প্রমাণ কর যে,

1.(a) $\tan^{-1}\frac{5}{6} - \tan^{-1}\frac{1}{11} = \tan^{-1}\frac{49}{71}$

$$\text{L.H.S.} = \tan^{-1}\frac{5}{6} - \tan^{-1}\frac{1}{11} = \tan^{-1}\frac{\frac{5}{6}-\frac{1}{11}}{1+\frac{5}{6}.\frac{1}{11}}$$

$$= \tan^{-1}(\frac{49}{66}\times\frac{66}{71}) = \tan^{-1}\frac{49}{71} = \text{R.H.S. (Proved)}$$

1(b) $\tan^{-1}\frac{5}{7} + \cot^{-1}\frac{8}{5} = \cot^{-1}\frac{31}{75}$

$$\text{L.H.S.} = \tan^{-1}\frac{5}{7} + \cot^{-1}\frac{8}{5}$$

$$= \tan^{-1}\frac{5}{7} + \tan^{-1}\frac{5}{8} = \tan^{-1}\frac{\frac{5}{7}+\frac{5}{8}}{1-\frac{5}{7}.\frac{5}{8}}$$

$$= \tan^{-1}(\frac{40+35}{56}\times\frac{56}{56-25}) = \tan^{-1}\frac{75}{31}$$

$$= \cot^{-1}\frac{31}{75} = \text{R.H.S. (Proved)}$$

1(c) $\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{5} = \tan^{-1}\frac{27}{11}$

Proof: $\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{\sqrt{5^2-3^2}} = \tan^{-1}\frac{3}{4}$

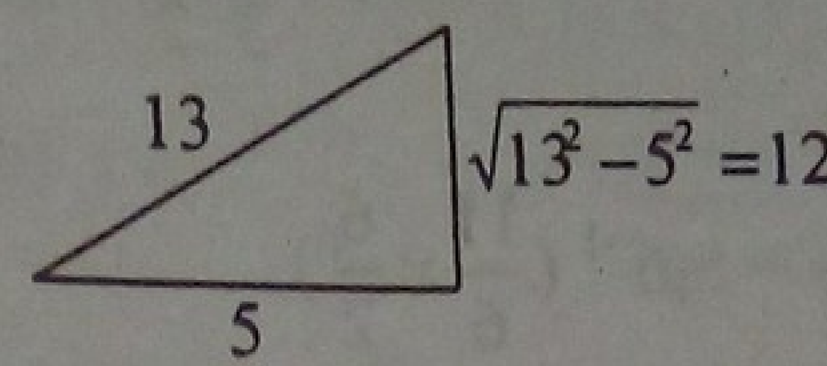
$$\text{L.H.S.} = \cot^{-1}\frac{5}{3} + \sin^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{4} = \tan^{-1}\frac{\frac{3}{5}+\frac{3}{4}}{1-\frac{3}{5}.\frac{3}{4}}$$

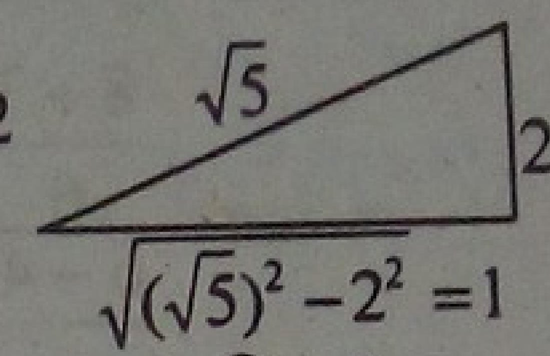
$$= \tan^{-1}(\frac{12+15}{20}\times\frac{20}{20-9})$$

$$= \tan^{-1}\frac{27}{11} = \text{R.H.S. (Proved)}$$

1(d) $\sec^{-1}\frac{13}{5} - \text{cosec}^{-1}\frac{\sqrt{5}}{2} = \tan^{-1}\frac{2}{29}$ [চ.'০২]



চিত্র-1



চিত্র-2

চিত্র-1 হতে, $\sec^{-1}\frac{13}{5} = \tan^{-1}\frac{12}{5}$ এবং

চিত্র -2 হতে, $\text{cosec}^{-1}\frac{\sqrt{5}}{2} = \tan^{-1}2$

$$\text{L.H.S.} = \sec^{-1}\frac{13}{5} - \text{cosec}^{-1}\frac{\sqrt{5}}{2}$$

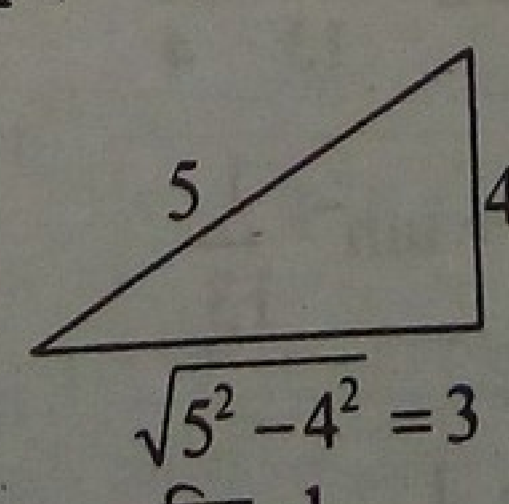
$$= \tan^{-1}\frac{12}{5} - \tan^{-1}2$$

$$= \tan^{-1}\frac{\frac{12}{5}-2}{1+\frac{12}{5}.2} = \tan^{-1}(\frac{12-10}{5}\times\frac{5}{5+24})$$

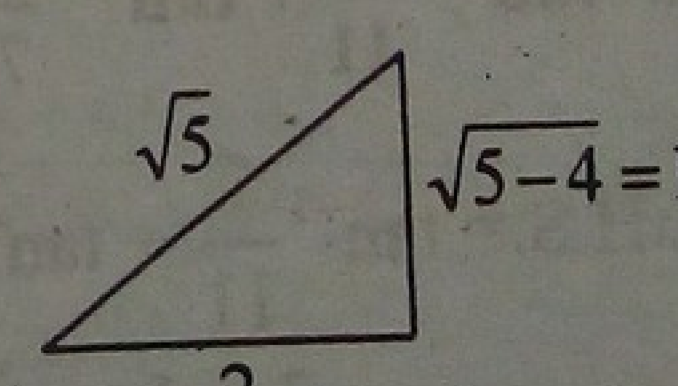
$$= \tan^{-1}\frac{2}{29} = \text{R.H.S. (Proved)}$$

1(e) $\sin^{-1}\frac{4}{5} + \cos^{-1}\frac{2}{\sqrt{5}} = \tan^{-1}\frac{11}{2}$

[কু. '০০,'০৫; সি.'০৫,'১২; চ.'০৫; দি.'০৯,'১৩; ব.'১০]



চিত্র-1



চিত্র-2

চিত্র-1 হতে, $\sin^{-1}\frac{4}{5} = \tan^{-1}\frac{4}{3}$ এবং

চিত্র -2 হতে, $\cos^{-1}\frac{2}{\sqrt{5}} = \tan^{-1}\frac{1}{2}$

$$\text{L.H.S.} = \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{2}{\sqrt{5}}$$

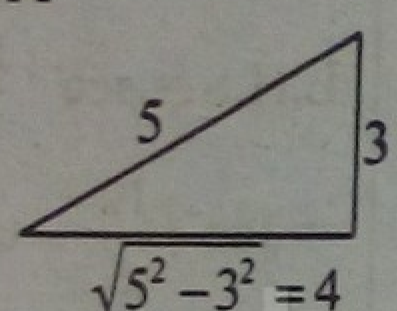
$$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{2} = \tan^{-1}\frac{\frac{4}{3}+\frac{1}{2}}{1-\frac{4}{3}.\frac{1}{2}}$$

$$= \tan^{-1}\frac{\frac{8+3}{6}}{\frac{6-4}{6}} = \tan^{-1}\left(\frac{11}{6}\times\frac{6}{2}\right)$$

$$= \tan^{-1}\frac{11}{2} = \text{R.H.S (Proved)}$$

1(f) $\cot^{-1}\frac{5}{3} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{27}{11}$ [বুয়েট'০৩-০৪]

পাশের চিত্র হতে আমরা পাই,



চিত্র-1

$$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$$

$$\text{L.H.S.} = \tan^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{4}$$

$$= \tan^{-1}\frac{\frac{3}{5}+\frac{3}{4}}{1-\frac{3}{5}\times\frac{3}{4}} = \tan^{-1}\frac{\frac{12+15}{20}}{\frac{20-9}{20}}$$

$$= \tan^{-1}\left(\frac{27}{20}\times\frac{20}{11}\right) = \tan^{-1}\frac{27}{11} = \text{R.H.S.}$$

2. $\tan^{-1}\frac{7}{11} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} = \frac{\pi}{4}$

$$\text{L.H.S.} = \tan^{-1}\frac{7}{11} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13}$$

$$= \tan^{-1}\frac{\frac{7}{11}+\frac{1}{7}}{1-\frac{7}{11}.\frac{1}{7}} + \tan^{-1}\frac{1}{13}$$

$$= \tan^{-1}\left(\frac{49+11}{22}\times\frac{22}{77-7}\right) + \tan^{-1}\frac{1}{13}$$

$$= \tan^{-1}\frac{60}{70} + \tan^{-1}\frac{1}{13} = \tan^{-1}\frac{6}{7} + \tan^{-1}\frac{1}{13}$$

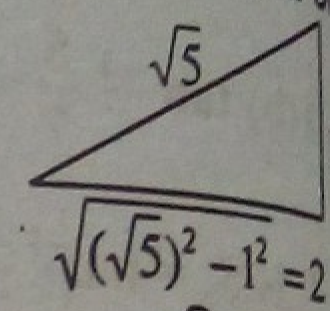
$$= \tan^{-1}\frac{\frac{6}{7}+\frac{1}{13}}{1-\frac{6}{7}.\frac{1}{13}} = \tan^{-1}\left(\frac{78+7}{91}\times\frac{91}{91-6}\right)$$

$$= \tan^{-1}\frac{85}{85} = \tan^{-1}1 = \frac{\pi}{4} = \text{R.H.S.(Proved)}$$

3(a) $4(\cot^{-1}3 + \text{cosec}^{-1}\sqrt{5}) = \pi$

[ব.'০২,'০৪; চ.'০৪; ঢা.'০৮]

পাশের চিত্র হতে আমরা পাই,



চিত্র

$$\text{cosec}^{-1}\frac{1}{\sqrt{5}} = \tan^{-1}\frac{1}{2}$$

L.H.S.

$$= 4(\cot^{-1}3 + \text{cosec}^{-1}\sqrt{5})$$

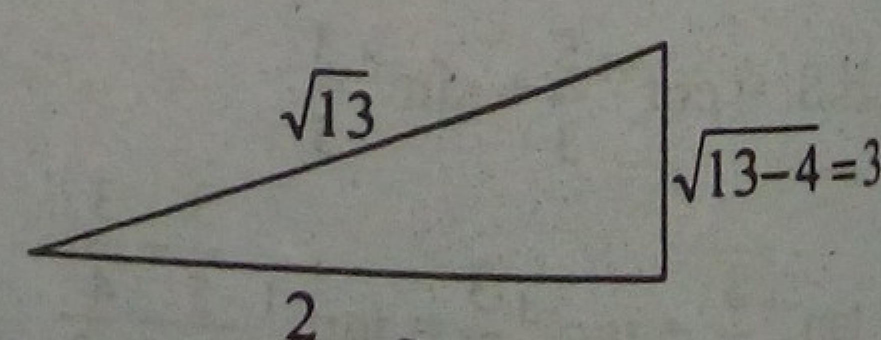
$$= 4\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}\right) = 4\tan^{-1}\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}$$

$$= 4.\tan^{-1}\left(\frac{3+2}{6}\times\frac{6}{6-1}\right) = 4\tan^{-1}\frac{5}{5}$$

$$= 4\tan^{-1}1 = 4.\frac{\pi}{4} = \pi = \text{R.H.S}$$

3(b) $\tan^{-1}\frac{2}{3} = \frac{\pi}{2} - \sec^{-1}\frac{\sqrt{13}}{2}$

[চ.'০৪; ঢা.'০২,'১১; য.'০৮; ব.'০৯; সি.'১০]



চিত্র

চিত্র হতে আমরা পাই, $\sec^{-1}\frac{\sqrt{13}}{2} = \tan^{-1}\frac{3}{2}$

এখন, $\tan^{-1}\frac{2}{3} + \sec^{-1}\frac{\sqrt{13}}{2} = \tan^{-1}\frac{2}{3} + \tan^{-1}\frac{3}{2}$

$$\Rightarrow \tan^{-1}\frac{2}{3} + \sec^{-1}\frac{\sqrt{13}}{2} = \tan^{-1}\frac{2}{3} + \cot^{-1}\frac{2}{3}$$

$$\Rightarrow \tan^{-1}\frac{2}{3} + \sec^{-1}\frac{\sqrt{13}}{2} = \frac{\pi}{2}$$

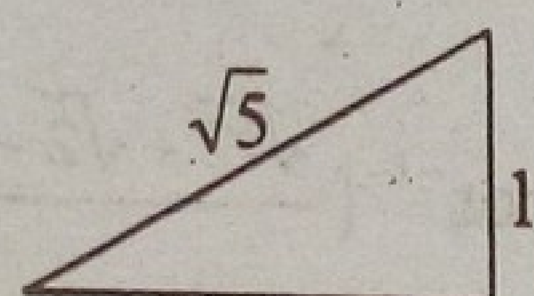
$$\therefore \tan^{-1}\frac{2}{3} = \frac{\pi}{2} - \sec^{-1}\frac{\sqrt{13}}{2}$$

3(c) $4\left(\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3\right) = \pi$

[ব.'০৭; ঢা.'০৯ ; য.'১১; দি.'১২]

