

## প্রশ্নমালা IX H

$$= \frac{x^x}{x \ln 10} + x^x \log_{10} x \left\{ x \frac{1}{x} + \ln x \right\}$$

$$= \frac{x^x}{x \ln 10} + x^x \log_{10} x \{1 + \ln x\}$$

## প্রশ্নমালা IX H

1.  $\frac{dy}{dx}$  নির্ণয় কর :

(a)  $x^a y^b = (x - y)^{a+b}$  [প্র.ভ.প. '০৬]

$$\therefore \ln(x^a y^b) = \ln(x - y)^{a+b}$$

$$\Rightarrow \ln(x^a) + \ln(y^b) = (a + b) \ln(x - y)$$

$$\Rightarrow a \ln x + b \ln y = (a + b) \ln(x - y)$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$a \cdot \frac{1}{x} + b \cdot \frac{1}{y} \frac{dy}{dx} = (a + b) \frac{1}{x - y} \left(1 - \frac{dy}{dx}\right)$$

$$\text{or, } \left(\frac{b}{y} + \frac{a + b}{x - y}\right) \frac{dy}{dx} = \frac{a + b}{x - y} - \frac{a}{x}$$

$$\text{or, } \frac{bx - by + ay + by}{y(x - y)} \cdot \frac{dy}{dx} = \frac{ax + bx - ax + ay}{x(x - y)}$$

$$\text{or, } \frac{bx + ay}{y(x - y)} \cdot \frac{dy}{dx} = \frac{bx + ay}{x(x - y)}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$

1(b)  $y = \sin(x + y)^2$  [রা. '০৪; কু. '০৭; য. '১১]

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} = \cos(x + y)^2 \cdot \frac{d}{dx} (x + y)^2$$

$$\Rightarrow \frac{dy}{dx} = \cos(x + y)^2 \cdot 2(x + y) \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \{1 - 2(x + y) \cos(x + y)^2\} \frac{dy}{dx} = 2(x + y) \cos(x + y)^2$$

$$\therefore \frac{dy}{dx} = \frac{2(x + y) \cos(x + y)^2}{1 - 2(x + y) \cos(x + y)^2}$$

1(c)  $x + y = \sin^{-1}(y/x)$

$$\Rightarrow \sin(x + y) = \frac{y}{x} \Rightarrow y = x \sin(x + y)$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} = x \cos(x + y) \left(1 + \frac{dy}{dx}\right) + \sin(x + y)$$

$$\Rightarrow \{1 - x \cos(x + y)\} \frac{dy}{dx} = x \cos(x + y) + \sin(x + y)$$

$$\therefore \frac{dy}{dx} = \frac{x \cos(x + y) + \sin(x + y)}{1 - x \cos(x + y)}$$

1. (d)  $x^2 = 5y^2 + \sin y$  [প্র.ভ.প. '০৬]

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$2x = 10y \frac{dy}{dx} + \cos y \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{2x}{10y + \cos y} \text{ (Ans.)}$$

1(e)  $(\cos x)^y = (\sin y)^x$  [প্র.ভ.প. '০৩]

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$(\cos x)^y \left[ y \frac{d}{dx} \{\ln(\cos x)\} + \ln(\cos x) \frac{dy}{dx} \right]$$

$$= (\sin y)^x \left[ x \frac{d}{dx} \{\ln(\sin y)\} + \ln(\sin y) \frac{d}{dx} (x) \right]$$

$$\Rightarrow \frac{y}{\cos x} (-\sin x) + \ln(\cos x) \frac{dy}{dx}$$

$$= \frac{x}{\sin y} (\cos y) \frac{dy}{dx} + \ln(\sin y) \cdot 1$$

$$[\because (\cos x)^y = (\sin y)^x]$$

$$\Rightarrow \{\ln(\cos x) - x \cot y\} \frac{dy}{dx} = \ln(\sin y) + y \tan x$$

$$\therefore \frac{dy}{dx} = \frac{\ln(\sin y) + y \tan x}{\ln(\cos x) - x \cot y}$$

1(f)  $\sqrt{x/y} + \sqrt{y/x} = 1$

$$\Rightarrow \frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} = 1 \Rightarrow x + y = \sqrt{xy}$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$1 + \frac{dy}{dx} = \frac{1}{2\sqrt{xy}} \left(x \frac{dy}{dx} + y \cdot 1\right)$$

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$$\Rightarrow \left(1 - \frac{\sqrt{x}}{2\sqrt{y}}\right) \frac{dy}{dx} = \frac{\sqrt{y}}{2\sqrt{x}} - 1$$

$$\Rightarrow \frac{2\sqrt{y} - \sqrt{x}}{2\sqrt{y}} \frac{dy}{dx} = \frac{\sqrt{y} - 2\sqrt{x}}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{\sqrt{y}(\sqrt{y} - 2\sqrt{x})}{\sqrt{x}(2\sqrt{y} - \sqrt{x})} \quad (\text{Ans.})$$

2.  $\frac{dy}{dx}$  নির্ণয় কর :

2(a)  $x^y = e^{x-y}$  [য.বো.'০৫]

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$x^y \left[ y \frac{d}{dx}(\ln x) + \ln x \frac{dy}{dx} \right] = e^{x-y} \left( 1 - \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{y}{x} + \ln x \frac{dy}{dx} = 1 - \frac{dy}{dx} \quad [\because x^y = e^{x-y}]$$

$$\Rightarrow (1 + \ln x) \frac{dy}{dx} = 1 - \frac{y}{x} = \frac{x-y}{x}$$

$$\therefore \frac{dy}{dx} = \frac{x-y}{x(1+\ln x)}$$

2(b)  $y + x = x^{-y}$  [রা.'১১; য.'১৩; প্র.ভ.প. '৯৫]

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} + 1 = x^{-y} \left[ -y \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(-y) \right]$$

$$\Rightarrow \frac{dy}{dx} + 1 = x^{-y} \left[ \frac{-y}{x} - \ln x \frac{dy}{dx} \right]$$

$$\Rightarrow (1 + x^{-y} \ln x) \frac{dy}{dx} = -1 - y \cdot x^{-y-1}$$

$$\therefore \frac{dy}{dx} = -\frac{1 + yx^{-y-1}}{1 + x^{-y} \ln x} \quad (\text{Ans.})$$

2(c)  $x^y + y^x = 1$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$x^y \left[ y \frac{d}{dx}(\ln x) + \ln x \frac{dy}{dx} \right] +$$

$$y^x \left[ x \frac{d}{dx}(\ln y) + \ln y \frac{d}{dx}(x) \right] = 0$$

$$\Rightarrow x^y \left[ \frac{y}{x} + \ln x \frac{dy}{dx} \right] + y^x \left[ \frac{x}{y} \frac{dy}{dx} + \ln y \cdot 1 \right] = 0$$

$$\Rightarrow (x^y \ln x + xy^{x-1}) \frac{dy}{dx} = - (x^{y-1}y + y^x \ln y)$$

$$\therefore \frac{dy}{dx} = -\frac{x^{y-1}y + y^x \ln y}{x^y \ln x + xy^{x-1}}$$

2(d)  $x^p y^q = (x+y)^{p+q}$

$$\therefore p \ln x + q \ln y = (p+q) \ln(x+y)$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{p+q}{x+y} \left( 1 + \frac{dy}{dx} \right)$$

$$\Rightarrow \left( \frac{q}{y} - \frac{p+q}{x+y} \right) \frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\Rightarrow \frac{qx + qy - py - qy}{y(x+y)} \frac{dy}{dx} = \frac{px + qx - px - py}{(x+y)x}$$

$$\Rightarrow \frac{qx - py}{y(x+y)} \frac{dy}{dx} = \frac{qx - py}{(x+y)x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} \quad (\text{Ans.})$$

2(e)  $y = x^{y^x} \therefore \ln y = y^x \ln x \dots (1)$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{y} \frac{dy}{dx} = y^x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(y^x)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = y^x \frac{1}{x} + \ln x \cdot y^x \left\{ \frac{x}{y} \frac{dy}{dx} + \ln y \right\}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{\ln y}{x \ln x} + \ln y \left\{ \frac{x}{y} \frac{dy}{dx} + \ln y \right\}$$

