ত্রিকোণমিতিক সমীকরণ

প্রশ্নমালা VII B

1. সমাধান কর ঃ

(a) $\sin x + \cos x = 1$ [ঢ.'০৫; সি.'০৯; চ.'১০]

উভয় পক্ষকে $\sqrt{1^2+1^2} = \sqrt{2}$ দ্বারা ভাগ করে পাই,

$$\Rightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin x \sin\frac{\pi}{4} + \cos x \cos\frac{\pi}{4} = \cos\frac{\pi}{4}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\therefore \quad x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, \text{ যেখানে } n \in \mathbb{Z}.$$

$$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$$

$$\therefore \quad x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{4} = 2n\pi + \frac{\pi}{2} = (4n+1)\frac{\pi}{2}$$

অথবা, $x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{4} = 2n\pi$; যেখানে $n \in \mathbb{Z}$.

(b) $\sin x + \cos x = \sqrt{2}$

[ঢা., য.'০২; ব.,রা.'০৩; দি.'১০; বুয়েট'০২-০৩]

উভয় পক্ষকে $\sqrt{1^2+1^2} = \sqrt{2}$ দ্বারা ভাগ করে পাই,

$$\Rightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow \cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4} = 1$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = 1 \quad \therefore x - \frac{\pi}{4} = 2n\pi, \quad n \in \mathbb{Z}.$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{4} = 2n\pi$$

$n = 0$ হলে, $x = \frac{\pi}{4}$; $n = 1$ হলে, $x = 2\pi + \frac{\pi}{4}$;

$n = -1$ হলে, $x = -2\pi + \frac{\pi}{4}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান, $x = \frac{\pi}{4}$

(c) $\sin x + \cos x = \frac{1}{\sqrt{2}}$ [য.'০২]

উভয় পক্ষকে $\sqrt{1^2+1^2} = \sqrt{2}$ দ্বারা ভাগ করে পাই,

$$\Rightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{2}$$

$$\Rightarrow \cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4} = \frac{1}{2}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}$$

$$\therefore \quad x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3}, \quad n \in \mathbb{Z}.$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{4}$$

$$\therefore \quad x = 2n\pi + \frac{\pi}{3} + \frac{\pi}{4} = 2n\pi + \frac{7\pi}{12}$$

অথবা, $x = 2n\pi - \frac{\pi}{3} + \frac{\pi}{4} = 2n\pi - \frac{\pi}{12}$

$\therefore$ নির্ণেয় সমাধান, $x = 2n\pi + \frac{7\pi}{12}, 2n\pi - \frac{\pi}{12}$; $n \in \mathbb{Z}$

1(d) $\cos x - \sin x = 1$

উভয় পক্ষকে $\sqrt{1^2+1^2} = \sqrt{2}$ দ্বারা ভাগ করে পাই,

$$\Rightarrow \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\therefore \quad x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, \quad n \in \mathbb{Z}$$

$\Rightarrow x = 2n\pi$ অথবা, $x = 2n\pi - 2.\frac{\pi}{4} = 2n\pi - \frac{\pi}{2}$

$\therefore$ নির্ণেয় সমাধান, $x = 2n\pi, (4n-1)\frac{\pi}{2}$

1(e) $\sin x + 2\cos x = 1$

উভয় পক্ষকে $\sqrt{1^2+2^2}=\sqrt{5}$ দ্বারা ভাগ করে পাই,

$\Rightarrow \frac{1}{\sqrt{5}}\sin x+\frac{2}{\sqrt{5}}\cos x=\frac{1}{\sqrt{5}}$

মনে করি, $\sin\theta=\frac{1}{\sqrt{5}}$ $\therefore \cos\theta=\frac{2}{\sqrt{5}}$ এবং

$\cos x\cos\theta+\sin x\sin\theta=\sin\theta$

$\Rightarrow \cos(x-\theta)=\cos(\frac{\pi}{2}-\theta)$

$\therefore\ x-\theta=2n\pi\pm(\frac{\pi}{2}-\theta)$, $n\in Z$.

$\Rightarrow x=2n\pi+\frac{\pi}{2}$

অথবা, $x=2n\pi-(\frac{\pi}{2}-2\theta)$, যেখানে $\alpha=\frac{\pi}{2}-2\theta$

$\therefore\ \sin\alpha=\sin(\frac{\pi}{2}-2\theta)=\cos2\theta$

$=\cos^2\theta-\sin^2\theta=\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$

নির্ণেয় সমাধান , $x=2n\pi+\frac{\pi}{2}$, $2n\pi-\alpha$; যেখানে

$n\in Z$ এবং $\sin\alpha=\frac{3}{5}$

1(f) $\cos\theta+2\sin\theta=1$ [চ.'০৪; বুয়েট'০৫-০৬]

উভয় পক্ষকে $\sqrt{1^2+2^2}=\sqrt{5}$ দ্বারা ভাগ করে পাই,

$\Rightarrow \frac{1}{\sqrt{5}}\cos x+\frac{2}{\sqrt{5}}\sin x=\frac{1}{\sqrt{5}}$

মনে করি , $\cos\alpha=\frac{1}{\sqrt{5}}$ $\therefore\ \sin\alpha=\frac{2}{\sqrt{5}}$ এবং

$\cos x\cos\alpha+\sin x\sin\alpha=\cos\alpha$

$\Rightarrow \cos(x-\alpha)=\cos\alpha$

$\therefore\ x-\alpha=2n\pi\pm\alpha$, $n\in Z$.

$\Rightarrow x=2n\pi$, $2n\pi+2\alpha$

$\therefore$ নির্ণেয় সমাধান , $x=2n\pi$, $2(n\pi+\alpha)$;

যেখানে $n\in Z$ এবং $\cos\alpha=\frac{1}{\sqrt{5}}$

2(a) $\sqrt{3}\cos x+\sin x=1$, $-2\pi<x<2\pi$

[বা.'০৬,'০৮; সি. '০৮; ব.'০৪; য.'০৭,'১০;কু.'০৮]

উভয় পক্ষকে $\sqrt{(\sqrt{3})^2+1^2}=2$ দ্বারা ভাগ করে পাই,

$\Rightarrow \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=\frac{1}{2}$

$\Rightarrow \cos x\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}=\cos\frac{\pi}{3}$

$\Rightarrow \cos(x-\frac{\pi}{6})=\cos\frac{\pi}{3}$

$\therefore\ x-\frac{\pi}{6}=2n\pi\pm\frac{\pi}{3}$, $n\in Z$.

$\Rightarrow x=2n\pi\pm\frac{\pi}{3}+\frac{\pi}{6}$

$\therefore\ x=2n\pi+\frac{\pi}{3}+\frac{\pi}{6}=2n\pi+\frac{\pi}{2}$

অথবা , $x=2n\pi-\frac{\pi}{3}+\frac{\pi}{6}=2n\pi-\frac{\pi}{6}$

$n=0$ হলে, $x=-\frac{\pi}{6}$, $\frac{\pi}{2}$

$n=-1$ হলে, $x=-2\pi+\frac{\pi}{2}=-\frac{3\pi}{2}$

অথবা, $x=-2\pi-\frac{\pi}{6}$

$n=1$ হলে, $x=2\pi+\frac{\pi}{2}$

অথবা, $x=2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান ,

$x=-\frac{3\pi}{2}$, $-\frac{\pi}{6}$, $\frac{\pi}{2}$, $\frac{11\pi}{6}$

2(b) $\sin x+\sqrt{3}\cos x=\sqrt{3}$

[কু .'০০ ; চ.'০৩ ; সি.'০৪ ; ব.'০৫,'১২]

উভয় পক্ষকে $\sqrt{1^2+(\sqrt{3})^2}=2$ দ্বারা ভাগ করে পাই,

$\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{3}}{2}$.

$\Rightarrow \cos x\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}=\cos\frac{\pi}{6}$

$\Rightarrow \cos(x-\frac{\pi}{6}) = \cos\frac{\pi}{6}$

$\therefore \quad x-\frac{\pi}{6} = 2n\pi \pm \frac{\pi}{6}$, $n\in\mathbb{Z}$.

$\Rightarrow x = 2n\pi$, $2n\pi + 2.\frac{\pi}{6}$

$\therefore$ নির্ণেয় সমাধান , $x = 2n\pi$, $2n\pi + \frac{\pi}{3}$; $n\in\mathbb{Z}$

2(c) $\sin x + \sqrt{3}\cos x = \sqrt{2}$ [রা.'০৫; ঢা.'০৭]

উভয় পক্ষকে $\sqrt{1^2+(\sqrt{3})^2} = 2$ দ্বারা ভাগ করে পাই ,

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6} = \cos\frac{\pi}{4}$

$\Rightarrow \cos(x-\frac{\pi}{6}) = \cos\frac{\pi}{4}$

$\therefore \quad x-\frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4}$, $n\in\mathbb{Z}$.

$\Rightarrow x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$

$\therefore \quad x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{6} = 2n\pi + \frac{5\pi}{12}$

অথবা , $x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{6} = 2n\pi - \frac{\pi}{12}$

$\therefore$ নির্ণেয় সমাধান, $x = 2n\pi - \frac{\pi}{12}$, $2n\pi + \frac{5\pi}{12}$; যেখানে $n\in\mathbb{Z}$.

2(d) $\cos x + \sqrt{3}\sin x = 2$

[কু.'০৬; ব.'০৮; য.'০৯,'১২]

উভয় পক্ষকে $\sqrt{1^2+(\sqrt{3})^2} = 2$ দ্বারা ভাগ করে পাই ,

$\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = 1$

$\Rightarrow \cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3} = 1$

$\Rightarrow \cos(x-\frac{\pi}{3}) = 1 \therefore x-\frac{\pi}{3} = 2n\pi$, $n\in\mathbb{Z}$.

$\Rightarrow x = (6n+1)\frac{\pi}{3}$

$\therefore$ নির্ণেয় সমাধান, $x = (6n+1)\frac{\pi}{3}$; যেখানে $n\in\mathbb{Z}$

3(a) $\sqrt{3}\sin x - \cos x = 2$, যখন $-2\pi < x < 2\pi$

[রা.'০৮,'১১; কু .'০৪; চ.'০৫; ঢা.'০৬,'০৯,'১৩; টেক্সটাইল'০৩-০৪; বুয়েট'০৭-০৮; চুয়েট'০৭-০৮]

উভয় পক্ষকে $\sqrt{(\sqrt{3})^2+1^2} = 2$ দ্বারা ভাগ করে পাই,

$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = 1$

$\Rightarrow \sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6} = 1$

$\Rightarrow \sin(x-\frac{\pi}{6}) = 1$

$\therefore \quad x-\frac{\pi}{6} = (4n+1)\frac{\pi}{2}$, যেখানে $n\in\mathbb{Z}$.

$\Rightarrow x = 2n\pi + \frac{\pi}{2} + \frac{\pi}{6}$

$= 2n\pi + \frac{3\pi+\pi}{6} = 2n\pi + \frac{2\pi}{3}$

$n = 0$ হলে, $x = \frac{2\pi}{3}$.

$n = -1$ হলে , $x = -2\pi + \frac{2\pi}{3} = -\frac{4\pi}{3}$.

$x = 1$ হলে , $x = 2\pi + \frac{2\pi}{3}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান, $x = -\frac{4\pi}{3}, \frac{2\pi}{3}$

3(b) $\sqrt{2}\cos x - \sqrt{2}\sin x = 1$, যখন $-\pi < x < \pi$

[কু, দি.'১৩]

উভয় পক্ষকে $\sqrt{(\sqrt{2})^2+(\sqrt{2})^2} = 2$ দ্বারা ভাগ করে পাই ,

$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{2}$

$\Rightarrow \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \frac{1}{2}$

$\Rightarrow \cos(x+\frac{\pi}{4}) = \cos\frac{\pi}{3}$

$\therefore x+\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3}$, $n\in\mathbb{Z}$.