পাশের চিত্র হতে আমরা পাই ,

$$\sin^{-1}\frac{1}{\sqrt{5}} = \tan^{-1}\frac{1}{2}$$



চিত্র

$$\text{L.H.S.} = 4\left(\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3\right)$$

$$= 4\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$$

$$= 4\tan^{-1}\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}} = 4.\tan^{-1}\left(\frac{3+2}{6}\times\frac{6}{6-1}\right)$$

$$= 4\tan^{-1}\frac{5}{5} = 4\tan^{-1}1 = 4.\frac{\pi}{4} = \pi = \text{R.H.S}$$

4(a) $\tan^{-1}\left(\frac{x\cos\theta}{1-x\sin\theta}\right) - \tan^{-1}\left(\frac{x-\sin\theta}{\cos\theta}\right) = \theta$

$$\text{L.H.S.} = \tan^{-1}\left(\frac{x\cos\theta}{1-x\sin\theta}\right) - \tan^{-1}\left(\frac{x-\sin\theta}{\cos\theta}\right)$$

$$= \tan^{-1}\frac{\frac{x\cos\theta}{1-x\sin\theta} - \frac{x-\sin\theta}{\cos x}}{1+\frac{x\cos\theta}{1-x\sin\theta}.\frac{x-\sin\theta}{\cos\theta}}$$

$$= \tan^{-1}\frac{x\cos^2\theta-(x-\sin\theta)(1-x\sin\theta)}{\cos\theta(1-x\sin\theta)+x\cos\theta(x-\sin\theta)}$$

$$= \tan^{-1}\frac{x\cos^2\theta-(x-x^2\sin\theta-\sin\theta+x\sin^2\theta)}{\cos\theta-x\sin\theta\cos\theta+x^2\cos\theta-x\sin\theta\cos\theta}$$

$$= \tan^{-1}\frac{x(1-\sin^2\theta)-x+x^2\sin\theta+\sin\theta-x\sin^2\theta}{x^2\cos\theta-2x\sin\theta\cos\theta+\cos\theta}$$

$$= \tan^{-1}\frac{x-x\sin^2\theta-x+x^2\sin\theta+\sin\theta-x\sin^2\theta}{(x^2-2x\sin\theta+1)\cos\theta}$$

$$= \tan^{-1}\frac{(x^2-2x\sin\theta+1)\sin\theta}{(x^2-2x\sin\theta+1)\cos\theta}$$

$$= \tan^{-1}\tan\theta = \theta = \text{R.H.S. (Proved)}$$

4(b) $\tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m-n}{m+n} = \frac{\pi}{4}$ [প্র.ভ.প.'১৭]

$$\text{L.H.S.} = \tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m-n}{m+n}$$

$$= \tan^{-1}\frac{\frac{m}{n}-\frac{m-n}{m+n}}{1+\frac{m}{n}.\frac{m-n}{m+n}}$$

$$= \tan^{-1}\frac{m^2+mn-mn+n^2}{mn+n^2+m^2-mn}$$

$$= \tan^{-1}\frac{m^2+n^2}{n^2+m^2} = \tan^{-1}1 = \frac{\pi}{4} = \text{R.H.S.}$$

4(c) $\tan^{-1}\frac{1}{m+n} - \tan^{-1}\frac{n}{m^2+mn+1} = \tan^{-1}\frac{1}{m}$

$$\text{L.H.S.} = \tan^{-1}\frac{1}{m+n} + \tan^{-1}\frac{n}{m^2+mn+1}$$

$$= \tan^{-1}\frac{\frac{1}{m+n}+\frac{n}{m^2+mn+1}}{1-\frac{1}{m+n}.\frac{n}{m^2+mn+1}}$$

$$= \tan^{-1}\frac{m^2+mn+1+mn+n^2}{m^3+m^2n+m+m^2n+mn^2+n-n}$$

$$= \tan^{-1}\frac{m^2+2mn+1+n^2}{m(m^2+2mn+1+n^2)}$$

$$= \tan^{-1}\frac{1}{m} = \text{R.H.S. (Proved)}$$

4(d) $\tan^{-1}\{(\sqrt{2}+1)\tan\alpha\} - \tan^{-1}\{(\sqrt{2}-1)\tan\alpha\} = \tan^{-1}(\sin 2\alpha)$

[কু.'০১; চ.'১০; দি.'১৩]

L.H.S. =

$\tan^{-1}\{(\sqrt{2}+1)\tan\alpha\} - \tan^{-1}\{(\sqrt{2}-1)\tan\alpha\}$

$= \tan^{-1}\dfrac{(\sqrt{2}+1)\tan\alpha - (\sqrt{2}-1)\tan\alpha}{1+(\sqrt{2}+1)\tan\alpha(\sqrt{2}-1)\tan\alpha}$

$= \tan^{-1}\dfrac{(\sqrt{2}+1-\sqrt{2}+1)\tan\alpha}{1+(2-1)\tan^2\alpha}$

$= \tan^{-1}\dfrac{2\tan\alpha}{1+\tan^2\alpha} = \tan^{-1}(\sin 2\alpha) = $ R.H.S.

4(e) $\tan^{-1}\dfrac{a^2-b^2}{1+a^2b^2} + \tan^{-1}\dfrac{b^2-c^2}{1+b^2c^2} + \tan^{-1}\dfrac{c^2-a^2}{1+c^2a^2} = 0$

L.H.S. $= \tan^{-1}\dfrac{a^2-b^2}{1+a^2b^2} + \tan^{-1}\dfrac{b^2-c^2}{1+b^2c^2} + \tan^{-1}\dfrac{c^2-a^2}{1+c^2a^2}$

$= \tan^{-1}a^2 - \tan^{-1}b^2 + \tan^{-1}b^2 - \tan^{-1}c^2 + \tan^{-1}c^2 - \tan^{-1}a^2 = 0 =$ R.H.S. (Proved)

5(a) $\sin^{-1}\dfrac{3}{5} + \sin^{-1}\dfrac{8}{17} = \sin^{-1}\dfrac{77}{85}$

L.H.S. $= \sin^{-1}\dfrac{3}{5} + \sin^{-1}\dfrac{8}{17}$

$= \sin^{-1}\left\{\dfrac{3}{5}\sqrt{1-\left(\dfrac{8}{17}\right)^2} + \dfrac{8}{17}\sqrt{1-\left(\dfrac{3}{5}\right)^2}\right\}$

$= \sin^{-1}\left\{\dfrac{3}{5}\sqrt{\dfrac{289-64}{289}} + \dfrac{8}{17}\sqrt{\dfrac{25-9}{25}}\right\}$

$= \sin^{-1}\left\{\dfrac{3}{5}.\dfrac{15}{17} + \dfrac{8}{17}.\dfrac{4}{5}\right\} = \sin^{-1}\dfrac{45+32}{85}$

$= \sin^{-1}\dfrac{77}{85} =$ R.H.S. (Proved)

5(b) $\cos^{-1}\sqrt{\dfrac{2}{3}} - \cos^{-1}\dfrac{\sqrt{6}+1}{2\sqrt{3}} = \dfrac{\pi}{6}$ [কু.'০২]

L.H.S. $= \cos^{-1}\sqrt{\dfrac{2}{3}} - \cos^{-1}\dfrac{\sqrt{6}+1}{2\sqrt{3}}$

$= \cos^{-1}\left\{\sqrt{\dfrac{2}{3}} \times \dfrac{\sqrt{6}+1}{2\sqrt{3}} + \sqrt{\left(1-\dfrac{2}{3}\right)\left(1-\dfrac{(\sqrt{6}+1)^2}{(2\sqrt{3})^2}\right)}\right\}$

$= \cos^{-1}\left\{\dfrac{\sqrt{2}(\sqrt{6}+1)}{6} + \sqrt{\dfrac{1}{3}.\dfrac{12-6-2\sqrt{6}-1^2}{12}}\right\}$

$= \cos^{-1}\left(\dfrac{\sqrt{12}+\sqrt{2}}{6} + \dfrac{\sqrt{5-2\sqrt{6}}}{6}\right)$

$= \cos^{-1}\left\{\dfrac{\sqrt{12}+\sqrt{2}-\sqrt{(\sqrt{3})^2-2.\sqrt{3}.\sqrt{2}+(\sqrt{2})^2}}{6}\right\}$

$= \cos^{-1}\left\{\dfrac{\sqrt{12}+\sqrt{2}+\sqrt{(\sqrt{3}-\sqrt{2})^2}}{6}\right\}$

$= \cos^{-1}\left(\dfrac{2\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}}{6}\right) = \cos^{-1}\dfrac{3\sqrt{3}}{6}$

$= \cos^{-1}\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{6} =$ R.H.S. (Proved)

5(c) $\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{65} = \dfrac{\pi}{2}$

[রুয়েট' ০৭-০৮]

L.H.S. $= \sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\left\{\dfrac{4}{5}\sqrt{1-\left(\dfrac{5}{13}\right)^2} + \dfrac{5}{13}\sqrt{1-\left(\dfrac{4}{5}\right)^2}\right\} + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\left\{\dfrac{4}{5}\sqrt{\dfrac{169-25}{169}} + \dfrac{5}{13}\sqrt{\dfrac{25-16}{25}}\right\} + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\left\{\dfrac{4}{5}.\dfrac{12}{13} + \dfrac{5}{13}.\dfrac{3}{5}\right\} + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\dfrac{48+15}{65} + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\dfrac{63}{65} + \sin^{-1}\dfrac{16}{65}$

$= \sin^{-1}\left\{\frac{63}{65}\sqrt{1-\left(\frac{16}{65}\right)^2}+\frac{16}{65}\sqrt{1-\left(\frac{63}{65}\right)^2}\right\}$

$= \sin^{-1}\left\{\frac{63}{65}\sqrt{\frac{4225-256}{4225}}+\frac{16}{65}\sqrt{\frac{4225-3969}{4225}}\right\}$

$= \sin^{-1}\left\{\frac{63}{65}\sqrt{\frac{3969}{4225}}+\frac{16}{65}\sqrt{\frac{256}{4225}}\right\}$

$= \sin^{-1}\left\{\frac{63}{65}.\frac{63}{65}+\frac{16}{65}.\frac{16}{65}\right\}$

$= \sin^{-1}\frac{3969+256}{4225}$

$= \sin^{-1}\frac{4225}{4225} = \sin^{-1}1 = \frac{\pi}{2} = \text{R.H.S}.$

5(d) $\sin^{-1}(\sqrt{2}\sin\theta) - \cos^{-1}(\sqrt{\cos 2\theta}) = 0$ **[প্র.ভ.প.'৯৯]**

L.H.S.$= \sin^{-1}(\sqrt{2}\sin\theta)-\cos^{-1}(\sqrt{\cos 2\theta})$

$= \sin^{-1}(\sqrt{2}\sin\theta)-\sin^{-1}(\sqrt{1-(\sqrt{\cos 2\theta})^2}$

$= \sin^{-1}(\sqrt{2}\sin\theta)-\sin^{-1}(\sqrt{1-\cos 2\theta}$

$= \sin^{-1}(\sqrt{2}\sin\theta)-\sin^{-1}(\sqrt{2\sin^2\theta})$

$= \sin^{-1}(\sqrt{2}\sin\theta)-\sin^{-1}(\sqrt{2}\sin\theta)$

$= 0 =$ R.H.S. (Proved)

6(a) $\tan^{-1}\frac{3}{4} - 2\tan^{-1}\frac{1}{5} = \cos^{-1}\frac{63}{65}$

L.H.S.$= \tan^{-1}\frac{3}{4} - 2\tan^{-1}\frac{1}{5}$

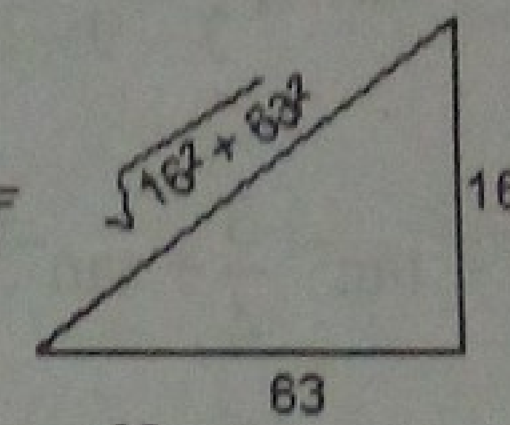
$= \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{2.\frac{1}{5}}{1-(\frac{1}{5})^2}$

$= \tan^{-1}\frac{3}{4} - \tan^{-1}(\frac{2}{5}\times\frac{25}{25-1})$

$= \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{10}{24} = \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{5}{12}$

$= \tan^{-1}\frac{\frac{3}{4}-\frac{5}{12}}{1+\frac{3}{4}\frac{5}{12}} = \tan^{-1}(\frac{36-20}{48}\times\frac{48}{48+15})$

$= \tan^{-1}\frac{16}{63} = \cos^{-1}\frac{63}{\sqrt{16^2+63^2}}$



[পাশের চিত্র হতে]

$= \cos^{-1}\frac{63}{\sqrt{256+3969}} = \cos^{-1}\frac{63}{\sqrt{4225}}$

$= \cos^{-1}\frac{63}{65} =$ R.H.S.

6(b) $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{4} = \tan^{-1}\frac{32}{43}$

L.H.S. $= 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{4}$

$= \tan^{-1}\frac{2.\frac{1}{5}}{1-(\frac{1}{5})^2} + \tan^{-1}\frac{1}{4}$

$= \tan^{-1}(\frac{2}{5}\times\frac{25}{25-1}) + \tan^{-1}\frac{1}{4}$

$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{1}{4} = \tan^{-1}\frac{\frac{5}{12}+\frac{1}{4}}{1-\frac{5}{12}.\frac{1}{4}}$

$= \tan^{-1}(\frac{20+12}{48}\times\frac{48}{48-5})$

$= \tan^{-1}\frac{32}{43} =$ R.H.S.

