Analytic Mechanics → Raines.

Plane Motion

Ex-173: Given that,

$$S = 15t^2 \text{ feet}.$$

and, Time, $t = 2$ sec.

we know,

$$a = \frac{d^2 s}{dt^2} = \cancel{15}\ \frac{d}{dt}(15t) \times 2$$

$$= 30 \text{ ft sec}^{-2} \text{ (Ans).}$$

Ex-174: Given that,

Acceleration, $a = 4t$ fps^2.

Initial speed $= 10$ fps.

Time interval, $t = 4$ sec.

We know, $a = \frac{dv}{dt}$.

$$\Rightarrow dv = a\,dt.$$

$$\Rightarrow \int_{10}^{v} dv = \int_{0}^{4} a\,dt.$$

$$\Rightarrow [v - 10] = \left[4 \cdot \frac{t^2}{2}\right]_0^4$$

$$= v - 10 = 2 \times 4^2$$

$$\Rightarrow v = 42 \text{ fps. (Ans)}$$

To find the distance, first integrate $dv = 4t\,dt$ as an indefinite integral and find,

$$v = 2t^2 + c, \quad \text{--- (1)}$$

Where c is the integration constant,

Now, when, $t = 0$ sec, $v = 10$ fps

$\therefore c = 10$

$\therefore (1) \Rightarrow v = 2t^2 + 10$

Again, we know,

$$v = \frac{ds}{dt}$$

$$\Rightarrow ds = v\,dt.$$

$$\Rightarrow \int_{0}^{s} ds = \int_{0}^{4} (2t^2 + 10)\,dt.$$

$$\Rightarrow s = \left[\frac{2t^3}{3} + 10t\right]_0^4.$$

$\therefore s = 82.67$ ft. (Ans.)

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Ex-175: Given,

$$\ddot{s} = a = 2s \quad \text{--- (i)}$$

we know that,

$$a\,ds = v\,dv.$$

Now,
$$\int v\,dv = \int a\,ds.$$

$$\Rightarrow \frac{v^2}{2} = 2 \cdot \frac{s^2}{2} + c.$$

$$\Rightarrow \frac{v^2}{2} = s^2 + c.$$

When, $v=0$, $s=0$,

$$\therefore c = 0.$$

$$\therefore v^2 = 2s^2. \quad \text{--- (ii)}$$

After 10 ft, the speed will be,

$$v^2 = 2 \times 10^2$$

$$\therefore v = \sqrt{2} \times 10 = 14.14 \text{ fps. (Ans.)}$$

Again from (11),

$$v^2 = 2s^2.$$

$\Rightarrow$ $v = \sqrt{2}\, s.$

$\Rightarrow$ $\frac{ds}{dt} = \sqrt{2}\, s.$ $\Rightarrow$ $\frac{ds}{s} = \sqrt{2}\, dt.$

$\Rightarrow$ $\int_0^{10} \frac{ds}{s} = \sqrt{2} \int_0^{t} dt$

$\Rightarrow$ $\left[\ln s\right]_0^{10} = \sqrt{2}\, t$

$\Rightarrow$ $(\ln 10 - \ln 0) = \sqrt{2}\, t.$

$\therefore t = 1.63$ sec. (Ans.)

Ex-178: Convert mph to fps.

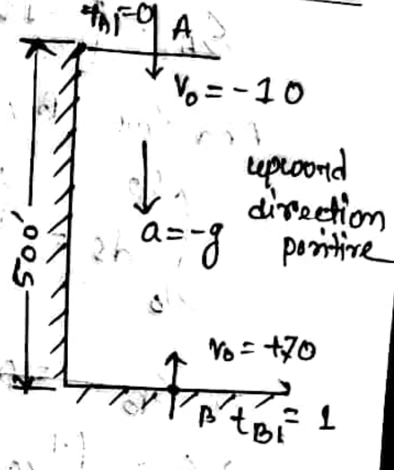
$$45 \text{ mph} = \frac{45 \times 5280}{3600} = 66 \text{ fps}.$$

$$15 \text{ mph} = \frac{15 \times 5280}{3600} = 22 \text{ fps}.$$

$$\therefore a = \frac{\Delta v}{t} = \frac{66 - 22}{1800} = 0.0244 \text{ fps}^2.$$

Ans

Ex-179: for body A,



$a = \frac{dv}{dt}$.

$\Rightarrow \int dv = \int a\,dt$.