[ (1) দ্বারা ]

$$\Rightarrow \left( \frac{1}{y} - \frac{x}{y} \ln y \right) \frac{dy}{dx} = \ln y \left( \frac{1}{x \ln x} + \ln y \right)$$

$$\Rightarrow \left( \frac{1 - x \ln y}{y} \right) \frac{dy}{dx} = \ln y \left( \frac{1 + x \ln x \ln y}{x \ln x} \right)$$

$$\therefore \frac{dy}{dx} = \frac{y \ln y (1 + x \ln x \ln y)}{x \ln x (1 - x \ln y)}$$

$$(f) y = \sqrt{x \sqrt{x \sqrt{x \dots \infty}}} = \sqrt{x \sqrt{x \sqrt{x \sqrt{x \dots \infty}}}}$$

$$\Rightarrow y = \sqrt{xy} \Rightarrow y^2 = xy \Rightarrow y = x$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} = 1 \text{ (Ans.)}$$

$$2.(g) \ln(xy) = x + y$$

[রা. '০৫; কু. '০৬]

$$\Rightarrow \ln x + \ln y = x + y$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{x} + \frac{1}{y} \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow y + x \frac{dy}{dx} = xy + xy \frac{dy}{dx}$$

$$\Rightarrow x(1-y) \frac{dy}{dx} = y(x-1)$$

$$\therefore \frac{dy}{dx} = \frac{y(x-1)}{x(1-y)} \text{ (Ans.)}$$

$$2.(h) \log_{10}(x^n y^n) = x^n + y^n \text{ [বুয়েট ০৭-০৮]}$$

$$\Rightarrow n \log_{10} x + n \log_{10} y = x^n + y^n$$

$$\Rightarrow n \log_{10} e \times \log_e x + n \log_{10} e \times \log_e y = x^n + y^n$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$n \frac{\log_{10} e}{x} + n \frac{\log_{10} e}{y} \frac{dy}{dx} = nx^{n-1} + ny^{n-1} \frac{dy}{dx}$$

$$\Rightarrow \left( \frac{\log_{10} e}{y} - y^{n-1} \right) \frac{dy}{dx} = x^{n-1} - \frac{\log_{10} e}{x}$$

$$\Rightarrow \frac{\log_{10} e - y^n}{y} \frac{dy}{dx} = \frac{x^n - \log_{10} e}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y(x^n - \log_{10} e)}{x(\log_{10} e - y^n)}$$

$$3.(a) \tan y = \sin x \text{ হলে, } \frac{dy}{dx} \text{ নির্ণয় কর।}$$

$$\text{প্রমাণ : } \tan y = \sin x$$

$$\Rightarrow y = \tan^{-1} \sin x$$

$$\therefore \frac{dy}{dx} = \frac{1}{1 + \sin^2 x} \times \cos x$$

$$= \frac{\cos x}{1 + \sin^2 x} \text{ (Ans.)}$$

$$3(b) x\sqrt{1+y} + y\sqrt{1+x} = 0 \text{ হলে, দেখাও যে,}$$

$$\frac{dy}{dx} = -\frac{1}{(1+x)^2}$$

[প্র.ভ.প. '০২, '০৪]

$$\text{প্রমাণ : } x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$$

$$\Rightarrow x^2(1+y) = y^2(1+x) \text{ [বর্গ করে।]}$$

$$\Rightarrow x^2 + x^2y = y^2 + xy^2$$

$$\Rightarrow x^2 - y^2 + xy(x-y) = 0$$

$$\Rightarrow (x-y)(x+y+xy) = 0$$

$$x+y+xy = 0 \text{ হলে, } (1+x)y = -x$$

$$\Rightarrow y = \frac{-x}{1+x} \therefore \frac{dy}{dx} = \frac{(1+x)(-1) + x(1)}{(1+x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1-x+x}{(1+x)^2} = -\frac{1}{(1+x)^2}$$

$$3.(c) x = a(t - \sin t) \text{ এবং } y = a(1 + \cos t) \text{ হলে,}$$

$$\text{দেখাও যে, } t = \frac{5\pi}{3} \text{ যখন } \frac{dy}{dx} = \sqrt{3}.$$

[প্র.ভ.প. '৮৫]

$$\text{প্রমাণ : } \frac{dx}{dt} = a(1 - \cos t), \frac{dy}{dt} = a(0 - \sin t)$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-a \sin t}{a(1 - \cos t)}$$

$$= \frac{-2 \sin \frac{t}{2} \cos \frac{t}{2}}{2 \sin^2 \frac{t}{2}} = -\cot \frac{t}{2}$$

$$\text{এখন, } \frac{dy}{dx} = \sqrt{3} \text{ হলে, } \cot \frac{t}{2} = -\sqrt{3}$$

$$\Rightarrow \tan \frac{t}{2} = -\frac{1}{\sqrt{3}} = -\tan \frac{\pi}{6} = \tan\left(\pi - \frac{\pi}{6}\right)$$

$$\Rightarrow \tan \frac{t}{2} = \tan \frac{5\pi}{6} \therefore \frac{t}{2} = \frac{5\pi}{6} \Rightarrow t = \frac{5\pi}{3}$$

$$3(d) f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x} \text{ হলে, প্রমাণ কর যে,}$$

$$f'(0) = \left(2 \ln \frac{a}{b} + \frac{b^2 - a^2}{ab}\right) \left(\frac{a}{b}\right)^{a+b}$$

প্রমাণ:  $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x} \therefore f(0) = \left(\frac{a}{b}\right)^{a+b}$

এবং  $\ln\{f(x)\} = (a+b+2x)\{\ln(a+x) - \ln(b+x)\}$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{f(x)} f'(x) = (a+b+2x) \left\{ \frac{1}{a+x} - \frac{1}{b+x} \right\} + \{\ln(a+x) - \ln(b+x)\} 2$$

$$\therefore f'(0) = f(0) \left[ (a+b) \left( \frac{1}{a} - \frac{1}{b} \right) + 2(\ln a - \ln b) \right]$$

$$\Rightarrow f'(0) = \left(\frac{a}{b}\right)^{a+b} \left[ (a+b) \left( \frac{b-a}{ab} \right) + 2 \ln \frac{a}{b} \right]$$

$$\therefore f'(0) = \left( 2 \ln \frac{a}{b} + \frac{b^2 - a^2}{ab} \right) \left(\frac{a}{b}\right)^{a+b}$$

(e)  $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x \dots \infty}}}$  হলে,

প্রমাণ কর যে,  $(2y-1) \frac{dy}{dx} + \sin x = 0$ .

প্রমাণ:  $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x \dots \infty}}}$

$$\Rightarrow y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x \dots \infty}}}}$$

$$\Rightarrow y = \sqrt{\cos x + y} \Rightarrow y^2 = \cos x + y$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$2y \frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\Rightarrow (2y-1) \frac{dy}{dx} + \sin x = 0$$

3(f)  $x^{y^n} = y^{x^n}$  হলে দেখাও যে,

$$\frac{dy}{dx} = \frac{y^{n+1}(n \ln x - 1)}{x^{n+1}(n \ln y - 1)} \quad [\text{বুয়েট } ০৮-০৯]$$

প্রমাণ:  $x^{y^n} = y^{x^n} \therefore y^n \ln x = x^n \ln y \dots (1)$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{y^n}{x} + \ln x \cdot (ny^{n-1}) \frac{dy}{dx} = \frac{x^n}{y} \frac{dy}{dx} + \ln y \cdot nx^{n-1}$$

$$\Rightarrow y^{n+1} + x \ln x \cdot ny^n \frac{dy}{dx} = x^{n+1} \frac{dy}{dx} + y \ln y \cdot nx^n$$

$$\Rightarrow (nx \ln x \cdot y^n - x^{n+1}) \frac{dy}{dx} = y \ln y \cdot nx^n - y^{n+1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{nyx^n \ln y - y^{n+1}}{nxy^n \ln x - x^{n+1}}$$

$$= \frac{ny \cdot y^n \ln x - y^{n+1}}{nx \cdot x^n \ln y - x^{n+1}} \quad [(1) \text{ দ্বারা}]$$

$$= \frac{y^{n+1}(n \ln x - 1)}{x^{n+1}(n \ln y - 1)}$$

সম্ভাব্য ধাপসহ প্রশ্ন:

1. (a) মূল নিয়মে  $x=2$  -তে  $\sqrt[3]{x}$  এর অন্তরক সহগ নির্ণয়।

মনে করি,  $f(x) = \sqrt[3]{x} = x^{1/3}$

$$\therefore f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{x^{1/3} - 2^{1/3}}{x - 2} \quad (S)$$

$$= \frac{1}{3} \times 2^{3^{-1}} \quad [ \because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} ] \quad (S)$$

$$= \frac{1}{3} \times 2^{-\frac{2}{3}} = \frac{1}{3} \times 4^{-\frac{1}{3}} = \frac{1}{3 \sqrt[3]{4}}$$

(b) মূল নিয়মে  $x=a$  -তে  $\cos^2 x$  এর অন্তরক সহগ নির্ণয়।

মনে করি,  $f(x) = \cos^2 x \therefore f(a) = \cos^2 a$

$$\therefore f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad (S)$$

$$= \lim_{x \rightarrow a} \frac{\cos^2 x - \cos^2 a}{x - a}$$

$$= \lim_{x \rightarrow a} \frac{\sin(x+a) \sin(a-x)}{x - a} \quad (S)$$

$$[ \because \cos^2 B - \cos^2 A = \sin(A+B) \sin(A-B) ]$$

$$= - \lim_{x \rightarrow a} \frac{\sin(x-a)}{x-a} \cdot \lim_{x \rightarrow a} \sin(x+a) \quad (S)$$

$$= -1 \cdot \sin(a+a) = -\sin 2a \quad (\text{Ans.}) \quad (S)$$

$x$  এর সাপেক্ষে নিম্নের ফাংশনগুলির অন্তরক সহগ নির্ণয় কর :

