MECHANICS OF MATERIALS –I
SHEAR STRESS IN BEAM

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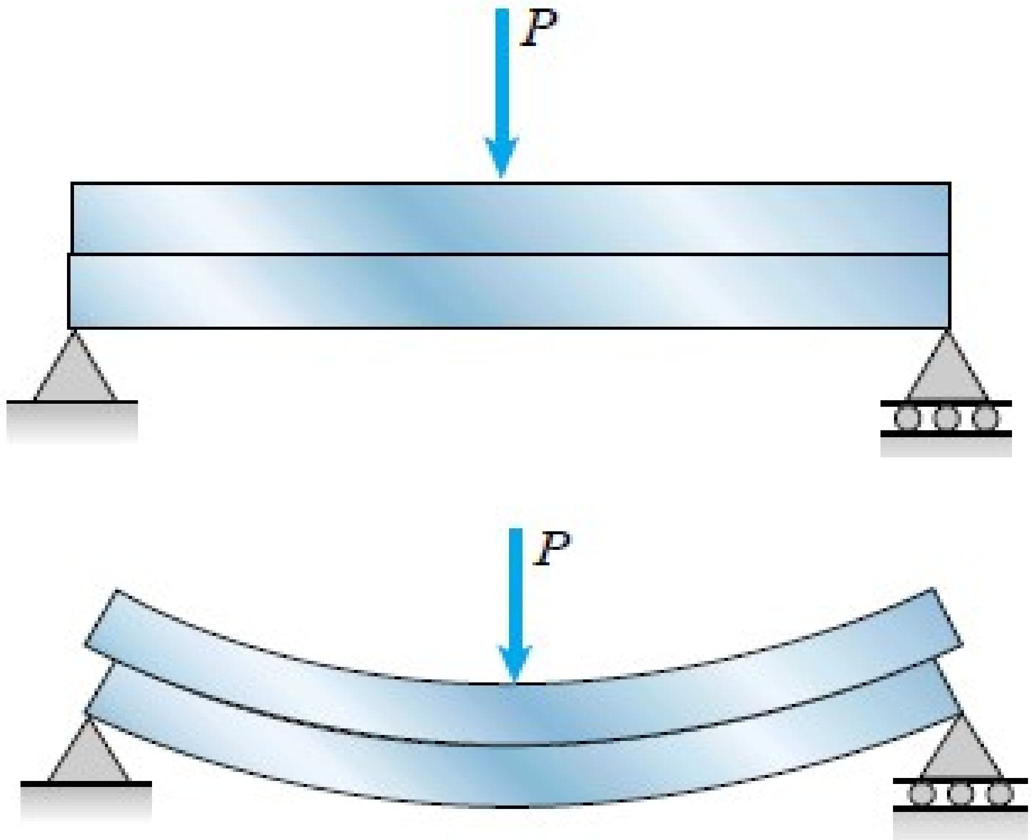
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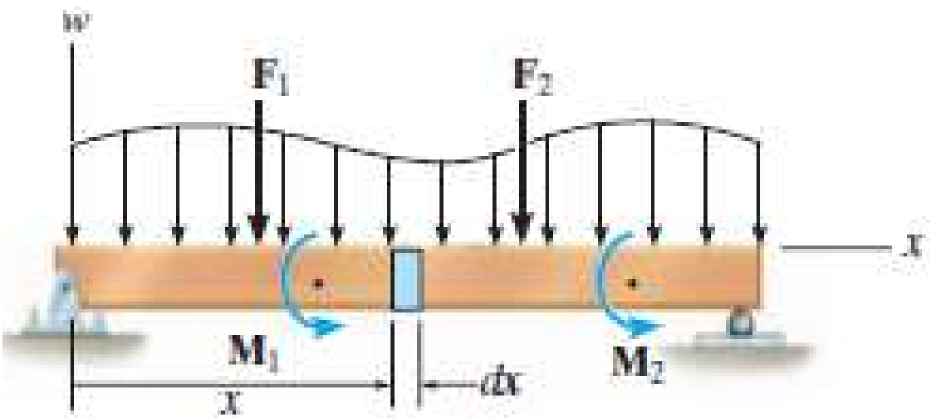
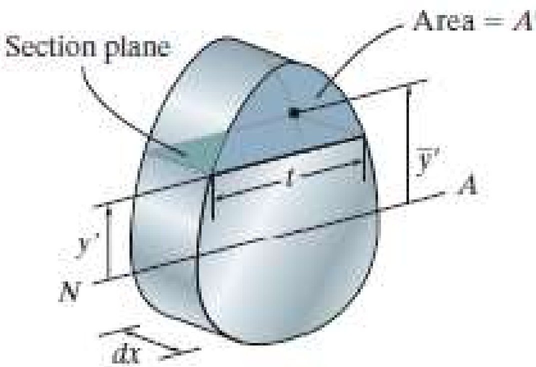
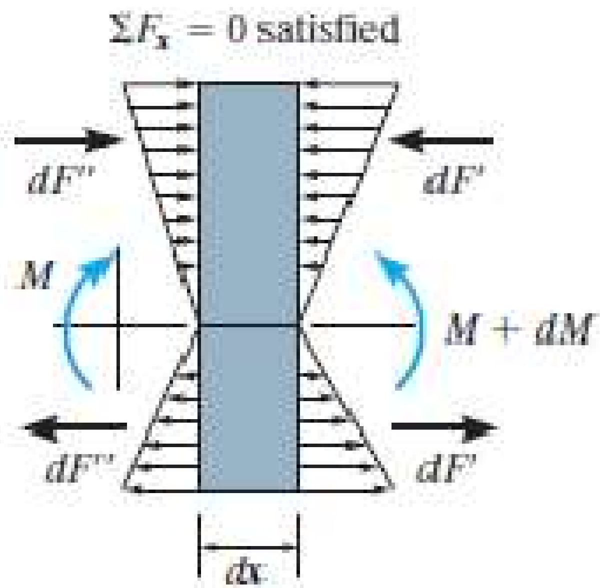
Shear Formula



