7(c) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ [রা.'১০; কু.'১১; ব.'১২]

$= \tan^{-1}\sqrt{\frac{2\sin^2(x/2)}{2\cos^2(x/2)}} = \tan^{-1}\sqrt{\tan^2\frac{x}{2}}$

$= \tan^{-1}\tan\frac{x}{2} = \frac{x}{2}$

$\therefore \frac{d}{dx}(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}) = \frac{d}{dx}(\frac{x}{2}) = \frac{1}{2}$

7(d) $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

[ব.'০২; চ. '০৮; রা. '০৯,'১১; দি.'০৯,'১১]

ধরি, $y = \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ এবং $x = \cos\theta$

তাহলে, $\theta = \cos^{-1}x$ এবং

$y = \sin\left(2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)$

$= \sin\left(2\tan^{-1}\sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}}\right)$

$= \sin(2\tan^{-1}\tan\frac{\theta}{2}) = \sin(2.\frac{\theta}{2}) = \sin\theta$

$= \sin(\cos^{-1}x) = \sin\sin^{-1}\sqrt{1-x^2}$

$= \sqrt{1-x^2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sqrt{1-x^2}) = \frac{1}{2\sqrt{1-x^2}}.(-2x)$

$\therefore \frac{d}{dx}\{\sin(2\tan^{-1}\sqrt{\frac{1-x}{1+x}})\} = \frac{-x}{\sqrt{1-x^2}}$

প্রশ্নমালা IX G

$\frac{dy}{dx}$ নির্ণয় কর ঃ **1. (a)** $x = \sqrt{t}$, $y = t - \frac{1}{\sqrt{t}}$

$\frac{dx}{dt} = \frac{d}{dt}(\sqrt{t}) = \frac{1}{2\sqrt{t}}$ এবং

$\frac{dy}{dt} = \frac{d}{dt}(t - \frac{1}{\sqrt{t}}) = \frac{d}{dt}(t - t^{-\frac{1}{2}})$

$= 1 - (-\frac{1}{2})t^{-\frac{1}{2}-1} = 1 + \frac{1}{2t\sqrt{t}}$

$= \frac{1}{2\sqrt{t}}(2\sqrt{t} + \frac{1}{t})$

$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1}{2\sqrt{t}}(2\sqrt{t} + \frac{1}{t}) \times \frac{2\sqrt{t}}{1}$

$= 2\sqrt{t} + \frac{1}{t}$

1.(b) $x = \frac{3at}{1+t^3}$ ……(1), $y = \frac{3at^2}{1+t^3}$ …(2)

(2) ÷ (1) $\Rightarrow \frac{y}{x} = t$

(1) হতে পাই, $x = \frac{3a\frac{y}{x}}{1+(\frac{y}{x})^3} = \frac{3ay}{x} \times \frac{x^3}{x^3+y^3}$

$\Rightarrow x = \frac{3ax^2y}{x^3+y^3} \Rightarrow x^3 + y^3 = 3axy$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$3x^2 + 3y^2\frac{dy}{dx} = 3a(x\frac{dy}{dx} + y)$

$\Rightarrow (y^2 - ax)\frac{dy}{dx} = ay - x^2 \therefore \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$

1(c) $x = a(\cos\phi + \phi\sin\phi)$, $y = a(\sin\phi - \phi\cos\phi)$

$\frac{dx}{d\phi} = \frac{d}{d\phi}\{a(\cos\phi + \phi\sin\phi)\}$

$= a(-\sin\phi + \phi\cos\phi + \sin\phi) = a\phi\cos\phi$

$\frac{dy}{d\phi} = \frac{d}{d\phi}\{a(\sin\phi - \phi\cos\phi)\}$

$= a(\cos\phi + \phi\sin\phi - \cos\phi) = a\phi\sin\phi$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\phi}}{\frac{dx}{d\phi}} = \frac{a\phi\sin\phi}{a\phi\cos\phi} = \tan\phi$