2.  $\frac{d}{dx} (5x^3 + 3x^2 - 4x - 9)$

$$= 5 \frac{d}{dx} (x^3) + 3 \frac{d}{dx} (x^2) - 4 \frac{d}{dx} (x) - \frac{d}{dx} (9) \quad (S)$$

$$= 5(3x^2) + 3(2x) - 4 - 0$$

$$= 15x^2 + 6x - 4 \text{ (Ans.)} \quad (১)$$

$$3. \frac{d}{dx} (2x^3 - 4x^{\frac{5}{2}} + \frac{7}{2}x^{\frac{2}{3}} + 7)$$

$$= 2 \frac{d}{dx} (x^3) - 4 \frac{d}{dx} (x^{\frac{5}{2}}) + \frac{7}{2} \frac{d}{dx} (x^{\frac{2}{3}}) + \frac{d}{dx} (7) \quad (১)$$

$$= 2(3x^2) - 4(\frac{5}{2}x^{\frac{5}{2}-1}) + \frac{7}{2}(-\frac{2}{3}x^{\frac{2}{3}-1}) + 0$$

$$= 6x^2 - 10x^{\frac{3}{2}} - \frac{7}{3}x^{-\frac{1}{3}} \text{ (Ans.)} \quad (১)$$

4.  $(2x)^n - b^n$  [চ. '০২]

$$(2x)^n - b^n = 2^n x^n - b^n$$

$$\therefore \frac{d}{dx} \{ (2x)^n - b^n \} = 2^n \frac{d}{dx} (x^n) - \frac{d}{dx} (b^n) \quad (১)$$

$$= 2^n n x^{n-1} - 0 = 2^n n x^{n-1} \quad (১)$$

5(a)  $x^2 \log_a x + 7e^x \cos x$  [সি. '০৪]

$$\frac{d}{dx} (x^2 \log_a x + 7e^x \cos x) = x^2 \frac{d}{dx} (\log_a x)$$

$$+ \log_a x \frac{d}{dx} (x^2) + 7 \{ e^x \frac{d}{dx} (\cos x) +$$

$$\cos x \frac{d}{dx} (e^x) \} \quad (১) + (১)$$

$$= x^2 \frac{1}{x \ln a} + \log_a x (2x) +$$

$$7 \{ e^x (-\sin x) + \cos x \cdot e^x \} \quad (১) + (১)$$

$$= x \left( \frac{1}{\ln a} + 2 \log_a x \right) + 7e^x (\cos x - \sin x)$$

5(b)  $\sin^2 2x + e^{2 \ln(\cos 2x)}$  [প্র.ভ.প. '১৩]

$$\sin^2 2x + e^{2 \ln(\cos 2x)} = \sin^2 2x + e^{\ln(\cos 2x)^2}$$

$$= \sin^2 2x + (\cos 2x)^2 \quad (১)$$

$$= \sin^2 2x + \cos^2 2x = 1$$

$$\therefore \frac{d}{dx} \{ \sin^2 2x + e^{2 \ln(\cos 2x)} \} = \frac{d}{dx} (1) = 0 \quad (১)$$

5(c)  $5e^x \ln x$  [য. '০৪]

মনে করি,  $y = 5e^x \ln x$

$$\therefore \frac{dy}{dx} = 5 \left\{ e^x \frac{d}{dx} (\ln x) + \ln x \frac{d}{dx} (e^x) \right\} \quad (১)$$

$$= 5 \left\{ e^x \cdot \frac{1}{x} + \ln x (e^x) \right\} \quad (১)$$

$$\therefore \frac{d}{dx} (5e^x \ln x) = 5e^x \left( \frac{1}{x} + \ln x \right)$$

6.(a)  $\frac{d}{dx} \left( \frac{x^n + \tan x}{e^x - \cot x} \right) =$

$$\frac{1}{(e^x - \cot x)^2} \left\{ (e^x - \cot x) \frac{d}{dx} (x^n + \tan x) \right.$$

$$\left. - (x^n + \tan x) \frac{d}{dx} (e^x - \cot x) \right\} \quad (১)$$

$$= \frac{1}{(e^x - \cot x)^2} \left\{ (e^x - \cot x)(nx^{n-1} + \sec^2 x) \right.$$

$$\left. - (x^n + \tan x)(e^x + \operatorname{cosec}^2 x) \right\} \quad (১) + (১)$$

6(b)  $\frac{d}{dx} \left( \frac{1 - \cos x}{1 + \cos x} \right)$

$$= \frac{(1 + \cos x) \frac{d}{dx} (1 - \cos x) - (1 - \cos x) \frac{d}{dx} (1 + \cos x)}{(1 + \cos x)^2} \quad (১)$$

$$= \frac{(1 + \cos x)(\sin x) - (1 - \cos x)(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\sin x(1 + \cos x + 1 - \cos x)}{(1 + \cos x)^2}$$

$$= \frac{2 \sin x}{(1 + \cos x)^2} \quad (১)$$

6(c)  $\frac{x \sin x}{x + \cos x}$  [রা. '০০]

$$\frac{d}{dx} \left( \frac{x \sin x}{x + \cos x} \right) = \frac{1}{(x + \cos x)^2} [(x + \cos x)$$

$$\frac{d}{dx} (x \sin x) - x \sin x \frac{d}{dx} (x + \cos x)] \quad (১)$$

$$= \frac{1}{(x + \cos x)^2} [(x + \cos x)(x \cos x + \sin x) \cdot 1$$

$$- x \sin x (1 - \sin x)] \quad (১) + (১)$$

$$= \frac{1}{(x + \cos x)^2} [(x^2 \cos x + x \sin x + x \cos^2 x + \cos x \sin x - x \sin x + x \sin^2 x)]$$

$$= \frac{x(\sin^2 x + \cos^2 x) + x^2 \cos x + \cos x \sin x}{(x + \cos x)^2}$$

$$= \frac{x + (x^2 + \sin x) \cos x}{(x + \cos x)^2} \text{ (Ans.)} \quad (১)$$

6.(d)  $\frac{\sin^2 x}{1 + \cos x}$

$$\frac{\sin^2 x}{1 + \cos x} = \frac{1 - \cos^2 x}{1 + \cos x} = \frac{(1 - \cos x)(1 + \cos x)}{1 + \cos x}$$

$$= 1 - \cos x \quad (১)$$

$$\therefore \frac{d}{dx} \left( \frac{\sin^2 x}{1 + \cos x} \right) = \sin x \quad (১)$$

6(e)  $\frac{\cos x}{1 + \sin^2 x}$  [স. '০৯]

$$\frac{d}{dx} \left( \frac{\cos x}{1 + \sin^2 x} \right) =$$

$$\frac{(1 + \sin^2 x) \frac{d}{dx} (\cos x) - \cos x \frac{d}{dx} (1 + \sin^2 x)}{(1 + \sin^2 x)^2} \quad (১)$$

$$= \frac{(1 + \sin^2 x)(-\sin x) - \cos x(2 \sin x \cos x)}{(1 + \sin^2 x)^2} \quad (২)$$

$$= \frac{-\sin x(1 + \sin^2 x + 2 \cos^2 x)}{(1 + \sin^2 x)^2}$$

$$= \frac{-\sin x(2 + \cos^2 x)}{(1 + \sin^2 x)^2} \quad (১)$$

7. (a) ধরি,  $y = (x + \sqrt{1 + x^2})^n$

$$\therefore \frac{dy}{dx} = n(x + \sqrt{1 + x^2})^{n-1} \frac{d}{dx} (x + \sqrt{1 + x^2}) \quad (১)$$

$$= n(x + \sqrt{1 + x^2})^{n-1} \left\{ 1 + \frac{1}{2\sqrt{1 + x^2}} \cdot 2x \right\} \quad (২)$$

$$= n(x + \sqrt{1 + x^2})^{n-1} \frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}}$$

$$\therefore \frac{d}{dx} ((x + \sqrt{1 + x^2})^n) = \frac{n(x + \sqrt{1 + x^2})^n}{\sqrt{1 + x^2}} \quad (১)$$

7(b)  $\frac{d}{dx} \{ \operatorname{cosec}(e^{x^2}) \}$

$$= \frac{d\{\operatorname{cosec}(e^{x^2})\}}{d(e^{x^2})} \cdot \frac{d(e^{x^2})}{d(x^2)} \cdot \frac{d(x^2)}{dx} \quad (১)$$

$$= -\operatorname{cosec}(e^{x^2}) \cot(e^{x^2}) \cdot (e^{x^2}) \cdot 2x \quad (২)$$

$$= -2x e^{x^2} \operatorname{cosec}(e^{x^2}) \cot(e^{x^2}) \text{ [সি. '০৯]} \text{ (Ans.)}$$