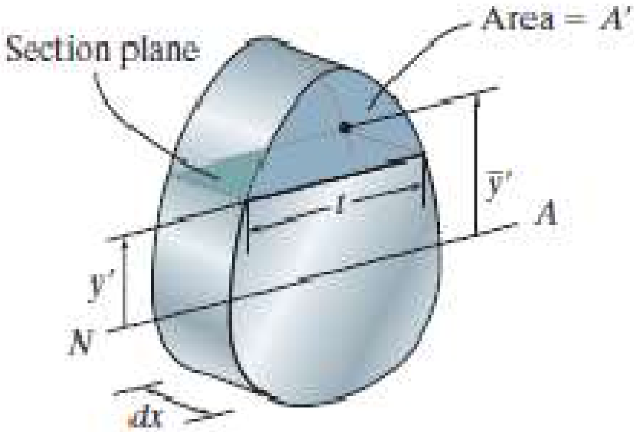


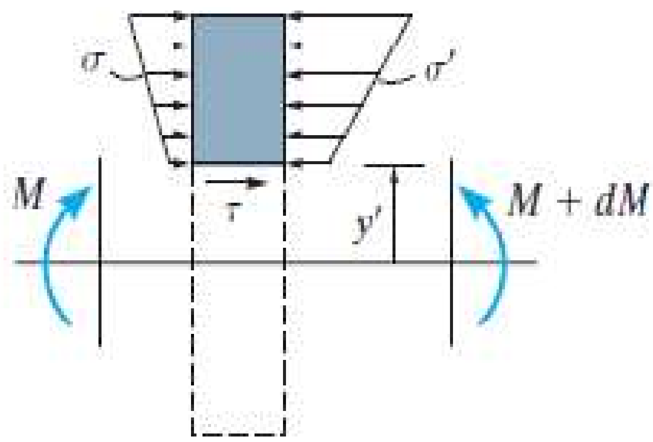


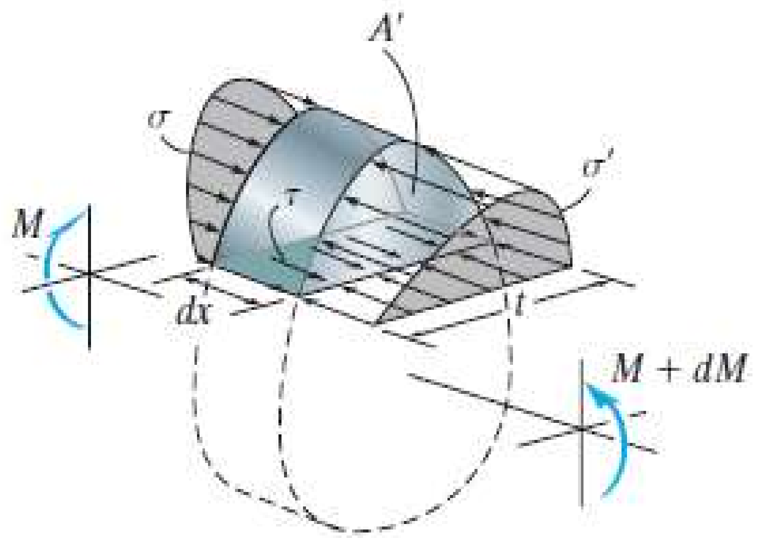
Shear Formula

- Consider a beam loaded as shown in Figure 1. Also consider an element of length dx. The vertical forces {V, V+dV, w(x)} are ignored because they are not involved in horizontal force summation.
- Now consider the shaded top portion of the element that has been sectioned at $y^{/}$ from the neutral axis.
- This section has a width t at the section and cross sectional area $A^{/}$.
- Because the resultant moments on each side of the element differ by *dM*, it can be seen in Figure that horizontal force equilibrium will not be satisfied *unless* a longitudinal shear stress acts over the bottom face of the segment

Shear Formula







Shear Formula

- Applying the equation of horizontal force equilibrium,

$$\sum F_x = \int_{A'} \sigma' \, dA' - \int_{A'} \sigma \, dA' - \tau(t dx) = 0$$

$$\therefore \int_{A'} \frac{(M + dM)}{I} y \, dA' - \int_{A'} \frac{M}{I} y \, dA' - \tau(t dx) = 0$$

$$\frac{dM}{I} \int_{A'} y \, dA' = \tau(t dx)$$

$$\tau = \frac{dM}{dx} \frac{1}{It} \int_{A'} y \, dA'$$

Shear Formula

$$\tau = \frac{dM}{dx}\frac{1}{It}\int_{A'} y\,dA'$$

$$\tau = \frac{dM}{dx}\frac{1}{It}\bar{y}'A'$$

$$\tau = \frac{VQ}{Ib}$$

$$\bar{y} = \frac{\int y\,dA}{A}$$

Shear formula

τ = the shear stress in the member at the point located a distance y' from the neutral axis. This stress is assumed to be constant and therefore *averaged* across the width t of the member

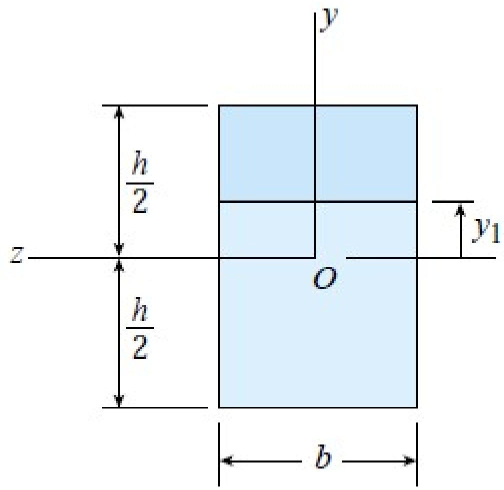
V = the internal resultant shear force, determined from the method of sections and the equations of equilibrium

I = the moment of inertia of the *entire* cross-sectional area calculated about the neutral axis

t = the width of the member's cross-sectional area, measured at the point where τ is to be determined

$Q = \overline{y}' A'$, where A' is the area of the top (or bottom) portion of the member's cross-sectional area, above (or below) the section plane where t is measured, and $\overline{y}'$ is the distance from the neutral axis to the centroid of A'

Distribution of Shear Stresses in a Rectangular Beam



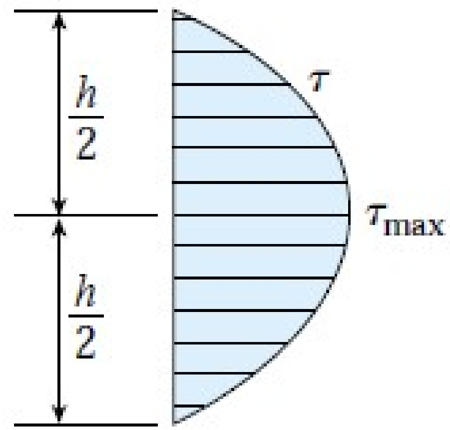
- The first moment Q *of the* shaded part of the cross-sectional area is obtained by multiplying the area by the distance from its own centroid to the neutral axis

$$Q = b\left(\frac{h}{2} - y_1\right)\left(y_1 + \frac{h/2 - y_1}{2}\right) = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