$\Rightarrow \int_{-10}^{v} dv = \int_0^t -g\,dt$

$\Rightarrow v+10 = -gt \quad [\because v_0 = -10]$. —— (1)

for body B,

$\int_{70}^{v} dv = -\int_1^t g\,dt$

$\Rightarrow v - 70 = -gt + g$. —— (ii)

Again, we know,

$v = \frac{ds}{dt}$

$\Rightarrow ds = v\,dt$.

$\Rightarrow \int_0^{S_A} ds = \int_0^t (-10 - gt)\,dt$

$$S_A = \left[-10t - \frac{gt^2}{2}\right]_0 \quad \text{——— (iii)}$$

And for B,

$$\int_0^{S_B} ds = \int_0^t (70 - gt + g)\, dt$$

$$\cancel{\Rightarrow S_B = 70t - \frac{gt^2}{2} + gt}$$

$$\Rightarrow S_B = \int_0^t 70\,dt - \int_0^t gt\,dt + \int_0^t g\, dt$$

$$= 70t - 70 - \frac{gt^2}{2} + \frac{g}{2} + gt - g \quad \text{——— (iv)}$$

$$S_B = 102.2t - \frac{gt^2}{2} - 86.1$$

When the bodies pass, they will be at that instant at the same elevation, $\therefore S_A = S_B$.

$$\cancel{10t - \frac{gt^2}{2} = 70t - 70 - g\; 102.2t - \frac{gt^2}{2} - 86.1}$$

$$\Rightarrow \cancel{102.2t = 8\ 76.1\ 102.2t = 86.1}$$

$$\Rightarrow \cancel{to} = 500 - 10t - \cancel{\frac{gt^2}{2}} = 102.2t - \cancel{\frac{gt^2}{2}} - 86.1$$

$$\Rightarrow 112.2t = 586.1$$

$$\therefore t = 5.22 \text{ sec.} \quad \text{Ans.}$$

$\therefore S_A = S_B = \cancel{10 - g} \times 500 - 10 \times 5.22 - g \times \frac{(5.22)^2}{2}$

$= 9.1$ ft. (Ans.)

(c) when, $t = 5.22$ sec.

$\therefore V_B = 70 - 32.2 \times 5.22 + 32.2$.

$= -65.88$ fps. (Ans.).

The negative answer in this case shows that B is moving downword, B is returning from its highest point. for another verification that B is moving down.

Ex-18 I. Given that,

$a = (3t - 12)$ fps^2.

we know,

$a = \frac{dv}{dt} = 3t - 12$

$\Rightarrow dv = (3t - 12)\,dt$.

$\Rightarrow \int_{15}^{v} dv = \int_{0}^{10} (3t - 12)\,dt$

$\Rightarrow v - 15 = \left[\frac{3t^2}{2} - 12t\right]_0$

$\Rightarrow v - 15 = 30$

$\therefore v = 15$ fps.

b) To find the displacement in a second integral make the first integration indefinite,

$$v = \frac{3t^2}{2} - 12t + c.$$

when, $t = 0$, $v = 15$ fps.

$\therefore c = 15$.

$$\therefore v = \frac{3t^2}{2} - 12t + 15.$$

$$\therefore \frac{ds}{dt} = \left(\frac{3t^2}{2} - 12t + 15\right).$$

$$\Rightarrow \int_0^s ds = \int_0^{14} \left(\frac{3t^2}{2} - 12t + 15\right) dt.$$

$$\Rightarrow s = \left[\frac{3t^3}{3 \times 2} - \frac{12t^2}{2} - 15t\right]_0^{14}$$

$$s = -14 \text{ ft}.$$

Assuming that the positive sense toward the right, the negative sign for x show that the displacement is to the left from the origin

Ex-186:

a) Given that $\theta = 0.1t^3 - 0.3t^2 + 0.8t$.

The angular displacement is obtained directly from the given law by substituting, $t=6$,

$\theta = 0.1\times6^3 - 0.3\times6^2 + 0.8\times6$.

$= 15.6$ rad $= \frac{15.6}{2\pi} = 2.48$ rev. (Ans.)

b) By differentiating θ with respect to t, we get,

$\omega = \frac{d\theta}{dt} = 0.1\times3t^2 - 0.3\times2t + 0.8$

After 6 sec, the angular velocity will be,

$\omega = 0.3\times6^2 - 0.6\times6 + 0.8$

$= 8$ rad per sec.

$= \frac{8\times60}{2\pi} = 76.4$ rpm.

