জটিল সংখ্যা — প্রশ্নমালা III

1.(i) নিচের জটিল সংখ্যাগুলো পোলার আকারে প্রকাশ কর । অতপর প্রতিক্ষেত্রে মডুলাস এবং আর্গুমেন্ট নির্ণয় কর :

(a) দেওয়া আছে, $1 + \sqrt{3}\,i$

[চ.'০০; রুয়েট'০৬-০৭,১০-১১]

মনে করি , $r\cos\theta = 1$ এবং $r\sin\theta = \sqrt{3}$

$\therefore\ r = \sqrt{1^2 + (\sqrt{3})^2}$ $\quad [\because r = \sqrt{x^2+y^2}\,]$

$= \sqrt{1+3} = \sqrt{4} = 2$

$\Rightarrow \theta = \tan^{-1}\frac{\sqrt{3}}{1}$ $\quad [\because \theta = \tan^{-1}\frac{y}{x}]$

$= \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore\ 1 + \sqrt{3}\,i = r\cos\theta + i\,r\sin\theta$

$= r(\cos\theta + i\sin\theta) = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

$\therefore$ মডুলাস $= 2$ এবং আর্গুমেন্ট $= \frac{\pi}{3}$

1(b) দেওয়া আছে, $3 - 5\,i$ [চ.'০০]

মনে করি, $r\cos\theta = 3$ এবং $r\sin\theta = -5$

$\therefore\ r = \sqrt{3^2 + (-5)^2}$ $\quad [\because r = \sqrt{x^2+y^2}\,]$

$= \sqrt{9+25} = \sqrt{34}$

$\therefore\ \theta = \tan^{-1}(\frac{-5}{3})$ $\quad [\because \theta = \tan^{-1}\frac{y}{x}]$

$= -\tan^{-1}\frac{5}{3}$ $\quad [\because \tan^{-1}\frac{-y}{x} = -\tan^{-1}\frac{y}{x}]$

$\therefore 3 - 5i = r\cos\theta + i\,r\sin\theta$

$= \sqrt{34}\,(\cos\theta + i\sin\theta)$,

যেখানে $\theta = -\tan^{-1}\frac{5}{3}$

$\therefore$ মডুলাস $= \sqrt{34}$ এবং আর্গুমেন্ট $= \tan^{-1}(-\frac{5}{3})$

1(c) দেওয়া আছে, $-\sqrt{3} + i$ [বুয়েট'১১-১২]

মনে করি , $r\cos\theta = -\sqrt{3}$ এবং $r\sin\theta = 1$

$\therefore\ r = \sqrt{(-\sqrt{3})^2 + 1^2}$ $\quad [\because r = \sqrt{x^2+y^2}\,]$

$= \sqrt{3+1} = \sqrt{4} = 2$

$\theta = \tan^{-1}\frac{1}{-\sqrt{3}}$ $\quad [\because \theta = \tan^{-1}\frac{y}{x}]$

$= \pi - \tan^{-1}\frac{1}{\sqrt{3}}$ $\quad [\because \tan^{-1}\frac{y}{-x} = \pi - \tan^{-1}\frac{y}{x}]$

$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\therefore\ -\sqrt{3} + i = r\cos\theta + i\,r\sin\theta$

$= 2\,(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$

$\therefore$ মডুলাস $= 2$ এবং আর্গুমেন্ট $= \frac{5\pi}{6}$

1(d) দেওয়া আছে, $-8 - 6i$

মনে করি, $r\cos\theta = -8$ এবং $r\sin\theta = -6$

$\therefore\ r = \sqrt{(-8)^2 + (-6)^2}$ $\quad [\because r = \sqrt{x^2+y^2}\,]$

$= \sqrt{64+36} = 10$ $\quad [\because \theta = \tan^{-1}\frac{y}{x}]$

$\therefore\ \theta = \tan^{-1}(\frac{-6}{-8})$

$= \tan^{-1}\frac{3}{4} - \pi$ $\quad [\because \tan^{-1}\frac{-y}{-x} = \tan^{-1}\frac{y}{x} - \pi]$

$\therefore\ -8 - 6i = r\cos\theta + i\,r\sin\theta$

$= 10(\cos\theta + i\sin\theta)$,

যেখানে $\theta = \tan^{-1}\frac{3}{4} - \pi$

$\therefore$ মডুলাস = 10 এবং আর্গুমেন্ট = $\tan^{-1}(\frac{3}{4})$

1(e) $4+3i$ [য.'০০; কুয়েট'০৪-০৫,১২-১৩]

মনে করি, $r\cos\theta = 4$ এবং $r\sin\theta = 3$

$\therefore r = \sqrt{4^2+3^2} = 5$ $[\because r = \sqrt{x^2+y^2}]$

$\therefore \theta = \tan^{-1}(\frac{3}{4})$ $[\because \theta = \tan^{-1}\frac{y}{x}]$

$\therefore 4+3i = r\cos\theta + i\, r\sin\theta$

$= 5(\cos\theta + i\sin\theta)$, যেখানে $\theta = \tan^{-1}(\frac{3}{4})$

$\therefore$ মডুলাস = 5 এবং আর্গুমেন্ট = $\tan^{-1}(\frac{3}{4})$

1(i) (f) $-1+\sqrt{3}\,i$ [য.'০২]

মনে করি, $r\cos\theta = -1$ এবং $r\sin\theta = \sqrt{3}$

$\therefore r = \sqrt{(-1)^2+(\sqrt{3})^2} = 2,$ $[\because r = \sqrt{x^2+y^2}]$

এবং $\theta = \tan^{-1}(\frac{\sqrt{3}}{-1})$ $[\because \theta = \tan^{-1}\frac{y}{x}]$

$= \pi - \tan^{-1}(\frac{\sqrt{3}}{1}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore -1+\sqrt{3}\,i = r\cos\theta + i\, r\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$= 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$

$\therefore$ মডুলাস = 2 এবং আর্গুমেন্ট = $\frac{2\pi}{3}$.

1(g) $(2\sqrt{3}-2i)(-2\sqrt{3}+6i)$ [কুয়েট'১২-১৩]

$= (2\sqrt{3})(-2\sqrt{3}) - (2i)(6i) + \{12\sqrt{3}+4\sqrt{3}\}i$

$= -12 - 12i^2 + 16\sqrt{3}\,i$

$= -12 + 12 + 16\sqrt{3}\,i = 0 + 16\sqrt{3}\,i$

মনে করি, $r\cos\theta = 0$ এবং $r\sin\theta = 16\sqrt{3}$

$\therefore r = \sqrt{0^2+(16\sqrt{3})^2}$, $[\because r = \sqrt{x^2+y^2}]$

$= 16\sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\frac{16\sqrt{3}}{0}$ $[\because \theta = \tan^{-1}\frac{y}{x}]$

$= \tan^{-1}(\tan\frac{\pi}{2}) = \frac{\pi}{2}$

$\therefore (2\sqrt{3}-2i)(-2\sqrt{3}+6i) = 0 + 16\sqrt{3}\,i$

$= r\cos\theta + i\, r\sin\theta$

$= 16\sqrt{3}(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

$\therefore$ মডুলাস = $16\sqrt{3}$ এবং আর্গুমেন্ট = $\frac{\pi}{2}$

1.(ii) আর্গুমেন্ট নির্ণয় কর : $1 - \frac{i}{1-\frac{1}{1+i}}$

সমাধান : $1 - \frac{i}{1-\frac{1}{1+i}} = 1 - \frac{i}{\frac{1+i-1}{1+i}}$

$= 1 - \frac{i(1+i)}{i} = 1-(1+i) = -i = 0 - 1.\,i$

$\therefore$ নির্ণেয় আর্গুমেন্ট = $\tan^{-1}\frac{-1}{0}$ $[\because \theta = \tan^{-1}\frac{y}{x}]$

$= -\tan^{-1}\frac{1}{0}$ $[\because \tan^{-1}\frac{-y}{x} = -\tan^{-1}\frac{y}{x}]$

$= -\cot^{-1}\frac{0}{1}$ $[\because \tan^{-1}\frac{y}{x} = \cot^{-1}\frac{x}{y}]$

$= -\cot^{-1} 0 = -\frac{\pi}{2}$ (Ans.)

2. A + i B আকারে প্রকাশ কর :

(a) $\frac{3+2i}{5-7i} = \frac{(3+2i)(5+7i)}{(5-7i)(5+7i)}$

$= \frac{15+21i+10i+14i^2}{5^2-7^2i^2} = \frac{15+31i-14}{25+49}$

$= \frac{1+31i}{74} = \frac{1}{74} + \frac{31}{74}i$, যা A + iB আকারে প্রকাশ করা হল ।

2(b) $\frac{1+5i}{1-2i} + \frac{5-3i}{2+3i}$

$= \frac{(1+5i)(1+2i)}{(1+2i)(1-2i)} + \frac{(5-3i)(2-3i)}{(2+3i)(2-3i)}$

$= \frac{1+7i-10}{1+4} + \frac{10-21i-9}{4+9}$

$= \frac{-9+7i}{5} + \frac{1-21i}{13} = \frac{-117+91i+5-105i}{65}$

$= (-\frac{112}{65}) + (-\frac{14}{65})i$, যা A + iB আকারে প্রকাশ করা হল ।

3. বর্গমূল নির্ণয় কর ঃ

(a) $7 - 30\sqrt{-2}$ [চ.'০১;ব.'১২]

$= 25 - 18 - 30\,i\,\sqrt{2}$

$= 25 - 30i\sqrt{2} + 18\,i^2$

$= 5^2 - 2.5.3\sqrt{2}\,i + (3\sqrt{2}\,i)^2$

$= (5 - 3\sqrt{2}\,i)^2$

$\therefore 7 - 30\sqrt{-2}$ এর বর্গমূল $= \pm\sqrt{(5-3\sqrt{2}i)^2}$

$= \pm(5 - 3\sqrt{2}\,i)$

3(b) $-7 + 24\,i = 9 - 16 + 24\,i$ [ঢা.'১২]

$= 3^2 + (4\,i)^2 + 2.3.4\,i = (3+4\,i)^2$

$\therefore -7 + 24\,i$ এর বর্গমূল $= \pm\sqrt{(3+4i)^2}$

$= \pm(3 + 4\,i)$

3(c) $2\,i$ [ঢা.'০১; চ.'০৩ ; সি.'০৫; রা.'০৬,'০৮]

$= 1 - 1 + 2\,i = 1 + i^2 + 2.1.\,i = (1 + i)^2$

$\therefore$ $2\,i$ এর বর্গমূল $= \pm\sqrt{(1+i)^2} = \pm(1 + i)$

3(d) $1 + i$ [চ.'০৫]

$= \frac{1}{2}(2 + 2\,i) = \frac{1}{2}\{2 + 2\sqrt{1}\,i\}$

$= \frac{1}{2}\{(\sqrt{2}+1) - (\sqrt{2}-1) + 2\sqrt{(\sqrt{2}+1)(\sqrt{2}-1)}\,i\}$

$= \frac{1}{2}\{(\sqrt{2}+1) + (\sqrt{2}-1)\,i^2 + 2(\sqrt{2}+1)^{\frac{1}{2}}(\sqrt{2}-1)^{\frac{1}{2}}\,i\}$

$= \frac{1}{2}\Big[\Big\{(\sqrt{2}+1)^{\frac{1}{2}}\Big\}^2 + \Big\{(\sqrt{2}-1)^{\frac{1}{2}}i\Big\}^2 + 2(\sqrt{2}+1)^{\frac{1}{2}}(\sqrt{2}-1)^{\frac{1}{2}}i\Big]$

$= \frac{1}{2}\Big\{(\sqrt{2}+1)^{\frac{1}{2}} + (\sqrt{2}-1)^{\frac{1}{2}}i\Big\}^2$

$\therefore$ $1 + i$ এর বর্গমূল

$= \pm\frac{1}{\sqrt{2}}\Big\{(\sqrt{2}+1)^{\frac{1}{2}} + (\sqrt{2}-1)^{\frac{1}{2}}i\Big\}$

3(e) $2 + i\sqrt{x^2 - 4}$

$= \frac{1}{2}\{4 + 2\,i\,\sqrt{(x+2)(x-2)}\}$

$= \frac{1}{2}\{(x+2) - (x-2) + 2i\sqrt{(x+2)(x-2)}\}$

$= \frac{1}{2}\{(x+2) + (x-2)i^2 + 2i\sqrt{(x+2)(x-2)}\}$

$= \frac{1}{2}\{(\sqrt{x+2})^2 + (i\sqrt{x-2})^2 +$

$2.\sqrt{x+2}.i\sqrt{x-2}\}$

$= \frac{1}{2}\{\sqrt{x+2} + i\sqrt{x-2}\}^2$

$\therefore 2 + i\sqrt{x^2-4}$ এর বর্গমূল =

$\pm\frac{1}{\sqrt{2}}(\sqrt{x+2} + i\sqrt{x-2})$

3(f) $x + i\sqrt{1-x^2}$ [ব.'০৩]

