

Matrices & Determinants

Chapter - 010 - 10th Paper

01) ಅನಿರ್ದೇಶನ ನಿರ್ಧಾರ:

ಢ್ನಿ ಡಿಟರ್ಮಿಂಟ್:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}; \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

ಅನಿರ್ದೇಶನ ನಿರ್ಧಾರ:

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; \quad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}; \quad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$x = \frac{D_x}{D}; \quad y = \frac{D_y}{D}; \quad z = \frac{D_z}{D}$$

Chapter-02

① • $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$ [$\theta = 90^\circ$]

• $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ [$\theta = 0^\circ$]

② • $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ [$\theta = 0^\circ$]

• $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$ [$\theta = 90^\circ$]

• $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$

③ a & b দুইটি vector - অর্থাৎ $a+b$ এর
মagnitude বা দৈর্ঘ্য কত? এটা vector,

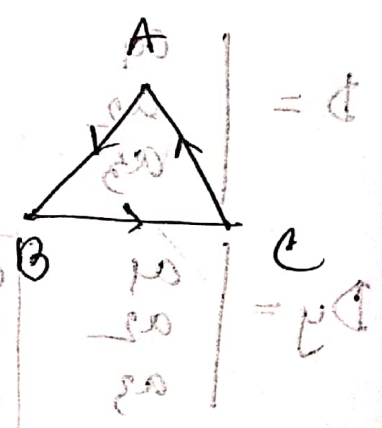
$$a+b = (x_1i + y_1j + z_1k) + (x_2i + y_2j + z_2k)$$

$$= (x_1+x_2)i + (y_1+y_2)j + (z_1+z_2)k$$

$$|a+b| = \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2 + (z_1+z_2)^2}$$

④ ত্রিভুজ সূত্র:

• $\vec{AB} + \vec{BC} + \vec{CA} = 0$



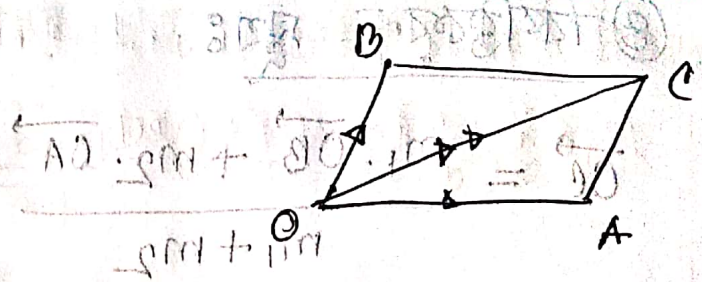
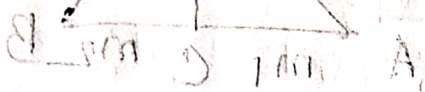
$\Rightarrow \vec{AB} + \vec{BC} = -\vec{CA}$

$\Rightarrow \vec{AB} + \vec{BC} = \vec{AC}$

$\frac{AB}{AC} = \frac{BC}{AC} = \frac{CA}{AC} = 1$

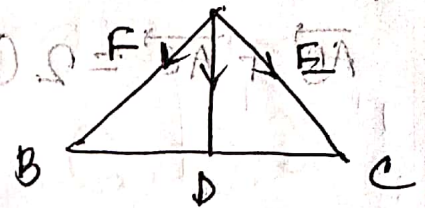
০৬) ত্রিভুজের সূত্র:

$$\vec{OA} + \vec{OB} = \vec{OC}$$



০৬) মধ্যমা সূত্র:

কোনো ত্রিভুজের মধ্যমা মধ্যমস্থ বাহুদ্বয়ের যোগফল উক্ত মধ্যমার দ্বিগুন।

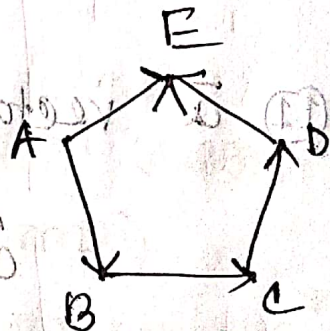


$$\text{① } \vec{AB} + \vec{AC} = 2\vec{AD}$$

$$\text{② } AB^2 + AC^2 = 2(AD^2 + BD^2)$$

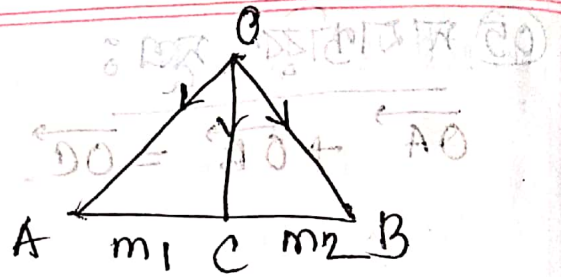
০৭) n ভুজের কোনো একভুজের (n-1) অন্যান্য বাহু একত্রে একে সূচি ২নে, অবশিষ্ট বাহুটি বিপরীতক্রমে একে মর্ষি প্রকাশ কর।

