$\therefore \frac{d}{dx}(2x^\circ \cos 3x^\circ) = \frac{\pi}{90}[x(-\sin\frac{3\pi x}{180}).\frac{d}{dx}(\frac{3\pi x}{180}) + \cos\frac{3\pi x}{180}\frac{d}{dx}(x)]$

$= \frac{\pi}{90}[x(-\sin 3x^\circ).(\frac{3\pi}{180}) + \cos 3x^\circ .1]$

$= \frac{\pi}{90}(\cos 3x^\circ - \frac{\pi}{60}x\sin 3x^\circ)$

প্রশ্নমালা IX F

1. (a) $\sqrt{\sin^{-1}x^5}$ [র.'০৪, '০৬]

$\frac{d}{dx}(\sqrt{\sin^{-1}x^5}) = \frac{1}{2\sqrt{\sin^{-1}x^5}}\frac{d}{dx}(\sin^{-1}x^5)$

$= \frac{1}{2\sqrt{\sin^{-1}x^5}}\frac{1}{\sqrt{1-(x^5)^2}}\frac{d}{dx}(x^5)$

$= \frac{1}{2\sqrt{\sin^{-1}x^5}\sqrt{1-x^{10}}}(5x^4)$

$= \frac{5x^4}{2\sqrt{\sin^{-1}x^5}\sqrt{1-x^{10}}}$

1.(b) $\tan^{-1}(\sin e^x)$ [চ. '০৫; ব. '০৫;য.'০৯]

$\frac{d}{dx}\{\tan^{-1}(\sin e^x)\} = \frac{d\{\tan^{-1}(\sin e^x)\}}{d(\sin e^x)}\frac{d(\sin e^x)}{d(e^x)}\frac{d(e^x)}{dx}$

$= \frac{1}{1+(\sin e^x)^2}(\cos e^x).e^x = \frac{e^x\cos e^x}{1+\sin^2 e^x}$

1(c) $\sin^{-1}(\sin e^x) = e^x$ [চ.'০৪]

$\frac{d}{dx}\{\sin^{-1}(\sin e^x)\} = \frac{d}{dx}(e^x) = e^x$

1(d) $\frac{d}{dx}(\sin^{-1}\sqrt{xe^x})$

$= \frac{1}{\sqrt{1-(\sqrt{xe^x})^2}}\frac{d}{dx}(\sqrt{xe^x})$

$= \frac{1}{\sqrt{1-xe^x}}\frac{1}{2\sqrt{xe^x}}\frac{d}{dx}(xe^x)$

$= \frac{1}{2\sqrt{xe^x(1-xe^x)}}(xe^x+e^x)$

$= \frac{e^x(1+x)}{2\sqrt{xe^x(1-xe^x)}}$ (Ans.)

1(e) $\sin^{-1}(\tan^{-1}x)$ [সি.'০১]

$\frac{d}{dx}\{\sin^{-1}(\tan^{-1}x)\}$

$= \frac{1}{\sqrt{1-(\tan^{-1}x)^2}}\frac{d}{dx}(\tan^{-1}x)$

$= \frac{1}{\sqrt{1-(\tan^{-1}x)^2}}\frac{1}{1+x^2}$

$= \frac{1}{(1+x^2)\sqrt{1-(\tan^{-1}x)^2}}$

1(f) $\frac{d}{dx}\{\tan^{-1}(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2})\}$

$= \frac{1}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\sqrt{\frac{a-b}{a+b}}\frac{d}{dx}(\tan\frac{x}{2})$

$= \frac{1}{1+\frac{(a-b)\sin^2(x/2)}{(a+b)\cos^2(x/2)}}\sqrt{\frac{a-b}{a+b}}\sec^2\frac{x}{2}.\frac{1}{2}$

$= \frac{(a+b)\cos^2(x/2)}{a(\cos^2\frac{x}{2}+\sin^2\frac{x}{2})+b(\cos^2\frac{x}{2}-\sin^2\frac{x}{2})}.\frac{1}{2}\frac{\sqrt{a-b}}{\sqrt{a+b}}\frac{1}{\cos^2(x/2)}$