Same result can be obtained by integration

$$Q = \int y\, dA = \int_{y_1}^{h/2} yb\, dy = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

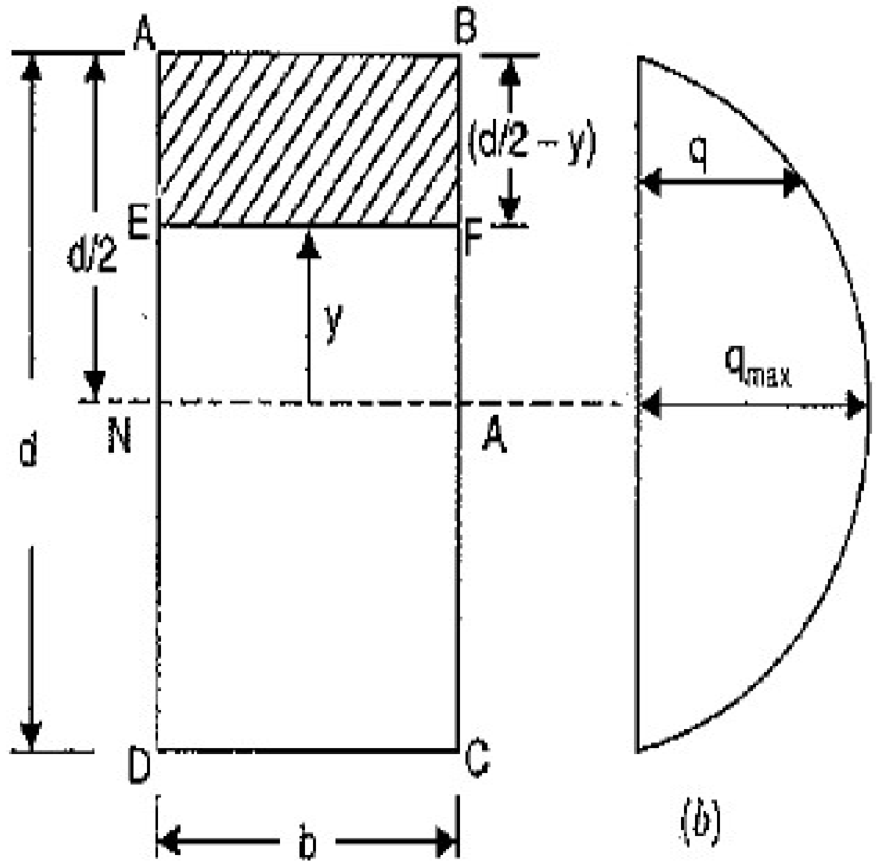


This equation shows that the shear stresses in a rectangular beam vary quadratically with the distance y_1 *from the neutral axis.*

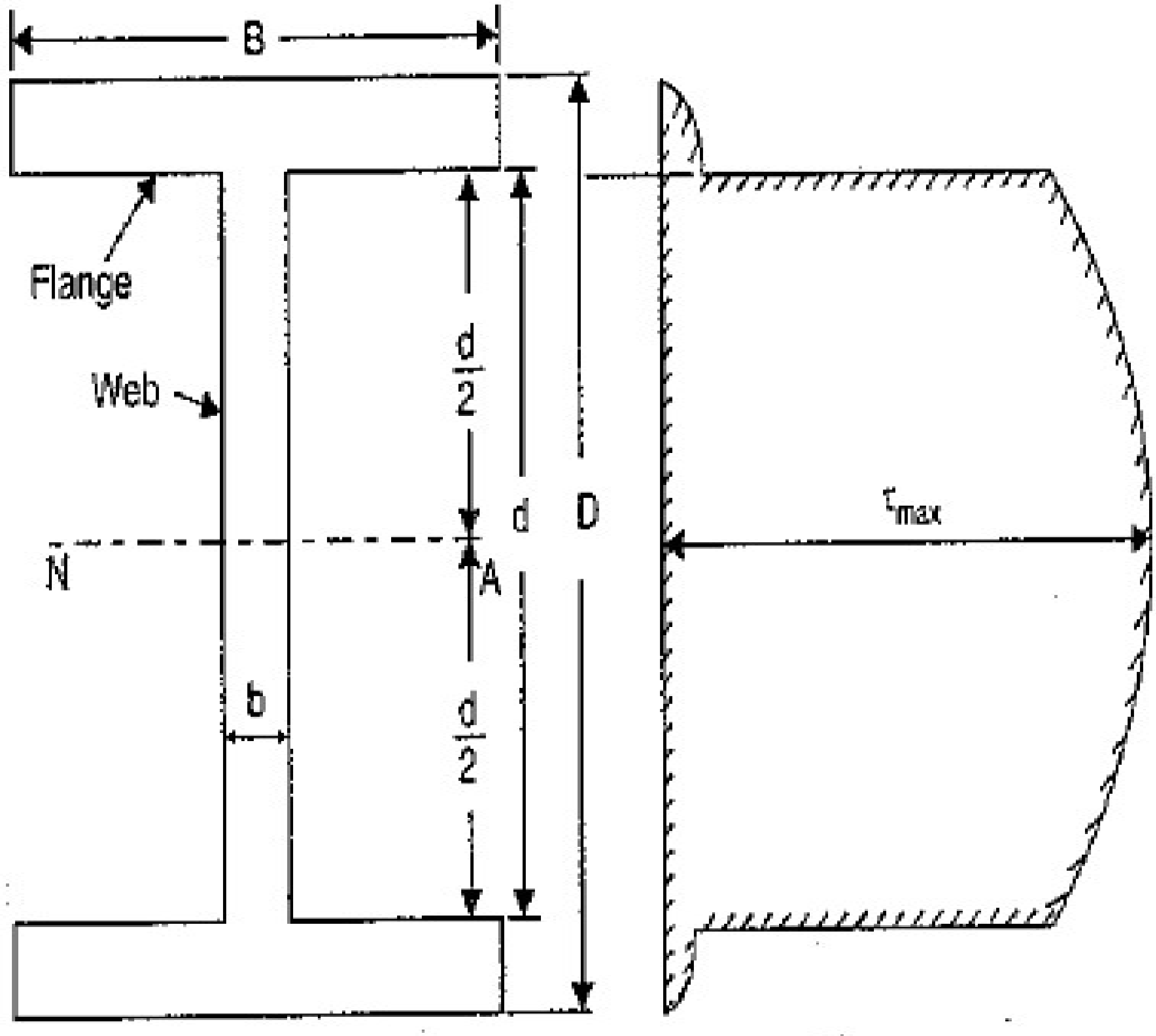
$$\tau_{\max} = \frac{Vh^2}{8I} = \frac{3V}{2A}$$

A = bh is the cross-sectional area.