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3} - \frac{\pi}{4}$

$\therefore x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi + \frac{\pi}{12}$

অথবা , $x = 2n\pi - \frac{\pi}{3} - \frac{\pi}{4} = 2n\pi - \frac{7\pi}{12}$

$n=0$ হলে , $x = \frac{\pi}{12}$, $-\frac{7\pi}{12}$

$n=-1$ হলে , $x = -2\pi + \frac{\pi}{12}$, $-2\pi - \frac{7\pi}{12}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান, $x = -\frac{7\pi}{12}, \frac{\pi}{12}$

3(c) $\sin x + \cos x + \sqrt{2} = 0$

উভয় পক্ষকে $\sqrt{1^2+1^2} = \sqrt{2}$ দ্বারা ভাগ করে পাই ,

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x + \frac{\sqrt{2}}{\sqrt{2}} = 0$

$\Rightarrow \sin x \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos x = -1$

$\Rightarrow \sin(x+\frac{\pi}{4}) = -1$

$\therefore x+\frac{\pi}{4} = (4n-1)\frac{\pi}{2}$, $n\in\mathbb{Z}$

$\Rightarrow x = 2n\pi - \frac{\pi}{2} - \frac{\pi}{4} = 2n\pi - \frac{3\pi}{4} = (8n-3)\frac{\pi}{4}$

$\therefore$ নির্ণেয় সমাধান, $x = (8n-3)\frac{\pi}{4}$; যেখানে $n\in\mathbb{Z}$

3(d) $\cos x + \sqrt{3}\sin x = \sqrt{2}$ [বা.'০১; কু.'০২,'০৬; ব.'০২,'০৮; য.'০৯]

উভয় পক্ষকে $\sqrt{(1)^2+(\sqrt{3})^2} = 2$ দ্বারা ভাগ করে পাই

$\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{\sqrt{2}}{2}$

$\Rightarrow \cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3} = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos(x-\frac{\pi}{3}) = \cos\frac{\pi}{4}$

$\therefore x-\frac{\pi}{3} = 2n\pi \pm \frac{\pi}{4}$, $n\in\mathbb{Z}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{3}$

$\therefore x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{3} = 2n\pi + \frac{7\pi}{12}$

অথবা , $x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{3} = 2n\pi + \frac{\pi}{12}$

$\therefore$ নির্ণেয় সমাধান, $x = 2n\pi + \frac{\pi}{12}, 2n\pi + \frac{7\pi}{12}$; $n\in\mathbb{Z}$

4(a) $\sqrt{2}\sec x + \tan x = 1$ [বা. '০২]

$\Rightarrow \sqrt{2}\frac{1}{\cos x} + \frac{\sin x}{\cos x} = 1$

$\Rightarrow \sqrt{2} + \sin x = \cos x \Rightarrow \cos x - \sin x = \sqrt{2}$

উভয় পক্ষকে $\sqrt{1^2+1^2} = \sqrt{2}$ দ্বারা ভাগ করে পাই ,

$\Rightarrow \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = 1$

$\Rightarrow \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = 1$

$\Rightarrow \cos(x+\frac{\pi}{4}) = 1 \quad \therefore x+\frac{\pi}{4} = 2n\pi$, $n\in\mathbb{Z}$

$\Rightarrow x = 2n\pi - \frac{\pi}{4}$

$\therefore$ নির্ণেয় সমাধান, $x = 2n\pi - \frac{\pi}{4}$; যেখানে $n\in\mathbb{Z}$.

4(b) $\operatorname{cosec} x + \cot x = \sqrt{3}$

$\Rightarrow \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \sqrt{3} \Rightarrow \cos x - \sqrt{3}\sin x = -1$

$\Rightarrow \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = -\frac{1}{2}$ [উভয় পক্ষকে $\sqrt{(\sqrt{3})^2+(1)^2} = 2$ দ্বারা ভাগ করে।]

$\Rightarrow \cos x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3} = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\Rightarrow \cos(x+\frac{\pi}{3}) = \cos\frac{2\pi}{3}$

$\therefore x + \frac{\pi}{3} = 2n\pi \pm \frac{2\pi}{3}$, $n\in\mathbb{Z}$

$\Rightarrow x = 2n\pi + \frac{2\pi}{3} - \frac{\pi}{3} = 2n\pi + \frac{\pi}{3}$

অথবা , $x = 2n\pi - \frac{2\pi}{3} - \frac{\pi}{3} = (2n-1)\pi$

কিন্তু $x = (2n-1)\pi$ এর জন্য প্রদত্ত সমীকরণ সিদ্ধ হয় না।

$\therefore$ নির্ণেয় সমাধান , $x = 2n\pi + \frac{\pi}{3}$; যেখানে $n\in\mathbb{Z}$

বিকল্প পদ্ধতি : $\operatorname{cosec} x + \cot x = \sqrt{3}$

$\Rightarrow \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \sqrt{3} \Rightarrow \frac{1+\cos x}{\sin x} = \sqrt{3}$

$\Rightarrow \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \sqrt{3} \Rightarrow \cot\frac{x}{2} = \sqrt{3}$

$\Rightarrow \tan\frac{x}{2} = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$\therefore \frac{x}{2} = n\pi + \frac{\pi}{6}$, $n\in\mathbb{Z}$.

$\Rightarrow x = 2n\pi + \frac{\pi}{3}$

$\therefore$ নির্ণেয় সমাধান , $x = 2n\pi + \frac{\pi}{3}$; যেখানে $n\in\mathbb{Z}$

4(c) $2\cos x + 3\sin x = 1$

প্রদত্ত সমীকরণের উভয় পক্ষকে $\sqrt{(2)^2+(3)^2} = \sqrt{13}$ দ্বারা ভাগ করে পাই ,

$\frac{2}{\sqrt{13}}\cos x + \frac{3}{\sqrt{13}}\sin x = \frac{1}{\sqrt{13}}$

মনে করি , $\cos\alpha = \frac{1}{\sqrt{13}}$ এবং $\cos\theta = \frac{2}{\sqrt{13}}$

$\therefore \sin\theta = \frac{3}{\sqrt{13}}$

$\therefore \cos x\cos\theta + \sin x\sin\theta = \cos\alpha$

$\Rightarrow \cos(x-\theta) = \cos\alpha$

$\therefore x - \theta = 2n\pi \pm \alpha$, $n\in\mathbb{Z}$

$\Rightarrow x = 2n\pi \pm \alpha + \theta$

$\therefore$ নির্ণেয় সমাধান , $x = 2n\pi \pm \alpha + \theta$; যেখানে $\cos\theta = \frac{2}{\sqrt{13}}$, $\cos\alpha = \frac{1}{\sqrt{13}}$ এবং $n\in\mathbb{Z}$.

4(d) $\operatorname{cosec}\theta + \cot\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{1}{\sqrt{2}} \Rightarrow \frac{1+\cos x}{\sin x} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{1}{\sqrt{2}} \Rightarrow \cot\frac{x}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \tan\frac{x}{2} = \sqrt{2} = \tan\alpha$ (ধরি) ,

যেখানে $\tan\alpha = \sqrt{2}$

$\Rightarrow \frac{x}{2} = n\pi + \alpha \Rightarrow x = 2(n\pi+\alpha)$

$\therefore$ নির্ণেয় সমাধান , $x = 2(n\pi+\alpha)$; যেখানে $\tan\alpha = \sqrt{2}$ এবং $n\in\mathbb{Z}$

5. সমাধান :

(a) $6\cos^2\theta + \sin\theta = 5$

$\Rightarrow 6(1-\sin^2\theta) + \sin\theta = 5$

$\Rightarrow 6 - 6\sin^2\theta + \sin\theta = 5$

$\Rightarrow 6\sin^2\theta - \sin\theta - 1 = 0$

$\Rightarrow 6\sin^2\theta - 3\sin\theta + 2\sin\theta - 1 = 0$

$\Rightarrow 3\sin\theta(2\sin\theta-1) + 1(2\sin\theta-1) = 0$

$\Rightarrow (2\sin\theta-1)(3\sin\theta+1) = 0$

$2\sin\theta - 1 = 0$ হলে, $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \theta = n\pi + (-1)^n \frac{\pi}{6}$, যেখানে $n \in \mathbb{Z}$

$3\sin\theta + 1 = 0$ হলে , $\sin\theta = -\frac{1}{3} = \sin\alpha$ (ধরি)

$\therefore \theta = n\pi + (-1)^n\alpha$, $n\in\mathbb{Z}$

$\therefore$ নির্ণেয় সমাধান , $x = n\pi + (-1)^n \frac{\pi}{6}$,

$n\pi + (-1)^n\alpha$; যেখানে $\sin\alpha = -\frac{1}{3}$ এবং $n\in\mathbb{Z}$

5(b) $4(\sin^2\theta + \cos\theta) = 5, -2\pi < \theta < 2\pi$ [সি.'০৭ ; রা.,ব.,সি.'১০; চ.'১১; দি.'১২]

$\Rightarrow 4(1 - \cos^2\theta + \cos\theta) = 5$

$\Rightarrow 4 - 4\cos^2\theta + 4\cos\theta = 5$

$\Rightarrow 4\cos^2\theta - 4\cos\theta + 1 \Rightarrow (2\cos\theta - 1)^2 = 0$

$\Rightarrow 2\cos\theta - 1 = 0$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \therefore \theta = 2n\pi \pm \frac{\pi}{3}$, $n\in\mathbb{Z}$.

$n = 0$ হলে, $\theta = \pm\frac{\pi}{3}$

$n = -1$ হলে , $\theta = -2\pi \pm \frac{\pi}{3} = -\frac{7\pi}{3}, -\frac{5\pi}{3}$

$n = 1$ হলে , $\theta = 2\pi \pm \frac{\pi}{3} = \frac{7\pi}{3}, \frac{5\pi}{3}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান, $\theta = \pm\frac{\pi}{3}, \pm\frac{5\pi}{3}$

5(c) $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$

$\Rightarrow 2 - 2\sin^2 - 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$

$\Rightarrow 2\sin^2\theta + 4\sin\theta - \sin\theta - 2 = 0$

$\Rightarrow 2\sin\theta(\sin\theta + 2) - 1(\sin\theta + 2)$

$\Rightarrow (\sin\theta + 2)(2\sin\theta - 1) = 0$

কিন্তু $\sin\theta + 2 \neq 0$ অর্থাৎ $\sin\theta \neq -2$, যেহেতু $-1 \le \sin\theta \le 1$

$\therefore 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \theta = n\pi + (-1)^n \frac{\pi}{6}$, $n\in\mathbb{Z}$

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi + (-1)^n \frac{\pi}{6}$, $n\in\mathbb{Z}$

5(d) $8\sin^2\theta - 2\cos\theta = 5$

$\Rightarrow 8(1 - \cos^2\theta) - 2\cos\theta = 5$

$\Rightarrow 8\cos^2\theta + 2\cos\theta + 5 - 8 = 0$

$\Rightarrow 8\cos^2\theta + 2\cos\theta - 3 = 0$

$\Rightarrow 8\cos^2\theta + 6\cos\theta - 4\cos\theta - 3 = 0$

$\Rightarrow 2\cos\theta(4\cos\theta + 3) - 1(4\cos\theta + 3) = 0$

$\Rightarrow (4\cos\theta + 3)(2\cos\theta - 1) = 0$

$2\cos\theta - 1 = 0$ হলে , $\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}$, $n\in\mathbb{Z}$

$4\cos\theta + 3 = 0$ হলে, $\cos\theta = -\frac{3}{4} = \cos\alpha$ (ধরি)

$\therefore \theta = 2n\pi \pm \alpha$, $n\in\mathbb{Z}$

$\therefore$ নির্ণেয় সমাধান , $\theta = 2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \alpha$;

যেখানে $\cos\alpha = -\frac{3}{4}$ এবং $n\in\mathbb{Z}$.

5(e) $\tan^2\theta - 2\sqrt{3}\sec\theta + 4 = 0$ [কু.'০৫]

$\Rightarrow \sec^2\theta - 1 - 2\sqrt{3}\sec\theta + 4 = 0$

$\Rightarrow \sec^2\theta - 2\sqrt{3}\sec\theta + 3 = 0$

$\Rightarrow (\sec\theta - \sqrt{3})^2 = 0 \Rightarrow \sec\theta - \sqrt{3} = 0$

$\Rightarrow \sec\theta = \sqrt{3} = \sec\alpha$ (ধরি)

$\therefore \theta = 2n\pi \pm \alpha$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = 2n\pi \pm \alpha$; যেখানে $\sec\alpha = \sqrt{3}$ এবং $n\in\mathbb{Z}$.