$= \frac{\sqrt{(a-b)(a+b)}}{2(a+b\cos x)} = \frac{\sqrt{a^2-b^2}}{2(a+b\cos x)}$

1.(g) $\frac{d}{dx}\{\sin^{-1}(\frac{a+b\cos x}{b+a\cos x})\}$

$= \frac{1}{\sqrt{1-(\frac{a+b\cos x}{b+a\cos x})^2}}$

$$\frac{(b+a\cos x)(-b\sin x)-(a+b\cos x)(-a\sin x)}{(b+a\cos x)^2}$$

$$=\frac{b+a\cos x}{\sqrt{(b+a\cos x)^2-(a+b\cos x)^2}}$$

$$\frac{(-b^2+a^2)\sin x}{(b+a\cos x)^2}$$

$$=\frac{(a^2-b^2)\sin x}{(b+a\cos x)\sqrt{b^2+a^2\cos^2 x-a^2-b^2\cos^2 x}}$$

$$=\frac{-(b^2-a^2)\sin x}{(b+a\cos x)\sqrt{(b^2-a^2)(1-\cos^2 x)}}$$

$$=\frac{-(b^2-a^2)\sin x}{(b+a\cos x)\sqrt{(b^2-a^2)\sin^2 x}}$$

$$=\frac{-\sqrt{b^2-a^2}}{b+a\cos x}$$

1(h) ধরি, $y=\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)=-\sec^{-1}\frac{1+x^2}{1-x^2}$

$$=-\cos^{-1}\frac{1-x^2}{1+x^2}=-2\tan^{-1}x$$

$$\therefore \frac{dy}{dx}=-2\frac{d}{dx}(\tan^{-1}x)=\frac{-2}{1+x^2}$$

2. (a) $x\sin^{-1}x$ [সি.'০১]

$$\frac{d}{dx}(x\sin^{-1}x)=x\frac{d}{dx}(\sin^{-1}x)+\sin^{-1}x\frac{d}{dx}(x)$$

$$=x\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x.1$$

$$=\frac{x}{\sqrt{1-x^2}}+\sin^{-1}x$$

2(b) $x^2\sin^{-1}(1-x)$ [রা.'০৬; ব.'০৮; ঢা.'১৪]

$$\frac{d}{dx}\{x^2\sin^{-1}(1-x)\}$$

$$=x^2\frac{d}{dx}\{\sin^{-1}(1-x)\}+\sin^{-1}(1-x)\frac{d}{dx}(x^2)$$

$$=x^2\frac{1}{\sqrt{1-(1-x)^2}}(-1)+\sin^{-1}(1-x).2x$$

$$=-\frac{x^2}{\sqrt{1-1+2x-x^2}}+2x\sin^{-1}(1-x)$$

$$=2x\sin^{-1}(1-x)-\frac{x^2}{\sqrt{2x-x^2}}$$

2(c) $\frac{d}{dx}\{e^x\sin^{-1}x\}$ [য.'০৪]

$$=e^x\frac{d}{dx}(\sin^{-1}x)+\sin^{-1}x\frac{d}{dx}(e^x)$$

$$=e^x\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x.e^x$$

$$=e^x\left(\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x\right)$$

2.(d) $\tan^{-1}\left(\frac{x^2}{e^x}\right)+\tan^{-1}\left(\frac{e^x}{x^2}\right)$

$$=\tan^{-1}\frac{\frac{x^2}{e^x}+\frac{e^x}{x^2}}{1-\frac{x^2}{e^x}.\frac{e^x}{x^2}}=\tan^{-1}\frac{\frac{x^2}{e^x}+\frac{e^x}{x^2}}{1-1}$$

$$=\cot^{-1}\frac{1-1}{\frac{x^2}{e^x}+\frac{e^x}{x^2}}=\cot^{-1}0=\frac{\pi}{2}$$

$$\therefore \frac{d}{dx}\left\{\tan^{-1}\left(\frac{x^2}{e^x}\right)+\tan^{-1}\left(\frac{e^x}{x^2}\right)\right\}=\frac{d}{dx}\left(\frac{\pi}{2}\right)=0$$

2(e) $\frac{d}{dx}(\tan x\sin^{-1}x)$ [ঢা.'০৫]

$$=\tan x\frac{d}{dx}(\sin^{-1}x)+\sin^{-1}x\frac{d}{dx}(\tan x)$$

$$=\tan x\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x.(\sec^2 x)$$

$$=\frac{\tan x}{\sqrt{1-x^2}}+\sec^2 x\sin^{-1}x$$

2(f) $(x^2+1)\tan^{-1}x-x$ [ঢা.,য.'১১; কু.,দি.'১২]