8(a)  $\log_x 5$  [প্র.ভ.প. '৮৮]

$$\log_x 5 = \log_x e \times \log_e 5 = \ln 5 \frac{1}{\log_e x} \quad (১)$$

$$= \ln 5 \frac{1}{\ln x} = \ln 5 (\ln x)^{-1}$$

$$\therefore \frac{d}{dx} (\log_x a) = \ln 5 \{-1(\ln x)^{-2} \frac{d}{dx} (\ln x)\}$$

$$= -\ln 5 \frac{1}{(\ln x)^2} \cdot \frac{1}{x} = -\frac{\ln 5}{x(\ln x)^2} \quad (১)$$

8(b)  $\ln(\sin e^{x^2})$  [প্র.ভ.প. '৯৫]

$$\frac{d}{dx} \{ \ln(\sin e^{x^2}) \}$$

$$= \frac{1}{\sin(e^{x^2})} \{ \cos(e^{x^2}) \} e^{x^2} \cdot 2x \quad (২)$$

$$= 2x e^{x^2} \cot(e^{x^2})$$

8(c)  $\frac{d}{dx} \{ \ln(\tan \frac{x}{2}) \}$

$$= \frac{d\{\ln(\tan \frac{x}{2})\}}{d(\tan \frac{x}{2})} \cdot \frac{d(\tan \frac{x}{2})}{d(\frac{x}{2})} \cdot \frac{d(\frac{x}{2})}{dx} \quad (১)$$

$$= \frac{1}{\tan \frac{x}{2}} \sec^2 \frac{x}{2} \cdot \frac{1}{2} = \frac{1}{2} \frac{\cos(x/2)}{\sin(x/2) \cos^2(x/2)} \quad (১)$$

$$= \frac{1}{2 \sin(x/2) \cos(x/2)} = \frac{1}{\sin x} = \operatorname{cosec} x \quad (১)$$

$$9. (a) \frac{d}{dx} \{ \ln(ax^2 + bx + c) \}$$

$$= \frac{1}{ax^2 + bx + c} \frac{d}{dx} (ax^2 + bx + c) \quad (S)$$

$$= \frac{2ax + b}{ax^2 + bx + c} \quad (\text{Ans.}) \quad (S)$$

$$9(b) \frac{d}{dx} \{ \ln(x + \sqrt{x^2 \pm a^2}) \}$$

$$= \frac{1}{x + \sqrt{x^2 \pm a^2}} \frac{d}{dx} (x + \sqrt{x^2 \pm a^2}) \quad (S)$$

$$= \frac{1}{x + \sqrt{x^2 \pm a^2}} \left\{ 1 + \frac{1}{2\sqrt{x^2 \pm a^2}} (2x) \right\} \quad (S)$$

$$= \frac{1}{x + \sqrt{x^2 \pm a^2}} \left\{ \frac{\sqrt{x^2 \pm a^2} + x}{\sqrt{x^2 \pm a^2}} \right\}$$

$$= \frac{1}{\sqrt{x^2 \pm a^2}} \quad (\text{Ans.}) \quad (S)$$

$$9(c) \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$= \ln(\sqrt{x+1}-1) - \ln(\sqrt{x+1}+1) \quad (S)$$

$$\therefore \frac{d}{dx} \left\{ \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} \right\}$$

$$= \frac{1}{\sqrt{x+1}-1} \cdot \frac{1}{2\sqrt{x+1}} - \frac{1}{\sqrt{x+1}+1} \cdot \frac{1}{2\sqrt{x+1}} \quad (S)$$

$$= \frac{\sqrt{x+1}+1 - \sqrt{x+1}-1}{2\sqrt{x+1}(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$$

$$= \frac{2}{2\sqrt{x+1}(x+1-1)} = \frac{1}{x\sqrt{x+1}} \quad (\text{Ans.}) \quad (S)$$

$$10. (a) \left( \frac{\sin 2x}{1 + \cos 2x} \right)^2 = \left( \frac{2 \sin x \cos x}{2 \cos^2 x} \right)^2 \quad (S)$$

$$= \left( \frac{\sin x}{\cos x} \right)^2 = \tan^2 x$$

$$\therefore \frac{d}{dx} \left( \frac{\sin 2x}{1 + \cos 2x} \right)^2 = 2 \tan x \frac{d}{dx} (\tan x) \quad (S)$$

$$= 2 \tan x \cdot \sec^2 x \quad (S)$$

$$10. (b) \left[ \frac{x}{\sqrt{1-x^2}} \right]^n \quad [\text{প্র.ভ.প.'০৫}]$$

$$\frac{d}{dx} \left[ \frac{x}{\sqrt{1-x^2}} \right]^n = n \left[ \frac{x}{\sqrt{1-x^2}} \right]^{n-1}$$

$$\frac{\sqrt{1-x^2} \cdot 1 - x \cdot \frac{1}{2\sqrt{1-x^2}} (-2x)}{(\sqrt{1-x^2})^2} \quad (S)$$

$$= n \left[ \frac{x}{\sqrt{1-x^2}} \right]^{n-1} \frac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}} \quad (S)$$

$$= n \left[ \frac{x}{\sqrt{1-x^2}} \right]^{n-1} \frac{1}{(1-x^2)^{3/2}} \quad (S)$$

$$10(c) \frac{d}{dx} \{ x \ln x \ln(\ln x) \}$$

$$= x \ln x \frac{d}{dx} \{ \ln(\ln x) \} + x \ln(\ln x) \frac{d}{dx} (x)$$

$$+ \ln x \ln(\ln x) \frac{d}{dx} (x) \quad (S)$$

$$= x \ln x \frac{1}{\ln x} \cdot \frac{1}{x} + x \ln(\ln x) \frac{1}{x} +$$

$$\ln x \ln(\ln x) \cdot 1 \quad (S)$$

$$= 1 + \ln(\ln x)(1 + \ln x) \quad (S)$$

$$10(d) \frac{d}{dx} (\sin x \sin 2x \sin 3x)$$

$$= \sin x \sin 2x \frac{d}{dx} (\sin 3x) + \sin x \sin 3x$$

$$\frac{d}{dx} (\sin 2x) + \sin 2x \sin 3x \frac{d}{dx} (\sin x) \quad (S)$$

$$= \sin x \sin 2x (\cos 3x) \cdot 3 + \sin x \sin 3x (\cos 2x) \cdot 2 + \sin 2x \sin 3x (\cos x) \cdot 1 \quad (S)$$

$$= 3 \sin x \sin 2x \cos 3x + 2 \sin x \sin 3x \cos 2x$$

$$+ \sin 2x \sin 3x \cos x \quad (S)$$

$$11(a) \frac{d}{dx} (e^{\sqrt{x}} + e^{-\sqrt{x}})$$

$$= e^{\sqrt{x}} \frac{d}{dx} (\sqrt{x}) + e^{-\sqrt{x}} \frac{d}{dx} (-\sqrt{x}) \quad (S)$$

$$= e^{\sqrt{x}} \frac{1}{2\sqrt{x}} - e^{-\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}} \quad (১)$$

11(b)  $\frac{d}{dx}(e^{-x} + e^{\frac{1}{x}})$

$$= e^{-x} \frac{d}{dx}(-x) + e^{\frac{1}{x}} \frac{d}{dx}\left(\frac{1}{x}\right) \quad (১)$$

$$= -e^{-x} \cdot 1 + e^{\frac{1}{x}} \left(-\frac{1}{x^2}\right) = -\left(e^{-x} + \frac{1}{x^2} e^{\frac{1}{x}}\right) \quad (১)$$

12(a) ধরি,  $y = \ln \sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{1}{2} \ln \frac{1+\sin x}{1-\sin x}$

$$= \frac{1}{2} \{\ln(1+\sin x) - \ln(1-\sin x)\} \quad (১)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \left\{ \frac{\cos x}{1+\sin x} - \frac{(-\cos x)}{1-\sin x} \right\} \quad (২)$$

$$= \frac{1}{2} \frac{\cos x(1-\sin x) + \cos x(1+\sin x)}{(1+\sin x)(1-\sin x)}$$

$$= \frac{1}{2} \frac{2 \cos x}{1-\sin^2 x} = \frac{\cos x}{\cos^2 x} = \sec x \quad (১)$$