1(d) $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$

$= \frac{1}{2\sqrt{a^{\sin^{-1}t}}}a^{\sin^{-1}t}\ln a\frac{1}{\sqrt{1-t^2}}$

$$= \frac{\ln a \sqrt{a^{\sin^{-1}t}}}{2\sqrt{1-t^2}} = \frac{x \ln a}{2\sqrt{1-t^2}}$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\sqrt{a^{\cos^{-1}t}}\right)$$

$$= \frac{1}{2\sqrt{a^{\cos^{-1}t}}} a^{\cos^{-1}t} \ln a \frac{1}{-\sqrt{1-t^2}}$$

$$= -\frac{\ln a \sqrt{a^{\cos^{-1}t}}}{2\sqrt{1-t^2}} = -\frac{y \ln a}{2\sqrt{1-t^2}}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\frac{y \ln a}{2\sqrt{1-t^2}} \times \frac{2\sqrt{1-t^2}}{x \ln a}$$

$$= -\frac{y}{x}$$

2. (a) $x^{\frac{1}{x}}$ [ব.'০৪; চ.'১৩;সি.'০৭,'০৯; ঢা., য.'০৮]

$$\frac{d}{dx}(x^{\frac{1}{x}}) = x^{\frac{1}{x}}\left[\frac{1}{x}\frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$\left[\because \frac{d}{dx}(u^v) = u^v\left\{v\frac{d}{dx}(\ln u) + \ln u \frac{dv}{dx}\right\}\right]$$

$$= x^{\frac{1}{x}}\left[\frac{1}{x}.\frac{1}{x} + \ln x \frac{d}{dx}(x^{-1})\right]$$

$$= x^{\frac{1}{x}}\left[\frac{1}{x^2} + \ln x.(-x^{-2})\right] = x^{\frac{1}{x}}\left(\frac{1}{x^2} - \frac{\ln x}{x^2}\right)$$

$$= x^{\frac{1}{x}}.\frac{1-\ln x}{x^2} = x^{\frac{1}{x}-2}(1-\ln x) \quad \text{(Ans.)}$$

2. (b) $\frac{d}{dx}(1+x)^x$ [ব.'১৩]

$$= (1+x)^x\left[x\frac{d}{dx}\{\ln(1+x)\} + \ln(1+x)\frac{d}{dx}(x)\right]$$

$$\left[\because \frac{d}{dx}(u^v) = u^v\left\{v\frac{d}{dx}(\ln u) + \ln u \frac{dv}{dx}\right\}\right]$$

$$= (1+x)^x\left[x\frac{1}{1+x} + \ln(1+x).1\right]$$

$$= (1+x)^x\left\{\frac{x}{1+x} + \ln(1+x)\right\}$$

2(c) $(1+x^2)^{2x}$ [য.'০৬]

$$\frac{d}{dx}\{(1+x^2)^{2x}\} = (1+x^2)^{2x}$$

$$\left[2x\frac{d}{dx}\{\ln(1+x^2)\} + \ln(1+x^2)\frac{d}{dx}(2x)\right]$$

$$= (1+x^2)^{2x}\left[\frac{2x}{1+x^2}(2x) + \ln(1+x^2).(2)\right]$$

$$= 2(1+x^2)^{2x}\left[\frac{2x^2}{1+x^2} + \ln(1+x^2)\right]$$

2(d) $(1+x^2)^{x^2}$ [সি.'০১]

$$\frac{d}{dx}(1+x^2)^{x^2} = (1+x^2)^{x^2}$$

$$\left[x^2\frac{d}{dx}\{\ln(1+x^2)\} + \ln(1+x^2)\frac{d}{dx}(x^2)\right]$$

$$= (1+x^2)^{x^2}\left[\frac{x^2}{1+x^2}(2x) + \ln(1+x^2).(2x)\right]$$

$$= 2x(1+x^2)^{x^2}\left[\frac{x^2}{1+x^2} + \ln(1+x^2)\right]$$

2(e) $(\sqrt{x})^{\sqrt{x}}$ [ব.'১২;চ.'১০; কু.'১১; প্র.ভ.প.'০৫]

