CE 2111
Mechanics of Materials-I
(Shear Force and Bending Moment)

Shear Force and Bending Moment

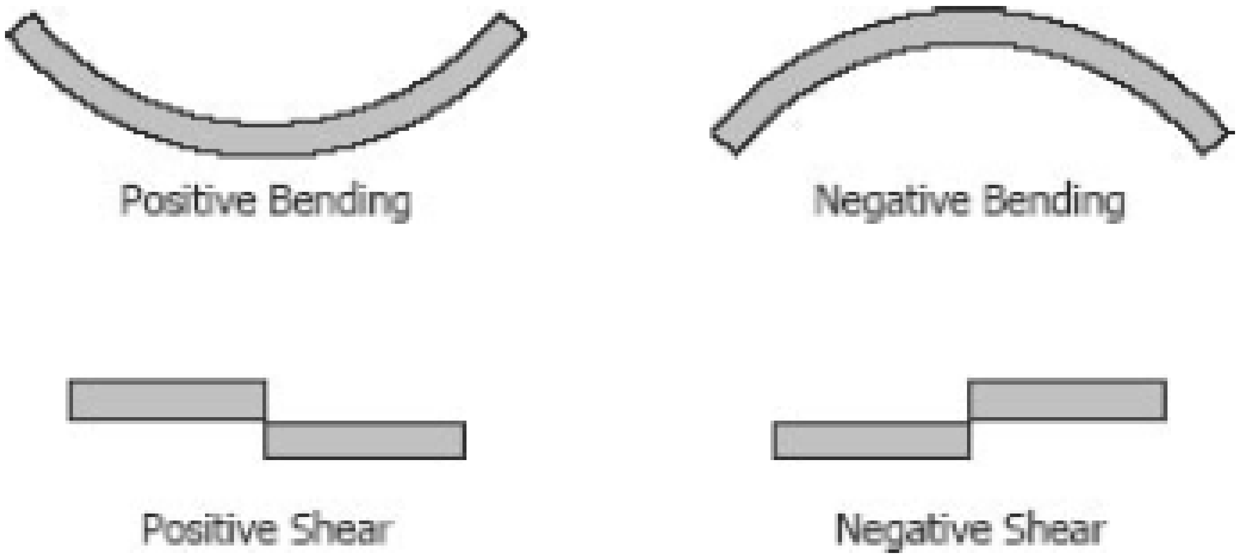
- The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force
- A shear force diagram (SFD) is one which shows the variation of shear force along the length of the beam.
- The algebraic sum of the moments of all the forces acting to the right or left of any section of a beam is known as bending moment

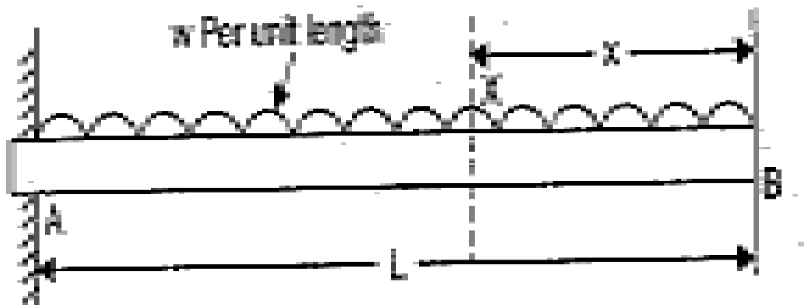
- A bending moment diagram (BMD) is one which shows the variation of bending moment along the length of the beam.

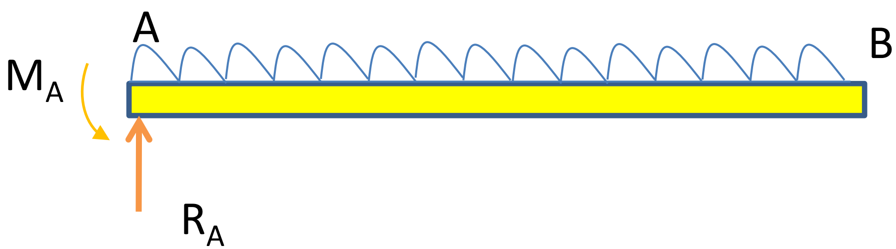
Guidelines for drawing SFD and BMD

- Consider left or right portion of the section
- Add the forces normal to the beam on one of the portion. If right portion of the section is chosen, a force on the right portion acting downward is positive while a force acting upward is downward
- The positive values of shear force and bending moments are plotted above the base line and negative values below the base line
- The shear force diagram increase or decrease suddenly i.e. by vertical straight line at a section where there is a vertical point load
- The shear force between any two vertical loads will be constant and hence the shear force diagram between two vertical loads will be horizontal
- Bending moment at any hinge or roller support, at the end of beam, is zero

SIGN CONVENTIONS





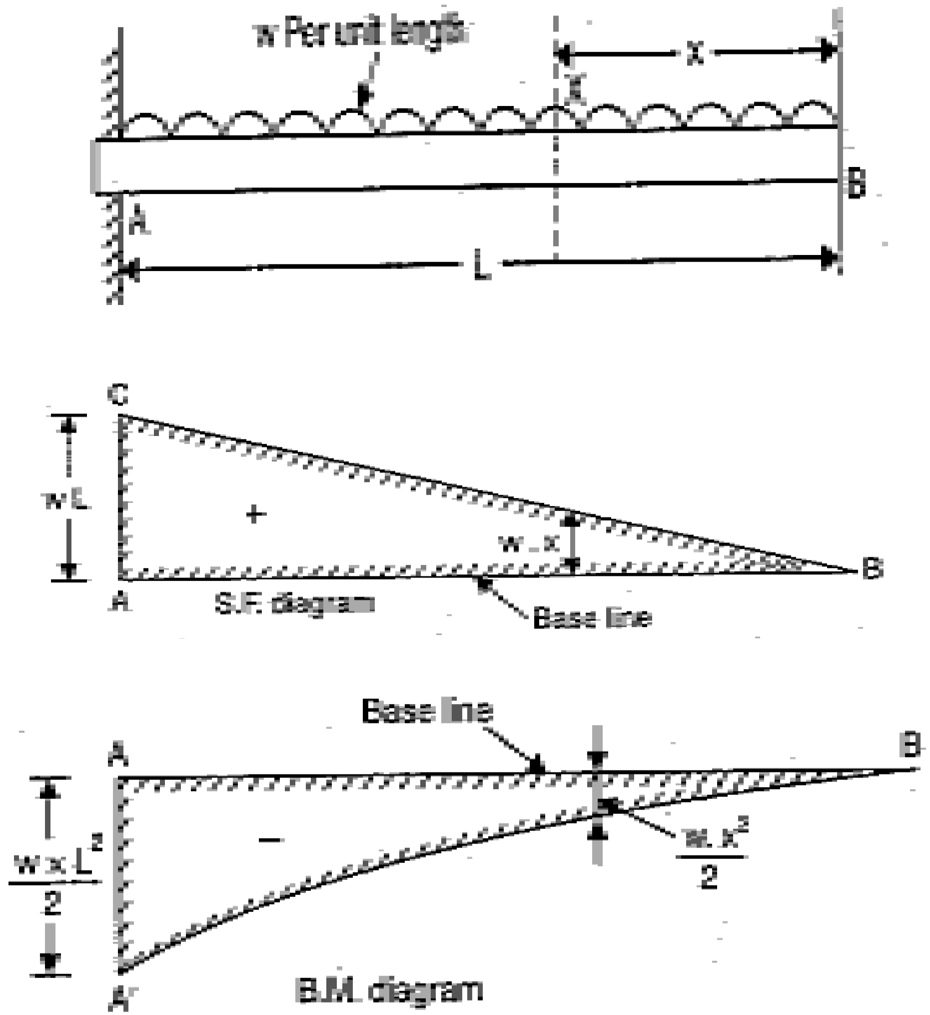


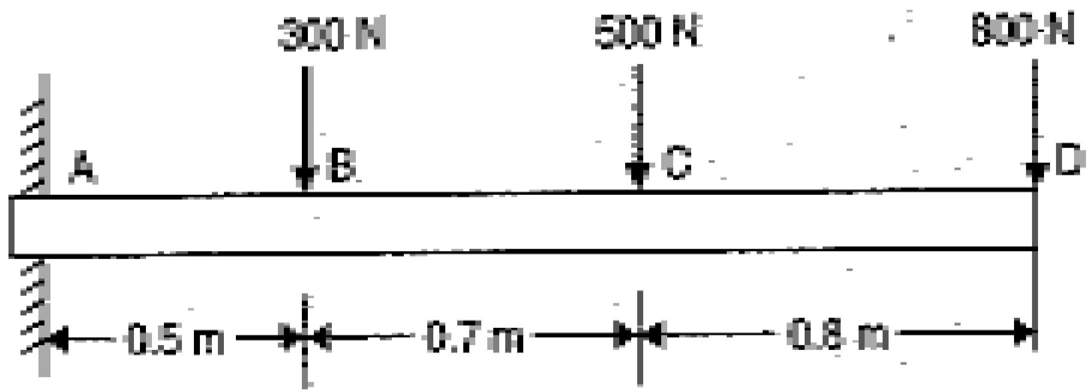
$R_A = wL$

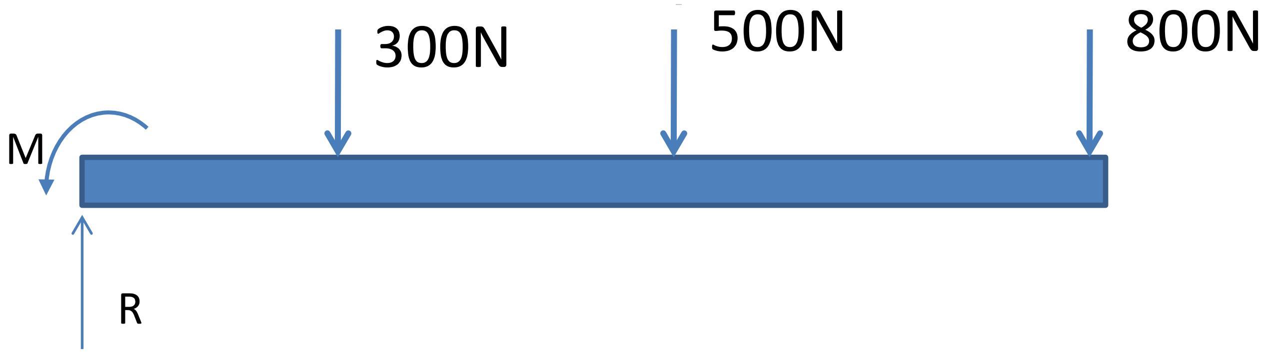
$M_A = wL^2/2$

Shear force at any section x from right end $V_x = wx$

Bending moment at any section x from right end $M_x = wx*x/2 = wx^2/2$

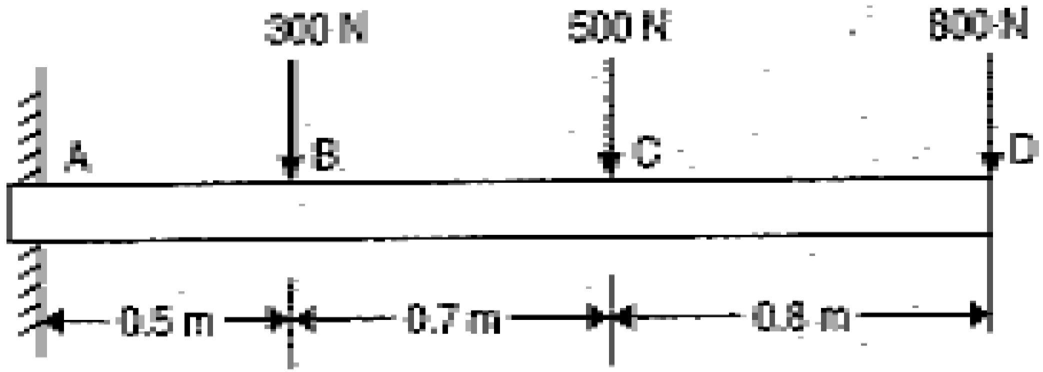

w Per unit length
x
X
A
B
L
C
w.L
+
w . x
B
A
S.F. diagram
Base line
Base line
A
B
w x L²
2
−
w. x²
2
A'
B.M. diagram