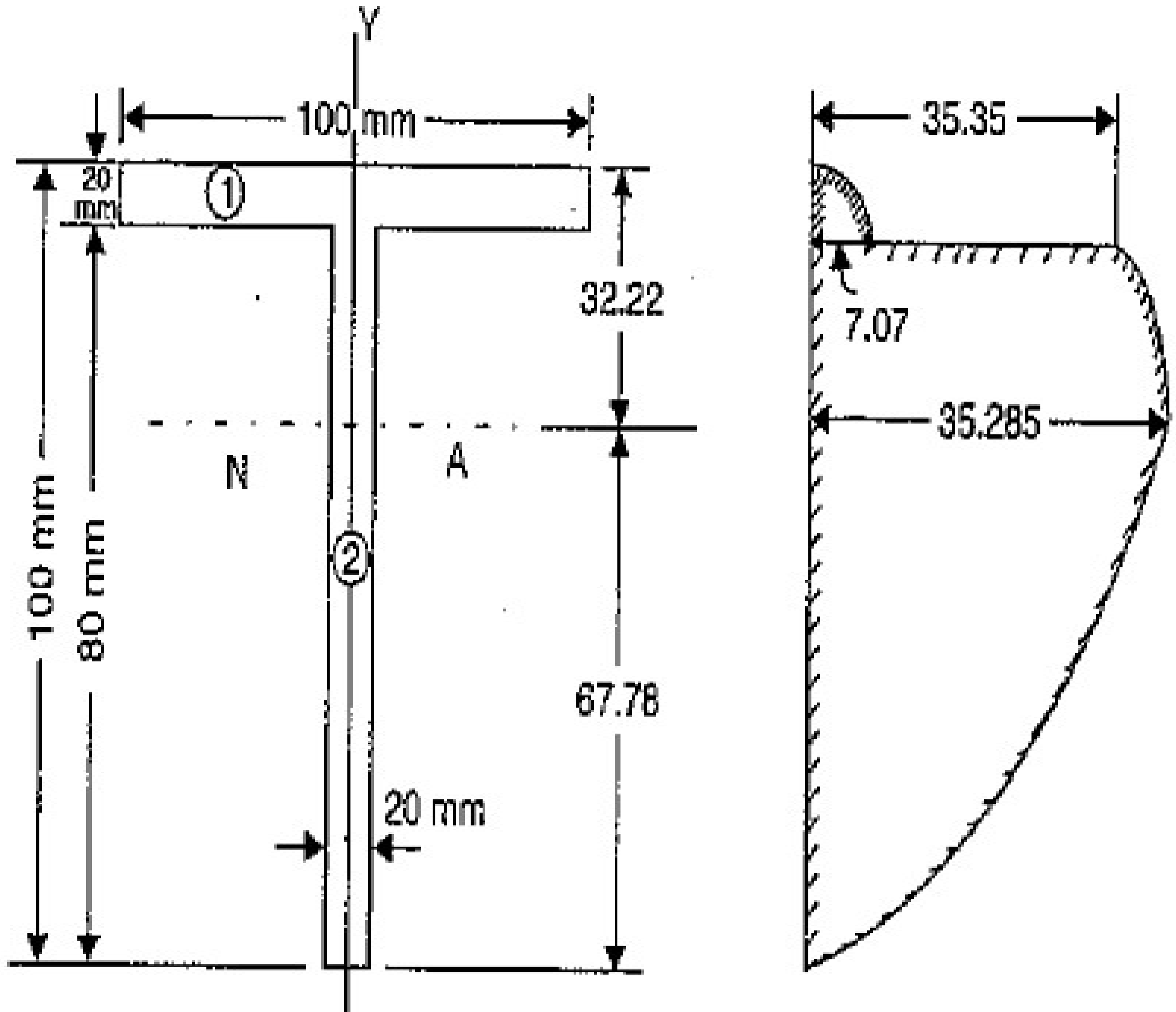
Thus, the maximum shear stress in a beam of rectangular cross section is 50% larger than the average shear stress *V/A.*



A
B
(d/2 – y)
q
E
F
d/2
y
q_{max}
d
N
A
D
C
b
(b)



B
Flange
Web
d/2
d
D
N
A
b
d/2
τ_{max}



(a)

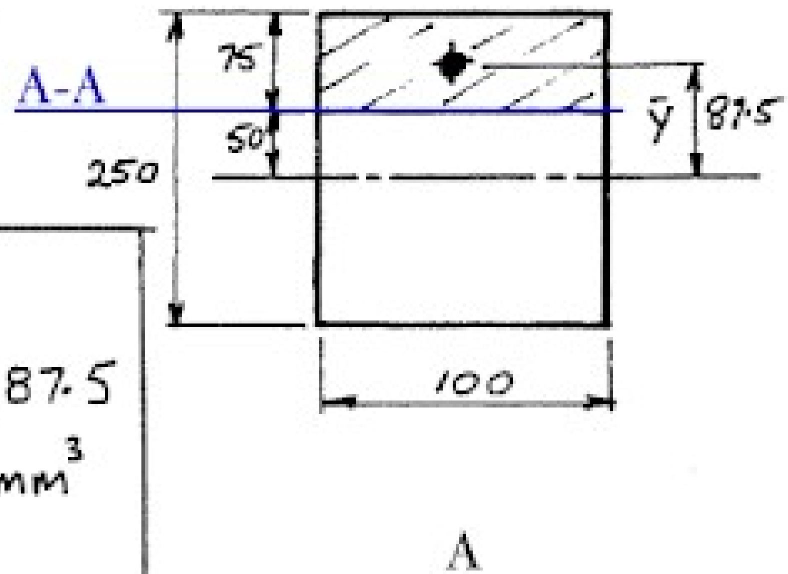
(b)