12(b) ধরি,  $y = \cos \frac{x^{-1}-x}{x^{-1}+x}$  [প্র.ভ.প. ৮৯]

$$\frac{dy}{dx} = -\sin \frac{x^{-1}-x}{x^{-1}+x} \frac{d}{dx} \left( \frac{x^{-1}-x}{x^{-1}+x} \right) \quad (১)$$

$$= -\sin \frac{x^{-1}-x}{x^{-1}+x} \frac{d}{dx} \left( \frac{1-x^2}{1+x^2} \right)$$

$$= -\sin \frac{x^{-1}-x}{x^{-1}+x} \frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2} \quad (২)$$

$$= -\sin \frac{x^{-1}-x}{x^{-1}+x} \frac{2x(-1-x^2-1+x^2)}{(1+x^2)^2}$$

$$= \frac{4x}{(1+x^2)^2} \sin \frac{x^{-1}-x}{x^{-1}+x} \quad (১)$$

12(c)  $e^{3x} \cos x^\circ = e^{3x} \cos \frac{\pi x}{180}$  (১)

$$\therefore \frac{d}{dx}(e^{3x} \cos x^\circ) = e^{3x} \cdot \left(-\sin \frac{\pi x}{180}\right) \frac{d}{dx} \left(\frac{\pi x}{180}\right)$$

$$+ \cos \frac{\pi x}{180} \cdot e^{3x} \frac{d}{dx}(3x) \quad (২)$$

$$= -e^{3x} \cdot \sin x^\circ \cdot \left(\frac{\pi}{180}\right) + \cos x^\circ \cdot e^{3x} \cdot 3$$

$$= e^{3x} \left(3 \cos x^\circ - \frac{\pi}{180} \sin x^\circ\right)$$

13(a)  $\frac{d}{dx} \{ \sin^{-1}(e^{\tan^{-1} x}) \}$

$$= \frac{1}{\sqrt{1-(e^{\tan^{-1} x})^2}} \frac{d}{dx}(e^{\tan^{-1} x})$$

$$= \frac{1}{\sqrt{1-e^{2 \tan^{-1} x}}} e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$= \frac{e^{\tan^{-1} x}}{(1+x^2)\sqrt{1-e^{2 \tan^{-1} x}}}$$

13(b)  $\frac{d}{dx} \left\{ \cos^{-1} \left( \frac{a+b \cos x}{b+a \cos x} \right) \right\}$

$$= -\frac{1}{\sqrt{1-\left(\frac{a+b \cos x}{b+a \cos x}\right)^2}}$$

$$\frac{(b+a \cos x)(-b \sin x) - (a+b \cos x)(-a \sin x)}{(b+a \cos x)^2} \quad (৩)$$

$$= -\frac{b+a \cos x}{\sqrt{(b+a \cos x)^2 - (a+b \cos x)^2}}$$

$$\frac{(-b^2+a^2) \sin x}{(b+a \cos x)^2}$$

$$= \frac{-(a^2-b^2) \sin x}{(b+a \cos x)\sqrt{b^2+a^2 \cos^2 x - a^2 - b^2 \cos^2 x}}$$

$$= \frac{(b^2-a^2) \sin x}{(b+a \cos x)\sqrt{(b^2-a^2)(1-\cos^2 x)}}$$

$$= \frac{(b^2-a^2) \sin x}{(b+a \cos x)\sqrt{(b^2-a^2) \sin^2 x}}$$

$$= \frac{\sqrt{b^2-a^2}}{b+a \cos x} \quad (১)$$

$$13(c) \sin^{-1}\left(\frac{2x^{-1}}{x+x^{-1}}\right) = \sin^{-1}\left(\frac{2/x}{x+1/x}\right)$$

$$= \sin^{-1}\left(\frac{2}{x^2+1}\right) \quad (১)$$

$$\therefore \frac{d}{dx} \left\{ \sin^{-1}\left(\frac{2x^{-1}}{x+x^{-1}}\right) \right\}$$

$$= \frac{1}{\sqrt{1-\frac{4}{(x^2+1)^2}}} \cdot 2 \frac{d}{dx} (x^2+1)^{-1} \quad (২)$$

$$= \frac{x^2+1}{\sqrt{x^4+2x^2+1-4}} \cdot 2(-1)(x^2+1)^{-2} \cdot 2x \quad (৩)$$

$$= \frac{-4x(x^2+1)^{-1}}{\sqrt{x^4+2x^2-3}} = \frac{-4x}{(x^2+1)\sqrt{x^4+2x^2-3}} \quad (৪)$$

$$13(d) \frac{d}{dx} \{ \cos^{-1} x \ln(\sin^{-1} x) \} \quad [\text{প্র.ভ.প. '০৪}]$$

$$= \cos^{-1} x \frac{d}{dx} \{ \ln(\sin^{-1} x) \} + \ln(\sin^{-1} x) \frac{d}{dx} (\cos^{-1} x) \quad (১)$$

$$= \cos^{-1} x \frac{1}{\sin^{-1} x} \frac{1}{\sqrt{1-x^2}} + \frac{\ln(\sin^{-1} x)}{-\sqrt{1-x^2}} \quad (২)$$

$$= \frac{1}{\sqrt{1-x^2}} \left\{ \frac{\cos^{-1} x}{\sin^{-1} x} - \ln(\sin^{-1} x) \right\} \quad (৩)$$

$$13(e) \cot^{-1}\left(\frac{x^2}{e^x}\right) + \cot^{-1}\left(\frac{e^x}{x^2}\right) \quad [\text{প্র.ভ.প. '০৫}]$$

$$= \tan^{-1}\left(\frac{e^x}{x^2}\right) + \tan^{-1}\left(\frac{x^2}{e^x}\right)$$

$$= \tan^{-1} \frac{\frac{e^x}{x^2} + \frac{x^2}{e^x}}{1 - \frac{e^x}{x^2} \cdot \frac{x^2}{e^x}} = \tan^{-1} \frac{\frac{e^x}{x^2} + \frac{x^2}{e^x}}{1-1} \quad (১)$$

$$= \cot^{-1} \frac{1-1}{\frac{e^x}{x^2} + \frac{x^2}{e^x}} = \cot^{-1} 0 = \frac{\pi}{2}$$

$$\therefore \frac{d}{dx} \left\{ \cot^{-1}\left(\frac{x^2}{e^x}\right) + \cot^{-1}\left(\frac{e^x}{x^2}\right) \right\} = \frac{d}{dx} \left(\frac{\pi}{2}\right) = 0 \quad (২)$$

$$13(f) \tan^{-1} \frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{ax}} \quad [\text{প্র.ভ.প. '১৬}]$$

$$= \tan^{-1} \frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{x}\sqrt{a}} = \tan^{-1} \sqrt{x} + \tan^{-1} \sqrt{a} \quad (১)$$

$$\therefore \frac{d}{dx} \left\{ \tan^{-1} \frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{ax}} \right\}$$

$$= \frac{d}{dx} (\tan^{-1} \sqrt{x}) + \frac{d}{dx} (\tan^{-1} \sqrt{a}) \quad (২)$$

$$= \frac{1}{1+(\sqrt{x})^2} \frac{d}{dx} (\sqrt{x}) + 0 \quad (৩)$$

$$= \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)} \quad (৪)$$

$$14(a) \text{ ধরি, } y = \tan^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \text{ এবং}$$

$$x^2 = \cos \theta. \text{ তাহলে, } \theta = \cos^{-1} x^2 \text{ এবং} \quad (১)$$

$$y = \tan^{-1} \frac{\sqrt{1+\cos \theta} - \sqrt{1-\cos \theta}}{\sqrt{1+\cos \theta} + \sqrt{1-\cos \theta}}$$

$$= \tan^{-1} \frac{\sqrt{2 \cos^2(\theta/2)} - \sqrt{2 \sin^2(\theta/2)}}{\sqrt{2 \cos^2(\theta/2)} + \sqrt{2 \sin^2(\theta/2)}} \quad (২)$$

$$= \tan^{-1} \frac{\sqrt{2} \{ \cos(\theta/2) - \sin(\theta/2) \}}{\sqrt{2} \{ \cos(\theta/2) + \sin(\theta/2) \}}$$

$$= \tan^{-1} \frac{\cos(\theta/2) \{ 1 - \tan(\theta/2) \}}{\cos(\theta/2) \{ 1 + \tan(\theta/2) \}}$$

$$= \tan^{-1} \frac{1 - \tan(\theta/2)}{1 + \tan(\theta/2)} = \tan^{-1}(1) - \tan^{-1} \tan \frac{\theta}{2} \quad (৩)$$

$$= \frac{\pi}{4} - \frac{\theta}{2} = \frac{\pi}{4} - \frac{1}{2} \tan^{-1} x^2$$

$$\therefore \frac{dy}{dx} = 0 - \frac{1}{2} \left\{ -\frac{1}{1+(x^2)^2} \right\} (2x) = \frac{x}{\sqrt{1-x^4}} \quad (৪)$$

$$14. (b) \text{ ধরি, } y = \sec^{-1} \frac{1}{2x^2-1} \text{ এবং } x = \cos \theta$$

$$\text{তাহলে, } \theta = \cos^{-1} x \text{ এবং}$$

$$y = \sec^{-1} \frac{1}{2 \cos^2 \theta - 1} = \sec^{-1} \frac{1}{\cos 2\theta} \quad (১)$$

$$= \sec^{-1} \sec 2\theta = 2\theta = 2 \cos^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(2 \cos^{-1} x) = \frac{-2}{\sqrt{1-x^2}} \text{ (Ans.) (১)}$$