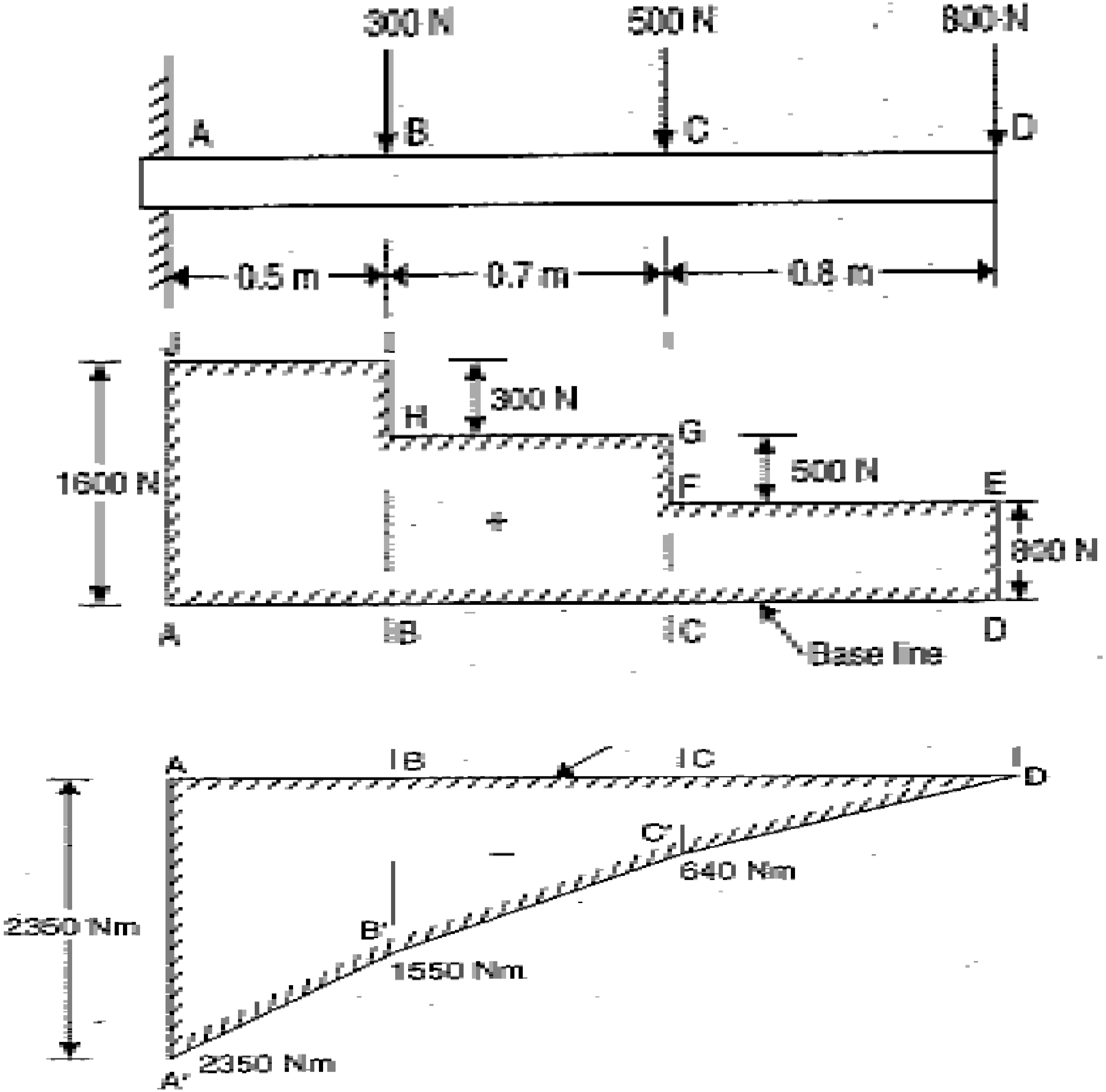


R = 1600 N

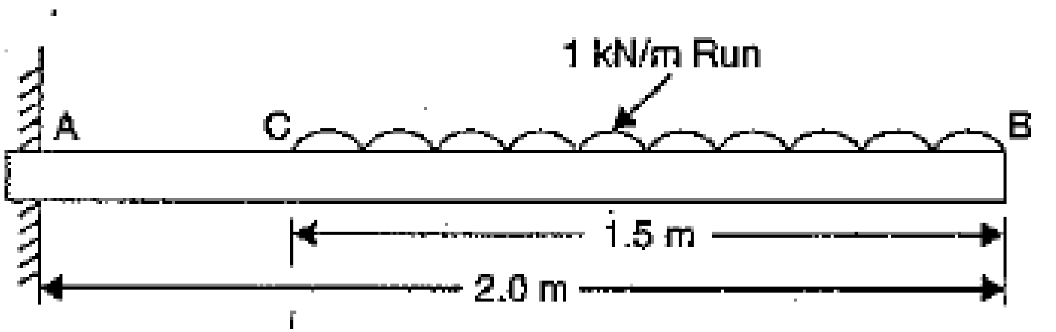
M = 2350 N-m

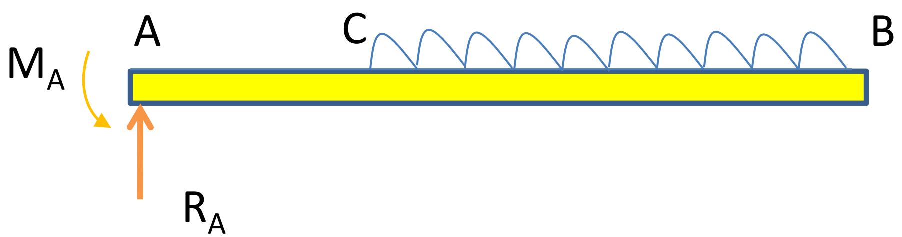


- Shear force at D V_D = 800N
- Shear force at C V_C = 800N + 500N = 1300N
- Shear force at B V_B = 800N+500N+300N = 1600N
- Shear force at A V_A = +1600N
- Bending moment at the free end D is zero
- Bending moment between D and C at a distance x from D M_x = -800*x
- At C, x = 0.8m M_C = -800*0.8 = 640N-m
- Bending moment between B and C at a distance x from D M_x = -800*x – 500(x-0.8)
- Bending moment between B and A at a distance x from D M_x = -800*x – 500(x-0.8) – 300(x-1.5)


300 N
500 N
800 N
A
B
C
D
0.5 m
0.7 m
0.8 m
1600 N
300 N
H
G
500 N
F
E
800 N
A
B
C
D
Base line
A
B
C
D
C'
640 Nm
2350 Nm
B'
1550 Nm
2350 Nm
A'

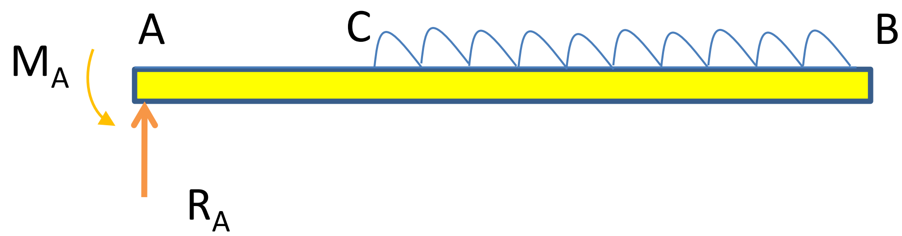
Draw SFD and BMD for the Cantilever Beam





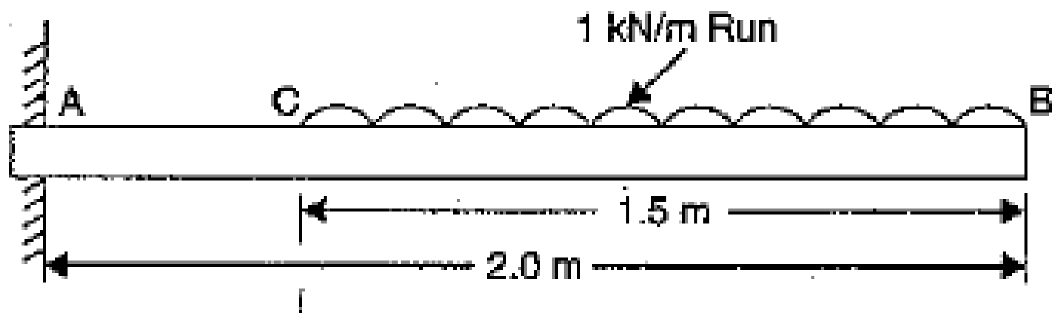
R_A = 1.5 kN

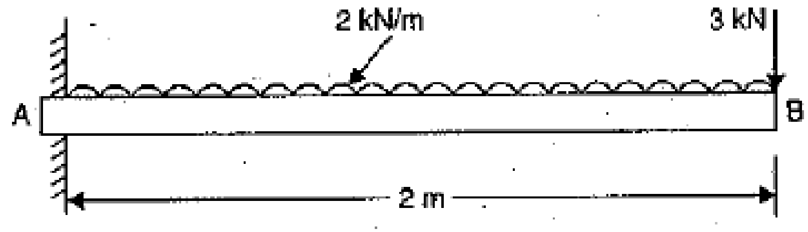
M_A = 1.875 kN-m

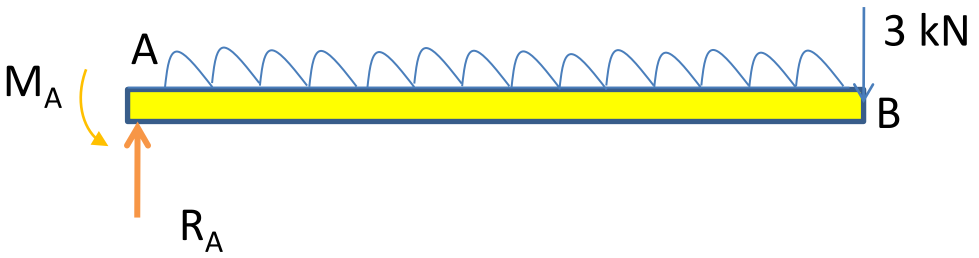


- Consider any section between C and B $V_x = 1.0*x$ kN
- At B; x =0 $V_B = 0$
- At C; x = 1.5 $V_C = 1.5$ kN
- Between A and C there is no load, the shear will remain constant
- Bending moment at any section between C and B, $M_x = -1.0*x*x/2 = -x^2/2$
- At B x =0 $M_B = 0$
- At C, x = 1.5 MC = -1.125 N-m
- Bending moment between A and C, M_x = -(Load due to UDL)* (x-0.75) [x =1.5-2.0m]

(a)

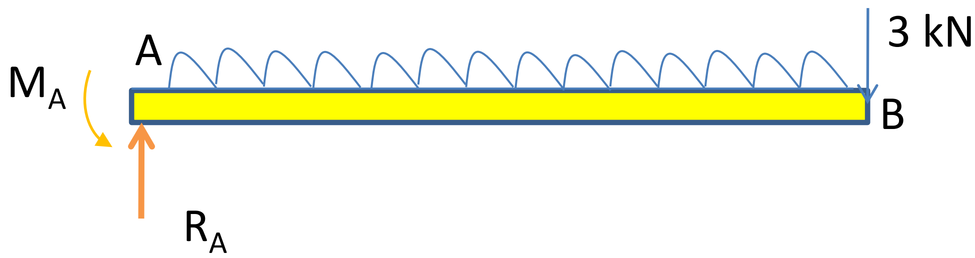






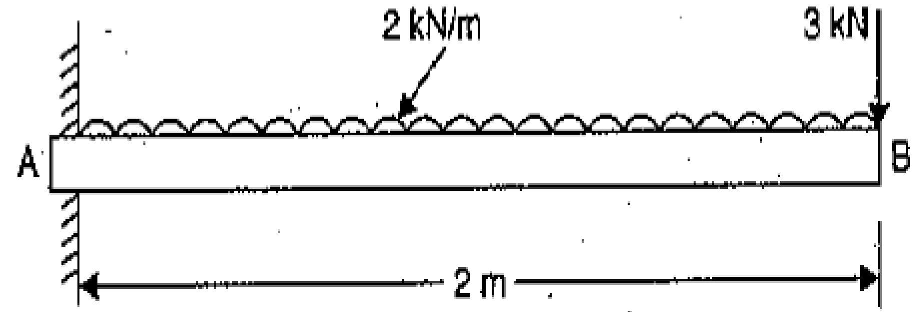
$R_A = 7$ kN

$M_A = 10$ kN-m

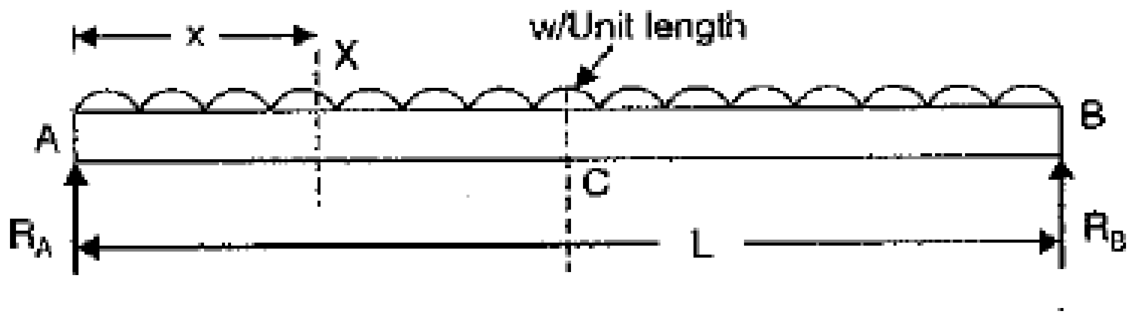


- Shear force at B V_B = 3.0 kN
- Shear force at any section x from free end
- Vx = 3.0 + w*x = 3.0 + 2*x
- At B x =0 V_B = 3.0N
- At A x = 2.0 V_A = 3.0 + 2*2 = 7 N
- Bending moment at any section from free end
- M_x = -(3*x + 2*x*x/2) = - $(3x + x^2)$
- At B x =0 M_B = 0
- At A, x = 2 M_A = -10 kN-m
- Bending moment between A and C, Mx = -(Load due to UDL)* (x-0.75) [x =1.5-2.0m]

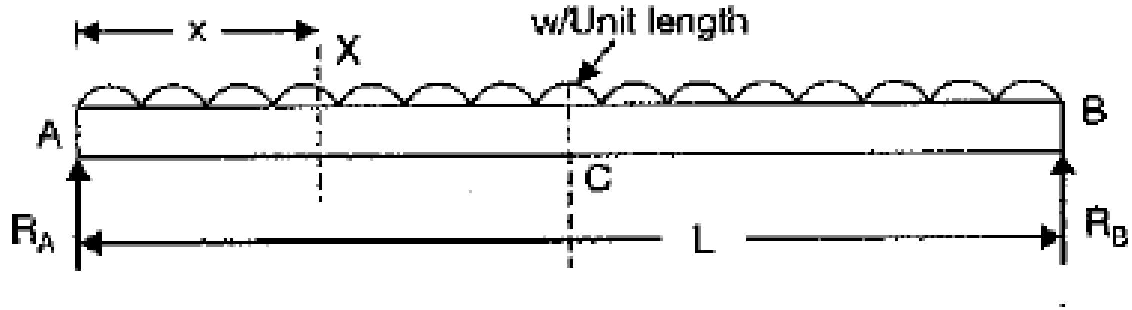
(a)



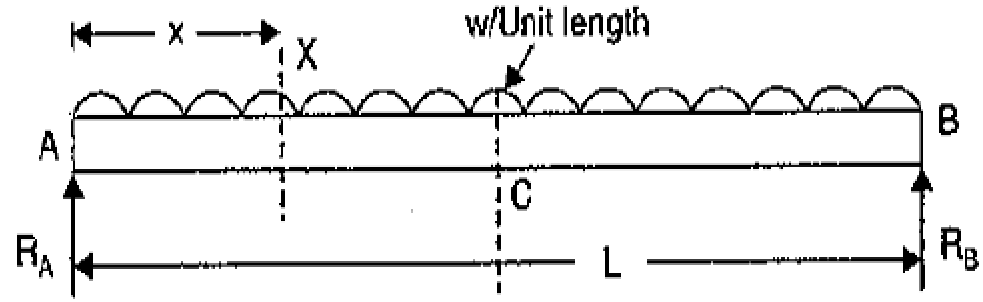
Draw SFD and BMD for the simple Supported Beam