Calculate the shear stress at AA

10 kN and I = 1.3 x 10^8 mm^4.

$$\tau = \frac{Q\,V}{I\,b}$$

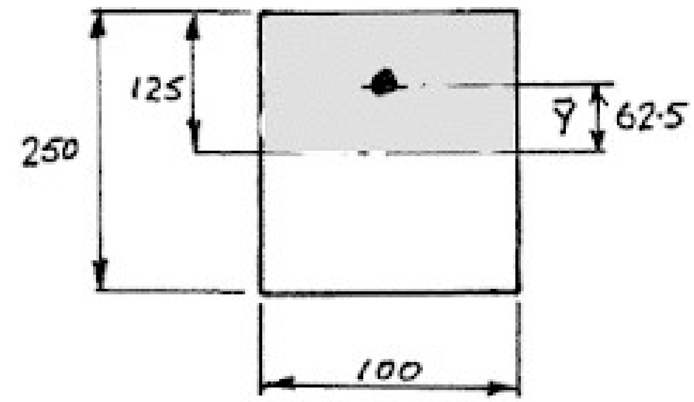
$$Q = A\bar{y}$$
$$= 100 \times 75 \times 87.5$$
$$= 6.56 \times 10^5\ mm^3$$



$$\tau_{A\text{-}A} = \frac{Q\,V}{I\,b}$$
$$= \frac{6.56 \times 10^5 \times 10 \times 10^3}{1.3 \times 10^8 \times 100}$$
$$= 0.5\ MPa$$

Calculate the shear stress at neutral axis

Vertical shear force = 10 000 N



$$Q = A\bar{y}$$

$$= 100 \times 125 \times 62.5$$

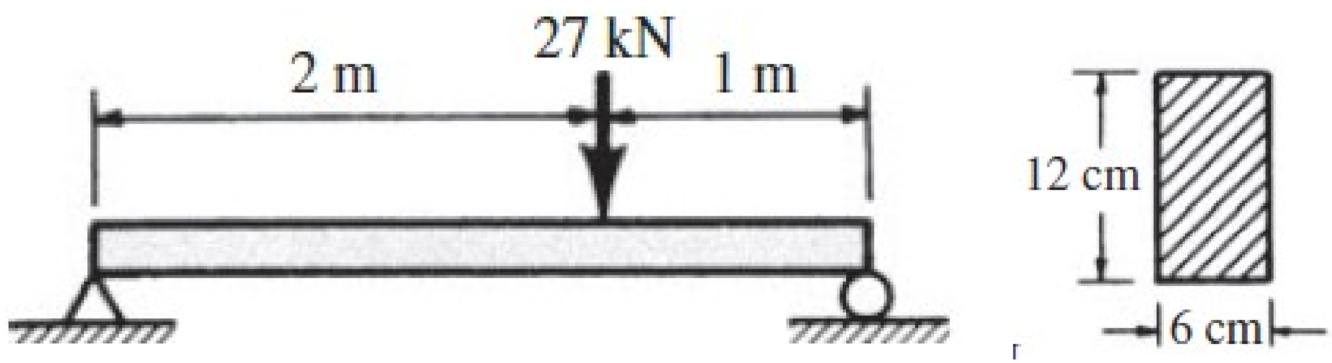
$$= 781\,250 \text{ mm}^3$$

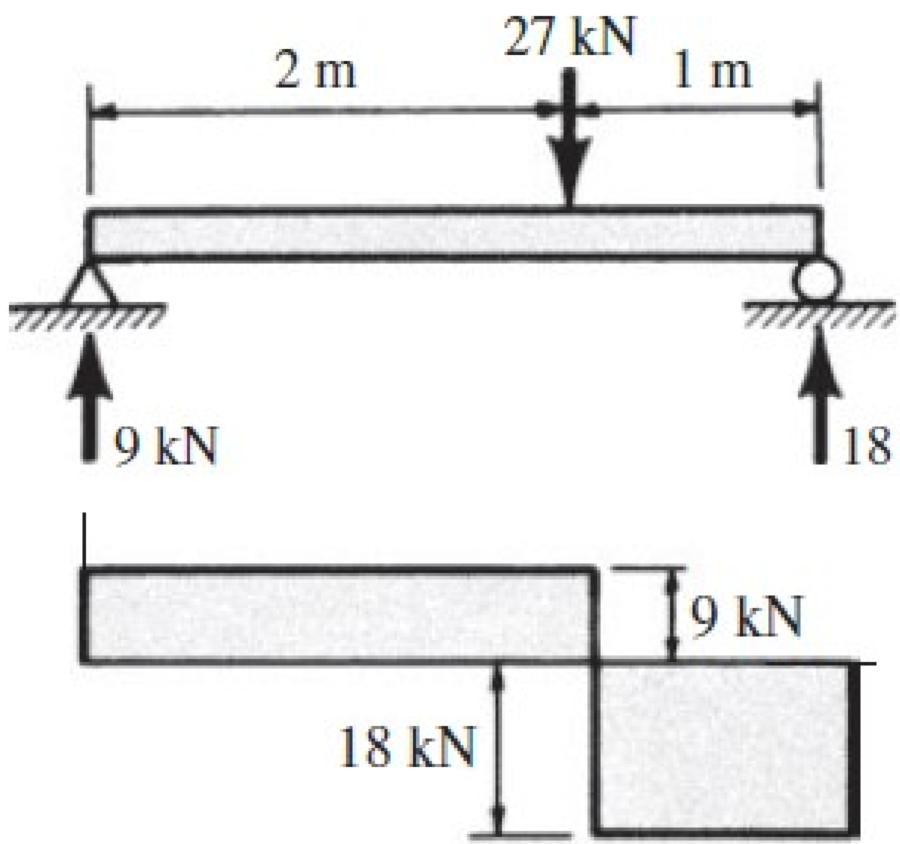
$$\tau_{N\text{-}N} = \frac{QV}{Ib}$$

$$= \frac{781\,250 \times 10 \times 10^3}{1.3 \times 10^8 \times 100}$$

$$= 0.6 \text{ MPa}$$

For the beam shown in figure below determine *(i) the shearing stress at a point 3 cm below the top of the beam* at a section 1 m to the right of the left reaction (ii) the maximum shearing stress due to the vertical shear *V*.





