Interference in wedge shaped film :

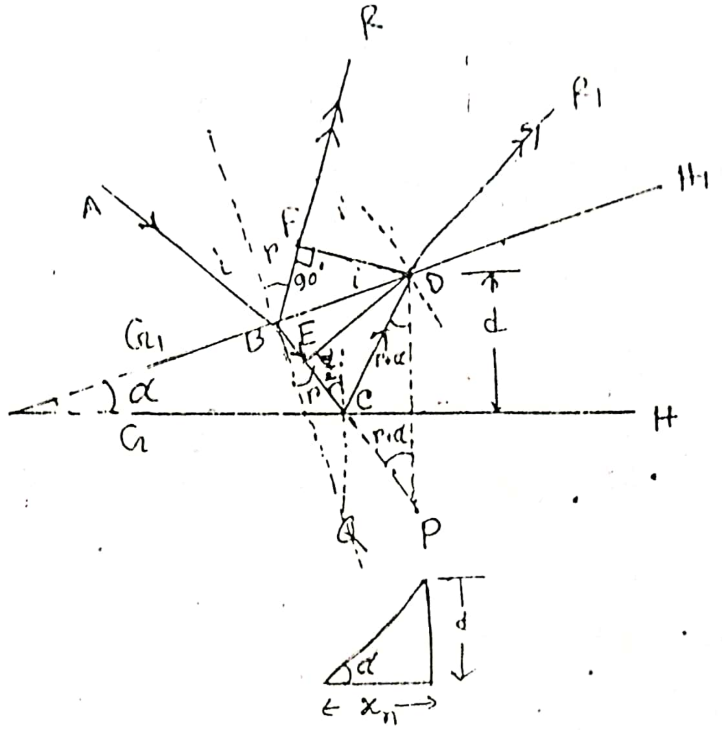

Fig. 1 : Interference in Wedge shaped film.

Let us consider two plane surfaces GH and G_1H_1 inclined at an angle α and enclosing a wedge shaped film (Fig. 1). The thickness of the film increases from G to H as shown in figure. Let μ be the refractive index of the material of the film. When this film is illuminated there is interference between two systems of rays, one reflected from the front surface and other obtained by internal reflection at the back surface.

The path difference x is given by.

$$x = \mu(BC + CD) - BF$$

$$= \mu(BE + EC + CD) - BF$$

But $\sin i = \frac{BF}{BD}$, $\sin r = \frac{BE}{BD}$, $\mu = \frac{\sin i}{\sin r} = \frac{BF}{BE}$

$$x = \mu (EC + CD) = \mu (EC + CP) = \mu EP$$

$$\cos(r+\alpha) = \frac{EP}{DP}$$

$$EP = 2d\cos(r+\alpha)$$

$$x = 2\mu d\cos(r+\alpha)$$

Due to reflection an additional phase difference $\lambda/2$ is introduced.

Hence path difference

$$x = 2\mu d\cos(r+\alpha) + \lambda/2$$

For constructive interference

$$2\mu d\cos(r+\alpha) + \lambda/2 = n\lambda$$

$$2\mu d\cos(r+\alpha) = (2n-1)\lambda/2 \quad \text{where, } n = 1, 2, 3 \ldots$$

For destructive interference.

$$2\mu d\cos(r+\alpha) + \lambda/2 = (2n+1)\lambda/2$$

$$2\mu d\cos(r+\alpha) = n\lambda \quad \text{where, } n = 0, 1, 2, 3 \ldots$$

~~Let~~ Spacing between two constructive bright band is obtained as follows.

For n^{th} maxima,

$$2\mu d\cos(r+\alpha) = (2n-1)\lambda/2$$

Let this band be obtained at a distance x_n from this edge as shown in figure. For normal incident $r = 0$.

Assuming, $\mu = 1$.

From the figure, $d = x_n \tan\alpha$.

$$2x_n \tan\alpha \cos\alpha = (2n-1)\lambda/2$$

$$2x_n \sin\alpha = (2n-1)\lambda/2$$

For $(n+1)^{th}$ maxima

$$2x_{n+1}\sin\alpha = (2n+1)\lambda/2$$

$$2(x_{n+1} - x_n)\sin\alpha = \lambda.$$

or fringe spacing $\beta = x_{n+1} - x_n = \frac{\lambda}{2\sin\alpha} = \frac{\lambda}{\alpha}.$

where α is small and measured in radians.