$= \frac{1}{2}\{2x + 2i\sqrt{(1+x)(1-x)}\}$

$= \frac{1}{2}\{(1+x)-(1-x) + 2i\sqrt{(1+x)(1-x)}\}$

$= \frac{1}{2}\{(\sqrt{1+x})^2 + (i\sqrt{1-x})^2$

$+ 2.\sqrt{1+x}.i\sqrt{1-x}\}$

$= \frac{1}{2}(\sqrt{1+x} + i\sqrt{1-x})^2$

$\therefore x + i\sqrt{1-x^2}$ এর বর্গমূল $= \pm\frac{1}{\sqrt{2}}(\sqrt{1+x} +$

$i\sqrt{1-x})$

3(g) $x + i\sqrt{x^4+x^2+1}$

$= \frac{1}{2}\{2x + i\,2\sqrt{(x^2)^2 + 2.x^2.1 + 1^2 - x^2}\}$

$= \frac{1}{2}\{2x + 2i\sqrt{(x^2+1)^2 - x^2}\}$

$= \frac{1}{2}\{2x + 2i\sqrt{(x^2-x+1)(x^2+x+1)}\}$

$= \frac{1}{2}\{(x^2+x+1) - (x^2-x+1) +$

$2i\sqrt{(x^2-x+1)(x^2+x+1)}\}$

$= \frac{1}{2}\{(\sqrt{x^2+x+1})^2 + (i\sqrt{x^2-x+1})^2 +$

$2.\sqrt{x^2+x+1}.i\sqrt{x^2-x+1}\}$

$= \frac{1}{2}(\sqrt{x^2+x+1} + i\sqrt{x^2-x+1})^2$

$\therefore x + i\sqrt{x^4+x^2+1}$ এর বর্গমূল

$= \pm\frac{1}{\sqrt{2}}(\sqrt{x^2+x+1} + i\sqrt{x^2-x+1})$

3(h) $\frac{1}{2}(-1+\sqrt{-3}) = \frac{1}{4}(-2+2\sqrt{3}i)$

$= \frac{1}{4}(1-3+2\sqrt{3}i)$

$= \frac{1}{4}\{1^2 + (i\sqrt{3})^2 + 2.1.i\sqrt{3}\}$

$= \frac{1}{4}(1+i\sqrt{3})^2 = \frac{1}{4}(1+\sqrt{-3})^2$

$\therefore \frac{1}{2}(-1+\sqrt{-3})$ এর বর্গমূল $= \pm\frac{1}{2}(1+\sqrt{-3})$

3(i) $1 - i\sqrt{x^2-1}$

$= \frac{1}{2}\{2-2i\sqrt{(x+1)(x-1)}\}$

$= \frac{1}{2}\{(x+1)-(x-1)-2i\sqrt{(x+1)(x-1)}\}$

$= \frac{1}{2}\{(\sqrt{x+1})^2 + (i\sqrt{x-1})^2 - 2.\sqrt{x+1}i\sqrt{x-1}\}$

$= \frac{1}{2}\{\sqrt{x+1} - i\sqrt{x-1}\}^2$

$\therefore 1 - i\sqrt{x^2-1}$ এর বর্গমূল

$= \pm\frac{1}{\sqrt{2}}(\sqrt{x+1} - i\sqrt{x-1})$

3(j) $i = \frac{1}{2}.2i = \frac{1}{2}(1-1+2i)$

$= \frac{1}{2}(1^2 + i^2 + 2.1.i) = \frac{1}{2}(1+i)^2$

$\therefore i$ এর বর্গমূল $= \pm\frac{1}{\sqrt{2}}(1+i)$

3(k) $-i = \frac{1}{2}(1-1-2i)$ [দি.'০৯]

$= \frac{1}{2}(1^2 + i^2 - 2.1.i) = \frac{1}{2}(1-i)^2$

$\therefore\ -i$ এর বর্গমূল $= \pm\frac{1}{\sqrt{2}}(1-i)$

3(l) $-2i$ [ব.'০৫]

$= 1 - 1 - 2i = 1 + i^2 - 2.1.i = (1-i)^2$

$\therefore\ -2i$ এর বর্গমূল $= \pm\sqrt{(1-i)^2} = \pm(1-i)$

3(m) $2i$ [দি.'০৯]

$= 1 - 1 + 2i = 1 + i^2 + 2.1.i = (1+i)^2$

$\therefore\ 2i$ এর বর্গমূল $= \pm\sqrt{(1+i)^2} = \pm(1+i)$

3(n) $4 - 4\sqrt{-1}$ [কুয়েট'১০-১১]

$= 4 - 4i = a + bi$ (ধরি) , যেখানে $a = 4,\ b = -4$

এখানে, $b < 0$.

নির্ণেয় বর্গমূল $= \pm\frac{1}{\sqrt{2}}\{(\sqrt{a^2+b^2}+a)^{\frac{1}{2}} - i(\sqrt{a^2+b^2}-a)^{\frac{1}{2}}\}$

$= \pm\frac{1}{\sqrt{2}}\{(\sqrt{4^2+(-4)^2}+4)^{\frac{1}{2}} - i(\sqrt{4^2+(-4)^2}-4)^{\frac{1}{2}}\}$

$= \pm\frac{1}{\sqrt{2}}\{(4\sqrt{2}+4)^{\frac{1}{2}} - i(4\sqrt{2}-4)^{\frac{1}{2}}\}$

$= \pm\frac{2}{\sqrt{2}}\{(\sqrt{2}+1)^{\frac{1}{2}} - i(\sqrt{2}-1)^{\frac{1}{2}}\}$

$= \pm\sqrt{2}\{(\sqrt{2}+1)^{\frac{1}{2}} - i(\sqrt{2}-1)^{\frac{1}{2}}\}$

3(o) $\frac{5+12i}{3-4i}$ [কুয়েট'০৯-১০]

$= \frac{(5+12i)(3+4i)}{(3-4i)(3+4i)}$

$= \frac{15+48i^2+(20+36)i}{9-(4i)^2}$

$= \frac{15-48+56i}{9+16} = \frac{-33+56i}{25}$

$= \frac{16-49+56i}{25} = \frac{4^2+(7i)^2+2.4.7i}{25}$

$= \frac{(4+7i)^2}{5^2}$

$\therefore$ নির্ণেয় বর্গমূল $= \pm\frac{4+7i}{5} = \pm(\frac{4}{5}+\frac{7}{5}i)$

4. মান নির্ণয় কর :

(a) $\sqrt[3]{-1}$ [ঢা.'০৪]

সমাধান : মনে করি , $x = \sqrt[3]{-1} \Rightarrow x^3 = -1$

$\Rightarrow x^3 + 1 = 0 \Rightarrow (x+1)(x^2 - x + 1) = 0$

$x + 1 = 0$ হলে , $x = -1$

$x^2 - x + 1 = 0$ হলে ,

$x = \frac{-(-1)\pm\sqrt{(-1)^2 - 4.1.1}}{2.1}$

$= \frac{1\pm\sqrt{1-4}}{2} = \frac{1\pm\sqrt{-3}}{2}$

$\therefore\ x = -1, \frac{1}{2}(1+\sqrt{-3}), \frac{1}{2}(1-\sqrt{-3})$.

$\therefore\ \sqrt[3]{-1} = -1, \frac{1}{2}(1+\sqrt{-3}), \frac{1}{2}(1-\sqrt{-3})$

4(b) $\sqrt[4]{-81}$ [রা.'০৪; ব.'০৮; ঢা.'১০;য.'১১]

মনে করি , $x = \sqrt[4]{-81} \Rightarrow x^4 = -81 = 81i^2$

$\Rightarrow (x^2)^2 = (9i)^2$

$\therefore\ x^2 = \pm 9i = \frac{9}{2}(\pm 2i)$

$= \frac{9}{2}(1^2 + i^2 \pm 2.1.i) = \frac{9}{2}(1 \pm i)^2$

$\therefore \; x = \pm \frac{3}{\sqrt{2}}(1 \pm i).$

সুতরাং, $\sqrt[4]{-81} = \pm \frac{3}{\sqrt{2}}(1 \pm i)$

4(c) $\sqrt[4]{1}$

মনে করি, $x = \sqrt[4]{1} \Rightarrow x^4 = 1 \Rightarrow x^4 = i^4$

$\Rightarrow (x^2)^2 - (i^2)^2 = 0$

$\Rightarrow (x^2 + i^2)(x^2 - i^2) = 0$

$x^2 + i^2 = 0$ হলে, $x^2 - 1 = 0$

$\Rightarrow x^2 = 1 \;\; \therefore x = \pm 1$

$x^2 - i^2 = 0$ হলে, $x^2 = i^2 \;\; \therefore x = \pm i$

$\therefore \; \sqrt[4]{1} = \pm 1, \pm i$

(d) $\sqrt[3]{i}$ [রা., কু.'০৫,'১২; ঢা.'০৪; চ.'০৯,'১৩; ব.'১০]

মনে করি, $x = \sqrt[3]{i} \Rightarrow x^3 = i = -i^3$

$\Rightarrow x^3 + i^3 = 0 \Rightarrow (x + i)(x^2 - xi + i^2) = 0$

$\Rightarrow (x + i)(x^2 - xi - 1) = 0$

$x + i = 0$ হলে, $x = -i$

$x^2 - xi - 1 = 0$ হলে,

$$x = \frac{-(-i) \pm \sqrt{(-i)^2 - 4.1.(-1)}}{2.1}$$

$$= \frac{i \pm \sqrt{-1+4}}{2} = \frac{i \pm \sqrt{3}}{2}$$

$\therefore \; \sqrt[3]{i} = -i, \; \frac{i \pm \sqrt{3}}{2}$

বিকল্প পদ্ধতি ঃ মনে করি, $x = \sqrt[3]{i}$

$\Rightarrow x^3 = i = (-i)^3$

$\therefore \; x = -i, \frac{1}{2}(-1 \pm \sqrt{-3})(-i)$

এখন, $\frac{1}{2}(-1 \pm \sqrt{3}\, i)(-i) = \frac{1}{2}(i \mp \sqrt{3})$

$\therefore \; \sqrt[3]{i} = -i, \frac{1}{2}(i \pm \sqrt{3})$

4(e) $\sqrt[3]{-i}$

[চ.'০১,'০৬; সি.'০৮; ব.,য.'১৩; কুয়েট'১১-১২]

মনে করি, $x = \sqrt[3]{-i} \Rightarrow x^3 = -i = i^3$

$\therefore \; x = i, \frac{1}{2}(-1 \pm i\sqrt{3}).\, i$

এখন, $\frac{1}{2}(-1 \pm i\sqrt{3}).\, i = \frac{1}{2}(-i \mp \sqrt{3})$

$\therefore \; \sqrt[3]{-i} = i, \; \frac{1}{2}(-i \pm \sqrt{3})$

4(f) $\sqrt[4]{-1}$

মনে করি, $x = \sqrt[4]{-1} \Rightarrow x^4 = -1 = i^2$

$\Rightarrow (x^2)^2 = i^2 \Rightarrow x^2 = \pm i = \frac{1}{2}(\pm 2i)$

$= \frac{1}{2}(1^2 + i^2 \pm 2i) = \frac{1}{2}(1 \pm i)^2$

$\therefore \; x = \pm \frac{1}{\sqrt{2}}(1 \pm i)$

4(g) $\sqrt[6]{64}$

ধরি, $x = \sqrt[6]{64} \Rightarrow x^6 = 64 \Rightarrow (x^3)^2 = 8^2$

$\therefore \; x^3 = \pm 8 = (\pm 2)^3$

$\therefore \; x = \pm 2, \pm 2\, \frac{1}{2}(-1 \pm i\sqrt{3})$

$\therefore \; x = \pm 2, \pm(1 \pm i\sqrt{3})$

$\therefore \; \sqrt[6]{64} = \pm 2, \; \pm(1 \pm i\sqrt{3})$

4(h) $\sqrt[4]{-144}$

[সি.'০২]

মনে করি, $x = \sqrt[4]{-144}$

$\Rightarrow x^4 = -144 = 144\, i^2 \Rightarrow (x^2)^2 = (12\, i)^2$

$\therefore \; x^2 = \pm 12\, i = 6(\pm 2\, i)$

$= 6(1^2 + i^2 \pm 2.1.\, i) = 6(1 \pm i)^2$

$\therefore\ x = \pm\sqrt{6}\,(1 \pm i)$.