মনে করি, $y=(x^2+1)\tan^{-1}x-x$

$\therefore \frac{dy}{dx} = (x^2+1)\frac{d}{dx}(\tan^{-1}x) + \tan^{-1}x\frac{d}{dx}(x^2+1) - \frac{d}{dx}(x)$

$= (x^2+1)\frac{1}{1+x^2} + \tan^{-1}x \times (2x) - 1$

$= 1 + 2x\tan^{-1}x - 1$

$\therefore \frac{d}{dx}\{(x^2+1)\tan^{-1}x - x\} = 2x\tan^{-1}x$

3.(a) $\tan^{-1}\frac{1-x}{1+x}$ [কু. '০৩]

$= \tan^{-1}\frac{1-x}{1+1.x} = \tan^{-1}(1) - \tan^{-1}x$

$= \frac{\pi}{4} - \tan^{-1}x$

$\therefore \frac{d}{dx}(\tan^{-1}\frac{1-x}{1+x}) = \frac{d}{dx}(\frac{\pi}{4} - \tan^{-1}x)$

$= 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$ (Ans.)

3(b) $\cot^{-1}\frac{1-x}{1+x}$ [চ. '০১, '১০; য. '০৫]

$= \tan^{-1}\frac{1+x}{1-x} = \tan^{-1}\frac{1+x}{1-1.x}$

$= \tan^{-1}(1) + \tan^{-1}x = \frac{\pi}{4} + \tan^{-1}x$

$\therefore \frac{d}{dx}\{\cot^{-1}\frac{1-x}{1+x}\} = \frac{d}{dx}(\frac{\pi}{4} + \tan^{-1}x)$

$= 0 + \frac{1}{1+x^2} = \frac{1}{1+x^2}$

3(c) $\tan^{-1}\frac{1-\sqrt{x}}{1+\sqrt{x}}$ [কু. '০০]

$= \tan^{-1}\frac{1-\sqrt{x}}{1+1.\sqrt{x}} = \tan^{-1}(1) - \tan^{-1}\sqrt{x}$

$= \frac{\pi}{4} - \tan^{-1}\sqrt{x}$

$\therefore \frac{d}{dx}\{\tan^{-1}\frac{1-\sqrt{x}}{1+\sqrt{x}}\} = \frac{d}{dx}(\frac{\pi}{4} - \tan^{-1}\sqrt{x})$

$= 0 - \frac{1}{1+(\sqrt{x})^2}\frac{d}{dx}(\sqrt{x})$

$= -\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}(1+x)}$

3(d) $\tan^{-1}\frac{a+bx}{a-bx}$ [য. '০২, '১১; চা. '০৯, '১১; ব. '০৯; চ. '১২; কু. '১৩ প্র.ভ.প. '০৬]

$= \tan^{-1}\frac{a(1+\frac{b}{a}x)}{a(1-\frac{b}{a}x)} = \tan^{-1}\frac{1+\frac{b}{a}x}{1-1.\frac{b}{a}x}$

$= \tan^{-1}(1) - \tan^{-1}(\frac{b}{a}x) = \frac{\pi}{4} - \tan^{-1}(\frac{b}{a}x)$

$\therefore \frac{d}{dx}\{\tan^{-1}\frac{a+bx}{a-bx}\} = \frac{d}{dx}\{\frac{\pi}{4} - \tan^{-1}(\frac{b}{a}x)\}$

$= 0 - \frac{1}{1+(\frac{b}{a}x)^2}\frac{d}{dx}(\frac{b}{a}x)$

$= \frac{a^2}{a^2+b^2x^2}\cdot\frac{b}{a} = \frac{ab}{a^2+b^2x^2}$

3(e) $\tan^{-1}\frac{a\cos x - b\sin x}{b\cos x + a\sin x}$ [প্র.ভ.প. '৯৬]

$= \tan^{-1}\frac{\frac{a\cos x}{b\cos x} - \frac{b\sin x}{b\cos x}}{\frac{b\cos x}{b\cos x} + \frac{a\sin x}{b\cos x}} = \tan^{-1}\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}.\tan x}$