$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} = \vec{AE}$$



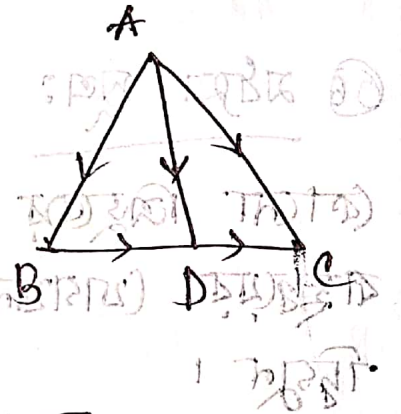
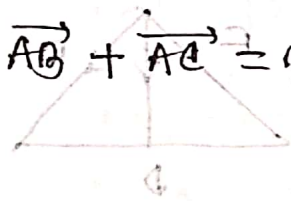
8) विभक्तिकरण सूत्रः

$$\vec{OC} = \frac{m_1 \cdot \vec{OB} + m_2 \cdot \vec{OA}}{m_1 + m_2}$$



9) अप्रामोला नियम के अनुसार:

$$\vec{AB} + \vec{AC} = 2(\vec{AD} + \vec{BD})$$



10) a एवं b का अदिक्लप,

$$b \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

11) a एवं b का अदिक्लप,

$$a \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

11) a vector वक्तव्य = एक vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

⑫ a vector ব্যবহার করে b vector এর উল্লম্ব

$$\frac{b - \frac{a \cdot b}{|a|^2} a}{|b - \frac{a \cdot b}{|a|^2} a|} = \frac{a \cdot b}{|a|^2} \cdot \frac{a}{|a|} = (\hat{a} \cdot \hat{b}) \hat{a}$$

[a vector ব্যবহার করে a নিজে হবে]

⑬ b vector ব্যবহার করে a এর উল্লম্ব

$$= \frac{a \cdot b}{|b|^2} \cdot b = (\hat{b} \cdot \hat{a}) \hat{b}$$

⑭ লম্ব হওয়ার কন্ডিশন : $a \cdot b = 0$

সমান্তরাল হওয়ার কন্ডিশন : $a \times b = 0$

⑮ মধ্যবর্তী কোণ θ হল,

$$\theta = \cos^{-1} \frac{a \cdot b}{|a||b|}$$

⑯ A (a) & B (b) বিদ্যুৎসূত্র সংযোগক (যেখানে R (r))

বিদ্যুৎ ছেদ হলে (m:n অনুপাত),

অনুবিভক্ত হলে R এর অক্ষীয় vector = $\frac{n\bar{a} + m\bar{b}}{m+n}$

বিশিষ্ট হলে = $\frac{n\bar{a} - m\bar{b}}{m+n}$

16) A(a), B(b), C(c) ΔABC এর ক্ষেত্রস্থ

$$\vec{a} \cdot \vec{b} = \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{c}}{3}$$

[এই সমীচরণটি ব্যবহার করে প্রমাণ করুন]

প্রমাণ: ΔABC-এর ক্ষেত্রস্থ D

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \frac{\vec{a} + \vec{b} + \vec{c}}{3} =$$

$$0 = \vec{a} \cdot \vec{c} \quad ; \quad \text{এই সমীচরণটি (১)}$$

$$0 = \vec{b} \cdot \vec{c} \quad ; \quad \text{এই সমীচরণটি (২)}$$

সুতরাং $\vec{a} \cdot \vec{c} = 0$ এবং $\vec{b} \cdot \vec{c} = 0$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \cos \theta = 0$$

∴ $\theta = 90^\circ$ অর্থাৎ $\vec{a} \perp \vec{b}$ ।

সুতরাং ΔABC-এর ক্ষেত্রস্থ D

$$\frac{m^2 - n^2}{m+n} = \text{কোনো মান}$$

১৩)

নামঘোষা

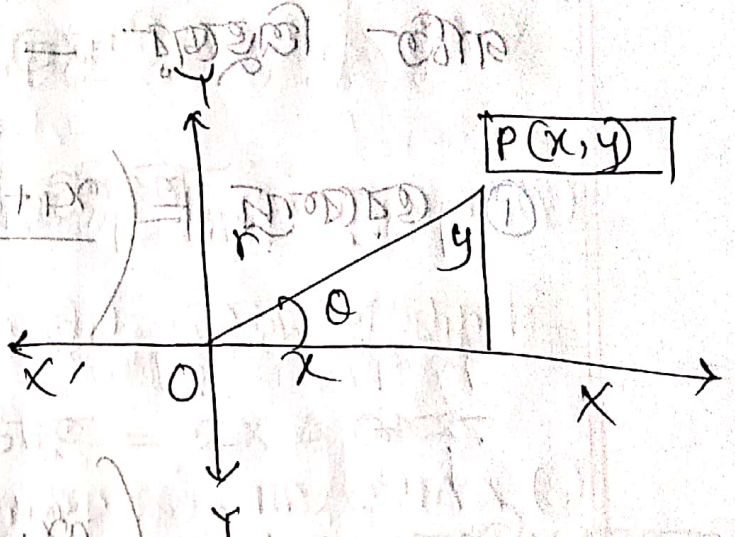
Chapter - 03

① • $x = r \cos \theta$

• $y = r \sin \theta$

• $r = \sqrt{x^2 + y^2}$

• $\theta = \tan^{-1} \left(\frac{y}{x} \right)$



② A(x₁, y₁) এবং B(x₂, y₂) বিন্দুদ্বয়ের মধ্যকার সরলরেখার সমীকরণ (x, y) বিন্দুটি m₁ ও m₂ মিশ্রিত করলে (→)

① অন্তর্বিচ্ছেদ করলে, $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$
 $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

② বাহ্যিকবিচ্ছেদ করলে, $x = \frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}$ ①

$y = \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}$

4 ক্ষেত্রফল (Area):

