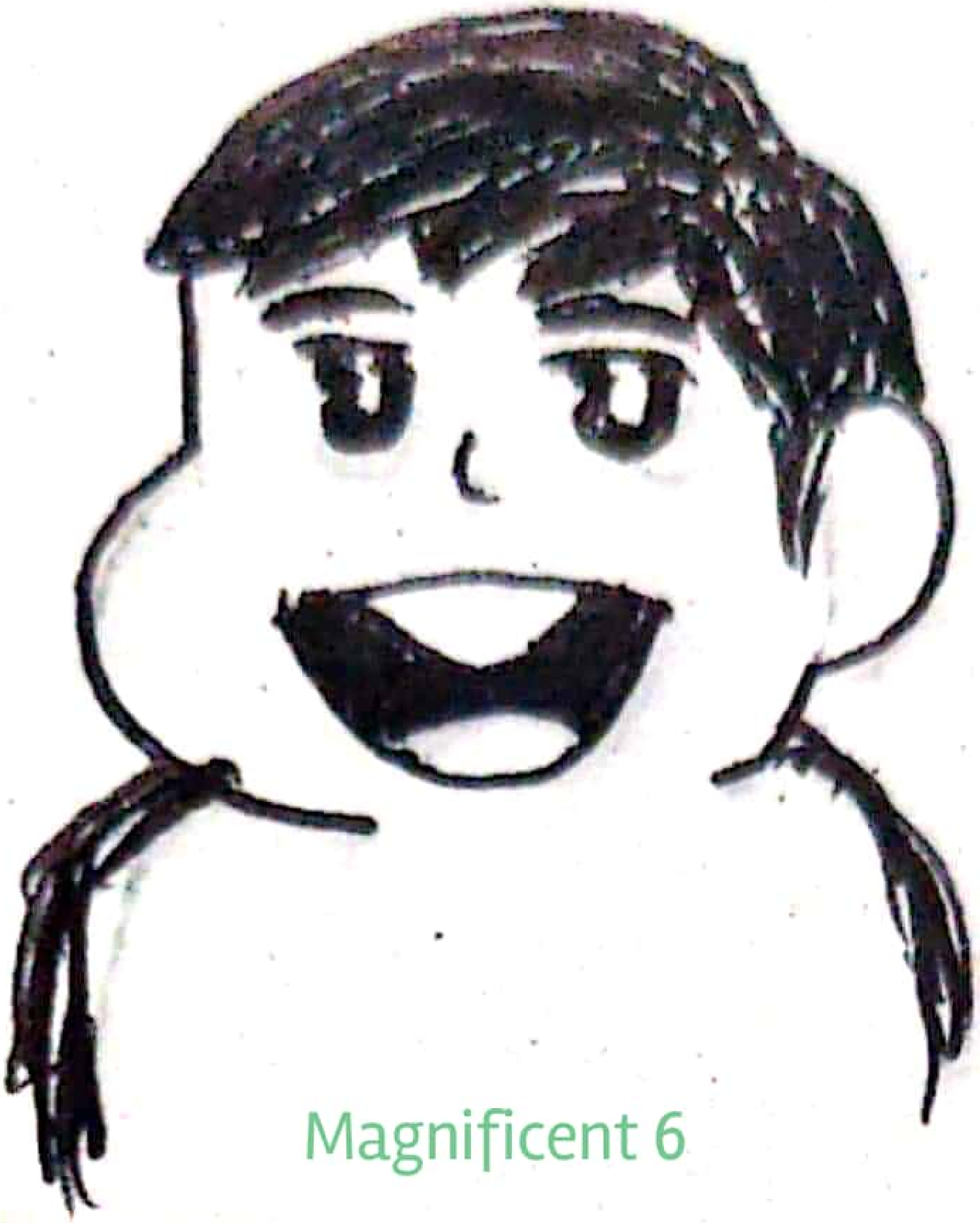


Magnificent 6

Centroids

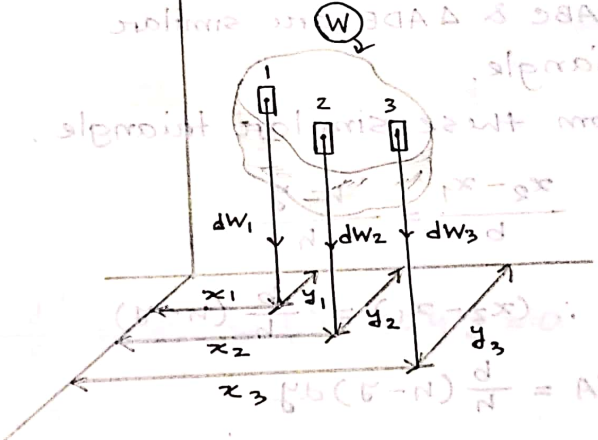
ভরকেন্দ্র

Moment, $W\bar{x} = dW_1x_1 + dW_2x_2 + \cdots \cdots + dW_nx_n$

$= \Sigma x\,dW$

$= \int x\,dW$

$\therefore W\bar{x} = \int x\,dW$



$$\therefore \quad \bar{x} = \frac{\int x\,dW}{W}$$

$$\bar{y} = \frac{\int y\,dW}{W}$$

$$\bar{z} = \frac{\int z\,dW}{W}$$

We know, $\rho = \frac{m}{V}$

$\therefore m = \rho V$

$$\bar{x} = \frac{\int x\,dm}{m}$$

$$\bar{y} = \frac{\int y\,dm}{m}$$

$$\bar{z} = \frac{\int z\,dm}{m}$$

$$\bar{x} = \frac{\int x\,dV}{V}$$

$$\bar{y} = \frac{\int y\,dV}{V}$$

$$\bar{z} = \frac{\int z\,dV}{V}$$

$$\bar{x} = \frac{\int x\,dA}{A}$$

$$\bar{y} = \frac{\int y\,dA}{A}$$

$$\bar{z} = \frac{\int z\,dA}{A}$$

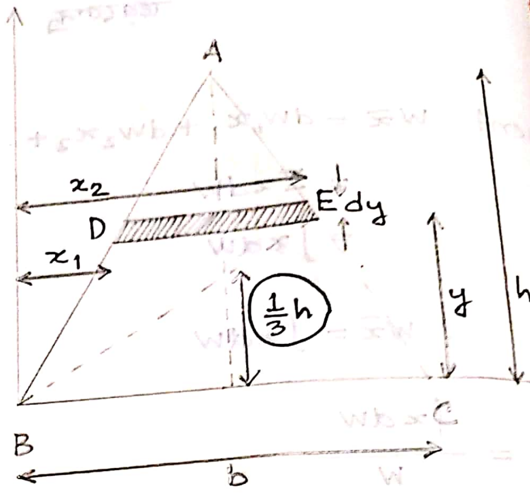
$$\bar{x} = \frac{\int x\,dL}{L}$$

$$\bar{y} = \frac{\int y\,dL}{L}$$

$$\bar{z} = \frac{\int z\,dL}{L}$$

Centroid of lines

Find the co-ordinates of the centroid of the area of a triangle



We know, $A\bar{y} = \int y\,dA$ ...... (i)

Here, $dA = (x_2 - x_1)dy$

ΔABC & ΔADE are similar triangle.

From these similar triangle,

$$\frac{x_2 - x_1}{b} = \frac{h-y}{h}$$

$$\therefore (x_2 - x_1) = \frac{b}{h}(h-y)$$

So, $dA = \frac{b}{h}(h-y)dy$

From (i),

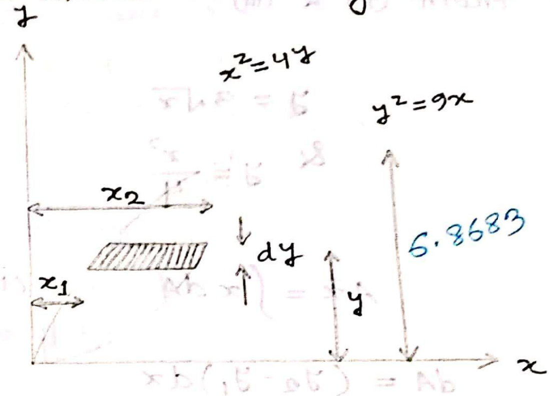
$$A\bar{y} = \int y\,dA$$
$$= \int_0^h y \cdot \frac{b}{h}(h-y)dy$$
$$= \int_0^h \left(by - \frac{by^2}{h}\right)dy$$
$$= \left[\frac{by^2}{2} - \frac{by^3}{3h}\right]_0^h$$
$$= \frac{bh^2}{2} - \frac{bh^3}{3h}$$
$$= \frac{bh^2}{2} - \frac{bh^2}{3}$$
$$= \frac{3bh^2 - 2bh^2}{6}$$
$$\therefore A\bar{y} = \frac{bh^2}{6} \text{ ...... (ii)}$$

$\frac{h}{3}$ proof করতে বলা হইছে

The area of triangle, $A = \frac{1}{2} \times b \times h$

$$\therefore A\bar{y} = \frac{bh^2}{6} \Rightarrow \bar{y} = \frac{bh^2}{6} \times \frac{1}{\frac{1}{2} \times b \times h} \qquad \therefore \bar{y} = \frac{h}{3}$$

* Find the co-ordinates of the centroid of an area formed by $y^2 = 9x$ and $x^2 = 4y$



We know, $A\bar{y} = \int y\,dA$ ......(i)

Here, $y^2 = 9x$ ......(ii)

$x^2 = 4y$ ......(iii)