সুতরাং, $\sqrt[4]{-144} = \pm\sqrt{6}\,(1 \pm i)$

5. দেখাও যে, (a) $\sqrt{i} + \sqrt{-i} = \sqrt{2}$

[য.'১২; কুয়েট'০৬-০৭,০৮-০৯; রুয়েট'১২-১৩]

প্রমাণ ঃ $\sqrt{i} + \sqrt{-i} = \sqrt{\frac{1}{2}.2i} + \sqrt{\frac{1}{2}.(-2i)}$

$= \sqrt{\frac{1}{2}.(1^2 + i^2 + 2.1.i)} + \sqrt{\frac{1}{2}.(1^2 + i^2 - 2.1.i)}$

$= \sqrt{\frac{1}{2}.(1+i)^2} + \sqrt{\frac{1}{2}.(1-i)^2}$

$= \frac{1}{\sqrt{2}}(1 + i + 1 - i) = \frac{2}{\sqrt{2}}$

$\therefore\ \sqrt{i} + \sqrt{-i} = \sqrt{2}$ (Showed)

5(b) $\sqrt[3]{i} + \sqrt[3]{-i} = 0$ [রুয়েট'০৫-০৬]

প্রমাণ ঃ $\sqrt[3]{i} + \sqrt[3]{-i} = \sqrt[3]{(-i)^3} + \sqrt[3]{(i)^3}$

$\sqrt[3]{i} = \sqrt[3]{(-i)^3} = -i\ ,\ -\frac{i}{2}(-1 \pm \sqrt{3}i)$

$\sqrt[3]{-i} = \sqrt[3]{(i)^3} = i\ ,\ \frac{i}{2}(-1 \pm \sqrt{3}i)$

$\therefore\ \sqrt[3]{i} + \sqrt[3]{-i} = 0$ (Showed)

6 (a) দেখাও যে, $\left|\frac{x-iy}{x+iy}\right| = 1$ [য.'০৪; কু.'০৭]

প্রমাণ ঃ $\left|\frac{x-iy}{x+iy}\right| = \frac{|x-iy|}{|x+iy|} = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$

$\therefore\ \left|\frac{x-iy}{x+iy}\right| = 1$ (Showed)

6(b) $\sqrt[3]{a+ib} = x + iy$ হলে দেখাও যে,

$\sqrt[3]{a-ib} = x - iy$ [কু.'০৩; য.'০৬; রা.'০৯; দি., সি.'১০; ঢা.'১৩; বুয়েট'০৩-০৪; টেক্সটাইল'০৩-০৪; রুয়েট'০৭-০৮]

এবং $-2(x^2 + y^2) = \frac{a}{x} - \frac{b}{y}$. [রা.'০১]

প্রমাণ ঃ দেওয়া আছে, $\sqrt[3]{a+ib} = x + iy$

$\therefore\ a + ib = x^3 + 3.x^2.iy + 3.x.(iy)^2 + (iy)^3$

$= x^3 + 3x^2yi - 3xy^2 - iy^3$

উভয় পক্ষ হতে বাস্তব ও কাল্পনিক অংশ সমীকৃত করে পাই, $a = x^3 - 3xy^2$, $b = 3x^2y - y^3$

এখন, $a - ib = x^3 - 3xy^2 - i(3x^2y - y^3)$

$= x^3 - 3xy^2 - 3x^2yi + iy^3$

$= x^3 + 3xy^2i^2 - 3x^2yi - i^3y^3$

$= x^3 - 3.x^2.iy + 3.x.(iy)^2 - (iy)^3$

$= (x - iy)^3$

$\therefore\ \sqrt[3]{a+ib} = x - iy$ (Showed)

এবং $\frac{a}{x} - \frac{b}{y} = \frac{x^3 - 3xy^2}{x} - \frac{3x^2y - y^3}{y}$

$= x^2 - 3y^2 - 3x^2 + y^2$

$= -2x^2 - 2y^2$

$\therefore\ -2(x^2 + y^2) = \frac{a}{x} - \frac{b}{y}$ (Showed)

6(c) $(a + ib)(c + id) = x + iy$ হলে দেখাও যে, $(a - ib)(c - id) = x - iy$ [য.'০৩; চ.'০৪]

প্রমাণ :দেওয়া আছে, $(a + ib)(c + id) = x + iy$

$\Rightarrow ac - bd + i(ad + bc) = x + iy$

উভয় পক্ষ হতে বাস্তব ও কাল্পনিক অংশ সমীকৃত করে পাই, $x = ac - bd$, $y = ad + bc$

এখন, $x - iy = ac - bd - i(ad + bc)$

$= ac - i\,ad - i\,bc - bd$

$= ac - i\,ad - i\,bc + bd\,i^2$

$= a(c - i\,d) - i\,b(c - i\,d)$

$= (a - i\,b)(c - i\,d)$

$\therefore\ (a - i\,b)(c - i\,d) = x - i\,y$ (Showed)

(d) $a^2 + b^2 = 1$ **হলে দেখাও যে, x এর একটি বাস্তব মান $\frac{1-ix}{1+ix} = a - ib$ সমীকরণকে সিদ্ধ করে ; যেখানে $a,\ b \in \mathbb{R}$. [কু. '০৮,'১০; রা.'০৬,'১২; সি.'০৮,'১২; য.'০৬,'০৮; ব.'০৭; ঢা.'০৭; দি.'১২; চ.'১২]**

প্রমাণ ঃ দেওয়া আছে , $a^2 + b^2 = 1 \cdots\cdots (1)$ এবং

$$\frac{1-ix}{1+ix} = a - ib$$

$$\Rightarrow 1 - i\,x = a + i\,ax - i\,b + bx$$

$$\Rightarrow (a\,i + b + i)x = 1 - a + i\,b$$

$$\Rightarrow x = \frac{(1-a) + ib}{b + (a+1)i}$$

$$= \frac{\{(1-a) + ib\}\{b - (a+1)i\}}{\{b + (a+1)i\}\{b - (a+1)i\}}$$

$$= \frac{b(1-a) + b(a+1) + \{b^2 - (1+a)(1-a)\}i}{b^2 + (a+1)^2}$$

$$= \frac{b - ab + ab + b - \{b^2 - 1 + a^2\}i}{b^2 + a^2 + 2a + 1}$$

$$= \frac{2b - 0.i}{1 + 2a + 1},\ [\because a^2 + b^2 = 1]$$

$$= \frac{2b}{2(a+1)} = \frac{b}{a+1}$$,যা বাস্তব সংখ্যা ।

6(e) $(1+x)^n = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ **হলে দেখাও যে ,** $(a_0 - a_2 + a_4 - \cdots)^2 + (a_1 - a_3 + a_5 - \cdots)^2 = a_0 + a_1 + a_2 + a_3 + \cdots + a_n$ **[ঢা.' ০১;ব.'০৩; কু.'১২]**

প্রমাণ ঃ দেওয়া আছে ,

$$(1+x)^n = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \text{ ---(1)}$$

(1) -এ $x = i$ বসিয়ে পাই ,

$$(1+i)^n = a_0 + a_1i + a_2i^2 + a_3i^3 + a_4i^4 + \cdots\cdots$$

$$\Rightarrow (1+i)^n = a_0 + a_1i - a_2 - a_3i + a_4 + a_5i + \cdots$$

$$\Rightarrow (1+i)^n = (a_0 - a_2 + a_4 - \cdots) + i(a_1 - a_3 + a_5 + \cdots) \text{ --- (2)}$$

(2)-এ i এর পরিবর্তে $-i$ লিখে পাই ,

$$(1-i)^n = (a_0 - a_2 + a_4 - \cdots\cdots) - i(a_1 - a_3 + a_5 + \cdots\cdots) \text{ ----- (3)}$$

(2) এবং (3) নং সমীকরণ গুণ করে পাই ,

$$(1-i^2)^n = \{(a_0 - a_2 + a_4 - \cdots\cdots) + i(a_1 - a_3 + a_5 + \cdots\cdots)\}\{(a_0 - a_2 + a_4 - \cdots) - i(a_1 - a_3 + a_5 + \cdots\cdots)\}$$

$$\Rightarrow (1+1)^n = (a_0 - a_2 + a_4 - \cdots)^2 - i^2(a_1 - a_3 + a_5 + \cdots)^2$$

$$\Rightarrow 2^n = (a_0 - a_2 + a_4 - \cdots)^2 + (a_1 - a_3 + a_5 + \cdots)^2 \text{ ------ (4)}$$

এখন, (1)-এ $x = 1$ বসিয়ে পাই ,

$$(1+1)^n = a_0 + a_1 + a_2 + \cdots\cdots + a_n$$

$$\Rightarrow 2^n = a_0 + a_1 + a_2 + \cdots\cdots + a_n \text{-------- (5)}$$

(4) এবং (5) হতে পাই ,

$$(a_0 - a_2 + a_4 - \cdots)^2 + (a_1 - a_3 + a_5 - \cdots)^2 = a_0 + a_1 + a_2 + \cdots\cdots + a_n$$

6(f) $x : y = (a + i\,b) : (c + i\,d)$ **হলে দেখাও যে,** $(c^2 + d^2)x^2 - 2(ac + bd)xy + (a^2 + b^2)y^2 = 0$ **[রা.'০৩; কু.'০৪; সি.'০৫,'০৯; য.'০৭; ব. '০৯; য.'১০; দি.'১২; বুয়েট'০৪]**

প্রমাণ ঃ দেওয়া আছে, $x : y = (a + i\,b) : (c + i\,d)$

$$\Rightarrow \frac{x}{y} = \frac{a+ib}{c+id} \Rightarrow cx + i\,dx = ay + i\,by$$

$$\Rightarrow cx - ay = i\,(by - dx)$$

$$\Rightarrow (cx - ay)^2 = i^2(by - dx)^2$$

$$\Rightarrow c^2x^2 - 2cx.ay + a^2y^2 = -(b^2y^2 - 2by.dx + d^2x^2)$$

$$\Rightarrow c^2x^2 - 2cx.ay + a^2y^2 +$$

$(b^2y^2 - 2by.dx + d^2x^2) = 0$

$\therefore (c^2+d^2)x^2 - 2(ac+bd)xy + (a^2+b^2)y^2 = 0$

6(g) $x = \dfrac{a+ib}{a-ib}$ **হলে প্রমাণ কর যে,** $(a^2+b^2)x^2 + a^2 + b^2 = 2(a^2-b^2)x$

প্রমাণ ঃ দেওয়া আছে , $x = \dfrac{a+ib}{a-ib}$

$\Rightarrow ax - i\,bx = a + i\,b \Rightarrow ax - a = i(b+bx)$

$\Rightarrow (ax-a)^2 = i^2(b+bx)^2$

$\Rightarrow a^2x^2 - 2a^2x + a^2 = -(b^2 + 2b^2x + b^2x^2)$

$\Rightarrow a^2x^2 - 2a^2x + a^2 + b^2 + 2b^2x + b^2x^2 = 0$

$\Rightarrow (a^2+b^2)x^2 + a^2 + b^2 - 2(a^2-b^2)x = 0$

$\therefore \quad (a^2+b^2)x^2 + a^2 + b^2 = 2(a^2-b^2)x$

6(h) $x = -1 + \sqrt{2}\,i$ **হলে দেখাও যে,**

$x^4 + 4x^3 + 6x^2 + 4x = 3$ [টেক্সটাইল'০৩-০৪]

প্রমাণ ঃ $x = -1 + \sqrt{2}\,i \Rightarrow x + 1 = \sqrt{2}\,i$

$\Rightarrow (x+1)^2 = 2.i^2 \Rightarrow x^2 + 2x + 1 = -2$

$\Rightarrow x^2 + 2x + 3 = 0$

$x^4 + 4x^3 + 6x^2 + 4x = x^2(x^2 + 2x + 3) +$
$2x(x^2 + 2x + 3) - 1(x^2 + 2x + 3) + 3$

$= x^2.0 + 2x.0 - 1.0 + 3 = 3$

$\therefore \quad x^4 + 4x^3 + 6x^2 + 4x = 3$ (Showed)

6(i) যদি $x = 2 + \sqrt{-3}$ হয় , তবে $3x^4 - 17x^3 + 41x^2 - 35x + 5$ এর মান নির্ণয় কর।

[বুয়েট'০১-০২]

সমাধান : $x = 2 + \sqrt{-3} \Rightarrow x - 2 = \sqrt{-3}$

$\Rightarrow (x-2)^2 = (\sqrt{-3})^2$

$\Rightarrow x^2 - 4x + 4 = -3 \Rightarrow x^2 - 4x + 7 = 0$

এখন, $3x^4 - 17x^3 + 41x^2 - 35x + 5$

$= 3x^3(x^2 - 4x + 7) - 5x^2(x^2 - 4x + 7) + 5$

$= 3x^3 \times 0 - 5x^2 \times 0 + 5 = 5$ (Ans.)