(i) $\text{আয়তক্ষেত্র} = \text{দৈর্ঘ্য} \times \text{প্রস্থ}$

(ii) $\text{ত্রুণ্ড} = \frac{1}{2} \times \text{কর্নদ্বয়ের গুণফল}$

(iii) $\text{বর্গক্ষেত্র} = (\text{বাহু})^2$

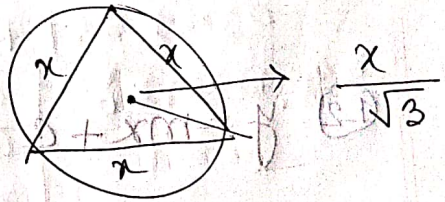
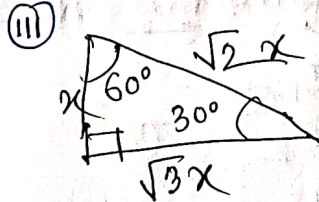
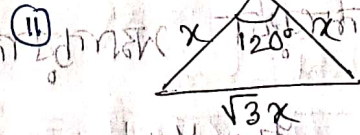
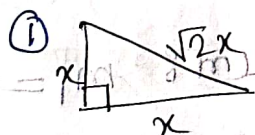
(iv) $\text{সামান্তরিকের} = \text{ভূমি} \times \text{উচ্চতা} = 2 \times \Delta \text{ ক্ষেত্র}$

(v) $\text{ত্রিভুজের ক্ষেত্রফল} = \frac{1}{2} \times \text{মধ্যস্থল বাহুর মধ্যস্থল} \times \text{তলের দূরত্ব}$

(vi) $\text{ত্রিভুজ} = \frac{1}{2} \times \text{ভূমি} \times \text{উচ্চতা} = \sqrt{s(s-a)(s-b)(s-c)}$

যেখানে, $s = \text{অর্ধপরিসর} = \frac{a+b+c}{2}$

5



⑥ $A(x_1, y_1)$ ਤੇ $B(x_2, y_2)$ ਦਾ ਦੂਰਾ $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (1)

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \text{ਦੂਰਾ} \quad (1)$$

⑦ ਦੋ ਬਿੰਦੂਆਂ $P_1(x_1, \theta_1)$ ਅਤੇ $P_2(x_2, \theta_2)$ ਦਾ ਦੂਰਾ

$$P_1 P_2 = \sqrt{x_1^2 + x_2^2 - 2x_1 x_2 \cos(\theta_2 - \theta_1)}$$

— ਦੂਰਾ $\Delta x \cdot \Delta y = \text{ਕੋਸਾਈਨ} \times \text{ਕੋਸਾਈਨ} = \text{ਕੋਸਾਈਨ}$ (VI)

⑧ ΔABC ਦਾ ਖੇਤਰ (ਕੋਆਰਡੀਨੇਟ) x, y

$$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

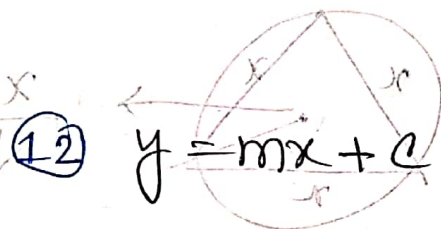
$$= \frac{1}{2} \{ (x_1 y_2 + x_2 y_3 + x_3 y_1) - (y_1 x_2 + y_2 x_3 + y_3 x_1) \} \quad (1)$$

$$= \frac{1}{2} \{ (x_1 y_2 + x_2 y_3 + x_3 y_1) - (y_1 x_2 + y_2 x_3 + y_3 x_1) \}$$

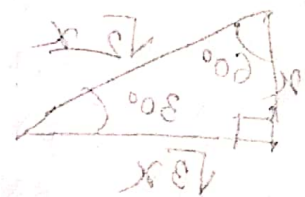
⑨ ਦੋ ਰੇਖੇ ਸਮਾਂਗੁਣੀਆਂ ਹੋਣ ਲਈ $m_1 \cdot m_2 = -1$ (2)

⑩ ਦੋ ਰੇਖੇ ਸਮਾਂਗੁਣੀਆਂ ਹੋਣ ਲਈ $m_1 = m_2$ (3)

⑪ ਢਲ, $m = \frac{y_1 - y_2}{x_1 - x_2}$ (VI)



⑫ $y = mx + c$

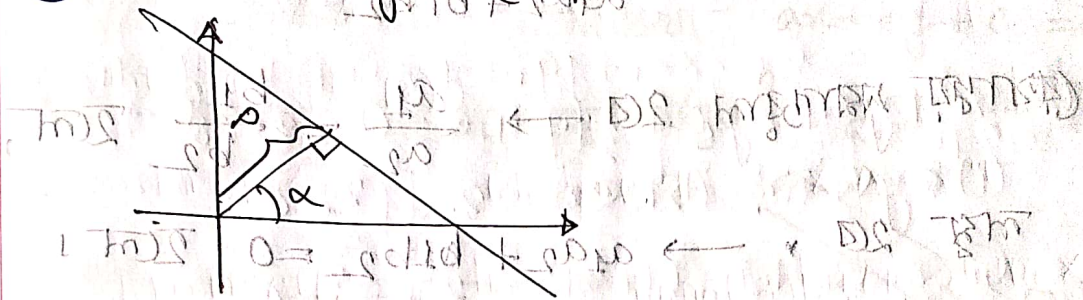


(13) $y - y_1 = m(x - x_1)$ $2x + 3y + 10 = 0$ (1)

(14) $\frac{y_1 - y_2}{x_1 - x_2} = m$

(15) $x/a + y/b = 1$

(16) $x \cos \alpha + y \sin \alpha = p$



(17) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Intersection point (x, y) is given by

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \right) \quad [a_1b_2 \neq a_2b_1]$$