$$14(c) \frac{d}{dx} \{ \sin^{-1}(\tan^{-1} x) \} \quad [\text{সি. '০১}]$$

$$= \frac{1}{\sqrt{1-(\tan^{-1} x)^2}} \frac{d}{dx}(\tan^{-1} x) \quad (১)$$

$$= \frac{1}{\sqrt{1-(\tan^{-1} x)^2}} \frac{1}{1+x^2} \quad (১)$$

$$= \frac{1}{(1+x^2)\sqrt{1-(\tan^{-1} x)^2}} \text{ (Ans.)}$$

$$14(d) \tan^{-1} \frac{\cos x - \sin x}{\cos x + \sin x} \quad [\text{প্র.ভ.প. '০৫}]$$

$$= \tan^{-1} \frac{\cos x(1 - \tan x)}{\cos x(1 + \tan x)} = \tan^{-1} \frac{1 - \tan x}{1 + \tan x}$$

$$= \tan^{-1} 1 - \tan^{-1}(\tan x) = \frac{\pi}{4} - x \quad (১)$$

$$\therefore \frac{d}{dx} \left\{ \tan^{-1} \frac{\cos x - \sin x}{\cos x + \sin x} \right\} = \frac{d}{dx} \left( \frac{\pi}{4} - x \right)$$

$$= 0 - 1 = -1 \text{ (Ans.)} \quad (১)$$

$\frac{dy}{dx}$  নির্ণয় কর :

$$15(a) x = a(\theta - \sin \theta), y = a(1 + \cos \theta) \quad [\text{প্র.ভ.প. '০৬}]$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta} \{ a(\theta - \sin \theta) \} = a(1 - \cos \theta) \quad (১)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta} \{ a(1 + \cos \theta) \} = a(0 - \sin \theta) \quad (১)$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-a \sin \theta}{a(1 - \cos \theta)} \quad (১)$$

$$= \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = -\cot \frac{\theta}{2} \quad (১)$$

$$15(b) \frac{d}{dx} (\sin x)^{\ln x} = (\sin x)^{\ln x}$$

$$= \left[ \ln x \frac{d}{dx} \{ \ln(\sin x) \} + \ln(\sin x) \frac{d}{dx} (\ln x) \right] (\sin x)^{\ln x}$$

$$= (\sin x)^{\ln x} \left[ \ln x \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot \frac{1}{x} \right]$$

$$= (\sin x)^{\ln x} \left[ \ln x \cdot \cot x + \frac{\ln(\sin x)}{x} \right] \quad (১)$$

$$15(c) \frac{d}{dx} (\sin x)^{\tan x} = (\sin x)^{\tan x}$$

$$\left[ \tan x \frac{d}{dx} \{ \ln(\sin x) \} + \ln(\sin x) \frac{d}{dx} (\tan x) \right] (\sin x)^{\tan x}$$

$$= (\sin x)^{\tan x} \left[ \frac{\sin x \cos x}{\cos x \sin x} + \ln(\sin x) \cdot \sec^2 x \right] \quad (১)$$

$$= (\sin x)^{\tan x} [1 + \sec^2 x \cdot \ln(\sin x)]$$

$$15(d) \frac{d}{dx} (\tan x)^{\ln x} = (\tan x)^{\ln x}$$

$$\left[ \ln x \frac{d}{dx} \{ \ln(\tan x) \} + \ln(\tan x) \frac{d}{dx} (\ln x) \right] (\tan x)^{\ln x}$$

$$= (\tan x)^{\ln x} \left[ \ln x \frac{1}{\tan x} \sec^2 x + \ln(\tan x) \cdot \frac{1}{x} \right] \quad (১)$$

$$= (\tan x)^{\ln x} \left[ \ln x \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} + \frac{\ln(\tan x)}{x} \right]$$

$$= (\tan x)^{\ln x} \left[ \ln x \frac{2}{2 \sin x \cos x} + \frac{\ln(\tan x)}{x} \right]$$

$$= (\tan x)^{\ln x} \left[ 2 \ln x \cdot \operatorname{cosec} 2x + \frac{\ln(\tan x)}{x} \right] \quad (১)$$

$$15(e) \frac{d}{dx} (\ln x)^{\ln x} = (\ln x)^{\ln x}$$

$$\left[ \ln x \frac{d}{dx} \{ \ln(\ln x) \} + \ln(\ln x) \frac{d}{dx} (\ln x) \right] (\ln x)^{\ln x}$$

$$= (\ln x)^{\ln x} \left[ \ln x \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) \cdot \frac{1}{x} \right]$$

$$= \frac{1}{x} (\ln x)^{\ln x} [1 + \ln(\ln x)] \quad (১)$$

$$15(f) \frac{d}{dx} (\ln x)^{\tan^{-1} x} = (\ln x)^{\tan^{-1} x}$$

$$\left[ \tan^{-1} x \frac{d}{dx} \{ \ln(\ln x) \} + \ln(\ln x) \frac{d}{dx} (\tan^{-1} x) \right] (\ln x)^{\tan^{-1} x}$$

$$= (\ln x)^{\tan^{-1} x} \left[ \tan^{-1} x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \frac{\ln(\ln x)}{1+x^2} \right] \quad (১)$$

$$= (\ln x)^{\tan^{-1} x} \left[ \frac{\tan^{-1} x}{x \ln x} + \frac{\ln(\ln x)}{1+x^2} \right]$$

$$(g) \frac{d}{dx} (\tan x)^{\cos^{-1} x} = (\tan x)^{\cos^{-1} x}$$

$$\left[ \cos^{-1} x \frac{d}{dx} \{ \ln(\tan x) \} + \ln(\tan x) \frac{d}{dx} (\cos^{-1} x) \right] \quad (১)$$

$$= (\tan x)^{\cos^{-1} x} \left[ \frac{\sec^2 x \cdot \cos^{-1} x}{\tan x} - \frac{\ln(\tan x)}{\sqrt{1-x^2}} \right] \quad (১)$$

$$(h) (\sin^{-1} x)^{\ln x} \quad [\text{প্র.ভ.প. '৯৬}]$$

$$\frac{d}{dx} (\sin^{-1} x)^{\ln x} = (\sin^{-1} x)^{\ln x}$$

$$\left[ \ln x \frac{d}{dx} \{ \ln(\sin^{-1} x) \} + \ln(\sin^{-1} x) \frac{d}{dx} (\ln x) \right] \quad (১)$$

$$= (\sin^{-1} x)^{\ln x} \left[ \frac{\ln x}{\sin^{-1} x \sqrt{1-x^2}} + \frac{\ln(\sin^{-1} x)}{x} \right] \quad (১)$$

$$= (\sin^{-1} x)^{\ln x} \left[ \frac{\ln x}{\sqrt{1-x^2} \sin^{-1} x} + \frac{\ln(\sin^{-1} x)}{x} \right]$$

$$16.(a) \frac{d}{dx} (x^x + x^{1/x}) = \frac{d}{dx} (x^x) + \frac{d}{dx} (x^{1/x}) \quad (১)$$

$$= x^x \left\{ x \frac{d}{dx} (\ln x) + \ln x \frac{d}{dx} (x) \right\} +$$

$$x^{1/x} \left\{ \frac{1}{x} \frac{d}{dx} (\ln x) + \ln x \frac{d}{dx} \left( \frac{1}{x} \right) \right\} \quad (১)$$

$$= x^x \left\{ x \cdot \frac{1}{x} + \ln x \cdot 1 \right\} + x^{1/x} \left\{ \frac{1}{x} \cdot \frac{1}{x} + \ln x \cdot \left( -\frac{1}{x^2} \right) \right\} \quad (১)$$

$$= x^x (1 + \ln x) + x^{1/x} \cdot \frac{1}{x^2} (1 - \ln x)$$

$$= x^x (1 + \ln x) + x^{\frac{1}{x}-2} (1 - \ln x) \quad (১)$$

$$16.(b) \frac{d}{dx} (x^x \cdot x^{\cos^{-1} x})$$

$$= x^x \frac{d}{dx} (x^{\cos^{-1} x}) + x^{\cos^{-1} x} \frac{d}{dx} (x^x) \quad (১)$$

$$= x^x \cdot x^{\cos^{-1} x} \left[ \cos^{-1} x \frac{d}{dx} (\ln x) \right]$$

$$+ \ln x \frac{d}{dx} (\cos^{-1} x)] + x^{\cos^{-1} x} \cdot x^x \left[ x \frac{d}{dx} (\ln x) + \ln x \frac{d}{dx} (x) \right] \quad (১)$$

$$= x^x \cdot x^{\cos^{-1} x} \left[ \frac{\cos^{-1} x}{x} + \frac{-\ln x}{\sqrt{1-x^2}} \right] + x^{\cos^{-1} x} \cdot x^x \left[ x \cdot \frac{1}{x} + \ln x \cdot 1 \right] \quad (১)$$

$$= x^x \cdot x^{\cos^{-1} x} \left[ \frac{\cos^{-1} x}{x} - \frac{\ln x}{\sqrt{1-x^2}} + 1 + \ln x \right] \quad (১)$$

$$17(a) x = y \cdot \ln(xy) \Rightarrow \frac{x}{y} = \ln x + \ln y$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{y \cdot 1 - x \cdot \frac{dy}{dx}}{y^2} = \frac{1}{x} + \frac{1}{y} \frac{dy}{dx} \quad (১)$$