6(j) যদি $x = 2 - i$ হয়, তবে $x^3 - 3x^2 + x + 10$ এর মান নির্ণয় কর। [টেক্সটাইল'০৬-০৭]

সমাধান : $x = 2 - i \Rightarrow x - 2 = -i$

$\Rightarrow x^2 - 4x + 4 = i^2 = -1$

$\Rightarrow x^2 - 4x + 5 = 0$

এখন, $x^3 - 3x^2 + x + 10$

$= x(x^2 - 4x + 5) + (x^2 - 4x + 5) + 5$

$= x \times 0 + 0 + 5 = 5$ (Ans.)

7(a) $x = 3 + 2i$ **এবং** $y = 3 - 2i$ **হলে দেখাও যে,** $x^2 + xy + y^2 = 23$ [য.'০৫]

প্রমাণ ঃ দেওয়া আছে , $x = 3 + 2i$

$\Rightarrow x^2 = 9 + 12i + 4i^2$

$= 9 + 12i - 4 = 5 + 12i$ এবং

$y = 3 - 2i \Rightarrow y^2 = 9 - 12i - 4 = 5 - 12i$

$xy = (3 + 2i)(3 - 2i)$

$= 9 - 4i^2 = 9 + 4 = 13$

$\therefore \quad x^2 + xy + y^2 = 5 + 12i + 5 - 12i + 13 = 23$

(b) $z = x + iy$ **এবং** $|2z - 1| = |z - 2|$ **হলে দেখাও যে ,** $x^2 + y^2 = 1$ [রা.'০৫; দি.'০৯; ব.'১০; ঢা.'১১]

প্রমাণ ঃ দেওয়া আছে, $|2z - 1| = |z - 2|$

$\Rightarrow |2(x + iy) - 1| = |x + iy - 2|$

$\Rightarrow |(2x - 1) + i.2y| = |(x - 2) + yi|$

$\Rightarrow \sqrt{(2x-1)^2 + 4y^2} = \sqrt{(x-2)^2 + y^2}$

$\Rightarrow 4x^2 - 4x + 1 + 4y^2 = x^2 - 4x + 4 + y^2$

$\Rightarrow 3x^2 + 3y^2 = 3$

$\therefore \quad x^2 + y^2 = 1$

7(c) $z = x + iy$ **এবং** $3|z - 1| = 2|z - 2|$ **হলে প্রমাণ কর যে ,** $5(x^2 + y^2) = 2x + 7$

[য.'০৮; চ.'১১,'১৩]

প্রমাণ ঃ দেওয়া আছে, $z = x + iy$ এবং

$$3|z-1| = 2|z-2|$$

$$\Rightarrow 3|x+iy-1| = 2|x+iy-2|$$

$$\Rightarrow 3|(x-1)+iy| = 2|(x-2)+iy|$$

$$\Rightarrow 3\sqrt{(x-1)^2+y^2} = 2\sqrt{(x-2)^2+y^2}$$

উভয় পক্ষকে বর্গ করে আমরা পাই,

$$9(x^2-2x+1+y^2) = 4(x^2-4x+4+y^2)$$

$$\Rightarrow 5(x^2+y^2) = -16x+16+18x-9$$

$$\therefore\ 5(x^2+y^2) = 2x+7\ \text{(Showed)}$$

7(d) $z = x+iy$ **হলে,** $|z-8|+|z+8| = 20$ **দ্বারা নির্দেশিত সঞ্চারপথের সমীকরণ নির্ণয় কর।**

[রা.'০২,'১০; য.'০৫]

প্রমাণ ঃ দেওয়া আছে, $|z-8|+|z+8| = 20$

$$\Rightarrow |x+iy-8|+|x+iy+8| = 20$$

$$\Rightarrow |(x-8)+iy|+|(x+8)+iy| = 20$$

$$\Rightarrow \sqrt{(x-8)^2+y^2}+\sqrt{(x+8)^2+y^2} = 20$$

$$\Rightarrow \sqrt{(x-8)^2+y^2} = 20-\sqrt{(x+8)^2+y^2}$$

উভয় পক্ষকে বর্গ করে পাই,

$$x^2-16x+64+y^2 = 400-40\sqrt{(x+8)^2+y^2}+x^2+16x+64+y^2$$

$$\Rightarrow 32x+400 = 40\sqrt{(x+8)^2+y^2}$$

$$\Rightarrow 4x+50 = 5\sqrt{x^2+16x+64+y^2}$$

আবার উভয় পক্ষকে বর্গ করে পাই,

$$16x^2+400x+2500 = 25(x^2+16x+64+y^2)$$
$$= 25x^2+400x+1600+25y^2$$

$\therefore\ 9x^2+25y^2 = 900$, ইহাই নির্ণেয় সঞ্চার পথের সমীকরণ।

8(a) $z_1 = 1+i\sqrt{3}$, $z_2 = \sqrt{3}-i$ **হলে দেখাও যে,** **(i)** $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

(ii) $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

প্রমাণঃ দেওয়া আছে, $z_1 = 1+i\sqrt{3}$, $z_2 = \sqrt{3}-i$

$$\therefore\ \arg z_1 = \arg(1+i\sqrt{3}) = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

এবং $\arg z_2 = \arg(\sqrt{3}-i) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

(i) $$z_1 z_2 = (1+i\sqrt{3})(\sqrt{3}-i)$$
$$= \sqrt{3}-i+3i-\sqrt{3}\,i^2$$
$$= \sqrt{3}+2i+\sqrt{3} = 2\sqrt{3}+2i$$

$$\therefore\ \arg(z_1 z_2) = \tan^{-1}\frac{2}{2\sqrt{3}} = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

এখন, $\arg z_1 + \arg z_2 = \frac{\pi}{3}-\frac{\pi}{6} = \frac{2\pi-\pi}{6} = \frac{\pi}{6}$

$$\therefore\ \arg(z_1 z_2) = \frac{\pi}{6} = \arg z_1 + \arg z_2$$

(ii) $$\frac{z_1}{z_2} = \frac{1+i\sqrt{3}}{\sqrt{3}-i} = \frac{(1+i\sqrt{3})(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$$
$$= \frac{\sqrt{3}-\sqrt{3}+i+3i}{3+1} = \frac{4i}{4} = i = 0+i$$

$$\therefore\ \arg\left(\frac{z_1}{z_2}\right) = \tan^{-1}\left(\frac{1}{0}\right) = \cot^{-1}\frac{0}{1}$$
$$= \cot^{-1}0 = \frac{\pi}{2}$$

এখন, $\arg z_1 - \arg z_2 = \frac{\pi}{3}+\frac{\pi}{6} = \frac{2\pi+\pi}{6}$
$$= \frac{3\pi}{6} = \frac{\pi}{2}$$

$\therefore \quad \arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2} = \arg z_1 - \arg z_2$ (Showed)

8(b) মান নির্ণয় কর ঃ

(i) $\sqrt{-3+\sqrt{-3+\sqrt{-3+\cdots\cdots\infty}}}$

[বুয়েট'১১-১২]

মনে করি , $y = \sqrt{-3+\sqrt{-3+\sqrt{-3+\cdots\cdots\infty}}}$

$\Rightarrow y = \sqrt{-3+\sqrt{-3+\sqrt{-3+\sqrt{-3\cdots\cdots\infty}}}}$

$\Rightarrow y = \sqrt{-3+y}$

$\Rightarrow y^2 = -3+y$ [উভয় পক্ষকে বর্গ করে।]

$\Rightarrow y^2 - y + 3 = 0$

$\Rightarrow y = \frac{-(-1)\pm\sqrt{(-1)^2-4.1.3}}{2.1}$

$= \frac{1\pm\sqrt{1-12}}{2} = \frac{1}{2}(1\pm\sqrt{-11})$

$\therefore \sqrt{-3+\sqrt{-3+\sqrt{-3+\cdots\cdots\infty}}} = \frac{1}{2}(1\pm\ i\sqrt{11})$

(ii) $\sqrt{-2+2\sqrt{-2+2\sqrt{-2+\cdots\cdots\infty}}}$

[কুয়েট'০৭-০৮]

মনে করি , $y = \sqrt{-2+2\sqrt{-2+2\sqrt{-2+\cdots\cdots\infty}}}$

$\Rightarrow y = \sqrt{-2+2\sqrt{-2+2\sqrt{-2+2\sqrt{-2+\cdots\cdots\infty}}}}$

$\Rightarrow y = \sqrt{-2+2y}$

$\Rightarrow y^2 = -2+2y \Rightarrow y^2 - 2y + 2 = 0$

$\Rightarrow y = \frac{-(-2)\pm\sqrt{(-2)^2-4.1.2}}{2.1}$

$= \frac{1}{2}(2\pm\sqrt{4-8}) = \frac{1}{2}(2\pm\sqrt{-4})$

$= \frac{2\pm2\sqrt{-1}}{2} = 1\pm i$

$\therefore \sqrt{-2+2\sqrt{-2+2\sqrt{-2+\cdots\cdots\infty}}} = 1\pm i$

(iii) $\sqrt{i+\sqrt{i+\sqrt{i+\ldots\ldots\ldots\infty}}}$ [বুয়েট'০৩-০৪]

মনে করি , $y = \sqrt{i+\sqrt{i+\sqrt{i+\ldots\ldots\ldots\infty}}}$

$\Rightarrow y = \sqrt{i+\sqrt{i+\sqrt{i+\sqrt{i+\ldots\ldots\ldots\infty}}}}$

$\Rightarrow y = \sqrt{i+y} \Rightarrow = y^2 = i+y$

$\Rightarrow y^2 - y - i = 0$

$\Rightarrow y = \frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-i)}}{2.1}$

$= \frac{1\pm\sqrt{1+4i}}{2}$

$\therefore \sqrt{i+\sqrt{i+\sqrt{i+\ldots\ldots\ldots\infty}}} = \frac{1\pm\sqrt{1+4i}}{2}$

8(c) প্রমাণ কর যে,

$|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2+|z_2|^2)$

প্রমাণঃ মনে করি , $z_1 = a+ib$ এবং $z_2 = c+id$

তাহলে, $|z_1| = \sqrt{a^2+b^2}$ এবং

$|z_2| = \sqrt{c^2+d^2}$

$\therefore |z_1|^2 = a^2+b^2$ এবং $|z_2|^2 = c^2+d^2$

L.H.S.$= |z_1+z_2|^2 + |z_1-z_2|^2$

$= |a+ib+c+id|^2 + |a+ib-c-id|^2$

$= (a+c)^2+(b+d)^2+(a-c)^2+(b-d)^2$

$= 2(a^2+c^2)+2(b^2+d^2)$

$= 2|z_1|^2 + 2|z_2|^2$

$= 2\{|z_1|^2 + |z_2|^2\} =$ R.H.S.

9. এককের কাল্পনিক ঘনমূল ω হলে দেখাও যে,

প্রমাণ ঃ

(a)

L.H.S.$= (1+\omega-\omega^2)(\omega+\omega^2-1)(\omega^2+1-\omega)$

[ব.'০৯; ঢা.'১২]

$= (-\omega^2-\omega^2)(-1-1)(-\omega-\omega)$

$[\because 1+\omega+\omega^2=0]$

$= (-2\omega^2)(-2)(-2\omega) = -8\omega^3$

$= -8.1 = -8 =$ R.H.S.

9(b) L.H.S $= (1-\omega+\omega^2)^2+(1+\omega-\omega^2)^2$

[কু. '০০]

$= (-\omega-\omega)^2+(-\omega^2-\omega^2)^2$

$[\because 1+\omega+\omega^2=0]$

$= (-2\omega)^2+(-2\omega^2)^2 = 4(\omega^2+\omega^4)$

$= 4(\omega^2+\omega) = 4(-1) = -4 =$ R.H.S.

9(c) L.H.S $= (x+y)^2+(x\omega+y\omega^2)^2+(x\omega^2+y\omega)^2$ [য.'০১; ঢা.'০৩; সি.'১১]

$= x^2+2xy+y^2+x^2\omega^2+2xy\omega^3+y^2\omega^4+x^2\omega^4+2xy\omega^3+y^2\omega^2$

$= x^2+2xy+y^2+x^2\omega^2+2xy.1+y^2\omega+x^2\omega+2xy.1+y^2\omega^2$

$= x^2(1+\omega^2+\omega)+6xy+y^2(1+\omega+\omega^2)$

$= x^2.0+6xy+y^2.0 = 6xy =$ R.H.S.