From (ii),

$$x = \frac{y^2}{9}$$

Now, from (iii),

$$\left(\frac{y^2}{9}\right)^2 = 4y \Rightarrow \frac{y^4}{81} = 4y \Rightarrow y^4 - 324y = 0$$

$$\Rightarrow y(y^3 - 324) = 0$$

$$\therefore y = 0 \text{ or } y = (324)^{\frac{1}{3}} = 6.8683$$

$$\therefore x = 0 \text{ or } x = 5.24$$

Now,
$$dA = (x_2 - x_1)\,dy$$

$$= \left(2\sqrt{y} - \frac{y^2}{9}\right)dy$$

$$\therefore \int dA = \int_0^{6.8683} \left(2\sqrt{y} - \frac{y^2}{9}\right)dy$$

$$\therefore A = 12 \text{ square unit}$$

From (i),
$$A\bar{y} = \int y\,dA$$

$$= \int_0^{6.8683} y\left(2\sqrt{y} - \frac{y^2}{9}\right)dy$$

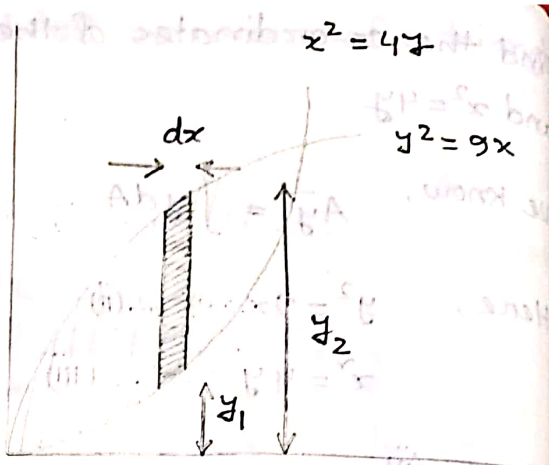
$$= \int_0^{6.8683} \left(2y^{\frac{3}{2}} - \frac{y^3}{9}\right)dy$$

$$\therefore A\bar{y} = 37.088$$

$$\therefore \bar{y} = \frac{37.088}{A} = \frac{37.088}{12} = 3.09$$

$\therefore \bar{y} = 3.09$ unit

From (ii) & (iii),



$$y = 3\sqrt{x}$$

$$\& \quad y = \frac{x^2}{4}$$

$$A\bar{x} = \int x \, dA \quad \ldots \ldots (iv)$$

$$dA = (y_2 - y_1)dx$$

$$= \left(\frac{x^2}{4} - 3\sqrt{x}\right)dx$$

$$\therefore \int dA = \int_0^{5.24} \left(\frac{x^2}{4} - 3\sqrt{x}\right)dx$$

$$\therefore A = -12 \text{ sq. unit}$$

$\therefore A = 12$ sq. unit (negative sign has no significance here)

$$\therefore A\bar{x} = \int_0^{5.24} x\,dA = \int_0^{5.24} x\left(\frac{x^2}{4} - 3\sqrt{x}\right)dx = \int_0^{5.24}\left(\frac{x^3}{4} - 3x^{\frac{3}{2}}\right)$$

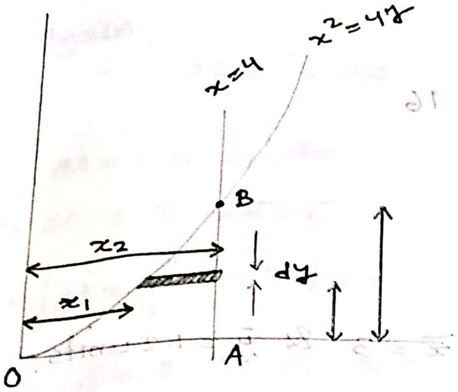
$$\therefore A\bar{x} = 28.304$$

$$\therefore \bar{x} = 2.358$$

The co-ordinates of the centroid are $\bar{x} = 2.358$ & $\bar{y} = 3.0$ unit.

Practice problems:

* Locate the centroid of the area included between the parabola $x^2 = 4y$, the line $x = 4$ and x-axis.



Given equations are

$x = 4 \quad \ldots\ldots (i)$

$x^2 = 4y \quad \ldots\ldots (ii)$

From (ii),

$x^2 = 4y$

$\Rightarrow 4^2 = 4y$

$\Rightarrow y = 4$

$\therefore$ The intersection point $B(4,4)$

we know,

$A\bar{y} = \int y\,dA$

Now, $dA = (x_2 - x_1)dy = (2\sqrt{y} - 4)dy$

$\therefore \int dA = \int_0^4 (2\sqrt{y} - 4)dy = \left[\frac{2y^{3/2}}{\frac{3}{2}} - 4y\right]_0^4 = -5.33$

$\therefore A = -5.33 = 5.33$ (Negative sign has no significance here)

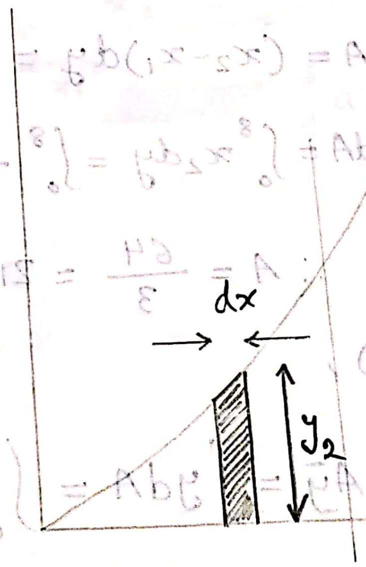
Now, $A\bar{y} = \int y\,dA = \int_0^4 \left(2y^{3/2} - 4y\right)dy = 6.4$

$\Rightarrow A\bar{y} = 6.4$

$\therefore \bar{y} = \frac{6.4}{5.33} = 1.2$

Again, we know,

$A\bar{x} = \int x\,dA \quad \ldots\ldots (iii)$



$dA = (y_2 - y_1)dx$

$\Rightarrow dA = (y_2 - 0)dx$

$\Rightarrow \int dA = \int_0^4 \frac{x^2}{4}dx = \frac{16}{3} = 5.33$

$\therefore A = 5.33$ sq. unit

$\therefore A\bar{x} = \int x\,dA$

$= \int_0^4 x \cdot \frac{x^2}{4}\,dx = \int_0^4 \frac{x^3}{4}\,dx = 16$

$\Rightarrow A\bar{x} = 16$

$\therefore \bar{x} = \frac{16}{\frac{16}{3}} = 3$ units

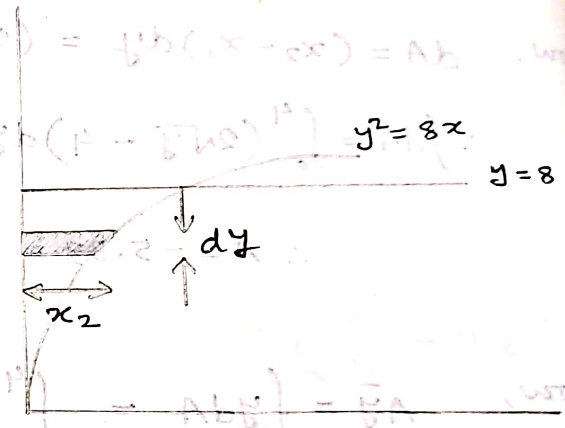
$\therefore$ The co-ordinates of the centroid are $\bar{x} = 3$ & $\bar{y} = 1.2$ units.

* Locate the centroid of the area included between the parabola $y^2 = 8x$, the y-axis and the line $y = 8$ inch.

Given equations are -

$y^2 = 8x$ ... (i)

$y = 8$ ... (ii)



From (i), $y^2 = 8x$

$\Rightarrow 8^2 = 8x$

$\therefore x = 8$

$\therefore$ The intersection point is $(8, 8)$

We know,

$A\bar{y} = \int y\,dA$ ... ... (iii)

Now, $dA = (x_2 - x_1)dy = (x_2 - 0)dy = x_2\,dy$

$\Rightarrow \int dA = \int_0^8 x_2\,dy = \int_0^8 \frac{y^2}{8}\,dy = \frac{64}{3}$

$\therefore A = \frac{64}{3} = 21.33$ sq. inch

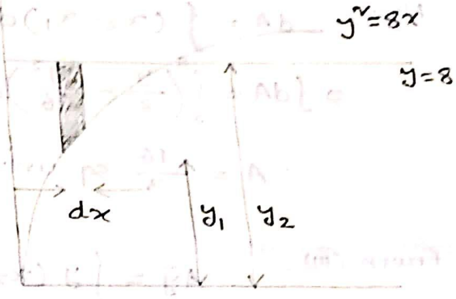
From (iii),

$A\bar{y} = \int y\,dA = \int_0^8 \frac{y^3}{8}\,dy = 128$

$\Rightarrow A\bar{y} = 128 \Rightarrow \bar{y} = \frac{128}{\frac{64}{3}} = 6$ $\therefore \bar{y} = 6$ inch

Again,

$$A\bar{x} = \int x\,dA \quad \ldots\ldots (iv)$$



$dA = (y_2 - y_1)\,dx$

$\Rightarrow dA = 8 - 2\sqrt{2x}$

$\Rightarrow \int dA = \int_0^8 8 - 2\sqrt{2x}$

$\therefore A = 21.33$ sq. inch

Now, $A\bar{x} = \int x\,dA$

$$\Rightarrow A\bar{x} = \int_0^8 \left(8x - 2\sqrt{2}\,x^{3/2}\right)dx$$

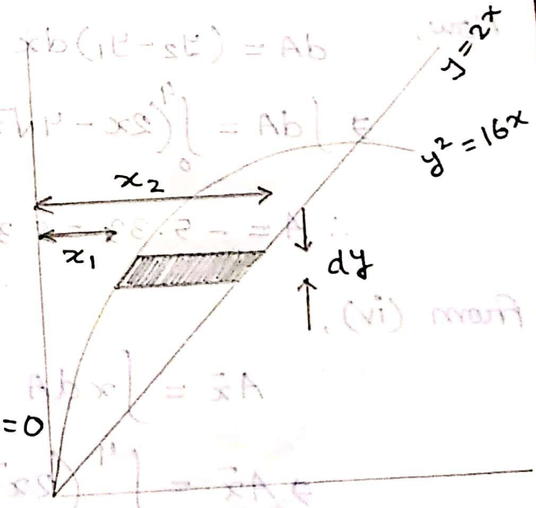
$\Rightarrow A\bar{x} = 51.2$

$$\Rightarrow \bar{x} = \frac{51.2}{21.33} = 2.4 \text{ inch}$$

$\therefore$ The co-ordinates of the centroid are $\bar{x} = 2.4$ & $\bar{y} = 6$ inch.