5(f) $4\cos^2\theta + 6\sin^2\theta = 5$

$\Rightarrow 4\cos^2\theta + 6\sin^2\theta = 5(\sin^2\theta + \cos^2\theta)$

$\Rightarrow \sin^2\theta = \cos^2\theta \Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm 1 = \tan(\pm\frac{\pi}{4})$

$\therefore \theta = n\pi \pm \frac{\pi}{4}$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi \pm \frac{\pi}{4}$; যেখানে $n\in\mathbb{Z}$.

(g) $2\sin^2 x = 3\cos x$, যখন $0<\theta<2\pi$ [য.'০৯]

$\Rightarrow 2(1-\cos^2 x) = 3\cos x$

$\Rightarrow 2\cos^2 x + 3\cos x - 2 = 0$

$\Rightarrow 2\cos^2 x + 3\cos x - 2 = 0$

$\Rightarrow 2\cos^2 x + 4\cos x - \cos x - 2 = 0$

$\Rightarrow 2\cos x(\cos x + 2) - 1(\cos x + 2) = 0$

$\Rightarrow (\cos x + 2)(2\cos x - 1) = 0$

$\therefore 2\cos x - 1 = 0 \quad [\because \cos x + 2 \neq 0]$

$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore x = 2n\pi \pm \frac{\pi}{3}$; যেখানে $n\in\mathbb{Z}$.

$n = 0$ হলে, $\theta = \frac{\pi}{3}$

$n = 1$ হলে, $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ এবং

$\theta = 2\pi + \frac{\pi}{3}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান , $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

5(h) $\sin^2 2x - 3\cos^2 x = 0$ [কু.'০৯]

$\Rightarrow (2\sin x\cos x)^2 - 3\cos^2 x = 0$

$\Rightarrow 4\sin^2 x\cos^2 x - 3\cos^2 x = 0$

$\Rightarrow (4\sin^2 x - 3)\cos^2 x = 0$

$\therefore \cos^2 x = 0$ হলে, $\cos x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$, যেখানে $n\in\mathbb{Z}$.

$4\sin^2 x - 3 = 0$ হলে, $\sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$\Rightarrow x = n\pi + (-1)^n\frac{\pi}{3}$, $n\in\mathbb{Z}$.

$\therefore x = (2n+1)\frac{\pi}{2}$, $n\pi + (-1)^n\frac{\pi}{3}$, $n\in\mathbb{Z}$.

(a) $5\tan^2\theta - \sec^2\theta = 11$

$\Rightarrow 5\tan^2\theta - (1 + \tan^2\theta) = 11$

$\Rightarrow 4\tan^2\theta = 12 \Rightarrow \tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3} = \tan(\pm\frac{\pi}{3})$

$\therefore \theta = n\pi \pm \frac{\pi}{3}$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi \pm \frac{\pi}{3}$; যেখানে $n\in\mathbb{Z}$.

6(b) $\cot^2\theta + \operatorname{cosec}^2\theta = 3$

$\Rightarrow \cot^2\theta + 1 + \cot^2\theta = 3 \Rightarrow 2\cot^2\theta = 2$

$\Rightarrow \cot^2\theta = 1 \Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm 1 = \tan(\pm\frac{\pi}{4})$

$\therefore \theta = n\pi \pm \frac{\pi}{4}$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi \pm \frac{\pi}{4}$; যেখানে $n\in\mathbb{Z}$

6(c) $\tan^2\theta + \sec^2\theta = 3$

$\Rightarrow \tan^2\theta + 1 + \tan^2\theta = 3 \Rightarrow 2\tan^2\theta = 2$

$\Rightarrow \tan^2\theta = 1 \Rightarrow \tan\theta = \pm 1$

$\Rightarrow \tan\theta = \tan(\pm\frac{\pi}{4})$

$\therefore \theta = n\pi \pm \frac{\pi}{4}$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi \pm \frac{\pi}{4}$; যেখানে $n\in\mathbb{Z}$.

(d) $\sec^2\frac{\theta}{2} - 2\sqrt{2}\tan\frac{\theta}{2} = 0$; $0<\theta<2\pi$ [কু.'০৩; সি.'০৩; ব.'১৩]

$\Rightarrow \sec^2\frac{\theta}{2} = 2\sqrt{2}\tan\frac{\theta}{2}$

উভয় পক্ষকে $\cos^2\frac{\theta}{2}$ দ্বারা গুণ করে আমরা পাই ,

$1 = 2\sqrt{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sqrt{2}\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\therefore \theta = n\pi + (-1)^n\frac{\pi}{4}$, $n\in\mathbb{Z}$.

$n=0$ হলে , $\theta = \frac{\pi}{4}$

$n=-1$ হলে , $\theta = -\pi - \frac{\pi}{4} = -\frac{5\pi}{4}$

$n=1$ হলে , $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$n=-2$ হলে , $\theta = -2\pi + \frac{\pi}{4} = -\frac{7\pi}{4}$

$n=2$ হলে , $\theta = 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান , $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

5(e) $\tan^2\theta - 3\,\mathrm{cosec}^2\theta + 1 = 0$

[কুয়েট '০৩-০৪; টেক্সটাইল '০৫-০৬]

$\Rightarrow \sec^2\theta - 3\mathrm{cosec}^2\theta = 0$

$\Rightarrow \sec^2\theta = 3\,\mathrm{cosec}^2\theta \Rightarrow \frac{1}{\cos^2\theta} = 3\frac{1}{\sin^2\theta}$

$\Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \pm\sqrt{3} = \tan(\pm\frac{\pi}{3})$

$\therefore \theta = n\pi \pm \frac{\pi}{3}$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi \pm \frac{\pi}{3}$; যেখানে $n\in\mathbb{Z}$.

7(a) $4\sin^2\theta + \sqrt{3} = 2(1+\sqrt{3})\sin\theta$

$\Rightarrow 4\sin^2\theta - 2(1+\sqrt{3})\sin\theta + \sqrt{3} = 0$

$\Rightarrow 4\sin^2\theta - 2\sin\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$

$\Rightarrow 2\sin\theta(2\sin\theta - 1) - \sqrt{3}(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(2\sin\theta - \sqrt{3}) = 0$

$2\sin\theta - 1 = 0$ হলে , $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \theta = n\pi + (-1)^n\frac{\pi}{6}$

$2\sin\theta - \sqrt{3} = 0$ হলে , $\sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$\therefore \theta = n\pi + (-1)^n\frac{\pi}{3}$

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi + (-1)^n\frac{\pi}{3}$,

$n\pi + (-1)^n\frac{\pi}{6}$; যেখানে $n\in\mathbb{Z}$

(b) $1 + \sqrt{3}\tan^2\theta = (1+\sqrt{3})\tan\theta,\ 0<\theta<2\pi$

[সি.'০১; য.'০৫; প্র.ভ.প.'০৩]

$\Rightarrow \sqrt{3}\tan^2\theta - \sqrt{3}\tan\theta - \tan\theta + 1 = 0$

$\Rightarrow \sqrt{3}\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$\Rightarrow (\tan\theta - 1)(\sqrt{3}\tan\theta - 1) = 0$

$\tan\theta - 1 = 0$ হলে, $\tan\theta = 1 = \tan\frac{\pi}{4}$

$\therefore \theta = n\pi + \frac{\pi}{4}$, $n\in\mathbb{Z}$

$\sqrt{3}\tan\theta - 1 = 0$ হলে, $\tan\theta = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$\therefore \theta = n\pi + \frac{\pi}{6}$, $n\in\mathbb{Z}$.

$n=0$ হলে , $\theta = \frac{\pi}{4}, \frac{\pi}{6}$

$n=1$ হলে , $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

অথবা , $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান ,

$\theta = \frac{\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}$

7(c) $\sqrt{3}\cot^2\theta + 4\cot\theta + \sqrt{3} = 0$

$\Rightarrow \sqrt{3}\cot^2\theta + 3\cot\theta + \cot\theta + \sqrt{3} = 0$

$\sqrt{3}\cot\theta(\cot\theta + \sqrt{3}) + 1(\cot\theta + \sqrt{3}) = 0$

$\Rightarrow (\cot\theta + \sqrt{3})(\sqrt{3}\cot\theta + 1) = 0$

$\cot\theta + \sqrt{3} = 0$ হলে, $\cot\theta = -\sqrt{3}$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}} = -\tan\frac{\pi}{6} = \tan(\pi - \frac{\pi}{6})$

$\Rightarrow \tan\theta = \tan\frac{5\pi}{6} \quad \therefore \theta = n\pi + \frac{5\pi}{6}, n\in Z$

$\sqrt{3}\cot\theta + 1 = 0$ হলে, $\cot\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan(\pi - \frac{\pi}{3})$

$\Rightarrow \tan\theta = \tan\frac{2\pi}{3} \quad \therefore \theta = n\pi + \frac{2\pi}{3}, n\in Z.$

$\therefore$ নির্ণেয় সমাধান, $\theta = n\pi + \frac{2\pi}{3}, n\pi + \frac{5\pi}{6}$;
যেখানে $n\in Z$.

7(d) $\sin\theta + \operatorname{cosec}\theta = \frac{3}{\sqrt{2}}$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = \frac{3}{\sqrt{2}}$

$\Rightarrow \sqrt{2}\sin^2\theta + \sqrt{2} = 3\sin\theta$

$\Rightarrow \sqrt{2}\sin^2\theta - 3\sin\theta + \sqrt{2} = 0$

$\Rightarrow \sqrt{2}\sin^2\theta - 2\sin\theta - \sin\theta + \sqrt{2} = 0$

$\Rightarrow \sqrt{2}\sin\theta(\sin\theta - \sqrt{2}) - 1(\sin\theta - \sqrt{2}) = 0$

$\Rightarrow (\sin\theta - \sqrt{2})(\sqrt{2}\sin\theta - 1) = 0$

$\therefore \sin\theta - \sqrt{2} = 0$ অথবা, $\sqrt{2}\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \sqrt{2}$ অথবা, $\sin\theta = \frac{1}{\sqrt{2}}$

কিন্তু $\sqrt{2} > 1$ বলে $\sin\theta \neq \sqrt{2}$.

$\therefore \sin\theta = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4}, n\in Z.$

$\therefore$ নির্ণেয় সমাধান, $\theta = n\pi + (-1)^n\frac{\pi}{4}$; যেখানে $n\in Z$

7(e) $\cos\theta + \sec\theta = \frac{5}{2}$

$\Rightarrow \cos\theta + \frac{1}{\cos\theta} = \frac{5}{2}$

$\Rightarrow 2\cos^2\theta + 2 = 5\cos\theta$

$\Rightarrow 2\cos^2\theta - 5\cos\theta + 2 = 0$

$\Rightarrow 2\cos^2\theta - 4\cos\theta - \cos\theta + 2 = 0$

$\Rightarrow 2\cos\theta(\cos\theta - 2) - 1(\cos\theta - 2) = 0$

$\Rightarrow (\cos\theta - 2)(2\cos\theta - 1) = 0$

$\therefore \cos\theta = 2$ অথবা, $\cos\theta = \frac{1}{2}$

কিন্তু $-1 \le \cos\theta \le 1$ বলে $\cos\theta \neq 2$

$\therefore \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n\in Z$

$\therefore$ নির্ণেয় সমাধান, $\theta = 2n\pi \pm \frac{\pi}{3}$, যেখানে $n\in Z$.

8(a) $\tan^2\theta + \cot^2\theta = 2$ [য.'০০; চ.'০১; ব.'০৭]

$\Rightarrow \tan^2\theta + \frac{1}{\tan^2\theta} = 2$

$\Rightarrow \tan^4\theta - 2\tan^2\theta + 1 = 0 \Rightarrow (\tan^2\theta - 1)^2 = 0$

$\Rightarrow \tan^2\theta - 1 = 0 \Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm 1 = \tan(\pm\frac{\pi}{4})$

$\therefore \theta = n\pi \pm \frac{\pi}{4}, n\in Z.$

$\therefore$ নির্ণেয় সমাধান, $\theta = n\pi \pm \frac{\pi}{4}$, যেখানে $n\in Z$.