9(d) L.H.S.$= (1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)(1-\omega^8+\omega^{16})$

[ব.'০৫; রা.'০৮; য.'১১]

$= (1-\omega+\omega^2)(1-\omega^2+\omega)(1-\omega+\omega^2)(1-\omega^2+\omega)$. $[\because \omega^n = 1, \omega, \omega^2$ হবে যদি n কে 3 দ্বারা ভাগ করলে ভাগশেষ যথাক্রমে 0, 1 . 2 হয় ।]

$= (1-\omega+\omega^2)^2(1-\omega^2+\omega)^2$

$= \{(-\omega-\omega)(-\omega^2-\omega^2)\}^2$

$[\because 1+\omega+\omega^2=0]$

$= \{(-2\omega)(-2\omega^2)\}^2 = (4\omega^3)^2$

$= (4.1)^2 = 16 =$ R.H.S.

9 (e) $(-1+\sqrt{-3})^4+(-1-\sqrt{-3})^4 = -16$

[ব.'০৬; কু. '০৮,'১০,'১৩; য.'০৪,'১৩; ঢা. '০৯; চ.'১০ সি.'১২; রা.'১২]

প্রমাণ ঃ আমরা জানি , $\omega = \frac{1}{2}(-1+\sqrt{-3})$ হলে

$\omega^2 = \frac{1}{2}(-1-\sqrt{-3})$ হবে ।

$\therefore -1+\sqrt{-3} = 2\omega$, $-1-\sqrt{-3} = 2\omega^2$

L.H.S. $= (-1+\sqrt{-3})^4+(-1-\sqrt{-3})^4$

$= (2\omega)^4+(2\omega^2)^4 = 16(\omega^4+\omega^8)$

$= 16(\omega+\omega^2) = 16(-1) = -16 =$ R.H.S

9(f) L.H.S.$= (1-\omega+\omega^2)(1+\omega-\omega^2)$

$= (-\omega-\omega)(-\omega^2-\omega^2)$ $[\because 1+\omega+\omega^2=0]$

$= (-2\omega)(-2\omega^2) = 4\omega^3 = 4.1 = 4 =$ R.H.S.

9(g) L.H.S.$= (1+\omega-\omega^2)^3-(1-\omega+\omega^2)^3$

$= (-\omega^2-\omega^2)^3-(-\omega-\omega)^3$

$[\because 1+\omega+\omega^2=0]$

$= (-2\omega^2)^3-(-2\omega)^3 = -8\omega^6+8\omega^3$

$= -8.1+8.1 = 0 =$ R.H.S.

(h) L.H.S.$= (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$= (1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)$

$= (1+\omega)^2(1+\omega^2)^2 = (1+\omega^2+\omega+\omega^3)^2$

$= (0+1)^2 = 1 =$ R.H.S.

9(i) L.H.S. $= (1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)\cdots$ 2n উৎপাদক পর্যন্ত

[ব.'১১; দি.'১৩]

$= (1-\omega+\omega^2)(1-\omega^2+\omega)(1-\omega+\omega^2)$

$(1-\omega^2+\omega)\cdots\cdots$ 2n উৎপাদক পর্যন্ত

$= \{(1-\omega+\omega^2)(1-\omega+\omega^2)\cdots\cdots$ n উৎপাদক পর্যন্ত $\}\{(1-\omega^2+\omega)(1-\omega^2+\omega)\cdots$ $\cdots$ n উৎপাদক পর্যন্ত $\}$

$= (1-\omega+\omega^2)^n(1-\omega^2+\omega)^n$

$= (-\omega-\omega)^n(-\omega^2-\omega^2)^n$

$= \{(-2\omega)(-2\omega^2)\}^n = (4\omega^3)^n$

$= (2^2.1)^n = 2^{2n} =$ R.H.S.

10(a) উৎপাদকে বিশ্লেষণ কর ঃ

(i) $a^2+ab+b^2 = a^2+ab.1+b^2.1$

$= a^2+ab(-\omega-\omega^2)+b^2\omega^3$

$[\because 1+\omega+\omega^2=0$ এবং $\omega^3=1]$

$= a^2-ab\omega-ab\omega^2+b^2\omega^3$

$= a(a-b\omega)-b\omega^2(a-b\omega)$

$= (a-b\omega)(a-b\omega^2)$ (Ans.)

(ii) $a^2-ab+b^2 = a^2+ab(-1)+b^2.1$

$= a^2+ab(\omega+\omega^2)+b^2.\omega^3$

$[\because 1+\omega+\omega^2=0$ and $\omega^3=1]$

$= a^2+ab\omega+ab\omega^2+b^2\omega^3$

$= a(a+b\omega)+b\omega^2(a+b\omega)$

$= (a+b\omega)(a+b\omega^2)$ (Ans.)

(iii) $a^2+b^2+c^2-ab-bc-ca$

$= a^2+b^2\omega^3+c^2\omega^3+ab(\omega+\omega^2)+$ $bc(\omega^4+\omega^2)+ca(\omega+\omega^2)$

$= a^2+b^2\omega^3+c^2\omega^3+ab\omega+ab\omega^2+$ $bc\omega^4+bc\omega^2+ca\omega+ca\omega^2$

$= (a^2+ab\omega+ca\omega^2)+(ab\omega^2+b^2\omega^3+$ $bc\omega^4)+(ca\omega+bc\omega^2+c^2\omega^3)$

$= a(a+b\omega+c\omega^2)+b\omega^2(a+b\omega+c\omega^2)$ $+c\omega(a+b\omega+c\omega^2)$

$= (a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$ (Ans.)

(iv) $a^3+b^3+c^3-3abc$

$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= (a+b+c)\{a^2+b^2\omega^3+c^2\omega^3+$ $ab(\omega+\omega^2)+bc(\omega^4+\omega^2)+ca(\omega+\omega^2)\}$

$= (a+b+c)\{a^2+b^2\omega^3+c^2\omega^3+ab\omega+$ $ab\omega^2+bc\omega^4+bc\omega^2+ca\omega+ca\omega^2\}$

$= (a+b+c)\{(a^2+ab\omega+ca\omega^2)+(ab\omega^2+$ $b^2\omega^3+bc\omega^4)+(ca\omega+bc\omega^2+c^2\omega^3)\}$

$= (a+b+c)\{a(a+b\omega+c\omega^2)+$ $b\omega^2(a+b\omega+c\omega^2)+c\omega(a+b\omega+c\omega^2)\}$

$= (a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$ (Ans.)

10(b) $p=\frac{1}{2}(-1+\sqrt{-3})$, $q=\frac{1}{2}(-1-\sqrt{-3})$ **হলে দেখাও যে, (*i*)** $p^3+p^{-3}=2$

(*ii*) $q^3+q^{-3}=2$ **(*iii*)** $(1-p)(1-q)=3$

প্রমাণ ঃ দেওয়া আছে, $p=\frac{1}{2}(-1+\sqrt{-3})$,

$q=\frac{1}{2}(-1-\sqrt{-3})$

আমরা জানি,

$\omega=\frac{1}{2}(-1+\sqrt{-3})$ হলে $\omega^2=\frac{1}{2}(-1-\sqrt{-3})$ হবে ; যেখানে ω এককের কাল্পনিক ঘনমূল ।

$\therefore$ আমরা লিখতে পারি, $P=\omega$ এবং $q=\omega^2$

(i) $p^3+p^{-3}=\omega^3+\omega^{-3}$

$=\omega^3+\frac{1}{\omega^3}=1+\frac{1}{1}=1+1=2$

$\therefore p^3+p^{-3}=2$ (Showed)

(ii) $q^3 + q^{-3} = q^3 + \frac{1}{q^3} = (\omega^2)^3 + \frac{1}{(\omega^2)^3}$

$= \omega^6 + \frac{1}{\omega^6} = 1 + \frac{1}{1} = 1 + 1 = 2$

$\therefore\ q^3 + q^{-3} = 2$ (Showed)

(iii) $(1-p)(1-q) = (1-\omega)(1-\omega^2)$

$= 1 - (\omega + \omega^2) + \omega^3 = 1 + 1 + 1$

[$\because 1 + \omega + \omega^2 = 0$ এবং $\omega^3 = 1$]

$\therefore\ (1-p)(1-q) = 3$ (Showed)

(iv) $p^4 + p^2q^2 + q^4$ [চুয়েট'১১-১২]

$= \omega^4 + \omega^2(\omega^2)^2 + (\omega^2)^4$

$= \omega^4 + \omega^6 + \omega^8 = \omega^3.\omega + (\omega^3)^2 + (\omega^3)^2\omega^2$

$= 1.\omega + (1)^2 + (1)^2\omega^2 = \omega^2 + \omega + 1 = 0$

$\therefore\ p^4 + p^2q^2 + q^4 = 0$

11(a) $(a\omega^2 + b + c\omega)^3 + (a\omega + b + c\omega^2)^3 = 0$ **হলে দেখাও যে,** $a = \frac{1}{2}(b+c)$, **অথবা** $b = \frac{1}{2}(c+a)$, **অথবা** $c = \frac{1}{2}(a+b)$ [কু.'০২]

প্রমাণ ঃ দেওয়া আছে,

$(a\omega^2 + b + c\omega)^3 + (a\omega + b + c\omega^2)^3 = 0$

$\Rightarrow (a\omega^2 + b + c\omega)^3 = -(a\omega + b + c\omega^2)^3$

$\Rightarrow \frac{(a\omega^2 + b + c\omega)^3}{(a\omega + b + c\omega^2)^3} = -1$

$\Rightarrow \left(-\frac{a\omega^2 + b + c\omega}{a\omega + b + c\omega^2}\right)^3 = 1$

$\therefore\ -\frac{a\omega^2 + b + c\omega}{a\omega + b + c\omega^2} = 1$, অথবা ω, অথবা ω^2

[$\because x^3 = 1$ হলে $x = 1, \omega, \omega^2$]

যদি $-\frac{a\omega^2 + b + c\omega}{a\omega + b + c\omega^2} = 1$ হয়, তাহলে

$a\omega^2 + b + c\omega = -a\omega - b - c\omega^2$

$\Rightarrow 2b = (a + b)(-\omega - \omega^2)$

$\Rightarrow 2b = a + b \quad \therefore\ b = \frac{1}{2}(a + b)$

যদি $-\frac{a\omega^2 + b + c\omega}{a\omega + b + c\omega^2} = \omega$ হয়, তাহলে

$a\omega^2 + b + c\omega = -a\omega^2 - b\omega - c\omega^3$

$\Rightarrow a\omega^2 + b + c\omega = -a\omega^2 - b\omega - c$

$\Rightarrow 2a\omega^2 = (b + c)(-1 - \omega) = (b + c)\omega^2$

$\Rightarrow 2a = b + c \quad [\because \omega^2 \neq 0]$

$\therefore\ a = \frac{1}{2}(b + c)$

যদি $-\frac{a\omega^2 + b + c\omega}{a\omega + b + c\omega^2} = \omega^2$ হয়, তাহলে

$a\omega^2 + b + c\omega = -a\omega^3 - b\omega^2 - c\omega^4$

$\Rightarrow a\omega^2 + b + c\omega = -a - b\omega^2 - c\omega$

$\Rightarrow 2c\omega = (a + b)(-1 - \omega^2) = (a + b)\omega$

$\Rightarrow 2c = a + b \quad [\because \omega \neq 0]$

$\therefore\ c = \frac{1}{2}(a + b)$

$\therefore\ a = \frac{1}{2}(b + c)$, অথবা $b = \frac{1}{2}(c + a)$,

অথবা $c = \frac{1}{2}(a + b)$ (Showed)

11(b) $x = p + q$, $y = p\omega + q\omega^2$, $z = p\omega^2 + q\omega$ **হলে দেখাও যে,** $x^2 + y^2 + z^2 = 6pq$

[সি.'০৭,'১৩; চ.'০৭,'০৯; ঢা.'১০,'১৩; রা.'১১; চ.'১২]

প্রমাণ ঃ দেওয়া আছে, $x = p + q$,

$y = p\omega + q\omega^2$, $z = p\omega^2 + q\omega$

এখন, $x^2 + y^2 + z^2 = (p + q)^2 + (p\omega + q\omega^2)^2$

$+ (p\omega^2 + q\omega)^2$

$= p^2 + q^2 + 2pq + p^2\omega^2 + q^2\omega^4 + 2pq\omega^3 + p^2\omega^4 + q^2\omega^2 + 2pq\omega^3$

$= p^2 + q^2 + 2pq + p^2\omega^2 + q^2\omega + 2pq + p^2\omega + q^2\omega^2 + 2pq$

$= 6pq + p^2(1 + \omega + \omega^2) + q^2(1 + \omega + \omega^2)$

$= 6pq + p^2.0 + q^2.0$

$\therefore x^3 + y^3 + z^3 = 6pq$ (Showed)

11(c) প্রমাণ ঃ দেওয়া আছে, $x = p + q$, $y = p + \omega q$,$z = p + \omega^2 q$ দেখাতে হবে যে ,