$R_A = R_B = wL/2$

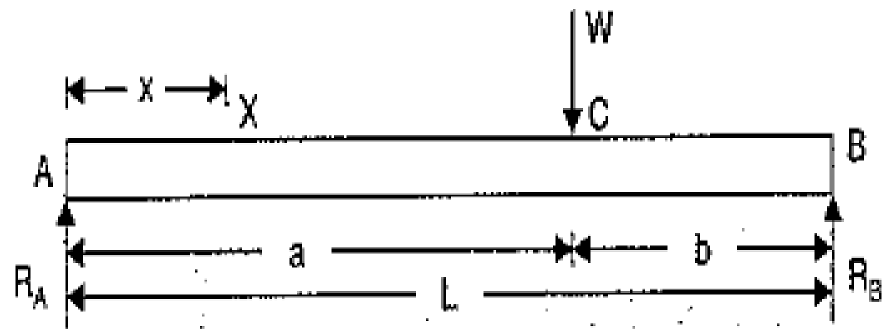


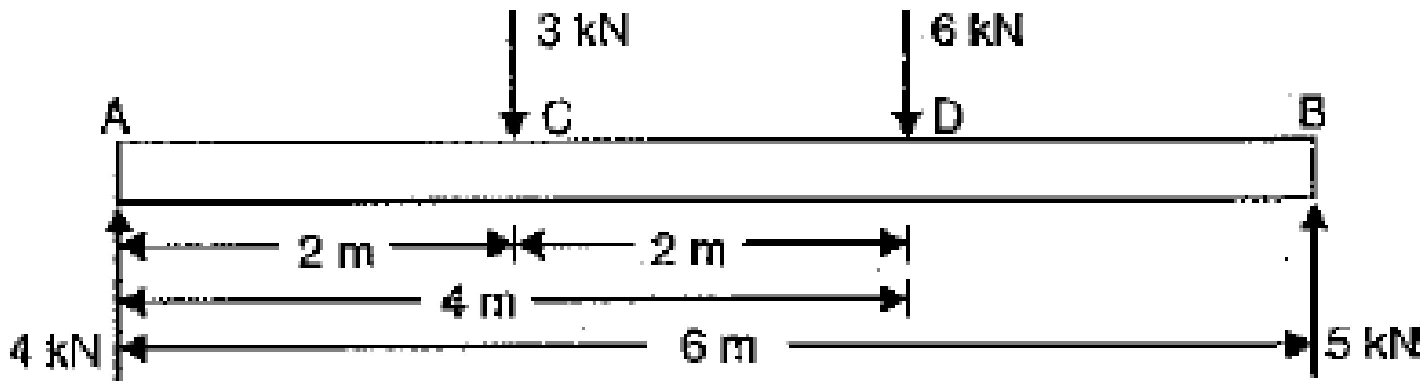
- Consider any section at distance x from left support
  $V_x = R_L - w*x = wL/2 - w*x$
- At A, $x=0 \Rightarrow V_A = wL/2$
- At B, $x=L \Rightarrow V_B = -wL/2$
- At C, $x = L/2 \Rightarrow Vc = 0$
- Bending moment at any section x from A
- $M_x = R_L*x - wx^2/2 = w(Lx-x^2)/2$
- At A, $x=0 \Rightarrow M_A = 0$
- At B, $x=L \Rightarrow M_B = 0$
- At C, $x = L/2 \Rightarrow M_c = wL^2/8$



(a)

(a)

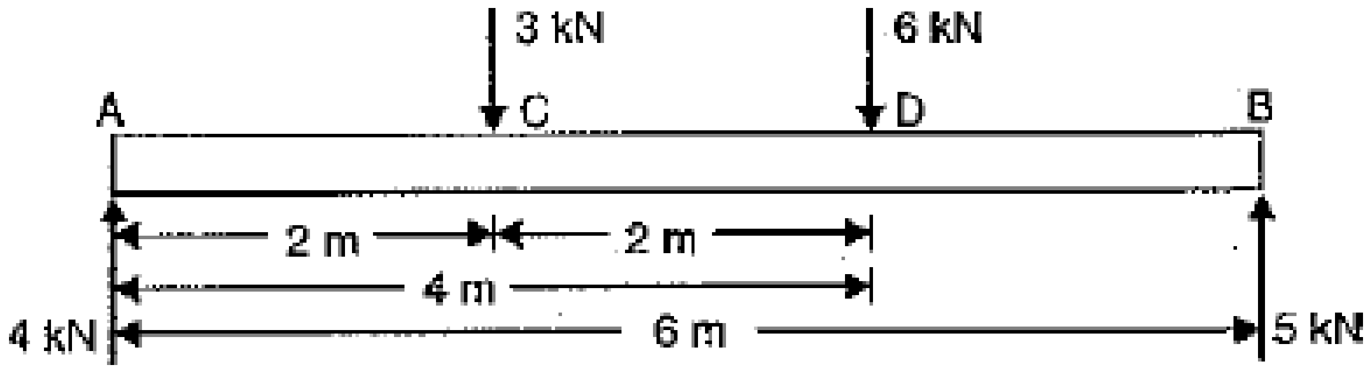




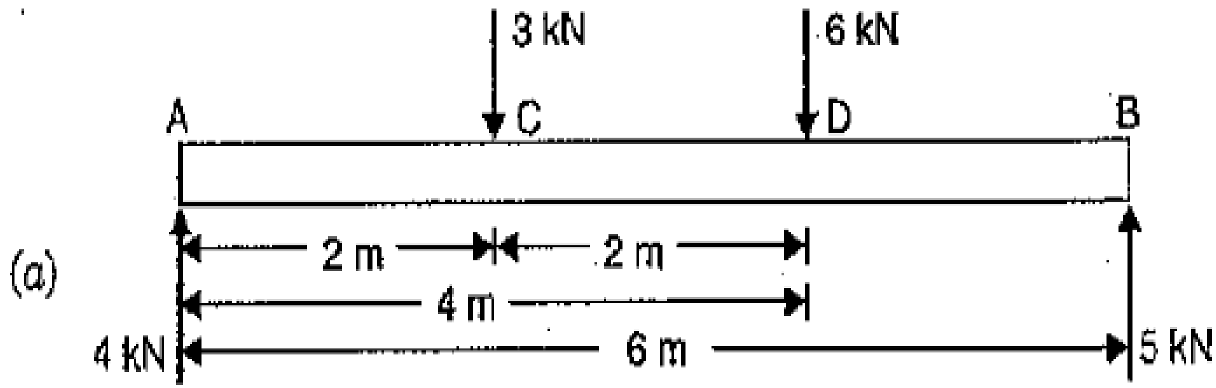
$$\sum M_A = 0; \qquad 3kN*2+6kN*4-R_B*6=0$$

R_B = 5 kN

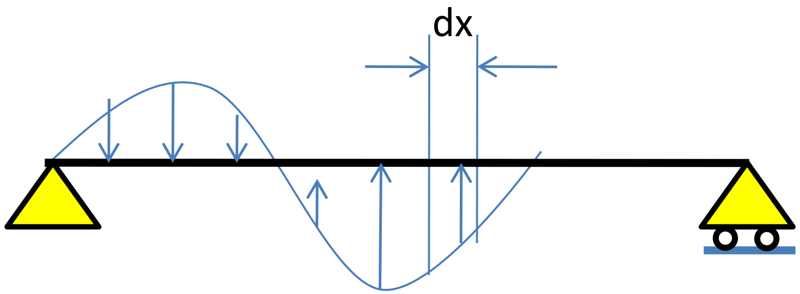
R_A = 4 kN

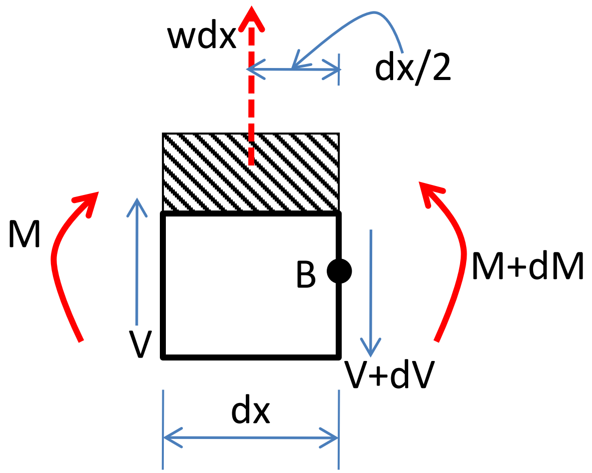


- Shear force at A = R_A = 4kN
- Shear force between A and C is constant and equal to 4 kN
- Shear force at C V_C = 4-3 = 1 kN
- Shear force at D V_D = 1-6 = -5 kN
- Shear force at B V_B = -5 kN
- **Bending moment at A M_A = 0**
- Bending moment at C $M_C = R_A *2 = 4*2 =8$ kN-m
- Bending moment at D $M_D = R_A *4 – 3*2 =10$ kN-m
- Bending moment at B M_B = 0


3 kN
6 kN
A
C
D
B
(a)
2 m
2 m
4 m
6 m
4 kN
5 kN

Relations among load, shear and moment





$\sum F_y = V + w\,dx - (V+dV) = 0$

$dV = w\,dx$

$$\int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} w\,dx$$

$V_2 - V_1 = \Delta V = (\text{Area})_{load}$

$\sum M_B = M + V\,dx + (w\,dx)\,dx/2 - (M + dM) = 0$

$dM = V\,dx$

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V\,dx$$

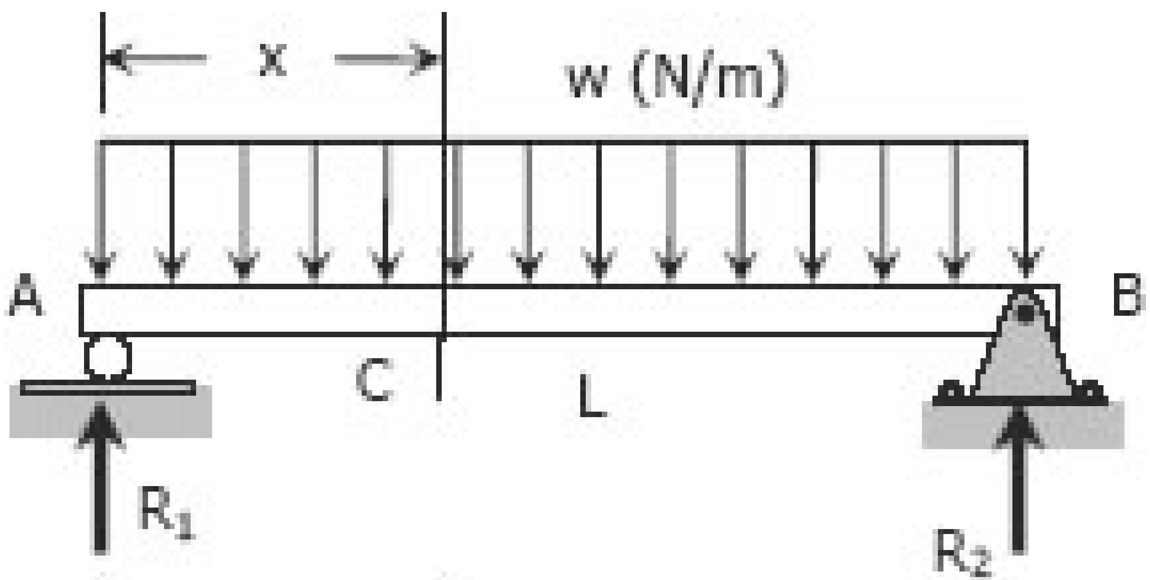
$M_2 - M_1 = \Delta M = (\text{Area})_{shear}$

$$w = \frac{dV}{dx} = slope\ of\ shear\ diagram$$

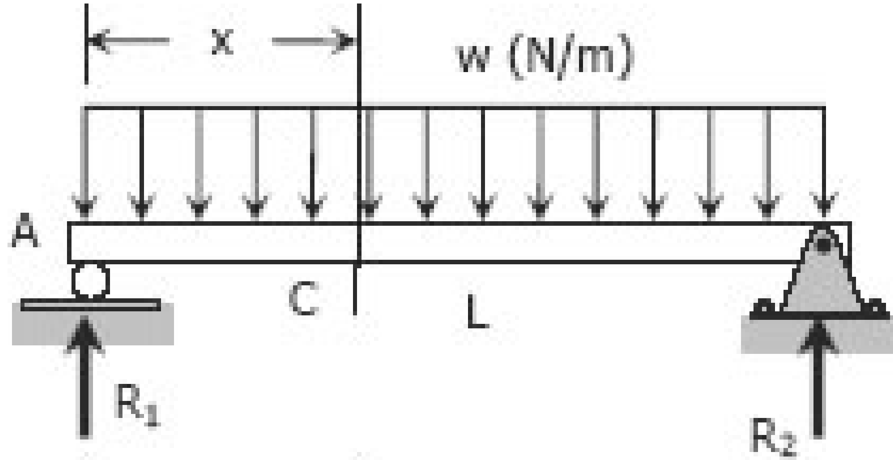
$$V = \frac{dM}{dx} = slope\ of\ momemt\ diagram$$

Relation among Load Shear and Moment

Consider a simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions R_1 and R_2.



$$R_1 = R_2 = wL/2$$



- $V_x = R_1 - wx = wL/2 - wx$
- $M_x = R_1 * x - w*x^2/2$
- $M_x = wLx/2 - w*x^2/2$

- Differentiate M with respect to x

$$\frac{dM}{dx} = \frac{wL}{2}\frac{dx}{dx} - \frac{w}{2}2x\frac{dx}{dx} \qquad \Rightarrow \frac{dM}{dx} = \frac{wL}{2} - wx$$

$$\therefore \frac{dM}{dx} = V$$

- Thus, the rate of change of the bending moment with respect to x is equal to the shearing force, or **the slope of the moment diagram at the given point is the shear at that point.**

- Differentiating V with respect to x

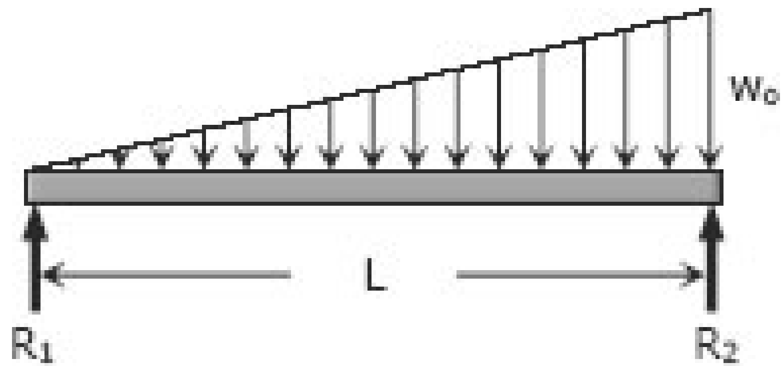
$$\frac{dV}{dx} = -w = Load$$

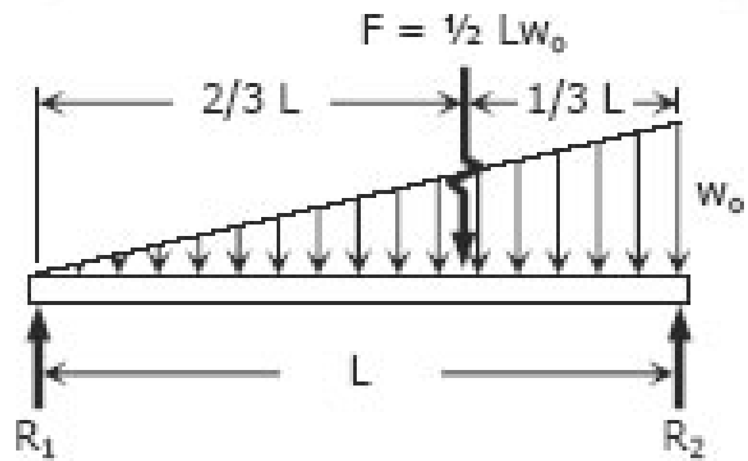
Thus, the rate of change of the shearing force with respect to x is equal to the load or **the slope of the shear diagram at a given point equals the load at that point.**