$= \tan^{-1}\frac{a}{b} - \tan^{-1}\tan x = \tan^{-1}\frac{a}{b} - x$

$\therefore \frac{d}{dx}\{\tan^{-1}\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\} = 0 - 1 = -1$

3(f) $\cot^{-1}\frac{1+x}{1-x}$. [চ. '০৬; সি. '০৪; রা. য. ০৭]

$= \tan^{-1}\frac{1-x}{1+x} = \tan^{-1}\frac{1-x}{1+1.x}$

$= \tan^{-1}(1) - \tan^{-1}x = \frac{\pi}{4} - \tan^{-1}x$

$\therefore \frac{d}{dx}\{\cot^{-1}\frac{1+x}{1-x}\} = \frac{d}{dx}(\frac{\pi}{4} - \tan^{-1}x)$

$$= 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2} \text{ ((Ans.)}$$

3(g) ধরি, $y = \cos^{-1}(\frac{1+x}{2})^{1/2}$ **[চ.'০৯]**

এবং $x = \cos\theta$. তাহলে, $\theta = \cos^{-1} x$ এবং

$$y = \cos^{-1}\{\frac{1}{2}(1+\cos\theta)\}^{1/2} = \cos^{-1}(\cos^2\frac{\theta}{2})^{1/2}$$

$$= \cos^{-1}\cos\frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2}\cos^{-1} x$$

$$\therefore \frac{d}{dx}\{\cos^{-1}(\frac{1+x}{2})^{1/2}\} = \frac{d}{dx}(\frac{1}{2}\cos^{-1} x)$$

$$= \frac{1}{-2\sqrt{1-x^2}}$$

3(h) $\tan^{-1}\frac{a+bx}{b-ax}$ **[ব.'১৩; বুয়েট.'০৯]**

$$= \tan^{-1}\frac{b(\frac{a}{b}+x)}{b(1-\frac{a}{b}.x)} = \tan^{-1}(\frac{a}{b}) + \tan^{-1}(x)$$

$$\therefore \frac{d}{dx}\{\tan^{-1}\frac{a+bx}{b-ax}\} = \frac{d}{dx}\{\tan^{-1}(\frac{a}{b})\} + \frac{d}{dx}\{\tan^{-1}(x)\}$$

$$= 0 + \frac{1}{1+x^2} = \frac{1}{1+x^2}$$

4.(a) ধরি, $y = \sin^{-1}\frac{1-x^2}{1+x^2}$ **[য.'০২,'১২,'১৪]**

এবং $x = \tan\theta$. তাহলে, $\theta = \tan^{-1} x$ এবং

$$y = \sin^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta} = \sin^{-1}\cos 2\theta$$

$$= \sin^{-1}\sin(\frac{\pi}{2}-2\theta) = \frac{\pi}{2}-2\theta$$

$$= \frac{\pi}{2} - 2\tan^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\frac{\pi}{2} - 2\tan^{-1} x) = 0 - 2\frac{1}{1+x^2}$$

$$\therefore \frac{d}{dx}(\sin^{-1}\frac{1-x^2}{1+x^2}) = \frac{-2}{1+x^2}$$

4(b) $\cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1} x$ **[য.'০৬; চ.'০৭]**

$$\therefore \frac{d}{dx}(\cos^{-1}\frac{1-x^2}{1+x^2}) = \frac{d}{dx}(2\tan^{-1} x)$$

$$= 2\frac{1}{1+x^2} = \frac{2}{1+x^2} \text{ (Ans.)}$$

4(c) $\sec^{-1}\frac{1+x^2}{1-x^2}$ **[য.'০৬; কু. ০৯; সি.'১০]**

$$= \cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1} x$$

$$\therefore \frac{d}{dx}(\sec^{-1}\frac{1+x^2}{1-x^2}) = \frac{d}{dx}(2\tan^{-1} x)$$

$$= 2\frac{1}{1+x^2} = \frac{2}{1+x^2} \text{ (Ans.)}$$

4(d) $\tan^{-1}\frac{4x}{1-4x^2}$ **[ব.'০৪]**

$$= \tan^{-1}\frac{2.2x}{1-(2x)^2} = 2\tan^{-1}(2x)$$

$$[\because \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1} x]$$

$$\therefore \frac{d}{dx}(\tan^{-1}\frac{4x}{1-4x^2}) = \frac{d}{dx}\{2\tan^{-1}(2x)\}$$