$x^3 + y^3 + z^3 = 3(p^3 + q^3)$

এখন , $x^3 + y^3 + z^3 = (p + q)^3 + (p + \omega q)^3 + (p + \omega^2 q)^3$

$= p^3 + q^3 + 3p^2q + 3pq^2 + p^3 + \omega^3q^3 + p^2q\omega + 3p\omega^2q^2 + p^3 + \omega^6q^3 + 3p^2q\omega^2 + pq^2\omega^4)$

$= p^3 + q^3 + 3p^2q + 3pq^2 + p^3 + q^3 + 3p^2q\omega + 3p\omega^2q^2 + p^3 + q^3 + 3p^2q\omega^2 + 3pq^2\omega$

$= 3(p^3 + q^3) + 3p^2q(1 + \omega + \omega^2) + 3pq^2(1 + \omega + \omega^2)$

$= 3(p^3 + q^3) + 3p^2q.0 + 3pq^2.0$

$\therefore x^3 + y^3 + z^3 = 3(p^3 + q^3)$

11(d) $p = \frac{1}{\sqrt{2}}(1 + \sqrt{-1})$ হলে দেখাও যে ,

$p^6 + p^4 + p^2 + 1 = 0$ [চ.'০৮,'১০; ব.'১১]

প্রমাণ ঃ দেওয়া আছে , $p = \frac{1}{\sqrt{2}}(1 + \sqrt{-1})$

$\Rightarrow \sqrt{2}\, p = 1 + \sqrt{-1} = 1 + i$ $[\because i = \sqrt{-1}]$

উভয় পক্ষকে বর্গ করে আমরা পাই,

$2p^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$

$\therefore p^2 = i$

এখন, $p^6 + p^4 + p^2 + 1 = (p^2)^3 + (p^2)^2 + p^2 + 1$

$= i^3 + i^2 + i + 1 = -i - 1 + i + 1$

$\therefore p^6 + p^4 + p^2 + 1 = 0$ (Showed)

11(e) $(a + b\omega + c\omega^2)^2 + (a\omega + b + c\omega^2)^2 + (a\omega + b\omega^2 + c)^2 = 0$ হলে দেখাও যে, $a = c$ অথবা, $b = \frac{1}{2}(a + c)$

প্রমাণ ঃ দেওয়া আছে , $(a + b\omega + c\omega^2)^2 + (a\omega + b + c\omega^2)^2 + (a\omega + b\omega^2 + c)^2 = 0$

$\Rightarrow a^2 + b^2\omega^2 + c^2\omega^4 + 2ab\omega + 2bc\omega^3 + 2ca\omega^2 + a^2\omega^2 + b^2 + c^2\omega^4 + 2ab\omega + 2bc\omega^2 + 2ca\omega^3 + a^2\omega^2 + b^2\omega^4 + c^2 + 2ab\omega^3 + 2bc\omega^2 + 2ca\omega = 0$

$\Rightarrow a^2(1 + 2\omega^2) + b^2(1 + \omega^2 + \omega^4) + c^2(1 + 2\omega^4) + 2ab(2\omega + \omega^3) + 2bc(\omega^3 + 2\omega^2) + 2ca(\omega^2 + \omega^3 + \omega) = 0$

$\Rightarrow a^2(1 + 2\omega^2) + b^2(1 + \omega^2 + \omega) + c^2(1+2\omega) + 2ab(2\omega + 1) + 2bc(1 + 2\omega^2) + 2ca(\omega^2 + 1 + \omega) = 0$

$\Rightarrow a^2(-\omega - \omega^2 + 2\omega^2) + b^2 \times 0 + c^2(-\omega - \omega^2 + 2\omega) + 2ab(2\omega - \omega^2 - \omega) + 2bc(-\omega - \omega^2 + 2\omega^2) + 2ca \times 0 = 0$

$\Rightarrow a^2(\omega^2 - \omega) + c^2(\omega - \omega^2) + 2ab(\omega - \omega^2) + 2bc(\omega^2 - \omega) = 0$

$\Rightarrow (\omega^2 - \omega)(a^2 - c^2 - 2ab + 2bc) = 0$

$\Rightarrow a^2 - c^2 - 2ab + 2bc = 0$ $[\because \omega^2 - \omega \neq 0]$

$\Rightarrow (a - c)(a + c) - 2b(a - c) = 0$

$\Rightarrow (a - c)(a + c - 2b) = 0$

$\therefore a - c = 0 \Rightarrow a = c$ অথবা , $a + c - 2b = 0$

$\Rightarrow b = \frac{1}{2}(a + c)$ (Showed)

বিকল্প পদ্ধতি ঃ $(a + b\omega + \omega^2)^2 + (a\omega + b + c\omega^2)^2 + (a\omega + b\omega^2 + c)^2 = 0$

$\Rightarrow (a\omega^3 + b\omega^4 + c\omega^2)^2 + (a\omega + b + c\omega^2)^2 + (a\omega + b\omega^2 + c)^2 = 0$

$\Rightarrow \{\omega^2(a\omega + b\omega^2 + c)\}^2 + (a\omega + b + c\omega^2)^2 + (a\omega + b\omega^2 + c)^2 = 0$

$\Rightarrow (\omega^4 + 1)(a\omega + b\omega^2 + c)^2 + (a\omega + b + c\omega^2)^2 = 0$

$\Rightarrow (\omega + 1)(a\omega + b\omega^2 + c)^2 + (a\omega + b + c\omega^2)^2 = 0$

$\Rightarrow -\omega^2(a\omega + b\omega^2 + c)^2 + (a\omega + b + c\omega^2)^2 = 0$

$\Rightarrow -(a\omega^2 + b\omega^3 + c\omega)^2 + (a\omega + b + c\omega^2)^2 = 0$

$\Rightarrow (a\omega + b + c\omega^2)^2 - (a\omega^2 + b + c\omega)^2 = 0$

$\Rightarrow (a\omega + b + c\omega^2 + a\omega^2 + b + c\omega)(a\omega + b + c\omega^2 - a\omega^2 - b - c\omega) = 0$

$\Rightarrow \{a(\omega + \omega^2) + 2b + c(\omega^2 + \omega)\}\{a(\omega - \omega^2) - c(\omega - \omega^2)\} = 0$

$\Rightarrow \{a(-1) + 2b + c(-1)\}(a - c)(\omega - \omega^2) = 0$

$\Rightarrow (2b - a - c)(a - c) = 0$

[$\because$ এখানে $\omega^2 - \omega \neq 0$]

$\therefore a = c$ অথবা $b = \frac{1}{2}(a + c)$

11(f) $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \cdots + a_{2n}x^{2n}$ হলে দেখাও যে, $a_0 + a_3 + a_6 + \cdots = 3^{n-1}$ [চ.'০৮]

প্রমাণ ঃ $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \cdots\cdots + a_{2n}x^{2n}$ ------- (1)

(1) -এ $x = 1$ বসিয়ে আমরা পাই ,

$(1 + 1 + 1)^n = a_0 + a_1 + a_2 + a_3 + \cdots\cdots$

$\Rightarrow 3^n = a_0 + a_1 + a_2 + a_3 + \text{------}$ (2)

(1) -এ $x = \omega$ বসিয়ে আমরা পাই ,

$(1 + \omega + \omega^2)^n = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4 + a_5\omega^5 + a_6\omega^6 + \cdots\cdots$

$\Rightarrow 0 = a_0 + a_1\omega + a_2\omega^2 + a_3 + a_4\omega + a_5\omega^2 + a_6 + \cdots\cdots$ ----- (3)

(1) -এ $x = \omega^2$ বসিয়ে আমরা পাই ,

$(1 + \omega^2 + \omega^4)^n = a_0 + a_1\omega^2 + a_2\omega^4 + a_3\omega^6 + a_4\omega^8 + a_5\omega^{10} + a_6\omega^{12} + \cdots\cdots$

$\Rightarrow (1 + \omega + \omega^2)^n = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega^2 + a_5\omega + a_6 + \cdots\cdots$

$\Rightarrow 0 = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega^2 + a_5\omega + a_6 + \cdots\cdots$ ------- (4)

(2), (3) এবং (4) যোগ করে পাই ,

$3^n = 3(a_0 + a_3 + a_6 + \cdots\cdots) + (a_1 + a_2 + a_4 + a_5 \cdots\cdots)(1 + \omega + \omega^2)$

$\Rightarrow 3^n = 3(a_0 + a_3 + a_6 + \cdots\cdots) + (a_1 + a_2 + a_4 + a_5 \cdots\cdots).0$

$\Rightarrow 3^n = 3(a_0 + a_3 + a_6 + \cdots\cdots)$

$\therefore a_0 + a_3 + a_6 + \cdots\cdots = 3^{n-1}$ (Showed)

12(a) $x + y + z = 0$ এবং এককের কাল্পনিক ঘনমূল ω হলে দেখাও যে, $(x + y\omega + z\omega^2)^3 + (x + y\omega^2 + z\omega)^3 = 27xyz$

[রা.'০২,'০৭,'১০ ;ঢা.'০৫,'০৭; সি.'১০; ব.'১৩]

প্রমাণ ঃ দেওয়া আছে, $x + y + z = 0$

মনে করি , $a = x + y\omega + z\omega^2$ এবং

$b = x + y\omega^2 + z\omega$

$\therefore a^3 + b^3 = (a + b)\ldots$

$= (a + b) \{a^2 + (\omega + \omega^2)ab + b^2\omega^3\}$

$= (a + b)(a + \omega b)(a + \omega^2 b)$

$= (x + y\omega + z\omega^2 + x + y\omega^2 + z\omega)$

$\{x + y\omega + z\omega^2 + \omega(x + y\omega^2 + z\omega)\}$

$\{x + y\omega + z\omega^2 + \omega^2(x + y\omega^2 + z\omega)\}$

$= \{2x + y(\omega + \omega^2) + z(\omega + \omega^2)\}$

$\{x + y\omega + z\omega^2 + x\omega + y + z\omega^2\}$

$\{x + y\omega + z\omega^2 + x\omega^2 + y\omega + z\}$

$= \{2x + y(-1) + z(-1)\}$

$\{x(1 + \omega) + y(1 + \omega) + 2z\omega^2\}$

$\{x(1 + \omega^2) + 2y\omega + z(1 + \omega^2)\}$

$= (2x - y - z)\{x(-\omega^2) + y(-\omega^2)$

$+ 2z\omega^2\}\{x(-\omega) + 2y\omega + z(-\omega)\}$

$= \{3x - (x + y + z)\}\{-(x + y + z)$

$+ 3z\}\omega^2 \{3y - (x + y + z)\}\omega$

$= \{3x - 0\}\{-0 + 3z\}\{3y - 0\}\omega^3$

$= 27xyz$ = R.H.S. (Proved)

12(b) প্রমাণ কর যে, $[\frac{1}{2}(-1 + i\sqrt{3})]^n + [\frac{1}{2}(-1 - i\sqrt{3})]^n = 2$ এবং -1, যখন n-এর মান যথাক্রমে 3 দ্বারা বিভাজ্য এবং 3 দ্বারা অবিভাজ্য ।

[কু.'০৬; ঢা.'০৮; রা., য. '০৯]

প্রমান ঃ আমরা জানি,

$\omega = \frac{1}{2}(-1 + i\sqrt{3})$ হলে, $\omega^2 = \frac{1}{2}(-1 - i\sqrt{3})$, যেখানে এককের কাল্পনিক ঘনমূল ω .

L.H.S. $= [\frac{1}{2}(-1 + i\sqrt{3})]^n + [\frac{1}{2}(-1 - i\sqrt{3})]^n$

$= \omega^n + (\omega^2)^n$

n −এর মান 3 দ্বারা বিভাজ্য হলে মনে করি,

$n = 3m$, যেখানে $m \in \mathbb{N}$.

$\therefore \omega^n + (\omega^2)^n = \omega^{3m} + (\omega^2)^{3m}$

$= (\omega^3)^m + (\omega^3)^{2m} = 1^m + 1^{2m} = 1 + 1 = 2$

n -এর মান 3 দ্বারা অবিভাজ্য হলে মনে করি, $n = 3m + 1$ অথবা $n = 3m + 2$, যেখানে $m \in \mathbb{N}$.

$n = 3m + 1$ হলে,

$\omega^n + (\omega^2)^n = \omega^{3m+1} + (\omega^2)^{3m+1}$

$= \omega^{3m}.\omega + \omega^{6m}.\omega^2 = \omega + \omega^2 = -1$

$n = 3m + 2$ হলে,

$\omega^n + (\omega^2)^n = \omega^{3m+2} + (\omega^2)^{3m+2}$

$= \omega^{3m}.\omega^2 + \omega^{6m}.\omega^4 = \omega^2 + \omega = -1$

$\therefore [\frac{1}{2}(-1 + i\sqrt{3})]^n + [\frac{1}{2}(-1 - i\sqrt{3})]^n =$ 2 অথবা −1, যখন n-এর মান 3 দ্বারা যথাক্রমে বিভাজ্য এবং 3 দ্বারা অবিভাজ্য ।

12(c) প্রমাণ কর যে, $(1 - \omega^{3n-1})(1 - \omega^{3n-2}) = 3$; যেখানে $n \in \mathbb{N}$.