PROPERTIES OF SHEAR AND MOMENT DIAGRAMS

- The following are some important properties of shear and moment diagrams:
- The area of the shear diagram to the left or to the right of the section is equal to the moment at that section.
- The slope of the moment diagram at a given point is the shear at that point.
- The slope of the shear diagram at a given point equals the load at that point.
- The maximum moment occurs at the point of zero shears.
- When the shear diagram is increasing, the moment diagram is concave upward.
- When the shear diagram is decreasing, the moment diagram is concave

Draw SFD and BMD



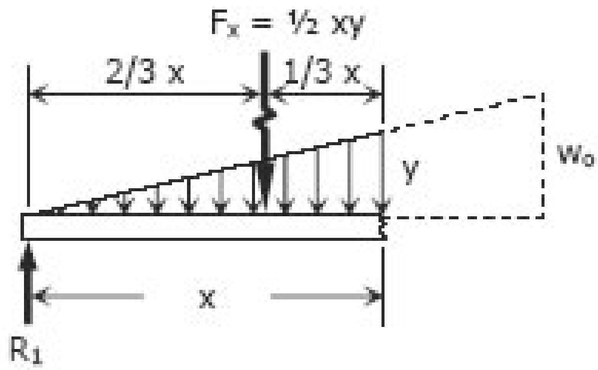


$\sum M_{R2} = 0$

$$R_1 = \frac{1}{6} L w_0$$

$\sum M_{R1} = 0$

$$R_2 = \frac{1}{3} L w_0$$



$$\frac{y}{x} = \frac{w_o}{L} \qquad y = \frac{w_o}{L}x$$

$$F_x = \tfrac{1}{2}xy = \frac{1}{2}x\left(\frac{w_o}{L}x\right) = \frac{w_o}{2L}x^2$$

$$V = R_1 - F_x = \tfrac{1}{6}Lw_o - \frac{w_o}{2L}x^2$$

$$M = R_1x - F_x\left(\tfrac{1}{3}x\right) = \tfrac{1}{6}Lw_ox - \frac{w_o}{6L}x^3$$

When x= 0; V = $1/6Lw_0 = R_1$

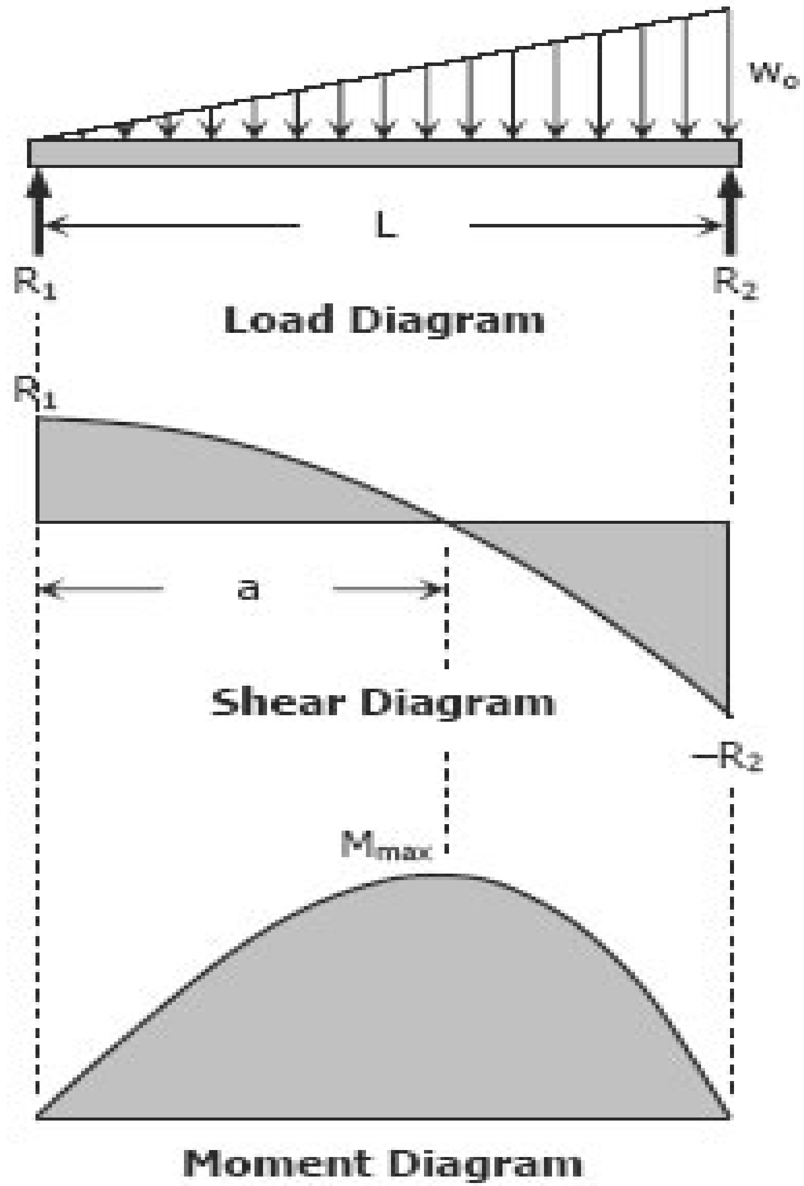
X= L; V= $-1/3Lw_0 = -R_2$

Location of zero shear from left end V= 0 i.e. a = 0.577L

When x= 0; M =0

X= L; M=0

Moment at zero shear $M_{max} = 0.064L^2w_0$

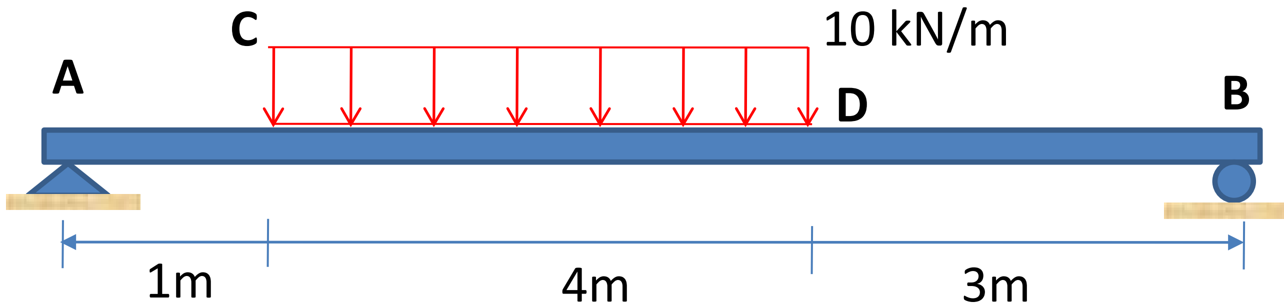


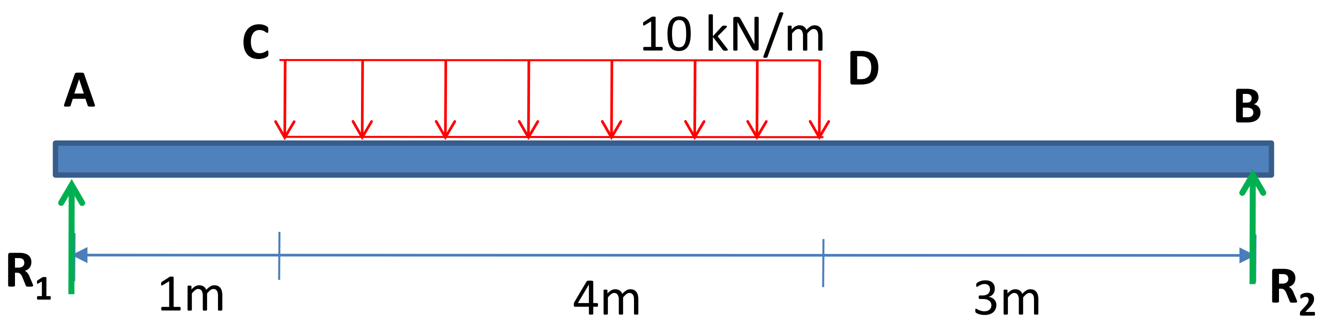
w₀
L
R1
R2
Load Diagram
R1
a
Shear Diagram
−R2
Mmax
Moment Diagram

Nature of Load, SF and BM

w(x)	Concentrated load	UDL	Triangular
V(x)	Straight Line, No slope	Sloped Straight Line	Parabolic
M(x)	Sloped Straight Line	Parabolic	Cubic

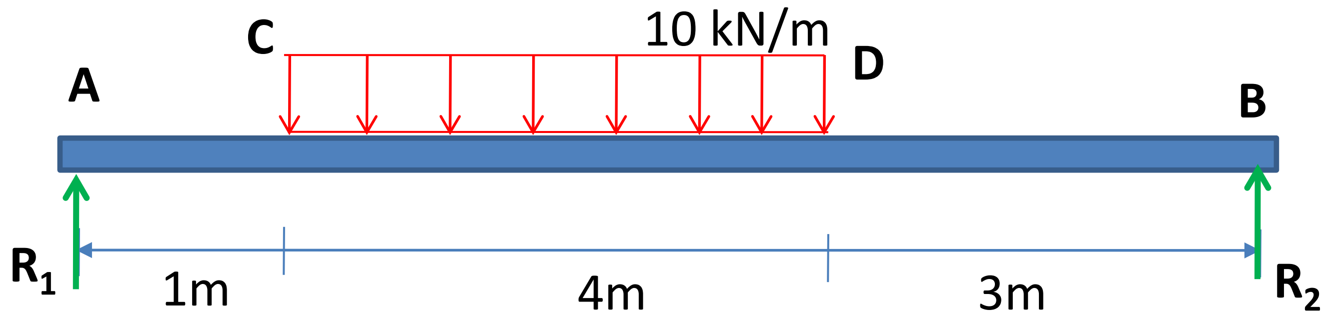
Draw the SFD and BMD for the simple supported beam shown in figure below



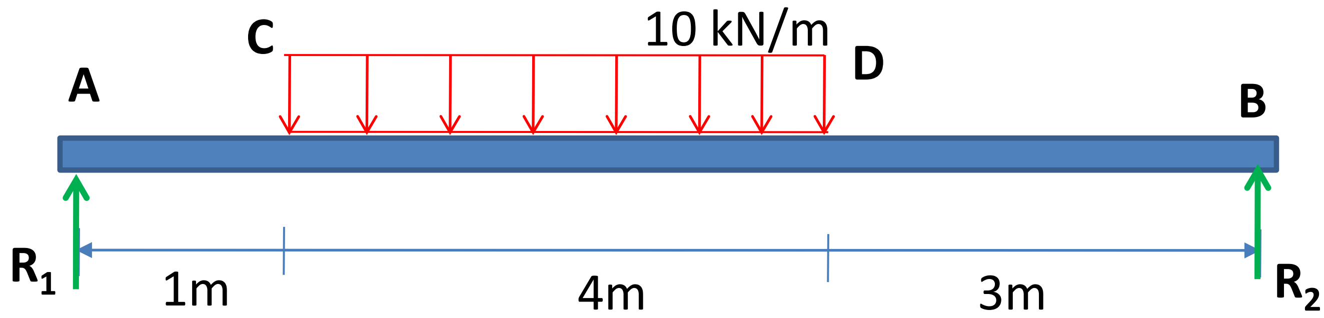


R_1 =25 kN

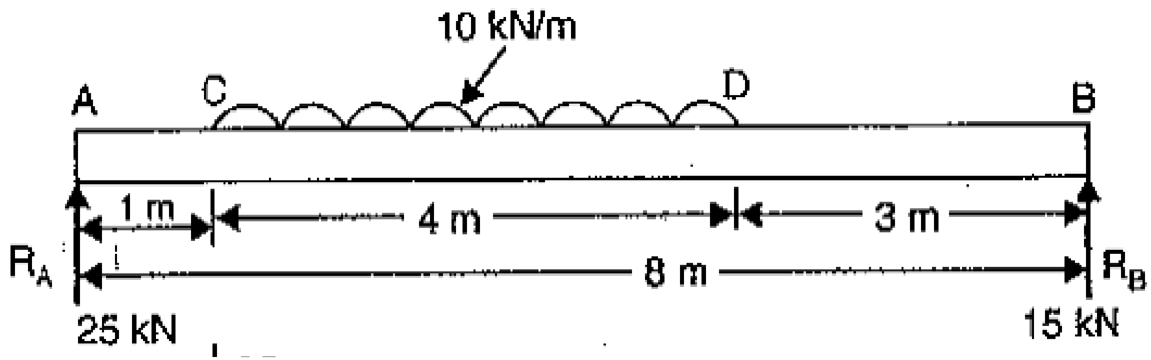
R_2 =15 kN



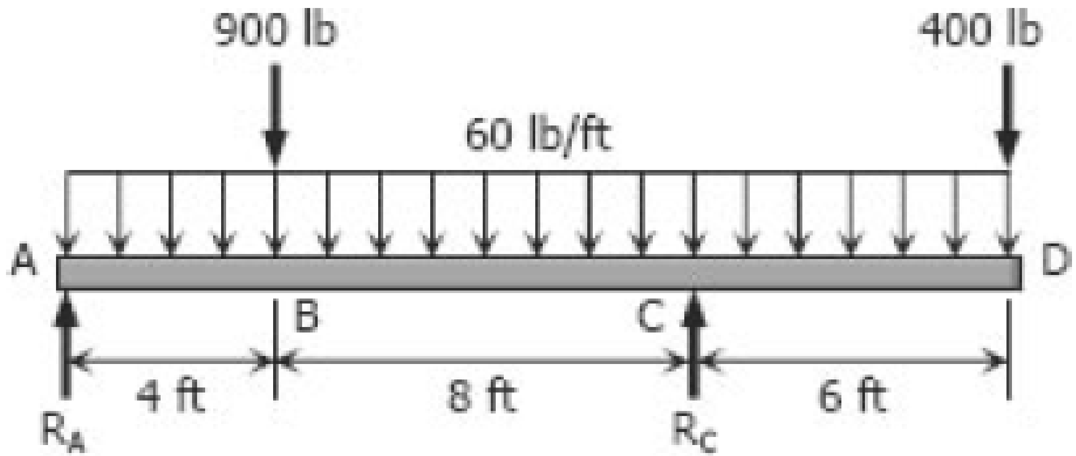
- Shear force at A $V_A = R_A = 25$ kN
- Shear force at C $V_C = 25$ kN
- Shear force at any section between C and D at a distance x from A
- $Vx = 25 - 10*(x - 1)$
- At D $x = 5$ and $V_D = 25 - 10*(5-1) = -15$ kN
- Shear force at B $V_B = -15$ kN
- Location of zero shear
- $0 = 25 - 10*(x-1)$
- $x = 35/10 = 3.5$ m from A