$$= 2\frac{1}{1+(2x)^2}.2 = \frac{4}{1+4x^2} \text{ (Ans.)}$$

4(e) $\tan^{-1}\frac{4\sqrt{x}}{1-4x}$

[চ.'০৯; রা.'০৬; সি.'০৯,'১২; ব.'১১; দি.'১৩]

$$= \tan^{-1}\frac{2.2\sqrt{x}}{1-(2\sqrt{x})^2} = 2\tan^{-1}(2\sqrt{x})$$

$$[\because \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1} x]$$

$$\therefore \frac{d}{dx}(\tan^{-1}\frac{4\sqrt{x}}{1-4x}) = \frac{d}{dx}\{2\tan^{-1}(2\sqrt{x})\}$$

$$= 2\frac{1}{1+(2\sqrt{x})^2}\frac{d}{dx}(2\sqrt{x})$$

$= \frac{2}{1+4x}.2.\frac{1}{2\sqrt{x}} = \frac{2}{\sqrt{x}(1+4x)}$ (Ans.)

4(f) $\sin^{-1}\frac{4x}{1+4x^2}$ [সি.'০২]

$= \sin^{-1}\frac{2.2x}{1+(2x)^2} = 2\tan^{-1}(2x)$.

$\therefore \frac{d}{dx}(\sin^{-1}\frac{4x}{1+4x^2}) = \frac{d}{dx}\{2\tan^{-1}(2x)\}$

$= 2\frac{1}{1+(2x)^2}\frac{d}{dx}(2x) = \frac{4}{1+4x^2}$ (Ans.)

4(g) $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

$\therefore \frac{d}{dx}(\sin^{-1}\frac{2x}{1+x^2}) = \frac{d}{dx}(2\tan^{-1}x)$

$= \frac{2}{1+x^2}$ (Ans.)

4(h) $\sin^{-1}\frac{6x}{1+9x^2}$ [ঢা.'০১]

$= \sin^{-1}\frac{2.3x}{1+(3x)^2} = 2\tan^{-1}(3x)$

$[\because \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x]$

$\therefore \frac{d}{dx}(\sin^{-1}\frac{6x}{1+9x^2}) = \frac{d}{dx}\{2\tan^{-1}(3x)\}$

$= 2\frac{1}{1+(3x)^2}\frac{d}{dx}(3x) = \frac{2}{1+9x^2}.3$

$= \frac{9}{1+9x^2}$ (Ans.)

4.(i) $\tan^{-1}\frac{2\sqrt{x}}{1-x}$ [চ'০৬,'১১; ঢা.'০৭; সি.'১১]

$= \tan^{-1}\frac{2\sqrt{x}}{1-(\sqrt{x})^2} = 2\tan^{-1}\sqrt{x}$

$\therefore \frac{d}{dx}(\tan^{-1}\frac{2\sqrt{x}}{1-x}) = \frac{d}{dx}\{2\tan^{-1}(\sqrt{x})\}$

$= 2\frac{1}{1+(\sqrt{x})^2}\frac{d}{dx}(\sqrt{x}) = \frac{2}{1+x}\frac{1}{2\sqrt{x}}$

$= \frac{1}{(1+x)\sqrt{x}}$ (Ans.)

5.(a) ধরি, $y = \cos^{-1}(2x\sqrt{1-x^2})$ [য.'০১,'১০; কু.'১০]

এবং $x = \sin\theta$. তাহলে, $\theta = \sin^{-1}x$ এবং

$y = \cos^{-1}(2\cos\theta\sqrt{1-\cos^2\theta})$

$= \cos^{-1}(2\cos\theta\sin\theta) = \cos^{-1}\sin 2\theta$

$= \cos^{-1}\cos(\frac{\pi}{2}-2\theta) = \frac{\pi}{2}-2\theta$

$= \frac{\pi}{2}-2\sin^{-1}x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\frac{\pi}{2}-2\sin^{-1}x)$

$= 0-2\frac{1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$ (Ans.)