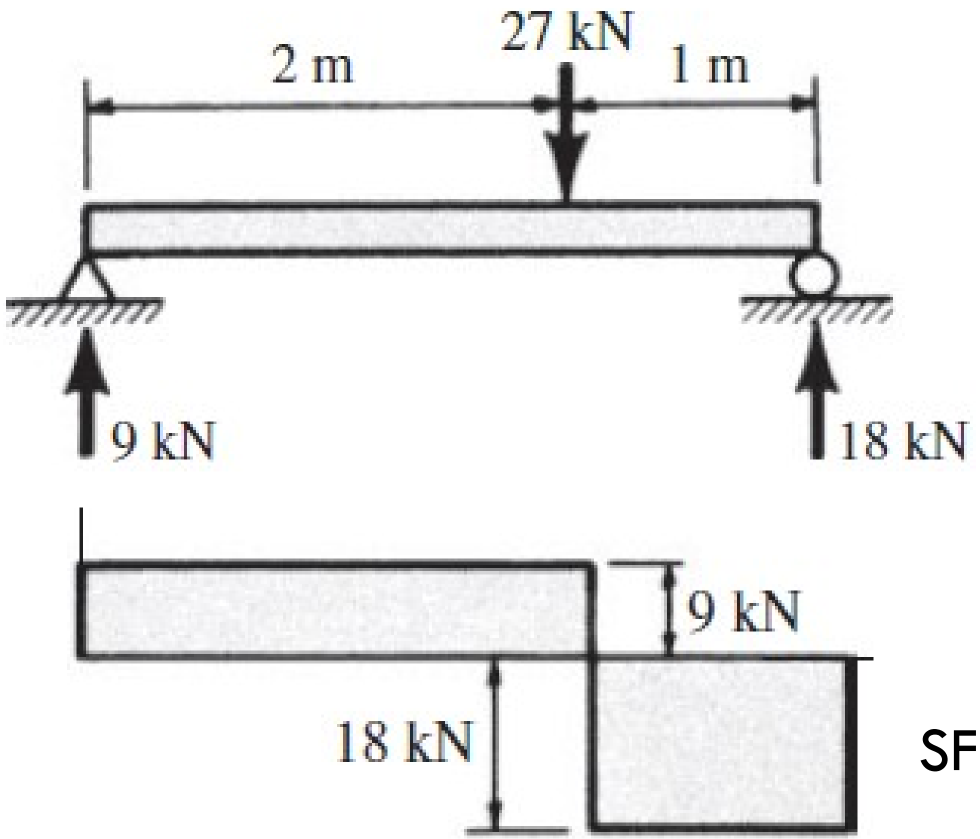
SFD

- From the shear diagram, the shearing force acting at a section 1 m to the right of the left reaction is 9000 N.
- The shearing stress *at any point in this section at a distan*

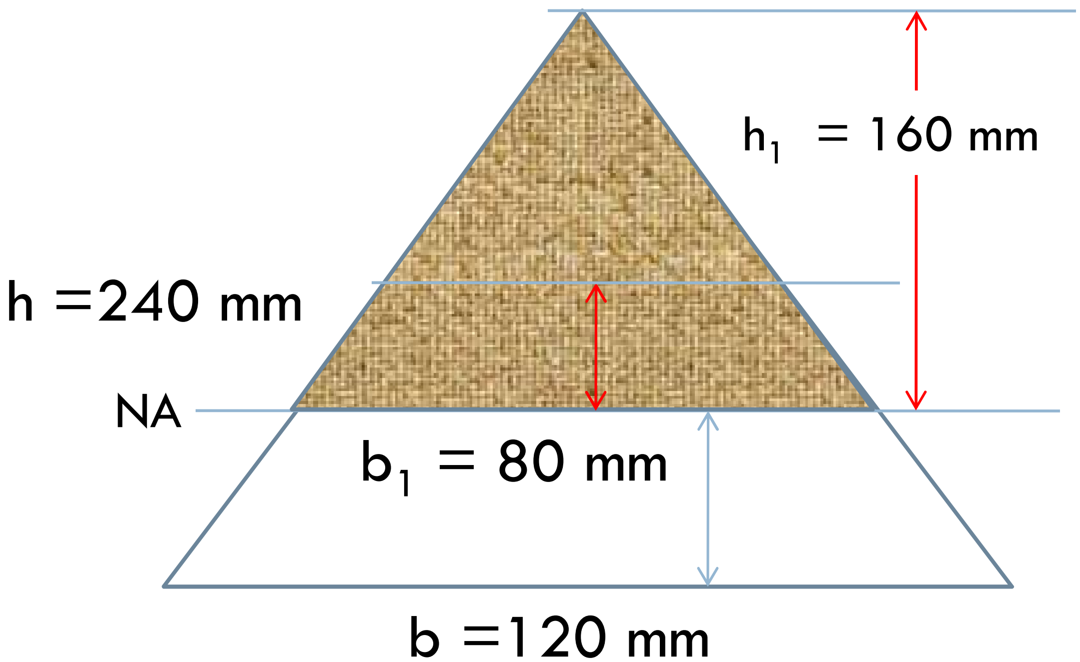
$$\tau = \frac{VQ}{Ib} = \frac{9000\ (0.06 \times 0.03) \times 0.045}{(0.06 \times 0.12^3/12) \times 0.06} = 1.41 \times 10^6 \text{ Pa}$$

- The maximum shearing stress due to the vertical

$$\tau_{\max} = \frac{VQ}{Ib} = \frac{18\,000\ (0.06 \times 0.06) \times 0.03}{(0.06 \times 0.12^3/12) \times 0.06} = 3.75 \times 10^6 \text{ Pa}$$