- Simply supported beam
- Bending moment at A and B is 0
- Bending moment at any section between C and D at a distance x from A
- $M_x = R_A*x - 10*(x-1)*(x-1)/2$
- $M_x = 25x - 5(x-1)^2$
- At C x = 1 $M_C = 25$ kN-m
- At D x = 5 $M_D = 25*5 - 5*(5-1)^2 = 45$ kN-m
- B.M. at zero shear $M_{max} = 25*3.5 - 5*(3.5-1)^2 = 56.25$ kN-m



10 kN/m
A
C
D
B
1 m
4 m
3 m
8 m
R_A
R_B
25 kN
15 kN



$\sum M_A = 0$

$12R_C = 4(900) + 18(400) + 9[(60)(18)]$

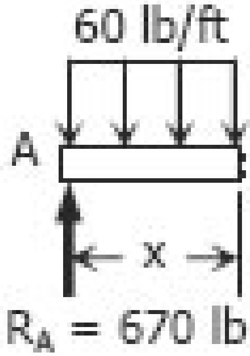
$R_C = 1710$ lb

$\sum M_C = 0$

$12R_A + 6(400) = 8(900) + 3[60(18)]$

$R_A = 670$ lb

Part AB



$$V_{AB} = 670 - 60x \text{ lb}$$

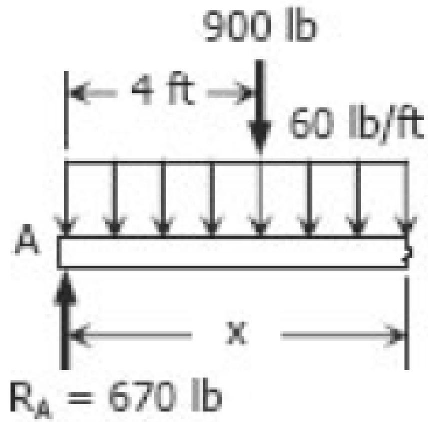
$$M_{AB} = 670x - 60x(x/2)$$

$$= 670x - 30x^2 \text{ lb-ft}$$

At x = 0 ft V_{AB} = 670 – 60*0 = 670 lb
At x = 4 ft V_{AB} = 670 – 60*4 = 430 lb

At x = 0 ft M_{AB} = 0
At x = 4 ft V_{AB} = 670*4 – 30*4^2 = 2200 lb-ft

Part BC



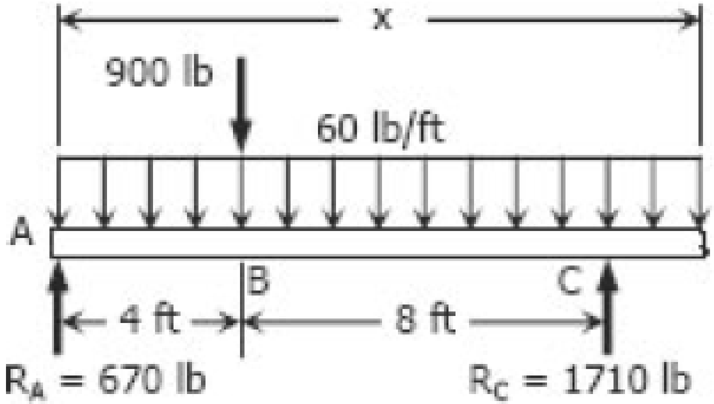
$$V_{BC} = 670 - 900 - 60x$$
$$= -230 - 60x \text{ lb}$$

$$M_{BC} = 670x - 900(x-4) - 60x(x/2)$$
$$= 3600 - 230x - 30x^2 \text{ lb-ft}$$

At x = 4 ft V_{BC} = -230 - 60*4 = -470 lb
At x = 12 ft V_{BC} = -230 – 60*12 = -950 lb

At x = 4 ft M_{BC} = 3600 - 230*4 – 30*4^2 = 2200 lb-ft
At x = 12 ft M_{BC} = 3600 - 230*12 – 30*12^2 = -3480 lb-ft
Location of zero bending moment
M_{BC} = 0 or x = 15.44 ft and 7.77 ft

Part CD

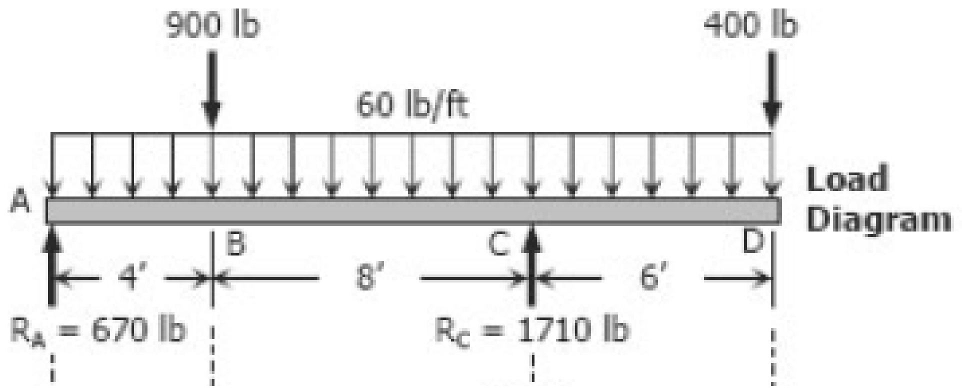


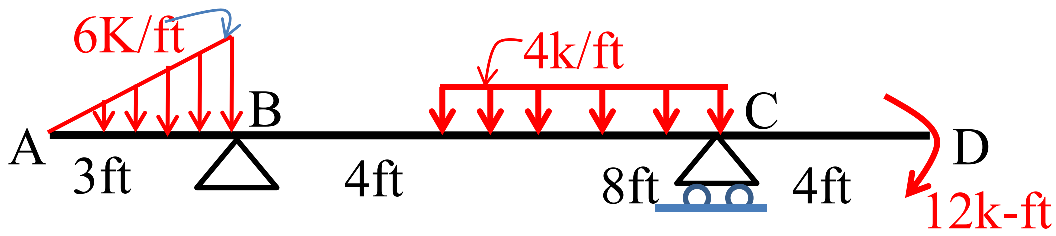
V_{CD} = 670 + 1710 – 900 – 60x
V_{CD} = 1480 -60x

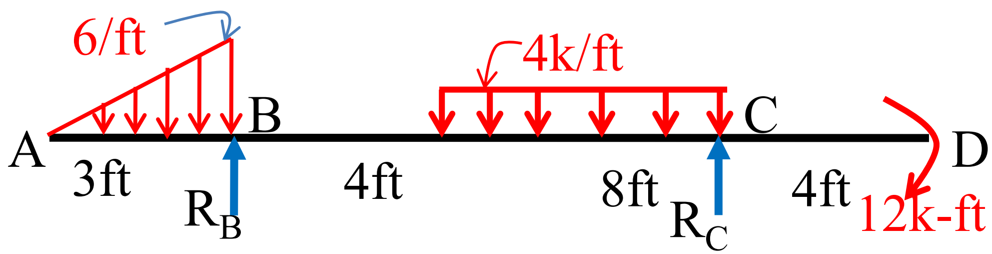
At x =12ft V_{CD} = 1480 -60 * 12 = 760 lb
At x = 18 ft V_{CD} = 1480 -60 * 18 = 400 lb

M_{CD} = 670x + 1710 (x-12) – 900(x-4) – 60x(x/2)
M_{CD} = -16920 + 1480x – $30x^2$

At x = 12ft M_{CD} = -16920 + 1480*12 – $30*12^2$ = -3480 lb-ft
At x = 18ft M_{CD} = -16920 + 1480*18 – $30*18^2$ = 0 lb-ft