** Michelson Interferometer

(see yourself from any book of optics)

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Newton's Rings :

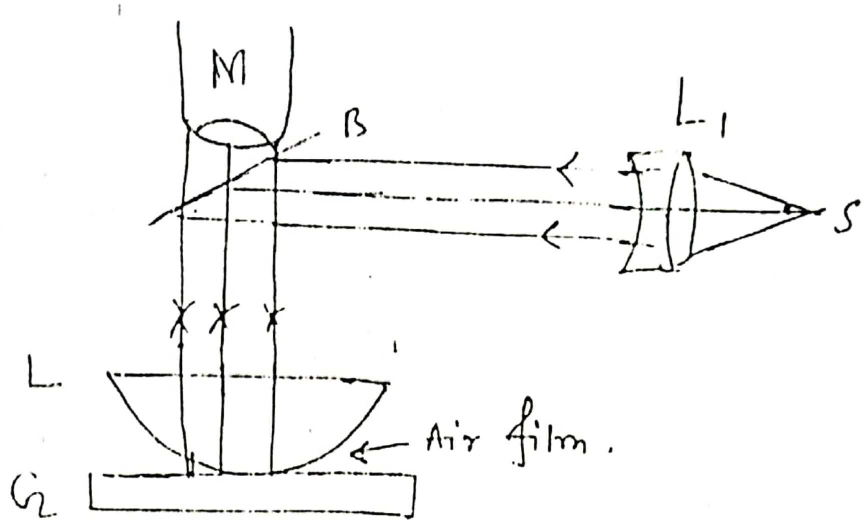

Fig. 1 : Experimental set up for viewing Newton's Ring

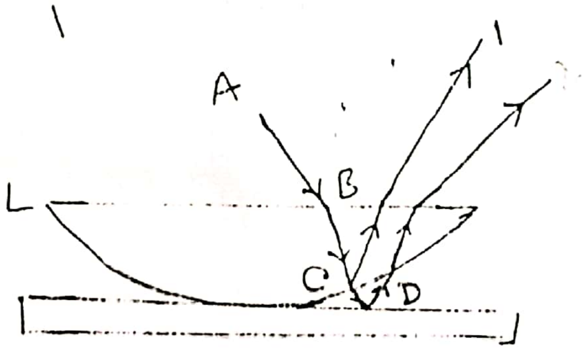

Fig. 1.1 : Interference in Newton's rings setup.

When a plano-convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. If monochromatic light is allowed to fall normally and viewed as shown in figure then alternate dark and bright circular fringes are observed. The fringes are circular because the air film has a circular symmetry. Newton's rings are formed because of the interference between the waves reflected from the top and bottom surfaces of the air film formed between the plates as shown in figure.

The path difference x between these rays (i.e., 1 and 2) is

$$2\mu d \cos(r+\alpha) = n\lambda$$ [Here α is very small compared with r]

$$2\mu d \cos r \pm n\lambda.$$

Since one of the rays is reflected from the denser medium additional path difference of $\lambda/2$ is introduced.

$$2\mu d \cos r + \lambda/2$$

Here r is angle of refraction. Since the rays are incident practically normal, so that angle r is nearly zero i.e. $\cos r = 1$.

then the path difference becomes.

$$2d + \lambda/2 \quad [\text{when } \mu = 1].$$

At the point of contact $d = 0$, the path difference is $\lambda/2$. Hence, the central spot is dark.

Now the condition for bright fringe is

$$2d + \lambda/2 = n\lambda$$

$$2d = \left(\frac{2n-1}{2}\right)\lambda.$$

[The minus sign has been selected on purpose since n cannot have a value of zero for bright fringes seen in reflected light]

where, $n = \cancel{0}, 1, 2, 3, \ldots$

and the condition for dark fringe is.

$$2d + \lambda/2 = (2n+1)\lambda/2$$

$$2d = n\lambda \quad \text{where, } n = 0, 1, 2, 3$$

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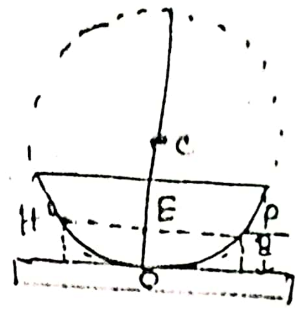
milarly, for a dark fringe

$$\frac{2r^2}{2R} = n\lambda \quad \text{or} \quad r^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R$$

$$D_n = 2\sqrt{n\lambda R}.$$



$EP \times EP = OE \times (2R - OE)$

$(2R-d)d = r^2$

$2Rd - d^2 = r^2$

$2Rd = r^2$

$2Rd = \left(\frac{D}{2}\right)^2$

$d = \left(\frac{r^2}{2R}\right)$

Thus, the diameter of the rings are proportional to the square roots of the natural numbers.

By measuring the diameter of the Newton's rings, it is possible to calculate the wavelength of light as follows. We have for the diameter of the nth dark fringe

$$D_n^2 = 4n\lambda R$$

Similarly diameter for $(n+p)^{th}$ dark fringe.

$$D_{n+p}^2 = 4(n+p)\lambda R$$

$$\therefore \; D_{n+p}^2 - D_n^2 = 4PR\lambda.$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}.$$

λ can be calculated using this formula.

Refractive index : $\mu = \dfrac{D_{n+p}^2 - D_n^2}{D_{n+p}'^2 - D_n'^2}.$

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