* Find the ~~centroids~~ co-ordinates of the centroid of the area bounded by the curve $y^2 = 16x$ and $y = 2x$.

Given equations are -



$y^2 = 16x \quad \ldots (i)$

$y = 2x \quad \ldots (ii)$

From (i),

$y^2 = 16x$

$\Rightarrow (2x)^2 = 16x \Rightarrow 4x^2 = 16x \Rightarrow 4x^2 - 16x = 0$

$\Rightarrow 4x(x-4) = 0$

$\therefore x = 0$ or $x = 4$

$\therefore y = 0$ or $y = 8$

We know, $A\bar{y} = \int y\, dA$ ......(iii)

Now, $dA = \int (x_2 - x_1)\, dy$

$$\Rightarrow \int dA = \int_0^8 \left(\frac{y}{2} - \frac{y^2}{16}\right) dy$$

$$\therefore A = \frac{16}{3} \text{ sq. units.}$$

From (iii),

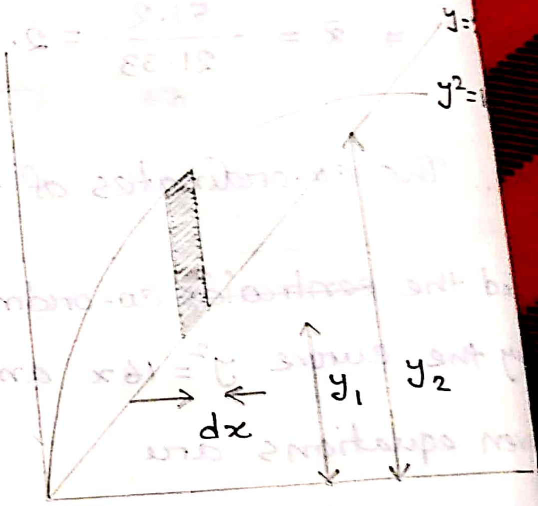
$$A\bar{y} = \int y\,(x_2 - x_1)\, dy$$

$$\Rightarrow A\bar{y} = \int_0^8 \left(\frac{y^2}{2} - \frac{y^3}{16}\right) dy$$

$$\Rightarrow A\bar{y} = \frac{64}{3}$$

$$\therefore \bar{y} = \frac{\frac{64}{3}}{\frac{16}{3}} = 4$$

$$\therefore \bar{y} = 4 \text{ units}$$



We know,

$$A\bar{x} = \int x\, dA \text{ ......(iv)}$$

Now, $dA = (y_2 - y_1)\, dx$

$$\Rightarrow \int dA = \int_0^4 (2x - 4\sqrt{x})\, dx$$

$$\therefore A = -5.33 = 5.33 \text{ sq. units}$$

From (iv),

$$A\bar{x} = \int x\, dA$$

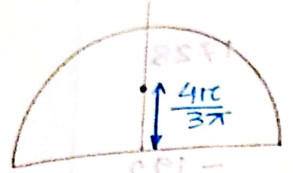
$$\Rightarrow A\bar{x} = \int_0^4 \left(2x^2 - 4x^{3/2}\right) dx$$

$$\Rightarrow A\bar{x} = 8.533$$

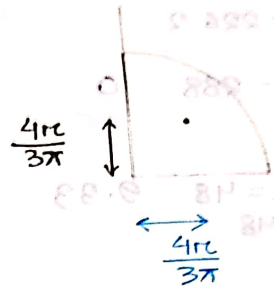
$$\therefore \bar{x} = \frac{8.533}{5.333} = 1.6 \text{ units}$$

$\therefore$ The co-ordinates of the centroid are $\bar{x} = 1.6$ units & $\bar{y} = 4$ units.

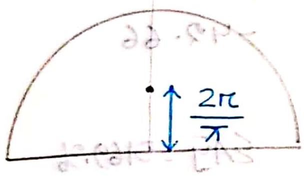
যেসব centroid গুলো মুখস্থ রাখা লাগবে :



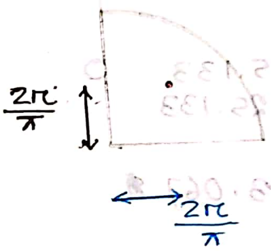
Half Circle (Area)



Quarter circle (Area)

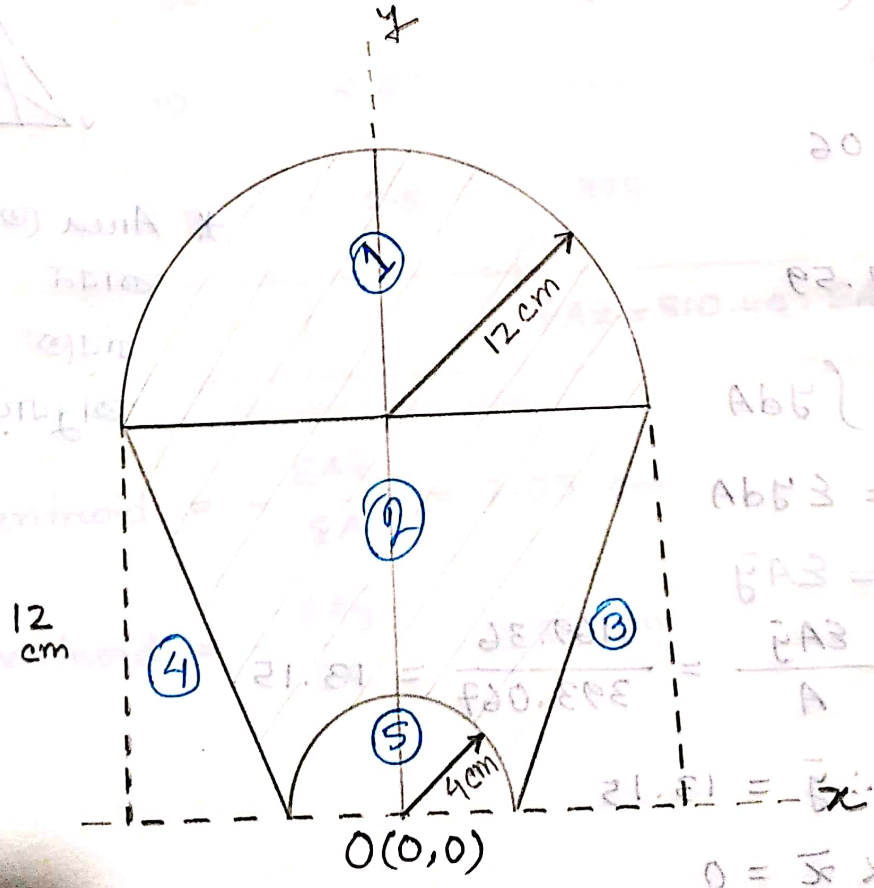


Half circle (Line)



Quarter circle (line)

\# Find out the co-ordinates of the centroid for the figure given below :



Components	Area, A (cm^2)	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
Half Circle ①	$\frac{\pi r^2}{2} = \frac{\pi \times 12^2}{2}$ $= 72\pi$ $= 226.2$	0	$12 + \frac{4r}{3\pi}$ $= 17.1$	0	3868.02
Rectangle ②	$24 \times 12 = 288$	0	$\frac{12}{2} = 6$	0	1728
Triangle ③	$\frac{1}{2} \times 8 \times 12 = 48$ $= -48$	9.33	4	-448	-192
Triangle ④	-48	-9.33	4	$+448$	-192
Halfcircle ⑤	$8\pi = 25.133$ $= -25.133$	0	$\frac{4 \times 4}{3\pi} = 1.69$	0	-42.66
	$\Sigma A = 393.067$			$\Sigma A\bar{x} = 0$	$\Sigma A\bar{y} = 5169.36$

$\bar{x}$ for triangle ③,

$$\bar{x} = \left\{\frac{2}{3} \times (12-4)\right\} + 4 = 9.33$$

Tips: যার angle কম তার থেকে $\frac{2}{3}$ অংশ

We know,

$$\Sigma A = 393.06$$

$$\Sigma A\bar{x} = 0$$

$$\Sigma A\bar{y} = 5167.59$$

* Area তে (−) ব্যবহার করার কারন অংশ বিয়োগ করতে হবে figure অনুযায়ী

Now,

$$A\bar{y} = \int y\,dA$$

$$\Rightarrow A\bar{y} = \Sigma y\,dA$$

$$\Rightarrow A\bar{y} = \Sigma A\bar{y}$$

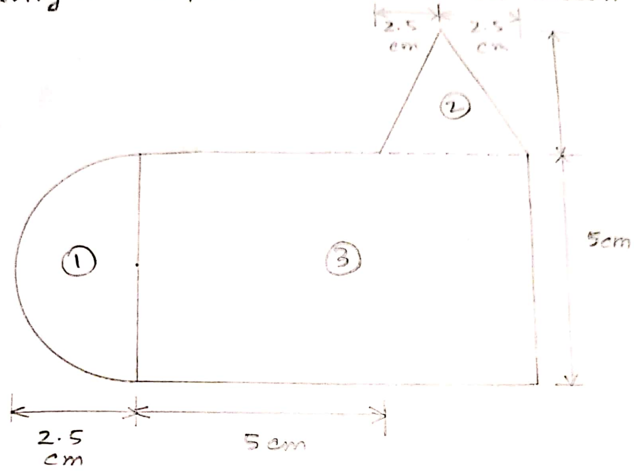
$$\Rightarrow \bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{5169.36}{393.067} = 13.15$$

$$\therefore \bar{y} = 13.15$$

$$\& \ \bar{x} = 0$$

Practice problems :