(18) $y = m_1x + c_1$ and $y = m_2x + c_2$

Angle between them is 2θ

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1m_2}$$

$\theta \rightarrow \oplus 2\theta$ acute angle
 $\theta \rightarrow \ominus 2\theta$ obtuse angle

(19) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Condition for perpendicular lines is $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$$\tan \theta = \pm \frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}$$

Condition for parallel lines is $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Condition for perpendicular lines is $a_1a_2 + b_1b_2 = 0$

(20) $ax + by + c = 0$ is a line. Another line is $ax + by + k = 0$

Condition for parallel lines is $\frac{a}{a} = \frac{b}{b}$

(21) $ax + by + c = 0$ is a line. Another line is $bx - ay + k = 0$

(22) $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$a_3x + b_3y + c_3 = 0$ is a line. Another line is $a_4x + b_4y + c_4 = 0$

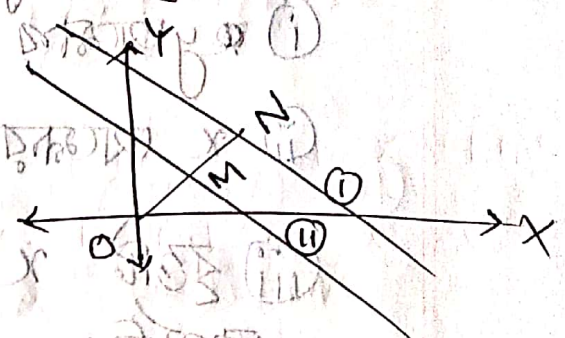
Condition for three lines to be concurrent is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

২২) $P(x_1, y_1)$ বিন্দু হতে $ax+by+c=0$ সরলরেখার উল্লম্ব
 অক্ষের দৈর্ঘ্য = $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$ (১)

২৩) দুইটি সরলরেখার সমান্তরাল সরলরেখা $ax+by+c=0$

এবং $ax+by+c_2=0$ এর মধ্যকার দূরত্ব নির্ণয়:

$$MN = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$



২৪) $a_1x + b_1y + c_1 = 0$ এবং $a_2x + b_2y + c_2 = 0$

দুই সরলরেখার অন্তর্ভুক্ত কোণের সমান্তরাল সরলরেখার

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

২৫) (x, y) হতে

$$\frac{|x^2 + y^2 - 4x + 6y - 12|}{\sqrt{1^2 + 1^2}} = r$$

Chapter-04

① কেন্দ্র $(0,0)$ হলে, $x^2 + y^2 = r^2$

② কেন্দ্র (h,k) হলে, $(x-h)^2 + (y-k)^2 = r^2$

③ $x^2 + y^2 + 2gx + 2fy + c = 0$ সমীকরণ -

(i) y অক্ষের ছেদাঙ্ক $= 2\sqrt{f^2 - c}$

(ii) x অক্ষের ছেদাঙ্ক $= 2\sqrt{g^2 - c} = 2g$

(iii) বৃত্তটি x অক্ষের স্পর্শক হলে, $g^2 = c$;

স্পর্শবিন্দু $(-g, 0)$

(iv) y অক্ষের স্পর্শক হলে, $f^2 = c$;

স্পর্শবিন্দু $(0, -f)$

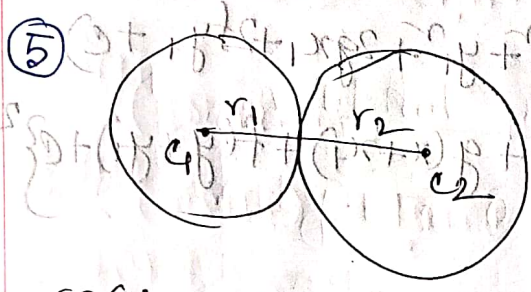
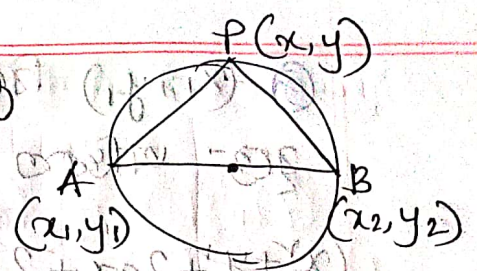
(v) উভয় অক্ষের স্পর্শক হলে $g^2 = f^2 = c$.

(vi) মূলবিন্দুসমী - হলে, $c = 0$

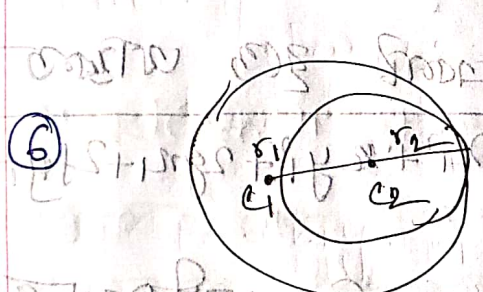
(vii) কেন্দ্র $(-g, -f)$ হলে,

$$r = \sqrt{g^2 + f^2 - c}$$

④ $(y-y_1)(y-y_2) + (x-x_1)(x-x_2) = 0$



একোত্র, $C_1C_2 = r_1 + r_2$



একোত্র, $C_1C_2 = r_1 - r_2$ [$r_1 > r_2$]

⑦ $y = mx + c$ ~~এক~~ $x^2 + y^2 = r^2$ ~~এক~~
 স্পর্শক রেখার সমীকরণ : $c = \pm r\sqrt{1+m^2}$

স্পর্শক রেখার সমীকরণ : $y = mx \pm r\sqrt{1+m^2}$

⑧ $x^2 + y^2 + 2gx + 2fy + c = 0$ ~~এক~~ (x_1, y_1)