6(c) $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$ **[রা.'০৩]**

L.H.S.$= 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{2.\frac{1}{3}}{1-(\frac{1}{3})^2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}(\frac{2}{3}\times\frac{9}{9-1}) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}(\frac{2}{3}\times\frac{9}{9-1}) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}.\frac{1}{7}}$

$= \tan^{-1}(\frac{21+4}{28}\times\frac{28}{28-3}) = \tan^{-1}\frac{25}{25}$

$= \tan^{-1}1 = \frac{\pi}{4}$ = R.H.S. (Proved)

6(d) $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

L.H.S. $= 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{8}$

$= \tan^{-1}\frac{2.\frac{1}{5}}{1-(\frac{1}{5})^2} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{2.\frac{1}{8}}{1-(\frac{1}{8})^2}$

$= \tan^{-1}(\frac{2}{5}\times\frac{25}{25-1}) + \tan^{-1}\frac{1}{7} + \tan^{-1}(\frac{2}{8}\times\frac{64}{64-1})$

$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{16}{63}$

$= \tan^{-1}\frac{\frac{5}{12}+\frac{1}{7}}{1-\frac{5}{12}.\frac{1}{7}} + \tan^{-1}\frac{16}{63}$

$= \tan^{-1}(\frac{35+12}{84}\times\frac{84}{84-5}) + \tan^{-1}\frac{16}{63}$

$= \tan^{-1}\frac{47}{79} + \tan^{-1}\frac{16}{63} = \tan^{-1}\frac{\frac{47}{79}+\frac{16}{63}}{1-\frac{47}{79}\frac{16}{63}}$

$= \tan^{-1}(\frac{2961+1264}{4977}\times\frac{4977}{4977-752})$

$= \tan^{-1}\frac{4225}{4225} = \tan^{-1}1 = \frac{\pi}{4}$

6(e) $\tan(2\tan^{-1}x) = 2\tan(\tan^{-1}x + \tan^{-1}x^3)$ [রা.'০১; ব.'০৫; কু.'০৭]

R.H.S. $= 2\tan(\tan^{-1}x + \tan^{-1}x^3)$

$= 2\tan.\tan^{-1}\frac{x+x^3}{1-x.x^3} = 2\frac{x(1+x^2).}{(1+x^2)(1-x^2)}$

$= \frac{2x}{1-x^2} = \tan\tan^{-1}\frac{2x}{1-x^2}$

$= \tan(2\tan^{-1}x)$ = L.H.S. = (Proved)

7(a) $4\sin^{-1}\frac{1}{\sqrt{17}} - \tan^{-1}\frac{79}{401} = \frac{\pi}{4}$

Proof : $4\sin^{-1}\frac{1}{\sqrt{17}} = 4\tan^{-1}\frac{1}{\sqrt{(\sqrt{17})^2-1^2}}$

$= 4\tan^{-1}\frac{1}{4} = 2\tan^{-1}\frac{2\frac{1}{4}}{1-(\frac{1}{4})^2}$

$= 2\tan^{-1}(\frac{1}{2}\times\frac{16}{16-1}) = 2\tan^{-1}\frac{8}{15}$

$= \tan^{-1}\frac{2\frac{8}{15}}{1-(\frac{8}{15})^2} = \tan^{-1}(\frac{16}{15}\times\frac{225}{225-64})$

$= \tan^{-1}\frac{16.15}{161} = \tan^{-1}\frac{240}{161}$

L.H.S. $= 4\sin^{-1}\frac{1}{\sqrt{17}} - \tan^{-1}\frac{29}{40}$

$= \tan^{-1}\frac{240}{161} - \tan^{-1}\frac{79}{401} = \tan^{-1}\frac{\frac{240}{161}-\frac{79}{401}}{1+\frac{240}{161}.\frac{79}{401}}$

$= \tan^{-1}(\frac{96240-12719}{64561}\times\frac{64561}{64561+18960})$

$= \tan^{-1}\frac{83521}{83521} = \tan^{-1}1 = \frac{\pi}{4} = $ R.H.S.

7(b) $\cos(2\tan^{-1}\frac{1}{7}) = \sin(4\tan^{-1}\frac{1}{2})$

[রা.'০৩,'১৩]

L.H.S. $= \cos(2\tan^{-1}\frac{1}{7}) = \cos\cos^{-1}\frac{1-(\frac{1}{7})^2}{1+(\frac{1}{7})^2}$

$= \frac{49-1}{49+1} = \frac{48}{50} = \frac{24}{25}$

R.H.S. $= \sin(4\tan^{-1}\frac{1}{2}) = \sin 2(2\tan^{-1}\frac{1}{2})$

$= 2\sin(2\tan^{-1}\frac{1}{2})\cos(2\tan^{-1}\frac{1}{2})$

$= 2\sin(\sin^{-1}\frac{2\frac{1}{2}}{1+(\frac{1}{2})^2}).\cos\cos^{-1}\frac{1-(\frac{1}{2})^2}{1+(\frac{1}{2})^2}$

$= 2.1\times\frac{4}{4+1}.\frac{4-1}{4}\times\frac{4}{4+1} = 2.\frac{4}{5}.\frac{3}{5} = \frac{24}{25}$

$\therefore$ L.H.S. = R.H.S. (Proved)

8(a) $2\tan^{-1}\{\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\} = \cos^{-1}\frac{b+a\cos\theta}{a+b\cos\theta}$

[ঢা.'০৫; কু.'০৮; দি.'০৯]

L.H.S. $= 2\tan^{-1}\{\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\}$

$= 2\tan^{-1}\{\frac{\sqrt{a-b}\sin\frac{\theta}{2}}{\sqrt{a+b}\cos\frac{\theta}{2}}\}$

$= \cos^{-1}\frac{1-\frac{(a-b)\sin^2\frac{\theta}{2}}{(a+b)\cos^2\frac{\theta}{2}}}{1+\frac{(a-b)\sin^2\frac{\theta}{2}}{(a+b)\cos^2\frac{\theta}{2}}}$

$= \cos^{-1}\frac{(a+b)\cos^2\frac{\theta}{2}-(a-b)\sin^2\frac{\theta}{2}}{(a+b)\cos^2\frac{\theta}{2}+(a-b)\sin^2\frac{\theta}{2}}$

$= \cos^{-1}\frac{a(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2})+b(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2})}{a(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2})+b(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2})}$

$= \cos^{-1}\frac{a\cos\theta+b}{a+b\cos\theta} = \cos^{-1}\frac{b+a\cos\theta}{a+b\cos\theta}$

8(b) $2\tan^{-1}\left(\sqrt{\frac{a}{b}}\tan\frac{\theta}{2}\right)$

$= \sin^{-1}\frac{2\sqrt{ab}\sin\theta}{b+a+(b-a)\cos\theta}$

[ঢা.'০৫; কু.'০৮; দি.'০৯; সি.'১৩; টেক্সটাইল'০০-০১]

L.H.S.

$= 2\tan^{-1}\left(\sqrt{\frac{a}{b}}\tan\frac{\theta}{2}\right) = \sin^{-1}\frac{2\frac{\sqrt{a}}{\sqrt{b}}\tan\frac{\theta}{2}}{1+\frac{a}{b}\tan^2\frac{\theta}{2}}$

$= \sin^{-1}\frac{\frac{2\sqrt{a}\sin\theta/2}{\sqrt{b}\cos\theta/2}}{1+\frac{a\sin^2\theta/2}{b\cos^2\theta/2}}$

$$= \sin^{-1}\left(\frac{2\sqrt{a}\sin\theta/2}{\sqrt{b}\cos\theta/2} \times \frac{b\cos^2\theta/2}{b\cos^2\theta/2 + a\sin^2\theta/2}\right)$$

$$= \sin^{-1}\frac{2\sqrt{a}\sin\theta/2 . \sqrt{b}\cos\theta/2}{b\frac{1}{2}(1+\cos\theta) + a\frac{1}{2}(1-\cos\theta)}$$

$$= \sin^{-1}\frac{2\sqrt{ab}(2\sin\theta/2 . \cos\theta/2)}{b+b\cos\theta+a-a\cos\theta)}$$

$$= \sin^{-1}\frac{2\sqrt{ab}\sin\theta}{b+a+(b-a)\cos\theta} = \text{R.H.S.}$$

8(c) $2\tan^{-1}\left\{\tan\frac{\alpha}{2}\ \tan(\frac{\pi}{4}-\frac{\beta}{2})\right\}$

$$= \tan^{-1}\frac{\sin\alpha\ \cos\beta}{\sin\beta+\cos\alpha}$$

$$\text{L.H.S.} = 2\tan^{-1}\{\tan\frac{\alpha}{2}\ \tan(\frac{\pi}{4}-\frac{\beta}{2})\}$$

$$= 2\tan^{-1}\left\{\frac{\sin\frac{\alpha}{2}\sin(\frac{\pi}{4}-\frac{\beta}{2})}{\cos\frac{\alpha}{2}\cos(\frac{\pi}{4}-\frac{\beta}{2})}\right\}$$

$$= \tan^{-1}\frac{2\dfrac{\sin\frac{\alpha}{2}\sin(\frac{\pi}{4}-\frac{\beta}{2})}{\cos\frac{\alpha}{2}\cos(\frac{\pi}{4}-\frac{\beta}{2})}}{1-\dfrac{\sin^2\frac{\alpha}{2}\sin^2(\frac{\pi}{4}-\frac{\beta}{2})}{\cos^2\frac{\alpha}{2}\cos^2(\frac{\pi}{4}-\frac{\beta}{2})}}$$

$$= \tan^{-1}\frac{2\sin\frac{\alpha}{2}\sin(\frac{\pi}{4}-\frac{\beta}{2})\cos\frac{\alpha}{2}\cos(\frac{\pi}{4}-\frac{\beta}{2})}{\cos^2\frac{\alpha}{2}\cos^2(\frac{\pi}{4}-\frac{\beta}{2})-\sin^2\frac{\alpha}{2}\sin^2(\frac{\pi}{4}-\frac{\beta}{2})}$$

এখন , $2\sin\frac{\alpha}{2}\sin(\frac{\pi}{4}-\frac{\beta}{2}\cos\frac{\alpha}{2}\cos(\frac{\pi}{4}-\frac{\beta}{2})$

$$= \frac{1}{2}\sin\alpha\sin 2(\frac{\pi}{4}-\frac{\beta}{2}) = \frac{1}{2}\sin\alpha\ \sin(\frac{\pi}{2}-\beta)$$

$= \frac{1}{2}\sin\alpha\cos\beta$ এবং

$$\cos^2\frac{\alpha}{2}\cos^2(\frac{\pi}{4}-\frac{\beta}{2})-\sin^2\frac{\alpha}{2}\sin^2(\frac{\pi}{4}-\frac{\beta}{2})$$

$$= \{\cos\frac{\alpha}{2}\cos(\frac{\pi}{4}-\frac{\beta}{2})+\sin\frac{\alpha}{2}\sin(\frac{\pi}{4}-\frac{\beta}{2})\}$$

$$\{\cos\frac{\alpha}{2}\cos(\frac{\pi}{4}-\frac{\beta}{2})-\sin\frac{\alpha}{2}\sin(\frac{\pi}{4}-\frac{\beta}{2})\}$$

$$= \cos(\frac{\alpha}{2}+\frac{\beta}{2}-\frac{\pi}{4}).\cos(\frac{\alpha}{2}-\frac{\beta}{2}+\frac{\pi}{4})$$

$$= \cos\{\frac{\alpha}{2}+(\frac{\beta}{2}-\frac{\pi}{4})\}.\cos\{\frac{\alpha}{2}-(\frac{\beta}{2}-\frac{\pi}{4})\}$$

$$= \frac{1}{2}\{\cos 2.\frac{\alpha}{2}+\cos 2.(\frac{\beta}{2}-\frac{\pi}{4})\}$$

$$= \frac{1}{2}\{\cos\alpha+\cos(\beta-\frac{\pi}{2})\}$$

$$= \frac{1}{2}\{\cos\alpha+\sin\beta)$$

$$\therefore \text{L.H.S.} = \tan^{-1}\frac{\frac{1}{2}\sin\alpha\cos\beta}{\frac{1}{2}(\sin\beta+\cos\alpha)}$$

$$= \tan^{-1}\frac{\sin\alpha\cos\beta}{\sin\beta+\cos\alpha} = \text{R.H.S.}$$

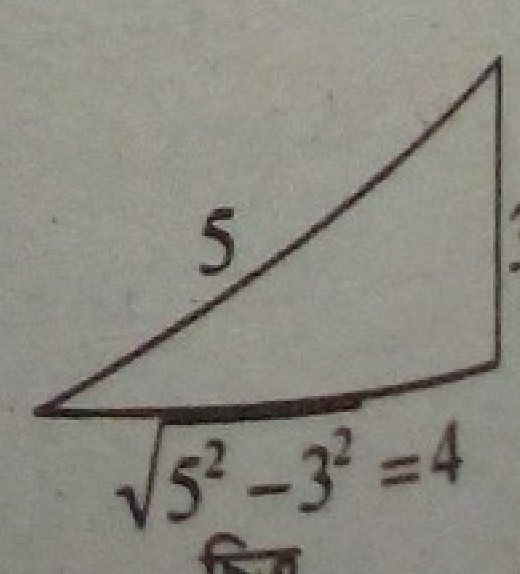
(Proved)

9(a) $\sin^{-1}\frac{3}{5}+\frac{1}{2}\cos^{-1}\frac{5}{13}-\cot^{-1}2 = \tan^{-1}\frac{28}{29}$

[রা.'০০,'০৫; কু.'১২; ব., সি., চ.'১৩]

পাশের চিত্র হতে আমরা পাই ,

$$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$$



চিত্র

মনে করি, $\frac{1}{2}\cos^{-1}\frac{5}{13} = \theta$

$\cos 2\theta = \frac{5}{13}$

ন, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$

$= \frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\theta}{\sqrt{1-\cos^2 2\theta}}$

$= \frac{1-\frac{5}{13}}{\sqrt{1-\frac{25}{169}}} = \frac{\frac{13-5}{13}}{\sqrt{\frac{169-25}{169}}} = \frac{\frac{8}{13}}{\frac{12}{13}}$.

$= \frac{8}{13} \times \frac{13}{12} = \frac{2}{3}$

$\theta = \tan^{-1}\frac{2}{3} \Rightarrow \frac{1}{2}\cos^{-1}\frac{5}{13} = \tan^{-1}\frac{2}{3}$

ন, L.H.S. $= \sin^{-1}\frac{3}{5} + \frac{1}{2}\cos^{-1}\frac{5}{13} - \cot^{-1}2$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3} - \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}.\frac{2}{3}} - \tan^{-1}\frac{1}{2}$

$= \tan^{-1}(\frac{9+8}{12} \times \frac{12}{12-6}) - \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{17}{6} - \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{\frac{17}{6}-\frac{1}{2}}{1+\frac{17}{6}.\frac{1}{2}} = \tan^{-1}(\frac{34-6}{12} \times \frac{12}{12+17})$

$= \tan^{-1}\frac{28}{29} =$ R.H.S. (Proved)

9(b) $\tan\{\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\}$

$= \frac{2x}{1-x^2}$

L.H.S. $= \tan\{\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\}$

$= \tan\{\tan^{-1}x + \tan^{-1}x) = \tan(2\tan^{-1})$

$= \tan\tan^{-1}\frac{2x}{1-x^2} = \frac{2x}{1-x^2} =$ R.H.S.