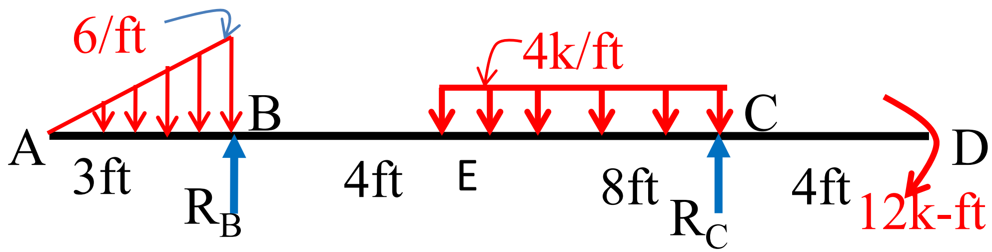

900 lb
400 lb
60 lb/ft
A
B
C
D
4′
8′
6′
Load Diagram
R_A = 670 lb
R_C = 1710 lb





$$\sum M_B = -\tfrac{1}{2}*3*6*(\tfrac{1}{3}*3)+4*8(4+4)+12 - R_C*12=0$$

R_B= 19.42k, R_C= 21.58K



$$4\frac{x^2}{2} - 21.58x + 12 = 0$$

X=0.59ft

R_B= 19.42k, R_C= 21.58K

Shear at D V_D = 0
Shear at C V_C = -R_C = -21.58K

Shear at E V_E = -21.58 + 4*8 = 10.42K

Shear at B V_B = -21.58 + 4*8 – 19.42 = -9 K

Shear at A = 0

Location of zero shear between C and E

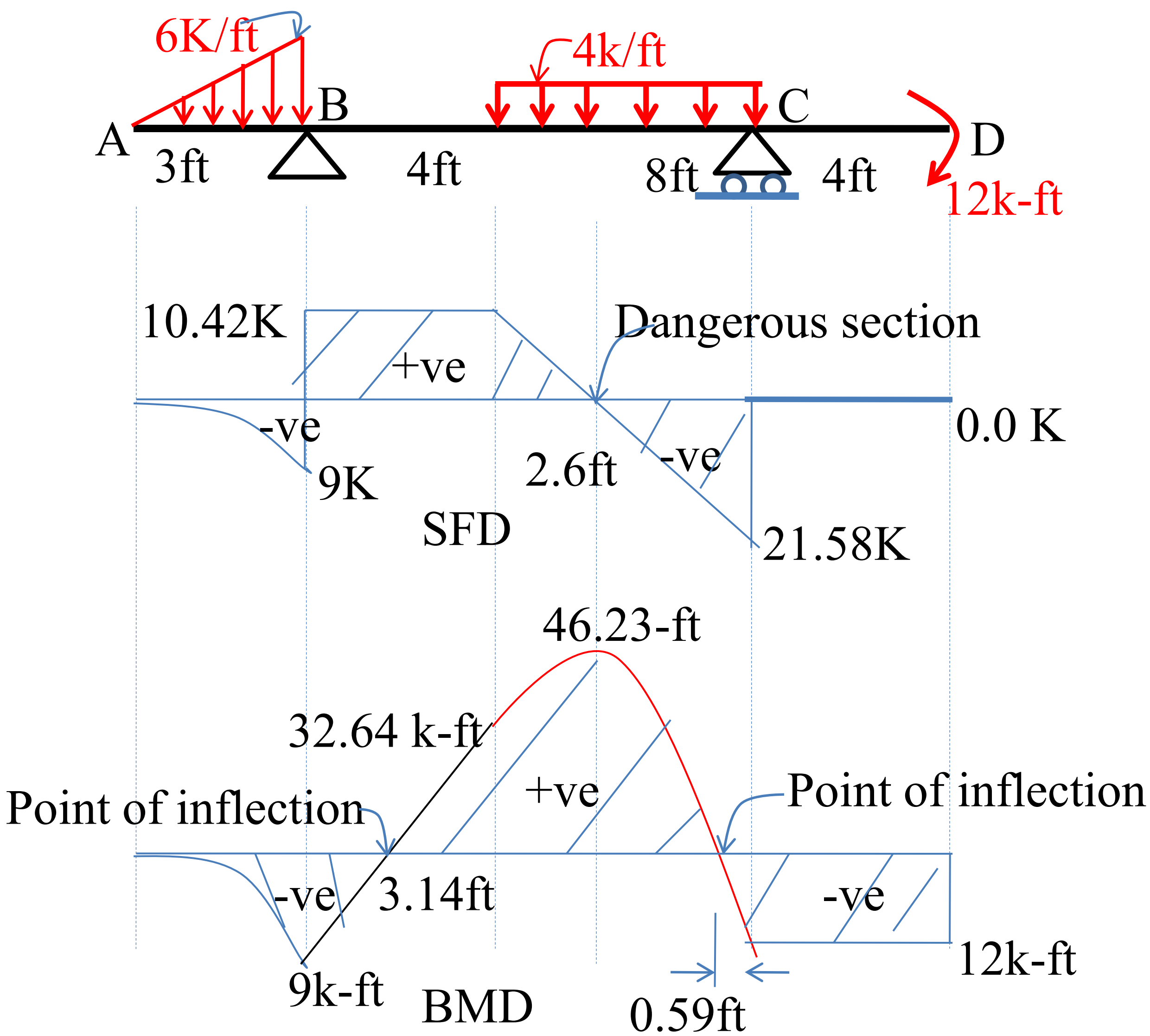
-21.58 + 4*x = 0; x = 5.4 ft from C

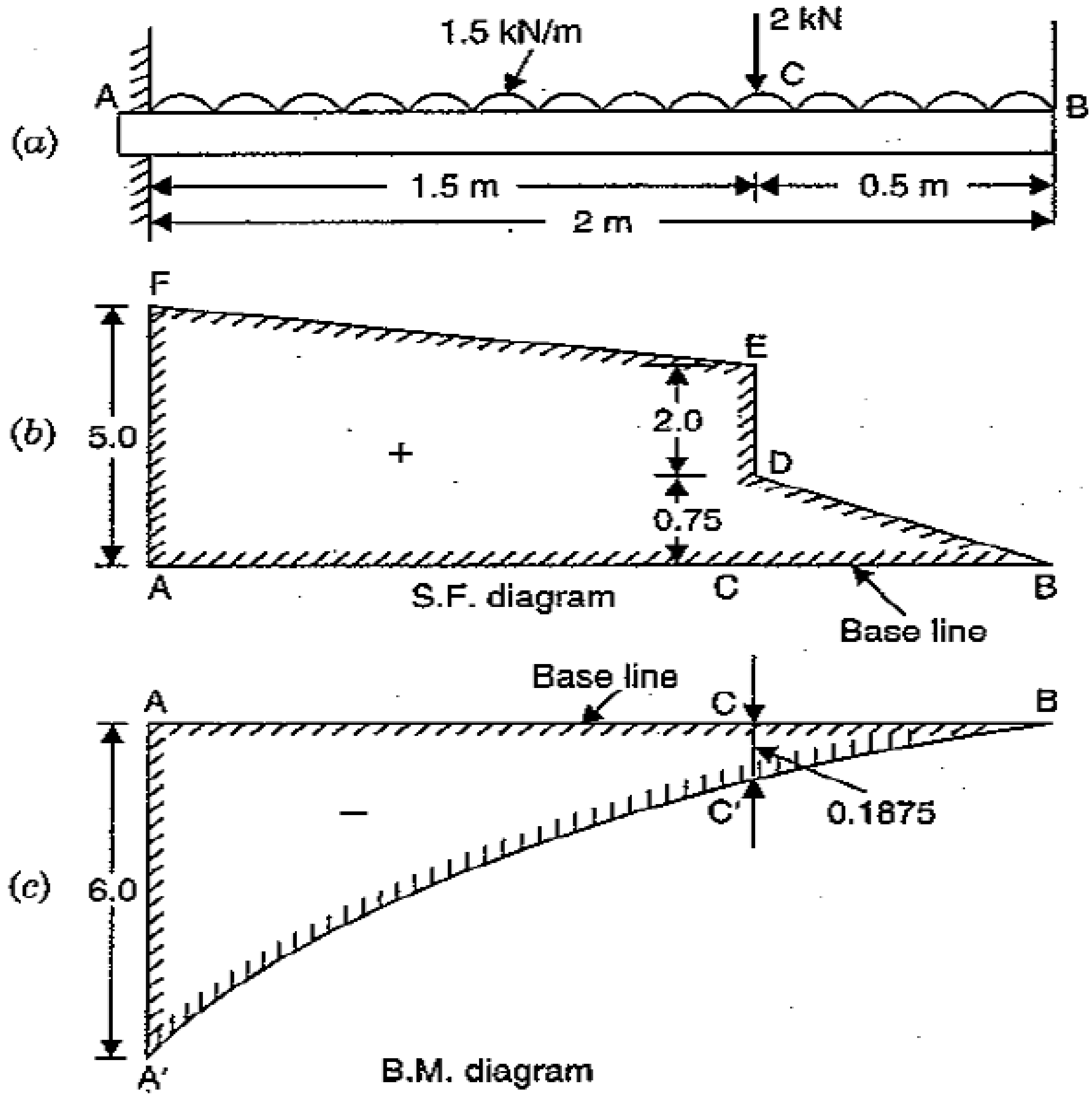
Bending moment at D M_D = -12 k-ft
Bending moment at C M_C = -12 k-ft

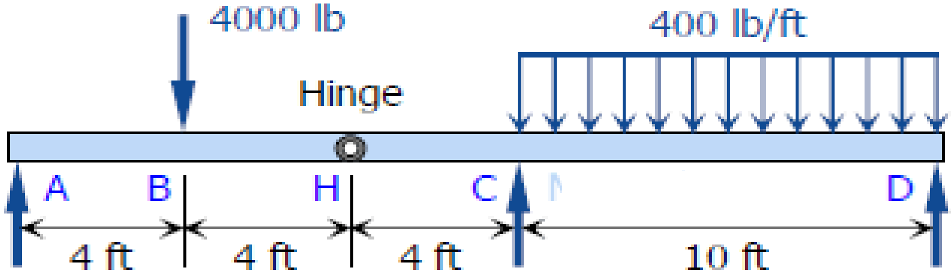
Bending moment at E M_E = -12 + (21.58*8) – 4*8*8/2 = 32.64 k-ft

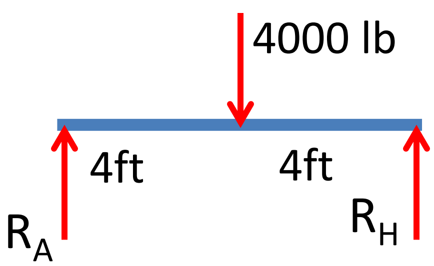
Bending moment at B M_B = -12 + (21.58*12) – 4*8* (4+8/2) = - 9 k-ft

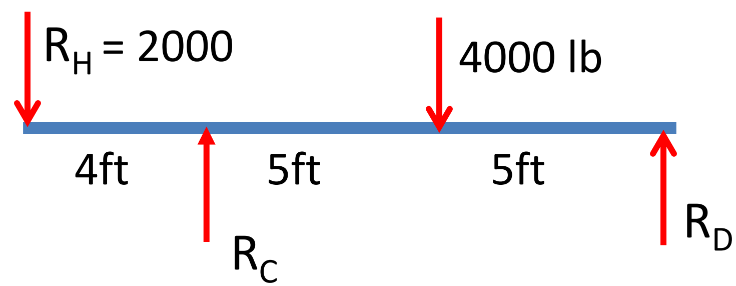
Maximum moment M_{max} = -12 + 21.58*5.4 – 4*5.4*5.4/2 = 46.2 k-ft


6K/ft
4k/ft
A
B
C
D
3ft
4ft
8ft
4ft
12k-ft
10.42K
+ve
Dangerous section
-ve
9K
2.6ft
-ve
0.0 K
SFD
21.58K
46.23-ft
32.64 k-ft
+ve
Point of inflection
Point of inflection
-ve
3.14ft
-ve
12k-ft
9k-ft
BMD
0.59ft


1.5 kN/m
2 kN
A
C
B
(a)
1.5 m
0.5 m
2 m
F
E
2.0
(b)
5.0
+
D
0.75
A
S.F. diagram
C
B
Base line
Base line
A
C
B
−
0.1875
C′
(c)
6.0
A′
B.M. diagram