$$\frac{d}{dx}\{(\sqrt{x})^{\sqrt{x}}\}$$

$$= (\sqrt{x})^{\sqrt{x}}\left[\sqrt{x}\frac{d}{dx}(\ln\sqrt{x}) + \ln\sqrt{x}\frac{d}{dx}(\sqrt{x})\right]$$

$$= (\sqrt{x})^{\sqrt{x}}\left[\sqrt{x}.\frac{1}{\sqrt{x}}.\frac{1}{2\sqrt{x}} + \ln\sqrt{x}.\frac{1}{2\sqrt{x}}\right]$$

$$= (\sqrt{x})^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\ln\sqrt{x}\right]$$

$$= (\sqrt{x})^{\sqrt{x}}\left[\frac{1+\ln\sqrt{x}}{2\sqrt{x}}\right] \quad \text{(Ans.)}$$

2(f) ধরি, $y = x^{\ln x}$ [রা.'০২; কু.'০৮; সি.'১১]

$$\therefore \frac{dy}{dx} = x^{\ln x}\left[\ln x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\ln x)\right]$$

$$\left[\because \frac{d}{dx}(u^v) = u^v\left[v\frac{d}{dx}(\ln u) + \ln u\frac{dv}{dx}\right]\right.$$

$$= x^{\ln x}\left[2\ln x.\frac{1}{x}\right] = \frac{2\ln x}{x}x^{\ln x}$$

অর্থাৎ, $\frac{d}{dx}(x^{\ln x}) = \frac{2\ln x}{x}x^{\ln x}$

2(g) $\frac{d}{dx}(\sin^{-1}x)^x = (\sin^{-1}x)^x$

$$[x\frac{d}{dx}\{\ln(\sin^{-1}x)\} + \ln(\sin^{-1}x)\frac{d}{dx}(x)]$$

$$= (\sin^{-1}x)^x [x\frac{1}{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}} + \ln(\sin^{-1}x).1]$$

$$= (\sin^{-1}x)^x [\frac{x}{\sqrt{1-x^2}\sin^{-1}x} + \ln(\sin^{-1}x)]$$

2(h) $\frac{d}{dx}(\sin x)^x$ [য.'০৭]

$$= (\sin x)^x [x\frac{d}{dx}\{\ln(\sin x)\} + \ln(\sin x)\frac{d}{dx}(x)]$$

$$= (\sin x)^x [x\frac{1}{\sin x}.\cos x + \ln(\sin x).1]$$

$$= (\sin x)^x [x\cot x + \ln(\sin x)]$$

2(i) $\frac{d}{dx}(lnx)^x$

$$= (lnx)^x [x\frac{d}{dx}\{\ln(\ln x)\} + \ln(\ln x)\frac{d}{dx}(x)]$$

$$= (lnx)^x [x\frac{1}{\ln x}.\frac{1}{x} + \ln(\ln x).1]$$

$$= (lnx)^x [\frac{1}{\ln x} + \ln(\ln x)]$$

2 (j) $\frac{d}{dx}(\log_{10}x)^x = (\log_{10}x)^x$

$$[x\frac{d}{dx}\{\ln(\log_{10}x)\} + \ln(\log_{10}x)\frac{d}{dx}(x)]$$

$$= (\log_{10}x)^x [x\frac{1}{\log_{10}x}.\frac{1}{x\ln 10} + \ln(\log_0 x).1]$$

$$= (\log_{10}x)^x [\frac{1}{\ln 10\log_{10}x} + \ln(\log_{10}x)]$$

2(k) $x^{\cos^{-1}x}$ [কু.'১০,'১৩; সি. ' ০৬,'০৮; ঢা. '১০, '১৩; রা. '০৫, '০৭; ব.'০৬,'১০; দি.'০৯; য.'১০]