বিদ্যুৎ কেন্দ্রের সমীকরণ $x^2 + y^2 + g(x+x_1) + f(y+y_1) + c = 0$

9) (x_1, y_1) बिन्दु को एक वृत्त $x^2 + y^2 + 2gx + 2fy + c = 0$

वृत्त-अन्तर्गत मन्कलको मन्कीकरण :

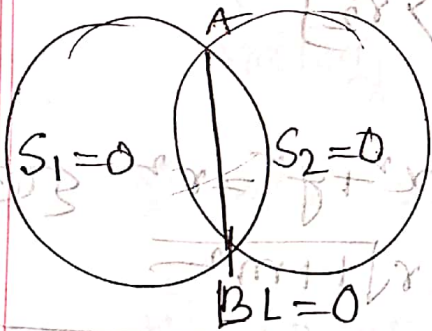
$$(x^2 + y^2 + 2gx + 2fy + c)(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c)$$

$$= \{xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c\}^2$$

अर्थात्, $SS_1 = T^2$

10) (x_1, y_1) बिन्दु को एक वृत्त-वृत्त अन्तर्गत मन्कलको मन्कीकरण $\text{दूरी} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

11) दुई वृत्त मन्कीकरण को एक मन्कीकरण



$$S_1 - S_2 = 2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$$

$$(15x) \text{ को } 0 = 2 + y + 2 + 2 + y + 2$$

$$(15x) + (15x) + (15x) + (15x) = 15x$$

$$0 = 2 +$$

বিন্যাস ও সমাবেশ

Chapter-05

① $n P_n = n! = \lim_{n \rightarrow \infty} n P_{n-1}$

② $n P_r = \frac{n!}{(n-r)!}$

③ $n P_r = n \cdot (n-1) P_{r-1}$

④ সবগুলো বিকল্প নিয়ে n বিকল্প সংখ্যক একজাতীয়
 প্রশ্নের ক্ষেত্রে বিন্যাস : n সংখ্যক "কিনিয়ে" মর্মে
 p সংখ্যক একজাতীয়, q সংখ্যক একজাতীয়, r
 সংখ্যক একজাতীয় - ২নে সবগুলো নিয়ে বিন্যাস
 সংখ্যক = $\frac{n!}{p! q! r!}$

⑤ n সংখ্যক নির্দিষ্ট কিনিয়কে একসাথে রাখা
 n সংখ্যক বিভিন্ন কিনিয় একসাথে নিয়ে বিন্যাস