9(c) $\frac{1}{2}\cos^{-1}\frac{3}{5} = \frac{\pi}{4} - \tan^{-1}\frac{1}{3}$ [প্র.ভ.প.১৩]

R.H.S. $= \frac{\pi}{4} - \tan^{-1}\frac{1}{3} = \tan^{-1}(1) - \tan^{-1}\frac{1}{3}$

$= \tan^{-1}\frac{1-\frac{1}{3}}{1+1.\frac{1}{3}} = \tan^{-1}(\frac{2}{3}\times\frac{3}{4}) = \tan^{-1}\frac{1}{2}$

$= \frac{1}{2}.2\tan^{-1}\frac{1}{2} = \frac{1}{2}.\cos^{-1}\frac{1-\frac{1}{4}}{1+\frac{1}{4}}$

$= \frac{1}{2}\cos^{-1}(\frac{3}{4}\times\frac{4}{5}) = \frac{1}{2}\cos^{-1}\frac{3}{5} =$ L.H.S.

10(a) $\cos^{-1}x = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$

$= 2\sin^{-1}\sqrt{\frac{1-x}{2}}$ [কুয়েট'০৪-০৫]

মনে করি, $\cos^{-1}x = \theta$. তাহলে $\cos\theta = x$

$\Rightarrow 1 - 2\sin^2\frac{\theta}{2} = x \Rightarrow 2\sin^2\frac{\theta}{2} = 1 - x$

$\Rightarrow \sin^2\frac{\theta}{2} = \frac{1-x}{2} \Rightarrow \sin\frac{\theta}{2} = \sqrt{\frac{1-x}{2}}$

$\Rightarrow \frac{\theta}{2} = \sin^{-1}\sqrt{\frac{1-x}{2}} \Rightarrow \theta = 2\sin^{-1}\sqrt{\frac{1-x}{2}}$

$\therefore \cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}}$ ……… (1)

আবার, $\cos\theta = x$

$\Rightarrow 2\cos^2\frac{\theta}{2} - 1 = x \Rightarrow 2\cos^2\frac{\theta}{2} = 1 + x$

$\Rightarrow \cos^2\frac{\theta}{2} = \frac{1+x}{2} \Rightarrow \cos\frac{\theta}{2} = \sqrt{\frac{1+x}{2}}$

$\Rightarrow \frac{\theta}{2} = \cos^{-1}\sqrt{\frac{1+x}{2}} \Rightarrow \theta = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$

$\therefore \cos^{-1}x = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$ ……… (2)

∴ (1) ও (2) হতে পাই,

$\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$

10(b) $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\frac{1-x}{1+x}$

[কু.'০৮; টেস্টটাইম' ০৮-০৯]

L.H.S. $= \tan^{-1}\sqrt{x} = \frac{1}{2}.2\tan^{-1}\sqrt{x}$

$= \frac{1}{2}\cos^{-1}\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2} = \frac{1}{2}\cos^{-1}\frac{1-x}{1+x} =$ R.H.S.

10(c) $\cos^{-1}\left\{1+\cos(2\tan^{-1}\frac{x}{a})\right\}^{\frac{1}{2}}$

$= \sin^{-1}\sqrt{\frac{x^2-a^2}{x^2+a^2}}$

L.H.S. $= \cos^{-1}\{1+\cos(2\tan^{-1}\frac{x}{a})\}^{\frac{1}{2}}$

$= \sin^{-1}\sqrt{1-\{1+\cos(2\tan^{-1}\frac{x}{a})\}}$

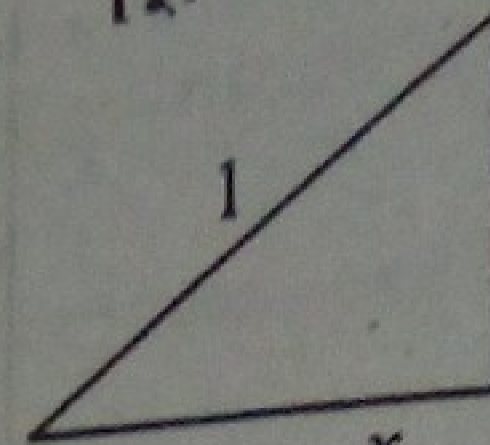
$= \sin^{-1}\sqrt{1-1-\cos(2\tan^{-1}\frac{x}{a})}$

$= \sin^{-1}\sqrt{-\cos\cos^{-1}\frac{1-(\frac{x}{a})^2}{1+(\frac{x}{a})^2}}$

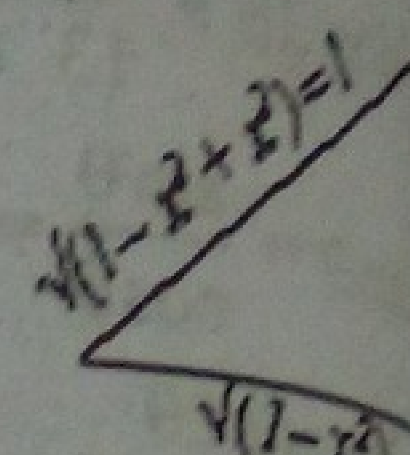
$= \sin^{-1}\sqrt{-\frac{a^2-x^2}{a^2+x^2}} = \sin^{-1}\sqrt{\frac{x^2-a^2}{x^2+a^2}} =$ R.H.S.

(a) $\sin\cot^{-1}\tan\cos^{-1}x = x$

[কু.'০৫; ঢা.'১২; ব.'০৯; সি.'০৭; রা.'০৮,'১১; ...]



চিত্র -1



চিত্র - 2

L.H.S. $= \sin\cot^{-1}\tan\cos^{-1}x$

$= \sin\cot^{-1}\tan\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ [চিত্র -1 হতে]

$= \sin\cot^{-1}\frac{\sqrt{1-x^2}}{x}$ [চিত্র -2 হতে]

$= \sin\sin^{-1}\frac{x}{1} = \sin\sin^{-1}x = x =$ R.H.S.

11(b) $\cot\cos^{-1}\sin\tan^{-1}x = x$

[ব.'০১,'০২,'১১ ; চ.'০৩; সি.'০...]

L.H.S. $= \cot\cos^{-1}\sin\tan^{-1}x$

$= \cot\cos^{-1}\sin\sin^{-1}\frac{x}{\sqrt{1+x^2}}$

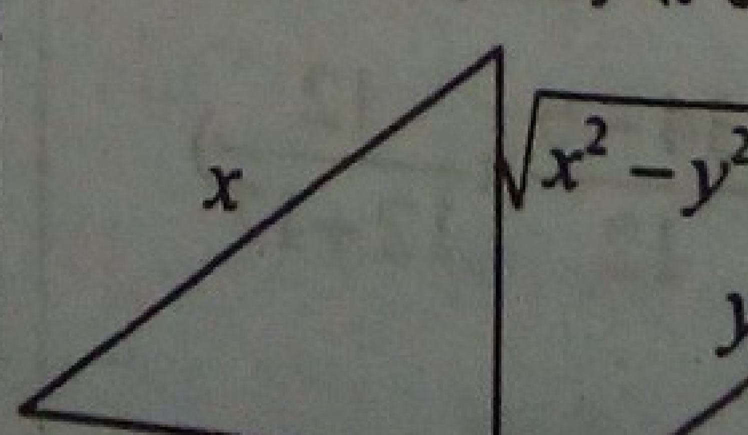
$= \cot\cos^{-1}\frac{x}{\sqrt{1+x^2}}$

$= \cot\cot^{-1}\frac{x}{\sqrt{(\sqrt{1+x^2})^2-x^2}} = \cot\cot^{-1}\frac{x}{1}$

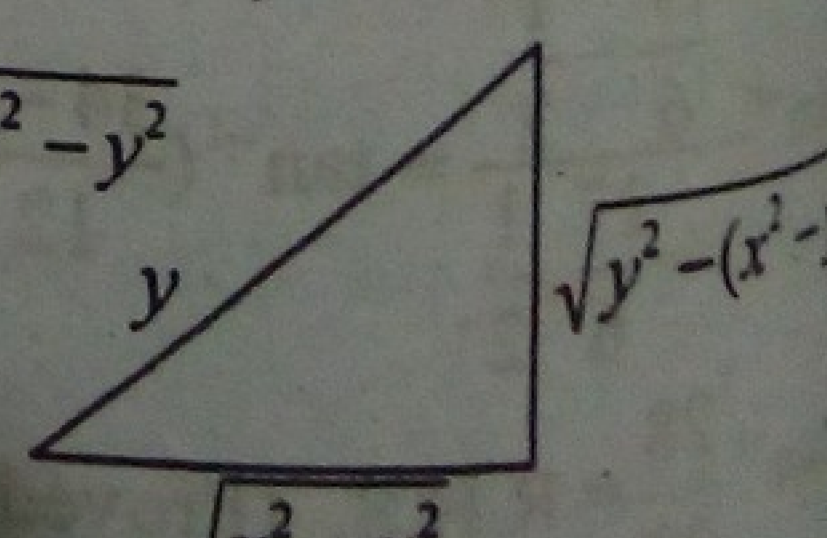
$= x =$ R.H.S. (Proved)

11(c) $\sin\cos^{-1}\tan\sec^{-1}\frac{x}{y} = \frac{\sqrt{2y^2-x^2}}{y}$

[য.'০১,'০৯; ব.'০৪ ; সি.'০৫, দি.'০৯; ঢা.'১...]



চিত্র–১



চিত্র–২

L.H.S. $= \sin\cos^{-1}\tan\sec^{-1}\frac{x}{y}$

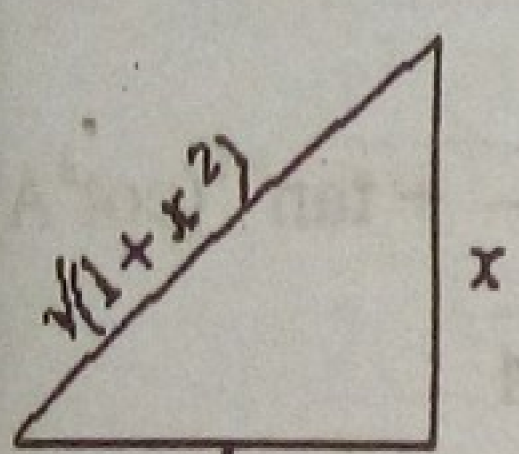
$= \sin\cos^{-1}\tan\tan^{-1}\dfrac{\sqrt{x^2-y^2}}{y}$ [চিত্র-১ হতে]

$= \sin\cos^{-1}\dfrac{\sqrt{x^2-y^2}}{y}$

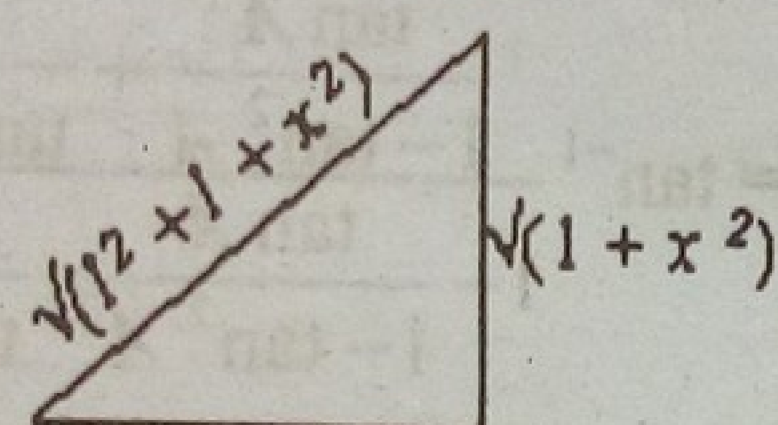
$= \sin\sin^{-1}\dfrac{\sqrt{2y^2-x^2}}{y}$ [চিত্র-২ হতে]

$= \dfrac{\sqrt{2y^2-x^2}}{y}$ = R.H.S. (Proved)

11(d) $\sin\cot^{-1}\cos\tan^{-1}x = \sqrt{\dfrac{1+x^2}{2+x^2}}$



চিত্র - 1



চিত্র -2

L.H.S.= $\sin\cot^{-1}\cos\tan^{-1}x$

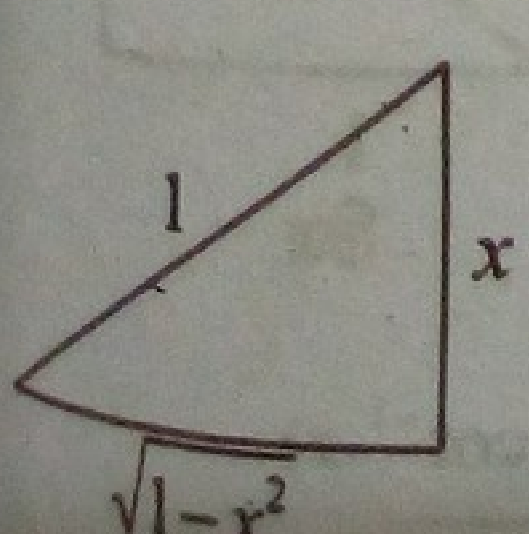
$= \sin\cot^{-1}\cos\cos^{-1}\dfrac{1}{\sqrt{1+x^2}}$ [চিত্র -1 হতে]

$= \sin\cot^{-1}\dfrac{1}{\sqrt{1+x^2}}$

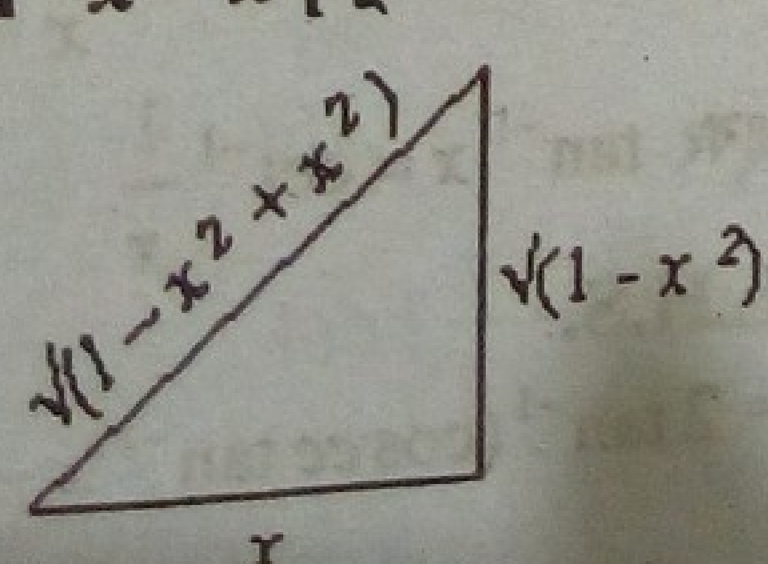
$= \sin\sin^{-1}\dfrac{\sqrt{1+x^2}}{\sqrt{1+x^2+1}}$ [চিত্র -2 হতে]

$= \dfrac{\sqrt{1+x^2}}{\sqrt{1+x^2+1}} = \sqrt{\dfrac{1+x^2}{2+x^2}}$ = R.H.S. (Proved)

11(e) $\cos\tan^{-1}\cot\sin^{-1}x = x$ [কু.'০৩; য.'১২]



চিত্র -1



চিত্র -2

L.H.S.= $\cos\tan^{-1}\cot\sin^{-1}x$

$= \cos\tan^{-1}\cot\cot^{-1}\dfrac{\sqrt{1-x^2}}{x}$ [চিত্র -1 হতে]

$= \cos\tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$

$= \cos\cos^{-1}\dfrac{x}{\sqrt{1-x^2+x^2}}$ [চিত্র -2 হতে]

$= \dfrac{x}{\sqrt{1}} = x$ = R.H.S. (Proved).