$$\frac{d}{dx}(x^{\cos^{-1}x})$$

$$= x^{\cos^{-1}x}[\cos^{-1}x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\cos^{-1}x)]$$

$$= x^{\cos^{-1}x}[\cos^{-1}x.\frac{1}{x} + \ln x\frac{-1}{\sqrt{1-x^2}}]$$

$$= x^{\cos^{-1}x}[\frac{\cos^{-1}x}{x} - \frac{\ln x}{\sqrt{1-x^2}}]$$

2(l) $\frac{d}{dx}(x^{-1/x})$ [বুয়েট''০৭]

$$= x^{-1/x}[-\frac{1}{x}\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(-\frac{1}{x})]$$

$$= x^{-1/x}[-\frac{1}{x}.\frac{1}{x} + \ln x\{-(-\frac{1}{x^2})\}]$$

$$= x^{-1/x} \times \frac{1}{x^2}(\ln x - 1) = \frac{1}{x^{2+1/x}}(\ln x - 1)$$

3(a) $\frac{d}{dx}(e^{x^x}) = e^{x^x}\frac{d}{dx}(x^x)$

$$= e^{x^x}x^x[x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x)]$$

$$= e^{x^x}.x^x\{x.\frac{1}{x} + \ln x.1\}$$

$$= e^{x^x}.x^x(1 + \ln x)$$

3(b) $\frac{d}{dx}(x^{e^x})$

$$= x^{e^x}[e^x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(e^x)]$$

$$= x^{e^x}[e^x\frac{1}{x} + \ln x.e^x]$$

$$= x^{e^x}e^x(\frac{1}{x} + \ln x)$$

(c) $\frac{d}{dx}(a^{a^x})$ [দি.'১২]

$$= a^{a^x}.\ln a.\frac{d}{dx}(a^x)$$

$$= a^{a^x}\ln a.a^x.\ln a = a^{a^x}a^x(\ln a)^2$$

3(d) $(\cot x)^{\tan x}$ [চ.'০৫; ব., দি.'০৯; য.'১২]

$$\frac{d}{dx}(\cot x)^{\tan x} = (\cot x)^{\tan x}$$

$$[\tan x\frac{d}{dx}\{\ln(\cot x)\} + \ln(\cot x)\frac{d}{dx}(\tan x)]$$

$= (\cot x)^{\tan x}\left[\frac{\tan x}{\cot x}(-\operatorname{cosec}^2 x) + \ln(\cot x).(\sec^2 x)\right]$

$= (\cot x)^{\tan x}\left[-\frac{\sin^2 x}{\cos^2 x}.\frac{1}{\sin^2 x} + \ln(\cot x).(\sec^2 x)\right]$

$= (\cot x)^{\tan x}[-\sec^2 x + \ln(\cot x).(\sec^2 x)]$

$= (\cot x)^{\tan x}.\sec^2 x\,[\ln(\cot x) - 1]$

4. (a) x^{x^x} **[রা.'০৬,'০৮; য.'১১; প্র.ভ.প.'০৫]**

$\frac{d}{dx}(x^{x^x}) = x^{x^x}\left[x^x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x^x)\right]$

$= x^{x^x}\left[x^x.\frac{1}{x} + \ln x.\,x^x\{x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x)\}\right]$

$= x^{x^x}.\,x^x\left[\frac{1}{x} + \ln x.\{x.\frac{1}{x} + \ln x.1\}\right]$

$= x^{x^x}.x^x\left[\frac{1}{x} + \ln x.(1 + \ln x\}\right]$

4(b) $(x^x)^x$ **[য.,মা.'০৯; কু.'০৫; ঢা.,ব.,দি.'১১; রা.'১২]**

$(x^x)^x = x^{x^2}$

$\therefore \frac{d}{dx}(x^x)^x = x^{x^2}\left[x^2\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x^2)\right]$