সংখ্যক = $(n-p+1) \times P$
 একসাথে না নিয়ে : $n - (n-p+1) \times P$

Permutation and Combination

⑨ $n P_r = n C_r \times r!$

⑩ $n C_r = \frac{n!}{r!(n-r)!}$ $n > r$

⑪ $n P_r = n C_r$ यदि $r=1$ या $r=n$

⑫ $n C_0 = 1, n C_1 = n, n C_n = 1$

⑬ $n C_r = n C_{n-r}$

⑭ $n C_x = n C_y$ यदि $x=y$ या $x+y=n$

⑮ $n C_r + n C_{r-1} = n C_r$

①	②	③	④	⑤	⑥
1	1	1	1	1	1
0	1	1	1	1	1
0	1	2	1	1	1
0	1	3	3	1	1
0	1	6	6	4	1
0	1	10	10	10	1
0	1	15	15	15	6
0	1	21	21	21	15
0	1	28	28	28	35
0	1	36	36	36	62
0	1	45	45	45	126
0	1	55	55	55	252
0	1	66	66	66	504
0	1	78	78	78	1008
0	1	91	91	91	2016
0	1	105	105	105	4032
0	1	120	120	120	8064
0	1	136	136	136	16128
0	1	153	153	153	32256
0	1	171	171	171	64512
0	1	190	190	190	129024
0	1	210	210	210	258048
0	1	231	231	231	516096
0	1	253	253	253	1032192
0	1	276	276	276	2064384
0	1	300	300	300	4128768
0	1	325	325	325	8257536
0	1	351	351	351	16515072
0	1	378	378	378	33030144
0	1	406	406	406	66060288
0	1	435	435	435	132120576
0	1	465	465	465	264241152
0	1	496	496	496	528482304
0	1	528	528	528	1056964608
0	1	561	561	561	2113929216
0	1	595	595	595	4227858432
0	1	630	630	630	8455716864
0	1	666	666	666	16911433728
0	1	703	703	703	33822867456
0	1	741	741	741	67645734912
0	1	780	780	780	135291469824
0	1	820	820	820	270582939648
0	1	861	861	861	541165879296
0	1	903	903	903	1082331758592
0	1	946	946	946	2164663517184
0	1	990	990	990	4329327034368
0	1	1035	1035	1035	8658654068736
0	1	1081	1081	1081	17317308137472
0	1	1128	1128	1128	34634616274944
0	1	1176	1176	1176	69269232549888
0	1	1225	1225	1225	138538465099776
0	1	1275	1275	1275	277076930199552
0	1	1326	1326	1326	554153860399104
0	1	1378	1378	1378	1108307720798208
0	1	1431	1431	1431	2216615441596416
0	1	1485	1485	1485	4433230883192832
0	1	1540	1540	1540	8866461766385664
0	1	1596	1596	1596	17732923532771328
0	1	1653	1653	1653	35465847065542656
0	1	1711	1711	1711	70931694131085312
0	1	1770	1770	1770	141863388262170624
0	1	1830	1830	1830	283726776524341248
0	1	1891	1891	1891	567453553048682496
0	1	1953	1953	1953	1134907106097364992
0	1	2016	2016	2016	2269814212194729984
0	1	2080	2080	2080	4539628424389459968
0	1	2145	2145	2145	9079256848778919936
0	1	2211	2211	2211	18158513697557839872
0	1	2278	2278	2278	36317027395115679744
0	1	2346	2346	2346	72634054790231359488
0	1	2415	2415	2415	145268109580462718976
0	1	2485	2485	2485	290536219160925437952
0	1	2556	2556	2556	581072438321850875904
0	1	2628	2628	2628	1162144876643701751808
0	1	2701	2701	2701	2324289753287403503616
0	1	2775	2775	2775	4648579506574807007232
0	1	2850	2850	2850	9297159013149614014464
0	1	2926	2926	2926	18594318026299228028928
0	1	3003	3003	3003	37188636052598456057856
0	1	3081	3081	3081	74377272105196912115712
0	1	3160	3160	3160	148754544210393824231424
0	1	3240	3240	3240	297509088420787648462848
0	1	3321	3321	3321	595018176841575296925696
0	1	3403	3403	3403	1190036353683150593851392
0	1	3486	3486	3486	2380072707366301187702784
0	1	3570	3570	3570	4760145414732602375405568
0	1	3655	3655	3655	9520290829465204750811136
0	1	3741	3741	3741	19040581658930409501622272
0	1	3828	3828	3828	38081163317860819003244544
0	1	3916	3916	3916	76162326635721638006489088
0	1	4005	4005	4005	152324653271443276012978176
0	1	4095	4095	4095	304649306542886552025956352
0	1	4186	4186	4186	609298613085773104051912704
0	1	4278	4278	4278	1218597226171546208103825408
0	1	4371	4371	4371	2437194452343092416207650816
0	1	4465	4465	4465	4874388904686184832415301632
0	1	4560	4560	4560	9748777809372369664830603264
0	1	4656	4656	4656	19497555618744739329661206528
0	1	4753	4753	4753	38995111237489478659322413056
0	1	4851	4851	4851	77990222474978957318644826112
0	1	4950	4950	4950	155980444949957914637289652224
0	1	5050	5050	5050	311960889899915829274579304448
0	1	5151	5151	5151	623921779799831658549158608896
0	1	5253	5253	5253	1247843559599663317098317217792
0	1	5356	5356	5356	2495687119199326634196634435584
0	1	5460	5460	5460	4991374238398653268393268871168
0	1	5565	5565	5565	9982748476797306536786537742336
0	1	5671	5671	5671	19965496953594613073573075484672
0	1	5778	5778	5778	39930993907189226147146150969344
0	1	5886	5886	5886	79861987814378452294292301938688
0	1	5995	5995	5995	159723975628756904588584603877376
0	1	6105	6105	6105	319447951257513809177169207754752
0	1	6216	6216	6216	638895902515027618354338415509504
0	1	6328	6328	6328	1277791805030055236708676831019008
0	1	6441	6441	6441	2555583610060110473417353662038016
0	1	6555	6555	6555	5111167220120220946834707324076032
0	1	6670	6670	6670	10222334440240441893669414648152064
0	1	6786	6786	6786	20444668880480883787338829296304128
0	1	6903	6903	6903	40889337760961767574677658592608256
0	1	7021	7021	7021	81778675521923535149355317185215504
0	1	7140	7140	7140	163557351043847070298710634370431008
0	1	7260	7260	7260	327114702087694140597421268740862016
0	1	7381	7381	7381	654229404175388281194842537481724032
0	1	7503	7503	7503	1308458808350776562389685074963448064
0	1	7626	7626	7626	2616917616701553124779370149926896128
0	1	7750	7750	7750	5233835233403106249558740299853792256
0	1	7875	7875	7875	10467670466806212499117480599707584512
0	1	8001	8001	8001	20935340933612424998234961199415169024
0	1	8128	8128	8128	41870681867224849996469922398830338048
0	1	8256	8256	8256	83741363734449699992939844797660676096
0	1	8385	8385	8385	167482727468899399985879689595321352192
0	1	8515	8515	8515	334965454937798799971759379190642704384
0	1	8646	8646	8646	669930909875597599943518758381285408768
0	1	8778	8778	8778	1339861819751195199887037516762570817536
0	1	8911	8911	8911	2679723639502390399774075033525141635072
0	1	9045	9045	9045	5359447279004780799548150067050283270144
0	1	9180	9180	9180	10718894558009561599096300134100566540288
0	1	9316	9316	9316	21437789116019123198192600268201133080576
0	1	9453	9453	9453	42875578232038246396385200536402266161152
0	1	9591	9591	9591	85751156464076492792770401072804532322304
0	1	9730	9730	9730	171502312928152985585540802145609064644608
0	1	9870	9870	9870	343004625856305971171081604291218129289216
0	1	10011	10011	10011	686009251712611942342163208582436258578432

Chapter - 6

① 1 π $\text{rad} = 180^\circ$
 $1^\circ = \frac{\pi}{180}$
 $1' = 60''$

② $\pi^c = 180^\circ$

$\frac{2}{\pi} = 1$ (विद्यमान) $\pi = \pi^c$; $1 = 1^c$

$1^\circ = \frac{\pi^c}{180}$

③ वृत्तखण्ड का क्षेत्रफल $A = \frac{1}{2} r^2 \theta$

④ $\sin^2 \theta + \cos^2 \theta = 1$
 $\sec^2 \theta - \tan^2 \theta = 1$
 $\text{cosec}^2 \theta - \cot^2 \theta = 1$

⑤

	0°	30°	45°	60°	90°	180°	270°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$ und.	0 und.	0 und.	
$\cot \theta$	und.	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0 und.	0 und.	
$\sec \theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2 und.	-1 und.		
$\text{cosec} \theta$	und.	2	$\sqrt{2}$	$2/\sqrt{3}$	1 und.	-1 und.	