11(f) $\cot\cos^{-1}\sin\tan^{-1}\dfrac{3}{4} = \dfrac{3}{4}$

[ঢা.'০২; প্র.ভ.প.'০৪]

L.H.S. $= \cot\cos^{-1}\sin\tan^{-1}\dfrac{3}{4}$

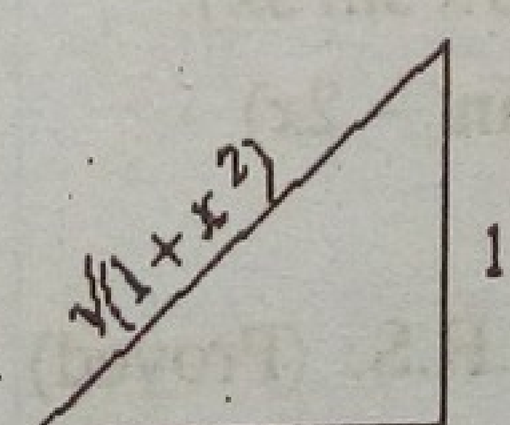
$= \cot\cos^{-1}\sin\sin^{-1}\dfrac{3}{\sqrt{4^2+3^2}}$

$= \cot\cos^{-1}\dfrac{3}{\sqrt{5^2}} = \cot\cos^{-1}\dfrac{3}{5}$

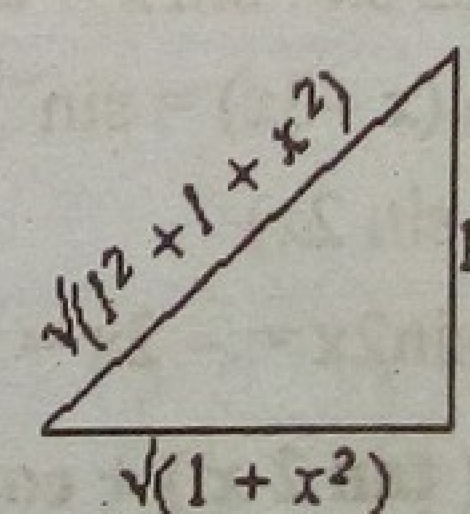
$= \cot\cot^{-1}\dfrac{3}{\sqrt{5^2-3^2}} = \dfrac{3}{4}$ = R.H.S. (Proved)

11(g) $\cos\tan^{-1}\sin\cot^{-1}x = \sqrt{\dfrac{1+x^2}{2+x^2}}$

[কু.'০২]



চিত্র - 1



চিত্র -2

L.H.S. $= \cos\tan^{-1}\sin\cot^{-1}x$

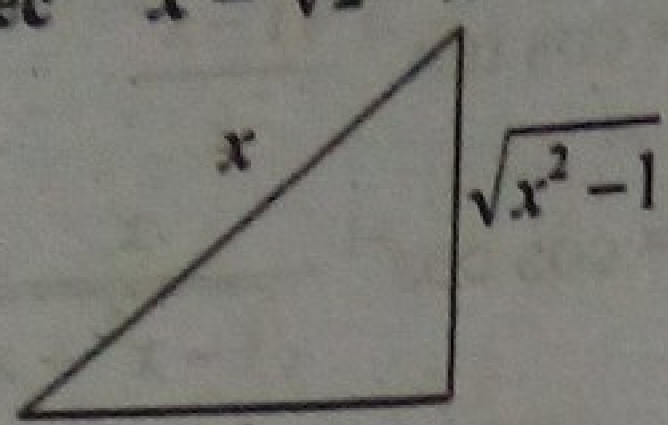
$= \cos\tan^{-1}\sin\sin^{-1}\dfrac{1}{\sqrt{1+x^2}}$ [চিত্র -1 হতে]

$= \cos\tan^{-1}\dfrac{1}{\sqrt{1+x^2}}$

$= \cos\cos^{-1}\dfrac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$ [চিত্র -2 হতে]

$= \sqrt{\dfrac{1+x^2}{2+x^2}}$ = R.H.S. (Proved)

12(a) $\sin \cos^{-1} \tan \sec^{-1} x = \sqrt{2-x^2}$



চিত্র

L.H.S.

$= \sin \cos^{-1} \tan \sec^{-1} x$

$= \sin \cos^{-1} \tan \tan^{-1} \dfrac{\sqrt{x^2-1}}{1}$ [চিত্র -1 হতে]

$= \sin \cos^{-1} \sqrt{x^2-1}$

$= \sin \sin^{-1} \sqrt{1-\left(\sqrt{x^2-1}\right)^2}$

$= \sqrt{1-x^2+1} = \sqrt{2-x^2}$ = R.H.S. (Proved)

12(b) $\sin^{-1}(-\cos x) + \sin^{-1}(\cos 3x) = -2x$

[ঢা.'০৩; সি.'০৯]

L.H.S. $= \sin^{-1}(-\cos x) + \sin^{-1}(\cos 3x)$

$= \sin^{-1}(\cos 3x) - \sin^{-1}(\cos x)$

$= \sin^{-1}\{\cos 3x \sqrt{1-\cos^2 x} - \cos x \sqrt{1-\cos^2 3x}\}$

$= \sin^{-1}(\cos 3x \sqrt{\sin^2 x} - \cos x \sqrt{\sin^2 3x})$

$= \sin^{-1}(\cos 3x \,.\, \sin x - \cos x \sin 3x)$

$= \sin^{-1} \sin (x-3x) = \sin^{-1} \sin(-2x)$

$= \sin^{-1}(-\sin 2x)$

$= -\sin^{-1} \sin 2x = -2x$ = R.H.S. (Proved)

(c) $\cot^{-1}(\tan 2x) + \cot^{-1}(-\tan 3x) = x$

[ব.'১১]

L.H.S. $= \cot^{-1}(\tan 2x) + \cot^{-1}(-\tan 3x)$

$= \cot^{-1}(\tan 2x) - \cot^{-1}(\tan 3x)$

$= \cot^{-1}\{\cot(\frac{\pi}{2} - 2x)\} - \cot^{-1}\{\cot(\frac{\pi}{2} - 3x)\}$

$= \frac{\pi}{2} - 2x - \frac{\pi}{2} + 3x = x$ = R.H.S. (Proved)

12(d) $\tan^{-1}(\frac{1}{2}\tan 2A) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A) = 0$

L.H.S. $= \tan^{-1}(\frac{1}{2}\tan 2A) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$

$= \tan^{-1}(\frac{1}{2}\dfrac{2\tan A}{1-\tan^2 A}) + \tan^{-1}\left(\dfrac{1}{\tan A}\right) + \tan^{-1}(\cot^3 A)$

$= \tan^{-1}(\dfrac{\tan A}{1-\tan^2 A}) + \tan^{-1}\left(\dfrac{1}{\tan A}\right) + \tan^{-1}(\cot^3 A)$

$= \tan^{-1} \dfrac{\dfrac{\tan A}{1-\tan^2 A} + \dfrac{1}{\tan A}}{1 - \dfrac{\tan A}{1-\tan^2 A} \times \dfrac{1}{\tan A}} + \tan^{-1}(\cot^3 A)$

$= \tan^{-1} \dfrac{\tan^2 A + 1 - \tan^2 A}{\tan A - \tan^3 A - \tan A} + \tan^{-1}(\cot^3 A)$

$= \tan^{-1}(\dfrac{1}{-\tan^3 A}) + \tan^{-1}(\cot^3 A)$

$= \tan^{-1}(-\cot^3 A) + \tan^{-1}(\cot^3 A)$

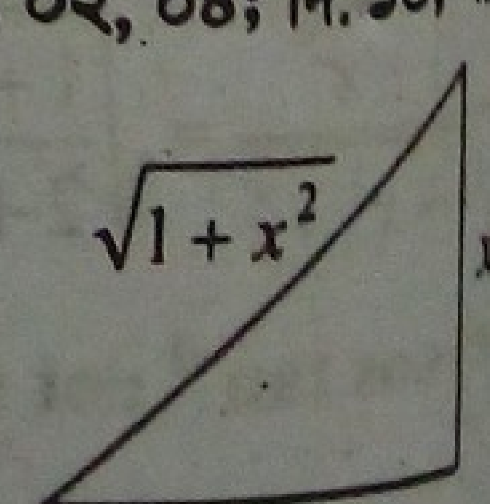
$= -\tan^{-1}(\cot^3 A) + \tan^{-1}(\cot^3 A)$

$= 0$ = R.H.S. (Proved)

12(e) $2\tan^{-1}(\text{cosec} \tan^{-1} x - \tan \cot^{-1} x) = \tan^{-1} x$ [রা.'০২,'১১; সি.'০২,'০৪; দি.'১০; ব.'১১]

চিত্র হতে পাই,

$\tan^{-1} x = \text{cosec}^{-1} \dfrac{\sqrt{1+x^2}}{x}$



চিত্র

এবং $\tan^{-1} x = \cot^{-1} \dfrac{1}{x}$

L.H.S.

$= 2\tan^{-1}(\text{cosec} \tan^{-1} x - \tan \cot^{-1} x)$

$= 2\tan^{-1}(\text{cosec}\, \text{cosec}^{-1} \dfrac{\sqrt{1+x^2}}{x} - \tan \tan^{-1} \dfrac{1}{x}$

$= 2\tan^{-1}\left(\frac{\sqrt{1+x^2}}{x}-\frac{1}{x}\right) = 2\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

$= \tan^{-1}\dfrac{2\frac{\sqrt{1+x^2}-1}{x}}{1-\frac{(\sqrt{1+x^2}-1)^2}{x^2}}$

$= \tan^{-1}\dfrac{\frac{2(\sqrt{1+x^2}-1)}{x}}{\frac{x^2-(1+x^2-2\sqrt{1+x^2}+1)}{x^2}}$

$= \tan^{-1}\dfrac{\frac{2(\sqrt{1+x^2}-1)}{x}}{\frac{x^2-1-x^2+2\sqrt{1+x^2}-1}{x^2}}$

$= \tan^{-1}\dfrac{\frac{2(\sqrt{1+x^2}-1)}{x}}{\frac{2(\sqrt{1+x^2}-1)}{x^2}}$

$= \tan^{-1}\left\{\frac{2(\sqrt{1+x^2}-1)}{x}\times\frac{x^2}{2(\sqrt{1+x^2}-1)}\right\}$

$= \tan^{-1}x = \text{R.H.S. (Proved)}$

12(f) $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

মনে করি, $\cos^{-1}\frac{a}{b}=x \Rightarrow \frac{a}{b}=\cos x$

L.H.S. $= \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

$= \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$= \dfrac{\sin(\frac{\pi}{4}+\frac{x}{2})}{\cos(\frac{\pi}{4}+\frac{x}{2})}+\dfrac{\sin(\frac{\pi}{4}-\frac{x}{2})}{\cos(\frac{\pi}{4}-\frac{x}{2})}$

$= \dfrac{\sin(\frac{\pi}{4}+\frac{x}{2})\cos(\frac{\pi}{4}-\frac{x}{2})+\sin(\frac{\pi}{4}-\frac{x}{2})\cos(\frac{\pi}{4}+\frac{x}{2})}{\cos(\frac{\pi}{4}+\frac{x}{2})\cos(\frac{\pi}{4}-\frac{x}{2})}$

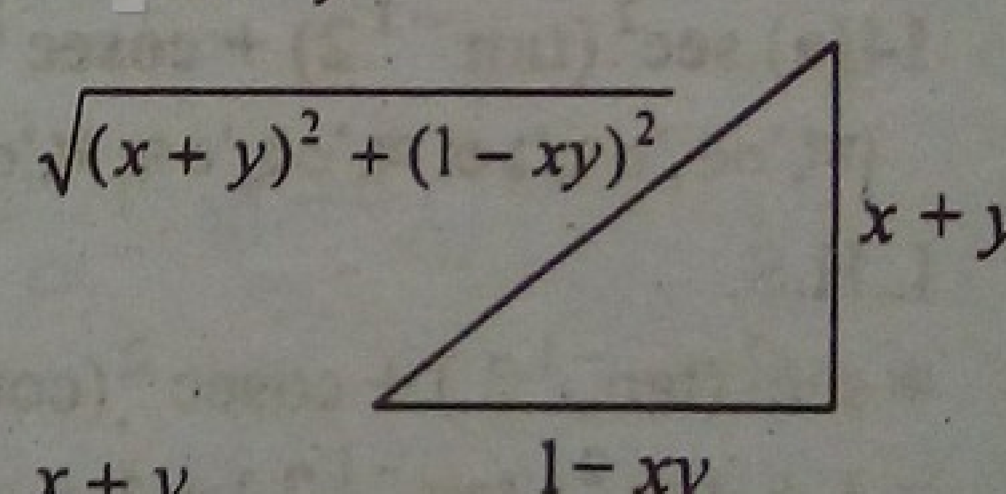
$= \dfrac{2\sin(\frac{\pi}{4}+\frac{x}{2}+\frac{\pi}{4}-\frac{x}{2})}{2\cos(\frac{\pi}{4}+\frac{x}{2})\cos(\frac{\pi}{4}-\frac{x}{2})} = \dfrac{2\sin 2.\frac{\pi}{4}}{\cos 2.\frac{\pi}{4}+\cos 2.\frac{x}{2}}$