8(b) $\sec^2\theta + 3\operatorname{cosec}^2\theta = 8$

$\Rightarrow 1 + \tan^2\theta + 3(1 + \cot^2\theta) = 8$

$\Rightarrow \tan^2\theta + 3\frac{1}{\tan^2\theta} = 4$

$\Rightarrow \tan^4\theta - 4\tan^2\theta + 3 = 0$

$\Rightarrow \tan^4\theta - 3\tan^2\theta - \tan^2\theta + 3 = 0$

$\Rightarrow \tan^2\theta(\tan^2\theta - 3) - 1(\tan^2\theta - 3) = 0$

$\Rightarrow (\tan^2\theta - 3)(\tan^2\theta - 1) = 0$

$\tan^2\theta - 3 = 0$ হলে, $\tan^2\theta = 3$

$\Rightarrow \tan\theta = \pm\sqrt{3} = \tan(\pm\frac{\pi}{3})$

$\therefore \theta = n\pi \pm \frac{\pi}{3}$, $n \in Z$.

$\tan^2\theta - 1 = 0$ হলে , $\tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm 1 = \tan(\pm\frac{\pi}{4})$

$\therefore \theta = n\pi \pm \frac{\pi}{4}$, $n \in Z$.

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi \pm \frac{\pi}{4}$, $n\pi \pm \frac{\pi}{3}$; যেখানে $n \in Z$.

9(a) $\frac{\cos\theta}{1+\sin\theta} + \tan\theta = 2$ [রা.'০২; সি.'১৩]

$\Rightarrow \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta} = 2$

$\Rightarrow \cos^2\theta + \sin\theta + \sin^2\theta = 2\cos\theta(1 + \sin\theta)$

$\Rightarrow 1 + \sin\theta - 2\cos\theta(1 + \sin\theta) = 0$

$\Rightarrow (1 + \sin\theta)(1 - 2\cos\theta) = 0$

$\therefore 1 + \sin\theta = 0$ অথবা, $1 - 2\cos\theta = 0$.

কিন্তু , এখানে $1 + \sin\theta \neq 0$.

$\therefore 1 - 2\cos\theta = 0 \Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}$, $n \in Z$.

$\therefore$ নির্ণেয় সমাধান , $\theta = 2n\pi \pm \frac{\pi}{3}$; যেখানে $n \in Z$.

9(b) $\cot\theta - \tan\theta = 2$ [চ.'০২]

$\Rightarrow \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = 2$

$\Rightarrow \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = 2$

$\Rightarrow \cos 2\theta = 2\sin\theta\cos\theta$

$\Rightarrow \cos 2\theta = \sin 2\theta \Rightarrow \tan 2\theta = 1 = \tan\frac{\pi}{4}$

$\therefore 2\theta = n\pi + \frac{\pi}{4}$, $n \in Z$.

$\Rightarrow \theta = (4n + 1)\frac{\pi}{8}$

$\therefore$ নির্ণেয় সমাধান, $\theta = (4n + 1)\frac{\pi}{8}$; যেখানে $n \in Z$.

9(c) $\operatorname{cosec}\theta \cot\theta = 2\sqrt{3}$

$\Rightarrow \frac{1}{\sin\theta}\frac{\cos\theta}{\sin\theta} = 2\sqrt{3}$

$\Rightarrow 2\sqrt{3}\sin^2\theta - \cos\theta = 0$

$\Rightarrow 2\sqrt{3}(1 - \cos^2\theta) - \cos\theta = 0$

$\Rightarrow 2\sqrt{3} - 2\sqrt{3}\cos^2\theta - \cos\theta = 0$

$\Rightarrow 2\sqrt{3}\cos^2\theta + \cos\theta - 2\sqrt{3} = 0$

$\Rightarrow 2\sqrt{3}\cos^2\theta + 4\cos\theta - 3\cos\theta - 2\sqrt{3} = 0$

$\Rightarrow 2\cos\theta(\sqrt{3}\cos\theta + 2) - \sqrt{3}(\sqrt{3}\cos\theta + 2) = 0$

$\Rightarrow (\sqrt{3}\cos\theta + 2)(2\cos\theta - \sqrt{3}) = 0$

$\therefore \sqrt{3}\cos\theta + 2 = 0$ অথবা, $2\cos\theta - \sqrt{3} = 0$.

$\Rightarrow \cos\theta = -\frac{2}{\sqrt{3}}$ অথবা , $\cos\theta = \frac{\sqrt{3}}{2}$.

$-\frac{2}{\sqrt{3}} < -1$ বলে $\cos\theta \neq -\frac{2}{\sqrt{3}}$.

$\therefore \cos\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$, $n \in Z$.

$\therefore$ নির্ণেয় সমাধান , $\theta = 2n\pi \pm \frac{\pi}{6}$, যেখানে $n \in Z$.

9(d) $3\tan\theta + \cot\theta = 5\operatorname{cosec}\theta$ [রা.'১৩]

$\Rightarrow 3.\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 5\frac{1}{\sin\theta}$

$\Rightarrow 3\sin^2\theta + \cos^2\theta = 5\cos\theta$

$\Rightarrow 3(1 - \cos^2\theta) + \cos^2\theta = 5\cos\theta$

$\Rightarrow 3 - 3\cos^2\theta + \cos^2\theta = 5\cos\theta$

$\Rightarrow 2\cos^2\theta + 5\cos\theta - 3 = 0$

$\Rightarrow 2\cos^2\theta + 6\cos\theta - \cos\theta - 3 = 0$

$\Rightarrow 2\cos\theta(\cos\theta + 3) - 1(\cos\theta + 3) = 0$

$\Rightarrow (\cos\theta + 3)(2\cos\theta - 1) = 0$

$\therefore 2\cos\theta - 1 = 0$ অথবা, $\cos\theta + 3 = 0$.

কিন্তু $-3 < -1$ বলে, $\cos\theta \neq -3$

$\therefore 2\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}$.

$\therefore$ নির্ণেয় সমাধান, $\theta = 2n\pi \pm \frac{\pi}{3}$, যেখানে $n \in \mathbb{Z}$.

9(e) $\sin\theta + \cos\theta = \sqrt{2\sin 2\theta}$ [ঢা.'০৪]

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\sin 2\theta$

$\Rightarrow 1 + \sin 2\theta = 2\sin 2\theta \Rightarrow \sin 2\theta = 1$

$\therefore 2\theta = (4n + 1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = (4n + 1)\frac{\pi}{4} = n\pi + \frac{\pi}{4}$.

$\therefore$ নির্ণেয় সমাধান, $\theta = (4n + 1)\frac{\pi}{4}$, যেখানে $n \in \mathbb{Z}$.

9(f) $\frac{\sqrt{3}}{\sin 2x} - \frac{1}{\cos 2x} = 4$ [বুয়েট'০৬-০৭]

$\Rightarrow \frac{\sqrt{3}\cos 2x - \sin 2x}{\sin 2x \cos 2x} = 4$

$\Rightarrow \frac{\sqrt{3}}{2}\cos 2x - \frac{1}{2}\sin 2x = 2\sin 2x \cos 2x$

$\Rightarrow \cos 2x \sin\frac{\pi}{3} - \sin 2x \cos\frac{\pi}{3} = \sin 4x$

$\Rightarrow \sin(\frac{\pi}{3} - 2x) = \sin 4x$

$\Rightarrow \sin 4x + \sin(2x - \frac{\pi}{3}) = 0$

$\Rightarrow 2\sin\frac{1}{2}(6x - \frac{\pi}{3})\cos\frac{1}{2}(2x + \frac{\pi}{3}) = 0$

$\Rightarrow \sin(3x - \frac{\pi}{6})\cos(x + \frac{\pi}{6}) = 0$

$\sin(3x - \frac{\pi}{6})$ হলে, $3x - \frac{\pi}{6} = n\pi$, $n \in \mathbb{Z}$

$\Rightarrow 3x = n\pi + \frac{\pi}{6} \quad \therefore x = (6n + 1)\frac{\pi}{18}$

$\cos(x + \frac{\pi}{6})$ হলে, $x + \frac{\pi}{6} = (2n + 1)\frac{\pi}{2}$, $n \in \mathbb{Z}$

$\Rightarrow x = n + \frac{\pi}{2} - \frac{\pi}{6} = n + \frac{\pi}{3} = (3n + 1)\frac{\pi}{3}$.

9(g) $2\sin\theta\tan\theta + 1 = \tan\theta + 2\sin\theta$ [চুয়েট'০৫-০৬]

$\Rightarrow 2\sin\theta \times \frac{\sin\theta}{\cos\theta} + 1 = \frac{\sin\theta}{\cos\theta} + 2\sin\theta$

$\Rightarrow 2\sin^2\theta + \cos\theta = \sin\theta + 2\sin\theta\cos\theta$

$\Rightarrow 2\sin^2\theta - 2\sin\theta\cos\theta + \cos\theta - \sin\theta = 0$

$\Rightarrow 2\sin\theta(\sin\theta - \cos\theta) - 1(\sin\theta - \cos\theta) = 0$

$\Rightarrow (\sin\theta - \cos\theta)(2\sin\theta - 1) = 0$

$\sin\theta - \cos\theta = 0$ হলে, $\sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4} \quad \therefore \theta = n\pi + \frac{\pi}{4}$, $n \in \mathbb{Z}$

$2\sin\theta - 1 = 0$ হলে, $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \theta = n\pi + (-1)^n\frac{\pi}{6}$, $n \in \mathbb{Z}$

10. সমাধান ঃ

(a) $\cos 6x + \cos 4x = \sin 3x + \sin x$ [ব.'০২]

$\Rightarrow 2\cos 5x\cos x = 2\sin 2x\cos x$

$\Rightarrow \cos x(\cos 5x - \sin 2x) = 0$

$\therefore \cos x = 0 \Rightarrow x = (2n + 1)\frac{\pi}{2}$, $n \in \mathbb{Z}$

অথবা, $\cos 5x - \sin 2x = 0$

$\Rightarrow \cos 5x = \sin 2x = \cos(\frac{\pi}{2} - 2x)$

$\therefore 5x = 2n\pi \pm (\frac{\pi}{2} - 2x)$, $n \in \mathbb{Z}$.

$\Rightarrow 10x = 4n\pi \pm (\pi - 4x)$

'+' চিহ্ন নিয়ে আমরা পাই , $10x = 4n\pi + \pi - 4x$

$\Rightarrow 14x = (4n+1)\pi \quad \therefore x = (4n+1)\frac{\pi}{14}$

'−' চিহ্ন নিয়ে আমরা পাই , $10x = 4n\pi - \pi + 4x$

$\Rightarrow 6x = (4n-1)\pi \quad \therefore x = (4n-1)\frac{\pi}{6}$

$\therefore$ নির্ণেয় সমাধান , $x = (2n+1)\frac{\pi}{2}$,

$(4n+1)\frac{\pi}{14}, (4n-1)\frac{\pi}{6}$; $n \in \mathbb{Z}$.

10(b) $\sin x + \sin 2x + \sin 3x = 0$ [ব.'০১,'১৩ ; রা.'০৭; কু,'১৩]

$\Rightarrow \sin 2x + (\sin x + \sin 3x) = 0$

$\Rightarrow \sin 2x + 2\sin 2x \cos x = 0$

$\Rightarrow \sin 2x(1 + 2\cos x) = 0$

$\sin 2x = 0$ হলে, $2x = n\pi \Rightarrow x = \frac{n\pi}{2}$, $n \in \mathbb{Z}$

$1 + 2\cos x = 0$ হলে, $\cos x = -\frac{1}{2} = -\cos\frac{\pi}{3}$

$\Rightarrow \cos x = \cos(\pi - \frac{\pi}{3}) = \cos\frac{2\pi}{3}$

$\therefore x = 2n\pi \pm \frac{2\pi}{3}$, $n \in \mathbb{Z}$

$\therefore$ নির্ণেয় সমাধান , $x = \frac{n\pi}{2}, 2n\pi \pm \frac{2\pi}{3}$; $n \in \mathbb{Z}$.

10(c) $\cos\theta - \cos 7\theta = \sin 4\theta$ [ঢা.'০৩, '১০; কু. '০৬; য.'০৮; দি.'০৯; ব.'১১]

$\Rightarrow 2\sin\frac{1}{2}(\theta + 7\theta)\sin\frac{1}{2}(7\theta - \theta) = \sin 4\theta$

$\Rightarrow 2\sin 4\theta \sin 3\theta - \sin 4\theta = 0$

$\Rightarrow \sin 4\theta(2\sin 3\theta - 1) = 0$

$\therefore \sin 4\theta = 0$

$\therefore 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$, $n \in \mathbb{Z}$.