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c) Differentiating ω with respect to t we get,

$$\alpha = \frac{d\omega}{dt} = 0.3 \times 2t - 0.6$$

After 6 sec, angular acceleration,

$$\alpha = 0.6 \times 6 - 0.6 = 3 \text{ rad per sec}^2.$$

Ex-188:

a) Given that,

$$\omega = 300 \text{ rpm}.$$

$$= \frac{300 \times 2\pi}{60} = 31.42 \text{ rad s}^{-1}$$

Angular accleration, $\alpha = -2 \text{ rad s}^{-2}$.

When the wheel will stop then $\omega = 0$,

According to the definition of angular acceleration,

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$\Rightarrow t = \frac{0 - 31.42}{-2}$$

$$= 15.707 \text{ sec}$$ Ans.

b) w

c) u

d)

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$v = u + at$.

(b) we know that,

$\omega = \omega_0 + \alpha t$,

$= 31.42 + (-2) \times 10$

$= 11.42$

$= \frac{11.42 \times 60}{2\pi}$

$= 109.05$ rpm.

(c) we know,

$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$.

$= 31.42 \times 10 - 2 \times \frac{1}{2} \times 10^2$

$\Rightarrow 214.2$

$\theta = 34.09$ rev. (Ans.).

| $\omega_0 = 31.42$ rads^{-1}. |
| $\alpha = -2$ rads^{-2}. |
| $t = 10$ sec. |

(d)

$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$.

$= 31.42 \times 15.708 + \frac{1}{2} \times (-2) \times (15.708)^2$

$= 247$ rad. (Ans).

| $\omega_0 = 31.42$ rads^{-1} |
| $t = 15.708$ sec. |
| $\alpha = -2$ rads^{-2} |

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(e) The angular displacement from 10 sec until the wheel stop is found as —

$247 - 214 = 33$ rad

$= \frac{33}{2\pi} = 5.25$ rev. (Ans.)

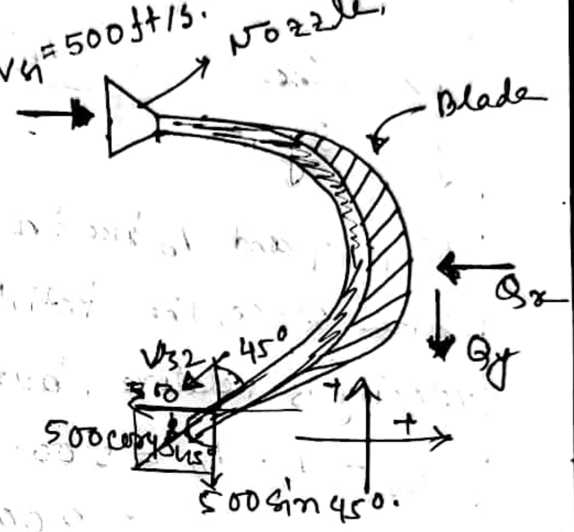
Faires

"Impulse and Momentum."

Ex → 303:

The stream moving, $V_1 = 500$ ft/s.

$m = \frac{1.5}{32.2}$ slugs/s.



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Now,

we know,

$R_x = m\bar{v}_2 - m\bar{v}_1$

$\therefore -Q_x = \frac{1.5}{32.2}(-500 \cos 45^\circ - 500)$.

$Q_x = 39.76$ lb.

And, $-Q_y = \frac{1.5}{32.2}(-500 \cos 45^\circ - 0)$.

$Q_y = 16.5$ lb

$\therefore R = \sqrt{Q_x^2 + Q_y^2} = 43.08$ lb. Ans

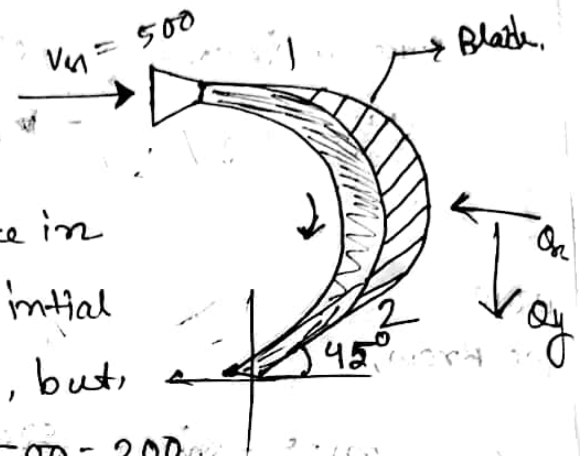
$\therefore \tan\theta = \frac{Q_y}{Q_x} \Rightarrow \theta = \tan^{-1}\left(\frac{16.5}{39.76}\right)$

$= 22.52^\circ$. Ans

Ex-304:

$R_x = -Q_x$

$R_y = -Q_y$.