$= \dfrac{2\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}+\cos x} = \dfrac{2}{0+\frac{a}{b}} = \dfrac{2b}{a} = \text{R.H.S.}$

13(a) $\tan^{-1}x+\tan^{-1}y = \sin^{-1}\dfrac{x+y}{\sqrt{(1+x^2)(1+y^2)}}$

L.H.S. $= \tan^{-1}x+\tan^{-1}y$

$= \tan^{-1}\dfrac{x+y}{1-xy}$



চিত্র

$= \sin^{-1}\dfrac{x+y}{\sqrt{(x+y)^2+(1-xy)^2}}$

$= \sin^{-1}\dfrac{x+y}{\sqrt{x^2+2xy+y^2+1-2xy+x^2y^2}}$

$= \sin^{-1}\dfrac{x+y}{\sqrt{x^2+y^2+1+x^2y^2}}$

$= \sin^{-1}\dfrac{x+y}{\sqrt{(1+x^2)+y^2(1+x^2)}}$

$= \sin^{-1}\dfrac{x+y}{\sqrt{(1+x^2)(1+y^2)}} = \text{R.H.S. (Proved)}$

(b) $\tan^{-1}x+\tan^{-1}y=\frac{1}{2}\sin^{-1}\frac{2(x+y)(1-xy)}{(1+x^2)(1+y^2)}$

L.H.S. $=\tan^{-1}x+\tan^{-1}y$

$=\tan^{-1}\frac{x+y}{1-xy}=\frac{1}{2}.2\tan^{-1}\frac{x+y}{1-xy}$

$=\frac{1}{2}\sin^{-1}\frac{2\frac{x+y}{1-xy}}{1+(\frac{x+y}{1-xy})^2}$

$=\frac{1}{2}\sin^{-1}\frac{2\frac{x+y}{1-xy}}{\frac{1-2xy+x^2y^2+x^2+2xy+y^2}{(1-xy)^2}}$

$=\frac{1}{2}\sin^{-1}\frac{2(x+y)(1-xy)}{1+x^2y^2+x^2+y^2}$

$=\frac{1}{2}\sin^{-1}\frac{2(x+y)(1-xy)}{(1+x^2)(1+y^2)}=$ R.H.S. (Proved)

14(a) $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=15$

[ঢা.'০৫,'০৭,'১৩; য.'০২,'০৭; ব.'০৫,'০৮,'১২; চ.'০৫]

L.H.S.

$=\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)$

$=1+\tan^2(\tan^{-1}2)+1+\cot^2(\cot^{-1}3)$

$=2+(\tan\tan^{-1}2)^2+(\cot\cot^{-1}3)^2$

$=2+2^2+3^2$

$=2+4+9=15=$ R.H.S. (Proved)

14(b) $\sin^2(\cos^{-1}\frac{1}{3})-\cos^2(\sin^{-1}\frac{1}{\sqrt{3}})=\frac{2}{9}$

[চ.'০৭; সি.'০২; য.'০৪; রা.'০৭; টেক্সটাইল'০৯-১০]

L.H.S. $=\sin^2(\cos^{-1}\frac{1}{3})-\cos^2(\sin^{-1}\frac{1}{\sqrt{3}})$

$=1-\cos^2(\cos^{-1}\frac{1}{3})-\{1-\sin^2(\sin^{-1}\frac{1}{\sqrt{3}})\}$

$=1-(\cos\cos^{-1}\frac{1}{3})^2-1+(\sin\sin^{-1}\frac{1}{\sqrt{3}})^2$

$=-(\frac{1}{3})^2+(\frac{1}{\sqrt{3}})^2=-\frac{1}{9}+\frac{1}{3}$

$=\frac{-1+3}{9}=\frac{2}{9}=$ R.HS. (Proved)

14(c) $\sec^2(\tan^{-1}4)+\tan^2(\sec^{-1}3)=25$

[ঢা.'০৩; কু.'১৩; রা.'১৩; চুয়েট'১১-১২]

L.H.S. $=\sec^2(\tan^{-1}4)+\tan^2(\sec^{-1}3)$

$=1+(\tan\tan^{-1}4)^2+(\sec\sec^{-1}3)^2-1$

$=4^2+3^2=16+9=25=$ R.H.S. (Proved)

14(d) $\sec^2(\cot^{-1}3)+\operatorname{cosec}^2(\tan^{-1}2)=2\frac{13}{36}$

L.H.S. $=\sec^2(\cot^{-1}3)+\operatorname{cosec}^2(\tan^{-1}2)$

$=\sec^2(\tan^{-1}\frac{1}{3})+\operatorname{cosec}^2(\cot^{-1}\frac{1}{2})$

$=1+\tan^2(\tan^{-1}\frac{1}{3})+1+\cot^2(\cot^{-1}\frac{1}{2})$

$=2+(\tan\tan^{-1}\frac{1}{3})^2+(\cot\cot^{-1}\frac{1}{2})^2$

$=2+(\frac{1}{3})^2+(\frac{1}{2})^2$

$=2+\frac{1}{9}+\frac{1}{4}=2+\frac{4+9}{36}=2\frac{13}{36}$

14(e) $\operatorname{cosec}^2(\tan^{-1}\frac{1}{2})-3\sec^2(\cot^{-1}\sqrt{3})=1$

[চ.'০১,'০৫]

L.H.S. $=\operatorname{cosec}^2(\tan^{-1}\frac{1}{2})-3\sec^2(\cot^{-1}\sqrt{3})$

$=\operatorname{cosec}^2(\cot^{-1}2)-3\sec^2(\tan^{-1}\frac{1}{\sqrt{3}})$

$=1+\cot^2(\cot^{-1}2)-3\{1+\tan^2(\tan^{-1}\frac{1}{\sqrt{3}})\}$

$=1+(\cot\cot^{-1}2)^2-3-3(\tan\tan^{-1}\frac{1}{\sqrt{3}})^2$

$=-2+2^2-3.\frac{1}{3}=-2+4-1=1=$ R.H.S

15(a) $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\theta$ হলে দেখাও যে,

$\frac{x^2}{a^2}-\frac{2xy\cos\theta}{ab}+\frac{y^2}{b^2}=\sin^2\theta$ [য.'০৩,'১৩; ঢা.'০৬,'১১; রা.'০৬,'১০; ব.'০৮; চ.'০৯; কু.'০৯; সি.'১১]

প্রমাণ ঃ দেওয়া আছে, $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\theta$

$$\Rightarrow \cos^{-1}\{\frac{x}{a}.\frac{y}{b}-\sqrt{1-\left(\frac{x}{a}\right)^2}\sqrt{1-\left(\frac{y}{b}\right)^2}\}=\theta$$

$$\Rightarrow \frac{xy}{ab}-\sqrt{1-\left(\frac{x}{a}\right)^2}\sqrt{1-\left(\frac{y}{b}\right)^2}=\cos\theta$$

$$\Rightarrow \frac{xy}{ab}-\cos\theta=\sqrt{1-\left(\frac{x}{a}\right)^2}\sqrt{1-\left(\frac{y}{b}\right)^2}$$

উভয় পক্ষকে বর্গ করে আমরা পাই,

$$\frac{x^2y^2}{a^2b^2}-2\frac{xy}{ab}\cos\theta+\cos^2\theta=(1-\frac{x^2}{a^2})(1-\frac{y^2}{b^2})$$

$$\Rightarrow \frac{x^2y^2}{a^2b^2}-2\frac{xy}{ab}\cos\theta+\cos^2\theta$$

$$=1-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{x^2y^2}{a^2b^2}-\cos^2\theta$$

$$\Rightarrow \frac{x^2}{a^2}-\frac{2xy\cos\theta}{ab}+\frac{y^2}{b^2}=1-\cos^2\theta$$

$$\therefore \frac{x^2}{a^2}-\frac{2xy\cos\theta}{ab}+\frac{y^2}{b^2}=\sin^2\theta \text{ (Showed)}$$

(b) $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=2\tan^{-1}x$ হলে দেখাও যে, $x=\frac{a-b}{1+ab}$ [সি.'০৩; য.'০৫; সি.'১১]

প্রমাণ ঃ দেওয়া আছে,

$$\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=2\tan^{-1}x$$

$$\Rightarrow 2\tan^{-1}a-2\tan^{-1}b=2\tan^{-1}x$$

$$\Rightarrow 2(\tan^{-1}a-\tan^{-1}b)=2\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\frac{a-b}{1+ab}=\tan^{-1}x$$

$$\therefore x=\frac{a-b}{1+ab} \text{ (Showed)}$$

15(c) $\tan^{-1}x+\frac{1}{2}\sec^{-1}\frac{1+y^2}{1-y^2}+\frac{1}{2}\text{cosec}^{-1}\frac{1+z^2}{2z}=\pi$ হলে দেখাও যে, $x+y+z=xyz$ [য.'০৪; চ.'১১]

প্রমাণ ঃ $\tan^{-1}x+\frac{1}{2}\sec^{-1}\frac{1+y^2}{1-y^2}+\frac{1}{2}\text{cosec}^{-1}\frac{1+z^2}{2z}=\pi$

$$\Rightarrow \tan^{-1}x+\frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}+\frac{1}{2}\sin^{-1}\frac{2z}{1+z^2}=\pi$$

$$\Rightarrow \tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$$

$$\Rightarrow \tan^{-1}\frac{x+y+z-xyz}{1-xy-yz-zx}=\pi$$

$$\Rightarrow \frac{x+y+z-xyz}{1-xy-yz-zx}=\tan\pi=0$$

$$\Rightarrow x+y+z-xyz=0\times(1-xy-yz-zx)=0$$

$$\therefore x+y+z=xyz \text{ (Showed).}$$

16(a) $\sin(\pi\cos\theta)=\cos(\pi\sin\theta)$ হলে দেখাও যে, $\theta=\pm\frac{1}{2}\sin^{-1}\frac{3}{4}$

[ঢা.,রা.,ব.'১০; সি.'১২; কু.'১৩]

প্রমাণ ঃ দেওয়া আছে, $\sin(\pi\cos\theta)=\cos(\pi\sin\theta)$

$$\Rightarrow \sin(\pi\cos\theta)=\sin(\frac{\pi}{2}\pm\pi\sin\theta)$$

$$\Rightarrow \pi\cos\theta=\frac{\pi}{2}\pm\pi\sin\theta \Rightarrow \cos\theta\mp\sin\theta=\frac{1}{2}$$

$$\Rightarrow \cos^2\theta+\sin^2\theta\mp 2\sin\theta\cos\theta=\frac{1}{4}$$

$\Rightarrow 1 \mp \sin 2\theta = \frac{1}{4} \Rightarrow (1 - \frac{1}{4}) = \pm \sin 2\theta$

$\Rightarrow \frac{3}{4} = \pm \sin 2\theta \Rightarrow \sin 2\theta = \pm \frac{3}{4}$

$\therefore\ 2\theta = \sin^{-1}(\pm \frac{3}{4}) = \pm \sin^{-1}\frac{3}{4}$

$\therefore\ \theta = \pm \frac{1}{2} \sin^{-1}\frac{3}{4}$ (Showed)

16(b) $\sin(\pi \cos\theta) = \cos(\pi \sin\theta)$ **হলে দেখাও যে,** $\theta = \pm \frac{\pi}{4} + \cos^{-1}\frac{1}{2\sqrt{2}}$ **[কু.'১২]**

প্রমাণ ঃ দেওয়া আছে, $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$

$\Rightarrow \sin(\pi\cos\theta) = \sin(\frac{\pi}{2} \pm \pi \sin\theta)$

$\Rightarrow \pi\cos\theta = \frac{\pi}{2} \pm \pi\sin\theta \Rightarrow \cos\theta \mp \sin\theta = \frac{1}{2}$

উভয় পক্ষকে $\sqrt{1^2 + 1^2} = \sqrt{2}$ ভাগ দ্বারা করে পাই,

$\frac{1}{\sqrt{2}}\cos\theta \mp \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\theta\cos\frac{\pi}{4} \mp \sin\theta\sin\frac{\pi}{4} = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos(\theta \mp \frac{\pi}{4}) = \frac{1}{2\sqrt{2}}$

$\therefore\ \theta \mp \frac{\pi}{4} = \cos^{-1}\frac{1}{2\sqrt{2}}$

$\Rightarrow \theta = \pm \frac{\pi}{4} + \cos^{-1}\frac{1}{2\sqrt{2}}$ (Showed).