808. Using the analytical method, determine the centre of gravity of the plano uniform lamino shown in the figure.



Components	Area, (cm^2) A	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
Half circle ①	9.82	1.061	2.5	10.42	24.55
Triangle ②	12.5	10	6.67	125	83.375
Rectangle ③	50	7.5	2.5	375	125
	$\Sigma A = 72.32$			$\Sigma A\bar{x} = 510.42$	$\Sigma A\bar{y} = 232.925$

$$\therefore \bar{x} \text{ of centroid} = \frac{\Sigma A\bar{x}}{\Sigma A} = 7.05 \text{ cm}$$

$$\bar{y} \text{ of centroid} = \frac{\Sigma A\bar{y}}{\Sigma A} = 3.22 \text{ cm}$$

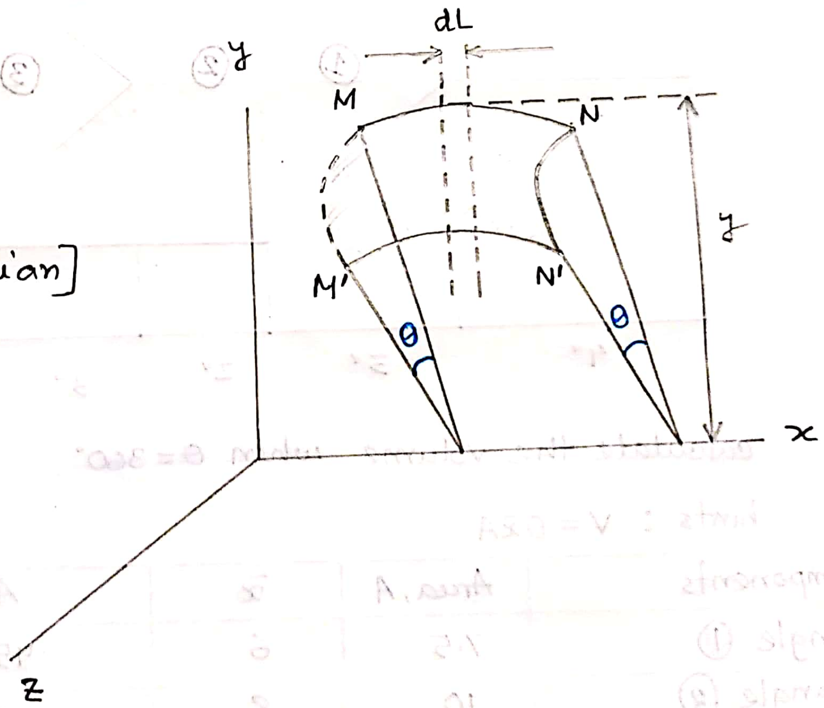
Pappus & Guldinus Theorem :

If a surface of revolution is generated by a plane curve revolving about any non-intersecting axis in its plane, the area of the generated surface is equal to the product of the length of the curve and the distance traveled by the centroid of the curve when the curve is being generated.

$$\int dA = \int \theta y \, dL$$

$$\Rightarrow A = \theta \int y \, dL$$

$$\therefore A = \theta y L \qquad [\theta = \text{radian}]$$



$$A = \theta \bar{y} L$$

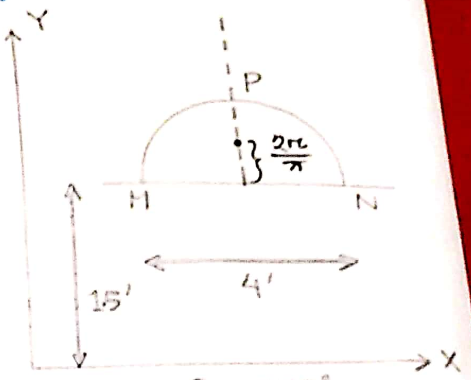
$$V = \theta \bar{y} A$$

Find out the area of generated surface :

#

Here, $r = 2'$

$\therefore \bar{y} = 1.5 + \frac{2 \times 2}{\pi} = 2.77$



$\theta = 270°$

$\therefore$ Area, $A = \theta \bar{y} L$

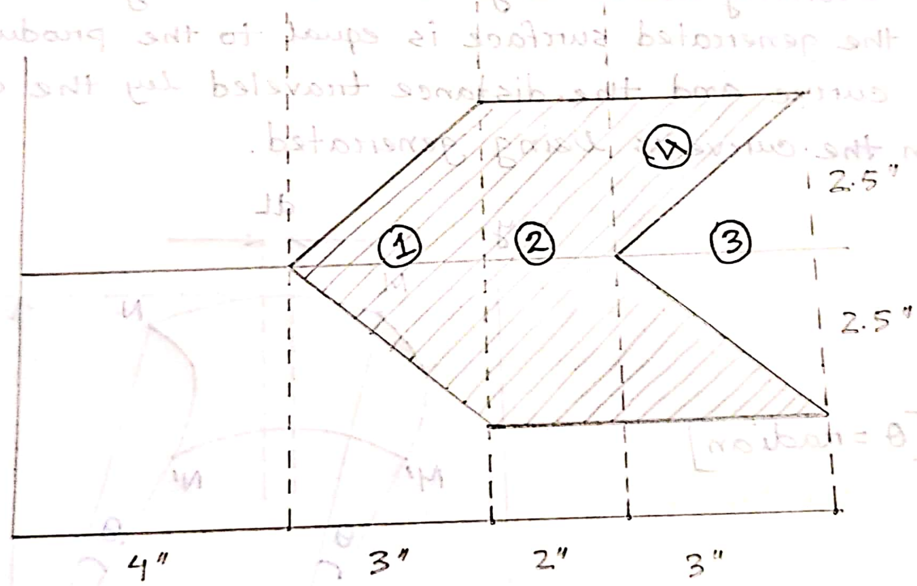
$= \left(270 \times \frac{\pi}{180}\right) \times 2.77 \times (\pi r)$

$= \frac{3\pi}{2} \times 2.77 \times 2\pi$

$= 82.02$ sq. ft

Full length of circle $= 2\pi r$
in this case (semi-circular)
length $= \frac{2\pi r}{2} = \pi r$

#



calculate the volume when $\theta = 360°$.

hints : $V = \theta \bar{x} A$

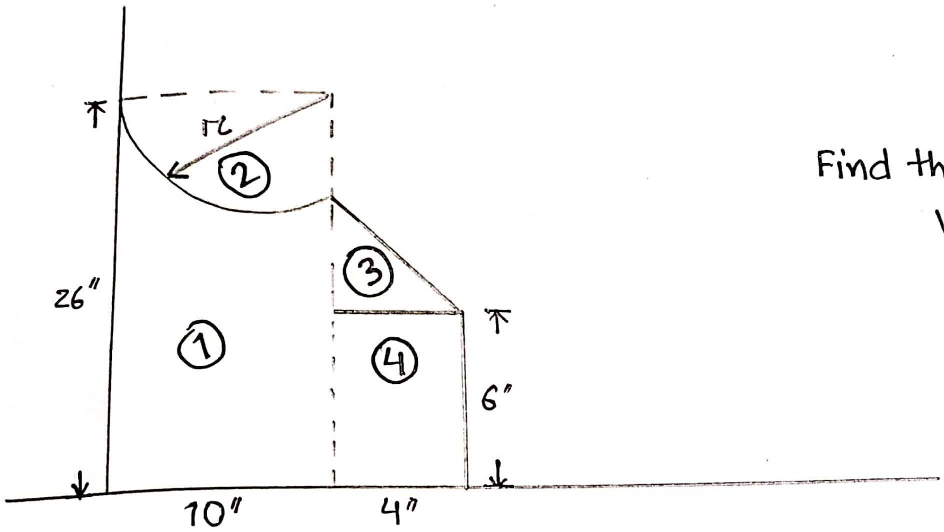
Components	Area, A	$\bar{x}$	$A\bar{x}$
Triangle 1	7.5	6	45
Rectangle 2	10	8	80
Rectangle 4	15	10.5	157.5
Triangle 3	−7.5	14	−105
	$\Sigma A = 25$		$\Sigma A\bar{x} = 177.5$

$$\bar{x} = \frac{\Sigma A\bar{x}}{A} = \frac{177.5}{25} = 7.1$$

Volume, $V = \theta \bar{x} A$

$$= \left(360 \times \frac{\pi}{180}\right) \times 7.1 \times 25 = 3.66 \times 10^{6} \text{ inch}^{3}$$

#



Find the area and Volume.

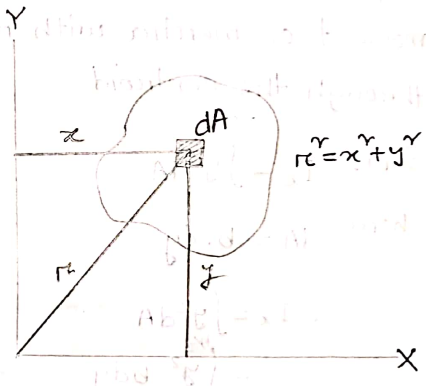
Moment of Inertia

Mathematical definition of moment of inertia :

$$\text{moment of inertia of an area} = \int a^2 dA$$

a = distance of an element of area dA from the axis

* If the axis is in the plane of the area, then it is called Rectangular moment of inertia, I
* If the axis is perpendicular to the area, then it is called Polar moment of inertia J



$r^2 = x^2 + y^2$

$$I_x = \int y^2 dA$$
$$I_y = \int x^2 dA$$

Polar moment of inertia :

$$J = \int r^2 dA = \int (x^2 + y^2)\, dA$$
$$= \int x^2 dA + \int y^2 dA$$

$$\therefore J = I_x + I_y$$

Radius of Gyration :

The radius of gyration, k, is a mathematical term.