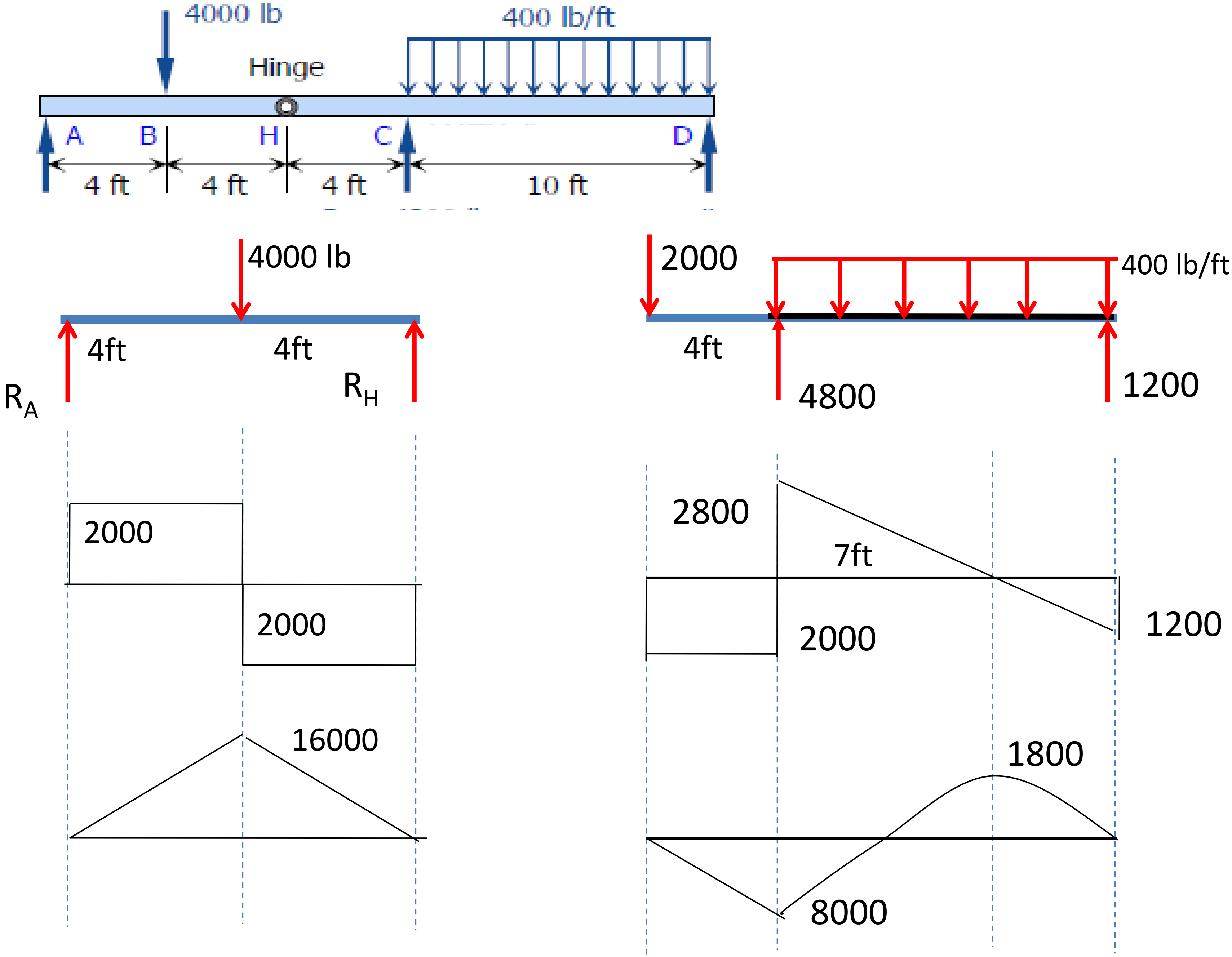
$\sum M_A = 0; 4000*4 - R_H*8 = 0$

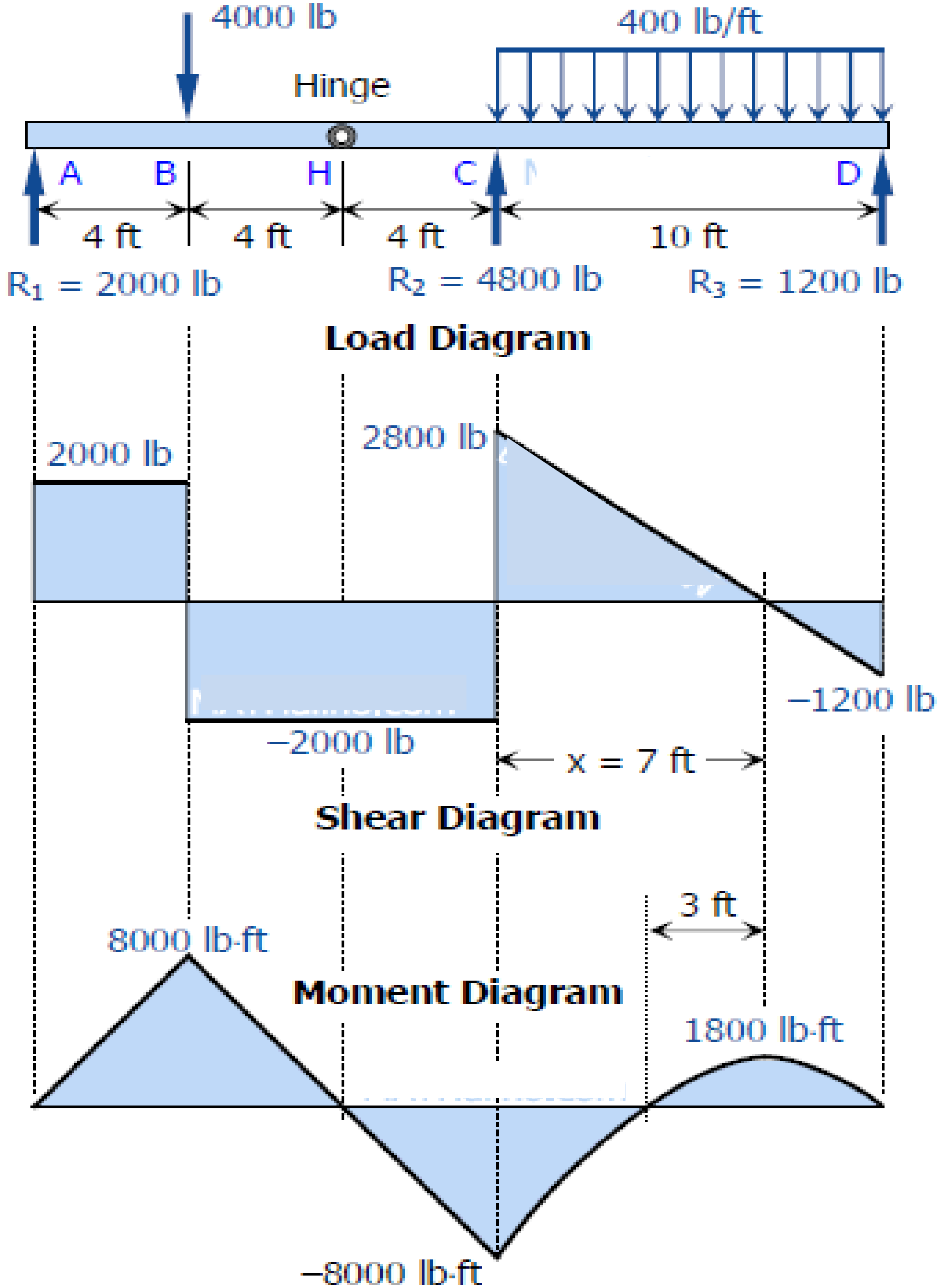
$R_H = 2000$ lb

$\sum M_D = 0; R_C*10 - 4000*5 - 2000*14 = 0$

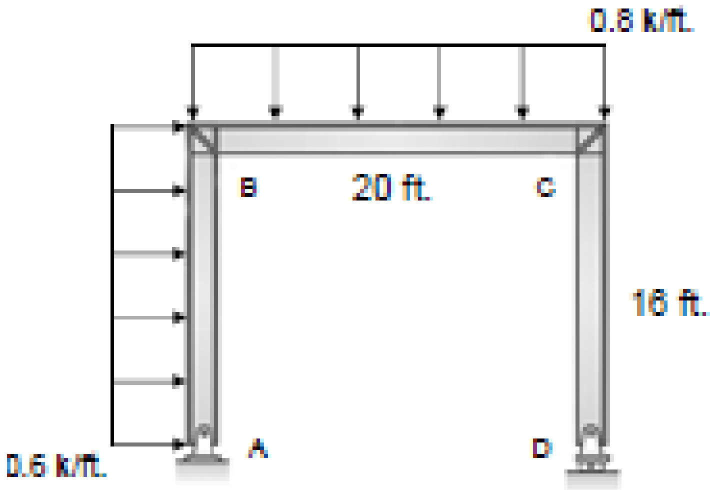
$R_C = 4800$ lb

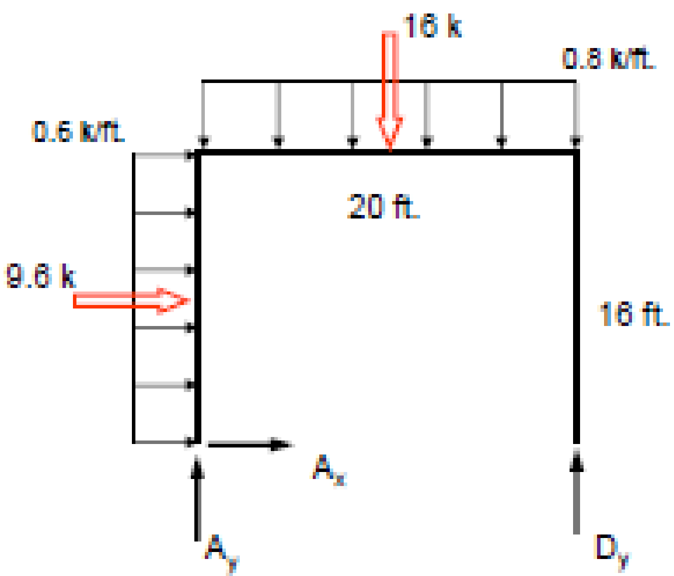
$R_D = 1200$ lb


4000 lb
400 lb/ft
Hinge
A
B
H
C
D
4 ft
4 ft
4 ft
10 ft
4000 lb
4ft
4ft
R_A
R_H
2000
400 lb/ft
4ft
4800
1200
2000
2000
16000
2800
7ft
2000
1200
1800
8000



4000 lb
400 lb/ft
Hinge
A
B
H
C
D
4 ft
4 ft
4 ft
10 ft
R₁ = 2000 lb
R₂ = 4800 lb
R₃ = 1200 lb
Load Diagram
2000 lb
2800 lb
−1200 lb
−2000 lb
x = 7 ft
Shear Diagram
3 ft
8000 lb-ft
Moment Diagram
1800 lb-ft
−8000 lb-ft



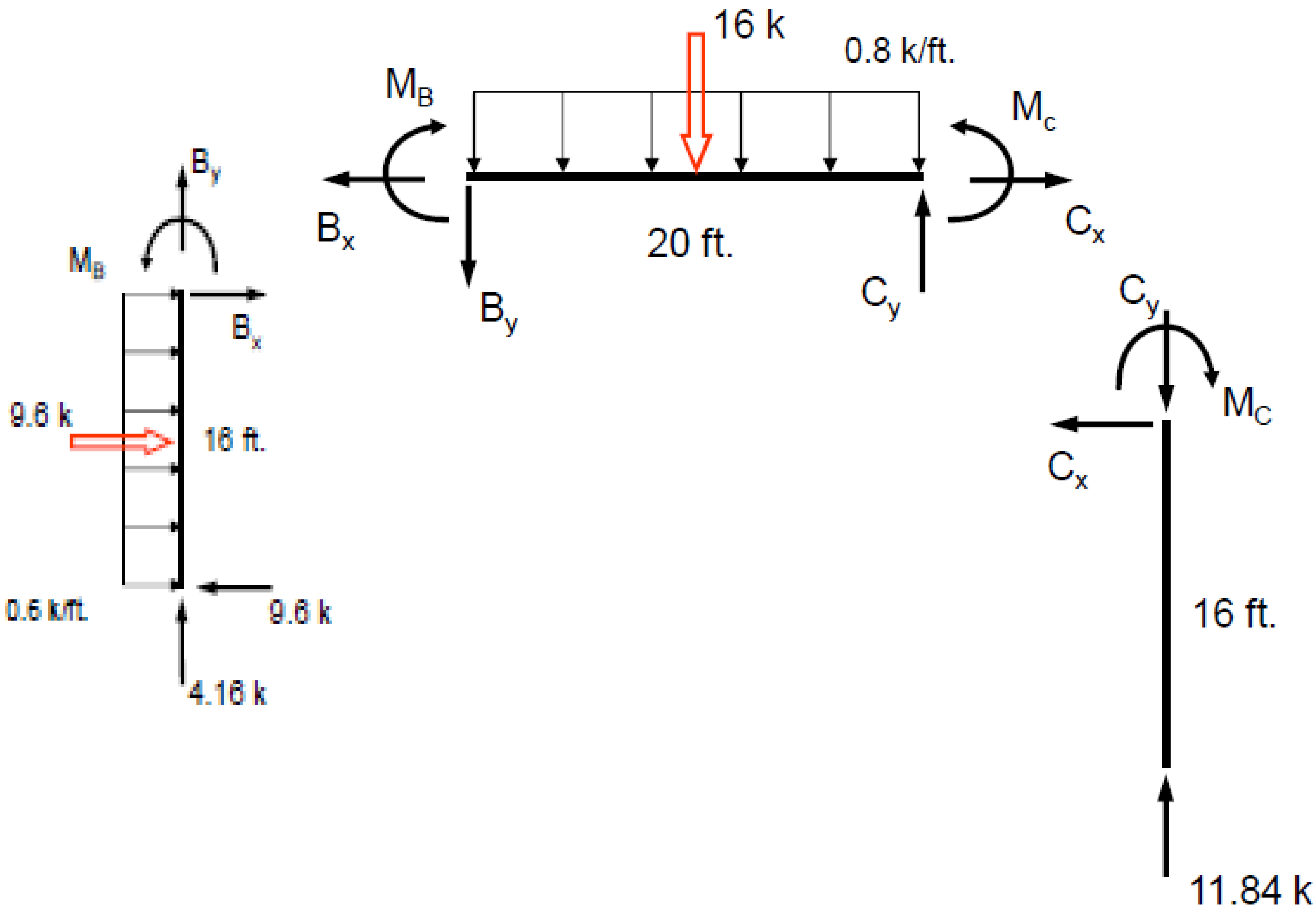


Free body diagram

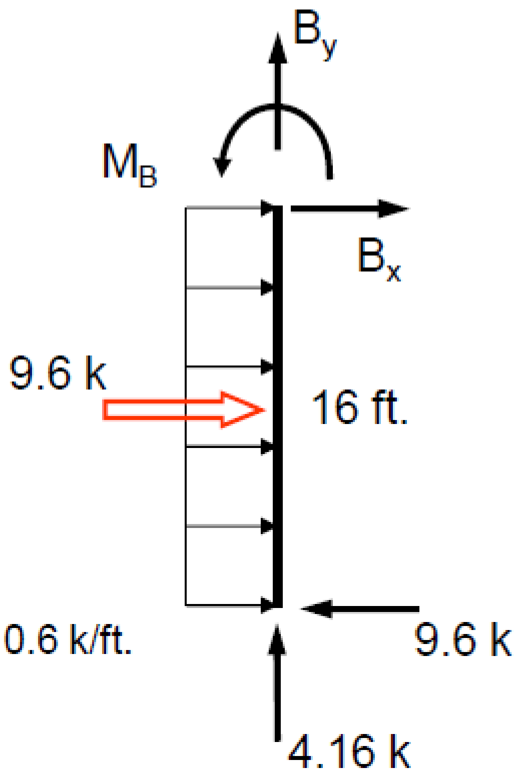
$\sum M_A = 0$; $D_y = 11.84$ k

$A_y = 4.16$ k

$A_x = -9.6$ k



16 k
0.8 k/ft.
M_B
M_c
B_y
B_x
20 ft.
C_x
B_y
C_y
M_B
B_x
9.6 k
16 ft.
0.5 k/ft.
9.6 k
4.16 k
C_y
M_C
C_x
16 ft.
11.84 k



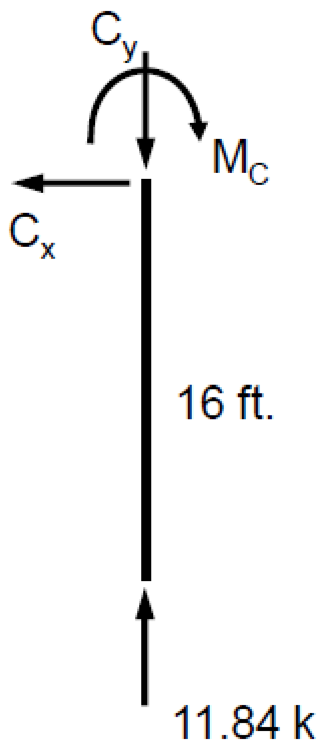
$$\sum M_B = 0 \; ; \; M_B = 76.8 \text{ k-ft}$$