সমস্ত কোণের ত্রিকোণমিতিক অনুসন্ধান

Chapter-7

① $\sin(-\theta) = -\sin\theta$ $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
 $\cos(-\theta) = \cos\theta$ $\sec(-\theta) = \sec\theta$
 $\tan(-\theta) = -\tan\theta$ $\cot(-\theta) = -\cot\theta$

② ত্রিকোণের সিনে সূত্রঃ

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

③ Cosine সূত্রঃ

(i) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

(ii) $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

(iii) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

④ Tangent Rule

(i) $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$

(ii) $\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$

(iii) $\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$

MATHS FORMULAE

(5) (i) $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$ (1)

(ii) $\sin \frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ca}}$

$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

(iii) $\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$ (2)

(iv) $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

(v) $\cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$

$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$ (3)

(vi) $\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(vii) $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(viii) $\tan \frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

(ix) $\tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$\frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$ (4)

$\frac{\sin \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$ (5)

$$\textcircled{6} \cdot \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cdot \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\textcircled{7} \cdot \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cdot \cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

$$\textcircled{8} \cdot 2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$\cdot 2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$\cdot 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$\cdot 2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\cdot \sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$

$$\cdot \cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

$$\textcircled{9} \cdot \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cdot \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cdot \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cdot \cos D - \cos C = 2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\bullet \quad 1 - \cos 2A = 2 \sin^2 A = (\sin A + \cos A)(\sin A - \cos A) \quad \textcircled{10}$$

$$\bullet \quad 1 + \cos 2A = 2 \cos^2 A = (\sin A + \cos A)(\sin A + \cos A)$$

$$\bullet \quad \frac{1 - \cos 2A}{1 + \cos 2A} = \tan^2 A = (\sin A + \cos A)(\sin A - \cos A) \quad \textcircled{11}$$

$$\textcircled{10} \quad \sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\bullet \quad \cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A$$

$$= 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\textcircled{11} \quad \tan 2A = \frac{1 - \cos 2A}{1 + \cos 2A} = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\bullet \quad \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\bullet \quad \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} \quad \textcircled{12}$$

$$\textcircled{12} \quad \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

trigonometri

• $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

• $\sin 1A = \frac{2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}}{2} = \frac{\sin A}{1}$ mül 1
0 ← x

• $\cos A = \frac{\cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}}{2}$ mül 2
0 ← x

$1 - x^2 = \frac{2 \cos^2 \frac{A}{2} - 1}{2}$

$\frac{1 - x^2}{2} = \frac{1 - 2 \sin^2 \frac{A}{2}}{2}$

• $1 + \cos A = \frac{2 \cos^2 \frac{A}{2}}{2} = \cos A$

• $1 - \cos A = \frac{2 \sin^2 \frac{A}{2}}{2} = \sin^2 \frac{A}{2}$ mül 3
0 ← x

• $\sin^3 A = \frac{1}{4} (3 \sin A - \sin 3A)$ mül 4
0 ← x

• $\cos^3 A = \frac{1}{4} (3 \cos A + \cos 3A)$ mül 5
0 ← x

⑬ • $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$ mül 6
0 ← x

• $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$ mül 7
0 ← x

$\frac{1-x}{1-x} = \frac{1-x}{1-x}$ mül 8
0 ← x

Differentiation

Chapter 9 - A note = A note.

① $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$

② $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = \frac{x}{\sin^{-1} x} = \frac{\tan x}{x} = \frac{x}{\tan x}$
 $= \frac{\tan^{-1} x}{x} = \frac{e^x - 1}{x} = \frac{\ln(1+x)}{x}$
 $= \frac{\cos x}{\cos x} = \frac{1}{\cos x} = 1$

③ $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$

④ $\lim_{x \rightarrow \infty} \frac{(1+x)^n - 1}{x} = n$

⑤ $\lim_{x \rightarrow \infty} \frac{x^n}{\ln x} = 0$

⑥ $\lim_{x \rightarrow \infty} \frac{a}{x^n} = 0$

⑦ $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$

$$\textcircled{8} \lim_{x \rightarrow 0} (1+x)^{1/x} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\textcircled{9} \lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

$$\textcircled{10} \lim_{x \rightarrow 0} x^r \cos \frac{1}{x^r} = 0$$

$$\textcircled{11} \frac{d}{dx} (c) = 0$$

$$\textcircled{12} \frac{d}{dx} cf(x) = c \frac{d}{dx} f(x)$$

$$\textcircled{13} \frac{d}{dx} (cx^n) = c \cdot \frac{d}{dx} x^n = cnx^{n-1}$$

$$\frac{d}{dx} \left(\frac{c}{x^n}\right) = c \frac{d}{dx} x^{-n} = -cnx^{-n-1} = -\frac{cn}{x^{n+1}}$$

$$\frac{d}{dx} (\sqrt{x}) = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\textcircled{14} \frac{d}{dx} (\sin x) = \cos x$$

$$\textcircled{15} \frac{d}{dx} (\cos x) = -\sin x$$

$$(16) \frac{d}{dx} (\tan x) = \sec^2 x \quad \text{mit } \infty \leftarrow x \quad (8)$$