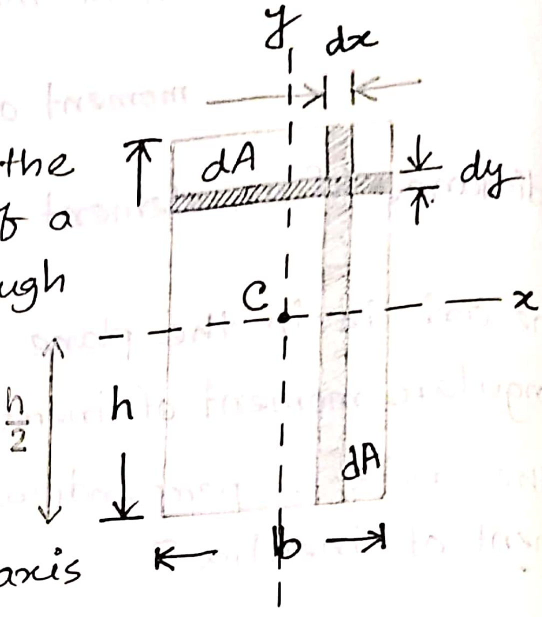
$$k = \left(\frac{I}{A}\right)^{1/2}; \quad k = \left(\frac{J}{A}\right)^{1/2}$$

$$\therefore I = k^2 A \quad ; \quad J = k^2 A$$

$$K_x = \sqrt{\frac{I_x}{A}}$$

$$K_y = \sqrt{\frac{I_y}{A}}$$

Example: (Rectangle) : ~~Derivation~~ Derive the expression for the moment of inertia of a rectangular area about an axis through the centroid and parallel to a side. Also find the radius of gyration with respect to this axis and the polar moment of inertia with respect to an axis through the centroid.



C = centroid

Soln: $I_x = \int y^2 dA$

here, $dA = b.dy$

$$\therefore I_x = \int y^2 dA$$
$$= \int_{-h/2}^{h/2} y^2 . b dy$$
$$= b \int_{-h/2}^{h/2} y^2 dy$$
$$= \frac{b}{3}\left[(h/2)^3 - (-\tfrac{h}{2})^3\right] = \frac{b}{3}\left[\frac{h^3}{8} + \frac{h^3}{8}\right] = \frac{b}{3} \times \frac{h^3}{4}$$

$$\therefore I_x = \frac{bh^3}{12}$$

$$I_y = \int x^2 dA = \int_{-b/2}^{b/2} x^2 . h dx = \frac{h}{3}\left[\frac{b^3}{4}\right] = \frac{hb^3}{12}$$

$\therefore$ Polar moment of inertia about the centroidal axis,

$$J_c = I_x + I_y = \frac{bh^3}{12} + \frac{hb^3}{12} = \frac{hb}{12}(h^2+b^2)$$

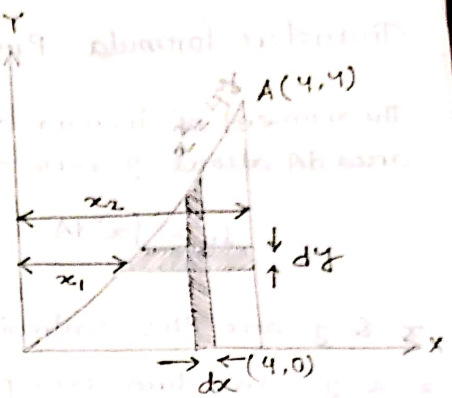
$$\therefore J_c = \frac{A}{12}(h^2+b^2)$$

$x^2 = 4y$ ---(i)

& $x = 4$ ---(ii)

From (i), $16 = 4y$

$\therefore y = 4$



$\therefore$ Intersection point A(4,4)

$$I_x = \int y^2 dA = \int y^2 (x_2 - x_1) dy = \int_0^4 y^2 (4 - 2\sqrt{y}) dy$$

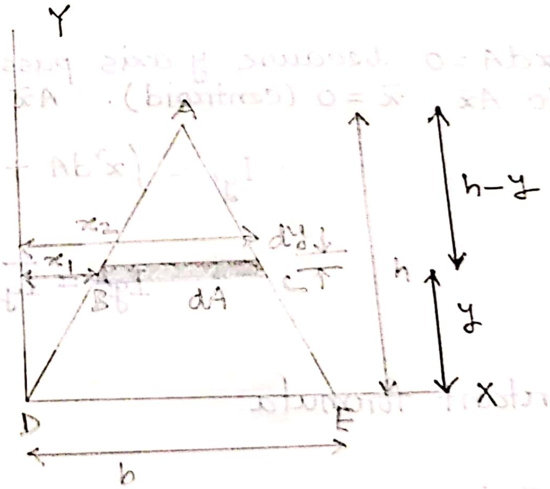
$$= \int_0^4 (4y^2 - 2y^{5/2}) dy = \left[\frac{4y^3}{3}\right]_0^4 - \left[\frac{2\times 2 y^{7/2}}{7}\right]_0^4$$

$$= 12.2 \text{ unit}^4$$

$$I_y = \int x^2 dA = \int_0^4 x^2 (y_2 - y_1) dx = \int_0^4 x^2 \left(\frac{x^2}{4} - 0\right) dx = 51.2 \text{ unit}^4$$

Example : Triangle : Find the moment of inertia of a triangular area about a base.

Soln: From similar triangle ABC & ADE,



$$\frac{x_2 - x_1}{b} = \frac{h-y}{h}$$

$$\therefore x_2 - x_1 = \frac{b(h-y)}{h}$$

$$I_x = \int y^2 dA = \int y^2 (x_2 - x_1) dy$$

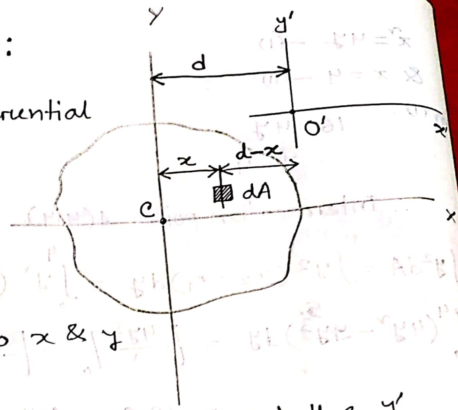
$$= \int_0^h y^2 \frac{b(h-y)}{h} dy = \frac{b}{h}\int_0^h (y^2 h - y^3) dy = \frac{b}{h}\left[\frac{y^3 h}{3} - \frac{y^4}{4}\right]_0^h$$

$$= \frac{b}{h} \times \frac{h^4}{12} = \frac{bh^3}{12} = \frac{h^2 . bh}{2\times 6}$$

$$\therefore I_x = \frac{h^2 A}{6} = \frac{Ah^2}{6}$$

The corresponding radius of gyration, $K_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{Ah^2}{6}}{A}} = \frac{h}{\sqrt{6}}$

Transfer formula - Parallel axis :



The moment of inertia of the differential area dA about y axis, $dI_y = x^2 dA$

$$I_y = \int x^2 dA$$

x & y are the centroidal axis.

x' & y' are two axis parallel to x & y axis.

The moment of inertia of the entire area A about the y' axis,

$$I_{y'} = \int (d-x)^2 dA$$
$$= \int (d^2 - 2dx + x^2) dA$$
$$= \int x^2 dA - 2d\int x dA + \int d^2 dA$$

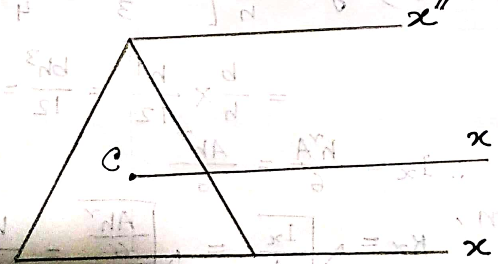
$\int x dA = 0$ because y axis passes through the centroid. $\int x dA$ is equal to $A\bar{x}$. $\bar{x} = 0$ (centroid). $\therefore A\bar{x} = 0 = \int x dA$

$$\therefore I_{y'} = \int x^2 dA + d^2 A = \bar{I}_y + Ad^2$$

$$I_{y'} = \bar{I}_y + Ad^2$$

Important formula

*** Triangle :

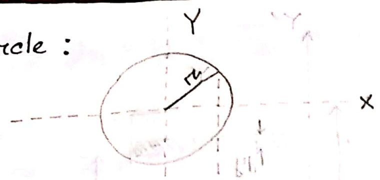


$$I_x = \frac{bh^3}{12}$$

$$I_{x'} = \frac{bh^3}{36}$$

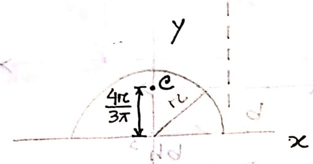
$$I_{x''} = \frac{bh^3}{84}$$

*** Circle :



$$\bar{I}_x = \bar{I}_y = \frac{\pi r^4}{4}$$

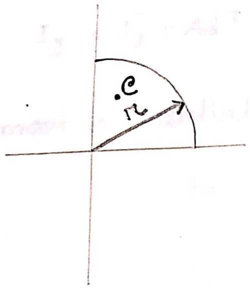
*** Half circle :