অথবা , $2\sin 3\theta - 1 = 0 \Rightarrow \sin 3\theta = \frac{1}{2}$

$\Rightarrow \sin 3\theta = \sin\frac{\pi}{6}$

$\therefore 3\theta = n\pi + (-1)^n\frac{\pi}{6}$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$

$\therefore$ নির্ণেয় সমাধান, $\theta = \frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$; $n \in \mathbb{Z}$

10(d) $\cos 2x + \sin x = 1$ [ব.'০১ ; ঢা.'০৪]

$\Rightarrow 1 - 2\sin^2 x + \sin x = 1$

$\Rightarrow 2\sin^2 x - \sin x = 0 \Rightarrow \sin x(2\sin x - 1) = 0$

$\sin x = 0$ হলে, $x = n\pi$, $n \in \mathbb{Z}$.

$2\sin x - 1 = 0$ হলে , $\sin x = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore x = n\pi + (-1)^n\frac{\pi}{6}$, $n \in \mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান, $x = n\pi, n\pi + (-1)^n\frac{\pi}{6}$; $n \in \mathbb{Z}$.

10(e) $2\sin\theta\tan\theta + 1 = \tan\theta + 2\sin\theta$ [য.'০৩; প্র.ভ.প.,'০৫]

$\Rightarrow 2\sin\theta\tan\theta + 1 - \tan\theta - 2\sin\theta = 0$

$\Rightarrow \tan\theta(2\sin x - 1) - 1(2\sin x - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(\tan\theta - 1) = 0$

$2\sin\theta - 1 = 0$ হলে , $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore x = n\pi + (-1)^n\frac{\pi}{6}$, $n \in \mathbb{Z}$.

$\tan\theta = 1$ হলে , $\tan\theta = 1 = \tan\frac{\pi}{4}$

$\therefore \theta = n\pi + \frac{\pi}{4}$, $n \in \mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান ,

$\theta = n\pi + (-1)^n\frac{\pi}{6}$, $n\pi + \frac{\pi}{4}$; $n \in \mathbb{Z}$.

10(f) $\sqrt{2}\cos 3\theta - \cos\theta = \cos 5\theta$ [সি.'০১,'১০; ঢা.'০৪; চ.'০৮; কু.'০৮; য.'১৩]

$\Rightarrow \sqrt{2}\cos 3\theta - (\cos\theta + \cos 5\theta) = 0$
$\Rightarrow \sqrt{2}\cos 3\theta - 2\cos 3\theta \cos 2\theta = 0$
$\Rightarrow \sqrt{2}\cos 3\theta (1 - \sqrt{2}\cos 2\theta) = 0$
$\sqrt{2}\cos 3\theta = 0$ হলে, $\cos 3\theta = 0$

$\therefore \quad 3\theta = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = (2n+1)\frac{\pi}{6}$

$1 - \sqrt{2}\cos 2\theta = 0$ হলে , $\cos 2\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos 2\theta = \cos\frac{\pi}{4} \quad \therefore 2\theta = 2n\pi \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = n\pi \pm \frac{\pi}{8}$.

$\therefore$ নির্ণেয় সমাধান ,

$\theta = (2n+1)\frac{\pi}{6}$, $n\pi \pm \frac{\pi}{8}$; যেখানে $n \in \mathbb{Z}$.

11(a) $\sin 5\theta + \sin\theta = \sin 3\theta$.
$\Rightarrow 2\sin 3\theta \cos 2\theta - \sin 3\theta = 0$
$\Rightarrow \sin 3\theta (2\cos 2\theta - 1) = 0$

$\sin 3\theta = 0$ হলে, $3\theta = n\pi \therefore \theta = \frac{1}{3}n\pi$, $n \in \mathbb{Z}$

$2\cos 2\theta - 1 = 0 \quad \cos 2\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore \quad 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$, $n \in \mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = \frac{1}{3}n\pi$, $n\pi \pm \frac{\pi}{6}$; $n \in \mathbb{Z}$.

11(b) $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$
$\Rightarrow (\cos 7\theta + \cos\theta) + (\cos 5\theta + \cos 3\theta) = 0$
$\Rightarrow 2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos\theta = 0$
$\Rightarrow 2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$
$\Rightarrow 2\cos 4\theta \cdot 2\cos 2\theta \cos\theta = 0$
$\Rightarrow \cos\theta \cos 2\theta \cos 4\theta = 0$

$\cos\theta = 0$ হলে , $\theta = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.

$\cos 2\theta = 0$ হলে , $2\theta = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = (2n+1)\frac{\pi}{4}$

$\cos 4\theta = 0$ হলে, $4\theta = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = (2n+1)\frac{\pi}{8}$

$n = 0$ ধরলে , $\theta = \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{8}$

$n = 1$ ধরলে , $\theta = \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{8}$

$n = 2$ ধরলে , $\theta = \frac{5\pi}{2}, \frac{5\pi}{4}, \frac{5\pi}{8}$

$n = 3$ ধরলে , $\theta = \frac{7\pi}{2}, \frac{7\pi}{4}, \frac{7\pi}{8}$

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান ,

$\theta = \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{8}, \frac{3\pi}{4}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$.

11(c) $2\sin 2\theta + 3\cos\theta = 0$
$\Rightarrow 2.2\sin\theta \cos\theta + 3\cos\theta = 0$
$\Rightarrow \cos\theta(4\sin\theta + 3) = 0$

$\cos\theta = 0$ হলে, $\theta = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.

$4\sin\theta + 3 = 0$ হলে, $\sin\theta = -\frac{3}{4} = \sin\alpha$ (ধরি)

$\therefore \quad \theta = n\pi + (-1)^n\alpha$, $n \in \mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = (2n+1)\frac{\pi}{2}$,
$n\pi + (-1)^n\alpha$; যখন $\sin\alpha = -\frac{3}{4}$ এবং $n \in \mathbb{Z}$.

11(d) $\sin 7\theta - \sqrt{3}\cos 4\theta = \sin\theta$
[য.'০৫; দি.'১০; প্র.ভ.প.'৯৬]
$\Rightarrow \sin 7\theta - \sin\theta - \sqrt{3}\cos 4x = 0$
$\Rightarrow 2\sin 3\theta \cos 4\theta - \sqrt{3}\cos 4x = 0$
$\Rightarrow \cos 4x (2\sin 3\theta - \sqrt{3}) = 0$

$\cos 4x = 0$ হলে , $4x = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$

$\Rightarrow x = (2n+1)\frac{\pi}{8}$

$2\sin 3\theta - \sqrt{3} = 0$ হলে , $\sin 3\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 3\theta = \sin\frac{\pi}{3}$

$\therefore 3\theta = n\pi + (-1)^n \frac{\pi}{3}$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = \frac{1}{3}\{ n\pi + (-1)^n \frac{\pi}{3} \}$

$\therefore$ নির্ণেয় সমাধান , $\theta = (2n+1)\frac{\pi}{8}$,

$\frac{1}{3}\{ n\pi + (-1)^n \frac{\pi}{3} \}$; $n \in \mathbb{Z}$.

11(e) $\sin 2\theta + 4\cos\theta = \sqrt{3}\sin\theta + 2\sqrt{3}$

$\Rightarrow 2\sin\theta\cos\theta + 4\cos\theta - \sqrt{3}(\sin\theta + 2) = 0$

$\Rightarrow 2\cos\theta(\sin\theta + 2) - \sqrt{3}(\sin\theta + 2) = 0$

$\Rightarrow (\sin\theta + 2)(2\cos\theta - \sqrt{3}) = 0$

$\therefore 2\cos\theta - \sqrt{3} = 0$ [$\because -2 < -1$, $\sin\theta + 2 \neq 0$]

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\therefore \theta = 2n\pi \pm \frac{\pi}{6}$, $n \in \mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = 2n\pi \pm \frac{\pi}{6}$, $n \in \mathbb{Z}$.

12(a) $\cos 2x + \cos x + 1 = 0$; $0° < x < 360°$ [ঢা.'১২]

$\Rightarrow 2\cos^2 x - 1 + \cos x + 1 = 0$

$\Rightarrow \cos x(2\cos x + 1) = 0$

$\cos x = 0$ হলে , $x = (2n+1)90°$, $n \in \mathbb{Z}$.

$2\cos x + 1 = 0$ হলে , $\cos x = -\frac{1}{2} = \cos 120°$

$\therefore x = n.360° \pm 120°$, $n \in \mathbb{Z}$.

$n = 0$ এর জন্য $x = 90°, \pm 120°$;

$n = 1$ এর জন্য $x = 270°, 240°, 480°$.

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান ,

$x = 90°, 120°, 240°, 270°$

(b) $\sec 4x - \sec 2x = 2$; $0° < x < 360°$

[য. '০১; ঢা.'০৮; চুয়েট'০৩-০৪]

$\Rightarrow \frac{1}{\cos 4x} - \frac{1}{\cos 2x} = 2$

$\Rightarrow \cos 2x - \cos 4x = 2\cos 4x \cos 2x$

$\Rightarrow \cos 2x - \cos 4x = \cos 6x + \cos 2x$

$\Rightarrow \cos 6x + \cos 4x = 0 \Rightarrow 2\cos 5x \cos x = 0$

$\cos x = 0$ হলে , $x = (2n+1)90°$, $n \in \mathbb{Z}$.

$n = 0, 1, 2$ বসিয়ে পাই , $x = 90°, 270°, 450°$

আবার , $\cos 5x = 0$ হলে , $5x = (2n+1)90°$

$\Rightarrow x = (2n+1)18°$, $n \in \mathbb{Z}$.

$n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ বসিয়ে পাই , $x = 18°, 54°, 90°, 126°, 162°, 198°, 234°, 270°, 306°, 342°, 378°$.

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান ,

$x = 18°, 54°, 90°, 126°, 162°, 198°, 234°, 270°, 306°, 342°$.

12(c) $\cos\theta + \cos 2\theta + \cos 3\theta = 0$ [ঢা.'০৫; কু.'১৩]

$\Rightarrow 2\cos 2\theta \cos\theta + \cos 2\theta = 0$

$\Rightarrow \cos 2\theta(2\cos\theta + 1) = 0$

এখন , $\cos 2\theta = 0$ হলে , $2\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{4}$, $n \in \mathbb{Z}$.

$2\cos\theta + 1 = 0$ হলে, $\cos\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\therefore \theta = 2n\pi \pm \frac{2\pi}{3}$, $n \in \mathbb{Z}$.

$\therefore \theta = (2n+1)\frac{\pi}{4}$, $2n\pi \pm \frac{2\pi}{3}$; $n \in \mathbb{Z}$.

12(d) $\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta = 0$ [সি.'০৩]

$\Rightarrow (\sin 3\theta + \sin 9\theta) + (\sin 5\theta + \sin 7\theta) = 0$

$\Rightarrow 2\sin 6\theta \cos 3\theta + 2\sin 6\theta \cos\theta = 0$

$\Rightarrow \sin 6\theta\,(\cos 3\theta + \cos\theta) = 0$

$\Rightarrow \sin 6\theta \,.2\cos 2\theta \cos\theta = 0$

$\Rightarrow \sin 6\theta \cos\theta \cos 2\theta = 0$

এখন, $\sin 6\theta = 0$ হলে $6\theta = n\pi \Rightarrow \theta = \frac{n\pi}{6}$, $n\in\mathbb{Z}$.

$\cos\theta = 0$ হলে , $\theta = (2n+1)\frac{\pi}{2}$, $n\in\mathbb{Z}$.

এবং $\cos 2\theta = 0$ হলে , $2\theta = (2n+1)\frac{\pi}{2}$, $n\in\mathbb{Z}$.

$\Rightarrow \theta = (2n+1)\frac{\pi}{4}$

$\therefore\ x = \frac{n\pi}{6}, (2n+1)\frac{\pi}{2}, (2n+1)\frac{\pi}{4}; n\in\mathbb{Z}$

(e) $\sin\theta + \sin 2\theta + \sin 3\theta = 1 + \cos\theta + \cos 2\theta$

[চ.'০৮; বুয়েট'০৮-০৯]

$\Rightarrow (\sin 3\theta + \sin\theta) + \sin 2\theta = (1+\cos 2\theta) + \cos\theta$

$\Rightarrow 2\sin 2\theta \cos\theta + 2\sin\theta\cos\theta = 2\cos^2\theta + \cos\theta$

$\Rightarrow \cos\theta\,(2.2\sin\theta\cos\theta + 2\sin\theta - 2\cos\theta - 1) = 0$

$\Rightarrow \cos\theta\,\{2\sin\theta\,(2\cos\theta+1) - 1(2\cos\theta - 1)\} = 0$

$\Rightarrow \cos\theta\,(2\cos\theta+1)\,(2\sin\theta - 1) = 0$

$\therefore\ \cos\theta = 0$ নিঃ , $\theta = (2n+1)\frac{\pi}{2}$, $n\in\mathbb{Z}$.

