\dfrac{1}{2\sqrt{x}} = \dfrac{\cos\sqrt{x}}{2\sqrt{x}}$

6.(a) ধরি, $y = x^2\sqrt{\dfrac{1+x}{1-x}}$ [রা.'০১]

$\therefore \ln y = 2\ln x + \dfrac{1}{2}[\ln(1+x) - \ln(1-x)]$

ইহাকে এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\dfrac{1}{y}\dfrac{dy}{dx} = 2.\dfrac{1}{x} + \dfrac{1}{2}[\dfrac{1}{1+x} - \dfrac{1}{1-x}(-1)]$

$\Rightarrow \dfrac{dy}{dx} = y[\dfrac{2}{x} + \dfrac{1}{2}\{\dfrac{1-x+1+x}{(1+x)(1-x)}\}]$

$\Rightarrow \dfrac{dy}{dx} = x^2\dfrac{\sqrt{1+x}}{\sqrt{1-x}}[\dfrac{2}{x} + \dfrac{1}{2}\{\dfrac{1-x+1+x}{(1+x)(1-x)}\}]$

$= 2x\sqrt{\dfrac{1+x}{1-x}} + \dfrac{x^2}{\sqrt{(1+x)}(1-x)^{3/2}}$

6(b) $\sqrt{e^{\sqrt{x}}}$ [কু.'০৪; ঢা.'০৬,'০৯; য.'১৩]

$\dfrac{d}{dx}(\sqrt{e^{\sqrt{x}}}\ ) = \dfrac{1}{2\sqrt{e^{\sqrt{x}}}}\ \dfrac{d}{dx}(e^{\sqrt{x}}\ )$

$= \dfrac{1}{2\sqrt{e^{\sqrt{x}}}}\ e^{\sqrt{x}}\ \dfrac{d}{dx}(\sqrt{x}\ )$

$= \dfrac{(e^{\sqrt{x}})^{1-\frac{1}{2}}}{2}.\dfrac{1}{2\sqrt{x}} = \dfrac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$ (Ans.)

6.(c) $\dfrac{1}{\sqrt{x+1}+\sqrt{x+2}}$ [চ.'০০]

$= \dfrac{\sqrt{x+1}-\sqrt{x+2}}{(\sqrt{x+1}+\sqrt{x+2})(\sqrt{x+1}-\sqrt{x+2})}$

$= \dfrac{\sqrt{x+1}-\sqrt{x+2}}{x+1-x-2} = \sqrt{x+2} - \sqrt{x+1}$

$$\therefore \frac{d}{dx}\left(\frac{1}{\sqrt{x+1}+\sqrt{x+2}}\right)=\frac{1}{2\sqrt{x+2}}-\frac{1}{2\sqrt{x+1}}$$

$$=-\frac{\sqrt{x+2}-\sqrt{x+1}}{2\sqrt{(x+2)(x+1)}} \text{ (Ans.)}$$

6(d) $\frac{d}{dx}\left\{\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}\right\}$ [কু.'০৯]

$$=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}\left[\frac{1}{(x+1)^2}\frac{d}{dx}(x+1)^2+\frac{1}{\sqrt{x-1}}\frac{d}{dx}(\sqrt{x-1})-\frac{1}{(x+4)^3}\frac{d}{dx}(x+4)^3-\frac{1}{e^x}\frac{d}{dx}(e^x)\right]$$

$$=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}\left[\frac{2(x+1)}{(x+1)^2}+\frac{1}{\sqrt{x-1}}\frac{1}{2\sqrt{x-1}}-\frac{3(x+4)^2}{(x+4)^3}-\frac{1}{e^x}(e^x)\right]$$

$$=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}\left[\frac{2}{x+1}+\frac{1}{2(x-1)}-\frac{3}{x+4}-1\right]$$