$$\Rightarrow xy - x^2 \frac{dy}{dx} = y^2 + xy \frac{dy}{dx}$$

$$\Rightarrow y(x-y) = x(x+y) \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{y(x-y)}{x(x+y)} \quad (১)$$

$$17(b) y = \cot(x+y) \Rightarrow \cot^{-1} y = x+y$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$-\frac{1}{1+y^2} \frac{dy}{dx} = 1 + \frac{dy}{dx} \quad (১)$$

$$\Rightarrow \left( -\frac{1}{1+y^2} - 1 \right) \frac{dy}{dx} = 1$$

$$\Rightarrow -\frac{1+1+y^2}{1+y^2} \frac{dy}{dx} = 1$$

$$\therefore \frac{dy}{dx} = -\frac{1+y^2}{2+y^2} \quad (\text{Ans.}) \quad (১)$$

$$17(c) y = \tan(x+y)$$

[প্র.ভ.প. '৯৯]

$$\Rightarrow \tan^{-1} y = x+y$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{1+y^2} \frac{dy}{dx} = 1 + \frac{dy}{dx} \quad (১)$$

$$\Rightarrow \left( \frac{1}{1+y^2} - 1 \right) \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{1-1-y^2}{1+y^2} \frac{dy}{dx} = 1 \quad \therefore \frac{dy}{dx} = -\frac{1+y^2}{y^2} \quad (২)$$

$$17. (d) \quad x^2 + y^2 = \sin(xy)$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$2x + 2y \frac{dy}{dx} = \cos(xy) \left( x \frac{dy}{dx} + y \right) \quad (১)$$

$$\Rightarrow \{2y - x \cos(xy)\} \frac{dy}{dx} = y \cos(xy) - 2x$$

$$\therefore \frac{dy}{dx} = \frac{y \cos(xy) - 2x}{2y - x \cos(xy)} \quad (২)$$

$$(e) \quad \cos y = x \cos(a+y) \Rightarrow x = \frac{\cos y}{\cos(a+y)}$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$1 = \frac{\cos(a+y)(-\sin y) \frac{dy}{dx} - \cos y \{-\sin(a+y)\} \frac{dy}{dx}}{\cos^2(a+y)} \quad (১)$$

$$1 = \frac{\{\sin(a+y) \cos y - \cos(a+y) \sin y\} \frac{dy}{dx}}{\cos^2(a+y)}$$

$$\cos^2(a+y) = \sin(a+y-y) \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a} \quad (\text{Ans.}) \quad (২)$$

$$17(f) \quad e^{2x} + 5y^3 = 3 \cos(xy) \quad [\text{প্র.ভ.প. '৯৫}]$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$e^{2x} \cdot 2 + 15y^2 \frac{dy}{dx} = 3 \{-\sin(xy)\} \frac{d}{dx}(xy) \quad (১)$$

$$\Rightarrow 2e^{2x} + 15y^2 \frac{dy}{dx} = -3 \sin(xy) \left( x \frac{dy}{dx} + y \right) \quad (২)$$

$$\Rightarrow \{15y^2 + 3x \sin(xy)\} \frac{dy}{dx} = -2e^{2x} + 3y \sin(xy)$$

$$\therefore \frac{dy}{dx} = \frac{2e^{2x} + 3y \sin(xy)}{15y^2 + 3x \sin(xy)} \quad (৩)$$

$$18(a) \quad y = x^y$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} = x^y \left[ y \frac{d}{dx}(\ln x) + \ln x \frac{dy}{dx} \right] \quad (১)$$

$$\Rightarrow \frac{dy}{dx} = y \left[ \frac{y}{x} + \ln x \frac{dy}{dx} \right] \quad [\because x^y = y]$$

$$\Rightarrow (1 - y \ln x) \frac{dy}{dx} = \frac{y^2}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y^2}{x(1 - y \ln x)} \quad (\text{Ans.}) \quad (২)$$

$$18(b) \quad x^y y^x = 1 \quad [\text{প্র.ভ.প. '০২}]$$

$$\therefore y \ln x + x \ln y = 0 \quad (১)$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$y \frac{1}{x} + \ln x \frac{dy}{dx} + x \cdot \frac{1}{y} \frac{dy}{dx} + \ln y = 0 \quad (২)$$

$$\Rightarrow y^2 + xy \ln x \frac{dy}{dx} + x^2 \frac{dy}{dx} + xy \ln y = 0$$

$$\Rightarrow (xy \ln x + x^2) \frac{dy}{dx} = -(xy \ln y + y^2)$$

$$\therefore \frac{dy}{dx} = -\frac{y(x \ln y + y)}{x(y \ln x + x)} \quad (৩)$$

$$18(c) \quad (\sin x)^{\cos y} + (\cos x)^{\sin y} = a$$

উভয় পক্ষকে  $x$  এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$(\sin x)^{\cos y} \left[ \cos y \frac{d}{dx} \{\ln(\sin x)\} + \ln(\sin x) \right]$$

$$\frac{d}{dx}(\cos y) + (\cos x)^{\sin y} \left[ \sin y \frac{d}{dx} \{\ln(\cos x)\} \right]$$

$$+ \ln(\cos x) \frac{d}{dx}(\sin y) = 0 \quad (১)$$

$$\Rightarrow (\sin x)^{\cos y} [\cos y \cot x + \ln(\sin x)$$

$$(-\sin y) \frac{dy}{dx}] + (\cos x)^{\sin y} [\sin y (-\tan x) +$$

$$\ln(\cos x) \cdot \cos y \frac{dy}{dx}] = 0 \quad (২)$$

$$\Rightarrow \{(\cos x)^{\sin y} \ln(\cos x) \cdot \cos y$$

$$\frac{d}{dx} \left\{ (\sin x)^{\cos y} \ln(\sin x) \sin y \right\} = (\cos x)^{\sin y} \sin y \tan x - (\sin x)^{\cos y} \cos y \cot x$$

$$\frac{dy}{dx} = \frac{(\cos x)^{\sin y} \sin y \tan x - (\sin x)^{\cos y} \cos y \cot x}{(\cos x)^{\sin y} \ln(\cos x) \cos y - (\sin x)^{\cos y} \ln(\sin x) \sin y} \quad (১)$$

19.  $y = \tan^{-1} \sqrt{\frac{1-x}{1+x}}$  হলে, দেখাও যে,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$$

প্রমাণ : ধরি,  $x = \cos \theta \Rightarrow \theta = \cos^{-1} x \quad (১)$

$$\therefore y = \tan^{-1} \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} = \tan^{-1} \sqrt{\frac{2 \sin^2 \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}} \quad (১)$$

$$= \tan^{-1} \tan \frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2} \cos^{-1} x \quad (১)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \frac{-1}{\sqrt{1-x^2}} = -\frac{1}{2\sqrt{1-x^2}} \quad (২)$$

20.  $x = 1$  বিন্দুতে  $y = x^2$  ফাংশনের অন্তরক আকার সমীকরণ থেকে  $dy$  এবং  $\delta y$  নির্ণয় কর যখন  $dx = \delta x = 2$ .

সমাধান : ধরি,  $f(x) = y = x^2$

$$\therefore \frac{dy}{dx} = 2x \Rightarrow dy = 2x dx \quad (১)$$

$$\Rightarrow dy = 2 \times 1 \times 2, [\because x = 1, dx = 2] \quad (১)$$

$$\Rightarrow dy = 4$$

আবার,  $\delta y = f(x + \delta x) - f(x)$

$$= f(1+2) - f(1) = f(3) - f(1)$$

$$= 3^2 - 1^2 = 9 - 1 = 8. \quad (১)$$

21.  $x = 3$  বিন্দুতে  $y = \frac{x^2}{3} + 1$  ফাংশনের অন্তরক আকার সমীকরণ থেকে  $dy$  এবং  $\delta y$  নির্ণয় কর যখন  $dx = \delta x = 3$ .