$2\cos\theta + 1 = 0 \Rightarrow \cos\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$ হলে,

$\theta = 2n\pi \pm \frac{2\pi}{3}$, $n\in\mathbb{Z}$.

$2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$ হলে , $\theta = n\pi + (-1)^n\frac{\pi}{6}$

13(a) $\cos 9\theta\cos 7\theta = \cos 5\theta\cos 3\theta$, $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ [ঢা.'১২]

$\Rightarrow 2\cos 9\theta\cos 7\theta = 2\cos 5\theta\cos 3\theta$

$\Rightarrow \cos 16\theta + \cos 2\theta = \cos 8\theta + \cos 2\theta$

$\Rightarrow \cos 16\theta - \cos 8\theta = 0$

$\Rightarrow 2\sin 12\theta\sin 4\theta = 0$

$\sin 4\theta = 0$ হলে, $4\theta = n\pi$, $n\in\mathbb{Z}$. $\therefore\ \theta = \frac{n\pi}{4}$

$\sin 12\theta = 0$ হলে, $12\theta = n\pi$, $n\in\mathbb{Z}$. $\therefore\ \theta = \frac{n\pi}{12}$

যখন $n = 0$, তখন $\theta = 0$

যখন $n = \pm 1$, তখন $\theta = \pm\frac{\pi}{4}, \pm\frac{\pi}{12}$

যখন $n = \pm 2$, তখন $\theta = \pm\frac{\pi}{2}, \pm\frac{\pi}{6}$

যখন $n = -3$, তখন $\theta = \pm\frac{3\pi}{4}, \pm\frac{\pi}{4}$

$\therefore$ প্রদত্ত সীমার মধ্যে , $\theta = 0, \pm\frac{\pi}{12}, \pm\frac{\pi}{6}$.

13(b) $4\cos x\cos 2x\cos 3x = 1$, $0 < x < \pi$

[দি.'০৮,'১১; কু .'১২ ; ঢা.'০৭; রা.'০৯,'১২; দি.'১১]

$\Rightarrow 2\cos 2x\,(2\cos 3x\cos x) = 1$

$\Rightarrow 2\cos 2x\,(\cos 4x + \cos 2x) = 1$

$\Rightarrow 2\cos 4x\cos 2x + 2\cos^2 2x - 1 = 0$

$\Rightarrow 2\cos 4x\cos 2x + \cos 4x = 0$

$\Rightarrow \cos 4x\,(2\cos 2x + 1) = 0$

$\cos 4x = 0$ হলে, $4x = (2n+1)\frac{\pi}{2}$, $n\in\mathbb{Z}$.

$\Rightarrow x = (2n+1)\frac{\pi}{8}$

$2\cos 2x + 1 = 0$ হলে , $\cos 2x = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\therefore\ 2x = 2n\pi \pm \frac{2\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3}$, $n\in\mathbb{Z}$.

$n = 0$ হলে , $x = \frac{\pi}{8}, \pm\frac{\pi}{3}$

$n = 1$ হলে , $x = \frac{3\pi}{8}, \frac{4\pi}{3}, \frac{2\pi}{3}$

$n = 2$ হলে , $x = \frac{5\pi}{8}, 2\pi \pm \frac{\pi}{3}$

$n = 3$ হলে , $x = \frac{7\pi}{8}, 3\pi \pm \frac{5\pi}{8}$.

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান ,

$$x = \frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{2\pi}{3}, \frac{5\pi}{8}, \frac{7\pi}{8}$$

13(c) $2\sin\theta \sin 3\theta = 1$ [চ.'০১; য. '০৮,'১৩; চ., সি., ব.'০৯; রা.'১০; দি.'১২]

$\Rightarrow \cos 2\theta - \cos 4\theta = 1$

$\Rightarrow \cos 2\theta - (1 + \cos 4\theta) = 0$

$\Rightarrow \cos 2\theta - 2\cos^2 2\theta = 0$

$\Rightarrow \cos 2\theta(1 - 2\cos 2\theta) = 0$

$\cos 2\theta = 0$ হলে, $2\theta = (2n+1)\frac{\pi}{2}$; $n \in \mathbb{Z}$.

$$\Rightarrow \theta = (2n+1)\frac{\pi}{4}$$

$1 - 2\cos 2\theta = 0$ হলে, $\cos 2\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$, $n \in \mathbb{Z}$.

$n = 0$ হলে, $\theta = \frac{\pi}{4}, \pm\frac{\pi}{6}$

$n = 1$ হলে, $\theta = \frac{3\pi}{4}, \frac{7\pi}{6}, \frac{5\pi}{6}$.

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান,

$$\theta = \frac{\pi}{4}, \frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}.$$

13(d) $4\sin\theta\cos\theta = 1 - 2\sin\theta + 2\cos\theta$, $0° < \theta < 180°$ [রা'০৩; সি.'১৩]

$\Rightarrow 4\sin\theta\cos\theta + 2\sin\theta - 1(1 + 2\cos\theta) = 0$

$\Rightarrow 2\sin\theta(2\cos\theta + 1) - 1(1 + 2\cos\theta) = 0$

$\Rightarrow (2\cos\theta + 1)(2\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0$ হলে, $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \theta = n \times 180° + (-1)^n 30°$, $n \in \mathbb{Z}$.

$2\cos\theta + 1 = 0$ হলে, $\cos\theta = -\frac{1}{2} = \cos 120°$

$\therefore \theta = n \times 360° \pm 120°$; $n \in \mathbb{Z}$.

$n = 0$ হলে, $\theta = 30°, \pm 120°$

$n = 1$ হলে, $\theta = 150°, 240°, 480°$.

$\therefore$ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান,

$\theta = 30°, 120°, 150°$.

13(e) $\sin 2\theta = \cos 3\theta$

$\Rightarrow \cos 3\theta = \sin 2\theta = \cos(\frac{\pi}{2} - 2\theta)$, $n \in \mathbb{Z}$.

$\therefore 3\theta = 2n\pi \pm (\frac{\pi}{2} - 2\theta)$

$\therefore 3\theta = 2n\pi + \frac{\pi}{2} - 2\theta \Rightarrow 5\theta = 2n\pi + \frac{\pi}{2}$

$$\Rightarrow \theta = (4n+1)\frac{\pi}{10}$$

অথবা, $3\theta = 2n\pi - \frac{\pi}{2} + 2\theta \Rightarrow \theta = (4n-1)\frac{\pi}{2}$

13(f) $\sin 3x \sin x = \cos 2x + \frac{1}{2}$

$\Rightarrow 2\sin 3x \sin x = 2\cos 2x + 1$

$\Rightarrow \cos 2x - \cos 4x - 2\cos 2x - 1 = 0$

$\Rightarrow -\cos 2x - (2\cos^2 2x - 1) - 1 = 0$

$\Rightarrow -\cos 2x - 2\cos^2 2x + 1 - 1 = 0$

$\Rightarrow 2\cos^2 2x + \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos 2x + 1) = 0$

$\therefore \cos 2x = 0$ হলে, $2x = (2n+1)\frac{\pi}{2}$; $n \in \mathbb{Z}$

$$\Rightarrow x = (2n+1)\frac{\pi}{4}$$

আবার, $2\cos 2x + 1 = 0 \Rightarrow \cos 2x = -\frac{1}{2}$.

$\Rightarrow \cos 2x = \cos\frac{2\pi}{3}$ হলে,

$$2x = 2n\pi \pm \frac{2\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3} \text{ ; } n \in \mathbb{Z}$$

14(a) $\cos^3 x - \cos x \sin x - \sin^3 x = 1$

$\Rightarrow (\cos x - \sin x)(\sin^2 x + \cos^2 x + \sin x \cos x) - (1 + \sin x \cos x) = 0$

$\Rightarrow (\cos x - \sin x)(1 + \sin x \cos x) - 1(1 + \sin x \cos x) = 0$

$\Rightarrow (1 + \sin x \cos x)(\cos x - \sin x - 1) = 0$

$\Rightarrow (1+\frac{1}{2}\sin 2x)(\cos x-\sin x-1)=0$

$\Rightarrow (2+\sin 2x)(\cos x-\sin x-1)=0$

$\therefore \cos x-\sin x-1=0$ [$\because -2<-1, \sin 2x\neq -2$]

$\Rightarrow \cos x-\sin x=1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$

$\Rightarrow \cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}=\cos\frac{\pi}{4}$

$\Rightarrow \cos(x+\frac{\pi}{4})=\cos\frac{\pi}{4}$

$\therefore x+\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$, $n\in\mathbb{Z}$.

$\Rightarrow x=2n\pi\pm\frac{\pi}{4}-\frac{\pi}{4}=2n\pi, 2n\pi-\frac{\pi}{2}$

$\therefore$ নির্ণেয় সমাধান , $x=2n\pi, 2n\pi-\frac{\pi}{2}$; $n\in\mathbb{Z}$.

14(b) $\cos^3 x\sin 3x+\sin^3 x\cos 3x=\frac{3}{4}$ [চ.'০২]

$\Rightarrow \cos^3 x(3\sin x-4\sin^3 x)+$

$\sin^3 x(4\cos^3 x-3\cos x)=\frac{3}{4}$

$\Rightarrow 4(3\sin x\cos^3 x-4\sin^3 x\cos^3 x+4\sin^3 x\cos^3 x-3\sin^3 x\cos x)=3$

$\Rightarrow 12\sin x\cos x(\cos^2 x-\sin^2 x)=3$

$\Rightarrow 2\sin 2x.\cos 2x=1\Rightarrow \sin 4x=1$

$\therefore 4x=(4n+1)\frac{\pi}{2}\Rightarrow x=(4n+1)\frac{\pi}{8}$; $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $x=(4n+1)\frac{\pi}{8}$, $n\in\mathbb{Z}$.

14(c) $\sin^2 2x-3\cos^2 x=0$ [কু.'১০; সি.'১১]

$\Rightarrow (2\sin x\cos x)^2-3\cos^2 x=0$

$\Rightarrow \cos^2 x(4\sin^2 x-3=0$

$\cos^2 x=0$ হলে , $\cos x=0$

$\therefore x=(2n+1)\frac{\pi}{2}$, $n\in\mathbb{Z}$.

$4\sin^2 x-3=0$ হলে , $\sin^2 x=\frac{3}{4}$

$\Rightarrow \sin x=\pm\frac{\sqrt{3}}{2}=\sin(\pm\frac{\pi}{3})$

$\therefore x=n\pi\pm(-1)^n\frac{\pi}{3}$

$\therefore$ নির্ণেয় সমাধান ,

$x=(2n+1)\frac{\pi}{2}$, $n\pi\pm(-1)^n\frac{\pi}{3}$; $n\in\mathbb{Z}$.

15(a) $\tan x+\tan 2x+\tan x\tan 2x=1$ [রা.'০২; ঢা.'০৯]

$\Rightarrow \tan 2x+\tan x=1-\tan 2x\tan x$

$\Rightarrow \frac{\tan 2x+\tan x}{1-\tan 2x\tan x}=1\Rightarrow \tan(2x+x)=1$

$\Rightarrow \tan 3x=\tan\frac{\pi}{4}$ $\therefore 3x=n\pi+\frac{\pi}{4}$, $n\in\mathbb{Z}$.

$\Rightarrow x=\frac{1}{3}(n\pi+\frac{\pi}{4})=(4n+1)\frac{\pi}{12}$

$\therefore$ নির্ণেয় সমাধান , $x=(4n+1)\frac{\pi}{12}$, $n\in\mathbb{Z}$.