SFD



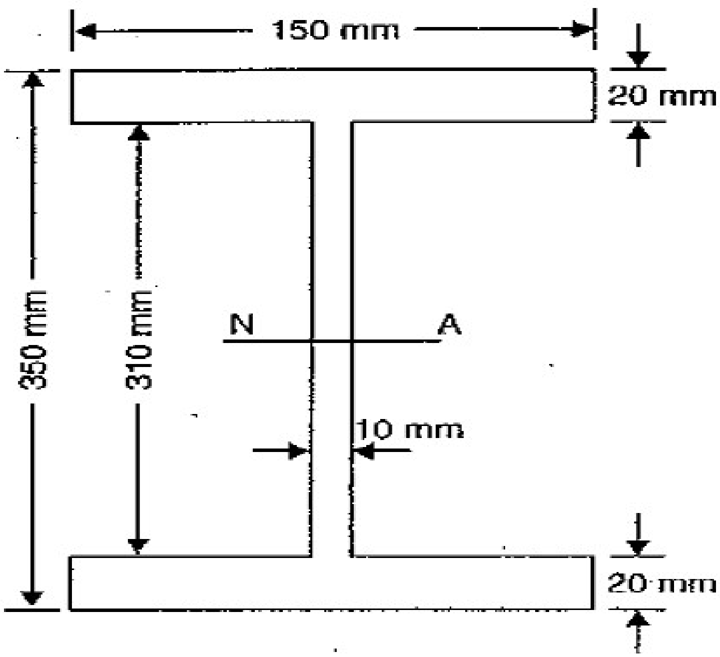
$$I = \frac{120 * 240^3}{36} mm^4$$

$I = 4608 \times 10^4\ mm^4$

$Q = Ay' = 0.5*80*160*(160/3)$

$$\tau_{\max} = \frac{VQ}{Ib} = \frac{18000 \times 341{,}333.33}{I \times 80}$$

Calculate the maximum shear stress and draw the shear stress distribution across the section. Determine the total shear resisted by the web. **V= 40 kN**

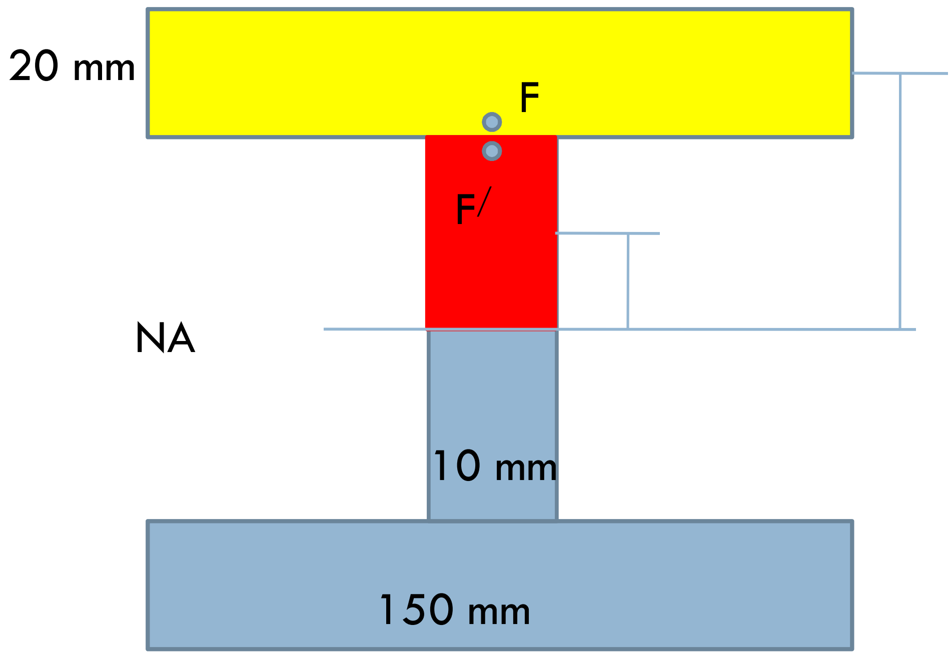


$$I = \frac{150 \times 350^3}{12} - \frac{140 \times 310^3}{12} \text{ mm}^4$$

$$= 535937500 - 347561666.6$$

$$= 188375833.4 \text{ mm}^4.$$

$$\tau_{max} = \frac{F \times A \times \bar{y}}{I \times b}$$



$Q = Ay^{/}$ = Moment of the area above the neutral axis about the neutral axis

$$= (150 \times 20) \times \left(\frac{310}{2} + \frac{20}{2}\right) + \left(\frac{310}{2} \times 10\right) \times \left(\frac{310}{2} \times \frac{1}{2}\right) \quad = 615125\ mm^3$$

$$\tau_{max} = \frac{40,000 \times 615125}{188375833.4 \times 10} = \mathbf{13.06\ N/mm^2}.$$

Shear stress at the upper edge of the flange is zero

Shear stress at the bottom edge of the flange

$Q_F =$

A

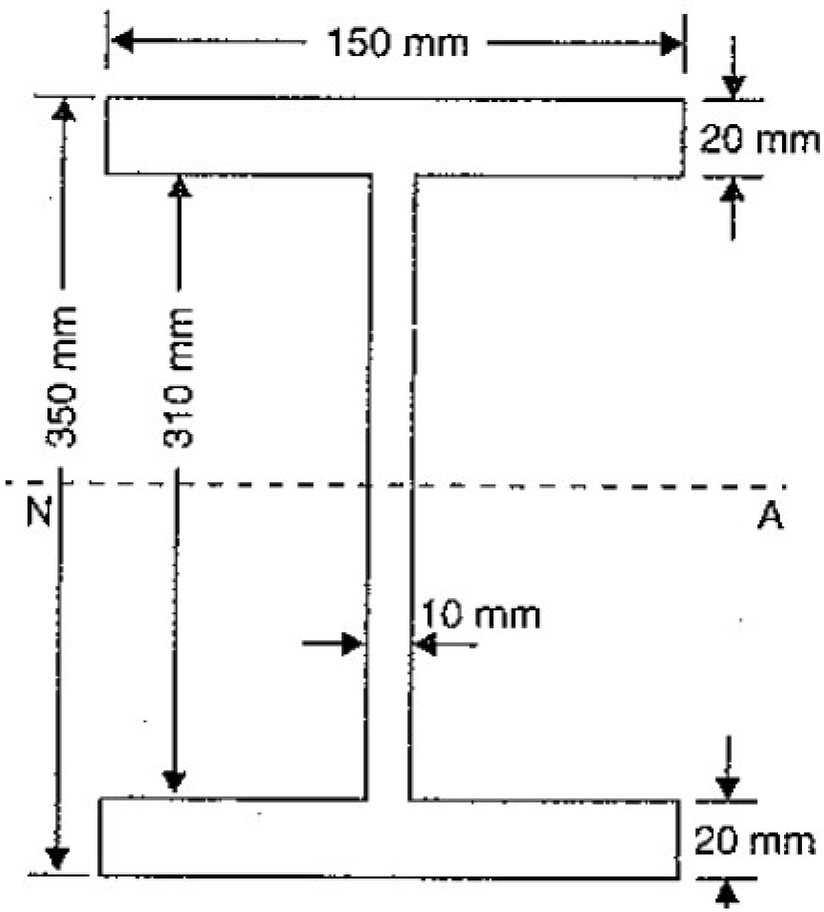
$$= (150 \times 20) \times \left(\frac{310}{2} + \frac{20}{2}\right) = 495000\ mm^3$$