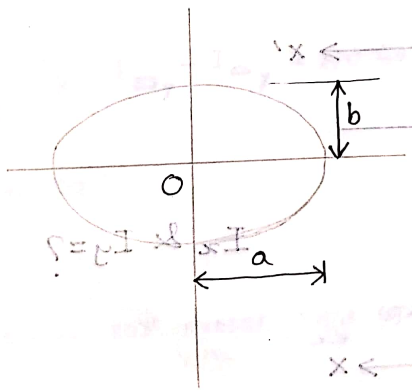
$$I_x = I_y = \frac{\pi r^4}{8}$$

*** Quarter circle :

$$I_x = I_y = \frac{\pi r^4}{16}$$



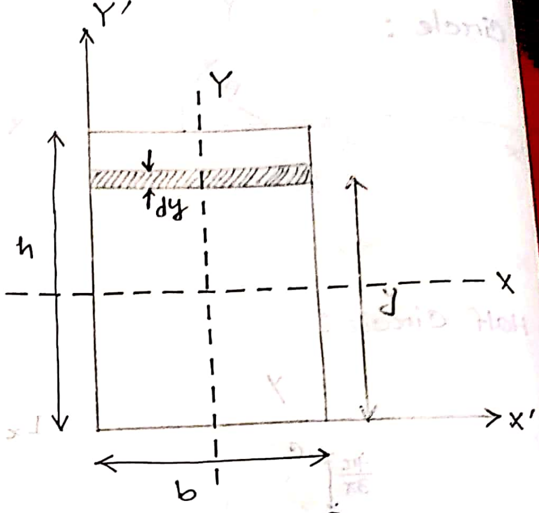
*** Ellipse :



$$I_x = \frac{\pi a b^3}{4}$$

$$I_y = \frac{\pi a^3 b}{4}$$

#



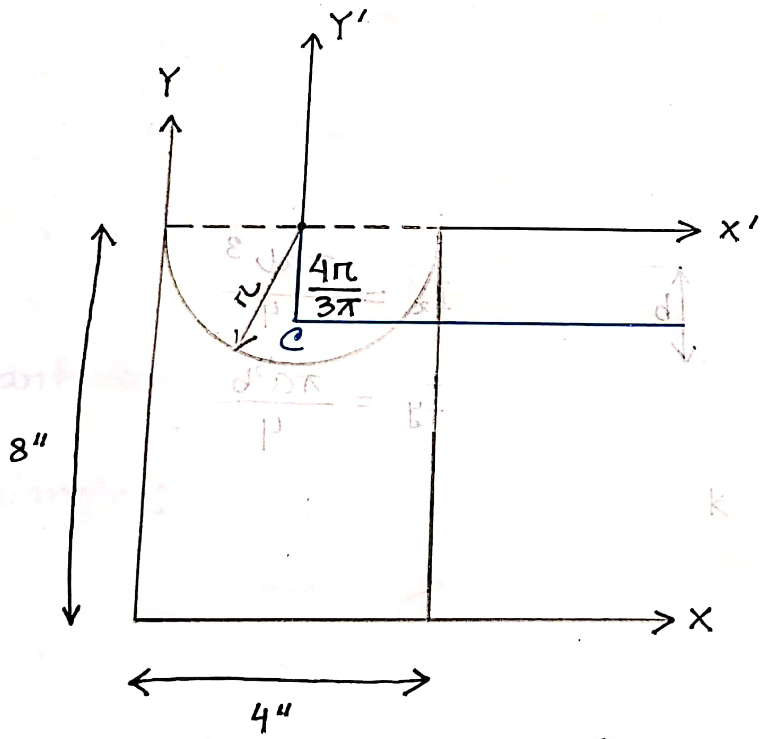
$$I_{x'} = \int y^2 dA = \int_0^h y^2\, b.dy$$
$$= b\left[\frac{y^3}{3}\right]_0^h$$
$$= \frac{bh^3}{3}$$

$$I_x = \frac{bh^3}{12}$$
$$I_y = \frac{hb^3}{12}$$

Transfere formula:

$$I_{x'} = \bar{I}_x + Ad^2$$
$$= \frac{bh^3}{12} + bh\cdot\left(\frac{h}{2}\right)^2$$
$$= \frac{bh^3}{12} + \frac{bh^3}{4}$$
$$= \frac{bh^3+3bh^3}{12} = \frac{bh^3}{3} \text{ unit}^4$$

#



I_x & $I_y = ?$

Sol^n:
Rectangle

$$I_x = \int y^2 dA = \frac{bh^3}{3} = \frac{4\,(8)^3}{3} = 682.67 \text{ inch}^4$$

$$I_y = \int x^2 dA = \frac{hb^3}{3} = \frac{8\,(4)^3}{3} = 170.67 \text{ inch}^4$$

Half circle : $I_{x'} = I_{y'} = \frac{\pi r^4}{8} = \frac{\pi \times 2^4}{8} = 6.28$ inch^4

$I_{x'} = \bar{I} + Ad^2$

$\Rightarrow 6.28 = \bar{I}_x + \frac{\pi r^2}{2} \cdot d^2$ $\left\{ d = \frac{4r}{3\pi} \right.$

$\Rightarrow 6.28 = \bar{I}_x + \frac{4\pi}{2} \cdot \frac{8^2}{9\pi^2}$

$\therefore \bar{I}_x = 1.75$ inch^4

Again, by applying transfer formula,

$I_x = \bar{I}_x + Ad^2$ $\left\{ r = 2 \right.$

$= 1.75 + \frac{\pi \cdot 2^2}{2} \cdot \left(8 - \frac{4r}{3\pi}\right)^2$

$= 323.06$ inch^4

$I_y = \bar{I}_y + Ad^2 = 6.28 + \frac{\pi \cdot 2^2}{2} \cdot 2^2 = 31.41$ inch^4

Moment of inertial of composite area :

$I_x = I_{\square x} - I_{\triangle x} = 682.67 - 323.06 = 359.61$ inch^4

$I_y = I_{\square y} - I_{\triangle y} = 170.67 - 31.4 = 139.27$ inch^4

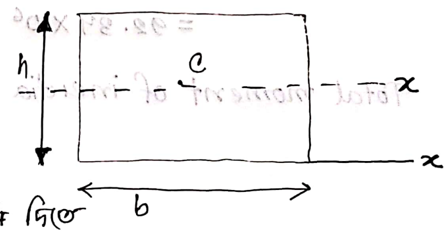
Moment of inertia :

$\hookrightarrow$ rotation

Moment/rotation কে resist করার জন্য যে resistance দেয়া হয়, সেটাই moment of inertia.

$\bar{I}_x = \frac{bh^3}{12}$

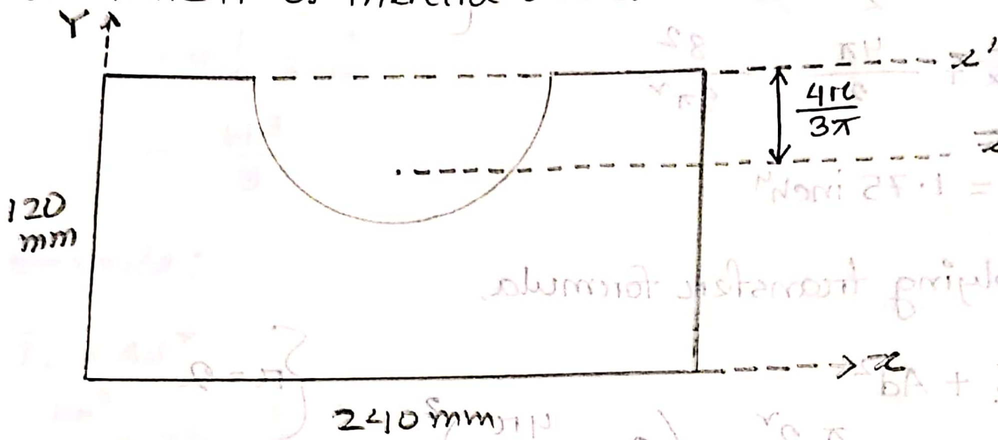
$I_{x'} = \frac{bh^3}{3}$



$I_{x'} > \bar{I}_x$, So, $I_{x'}$ এ বেশি resistance ~~বেশি~~ দিতে হবে rotation resist করার জন্য।

Moment of inertia of any axis = Moment of inertia of the centroid axis + Aa^2

Calculate moment of inertia about x-axis :



Sol^n :

For rectangle, $I_x = \frac{bh^3}{3} = \frac{240 \times 120^3}{3} = 138.2 \times 10^6 \ mm^4$

For semi circle,

$$I_{x'} = \frac{\pi r^4}{8} = 25.765 \times 10^6 \ mm^4$$

From x' to $\bar{x}$,

$$I_{x'} = I_{\bar{x}} + \frac{\pi r^2}{2} \cdot \left(\frac{4r}{3\pi}\right)^2$$

$$\therefore I_{\bar{x}} = 7.2 \times 10^6 \ mm^4$$

From $\bar{x}$ to x,

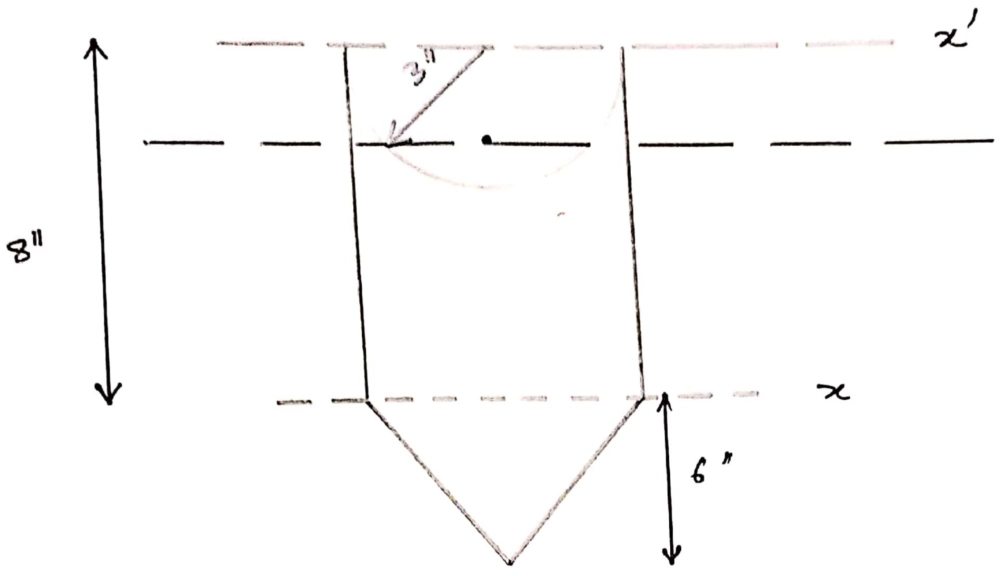
$$I_x = I_{\bar{x}} + Aa^2$$

$$= 7.2 \times 10^6 + \frac{\pi r^2}{2} \cdot \left(120 - \frac{4r}{3\pi}\right)^2$$

$$= 92.34 \times 10^6 \ mm^4$$

Total moment of inertia about x axis $= (138.2 \times 10^6) - (92.34 \times 10^6)$