$= x^{x^2}\left[x^2.\frac{1}{x} + \ln x.(2x)\right]$

$= x^{x^2}[x + 2x\ln x] = (x^x)^x.x[1 + 2\ln x]$

4(c) $\frac{d}{dx}(\sec x)^{x^x} = (\sec x)^{x^x}$

$\left[x^x\frac{d}{dx}\{\ln(\sec x)\} + \ln(\sec x)\frac{d}{dx}(x^x)\right]$

$= (\sec x)^{x^x}\left[x^x\frac{1}{\sec x}.\sec x\tan x + \ln(\sec x).x^x\{x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x)\}\right]$

$= (\sec x)^{x^x}.x^x[\tan x + \ln(\sec x)\{x.\frac{1}{x} + \ln x.1\}]$

$= (\sec x)^{x^x}.\,x^x[\tan x + (1 + \ln x)\ln(\sec x)]$

5.(a) $\frac{d}{dx}(x^x \ln x)$ **[কু. '০৪; দি.'১০; ব.'১২]**

$= x^x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x^x)$

$= x^x.\frac{1}{x} + \ln x.\;x^x\{x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x)\}$

$= x^x[\frac{1}{x} + \ln x.\;\{x.\frac{1}{x} + \ln x.1\}]$

$= x^x\{\frac{1}{x} + \ln x.\;(1 + \ln x)\}$

5(b) $\frac{d}{dx}(ax)^{bx}$

$= (ax)^{bx}\left[bx\frac{d}{dx}\{\ln(ax)\} + \ln(ax)\frac{d}{dx}(bx)\right]$

$= (ax)^{bx}\left[bx.\frac{1}{ax}.a + \ln(ax).b\right]$

$= (ax)^{bx}.b\,[1 + \ln(ax)]$

5(c) ধরি, $y = (xe^x)^{\sin x}$

$\therefore \ln y = \ln(xe^x)^{\sin x} = \sin x(\ln x + \ln e^x)$

$= \sin x(\ln x + x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{1}{y}\frac{dy}{dx} = \sin x(\frac{1}{x} + 1) + (\ln x + x)\cos x$

$\Rightarrow \frac{dy}{dx} = y[(\frac{1}{x} + 1)\sin x + (\ln x + x)\cos x]$

$= (xe^x)^{\sin x}[\sin x.(\frac{1}{x} + 1) + (\ln x + x)\cos x]$

5(d) $\frac{d}{dx}(e^{x^2} + x^{x^2})$ **[ঢা.'০৬,'১২]**

$= \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(x^{x^2})$

$= e^{x^2}(2x) + x^{x^2}\left[x^2\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x^2)\right]$

$= 2xe^{x^2} + x^{x^2}\left[\frac{x^2}{x} + \ln x.(2x)\right]$

$= 2xe^{x^2} + x^{x^2}[x + 2x\ln x]$

5(e) $\frac{d}{dx}\{(\tan x)^x + x^{\tan x}\}$

$= \frac{d}{dx}(\tan x)^x + \frac{d}{dx}(x^{\tan x})$

$= (\tan x)^x [x\frac{d}{dx}\{\ln(\tan x)\} + \ln(\tan x)\frac{d}{dx}(x)]$

$+ x^{\tan x}[\tan x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\tan x)]$

$= (\tan x)^x [x\frac{1}{\tan x}.\sec^2 x + \ln(\tan x).1]$

$+ x^{\tan x}[\tan x.\frac{1}{x} + \ln x.\sec^2 x]$

$= (\tan x)^x [x\frac{\cos x}{\sin x}.\frac{1}{\cos^2 x} + \ln(\tan x)]$

$+ x^{\tan x}[\frac{1}{x}\tan x + \sec^2 x \ln x]$

$= (\tan x)^x [2x\cos ec 2x + \ln(\tan x)]$

$+ x^{\tan x}[\frac{1}{x}\tan x + \sec^2 x \ln x]$

5.(f) $\frac{d}{dx}(x^{\ln x} + x^{\log x})$

$= \frac{d}{dx}(x^{\ln x}) + \frac{d}{dx}(x^{\ln x})$

$= x^{\ln x}[\ln x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\ln x)]$

$+ x^{\log x}[\log x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\log x)]$