5(b) ধরি, $y = \sin^{-1}\{2ax\sqrt{1-a^2x^2}\}$ [কু.'০৮; সি.'১৩]

এবং $ax = \sin\theta$. তাহলে, $\theta = \sin^{-1}(ax)$ এবং

$y = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$

$= \sin^{-1}\{2\sin\theta\cos\theta\} = \sin^{-1}\sin 2\theta$

$= 2\theta = 2\sin^{-1}(ax)$

$\therefore \frac{dy}{dx} = 2\frac{1}{\sqrt{1-(ax)^2}}\frac{d}{dx}(ax)$

$= \frac{2a}{\sqrt{1-a^2x^2}}$

5(c) ধরি, $y = \tan^{-1}\frac{4x}{\sqrt{1-4x^2}}$ [রা.'০২]

এবং $2x = \sin\theta$.

$\therefore 2\frac{dx}{d\theta} = \cos\theta$ এবং

$y = \tan^{-1}\frac{2\sin\theta}{\sqrt{1-\sin^2\theta}} = \tan^{-1}\frac{2\sin\theta}{\cos\theta}$

$= \tan^{-1}(2\tan\theta)$

$\therefore \frac{dy}{dx} = \frac{1}{1+(2\tan\theta)^2}\frac{d}{d\theta}(2\tan\theta).\frac{d\theta}{dx}$

$$= \frac{1}{1+4\tan^2\theta}.2\sec^2\theta.\frac{2}{\cos\theta}$$

$$= \frac{1}{1+\frac{4\sin^2\theta}{\cos^2\theta}}.\frac{2}{\cos^2\theta}.\frac{2}{\cos\theta}$$

$$= \frac{4}{(\cos^2\theta+4\sin^2\theta)\cos\theta}$$

$$= \frac{4}{(1+3\sin^2\theta)\sqrt{1-\sin^2\theta}}$$

$$= \frac{4}{\{1+3(2x)^2\}\sqrt{1-(2x)^2}}$$

$$= \frac{4}{(1+12x^2)\sqrt{1-4x^2}}$$

বিকল্প পদ্ধতি: ধরি, $y = \tan^{-1}\frac{4x}{\sqrt{1-4x^2}}$

$$\therefore \frac{dy}{dx} = \frac{1}{1+\left(\frac{4x}{\sqrt{1-4x^2}}\right)^2}.\frac{d}{dx}\left(\frac{4x}{\sqrt{1-4x^2}}\right)$$

$$= \frac{1}{1+\frac{16x^2}{1-4x^2}}.\frac{\sqrt{1-4x^2}(4)-4x\frac{-8x}{2\sqrt{1-4x^2}}}{(\sqrt{1-4x^2})^2}$$

$$= \frac{1-4x^2}{1-4x^2+16x^2}.\frac{4(1-4x^2)+16x^2}{(1-4x^2)\sqrt{1-4x^2}}$$

$$= \frac{4-16x^2+16x^2}{(1+12x^2)\sqrt{1-4x^2}} = \frac{4}{(1+12x^2)\sqrt{1-4x^2}}$$

5(d) ধরি, $y = \sin^{-1}\frac{x+\sqrt{1-x^2}}{\sqrt{2}}$ এবং

$x = \sin\theta$. তাহলে, $\theta = \sin^{-1}x$ এবং

$$y = \sin^{-1}\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}$$

$$= \sin^{-1}(\sin\theta.\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cos\theta)$$

$$= \sin^{-1}(\sin\theta.\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos\theta)$$

$$= \sin^{-1}\sin(\theta+\frac{\pi}{4}) = \theta+\frac{\pi}{4} = \sin^{-1}x+\frac{\pi}{4}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}x+\frac{\pi}{4}) = \frac{1}{\sqrt{1-x^2}} \text{ (Ans.)}$$

6.(a) ধরি, $y = \tan^{-1}\frac{1}{\sqrt{x^2-1}}$ [রা.'০৩]

এবং $x = \sec\theta$. তাহলে, $\theta = \sec^{-1}x$ এবং

$$y = \tan^{-1}\frac{1}{\sqrt{\sec^2\theta-1}} = \tan^{-1}\frac{1}{\sqrt{\tan^2\theta}}$$

$$= \tan^{-1}\frac{1}{\tan\theta} = \tan^{-1}\cot\theta = \tan^{-1}\tan(\frac{\pi}{2}-\theta) =$$

$$\frac{\pi}{2}-\theta = \frac{\pi}{2}-\sec^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\frac{\pi}{2}-\sec^{-1}x) = 0-\frac{1}{x\sqrt{x^2-1}}$$

অর্থাৎ, $\frac{d}{dx}(\tan^{-1}\frac{1}{\sqrt{x^2-1}}) = -\frac{1}{x\sqrt{x^2-1}}$

6.(b) $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$ [সি.'০৫,'০৭; প্র.ভ.প.'৯০]

ধরি, $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ এবং $x = \cos\theta$.