L.H.S. $= (1 - \omega^{3n-1})(1 - \omega^{3n-2})$

$= (1 - \frac{\omega^{3n}}{\omega})(1 - \frac{\omega^{3n}}{\omega^2})$

$= (1 - \frac{1}{\omega})(1 - \frac{1}{\omega^2})$ $[\because n \in \mathbb{N}, \omega^{3n} = 1]$

$= (1 - \omega^2)(1 - \omega)$ $[\because \omega^3 = 1 \Rightarrow \omega^2 = \frac{1}{\omega}]$

$= 1 - (\omega^2 + \omega) + \omega^3 = 1 - (-1) + 1$

$= 3$ = R.H.S. (Proved)

13(a) $\frac{(i+1)^2}{(i-1)^4}$ জটিল সংখ্যাটির আর্গুমেন্ট হবে-

[DU '12-13]

A. $-\frac{\pi}{2}$ **B.** $\frac{\pi}{2}$ **C.** $-\pi$ **D.** π

$Sol^n.: \frac{(i+1)^2}{(i-1)^4} = \frac{i^2+2i+1}{(i^2-2i+1)^2}$

$= \frac{-1+2i+1}{(-1-2i+1)^2} = \frac{2i}{(-2i)^2} = \frac{2i}{4i^2}$

$= \frac{i}{2(-1)} = -\frac{1}{2}i$

$\frac{(i+1)^2}{(i-1)^4}$ এর আর্গুমেন্ট $= \tan^{-1}\frac{-1/2}{0}$

$= -\tan^{-1}(\tan\frac{\pi}{2}) = -\frac{\pi}{2}$

13(b) $\sqrt{-16}\times\sqrt{-1}$ এর মান কত?

[Textile '10-11]

A. 4i **B.** 4ω **C.** −4 **D.** 4

$Sol^n.: \sqrt{-16}\times\sqrt{-1} = \sqrt{16i^2}\times\sqrt{i^2}$

$= 4i\times i = 4i^2 = 4(-1) = -4$

(c) $i^2 = -1$ হলে, $i + i^2 + i^3 + \cdots\cdots + i^{23} = ?$

A. i **B.** −i **C.** 1 **D.** −1

$Sol^n.: i + i^2 + i^3 + \cdots\cdots + i^{23}$

$= i\frac{1-i^{23}}{1-i} = i\frac{1-(i^2)^{11}i}{1-i} = i\frac{1-(-1)^{11}i}{1-i}$

$= i\frac{1-(-1)i}{1-i} = i\frac{1+i}{1-i} = i\frac{(1+i)^2}{(1-i)(1+i)}$

$= i\frac{2i}{2} = i^2 = -1$

14 সমাধানঃ দেওয়া আছে, $f(x) = x^2 + x + 1$

(a) $f(x) = x^2 + 2.x.\frac{1}{2} + (\frac{1}{2})^2 + 1 - \frac{1}{4}$

$= (x + \frac{1}{2})^2 + \frac{3}{4}$

$x + \frac{1}{2} = 0$ অর্থাৎ $x = -\frac{1}{2}$ হলে $f(x)$ এর মা[illegible] ক্ষুদ্রতম হবে। x এর এ বাস্তব মানের জন্য $f(x)$ এ[illegible] ক্ষুদ্রতম মান $\frac{3}{4}$.

14(b) $f(x) = 0$ হলে, $x^2 + x + 1 = 0$

$\Rightarrow x = \frac{-1\pm\sqrt{1-4.1}}{2.1} = \frac{-1\pm\sqrt{-3}}{2}$

$= \frac{-1\pm\sqrt{3}i}{2}$

$\therefore x = -\frac{1}{2}+\frac{\sqrt{3}}{2}i, \ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$; যাদের প্রত্যেকটি জটিল সংখ্যার সাধারণ আকার $x + iy$ এর অনুরূপ। সুতরাং, x এর দুইটি জটিল মান পাওয়া যায়।

এখন, $\{\frac{1}{2}(-1+\sqrt{-3})\}^2$

$= \frac{1}{4}\{1 + 2(-1)\sqrt{-3} + (\sqrt{-3})^2\}$

$= \frac{1}{4}\{1 - 2\sqrt{-3} - 3\}$

$= \frac{1}{4}\{-2 - 2\sqrt{-3}\} = \frac{1}{2}(-1-\sqrt{-3})$

আবার, $\{\frac{1}{2}(-1-\sqrt{-3})\}^2$

$= \frac{1}{4}\{1 + 2\sqrt{-3} - 3\}$

$= \frac{1}{4}(-2-2\sqrt{-3}) = \frac{1}{2}(-1+\sqrt{-3})$

∴ জটিল সংখ্যা দুইটির একটি অপরটির বর্গ।

14(c) $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ এর আর্গুমেন্ট $= \tan^{-1}\frac{\sqrt{3}/2}{-1/2}$

$= \pi - \tan^{-1}\sqrt{3} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$-\frac{1}{2}\mp\frac{\sqrt{3}}{2}i$ এর আর্গুমেন্ট $= \tan^{-1}\frac{-\sqrt{3}/2}{-1/2}$

$$= -\pi + \tan^{-1}\sqrt{3} = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$$

অতিরিক্ত প্রশ্ন (সমাধানসহ)

1 দেখাও যে,

(a) $(1-i)^{-2} - (1+i)^{-2} = i$

প্রমাণ ঃ $(1-i)^{-2} - (1+i)^{-2}$

$$= \frac{1}{(1-i)^2} + \frac{1}{(1+i)^2} = \frac{(1+i)^2 - (1-i)^2}{(1-i)^2(1+i)^2}$$

$$= \frac{4i}{\{(1-i)(1+i)\}^2} = \frac{4i}{(1^2 - i^2)^2}$$

$$= \frac{4i}{(1+1)^2} = \frac{4i}{4}$$

$\therefore\ (1-i)^{-1} - (1+i)^{-1} = i$ (Showed)

1(b) $(3+4i)^{-\frac{1}{2}} + (3-4i)^{-\frac{1}{2}} = \frac{4}{5}$

প্রমাণ ঃ $(3+4i)^{-\frac{1}{2}} + (3-4i)^{-\frac{1}{2}}$

$$= \frac{1}{\sqrt{3+4i}} + \frac{1}{\sqrt{3-4i}}$$

$$= \frac{1}{\sqrt{4-1+4i}} + \frac{1}{\sqrt{4-1-4i}}$$

$$= \frac{1}{\sqrt{2^2 + i^2 + 2.2.i}} + \frac{1}{\sqrt{2^2 + i^2 - 2.2.i}}$$

$$= \frac{1}{\sqrt{(2+i)^2}} + \frac{1}{\sqrt{(2-i)^2}} = \frac{1}{2+i} + \frac{1}{2-i}$$

$$= \frac{1}{(2+i)(2-i)} = \frac{2-i+2+i}{2^2 - i^2} = \frac{4}{4+1}$$

$\therefore\ (3+4i)^{-\frac{1}{2}} + (3-4i)^{-\frac{1}{2}} = \frac{4}{5}$ (Showed)

1(c) $(5+12i)^{-1/2} + (5-12i)^{-1/2} = \frac{6}{13}$

প্রমাণ ঃ $(5+12i)^{-\frac{1}{2}} + (5-12i)^{-\frac{1}{2}}$

$$= \frac{1}{\sqrt{5+12i}} + \frac{1}{\sqrt{5-12i}}$$

$$= \frac{1}{\sqrt{9-4+12i}} + \frac{1}{\sqrt{9-4-12i}}$$

$$= \frac{1}{\sqrt{3^2 + (2i)^2 + 2.3.2i}} + \frac{1}{\sqrt{3^2 + (2i)^2 - 2.3.2i}}$$

$$= \frac{1}{\sqrt{(3+2i)^2}} + \frac{1}{\sqrt{(3-2i)^2}} = \frac{1}{3+2i} + \frac{1}{3-2i}$$

$$= \frac{1}{(3+2i)(3-2i)} = \frac{3-2i+3+2i}{3^2 - (2i)^2}$$

$$= \frac{6}{9-4.i^2} = \frac{6}{9+4}$$

$\therefore\ (5+12i)^{-\frac{1}{2}} + (5-12i)^{-\frac{1}{2}} = \frac{6}{13}$ (Showed)

2.(a) $\frac{2+i}{2+3i} = x + iy$ হলে, দেখাও যে, $13(x^2 + y^2) = 5$.

প্রমাণ ঃ দেওয়া আছে, $\frac{2+i}{2+3i} = x + iy \cdots\cdots$ (i)

i কে $-i$ দ্বারা প্রতিস্থাপন করে পাই,

$\frac{2-i}{2-3i} = x - iy \cdots\cdots\cdots$ (ii)

$$\text{(i)} \times \text{(ii)} \Rightarrow \frac{2+i}{2+3i} \times \frac{2-i}{2-3i} = (x+iy)(x-iy)$$

$$\Rightarrow \frac{2^2 - i^2}{2^2 - (3i)^2} = x^2 - (iy)^2 \Rightarrow \frac{4+1}{4+9} = x^2 + y^2$$

$\therefore\ 13(x^2 + y^2) = 5$ (Showed)

2(b) $x + iy = \frac{2}{3 + \cos\theta + i\sin\theta}$ হলে দেখাও যে, $2(x^2 + y^2) = 3x - 1$

প্রমাণ ঃ দেওয়া আছে,

$$x + iy = \frac{2}{3 + \cos\theta + i\sin\theta}$$

$$= \frac{2(3+\cos\theta - i\sin\theta)}{\{(3+\cos\theta)+i\sin\theta\}\{(3+\cos\theta)-i\sin\theta\}}$$

$$= \frac{2(3+\cos\theta)-2i\sin\theta}{(3+\cos\theta)^2 - i^2\sin^2\theta}$$

$$= \frac{2(3+\cos\theta)-2i\sin\theta}{9+6\cos\theta+\cos^2\theta+\sin^2\theta}$$

$$\Rightarrow x+iy = \frac{2(3+\cos\theta)}{2(5+3\cos\theta)} + \frac{-2i\sin\theta}{2(5+3\cos\theta)}$$

$$\therefore x = \frac{3+\cos\theta}{5+3\cos\theta}, \quad y = \frac{-\sin\theta}{5+3\cos\theta}$$

এখন, $x^2+y^2 = \frac{9+6\cos\theta+\cos^2\theta}{(5+3\cos\theta)^2} + \frac{\sin^2\theta}{(5+3\cos\theta)^2}$

$$= \frac{9+6\cos\theta+\cos^2\theta+\sin^2\theta}{(5+3\cos\theta)^2}$$

$$= \frac{2(5+3\cos\theta)}{(5+3\cos\theta)^2} = \frac{2}{5+3\cos\theta}$$

$$= \frac{1}{2}\frac{3(3+\cos\theta)-(5+3\cos\theta)}{5+3\cos\theta}$$

$$= \frac{1}{2}\left\{\frac{3(3+\cos\theta)}{5+3\cos\theta} - \frac{5+3\cos\theta}{5+3\cos\theta}\right\}$$

$$= \frac{1}{2}(3x-1)$$

$$\therefore 2(x^2+y^2) = 3x-1$$

2(c) $13x^2 - 28xy + 17y^2 = 0$ হলে, $x : y$ কত ?