7.(a) $\frac{\ln(\cos x)}{x}$ [ঢা.'০৬; সি.'০৭;'০৯,'১১; ব.'১০]

$$\frac{d}{dx}\left\{\frac{\ln(\cos x)}{x}\right\}$$

$$=\frac{x\frac{d}{dx}\{\ln(\cos x)-\ln(\cos x)\frac{d}{dx}(x)}{x^2}$$

$$=\frac{x\frac{1}{\cos x}(-\sin x)-\ln(\cos x).1}{x^2}$$

$$=\frac{\{x\tan x+\ln(\cos x)}{x^2}$$

7(b) ধরি, $y=\frac{e^{-3x}(3x+5)}{7x-1}$ [য.'০৫]

$$\therefore \ln y=\ln e^{-3x}+\ln(3x+5)-\ln(7x-1)$$

$$=-3x+\ln(3x+5)-\ln(7x-1)$$

ইহাকে এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{y}\frac{dy}{dx}=-3+\frac{1}{3x+5}(3)-\frac{1}{7x-1}(7)$$

$$=\frac{-3(21x^2+32x-5)+21x-3-21x-35}{(3x+5)(7x-1)}$$

$$\Rightarrow \frac{dy}{dx}=y\frac{-63x^2-96x+15-38}{(3x+5)(7x-1)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-3x}(3x+5)}{7x-1}\cdot\frac{-(63x^2+96x+23)}{(3x+5)(7x-1)}$$

$$=\frac{-(63x^2+96x+23)e^{-3x}}{(7x-1)^2}$$

7. (c) $\frac{x^4}{\ln x}$ [ঢা.'০৪]

$$\frac{d}{dx}\left(\frac{x^4}{\ln x}\right)=\frac{\ln x\frac{d}{dx}(x^4)-x^4\frac{d}{dx}(\ln x)}{(\ln x)^2}$$

$$=\frac{\ln x(4x^3)-x^4\frac{1}{x}}{(\ln x)^2}=\frac{x^3(4\ln x-1)}{(\ln x)^2}$$

8. (a) $\cos x°$ [রা.'০৪]

$$\cos x°=\cos\frac{\pi x}{180}$$

$$\therefore \frac{d}{dx}(\cos x°)=-\sin\frac{\pi x}{180}.\frac{d}{dx}\left(\frac{\pi x}{180}\right)$$

$$=-\sin x°.\frac{\pi}{180}=-\frac{\pi}{180}\sin x°$$

8(b) $e^{5x}\sin x°$ [সি.'০২]

$$=e^{5x}\sin\frac{\pi x}{180}$$

$$\therefore \frac{d}{dx}\left(e^{5x}\sin\frac{\pi x}{180}\right)=e^{5x}.\cos\frac{\pi x}{180}\frac{d}{dx}\left(\frac{\pi x}{180}\right)+\sin\frac{\pi x}{180}.e^{5x}\frac{d}{dx}(5x)$$

$$=e^{5x}.\cos x°.\left(\frac{\pi}{180}\right)+\sin x°.e^{5x}.5$$

$$=e^{5x}\left(\frac{\pi}{180}\cos x°+5\sin x°\right)$$

8(c) $2x°\cos 3x°$ [চ.'০৩; য.'০৫; কু.'১০,'১৩; সি. '০৬,'০৮,'১১; ব., রা.'০৭,'১৪; দি.'০৯,'১১]

$$2x°\cos 3x°=2\frac{\pi x}{180}\cos\frac{3\pi x}{180}$$

$$\therefore \frac{d}{dx}(2\, x^\circ \cos\, 3x^\circ) = \frac{\pi}{90}[x\,(-\sin\frac{3\pi x}{180}).\frac{d}{dx}(\frac{3\pi x}{180}) + \cos\frac{3\pi x}{180}\,\frac{d}{dx}(x)\,]$$

$$= \frac{\pi}{90}[x\,(-\sin 3x^\circ\,).\,(\frac{3\pi}{180}) + \cos 3x^\circ .1\,]$$