Calculate moment of inertia about x-axis



For rectangle : $I_{x_1} = \frac{bh^3}{3} = \frac{6\times 8^3}{3} = 1024$ inch^4

For Triangle : $I_{x_2} = \frac{bh^3}{12} = \frac{6\times 6^3}{12} = 108$ inch^4

For semi-circle : $I_{x'} = \frac{\pi r^4}{8} = \frac{\pi\times 3^4}{8} = 31.81$ inch^4

$I_{x'} = \bar{I_x} + Aa^2$

$\Rightarrow \bar{I_x} = 31.8 - \frac{\pi\times 3^2}{2}\cdot\left(\frac{4\times 3}{3\pi}\right)^2 = 8.88$ inch^4

$I_{x_3} = \bar{I_x} + Aa^2 = 8.88 + \frac{\pi\cdot 3^2}{2}\cdot\left(8 - \frac{4\times 3}{3\pi}\right)^2 = 648.57$ inch^4

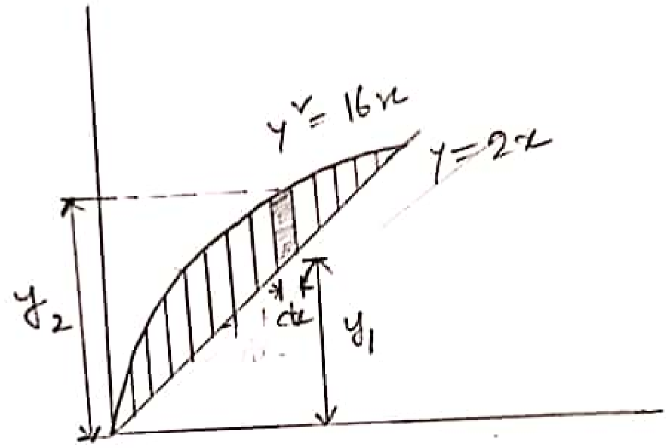
Total moment of inertia about x-axis $= I_{x_1} + I_{x_2} - I_{x_3}$

$\Rightarrow I_x = 1024 + 108 - 648.57$

$\therefore I_x = 483.43$ inch^4

36. Find the coordinates of the centroid of the area bounded by the curves $y^2 = 16x$ and $y = 2x$.

Soln:



we know that the coordinates of the centroid are,

$$\bar{x} = \frac{\int x\,dA}{A} \quad \text{——— (I)}$$

$$\bar{y} = \frac{\int y\,dA}{A} \quad \text{——— (II)}$$

The given,

$y^2 = 16x$ and $y = 2x$.

By solving the equations, the insection point is $(4, 8)$.

Now considering the element as a rectangular

$dA = (y_2 - y_1).dx$.

From the equations,

$y_2 = 4\sqrt{x}$ and $y_1 = 2x$.

Here, $A = \int_0^4 dA = \int_0^4 (4\sqrt{x} - 2x).dx.$

$$= \left[4.\frac{x^{3/2}}{3/2} - 2.\frac{x^2}{2}\right]_0^4$$

$\therefore A = \frac{16}{3}$ sq. units.

Now from eq^n (1) we get,

$$A\bar{x} = \int_0^4 (4\sqrt{x} - 2x).x.dx.$$

$$= \int_0^4 (4x^{3/2} - 2x^2) dx$$

$$= \left[4.\frac{x^{5/2}}{5/2} - 2.\frac{x^3}{3}\right]_0^4$$

$$A\bar{x} = \frac{128}{15}$$

$\therefore \bar{x} = \frac{128}{15} \div \frac{16}{3} = \frac{8}{5}$ units.

ain,

$$\bar{y} = \frac{\int y\, dA}{A}.$$

$A = (x_2 - x_1).dy.$

m eq^n.

$$x_2 = \frac{y^2}{16} \text{ and } x_1 = \frac{y}{2}.$$

Now, $A = \int_0^8 \left(\frac{y^2}{16} - \frac{y}{2} \right) dy$.

$$= \left[\frac{y^3}{16 \times 3} - \frac{y^2}{2 \times 2} \right]_0^8$$

$$A = \frac{16}{3} \text{ sq. units.}$$

$$\therefore A\bar{y} = \int_0^8 y \cdot dA = \int_0^8 \left(\frac{y^2}{2} - \frac{y^2}{16} \right) \cdot y \cdot dy$$

$$= \int_0^8 \left(\frac{y^2}{2} - \frac{y^3}{16} \right) dy$$

$$= \left[\frac{y^3}{2 \times 3} - \frac{y^4}{16 \times 4} \right]_0^8$$

$$\therefore A\bar{y} = \frac{64}{3}.$$

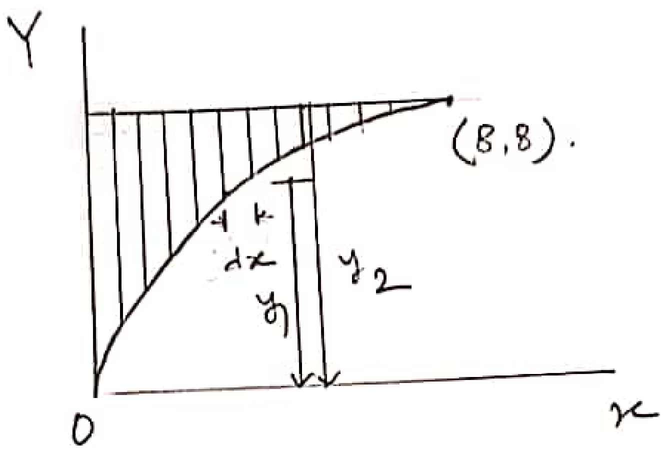
$$\therefore \bar{x} = \frac{64}{3} \div \frac{16}{3}.$$

$$\therefore \bar{y} = 4 \text{ units}$$

$\therefore$ The coordinates of the centroid are $\bar{x} = 4$ units and $\bar{y} = \bar{x} =$ [illegible] units.

685. Locate the centroid of the area included between the parabola $y^2 = 8x$, the y-axis and the line $y = 8$ in.

Soln:



We know the coordinates of the centroid are

$$\bar{x} = \frac{\int x\,dA}{A} \qquad \bar{y} = \frac{\int y\,dA}{A}.$$

The given eqn is $y^2 = 8x$ and $y = 8$.

By solving the equation, the insection point is (8,8)

Now consider the element as a rectangle,

$dA = (y_2 - y_1).dx.$

From. the equations,

$y_2 = 8,$

$y_1 = \sqrt{8x}$.

Now, $A = \int_0^8 dA = \int_0^8 (8 - \sqrt{8x})\, dx.$

$$= \left[8x - \sqrt{8} \cdot \frac{x^{3/2}}{3/2}\right]_0^8$$

$$= \frac{64}{3} \text{ sq. unit}$$

Now, From eq^n ① we get.

$$A\bar{x} = \int_0^8 (8 - \sqrt{8x})\, x\, dx$$

$$= \int_0^8 (8x - \sqrt{8} \cdot x^{3/2})\, dx$$

$$= \left[8 \cdot \frac{x^2}{2} - \sqrt{8} \cdot \frac{x^{5/2}}{5/2}\right]_0^8$$

$$= \frac{256}{5}$$

$$\therefore \bar{x} = \frac{256}{5} \div \frac{64}{3}$$

$$= 2{\cdot}4 \text{ unit.}$$

Again, $\bar{y} = \frac{\int y\, dA}{A}.$

$dA = (x_2 - x_1) \cdot dy,$

$= x_2\, dy.$

Now, $A = \int_0^8 dA = \int_0^8 \frac{y^2}{8} \cdot dy = \left[\frac{y^3}{8\times 3}\right]_0^8$

$= \frac{64}{3}$ sq. units.

Now, from eq^n (II), we get.

$A\bar{y} = \int_0^8 \cdot \frac{y^2}{8} \cdot y \cdot dy = \int_0^8 \frac{y^3}{8} dy$.

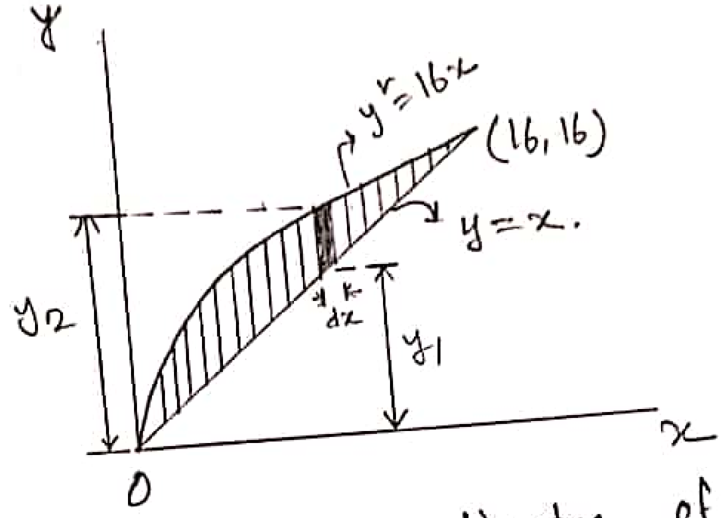
$= \left[\frac{y^4}{8\times 4}\right]_0^8 = 128$

$\therefore \bar{y} = 128 \cancel{\times} \div \frac{64}{3} = 6$ units.