তাহলে, $\theta = \cos^{-1}x$ এবং

$$y = \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan^{-1}\sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}}$$

$$= \tan^{-1}\sqrt{\tan^2\frac{\theta}{2}} = \tan^{-1}\tan\frac{\theta}{2}$$

$$= \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\cos^{-1}x) = \frac{1}{2}\frac{-1}{\sqrt{1-x^2}}$$

অর্থাৎ, $\frac{d}{dx}(\tan^{-1}\sqrt{\frac{1-x}{1+x}}) = \frac{-1}{2\sqrt{1-x^2}}$

6(c) $\sin^4\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ [বুয়েট,'০৯]

ধরি, $y = \sin^4\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ এবং $x = \cos\theta$

$\therefore\ y = \sin^4\left(\cot^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right)$

$= \sin^4\left(\cot^{-1}\sqrt{\frac{2\cos^2(\theta/2)}{2\sin^2(\theta/2)}}\right)$

$= \sin^4(\cot^{-1}\cot\frac{\theta}{2}) = \sin^4\frac{\theta}{2} = \{\frac{1}{2}(2\sin^2\frac{\theta}{2})\}^2$

$= \{\frac{1}{2}(1-\cos\theta)\}^2 = \frac{1}{4}(1-x)^2$

$\therefore\ \frac{dy}{dx} = \frac{1}{4}\times 2(1-x)\times(-1) = -\frac{1}{2}(1-x)$

6(d) $\tan(\sin^{-1}x)$ [চ.'০২,'০৯;কু.'০৮,'১১; রা.'০৮; ব.'০৯,'১২; ঢা.,য.,সি.'১০; ঢা.'১২; দি.'১৩]

$\frac{d}{dx}\{\tan(\sin^{-1}x)\} = \sec^2(\sin^{-1}x).\frac{d}{dx}(\sin^{-1}x)$

$= \frac{1}{\cos^2(\sin^{-1}x)}\cdot\frac{1}{\sqrt{1-x^2}}$

$= \frac{1}{1-\sin^2(\sin^{-1}x)}\cdot\frac{1}{\sqrt{1-x^2}}$

$= \frac{1}{1-\{\sin(\sin^{-1}x)\}^2}\cdot\frac{1}{\sqrt{1-x^2}}$

$= \frac{1}{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}} = \frac{1}{(1-x^2)^{3/2}}$ (Ans.)

7.(a) $\tan^{-1}(\sec x + \tan x)$ [সি.'১৪;য.'০৭; চ.'১৩]

$= \tan^{-1}(\frac{1}{\cos x}+\frac{\sin x}{\cos x}) = \tan^{-1}(\frac{1+\sin x}{\cos x})$

$= \tan^{-1}\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$

$= \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})}$

$= \tan^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$

$= \tan^{-1}\frac{\cos\frac{x}{2}(1+\tan\frac{x}{2})}{\cos\frac{x}{2}(1-\tan\frac{x}{2})} = \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

$= \tan^{-1}(1)+\tan^{-1}\tan(\frac{x}{2}) = \frac{\pi}{4}+\frac{x}{2}$

$\therefore\ \frac{d}{dx}\{\tan^{-1}(\sec x+\tan x)\} = \frac{d}{dx}(\frac{\pi}{4}+\frac{x}{2})$

$= \frac{1}{2}$ (Ans.)