(c) $\sin(\pi \text{cosec}\theta) = \cos(\frac{\pi}{2}\text{cosec}\,\theta)$ **হলে দেখাও যে,** $\theta = \sin^{-1}(\frac{3}{4n+1})$ **অথবা** $\theta = \sin^{-1}(\frac{1}{1-4n})$

প্রমাণ ঃ দেওয়া আছে,

$\sin(\pi\,\text{cosec}\,\theta) = \cos(\frac{\pi}{2}\text{cosec}\theta)$

$\Rightarrow \cos(\frac{\pi}{2}\text{cosec}\theta) = \cos\{\frac{\pi}{2} - \pi\,\text{cosec}\,\theta\}$

$\therefore\ \frac{\pi}{2}\text{cosec}\,\theta = 2n\pi \pm (\frac{\pi}{2} - \pi\,\text{cosec}\,\theta)$

$\therefore\ \frac{\pi}{2}\text{cosec}\,\theta = 2n\pi + \frac{\pi}{2} - \pi\,\text{cosec}\,\theta$

$\Rightarrow \frac{3}{2}\text{cosec}\,\theta = 2n + \frac{1}{2} = \frac{4n+1}{2}$

$\Rightarrow \text{cosec}\,\theta = \frac{4n+1}{3} \Rightarrow \sin\theta = \frac{3}{4n+1}$

$\Rightarrow \theta = \sin^{-1}\frac{3}{4n+1}$

অথবা, $\frac{\pi}{2}\text{cosec}\,\theta = 2n\pi - \frac{\pi}{2} + \pi\,\text{cosec}\,\theta$

$\Rightarrow \frac{1}{2}\text{cosec}\,\theta = 2n - \frac{1}{2} + \text{cosec}\,\theta$

$\Rightarrow \frac{1}{2}\text{cosec}\,\theta = \frac{1}{2} - 2n$

$\Rightarrow \text{cosec}\,\theta = 1 - 4n \Rightarrow \sin\theta = \frac{1}{1-4n}$

$\Rightarrow \theta = \sin^{-1}\frac{1}{1-4n}$

$\therefore\ \theta = \sin^{-1}\frac{3}{4n+1}$ অথবা $\theta = \sin^{-1}\frac{1}{1-4n}$

(Showed)

17(a) $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$ **হলে দেখাও যে,** $x\sqrt{1-y^2} + y\sqrt{1-x^2} = 1$

প্রমাণ ঃ দেওয়া আছে, $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-y^2}\} = \frac{\pi}{2}$

$\therefore\ x\sqrt{1-y^2} + y\sqrt{1-y^2} = \sin\frac{\pi}{2} = 1$

(b) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ **হলে দেখাও যে,** $x^2 + y^2 + z^2 + 2xyz = 1$ **[কু.'১২;প্র.ভ.প.'৯৯]**

প্রমাণ ঃ $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$

$\Rightarrow \cos^{-1}\{xy - \sqrt{(1-x^2)(1-y^2)}\} = \pi - \cos^{-1}z$

$\Rightarrow xy - \sqrt{(1-x^2)(1-y^2)} = \cos(\pi - \cos^{-1}z)$

$\Rightarrow xy - \sqrt{1-x^2-y^2+x^2y^2}$

$= -\cos(\cos^{-1}z) = -z$

$\Rightarrow \sqrt{1-x^2-y^2+x^2y^2} = xy + z$

$\Rightarrow 1-x^2-y^2+x^2y^2 = x^2y^2 + 2xyz + z^2$

$\therefore x^2+y^2+z^2+2xyz = 1$ (Showed)

17(c) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ **হলে দেখাও যে,** $xy + yz + zx = 1$

প্রমাণ ঃ দেওয়া আছে, $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\frac{x+y}{1-xy} = \frac{\pi}{2} - \tan^{-1}z$

$\Rightarrow \frac{x+y}{1-xy} = \tan\left(\frac{\pi}{2} - \tan^{-1}z\right)$

$\Rightarrow \frac{x+y}{1-xy} = \cot(\tan^{-1}z) = \cot(\cot^{-1}\frac{1}{z}) = \frac{1}{z}$

$\Rightarrow xz + yz = 1 - xy$

$\therefore xy + yz + zx = 1$ (Showed)

17(d) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ **হলে দেখাও যে,** $x + y + z = xyz$

প্রমাণ ঃ দেওয়া আছে, $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\Rightarrow \tan^{-1}\frac{x+y}{1-xy} = \pi - \tan^{-1}z$

$\Rightarrow \frac{x+y}{1-xy} = \tan(\pi - \tan^{-1}z)$

$= -\tan(\tan^{-1}z) = -z$

$\Rightarrow x + y = -z + xyz$

$\therefore x + y + z = xyz$ (Showed)

(e) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ **হলে দেখাও যে,** $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

প্রমাণ ঃ মনে করি, $\sin^{-1}x = A \Rightarrow \sin A = x$

$\sin^{-1}y = B \Rightarrow \sin B = y$ এবং

$\sin^{-1}z = C \Rightarrow \sin C = z$

$\therefore A + B + C = \pi$

এখন, $A + B + C = \pi$ হলে,

$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

$\Rightarrow 2\sin A \cos B + 2\sin B \cos B +$

$2\sin C \cos C = 4 \sin A \sin B \sin C$

$\Rightarrow \sin A\sqrt{1-\sin^2 A} + \sin B\sqrt{1-\sin^2 B}$

$+ \sin C\sqrt{1-\sin^2 C} = 2\sin A \sin B \sin C$

$\Rightarrow x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

18(a) $\sec\theta - \operatorname{cosec}\theta = \frac{4}{3}$ **হলে দেখাও যে,** $\theta = \frac{1}{2}\sin^{-1}\frac{3}{4}$

প্রমাণ ঃ দেওয়া আছে, $\sec\theta - \operatorname{cosec}\theta = \frac{4}{3}$

$\Rightarrow \frac{1}{\cos\theta} - \frac{1}{\sin\theta} = \frac{4}{3} \Rightarrow \frac{\sin\theta - \cos\theta}{\sin\theta\cos\theta} = \frac{4}{3}$

$\Rightarrow 3(\sin\theta - \cos\theta) = 2\sin 2\theta$

$\Rightarrow 9(\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$

$= 4\sin^2 2\theta$

$\Rightarrow 9(1 - \sin 2\theta) = 4\sin^2 2\theta$

$\Rightarrow 4\sin^2 2\theta + 9\sin 2\theta - 9 = 0$

$\Rightarrow 4\sin^2 2\theta + 12\sin 2\theta - 3\sin 2\theta - 9 = 0$

$\Rightarrow 4\sin 2\theta(\sin 2\theta + 3) - 3(\sin 2\theta + 3) = 0$

$\Rightarrow (\sin 2\theta + 3)(4\sin 2\theta - 3) = 0$

এখানে, $\sin 2\theta \neq -3$ [$\because -1 \leq \sin\theta \leq 1$]

$\therefore 4\sin 2\theta - 3 = 0 \Rightarrow \sin 2\theta = \frac{3}{4}$

$\Rightarrow 2\theta = \sin^{-1}\frac{3}{4} \quad \therefore \theta = \frac{1}{2}\sin^{-1}\frac{3}{4}$ (Showed)

18(b) $\tan(\theta-\alpha)\tan(\theta-\beta)=\tan^2\theta$ হলে দেখাও যে, $\theta=\frac{1}{2}\tan^{-1}\frac{2\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$.

প্রমাণ : দেওয়া আছে,

$\tan(\theta-\alpha)\tan(\theta-\beta)=\tan^2\theta$

$\Rightarrow \frac{\sin(\theta-\alpha)\sin(\theta-\beta)}{\cos(\theta-\alpha)\cos(\theta-\beta)}=\tan^2\theta$

$\Rightarrow \frac{\cos(\theta-\alpha)\cos(\theta-\beta)}{\sin(\theta-\alpha)\sin(\theta-\beta)}=\frac{1}{\tan^2\theta}$

$\Rightarrow \frac{\cos(\theta-\alpha)\cos(\theta-\beta)+\sin(\theta-\alpha)\sin(\theta-\beta)}{\cos(\theta-\alpha)\cos(\theta-\beta)-\sin(\theta-\alpha)\sin(\theta-\beta)}$

$=\frac{1+\tan^2\theta}{1-\tan^2\theta}$ [যোজন- বিয়োজন করে।]

$\Rightarrow \frac{\cos(\theta-\alpha-\theta+\beta)}{\cos(\theta-\alpha+\theta-\beta)}=\frac{1}{\cos2\theta}$

$\Rightarrow \frac{\cos(\alpha-\beta)}{\cos\{2\theta-(\alpha+\beta)\}}=\frac{1}{\cos2\theta}$

$\Rightarrow \cos2\theta\cos(\alpha+\beta)+\sin2\theta\sin(\alpha+\beta)$
$=\cos2\theta\cos(\alpha-\beta)$

$\Rightarrow \sin2\theta\sin(\alpha+\beta)=$
$\cos2\theta\{\cos(\alpha-\beta)-\cos(\alpha+\beta)\}$

$\Rightarrow \sin2\theta\sin(\alpha+\beta)=\cos2\theta\,.\,2\sin\alpha\sin\beta$

$\Rightarrow \tan2\theta=\frac{2\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$

$\Rightarrow 2\theta=\tan^{-1}\frac{2\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$

$\therefore\ \theta=\frac{1}{2}\tan^{-1}\frac{2\sin\alpha\sin\beta}{\sin(\alpha+\beta)}$ (Showed)

18(c) $A+B+C=\pi$, $A=\tan^{-1}2$ এবং $B=\tan^{-1}3$ হলে দেখাও যে, $C=\frac{\pi}{4}$

[চ.'০৭,'১৩; ব.'০৭; ঢা.'০৮]

প্রমাণ : দেওয়া আছে, $A+B+C=\pi$ ……(1),

$A=\tan^{-1}2\Rightarrow\tan A=2$ এবং

$B=\tan^{-1}3\Rightarrow\tan B=3$

এখন, $A+B=\pi-C$

$\Rightarrow \tan(A+B)=\tan(\pi-C)$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$

$\Rightarrow \frac{2+3}{1-2.3}=-\tan C\Rightarrow\frac{5}{-5}=-\tan C$

$\Rightarrow \tan C=\tan\frac{\pi}{4}\quad\therefore\ C=\frac{\pi}{4}$ (Showed)

19(a) দেখাও যে, $\tan^{-1}6+\tan^{-1}\frac{7}{5}=\frac{3\pi}{4}$

প্রমাণ : $\tan^{-1}6+\tan^{-1}\frac{7}{5}=\tan^{-1}\frac{6+\frac{7}{5}}{1-6\times\frac{7}{5}}$

$=\tan^{-1}\frac{37}{5-42}=\tan^{-1}\frac{37}{-37}=\pi-\tan^{-1}\frac{37}{37}$

$=\pi-\tan^{-1}1=\pi-\frac{\pi}{4}$

$\therefore\ \tan^{-1}6+\tan^{-1}\frac{7}{5}=\frac{3\pi}{4}$ (Showed)

19(b) প্রমাণ কর যে, $\tan^{-1}5+\tan^{-1}6+\tan^{-1}7+\tan^{-1}8=2\pi-\tan^{-1}\frac{8}{11}$

প্রমাণ : $\tan^{-1}5+\tan^{-1}6+\tan^{-1}7+\tan^{-1}8$

$=\tan^{-1}\frac{5+6}{1-5\times6}+\tan^{-1}\frac{7+8}{1-7\times8}$

$=\tan^{-1}\frac{11}{-29}+\tan^{-1}\frac{15}{-55}$

$=\pi-\tan^{-1}\frac{11}{29}+\pi-\tan^{-1}\frac{3}{11}$

$=2\pi-\tan^{-1}\frac{\frac{11}{29}+\frac{3}{11}}{1-\frac{11}{29}.\frac{3}{11}}=2\pi-\tan^{-1}\frac{\frac{121+87}{29.11}}{\frac{319-33}{29.11}}$

$= 2\pi - \tan^{-1}(\frac{208}{29.11} \times \frac{29.11}{286})$

$= 2\pi - \tan^{-1}\frac{8}{11} = \text{R.H.S.}$ (Proved)

20. সমাধান কর :

(a) $\sec^{-1}\frac{x}{2} - \sec^{-1}\frac{x}{3} = \sec^{-1}3 - \sec^{-1}2$

$\Rightarrow \cos^{-1}\frac{2}{x} - \cos^{-1}\frac{3}{x} = \cos^{-1}\frac{1}{3} - \cos^{-1}\frac{1}{2}$

$\Rightarrow \cos^{-1}\frac{2}{x} + \cos^{-1}\frac{1}{2} = \cos^{-1}\frac{1}{3} + \cos^{-1}\frac{3}{x}$

$\Rightarrow \cos^{-1}\{\frac{2}{x}.\frac{1}{2} - \sqrt{(1-\frac{4}{x^2})(1-\frac{1}{4})}\} =$

$\cos^{-1}\{\frac{1}{3}.\frac{3}{x} - \sqrt{(1-\frac{1}{9})(1-\frac{9}{x^2})}\}$

$\Rightarrow \frac{1}{x} - \sqrt{\frac{x^2-4}{x^2}.\frac{3}{4}} = \frac{1}{x} - \sqrt{\frac{8}{9}\frac{x^2-9}{x^2}}$

$\Rightarrow \sqrt{\frac{x^2-4}{x^2}.\frac{3}{4}} = \sqrt{\frac{8}{9}\frac{x^2-9}{x^2}}$

$\Rightarrow \frac{3x^2-12}{4x^2} = \frac{8x^2-72}{9x^2}$

$\Rightarrow 27x^2 - 108 = 32x^2 - 288$ [এখানে $x \neq 0$]

$\Rightarrow 5x^2 = 180 \Rightarrow x^2 = 36$

$\therefore x = 6$

[যেহেতু ঋণাত্মক মান সমীকরণকে সিদ্ধ করে না]

20(b) $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$ [বুয়েট'০৬-০৭]

$\Rightarrow 2\tan^{-1}\frac{1-x}{1+x} = \tan^{-1}x$

$\Rightarrow \tan^{-1}\frac{2\frac{1-x}{1+x}}{1-(\frac{1-x}{1+x})^2} = \tan^{-1}x$

$\Rightarrow \frac{2(1-x)(1+x)}{(1+x)^2-(1-x)^2} = x$

$\Rightarrow \frac{2(1-x^2)}{4x} = x \Rightarrow 2x^2 = 1 - x^2$

$\Rightarrow 3x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$ (Ans.)