$\therefore$ The coordinates of the included area's centroid is $\bar{x} = 2{\cdot}4$ and $\bar{y} = 6$ units.

587. Find the coordinates of the centroid of the area bounded by the curves $y^2=16x$ and $y=x$.

Soln:



we know that the coordinates of the centroid are,

$$\bar{x} = \frac{\int x\, dA}{A} \quad \text{------ (I)}$$

$$\bar{y} = \frac{\int y\, dA}{A} \quad \text{------ (II)}$$

the given equations,

$y^2 = 16x$ and $y=x.$

By solving the equations, the insection point (16,16).

Now considering the element as a rectangle

$dA = (y_2 - y_1)\,dx.$

From equations,

$y_2 = 4\sqrt{x}$ and $y_1 = x.$

Hence, $A = \int_0^{16} dA = \int_0^{16} (4\sqrt{x} - x)\, dx.$

$$= \left[4 \cdot \frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^{16}$$

$$= \frac{128}{3} \text{ sq. units.}$$

Now from eq^n (1) we get,

$$A\bar{x} = \int_0^{16} (4\sqrt{x} - x)\, x \, dx.$$

$$= \int_0^{16} (4x^{3/2} - x^2) \, dx$$

$$= \left[4 \cdot \frac{x^{5/2}}{5/2} - \frac{x^3}{3}\right]_0^{16}$$

$$A\bar{x} = \frac{4096}{15}$$

$$\therefore \bar{x} = \frac{4096}{15} \div \frac{128}{3}$$

$$= 6.4 \text{ units.}$$

Again, $\bar{y} = \dfrac{\int y\, dA}{A}.$

$$dA = (x_2 - x_1) \, dy.$$

From eq^n,

$x_2 = \dfrac{y^2}{16}$ and $x_1 = y.$

Now, $A = \int_0^{16} \left(\frac{y^2}{16} - y\right) dy.$

$$= \left[\frac{y^3}{16\times 3} - \frac{y^2}{2}\right]_0^{16}$$

$\therefore A = \frac{128}{3}$ sq. units.

$\therefore A\bar{y} = \int_0^{16} y\,dA = \int_0^{16} \left(\frac{y^2}{16} - y\right) y\,dy.$

$$= \int_0^{16} \left(\frac{y^3}{16} - y^2\right) dy.$$

$$= \left[\frac{y^4}{16\times 4} - \frac{y^3}{3}\right]_0^{16} = \frac{1024}{3}$$

$\therefore A\bar{y} = \frac{1024}{3}$

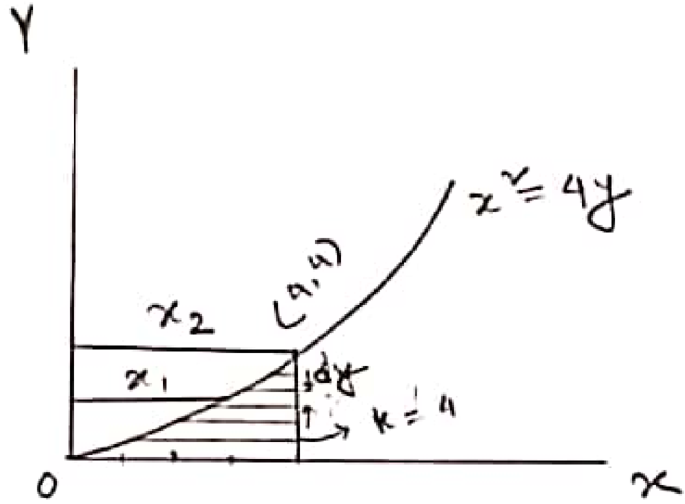
$\therefore \bar{y} = \frac{1024}{3} \div \frac{128}{3} = 8$ unit.

$\therefore$ the coordinates of the centroid are

$\bar{x} = 6.4$ and $\bar{y} = 8$.

682| Locate the centroid of the area included between the parabola $x^2 = 4y$, the line $x=4$ and x-axis.

Soln:



We know that the coordinates of the centroid are,

$$\bar{x} = \frac{\int x\, dA}{A}$$

and $$\bar{y} = \frac{\int y \cdot dA}{A}.$$

The given eqn is,

$$x^2 = 4y \text{ and } x = 4.$$

Solving the equations, the insection point is (4,4).

Now consider the element as a rectangle

$dA = (x_2 - x_1)\, dy$.

From the eq^n.

$x_2 = 4$,

and $x_1 = 2\sqrt{y}$.

Now, $\bar{y} = \dfrac{\int y\,dA}{A} = \dfrac{\int_0^4 y.(4-2\sqrt{y})dy}{A}$ ——— (1)

Here,

$A = \int_0^A dA = \int_0^4 (4-2\sqrt{y})\,dy$

$= \left[4y - \dfrac{2.y^{3/2}}{3/2}\right]_0^4$

$= \dfrac{16}{3}$ sq unit

Now, From eq^n (1), we can get,

$A\bar{y} = \int_0^4 (4y - 2y^{3/2})\,dy$

$= \left[4.\dfrac{y^2}{2} - \dfrac{2.y^{5/2}}{5/2}\right]_0^4$

$A\bar{y} = 32 - \dfrac{128}{5}$

$= \dfrac{32}{5}$.

$\therefore \bar{y} = \frac{32}{5} \div \frac{16}{3}$

$= \frac{6}{5}$ units.

Again, $\bar{x} = \frac{\int x \, dA}{A}$

$dA = (y_2 - y_1) dx$.

$\therefore A\bar{y} = \int (y_2 - y_1) x . dx$.

From eq^n,

$y_2 = 0$.

$y_1 = \frac{x^2}{4}$.

Now, $A = \int_0^4 dA = \int_0^4 (\frac{x^2}{4}) . dx$

$= \left[\frac{x^3}{4 \times 3}\right]_0^4$

$= \frac{16}{3}$ sq. units.

$\therefore A\bar{x} = \int_0^4 x . (\frac{x^2}{4}) dx$

$A\bar{x} = \int_0^4 \frac{x^3}{4} dx. \quad = \left[\frac{x^4}{4 * 4}\right]_0^4$

$A\bar{x} = 16$

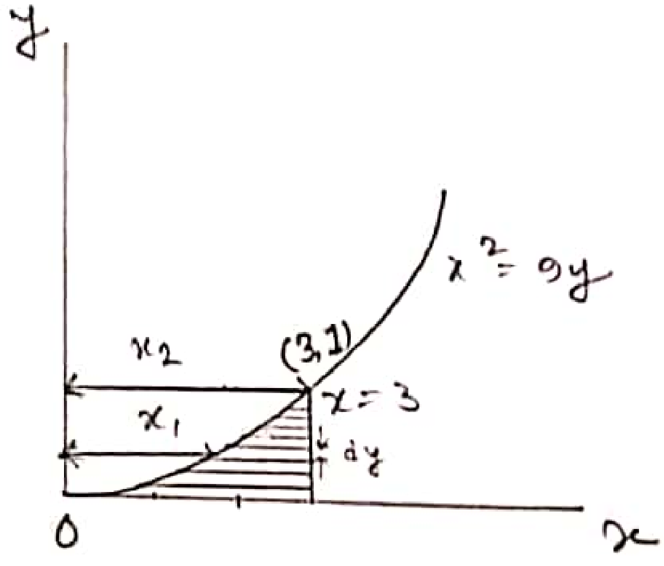
$\bar{x} = 16 \div \frac{16}{3} = 3$ units.

$\therefore$ The coordinates of the centroid are

$\bar{x} = 3$ unit and $\bar{y} = \frac{6}{5}$ unit.

Locate the centroid of the area bounded by a parabola $x^2 = 9y$, the x-axis and the line $x = 3$. The unit is in inch.

Soln:



We know that the co-ordinates of the centroid are,

$$\bar{x} = \frac{\int x \, dA}{A} \quad \text{——— (I)}$$

and

$$\bar{y} = \frac{\int y \cdot dA}{A} \quad \text{——— (II)}$$

The given eq^n is,

$$x^2 = 9y \quad \text{and} \quad x = 3.$$

By solving the equations the insection point $(3,1)$.

Now, considering the element as a rectangle.

$$dA = (x_2 - x_1).dy.$$

From the eq^n;

$$x_2 = 3.$$

$$x_1 = 3.\sqrt{y}.$$

Here, $A = \int_0^1 (3 - 3\sqrt{y}).dy$

$$= \left[3y - 3.\frac{y^{3/2}}{3/2}\right]_0^1$$

$$= \left[3 - \frac{3.1}{3/2}\right].$$

$$= 1 \text{ sq. unit}.$$

Now, from eq^n (ii), we get,

$$A\bar{y} = \int_0^1 (3 - 3\sqrt{y}).y.dy$$

$$= \int_0^1 (3y - 3y^{3/2})\,dy.$$

$$= \left[3.\frac{y^2}{2} - 3.\frac{y^{5/2}}{5/2}\right]_0^1.$$

$$A\bar{y} = \frac{3}{10}$$

$$\therefore \bar{y} = \frac{3}{10} \text{ unit.}$$

Again, $\bar{x} = \dfrac{\int x\, dA}{A}$.

$dA = (y_2 - y_1).dx.$

$\therefore A\bar{x} = \int (y_2 - y_1).x.dx.$

From. eqn.

$y_2 = 0$ and $y_1 = \dfrac{x^2}{9}$.

Now, $A = \displaystyle\int_0^3 \left(\frac{x^2}{9}\right).dx.$

$$= \left[\frac{x^3}{9\times 3}\right]_0^3.$$

$= 1$ sq unit

$\therefore A\bar{x} = \displaystyle\int_0^3 x.\left(\frac{x^2}{9}\right).dx. = \left[\frac{x^4}{9\times 4}\right]_0^3$

$= \frac{9}{4}$ unit.

$\therefore \bar{x} = \frac{9}{4}$ unit.