7(b) $\tan^{-1}\frac{\cos x}{1+\sin x}$ [ঢা.'০৫,'১৩]

$= \tan^{-1}\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})}{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}$

$= \tan^{-1}\frac{\cos\frac{x}{2}(1-\tan\frac{x}{2})}{\cos\frac{x}{2}(1+\tan\frac{x}{2})} = \tan^{-1}\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

$= \tan^{-1}(1)-\tan^{-1}\tan(\frac{x}{2}) = \frac{\pi}{4}-\frac{x}{2}$

$\therefore\ \frac{d}{dx}(\tan^{-1}\frac{\cos x}{1+\sin x}) = \frac{d}{dx}(\frac{\pi}{4}-\frac{x}{2})$

$= 0-\frac{1}{2} = -\frac{1}{2}$

7(c) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ [রা.'১০; কু.'১১; ব.'১২]

$= \tan^{-1}\sqrt{\frac{2\sin^2(x/2)}{2\cos^2(x/2)}} = \tan^{-1}\sqrt{\tan^2\frac{x}{2}}$

$= \tan^{-1}\tan\frac{x}{2} = \frac{x}{2}$

$\therefore \frac{d}{dx}(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}) = \frac{d}{dx}(\frac{x}{2}) = \frac{1}{2}$

7(d) $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

[ব.'০২; চ. '০৮; রা. '০৯,'১১; দি.'০৯,'১১]

ধরি, $y = \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ এবং $x = \cos\theta$

তাহলে, $\theta = \cos^{-1}x$ এবং

$y = \sin\left(2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)$

$= \sin\left(2\tan^{-1}\sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}}\right)$

$= \sin(2\tan^{-1}\tan\frac{\theta}{2}) = \sin(2.\frac{\theta}{2}) = \sin\theta$

$= \sin(\cos^{-1}x) = \sin\sin^{-1}\sqrt{1-x^2}$

$= \sqrt{1-x^2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sqrt{1-x^2}) = \frac{1}{2\sqrt{1-x^2}}.(-2x)$

$\therefore \frac{d}{dx}\{\sin(2\tan^{-1}\sqrt{\frac{1-x}{1+x}})\} = \frac{-x}{\sqrt{1-x^2}}$

প্রশ্নমালা IX G

$\frac{dy}{dx}$ নির্ণয় কর ঃ **1. (a)** $x = \sqrt{t}$, $y = t - \frac{1}{\sqrt{t}}$

$\frac{dx}{dt} = \frac{d}{dt}(\sqrt{t}) = \frac{1}{2\sqrt{t}}$ এবং

$\frac{dy}{dt} = \frac{d}{dt}(t - \frac{1}{\sqrt{t}}) = \frac{d}{dt}(t - t^{-\frac{1}{2}})$

$= 1 - (-\frac{1}{2})t^{-\frac{1}{2}-1} = 1 + \frac{1}{2t\sqrt{t}}$

$= \frac{1}{2\sqrt{t}}(2\sqrt{t} + \frac{1}{t})$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1}{2\sqrt{t}}(2\sqrt{t} + \frac{1}{t}) \times \frac{2\sqrt{t}}{1}$

$= 2\sqrt{t} + \frac{1}{t}$

1.(b) $x = \frac{3at}{1+t^3}$ ……(1), $y = \frac{3at^2}{1+t^3}$ …(2)

(2) ÷ (1) $\Rightarrow \frac{y}{x} = t$

(1) হতে পাই, $x = \frac{3a\frac{y}{x}}{1+(\frac{y}{x})^3} = \frac{3ay}{x} \times \frac{x^3}{x^3+y^3}$

$\Rightarrow x = \frac{3ax^2y}{x^3+y^3} \Rightarrow x^3 + y^3 = 3axy$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$3x^2 + 3y^2\frac{dy}{dx} = 3a(x\frac{dy}{dx} + y)$

$\Rightarrow (y^2 - ax)\frac{dy}{dx} = ay - x^2 \therefore \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$

1(c) $x = a(\cos\phi + \phi\sin\phi)$, $y = a(\sin\phi - \phi\cos\phi)$

$\frac{dx}{d\phi} = \frac{d}{d\phi}\{a(\cos\phi + \phi\sin\phi)\}$

$= a(-\sin\phi + \phi\cos\phi + \sin\phi) = a\phi\cos\phi$

$\frac{dy}{d\phi} = \frac{d}{d\phi}\{a(\sin\phi - \phi\cos\phi)\}$

$= a(\cos\phi + \phi\sin\phi - \cos\phi) = a\phi\sin\phi$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\phi}}{\frac{dx}{d\phi}} = \frac{a\phi\sin\phi}{a\phi\cos\phi} = \tan\phi$

1(d) $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$

$= \frac{1}{2\sqrt{a^{\sin^{-1}t}}}a^{\sin^{-1}t}\ln a\frac{1}{\sqrt{1-t^2}}$