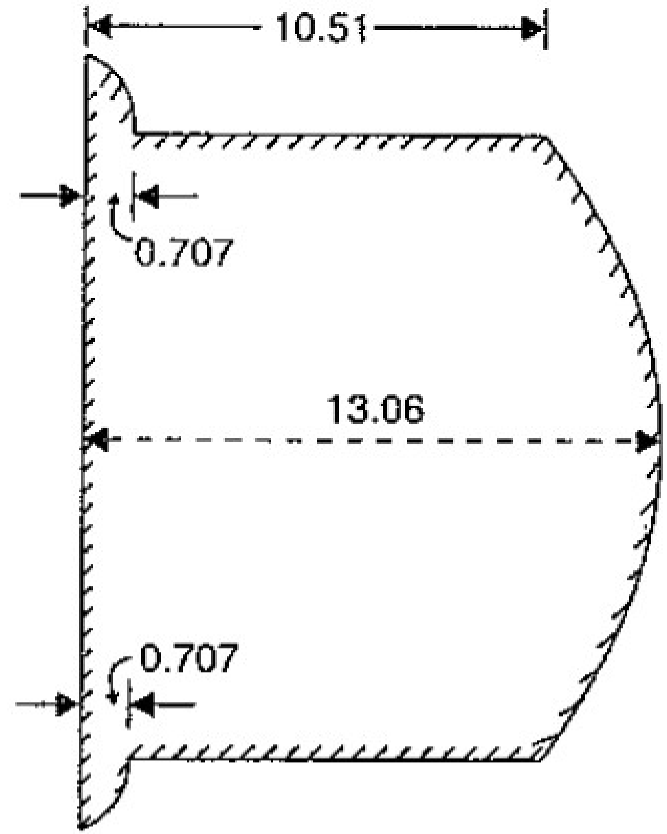
$$\tau_F = \frac{VQ}{Ib} = \frac{40000 * 495000}{188375833 * 150} = 0.707 N / mm^2$$

$Q_F^{/}$ =   = 495000 mm^3

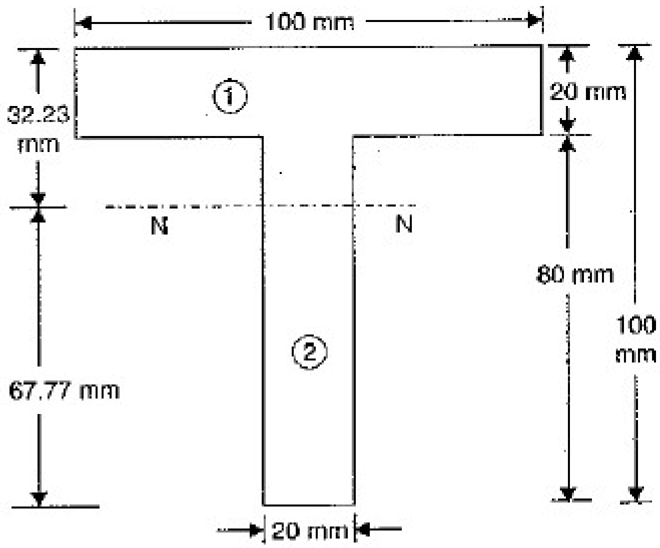
Q_F

$$\tau_{F^/} = \frac{VQ}{Ib} = \frac{40000 * 495000}{188375833 * 10} = 10.5 N / mm^2$$


150 mm
20 mm
350 mm
310 mm
N
A
10 mm
20 mm


10.51
0.707
13.06
0.707

Calculate maximum shear stress if the shear force is 50 kN



Shear stress $\tau = \dfrac{VQ}{Ib}$

Shear stress at the top edge of flange and bottom of is zero

$Q = Ay$

Shear stress in the flange at the junction of web

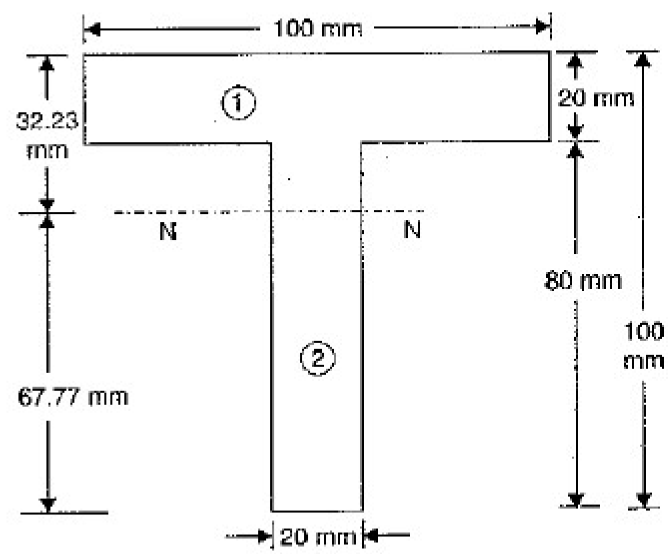
$$A = 100 \times 20 = 2000 \text{ mm}^2$$

$\bar{y}$ = Distance of C.G. of the area of flange from N.A.

$$= 32.22 - \frac{20}{2} = 22.22 \text{ mm}$$

b = width of flange = 100mm

$$\tau = \frac{50000 \times 2000 \times 22.22}{314.221 \times 10^4 \times 100} = 7.07 \text{ N/mm}^2.$$



Shear stress in the web at the junction of web and flange

$$\tau = \frac{50000 * 2000 * 22.22}{314.221 \times 10^4 * 20} = 35.35 N/mm^2$$

Maximum shear stress at the N.A.

$$Q = 20 \times 100 \times (32.22 - 10) + 20 \times (32.22 - 10) \times \frac{22.22}{2}$$

$$= 49377.284 \text{ mm}^3$$

$$\tau = \frac{50000 \times 49377.284}{314.221 \times 10^4 \times 20} = 39.285 \text{ N/mm}^2$$