$$(17) \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x \quad \text{mit } \infty \leftarrow x \quad (9)$$

$$(18) \frac{d}{dx} (\sec x) = \sec x \cdot \tan x \quad \text{mit } \infty \leftarrow x \quad (10)$$

$$(19) \frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x \quad \text{mit } \infty \leftarrow x \quad (11)$$

$$(20) \frac{d}{dx} (e^x) = e^x \quad \text{mit } \infty \leftarrow x \quad (12)$$

$$(21) \frac{d}{dx} (\ln x) = \frac{1}{x} \quad \text{mit } \infty \leftarrow x \quad (13)$$

$$(22) \frac{d}{dx} (a^x) = a^x \ln a \quad \text{mit } \infty \leftarrow x \quad (14)$$

$$(23) \frac{d}{dx} (e^{nx}) = n e^{nx} \quad \text{mit } \infty \leftarrow x \quad (15)$$

$$(24) \frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx} \quad \text{mit } \infty \leftarrow x \quad (16)$$

$$(25) \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad \text{mit } \infty \leftarrow x \quad (17)$$

$$(26) \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} \quad \text{mit } \infty \leftarrow x \quad (18)$$

$$(27) \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} = (uv) \frac{b}{xb} \quad (27)$$

$$(28) \frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} = (v+u) \frac{b}{xb} \quad (28)$$

$$(29) \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2} = x \quad (29)$$

$$(30) \frac{d}{dx} (\cot^{-1} x) = -\frac{1}{1+x^2} = x \quad (30)$$

$$(31) \frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} = (x+1) \quad (31)$$

$$(32) \frac{d}{dx} (\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$(33) \frac{d}{dx} (\log_a x) = \frac{1}{x} \cdot \log_a e$$

$$(34) \frac{d}{dx} (\sin mx) = m \cos mx$$

$$(35) \frac{d}{dx} (uvw) = vw \frac{du}{dx} + uv \frac{dv}{dx} + uv \frac{dw}{dx}$$

$$(36) \frac{d}{dx} (u^v) = u^v \frac{dv}{dx} + (v \ln u) \frac{du}{dx} \quad (36)$$

$$(37) \frac{d}{dx} (u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx} \quad (37)$$

$$(38) e^x = 1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{6} + \dots \quad (38)$$

$$(39) a^x = 1 + \frac{x}{1} (\ln a) + \frac{x^2}{2} (\ln a)^2 + \dots \quad (39)$$

$$(40) \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad (40)$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \quad (41)$$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + \dots \quad (42)$$

$$\frac{1}{1-x^3} = 1 + x^3 + x^6 + x^9 + \dots \quad (43)$$

$$\frac{1}{1-x^n} = 1 + x^n + x^{2n} + x^{3n} + \dots \quad (44)$$

Integration

Chapter 10

$$\textcircled{1} \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\textcircled{2} \int \frac{1}{x^n} dx = \frac{1}{(n-1)x^{n-1}} + C$$

$$\textcircled{3} \int \frac{1}{x} dx = \ln|x| + C$$

$$\textcircled{4} \int e^x dx = e^x + C$$

$$\textcircled{5} \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\textcircled{6} \int \cos x \cdot dx = \sin x + C$$

$$\textcircled{7} \int \sin x dx = -\cos x + C$$

$$\textcircled{8} \int \sec^2 x dx = \tan x + C$$

$$\textcircled{9} \int \operatorname{cosec}^2 x dx = -\cot x + C$$

$$\textcircled{10} \int \sec x \cdot \tan x \cdot dx = \sec x + C$$

$$(11) \int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + c$$

$$(12) \int \frac{1}{2\sqrt{x}} \, dx = \sqrt{x} + c \quad \left[\frac{x^{1+n}}{1+n} = x^b \cdot n \right] \quad (1)$$

$$(13) \int \frac{a+x}{a-x} \, dx = a \sin^{-1} \frac{x}{a} - \sqrt{a^2 - x^2} + c \quad (1)$$

$$(14) \int \frac{a-x}{a+x} \, dx = a \sin^{-1} \frac{x}{a} + \sqrt{a^2 - x^2} + c \quad (1)$$

$$(15) \int \tan x \, dx = -\ln(\cos x) + c = \ln(\sec x) + c \quad (1)$$

$$(16) \int \cot x \, dx = \ln(\sin x) + c = -\ln(\operatorname{cosec} x) + c \quad (1)$$

$$(17) \int \sec x \, dx = \ln \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + c = \ln |\sec x + \tan x| + c \quad (1)$$

$$(18) \int \operatorname{cosec} x \, dx = \ln \left| \tan \frac{x}{2} \right| + c = \ln |\operatorname{cosec} x + \cot x| + c \quad (1)$$

$$(19) \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \cdot \tan^{-1} \frac{x}{a} + c \quad (1)$$

$$(20) \int \frac{dx}{1+x^2} = \tan^{-1} x + c \quad (1)$$

$$\textcircled{21} \int \frac{dx}{\sqrt{a^2 + x^2}} = \sin^{-1} \frac{x}{a}$$

$$\textcircled{22} \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$$

~~$$\textcircled{23} \int \frac{dx}{\sqrt{x^2 + a^2}} =$$~~

$$\textcircled{23} \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$$

$$\textcircled{24} \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$\textcircled{25} \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln (x + \sqrt{x^2 + a^2}) + c$$

$$\textcircled{26} \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln (x + \sqrt{x^2 - a^2}) + c$$

$$\textcircled{27} \int uv dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

$$\textcircled{28} \int \ln x dx = x \ln x - x + c$$