As, V_{s1} and V_b are in same sense, the initial velocity is relative, but,

$V_{s2} = V_{s1} - V_b = 500 - 200$

$= 300$ fps.

Mass involve in 300 fps $= 1.5 \times \frac{300}{500}$

$= 0.9$.

$\therefore R_x = (-Q_x) = \frac{0.9}{32.2}(-300 \sin 45^\circ - 300)$.

$R_x = 14.31$ lb.

$R_y = -Q_y = \frac{0.9}{32.2}(-300 \cos 45^\circ - 0)$.

$= 5.93$ lb.

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(14.31)^2 + (5.99)^2}$

$= 15.5$ lb.

The horsepower is, $hp = \frac{Q_x V_b}{550}$

$= \frac{(14.31) \times 200}{550}$

$= 5.2$ hp. Ans.

The horsepower represented by the original kinetic energy of the jet is

$\frac{wv^2}{2g\,550} = \frac{1.5 \times (500)^2}{2 \times 32.2 \times 550}$

$= 10.59$ hp.

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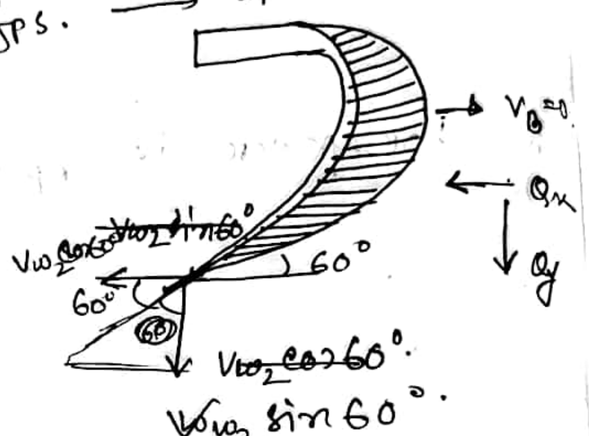
1580.

Here,

$V_{w_1} = V_{w_2} = 200$ fps.



Now,

$\cancel{-Q_x = \frac{5}{32.2}\left(-V_{w_2}\cos 60^\circ\right.}$

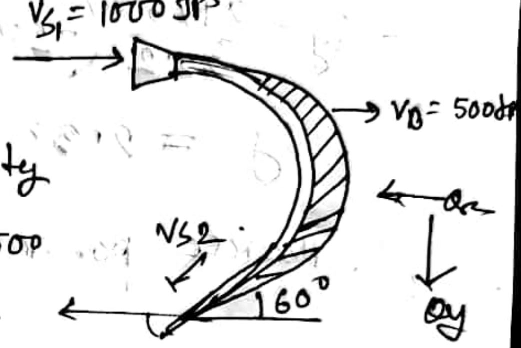
$-Q_x = \frac{5}{32.2}\left(-200\cos 60^\circ - 200\right).$

$Q_x = 46.58$ lb. Ans

And $-Q_y = \frac{5}{32.2}\left(-200\sin 60^\circ - 0\right).$

$Q_y = 26.9$ lb. (Ans)

Prob 15.62:



As V_{S_1} and V_B are in same sense, initial velocity is relative, $V_{S_2} = 1000 - 500$

$= 500$ fps.

Rate of flow $= A \times V_{S_1} \times w$.

$= \frac{1 \times 1}{12 \times 12} \times 1000 \times 0.05$.

$= 0.35$ lb/sec.

$\therefore$ striking blade, $w_p = $ Rate of flow $\times \frac{\text{Relative velocity}}{\text{Absolute velocity}}$

$= 0.35 \times \frac{500}{1000}$

$= 0.175$ lb/sec.

Now,

$-Q_x = \frac{0.175}{32.2}\left(-500\cos 60^\circ - 500\right)$.

$Q_x = 4.08$ lb.

$$-Q_y = \frac{0.175}{32.2}\left(-200\sin 60^\circ - 0\right)$$

$$Q_y = 2.35 \text{ lb.}$$

$\therefore$ Horsese power $= \frac{Q_x \times V_b}{550}$

$$= \frac{4.08 \times 500}{550}$$

$$= 3.71 \text{ H.P.}$$ Ans