15(b) $\sqrt{3}(\tan x+\tan 2x)+\tan x\tan 2x=1$ [রা. '০৪,'১১ ; সি.'০৭; য.'১১; চ.'১৩; দি.'১৩]

$\Rightarrow \sqrt{3}(\tan x+\tan 2x)=1-\tan 2x\tan x$

$\Rightarrow \frac{\tan 2x+\tan x}{1-\tan 2x\tan x}=\frac{1}{\sqrt{3}}\Rightarrow \tan(2x+x)=\frac{1}{\sqrt{3}}$

$\Rightarrow \tan 3x=\tan\frac{\pi}{6}$ $\therefore 3x=n\pi+\frac{\pi}{6}$, $n\in\mathbb{Z}$.

$\Rightarrow x=(6n+1)\frac{\pi}{18}$

$\therefore$ নির্ণেয় সমাধান, $x=(6n+1)\frac{\pi}{18}$, $n\in\mathbb{Z}$.

(c) $\tan\theta+\tan 2\theta+\tan 3\theta=\tan\theta\tan 2\theta\tan 3\theta$

$\Rightarrow \tan 2\theta+\tan\theta=\tan\theta\tan 2\theta\tan 3\theta-\tan 3\theta$

$\Rightarrow \tan 2\theta+\tan\theta=-\tan 3\theta(1-\tan 2\theta\tan\theta)$

$\Rightarrow \frac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}=-\tan 3\theta$

$\Rightarrow \tan 3\theta=-\tan 3\theta\Rightarrow 2\tan 3\theta=0$

$\Rightarrow \tan3\theta = 0 \therefore 3\theta = n\pi \Rightarrow \theta = \frac{1}{3}n\pi$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = \frac{1}{3}n\pi$, $n\in\mathbb{Z}$.

5(d) $\tan\theta + \tan2\theta + \sqrt{3}\tan\theta\tan2\theta = \sqrt{3}$

[রা.'০৬ ; য.'০৬; চ. '০৯,'১১; ব.'০৯]

$\Rightarrow \tan\theta + \tan2\theta = \sqrt{3}(1 - \tan\theta\tan2\theta)$

$\Rightarrow \frac{\tan2\theta + \tan\theta}{1 - \tan2\theta\tan\theta} = \sqrt{3} \Rightarrow \tan(2\theta+\theta) = \sqrt{3}$

$\Rightarrow \tan3\theta = \tan\frac{\pi}{3} \therefore 3\theta = n\pi + \frac{\pi}{3}$, $n\in\mathbb{Z}$.

$\Rightarrow \theta = (3n+1)\frac{\pi}{9}$

$\therefore$ নির্ণেয় সমাধান , $\theta = (3n+1)\frac{\pi}{9}$, $n\in\mathbb{Z}$.

(e) $\cot x + \cot2x + \cot3x = \cot x\cot2x\cot3x$

$\Rightarrow \cot x + \cot2x = \cot x\cot2x\cot3x - \cot3x$

$\Rightarrow \cot x + \cot2x = \cot3x(\cot2x\cot x - 1)$

$\Rightarrow \cot3x\frac{\cot x\cot2x - 1}{\cot x + \cot2x} = 1$

$\Rightarrow \cot3x.\cot3x = 1 \Rightarrow \tan^2 3x = 1$

$\Rightarrow \tan3x = \pm 1 = \tan(\pm\frac{\pi}{4})$

$\therefore 3x = n\pi \pm \frac{\pi}{4}$, $n\in\mathbb{Z}$. $\Rightarrow x = \frac{n\pi}{3} \pm \frac{\pi}{12}$.

$\therefore$ নির্ণেয় সমাধান , $x = \frac{n\pi}{3} \pm \frac{\pi}{12}$, $n\in\mathbb{Z}$.

16(a) $\tan(\frac{\pi}{4} + \theta) + \tan(\frac{\pi}{4} - \theta) = 4$

$\Rightarrow \frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta} + \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta} = 4$

$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta} = 4$

$\Rightarrow \frac{(1+\tan\theta)^2 + (1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)} = 4$

$\Rightarrow \frac{2(1+\tan^2)}{1-\tan^2\theta} = 4$

$\Rightarrow 1 + \tan^2\theta = 2 - 2\tan^2\theta \Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}} = \tan(\pm\frac{\pi}{6})$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$, $n\in\mathbb{Z}$.

$\therefore$ নির্ণেয় সমাধান , $\theta = n\pi \pm \frac{\pi}{6}$, $n\in\mathbb{Z}$.

16(b) $\tan\theta + \tan2\theta + \tan3\theta = 0$

$\Rightarrow \tan\theta + \tan2\theta + \tan(2\theta + \theta) = 0$

$\Rightarrow \tan\theta + \tan2\theta + \frac{\tan2\theta + \tan\theta}{1 - \tan2\theta\tan\theta} = 0$

$\Rightarrow (\tan\theta + \tan2\theta)(1 - \tan\theta\tan2\theta) + 1(\tan2\theta + \tan\theta) = 0$

$\Rightarrow (\tan2\theta + \tan\theta)(1 - \tan2\theta\tan\theta + 1) = 0$

$\Rightarrow (\tan2\theta + \tan\theta)(2 - \tan2\theta\tan\theta) = 0$

এখন , $\tan2\theta + \tan\theta = 0$ হলে ,

$\tan2\theta + \tan\theta = 0(1 - \tan2\theta\tan\theta)$

$\Rightarrow \frac{\tan2\theta + \tan\theta}{1 - \tan2\theta\tan\theta} = 0$

$\Rightarrow \tan(2\theta + \theta) = 0 \Rightarrow \tan3\theta = 0$

$\therefore 3\theta = n\pi \Rightarrow \theta = \frac{1}{3}n\pi$, $n\in\mathbb{Z}$.

আবার , $2 - \tan2\theta\tan\theta = 0$ হলে ,

$\Rightarrow 2 - \frac{2\tan\theta}{1-\tan^2\theta}\tan\theta = 0$

$\Rightarrow 2 - 2\tan^2\theta - 2\tan^2\theta = 0$

$\Rightarrow 4\tan^2\theta = 2 \Rightarrow 2\tan^2\theta = 1$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{2}} = \pm\tan\alpha = \tan(\pm\alpha)$,যেখানে

$\tan\alpha = \frac{1}{\sqrt{2}}$ $\therefore \theta = n\pi \pm \alpha$, $n\in\mathbb{Z}$.

∴ নির্ণেয় সমাধান , $\theta = \frac{1}{3}n\pi$, $n\pi \pm \alpha$;

যেখানে $\tan\alpha = \frac{1}{\sqrt{2}}$ এবং $n \in \mathbb{Z}$.

16(c) $\tan x + \tan 3x = 0$ [ব্র.'০৫]

$\Rightarrow \frac{\sin x}{\cos x} + \frac{\sin 3x}{\cos 3x} = 0$

$\Rightarrow \frac{\sin x\cos 3x + \sin 3x\cos x}{\cos x\cos x} = 0$

$\Rightarrow \sin(3x + x) = 0 \Rightarrow \sin 4x = 0$

$\therefore 4x = n\pi \Rightarrow x = \frac{n\pi}{4}$, $n \in \mathbb{Z}$.

∴ নির্ণেয় সমাধান, $\theta = \frac{n\pi}{4}$, $n \in \mathbb{Z}$.

(d) $\cos 7\theta = \cos 3\theta + \sin 5\theta$ [চ.'০৫ ; ঢা.'০৭]

$\Rightarrow \cos 3\theta - \cos 7\theta + \sin 5\theta = 0$

$\Rightarrow 2\sin 5\theta \sin 2\theta + \sin 5\theta = 0$

$\Rightarrow \sin 5\theta\,(2\sin 2\theta + 1) = 0$

$\sin 5\theta = 0$ হলে, $5\theta = n\pi$, $n \in \mathbb{Z}$.

$\Rightarrow \theta = \frac{n\pi}{5}$

$2\sin 2\theta + 1 = 0$ হলে , $\sin 2\theta = -\frac{1}{2} = -\sin(\frac{\pi}{6})$

$\Rightarrow \sin 2\theta = \sin(\pi + \frac{\pi}{6}) = \sin\frac{7\pi}{6}$

$\therefore 2\theta = n\pi + (-1)^n \frac{7\pi}{6}$

$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{7\pi}{12}$, $n \in \mathbb{Z}$.

∴ নির্ণেয় সমাধান ,

$\theta = \frac{n\pi}{5}, \frac{n\pi}{2} + (-1)^n \frac{7\pi}{12}$; $n \in \mathbb{Z}$

16(e) $\sin\theta - 2 = \cos 2\theta$, যখন $-2\pi \le \theta \le 2\pi$

[য.'০৬; চ.'১০; সি.'১২]

$\Rightarrow \sin\theta - 2 = 1 - 2\sin^2\theta$

$\Rightarrow 2\sin^2\theta + \sin\theta - 3 = 0$

$\Rightarrow 2\sin^2\theta + 3\sin\theta - 2\sin\theta - 3 = 0$

$\Rightarrow \sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\Rightarrow (\sin\theta - 1)(2\sin\theta + 3) = 0$

$-\frac{3}{2} < -1$ বলে , $\sin\theta \neq -\frac{3}{2}$ .i.e.$2\sin\theta + 3 \neq 0$

$\therefore \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = 1 \quad \therefore \theta = (4n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.

$n = 0, -1, 1$ ইত্যাদি বসিয়ে পাই ,

$\theta = \frac{\pi}{2}, -\frac{3\pi}{2}, \frac{5\pi}{2}$

∴ প্রদত্ত সীমার মধ্যে নির্ণেয় সমাধান ,

$\theta = \frac{\pi}{2}, -\frac{3\pi}{2}$

17(a) নিম্নের কোন সম্পর্কটি সত্য ?

A. $\sin^{-1}(\sin x) = x$, যখন $-1 \le x \le 1$

B. $\cos(\cos^{-1} x) = x$, যখন $0 \le x \le \pi$

C. $\sin(\sin^{-1} x) = x$, যখন $-1 \le x \le 1$

D $\cos^{-1}(\cos x) = x$, যখন $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(b) $\tan(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2})$ এর মান হবে.

A. $\frac{5}{6}$ B. 1 C. $\frac{\pi}{4}$ D. $-\frac{5}{6}$

সমাধান: $\tan(\tan^{-1}\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}})$

$= \tan\{\tan^{-1}(\frac{5}{6}\times\frac{5}{6})\} = \tan\{\tan^{-1}1\} = 1$

(c) যদি $\cot^2\theta + \text{cosec}\theta - 5 = 0$ হয়, তবে $0 < \theta < 90^0$ এর জন্য θ এর মান হবে-

A. 0^0 B. 30^0 C. 45^0 D. 60^0

সমাধান: $\theta = 30^0$ এর জন্য, $\cot^2\theta = 3, \text{cosec}\theta = 2$

18. $f(x) \equiv 8\sin x^0 + 15\cos x^0 \equiv 17\sin(x^0 + a^0)$, যেখানে $0 < a < 90$.

(a) এক দশমিক স্থান পর্যন্ত a এর মান নির্ণয় কর।

(b) সমাধান কর: $f(x) = 0$, যখন $0 < x < 360$.

(c) $0 < x < 360$ এর জন্য $y = f(x)$ এর লেখচিত্র অঙ্কন কর। এর সাহায্যে এর বিপরীত অন্বয়ের লেখচিত্র অঙ্কন কর।

সমাধান:

(a) $8 \sin x^0 + 15 \cos x^0$

$= 27\left(\frac{8}{\sqrt{8^2+15^2}} \sin x^0 + \frac{15}{\sqrt{8^2+15^2}} \cos x^0\right)$

$= 27(\cos\theta^0 \sin x^0 + \sin\theta^0 \cos x^0)$, যখন

$\sin\theta^0 = \frac{15}{\sqrt{8^2+15^2}}$ এবং $\cos\theta^0 = \frac{8}{\sqrt{8^2+15^2}}$

$\therefore\ 27\sin(x^0 + \theta^0) \equiv 17 \sin(x^0 + a^0)$

$\Rightarrow a^0 = \theta^0 = \sin^{-1}\frac{15}{\sqrt{8^2+15^2}} = \sin^{-1}\frac{15}{17}$

$\Rightarrow a^0 = 61{\cdot}9^0$ (প্রায়)

এক দশমিক স্থান পর্যন্ত a এর মান $61{\cdot}9$

(b) $f(x) = 0 \Rightarrow 17 \sin(x^0 + a^0) = 0$

$\Rightarrow \sin(x^0 + 61{\cdot}9^0) = 0$

$\Rightarrow x^0 + 61{\cdot}93^0 = n \times 180^0$, যেখানে $n = 1$ অথবা $n = 2$, $[\because 0 < x < 360]$

$n = 1$ হলে, $x^0 + 61{\cdot}9^0 = 180^0 \Rightarrow x^0 = 118{\cdot}1^0$

$\therefore\ x = 118{\cdot}1$

আবার, $n = 2$ হলে, $x^0 + 61{\cdot}9^0 = 360^0$

$\Rightarrow x^0 = 298{\cdot}1^0 \quad \therefore x = 298{\cdot}1$

$\therefore$ নির্ণেয় সমাধান, $x = 118{\cdot}1, 298{\cdot}1$.