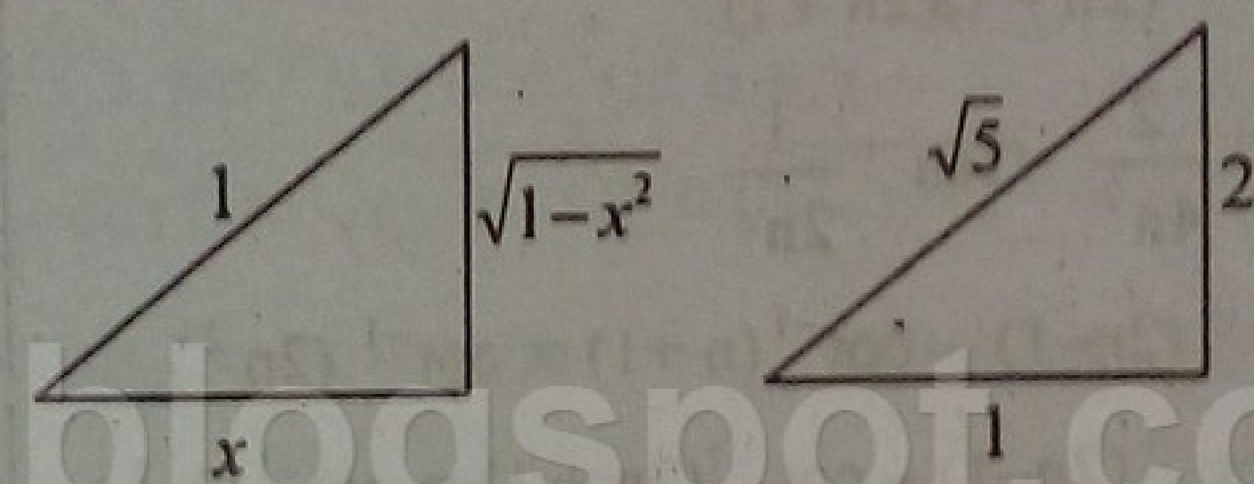
20(c) $\tan^{-1}x + 2\cot^{-1}x = \frac{2}{3}\pi$ [বুয়েট'১০-১১]

$\Rightarrow \tan^{-1}x + \cot^{-1}x + \cot^{-1}x = \frac{2}{3}\pi$

$\Rightarrow \frac{\pi}{2} + \cot^{-1}x = \frac{2}{3}\pi \Rightarrow \cot^{-1}x = \frac{2}{3}\pi - \frac{\pi}{2}$

$\Rightarrow \cot^{-1}x = \frac{2}{3}\pi - \frac{4\pi-3\pi}{6} = \frac{\pi}{6} \quad \therefore x = \sqrt{3}$

20(d) $\tan(\cos^{-1}x) = \sin(\tan^{-1}2)$ [বুয়েট'১২-১৩]



চিত্র হতে পাই, $\cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$ এবং

$\tan^{-1}2 = \sin^{-1}\frac{2}{\sqrt{5}}$

এখন, $\tan(\cos^{-1}x) = \sin(\tan^{-1}2)$

$\Rightarrow \tan(\tan^{-1}\frac{\sqrt{1-x^2}}{x}) = \sin(\sin^{-1}\frac{2}{\sqrt{5}})$

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5}$

$\Rightarrow 4x^2 = 5 - 5x^2 \Rightarrow 9x^2 = 5 \quad \therefore x = \pm\frac{\sqrt{5}}{3}$

অতিরিক্ত প্রশ্ন (সমাধানসহ)

1. দেখাও যে, $\cot^{-1}(2n-1) - \cot^{-1}(2n+1) = \cot^{-1}(2n^2)$. ইহার সাহায্যে দেখাও যে,

$\cot^{-1}(2.1^2)+\cot^{-1}(2.2^2)+\cot^{-1}(2.3^2)$
$=\cot^{-1}\frac{4}{3}$

প্রমাণ ঃ $\cot^{-1}(2n-1)-\cot^{-1}(n+1)$

$=\tan^{-1}\frac{1}{2n-1}-\tan^{-1}\frac{1}{2n+1}$

$=\tan^{-1}\dfrac{\frac{1}{2n-1}-\frac{1}{2n+1}}{1+\frac{1}{2n-1}.\frac{1}{2n+1}}$

$=\tan^{-1}\dfrac{\frac{(2n+1)-(2n-1)}{(2n-1)(2n+1)}}{\frac{4n^2-1+1}{(2n-1)(2n+1)}}$

$=\tan^{-1}\frac{2}{4n^2}=\tan^{-1}\frac{1}{2n^2}$

$\therefore\ \cot^{-1}(2n-1)-\cot^{-1}(n+1)=\cot^{-1}(2n^2)$.

এখন, $\cot^{-1}(2n-1)-\cot^{-1}(2n+1)=\cot^{-1}(2n^2)$

$n = 1, 2, 3, 4$ বসিয়ে আমরা পাই,

$\cot^{-1}1-\cot^{-1}3=\cot^{-1}(2.1^2)$

$\cot^{-1}3-\cot^{-1}5=\cot^{-1}(2.2^2)$

$\cot^{-1}5-\cot^{-1}7=\cot^{-1}(2.3^2)$

যোগ করে আমরা পাই, $\cot^{-1}1-\cot^{-1}7=$
$\cot^{-1}(2.1^2)+\cot^{-1}(2.2^2)+\cot^{-1}(2.3^2)$
$\Rightarrow \cot^{-1}(2.1^2)+\cot^{-1}(2.2^2)+\cot^{-1}(2.3^2)$

$=\tan^{-1}1-\tan^{-1}\frac{1}{7}=\tan^{-1}\dfrac{1-\frac{1}{7}}{1+1.\frac{1}{7}}=\tan^{-1}\frac{6}{8}$

$\therefore\ \cot^{-1}(2.1^2)+\cot^{-1}(2.2^2)+\cot^{-1}(2.3^2)$

$=\tan^{-1}\frac{3}{4}=\cot^{-1}\frac{4}{3}$

2. দেখাও যে, $\tan^{-1}(1+a)-\tan^{-1}a=\cot^{-1}(1+a+a^2)$.ইহার সাহায্যে দেখাও যে, $\cot^{-1}3+\cot^{-1}7+\cot^{-1}13+\cot^{-1}21=\cot^{-1}\frac{3}{2}$

প্রমাণ ঃ $\tan^{-1}(1+a)-\tan^{-1}a$

$=\tan^{-1}\frac{1+a-a}{1+(1+a).a}=\tan^{-1}\frac{1}{1+a+a^2}$

$\therefore\ \tan^{-1}(1+a)-\tan^{-1}a=\cot^{-1}(1+a+a^2)$

$n = 1, 2, 3, 4$ বসিয়ে আমরা পাই,

$\tan^{-1}2-\tan^{-1}1=\cot^{-1}3$

$\tan^{-1}3-\tan^{-1}2=\cot^{-1}7$

$\tan^{-1}4-\tan^{-1}3=\cot^{-1}13$

$\tan^{-1}5-\tan^{-1}4=\cot^{-1}21$

যোগ করে আমরা পাই, $\tan^{-1}5+\tan^{-1}1=$
$\cot^{-1}3+\cot^{-1}7+\cot^{-1}13+\cot^{-1}21$

3. দেখাও যে, $\cot^{-1}(1-a+a^2)=\tan^{-1}a-\tan^{-1}(a-1)$

প্রমাণ ঃ $\tan^{-1}a-\tan^{-1}(a-1)=\tan^{-1}\frac{a-a+1}{1+a(a-1)}$

$=\tan^{-1}\frac{1}{1-a+a^2}=\cot^{-1}(1-a+a^2)$

$\therefore\ \cot^{-1}(1-a+a^2)=\tan^{-1}a-\tan^{-1}(a-1)$

ভর্তি পরীক্ষার MCQ :

1. $\cos(\sin^{-1}\frac{1}{4}+\cos^{-1}\frac{1}{4})$ এর মান–[DU 00-01]

Sol^n.: $\cos 90° = 0$ [$\because \sin x+\cos x = 90°$]

2. $\sin[2(\sin^{-1}+\cos^{-1})]$ = a হলে, a এর মান কত৷ [DU 97-98]

Sol^n.: $\sin(2.90°)=a \Rightarrow a=0$

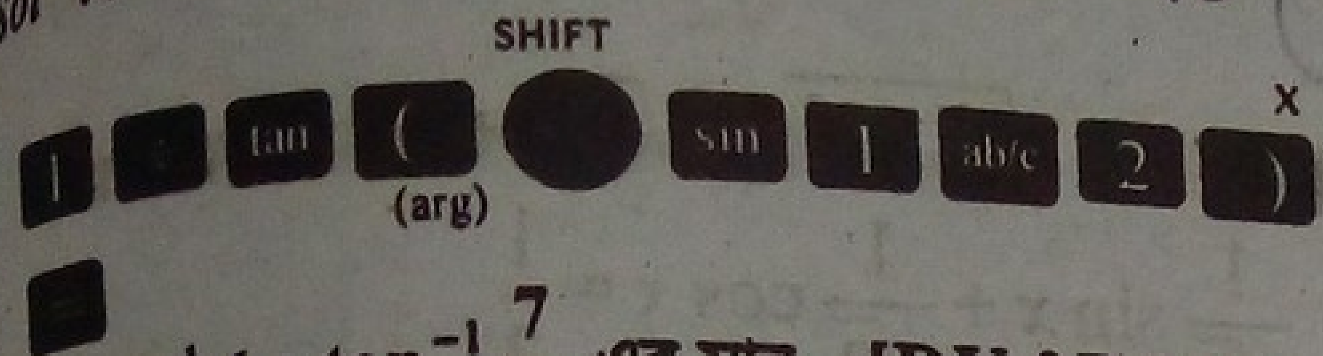
3. $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{2b}{1+b^2}$ এর মান– [DU 06-07]

Sol^n.: প্রদত্ত রাশি $=2\tan^{-1}a-(\frac{\pi}{2}-2\tan^{-1}b)$

$=2\tan^{-1}\frac{a+b}{1-ab}-\frac{\pi}{2}$

4. $\cot(\sin^{-1}\frac{1}{2})$ এর মান– [DU 98-99, 08-09]

Sol^n.: ক্যালকুলেটরের সাহায্যে, মান $= 1.73 = \sqrt{3}$



5. $\tan^{-1}6 + \tan^{-1}\frac{7}{5}$ এর মান– [DU 07-08]

Sol^n.: ক্যালকুলেটরের সাহায্যে, মান $= 135° = \frac{3\pi}{4}$

6. $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$ এর মান–

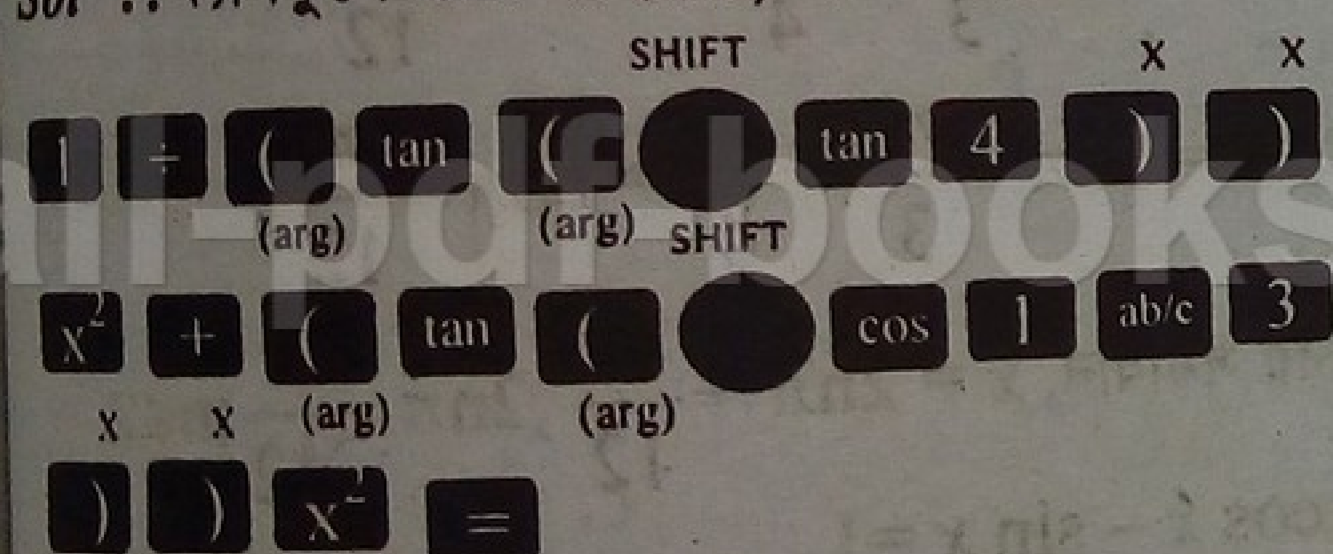
[DU 03-04, 06-07; Jt.U 08-09; RU 05-06]

Sol^n.: ক্যালকুলেটরের সাহায্যে, মান $= 180° = \pi$

7. $\sec^2(\tan^{-1}4) + \tan^2(\sec^{-1}3)$ এর মান–

[JU 06-07; EA 05-06]

Sol^n.: ক্যালকুলেটরের সাহায্যে, মান = 25



8. যদি $A + B + C = \pi$, $\tan^{-1}2 = A$ এবং

$\tan^{-1}3 = B$ হয় তবে $C = ?$

[IU 04-05; CU 06-07; RU 06-07]

Sol^n.: $C = 180° - (\tan^{-1}2 + \tan^{-1}3) = 45°$

9. $\sin\cot^{-1}\tan\cos^{-1}x = ?$ [CU 04-05; RU 06-07]

কৌশল: $fn_1\ (cofn_2)^{-1}\ fn_2\ (cofn_1)^{-1}\ x = x$

Sol^n.: কৌশল অনুযায়ী, মান = x

10. $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$ হলে x এর মান–

[RU 07-08]

A. 0 B. $\frac{1}{\sqrt{2}}$ C. $\sqrt{3}$ D. 1

Sol^n.: $x = \sqrt{3}$ বসালে সমীকরণটি সিদ্ধ হয়। Ans.C

11. $\arctan\left\{\sin\left(\arccos\frac{\sqrt{2}}{\sqrt{3}}\right)\right\}$ সমান-

DU 13-14

A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{6}$

Sol^n.: $\arctan\{\sin(\arcsin\frac{\sqrt{3-2}}{\sqrt{3}})$

$= \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

12. যদি $\sin^{-1}x = \theta$ হয়, তবে $\cos\theta$-এর মান কত?

Jagannat 13-14

A. $1 - x^2$ B. $\sqrt{1-x^2}$ C. $\frac{1}{\sqrt{1-x^2}}$ D. $\frac{x}{\sqrt{1-x^2}}$

Sol^n.: $\sin^{-1}x = \theta \Rightarrow \sin\theta = x$

$\therefore\ \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$

13. $\sin^{-1}x$ এর মান হবে- [Textile 13-14]

A. $\cot^{-1}\frac{\sqrt{1-x^2}}{x}$ B. $\cot^{-1}\frac{x}{\sqrt{1-x^2}}$

C. $\cot^{-1}\sqrt{1-x^2}$ D. $\cot^{-1}\frac{1}{\sqrt{1-x^2}}$

Sol^n.: $\sin^{-1}x = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$