$$= \frac{\pi}{90}(\cos 3x^\circ - \frac{\pi}{60}x \sin 3x^\circ)$$

প্রশ্নমালা IX F

1. (a) $\sqrt{\sin^{-1} x^5}$ **[রা.'০৪, '০৬]**

$$\frac{d}{dx}(\sqrt{\sin^{-1} x^5}) = \frac{1}{2\sqrt{\sin^{-1} x^5}}\,\frac{d}{dx}(\sin^{-1} x^5)$$

$$= \frac{1}{2\sqrt{\sin^{-1} x^5}}\,\frac{1}{\sqrt{1-(x^5)^2}}\,\frac{d}{dx}(x^5)$$

$$= \frac{1}{2\sqrt{\sin^{-1} x^5}\sqrt{1-x^{10}}}(5\,x^4)$$

$$= \frac{5x^4}{2\sqrt{\sin^{-1} x^5}\sqrt{1-x^{10}}}$$

1.(b) $\tan^{-1}(\sin e^x)$ **[চ. '০৫; ব. '০৫;য.'০৯]**

$$\frac{d}{dx}\{\tan^{-1}(\sin e^x)\} = \frac{d\{\tan^{-1}(\sin e^x)\}}{d(\sin e^x)}\,\frac{d(\sin e^x)}{d(e^x)}\,\frac{d(e^x)}{dx}$$

$$= \frac{1}{1+(\sin e^x)^2}(\cos e^x).\,e^x = \frac{e^x \cos e^x}{1+\sin^2 e^x}$$

1(c) $\sin^{-1}(\sin e^x) = e^x$ **[চ.'০৪]**

$$\frac{d}{dx}\{\sin^{-1}(\sin e^x)\} = \frac{d}{dx}(e^x) = e^x$$

1(d) $\frac{d}{dx}(\sin^{-1}\sqrt{xe^x})$

$$= \frac{1}{\sqrt{1-(\sqrt{xe^x})^2}}\,\frac{d}{dx}(\sqrt{xe^x})$$

$$= \frac{1}{\sqrt{1-xe^x}}\,\frac{1}{2\sqrt{xe^x}}\,\frac{d}{dx}(xe^x)$$

$$= \frac{1}{2\sqrt{xe^x(1-xe^x)}}(xe^x+e^x)$$

$$= \frac{e^x(1+x)}{2\sqrt{xe^x(1-xe^x)}} \text{ (Ans.)}$$

1(e) $\sin^{-1}(\tan^{-1}x)$ **[সি.'০১]**

$$\frac{d}{dx}\{\sin^{-1}(\tan^{-1}x)\}$$

$$= \frac{1}{\sqrt{1-(\tan^{-1}x)^2}}\,\frac{d}{dx}(\tan^{-1}x)$$

$$= \frac{1}{\sqrt{1-(\tan^{-1}x)^2}}\,\frac{1}{1+x^2}$$

$$= \frac{1}{(1+x^2)\sqrt{1-(\tan^{-1}x)^2}}$$

1(f) $\frac{d}{dx}\{\tan^{-1}(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2})\}$

$$= \frac{1}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\sqrt{\frac{a-b}{a+b}}\,\frac{d}{dx}(\tan\frac{x}{2})$$

$$= \frac{1}{1+\frac{(a-b)\sin^2(x/2)}{(a+b)\cos^2(x/2)}}\sqrt{\frac{a-b}{a+b}}\sec^2\frac{x}{2}.\frac{1}{2}$$

$$= \frac{(a+b)\cos^2(x/2)}{a(\cos^2\frac{x}{2}+\sin^2\frac{x}{2})+b(\cos^2\frac{x}{2}-\sin^2\frac{x}{2})}.\frac{1}{2}\frac{\sqrt{a-b}}{\sqrt{a+b}}\frac{1}{\cos^2(x/2)}$$

$$= \frac{\sqrt{(a-b)(a+b)}}{2(a+b\cos x)} = \frac{\sqrt{a^2-b^2}}{2(a+b\cos x)}$$

1.(g) $\frac{d}{dx}\{\sin^{-1}(\frac{a+b\cos x}{b+a\cos x})\}$

$$= \frac{1}{\sqrt{1-(\frac{a+b\cos x}{b+a\cos x})^2}}$$