সমাধানঃ দেওয়া আছে, $13x^2 - 28xy + 17y^2 = 0$

$$\Rightarrow x = \frac{28y \pm \sqrt{784y^2 - 4.13.17y^2}}{2.13}$$

$$= \frac{28 \pm \sqrt{784-884}}{26}y = \frac{28 \pm 10i}{26}y$$

$\therefore x : y = 14 \pm 5i : 13$

3.(a) $\frac{(1+i)^n}{(1-i)^{n-2}}$ এর মান নির্ণয় কর ।

সমাধানঃ $\frac{(1+i)^n}{(1-i)^{n-2}} = \frac{(1+i)^n(1-i)^2}{(1-i)^n}$

$$= \left(\frac{1+i}{1-i}\right)^n (1-2.1.i+i^2)$$

$$= \left\{\frac{(1+i)^2}{(1-i)(1+i)}\right\}^n (1-2i-1)$$

$$= \left\{\frac{1+2i+i^2}{1-i^2}\right\}^n (-2i) = \left\{\frac{1+2i-1}{1+1}\right\}^n (-2i)$$

$$= \left\{\frac{2i}{2}\right\}^n (-2i) = i^n(-2i) = -2i^{n+1} \text{ (Ans.)}$$

3(b) $3 + ix^2y$ এবং x^2+y+4 র পরস্পর অনুবন্ধী জটিল সংখ্যা । x এবং y এর বাস্তব মান নির্ণয় কর।

সমাধানঃ দেওয়া আছে, $3 + ix^2y$ এবং x^2+y+4i পরস্পর অনুবন্ধী জটিল সংখ্যা ।

$\therefore x^2 + y = 3 \Rightarrow x^2 = 3 - y \dots\dots (1)$ এবং

$x^2y = -4 \Rightarrow (3-y)y = -4$

$\therefore y^2 - 3y - 4 = 0$

$\Rightarrow (y-4)(y+1) = 0$

$\therefore y = 4$ অথবা, $y = -1$

$y = 4$ হলে, $x^2 = 3-4 = -1 \notin \mathbb{R}$

$y = -1$ হলে, $x^2 = 3+1 = 4 \Rightarrow x = \pm 2$

$\therefore x = \pm 2, y = -1$ (Ans.)

3(c) প্রমাণ কর যে, $\left\{\frac{1+(i)^{4n+1}}{1+(i)^{4n+3}}\right\}^{2(2m+1)} = -1$; যেখানে $m, n \in \mathbb{N}$.

$$\text{L.H.S.} = \left\{\frac{1+(i)^{4n+1}}{1+(i)^{4n+3}}\right\}^{2(2m+1)} = \left\{\frac{1+i}{1-i}\right\}^{2(2m+1)}$$

[$\because$ $n \in \mathbb{N}$ এবং $i^{4m+1} = i, i^{4m+3} = -i$]

$$= \left\{\frac{1+i}{1-i}\right\}^{4m+2} = \left\{\frac{(1+i)^2}{(1-i)(1+i)}\right\}^{4m+2}$$

$$= \left\{\frac{1+2i+i^2}{1-i^2}\right\}^{4m+2} = \left\{\frac{1+2i-1}{1+1}\right\}^{4m+2}$$

$$= \left\{\frac{2i}{2}\right\}^{4m+2} = i^{4m+2}$$

$= -1 =$ R.H.S. [$\because$ $m \in \mathbb{N}$ এবং $i^{4m+2} = -1$]

3(d) প্রমাণ কর যে, $(1 + i^{4n+1})(1 + i^{4n-1}) = 2$; যেখানে $n \in \mathbb{N}$.

L. H. S. $= (1+i^{4n+1})(1+i^{4n-1})$

$$= (1+i^{4n}.i)(1+\frac{i^{4n}}{i})$$

$$= (1+i)(1+\frac{1}{i}) \quad [\because n \in \mathbb{N} \text{ এবং } i^{4n} = 1]$$

$$= (1+i)(1+\frac{-i^2}{i}) = (1+i)(1-i)$$

$= 1^2 - i^2 = 1 + 1 = 2 =$ R.H.S. (Proved)

ভর্তি পরীক্ষার MCQ ঃ

1. $-2-2i$ জটিল রাশিটির আর্গুমেন্ট-[BUET 07-08; IU 08-09; CU 09-10,08-09; JU 09-10]

***Sol*^{n}.:** $-2-2i$ এর আর্গুমেন্ট $= \tan^{-1}\frac{-2}{-2} =$

$$-\pi + \tan^{-1}1 = -\pi + \frac{\pi}{4} = -\frac{3\pi}{4}$$

ক্যালকুলেটরের সাহায্যে, Mode 2 (Complex)

Shift ((arg) ((arg) − 2 − 2 ENG (i)) (x) =

Shift) (abc) (x) ((arg) − 2 − 2 ENG (i)) (x) =

-135^0 যা $-\frac{3\pi}{4}$ এর সমান।

$2\sqrt{2}$; যা প্রদত্ত জটিল রাশির মডুলাস।

2. $\frac{2+3i}{2-i} = P + Qi$ হলে এবং P ,Q বাস্তব সংখ্যা হলে Q = ? [BAU 07-08]

***Sol*^{n}.:** ক্যালকুলেটরের সাহায্যে,

((arg) 2 + 3 ENG (i)) (x) ÷ ((arg) 2 − ENG (i)) (x) = 1/5 (Real part)

[Shift = 8/5 (Imaginary part)

$$\therefore \quad Q = \frac{8}{5}$$

3. i^{-49} এর মান– [BUET 06-07; JU 09-10]

***Sol*^{n}.:** $i^{-49} = i^{-1} = -i$

4. $i^2 = -1$ হলে $\frac{i - i^{-1}}{i + 2i^{-1}}$ এর মান কত? [DU 07-08, Jt.U 08-09]

***Sol*^{n}.:** $\frac{i-i^{-1}}{i+2i^{-1}} = \frac{2i}{-i} = -2$

$[\because i^{-1} = -i, i^{-2} = -1, i^{-3} = i, i^{-4} = 1]$

অথবা, ক্যালকুলেটরের সাহায্যেও উত্তর পাওয়া যায়।

5. $i^2 = -1$ হলে $\frac{i+i^{-1}}{i-i^{-1}}$ এর মান কত?[DU05-06]

$Sol^n.: \frac{i+i^{-1}}{i-i^{-1}} = \frac{i-i}{i+i} = 0$

6. $\dfrac{i}{1-\dfrac{1}{1-\dfrac{1}{i}}}$ এর মান- **[DU 06-07; NU 08-09]**

$Sol^n.: \dfrac{i}{1-\dfrac{1}{1-\dfrac{1}{i}}} = \dfrac{i}{1-\dfrac{1}{1+i}} = \dfrac{i(1+i)}{1+i-1} = 1+i$

অথবা, ক্যালকুলেটরের সাহায্যেও উত্তর পাওয়া যায়।

7. $x = -1+i$ হলে x^3+3x^2+4x+7 এর মান- **[DU 05-06; RU 09-10]**

$Sol^n.:$ ক্যালকুলেটরের সাহায্যে,

[−] [1] [+] [ENG] [=]

[Ans] [Shift x^3] [+] [3] [Ans] [x^2] [+] [4] [Ans] [+] [7] [=] 5 [Shift] [=] 0

8. $17-20\sqrt{-2}$ এর বর্গমূল- **[RU 07-08]**

A. $\pm(5-2\sqrt{2}i)$ B. $\pm(4-3\sqrt{3}i)$

C. $\pm(4-3\sqrt{2}i)$ D. $\pm(5-3\sqrt{2}i)$

$Sol^n.: 17-20\sqrt{-2}$ এর বর্গমূল

$= \pm\sqrt{5^2-(2\sqrt{2})^2-2.5.2\sqrt{2}i} = \pm(5-2\sqrt{2}i)$

ক্যালকুলেটরের সাহায্যে, প্রদত্ত Option গুলোর মধ্যে যাকে বর্গ করে $17-20\sqrt{2}i$ বা $17-28.28i$ পাওয়া যাবে সেটিই উত্তর।

[((arg)] [5] [−] [2] [√] [2] [ENG (i)] [ ) (abc)] [x^2 (x^3)]

[=] 17 [Shift] [=] $-28.28i$

9. $7-24i$ এর বর্গমূল - **[BUET 09-10]**

$Sol^n.: 7-24i$ এর বর্গমূল

$= \pm\sqrt{4^2+(3i)^2-2.4.3i} = \pm(4-3i)$

10. $\sqrt{i}+\sqrt{-i}$ এর মান- **[BUET 05-06, KUET 08-09, 06-07; CU 07-08,05-06; RUET 12-13]**

$Sol^n.: \sqrt{i}+\sqrt{-i} = \frac{1}{\sqrt{2}}(\sqrt{2i}+\sqrt{-2i})$

$= \frac{1}{\sqrt{2}}\{\sqrt{(1+i)^2}+\sqrt{(1-i)^2}\} = \frac{1}{\sqrt{2}}\{1+i+1-i\}$

$= \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$

11. $\sqrt{-2}\sqrt{-1}$ এর মান- **[CU 08-09]**

$Sol^n.: \sqrt{-2}\sqrt{-1} = \sqrt{2i^2}\sqrt{i^2} = \sqrt{2}i.i$

$= \sqrt{2}i^2 = -\sqrt{2}$

12. $\sqrt[4]{-81}$ এর মান- **[BUET 08-09]**

$Sol^n.: \sqrt[4]{-81} = \sqrt[4]{(9i)^2} = \sqrt{\pm 9i}$

$= \sqrt{\frac{9}{2}(\pm 2i)} = \sqrt{\frac{9}{2}(1+i^2\pm 2i)}$

$= \sqrt{\frac{9}{2}(1\pm i)^2} = \pm\frac{3}{\sqrt{2}}(1\pm i)$

13. $\sqrt[4]{-169}$ এর মান- **[SU 06-07; RU 06-07]**

$Sol^n.: \sqrt[4]{-169} = \sqrt[4]{(13i)^2} = \sqrt{\pm 13i}$

$= \sqrt{\frac{13}{2}(\pm 2i)} = \sqrt{\frac{13}{2}(1+i^2\pm 2i)}$

$= \sqrt{\frac{26}{4}(1\pm i)^2} = \pm\frac{\sqrt{26}}{2}(1\pm i)$

14. $\sqrt[6]{-64}$ এর সম্ভাব্য মান- **[KU 07-08]**

$Sol^n.:$ ধরি, $\sqrt[6]{-64} = x \Rightarrow x^6 = -64 = (-4)^3$

$\therefore\ x^2 = -4, -4\,\frac{1}{2}(-1\pm\sqrt{3}i)$

$= -4,\ (2 \pm 2\sqrt{3}i) = (2i)^2,\ (\sqrt{3} \pm i)^2$

$\therefore\ x = \sqrt[6]{-64} = \pm 2i, \pm(\sqrt{3} \pm i)$

কৌশল : $\sqrt{-n + n\sqrt{-n + n\sqrt{-n + \cdots\cdots\infty}}}$

$= \dfrac{n \pm \sqrt{n^2 - 4n}}{2}$

15. $\sqrt{-2 + 2\sqrt{-2 + 2\sqrt{-2 + \cdots\infty}}}$ এর মান কত? **[JU 06-07; KUET 07-08]**

Solⁿ.: $\sqrt{-2 + 2\sqrt{-2 + 2\sqrt{-2 + \cdots\infty}}}$

$= \dfrac{2 \pm \sqrt{2^2 - 4.2}}{2} = 1 \pm i$

16. যদি ω এককের একটি কাল্পনিক জটিল ঘনমূল হয়, তবে $(1 - \omega + \omega^2)^2 + (1 + \omega - \omega^2)^2 =$

[DU 05-06, 01-02, 97-98; Jt.U 08-09]

Solⁿ.: $(1 - \omega + \omega^2)^2 + (1 + \omega - \omega^2)^2$

$= (-2\omega)^2 + (-2\omega^2)^2 = 4(\omega + \omega^2) = -4$

ক্যালকুলেটরের সাহায্যে,

((arg) − 1 + √ 3) (abc) [x] ÷ 2 =

((arg) 1 − Ans + Ans x² [x³]) (abc) [x] x² [x³] +

((arg) 1 + Ans − Ans x² [x³]) (abc) [x] x² [x³] =

17. যদি ω এককের একটি কাল্পনিক জটিল ঘনমূল হয়, তবে $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) =$

[Jt.U07-08; JU 09-10]

Solⁿ.: $(-2\omega^2)(-2\omega)(-2\omega^2)(-2\omega) = 16$

18. $x = \dfrac{1}{2}(-1 + \sqrt{3}i)$ হলে $x^{3(n+2)}$ এর মান কত?

[BAU 05-06]

Solⁿ.: এখানে, $x = \omega$ $[\because \omega^{3(n+2)} = 1$

19. $z_1 = 2 + i$ এবং $z_2 = 3 + i$ হলে $z_1\bar{z}_2$-এর মডুলাস- **DU 13-14**

A. 6 B. $5\sqrt{2}$ C. 7 D. $5\sqrt{3}$

Solⁿ.: $z_1\bar{z}_2 = (2 + i)(3 - i) = 6 - i^2 + i$
$= 7 + i$

$\therefore$ মডুলাস $= \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2}$

20. যদি $a = \dfrac{1 + i}{\sqrt{2}}$ হয়, তবে a^6 এর মান হবে-

BUET 11-12

A. −1 B. i C. 1 D. −i

Solⁿ.: $a^2 = \dfrac{1 + 2i + i^2}{2} = \dfrac{1 + 2i - 1}{2} = i$

$\therefore\ a^6 = (a^2)^3 = i^3 = -i$