সমাধান : ধরি,  $f(x) = y = \frac{x^2}{3} + 1$

$$\therefore \frac{dy}{dx} = \frac{2}{3}x \Rightarrow dy = \frac{2}{3}x dx \quad (১)$$

$$\Rightarrow dy = \frac{2}{3} \times 3 \times 3, [\because x = 3, dx = 3] \quad (১)$$

$$\therefore dy = 6$$

আবার,  $\delta y = f(x + \delta x) - f(x)$

$$= f(3+3) - f(3) = f(6) - f(3) \quad (১)$$

$$= \left(\frac{6^2}{3} + 1\right) - \left(\frac{3^2}{3} + 1\right)$$

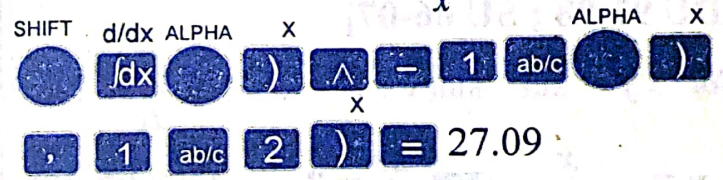
$$= 12 - 3 = 9 \quad (১)$$

ভর্তি পরীক্ষার MCQ :

1.  $y = x^{-\frac{1}{x}}$  হলে  $\frac{dy}{dx}$  এর মান- [BUET 07-08]

Sol<sup>n</sup> :  $\frac{dy}{dx} = x^{-\frac{1}{x}} \left[ -\frac{1}{x} \cdot \frac{1}{x} + \ln x \left( +\frac{1}{x^2} \right) \right]$

$$= x^{-\frac{1}{x}} \cdot \frac{1}{x^2} (\ln x - 1) = \frac{1}{x^{2+\frac{1}{x}}} (\ln x - 1)$$



Option গুলোতে  $x = \frac{1}{2}$  বসালে  $\frac{1}{x^{2+\frac{1}{x}}} (\ln x - 1)$

$$= 27.09 \text{ হয়।}$$

2.  $\frac{d}{dx} (\log_x e) = ?$  [DU 08-09]

Sol<sup>n</sup> :  $\frac{d}{dx} (\log_x e) = \frac{d}{dx} \left( \frac{1}{\ln x} \right) = -\frac{1}{x(\ln x)^2}$

3.  $\frac{d}{dx} \{ \ln(x + \sqrt{x^2 + a^2}) \} = ?$  [DU 07-08]

Sol<sup>n</sup> :  $\frac{d}{dx} \{ \ln(x + \sqrt{x^2 + a^2}) \}$

$$= \frac{1}{x + \sqrt{x^2 + a^2}} \cdot \left( 1 + \frac{2x}{2\sqrt{x^2 + a^2}} \right) = \frac{1}{\sqrt{x^2 + a^2}}$$

4.  $y = \sqrt{\sec x}$  হলে,  $\frac{dy}{dx} = ?$  [DU 00-01]

Sol<sup>n</sup>:  $\frac{dy}{dx} = \frac{1}{2\sqrt{\sec x}} \cdot \sec x \tan x$   
 $= \frac{\sqrt{\sec x} \tan x}{2} = \frac{y}{2} \tan x$

5.  $y = \cos \sqrt{x}$  হলে,  $\frac{dy}{dx} = ?$  [DU 03-04]

Sol<sup>n</sup>:  $\frac{dy}{dx} = -\sin \sqrt{x} \cdot \frac{1}{2\sqrt{x}} = -\frac{\sin \sqrt{x}}{2\sqrt{x}}$

6.  $f(x) = \sqrt{1-\sqrt{x}}$  হলে,  $\frac{df}{dx} = ?$  [DU 01-02]

Sol<sup>n</sup>:  $\frac{df}{dx} = \frac{1}{2\sqrt{1-\sqrt{x}}} \cdot \frac{-1}{2\sqrt{x}} = \frac{-1}{4\sqrt{x}\sqrt{1-\sqrt{x}}}$

7.  $y = \log_e(2x)^{1/3}$  হলে,  $\frac{dy}{dx} = ?$  [DU 98-99]

Sol<sup>n</sup>:  $\frac{dy}{dx} = \frac{1}{3} \frac{d}{dx} \{\log_e(2x)\} = \frac{1}{3 \cdot 2x} (2) = \frac{1}{3x}$

8.  $y = \sin^{-1} \sin(x+1)$  হলে,  $\frac{dy}{dx} = ?$

[DU 97-98 ; SU 06-07]

Sol<sup>n</sup>:  $y = \sin^{-1} \sin(x+1) = x+1 \therefore \frac{dy}{dx} = 1$

9.  $y = \frac{x}{\sqrt{x^2+1}}$  হলে,  $\frac{dy}{dx} = ?$  [NU 07-08]

Sol<sup>n</sup>:  $\frac{dy}{dx} = \frac{\sqrt{x^2+1} \cdot 1 - x \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{(\sqrt{x^2+1})^2}$   
 $= \frac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}} = \frac{1}{(x^2+1)^{3/2}}$

10.  $\frac{d}{dx}(a^x) = ?$  [KU, RU07-08; IU 02-03]

Sol<sup>n</sup>:  $\frac{d}{dx}(a^x) = a^x \ln a$

11.  $\frac{d}{dx}(\log_a m^2) = ?$

[CU 07-08]

Sol<sup>n</sup>:  $\frac{d}{dx}(\log_a m^2) = 0$

12.  $x = \frac{1}{2}$  হলে,  $\frac{d}{dx}(x^2 e^{2x} \log_e 2x) = ?$

[RU 07-08]

Sol<sup>n</sup>:  $\frac{d}{dx}(x^2 e^{2x} \log_e 2x)$

$= x^2 e^{2x} \cdot \frac{1}{2x} (2) + x^2 (e^{2x} \cdot 2) \log_e 2x$   
 $+ (2x) \cdot e^{2x} \log_e 2x$

$x = \frac{1}{2}$  হলে,  $\frac{d}{dx}(x^2 e^{2x} \log_e 2x)$

$= \frac{1}{4} e \cdot 2 + 0 + 0 = \frac{1}{2} e$

13.  $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$  হলে,  $\frac{dy}{dx} = ?$

[SU 06-07, 05-06; RU 03-04; IU 06-07]

Sol<sup>n</sup>:  $y = \sqrt{x+y} \Rightarrow y^2 = x+y$

$\therefore 2y \frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$

14.  $y = \cos^{-1} \frac{x-x^{-1}}{x+x^{-1}}$  হলে,  $\frac{dy}{dx} = ?$

[RU 06-07]

Sol<sup>n</sup>:  $y = \cos^{-1} \frac{x^2-1}{x^2+1} = -2 \tan^{-1} x$

$\therefore \frac{dy}{dx} = -\frac{2}{1+x^2}$

15.  $y = (\log_a x)(\log x)$  হলে,  $\frac{dy}{dx} = ?$  [RU 05-06]

Sol<sup>n</sup>:  $\frac{dy}{dx} = (\log_a x) \frac{1}{x \ln 10} + \frac{1}{x \ln a} (\log x)$

ie.  $\frac{dy}{dx} = (\log_a x) \frac{\log_a e}{x} + \frac{\log_{10} a}{x} (\log x)$

16.  $y = \tan^{-1} \frac{1+x}{1-x}$  হলে,  $\frac{dy}{dx} = ?$  [IU 05-06;

CU 02-03]

Sol<sup>n</sup>:  $y = \tan^{-1} \frac{1+x}{1-x} = \tan^{-1}(1) + \tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{1}{1+x^2}$

17.  $\tan y = \frac{2t}{1-t^2}$ ,  $\sin x = \frac{2t}{1+t^2}$

হলে,

20.  $x^2 + 3xy + 5y^2 = 1$  হলে,  $\frac{dy}{dx} = ?$

[DU 07-08]

$\frac{dy}{dx} = ?$  [SU 04-05; JU 06-07]

$Sol^n$ :  $y = \tan^{-1} \frac{2t}{1-t^2} = 2 \tan^{-1} t$ ,

$x = \sin^{-1} \frac{2t}{1+t^2} = 2 \tan^{-1} t \therefore y = x \Rightarrow \frac{dy}{dx} = 1$

18.  $x^y = e^{x-y}$  হলে,  $\frac{dy}{dx} = ?$  [SU 06-07]

$Sol^n$ :  $y \ln x = x - y \Rightarrow y = \frac{x}{1 + \ln x}$

$\therefore \frac{dy}{dx} = \frac{(1 + \ln x) \cdot 1 - x \cdot \frac{1}{x}}{(1 + \ln x)^2} = \frac{\ln x}{(1 + \ln x)^2}$

19.  $y = f(x)$  হলে,  $\frac{d}{dx}(e^y) = ?$  [CU 07-08]

$Sol^n$ :  $\frac{d}{dx}(e^y) = e^y \frac{dy}{dx}$

$Sol^n$ :  $2x + 3(x \frac{dy}{dx} + y \cdot 1) + 10y \frac{dy}{dx} = 0$

$\Rightarrow (3x + 10y) \frac{dy}{dx} = -(2x + 3y)$

$\therefore \frac{dy}{dx} = -\frac{2x + 3y}{3x + 10y}$

21.  $y = x^{\frac{1}{3}} + x^{-\frac{1}{3}}$  হলে,  $3(y^2 - 1) \frac{dy}{dx} = ?$

[DU 04-05]

$Sol^n$ :  $y^3 = x + x^{-1} + 3 \cdot x^{\frac{1}{3}} x^{-\frac{1}{3}} (x^{\frac{1}{3}} + x^{-\frac{1}{3}})$

$\Rightarrow y^3 = x + \frac{1}{x} + 3y$

$\therefore 3y^2 \frac{dy}{dx} = 1 - \frac{1}{x^2} + 3 \frac{dy}{dx}$

$\Rightarrow 3(y^2 - 1) \frac{dy}{dx} = 1 - \frac{1}{x^2}$  (Ans.)