$= x^{\ln x} 2\ln x.\frac{1}{x} + x^{\log x}[\log x.\frac{1}{x} + \ln x.\frac{1}{x\ln 10}]$

$= \frac{2\ln x}{x}.x^{\ln x} + x^{\log x}[\frac{\log x}{x} + \frac{\ln x}{x\ln 10}]$

5(g) $\frac{d}{dx}\{(\ln x)^x + (\log_{10} x)^x\}$

$= \frac{d}{dx}(\ln x)^x + \frac{d}{dx}(\log_{10} x)^x$

$= (\ln x)^x [x\frac{d}{dx}\{\ln(\ln x)\} + \ln(\ln x)\frac{d}{dx}(x)] +$

$(\log_{10} x)^x [x\frac{d}{dx}\{\ln(\log_{10} x)\} + \ln(\log_{10} x)\frac{d}{dx}(x)]$

$= (\ln x)^x [x\frac{1}{\ln x}.\frac{1}{x} + \ln(\ln x).1] +$

$(\log_{10} x)^x [x\frac{1}{\log_{10} x}.\frac{1}{x\ln 10} + \ln(\log_{10} x).1]$

$= (\ln x)^x [\frac{1}{\ln x} + \ln(\ln x)] +$

$(\log_{10} x)^x [\frac{1}{\ln 10\log_{10} x} + \ln(\log_{10} x)]$

5(h) $\frac{d}{dx}\{(\tan x)^{\cot x} + (\cot x)^{\tan x}\}$

$= \frac{d}{dx}(\tan x)^{\cot x} + \frac{d}{dx}(\cot x)^{\tan x}$

$= (\tan x)^{\cot x}[\cot x\frac{d}{dx}\{\ln(\tan x)\} + \ln(\tan x)$

$\frac{d}{dx}(\cot x)] + (\cot x)^{\tan x}[\tan x\frac{d}{dx}\{\ln(\cot x)\}$

$+ \ln(\cot x)\frac{d}{dx}(\tan x)]$

$= (\tan x)^{\cot x}[\frac{\cot x}{\tan x}\sec^2 x + \ln(\tan x).$

$(-\cos ec^2 x)] + (\cot x)^{\tan x}[\frac{\tan x}{\cot x}$

$(-\cos ec^2 x) + \ln(\cot x).(\sec^2 x)]$

$= (\tan x)^{\cot x}[\frac{\cos^2 x}{\sin^2 x}\frac{1}{\cos^2 x} - \ln(\tan x).$

$\cos ec^2 x] + (\cot x)^{\tan x}[-\frac{\sin^2 x}{\cos^2 x}.\frac{1}{\sin^2 x}$

$+ \ln(\cot x).(\sec^2 x)]$

$= (\tan x)^{\cot x}.\cos ec^2 x[1 - \ln(\tan x)]$

$+ (\cot x)^{\tan x}.\sec^2 x[\ln(\cot x) - 1]$

5(i) $\frac{d}{dx}(x^x \log_{10} x)$ [চ.'১২]

$= x^x\frac{d}{dx}(\log_{10} x) + \log_{10} x\frac{d}{dx}(x^x)$

$= x^x\frac{1}{x\ln 10} + \log_{10} x[x^x\{x\frac{d}{dx}(\ln x)$

$+ \ln x\frac{d}{dx}(x)\}]$

$$= \frac{x^x}{x\ln 10} + x^x \log_{10} x \left\{x\frac{1}{x} + \ln x\right\}$$

$$= \frac{x^x}{x\ln 10} + x^x \log_{10} x \{1 + \ln x\}$$

প্রশ্নমালা IX H

1. $\frac{dy}{dx}$ নির্ণয় কর ঃ

(a) $x^a y^b = (x-y)^{a+b}$ [প্র.ভ.প. '০৬]