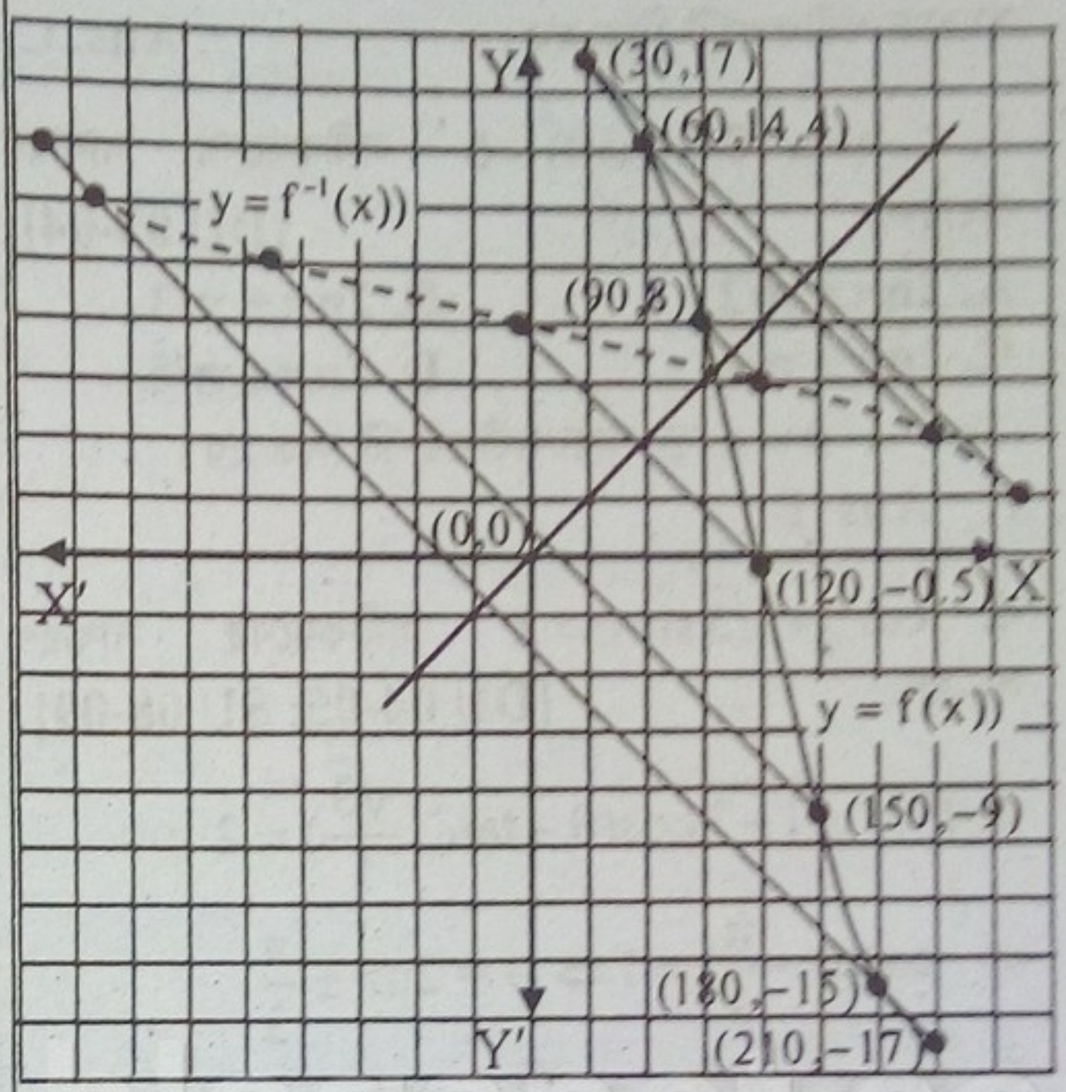
(c) $y = 8 \sin x^0 + 15 \cos x^0$

সমাধান ঃ নিচের তালিকায় $x \in [28 , 208]$ এর জন্য $y = 17 \sin(x^0 + 61{\cdot}9^0)$ এর প্রতিরূপী মান নির্ণয় করি :

x	30	60	90	120	150	180	210
y	17	14.4	8	-0.5	-9	-15	-17

একটি ছক কাগজে স্থানাঙ্কের অক্ষরেখা $X'OX$ ও YOY' আঁকি।

স্কেল নির্ধারণ ঃ x-অক্ষ বরাবর ছোট বর্গক্ষেত্রের এক বাহু =30 এবং y- অক্ষ বরাবর ছোট বর্গক্ষেত্রের 1 বাহু = 2



এখন নির্ধারিত স্কেল অনুযায়ী তালিকাভুক্ত বিন্দুগুলি ছক কাগজে স্থাপন করি। স্থাপিত বিন্দুগুলি মুক্ত হস্তে বক্রাকারে যোগ করে প্রদত্ত সীমা অনুযায়ী $y = f(x)$ এর লেখ অঙ্কন করা হল।

$y = x$ রেখা থেকে স্থাপিত বিন্দুগুলির সমদূরবর্তী বিন্দু ছক কাগজে স্থাপন করি। স্থাপিত বিন্দুগুলি মুক্ত হস্তে বক্রাকারে যোগ করে $y = f(x)$ এর বিপরীত অন্বয় $y = f^{-1}(x)$ এর লেখ অঙ্কন করা হল।

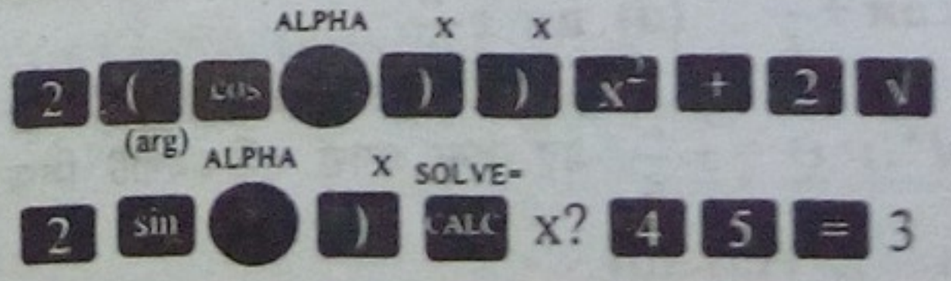
ভর্তি পরীক্ষার MCQ :

1. $2\cos^2\theta + 2\sqrt{2}\ \sin\theta = 3$ হলে θ এর মান –

[RU 06-07; Jt.U 08-09; DU 07-08;]

$A.\ 30^0$ $B.\ 45^0$ $C.\ 60^0$ $D.\ 135^0$

$Sol^n.:$ $\theta = 45^0$ দ্বারা সমীকরণটি সিদ্ধ হয়। Ans. B



2. $\cot x - \tan x = 2$ সমীকরণের সাধারণ সমাধান- **[DU 05-06; Jt.U 05-06]**

A. $\frac{n\pi}{4}$ B. $\frac{n\pi}{2}$ C. $\frac{(4n+1)\pi}{8}$ D. $\frac{(4n+1)\pi}{2}$

Sol^n.: Option গুলোতে n = 0 বসালে $C = \frac{\pi}{8}$ হয় যা দ্বারা সমীকরণটি সিদ্ধ হয়। ∴ **Ans. C**

3. $4(\sin^2\theta + \cos\theta) = 5$ সমীকরণের সাধারণ সমাধান- **[DU 03-04]**

A. $2n\pi \pm \pi/2$ **B.** $2n\pi \pm \pi/3$
C. $2n\pi \pm \pi/4$ **D.** $2n\pi \pm \pi/5$

Sol^n.: $\theta = \pi/3$ দ্বারা সমীকরণটি সিদ্ধ হয়।
∴ **Ans. B**

4. $\cos\theta + \sqrt{3}\sin\theta = 2$ সমীকরণের সাধারণ সমাধান- **[DU 04-05; SU 08-09]**

Sol^n.: $\sqrt{1+3}\cos(\theta - \tan^{-1}\frac{\sqrt{3}}{1}) = 2$

$\Rightarrow \cos(\theta - \frac{\pi}{3}) = 1 \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

5. $\sin x + \cos x = \sqrt{2}$, যখন $-\pi < \theta < \pi$ এর সমাধান কত? **[RU 06-07]**

Sol^n.: $\sqrt{2}\cos(x - \tan^{-1}1) = \sqrt{2}$

$\Rightarrow \cos(x - \frac{\pi}{4}) = 1 = \cos\frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$

6. $\cos\theta + \sqrt{3}\sin\theta < \sqrt{3}$; $0 < \theta < \pi/2$ সমীকরণের সমাধান- **[IU 07-08]**

A. $\theta > \pi/3$ **B.** $\theta < \pi/6$ C. $\theta > \pi/6$ D. $\theta < \pi/3$

Sol^n.: $0 < \theta < \pi/6$ এর জন্য প্রদত্ত অসমতাটি সত্য হয়। যেমন $\theta = 20°$ এর জন্য বামপক্ষ $= 1.53 < \sqrt{3}$

7. $\tan 2\theta \tan\theta = 1$ এর সমাধান - **[BUET 06-07]**

Sol^n.: **(a)** $2n\pi + \frac{\pi}{3}$ **(b)** $2n\pi \pm \frac{\pi}{3}$

(c) $n\pi + \frac{\pi}{6}$ **(d)** $n\pi \pm \frac{\pi}{6}$

Sol^n.: $\theta = \pm\frac{\pi}{6}$ এর জন্য প্রদত্ত সমীকরণটি সিদ্ধ হয়। ∴ Ans. (d)

8. যদি $\tan^2\theta + \sec\theta = -1$; $0 < \theta < 2\pi$ হয় তবে θ এর মান নির্ণয় কর। **[BUET 07-08]**

(a) π **(b)** $\frac{\pi}{2}$ **(c)** $\frac{\pi}{4}$ **(d)** $\frac{3\pi}{2}$

Sol^n.: $\theta = \pi$ এর জন্য প্রদত্ত সমীকরণটি সিদ্ধ হয়।
∴ Ans. (a)

9. $\cos\theta = -1$ হলে সাধারণ সমাধাত কত? **[Textile 13-14]**

A. 0 B. $2n\pi$ C. $(2n+1)\pi$ D. $n\pi$

Sol^n.: $\cos\theta = -1$ হলে $\theta = (2n+1)\pi$

10. $\tan 2x \tan x = 1$ হলে x এর দুইটি মানই কোন ক্ষেত্রে সঠিক নয়? **[Textile 13-14]**

A. $\frac{7\pi}{6}, \frac{5\pi}{6}$ B. $\frac{13\pi}{6}, \frac{15\pi}{6}$ C. $\frac{8\pi}{6}, \frac{9\pi}{6}$ D. $\frac{13\pi}{6}, \frac{11\pi}{6}$

Sol^n.: $\tan 2x \tan x = 1$

$\Rightarrow \sin 2x \sin x = \cos 2x \cos x \Rightarrow \cos 3x = 0$

$\Rightarrow 3x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{6}$

$n \in \mathbb{N}$ এর জন্য $x = \frac{8\pi}{6}$ সত্য নয়।

নিচের কোন সম্পর্কটি সত্য নয়? **[Textile 13-14]**

A. $\sin x = \sin(2n\pi + x)$ B. $\sin(\sin^{-1}x) = x$
C. $\sin^2 x = (-\sin x)^2$ D. $\sin^{-1}x = (\sin x)^{-1}$

Sol^n.: $\sin^{-1}x = (\sin x)^{-1}$ সম্পর্কটি সত্য নয়।