$\therefore ln(x^a y^b) = ln(x-y)^{a+b}$

$\Rightarrow ln(x^a) + ln(y^b) = (a+b)\, ln(x-y)$

$\Rightarrow a\, lnx + b\, ln\, y = (a+b)\, ln(x-y)$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$a.\frac{1}{x} + b.\frac{1}{y}\frac{dy}{dx} = (a+b)\frac{1}{x-y}(1 - \frac{dy}{dx})$$

or, $(\frac{b}{y} + \frac{a+b}{x-y})\frac{dy}{dx} = \frac{a+b}{x-y} - \frac{a}{x}$

or, $\frac{bx-by+ay+by}{y(x-y)} \cdot \frac{dy}{dx} = \frac{ax+bx-ax+ay}{x(x-y)}$

or, $\frac{bx+ay}{y(x-y)} \cdot \frac{dy}{dx} = \frac{bx+ay}{x(x-y)}$

$\therefore \frac{dy}{dx} = \frac{y}{x}$

1(b) $y = \sin(x+y)^2$ [রা.'০৪; কু.'০৭; য.'১১]

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} = \cos(x+y)^2 \frac{d}{dx}(x+y)^2$$

$$\Rightarrow \frac{dy}{dx} = \cos(x+y)^2 .2(x+y)(1 + \frac{dy}{dx})$$

$$\Rightarrow \{1 - 2(x+y)\cos(x+y)^2\}\frac{dy}{dx}$$

$$= 2(x+y)\cos(x+y)^2$$

$$\therefore \frac{dy}{dx} = \frac{2(x+y)\cos(x+y)^2}{1-2(x+y)\cos(x+y)^2}$$

1(c) $x + y = \sin^{-1}(y/x)$

$\Rightarrow \sin(x+y) = \frac{y}{x} \Rightarrow y = x\sin(x+y)$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} = x\cos(x+y)(1 + \frac{dy}{dx}) + \sin(x+y)$$

$$\Rightarrow \{1 - x\cos(x+y)\}\frac{dy}{dx} = x\cos(x+y) + \sin(x+y)$$

$$\therefore \frac{dy}{dx} = \frac{x\cos(x+y) + \sin(x+y)}{1 - x\cos(x+y)}$$

1. (d) $x^2 = 5y^2 + \sin y$ [প্র.ভ.প.'০৬]

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$2x = 10y\frac{dy}{dx} + \cos y\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{2x}{10y + \cos y} \text{ (Ans.)}$$

1(e) $(\cos x)^y = (\sin y)^x$ [প্র.ভ.প.'০৩]

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$(\cos x)^y [\, y\frac{d}{dx}\{\ln(\cos x)\} + \ln(\cos x)\frac{dy}{dx}]$$

$$= (\sin y)^x \, [\, x\frac{d}{dx}\{\ln(\sin y)\} + \ln(\sin y)\frac{d}{dx}(x)]$$

$$\Rightarrow \frac{y}{\cos x}(-\sin x) + \ln(\cos x)\frac{dy}{dx}$$

$$= \frac{x}{\sin y}(\cos y)\frac{dy}{dx} + \ln(\sin y).1$$

$$[\because (\cos x)^y = (\sin y)^x]$$

$$\Rightarrow \{\ln(\cos x) - x\cot y\}\frac{dy}{dx} = \ln(\sin y) + y\tan x$$

$$\therefore \frac{dy}{dx} = \frac{\ln(\sin y) + y\tan x}{\ln(\cos x) - x\cot y}$$

1(f) $\sqrt{x/y} + \sqrt{y/x} = 1$

$$\Rightarrow \frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} = 1 \Rightarrow x + y = \sqrt{xy}$$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$1 + \frac{dy}{dx} = \frac{1}{2\sqrt{xy}}(x\frac{dy}{dx} + y.1)$$