$$B_y = -4.16 \text{ k}$$

$$B_x = 0\text{k}$$

$$V_y = 9.6 - 0.6\,x$$

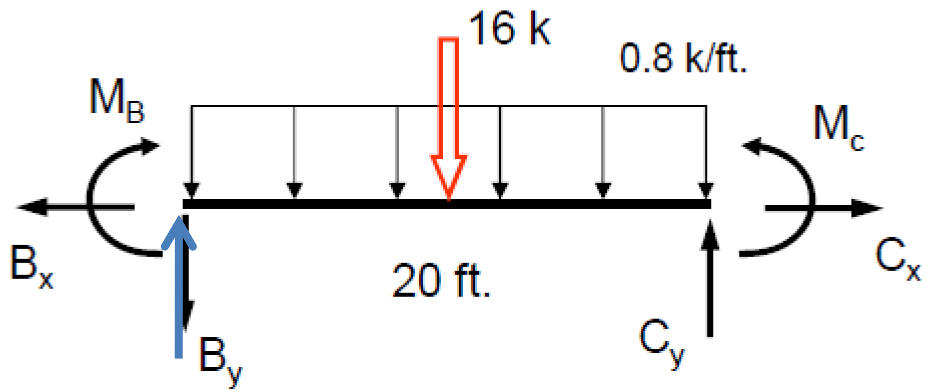
$$M_y = 9.6*x - (0.6\,x*x/2)$$



$$\sum M_C = 0 = -M_C$$

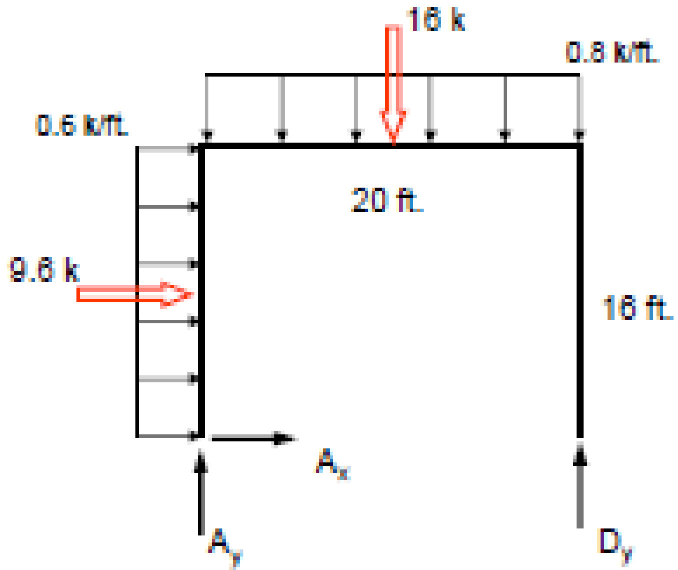
C_y = 11.84 k

C_x = 0k

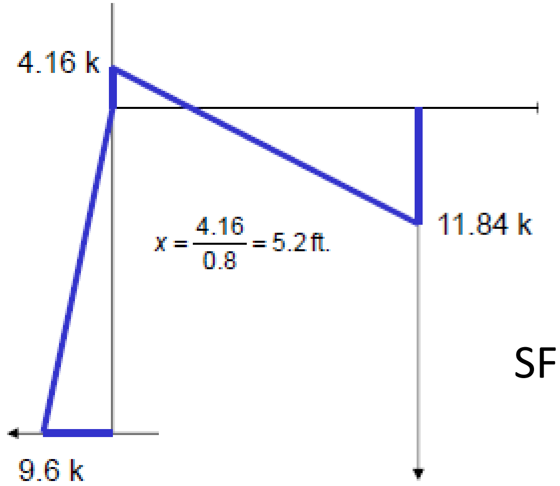


$$V_x = 4.16 - 0.8\,x$$

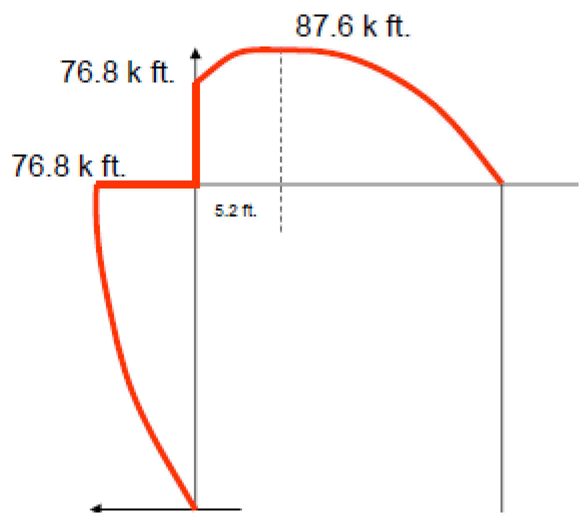
$$M_x = 4.16*x + 76.8 - (0.8\,x*x/2)$$



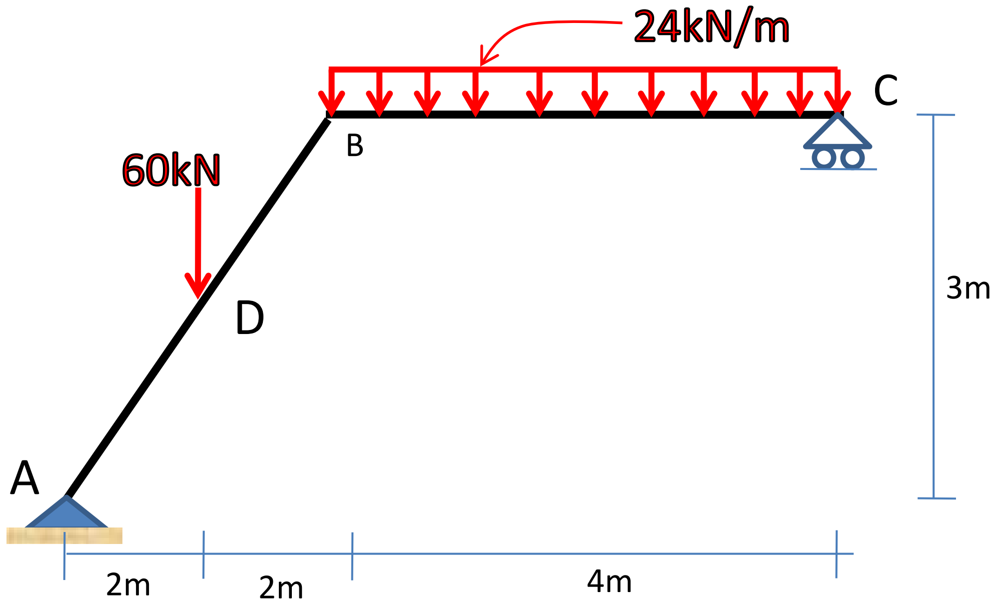
Free body diagram



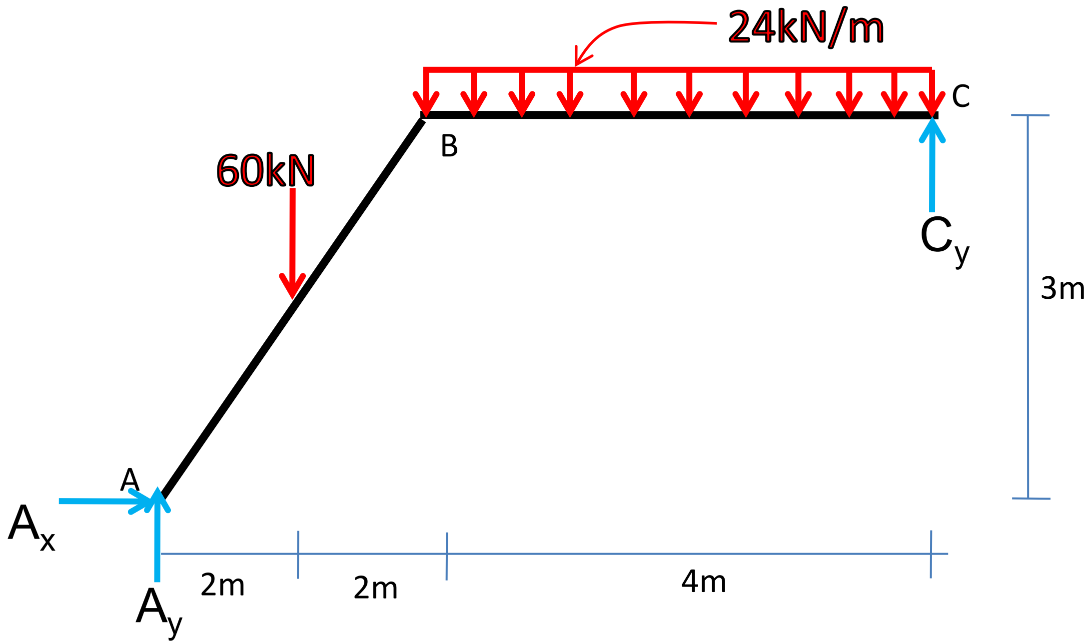
SFD



BMD

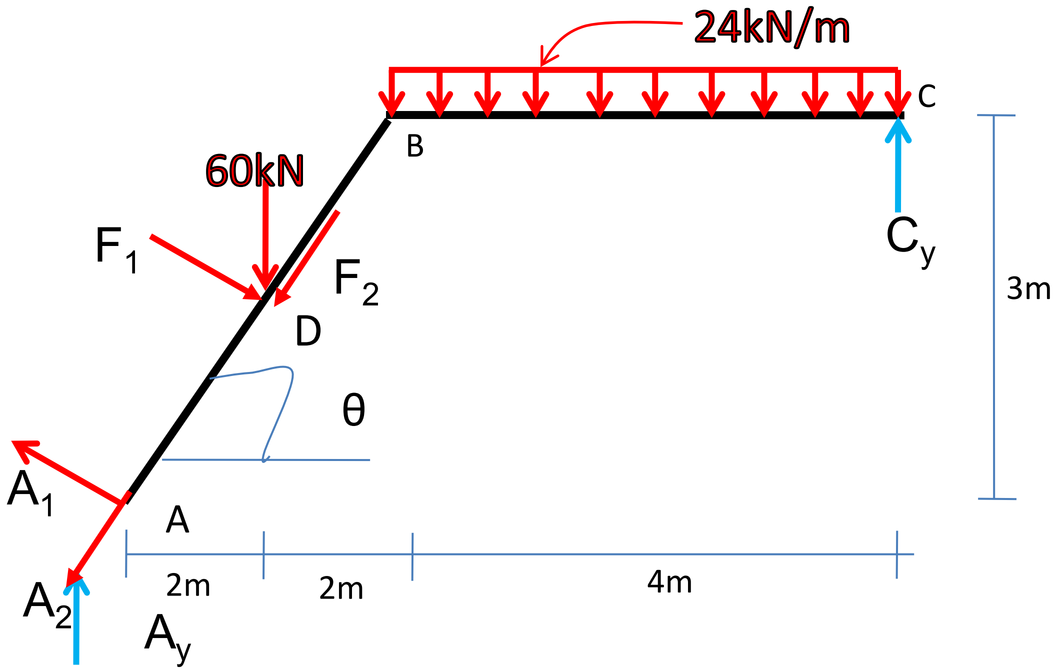


24kN/m
C
B
60kN
3m
D
A
2m
2m
4m



$$\sum M_A = 0; \quad 60*2 + 24*4*(4+4/2) - C_y*8 = 0 \qquad C_y = 87\ kN$$

$$\sum F_y = 0; \quad A_y = 69\ kN \qquad \sum F_x = 0; \quad A_x = 0\ kN$$



$\cos\theta = 4/5$

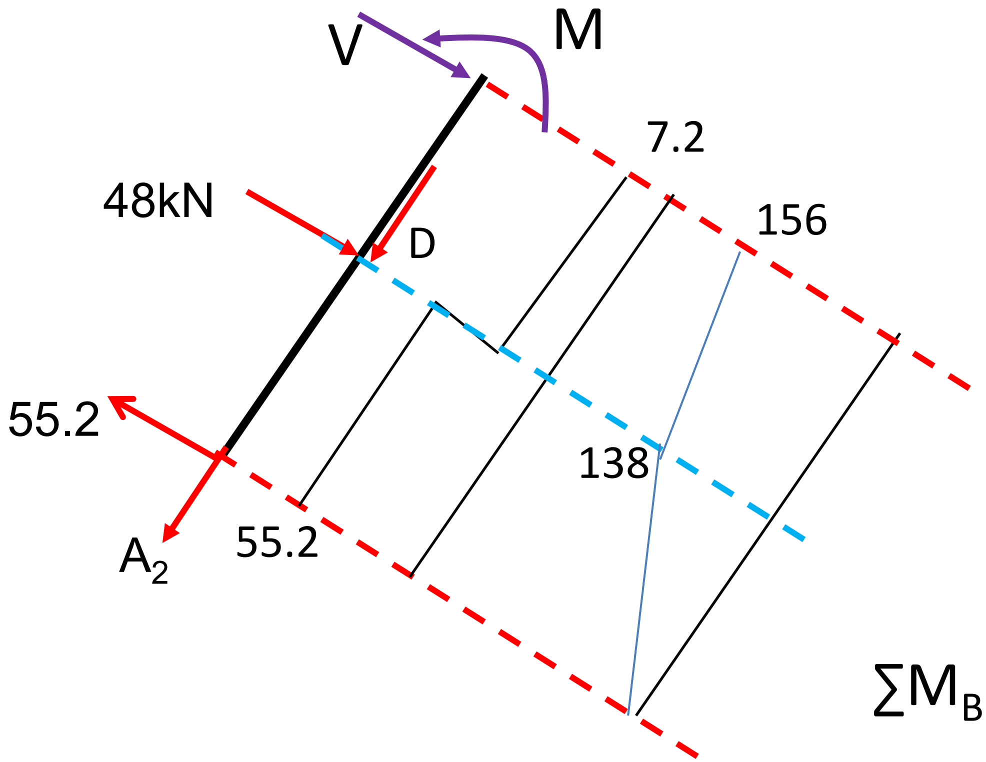
$\sin\theta = 3/5$

$$A_1 = A_y \cos\theta = 69 * 4/5 = 55.2 \text{ kN}$$

$$A_2 = -A_y \sin\theta = -69 * 3/5 = -41.4 \text{ kN}$$

$$F_1 = 60\cos\theta = 60 * 4/5 = 48 \text{ kN}$$

$$F_2 = -60 \sin\theta = -60 * 3/5 = -36 \text{ kN}$$



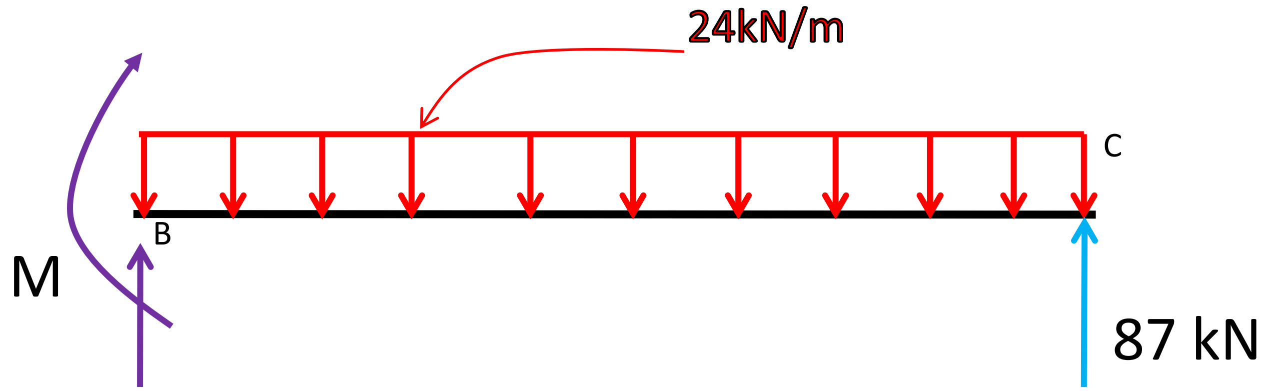
$\sum M_B = 0;\ 55.2*2.5 - 48*2.5 - M = 0$

$M = 156$ kN-m

$M_A = 0$

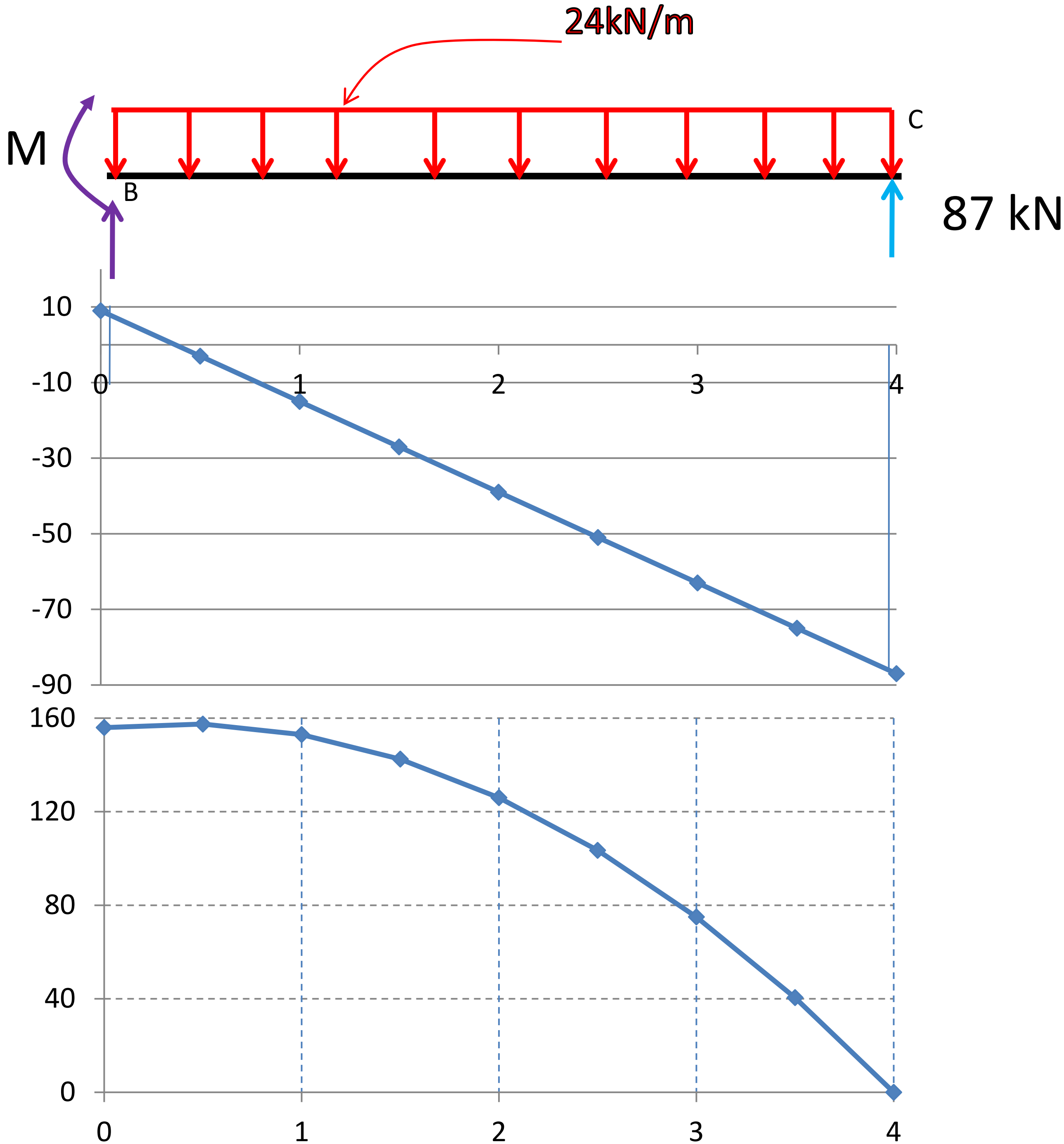
$M_D = 55.2*\ 2.5 = 138$ kN-m

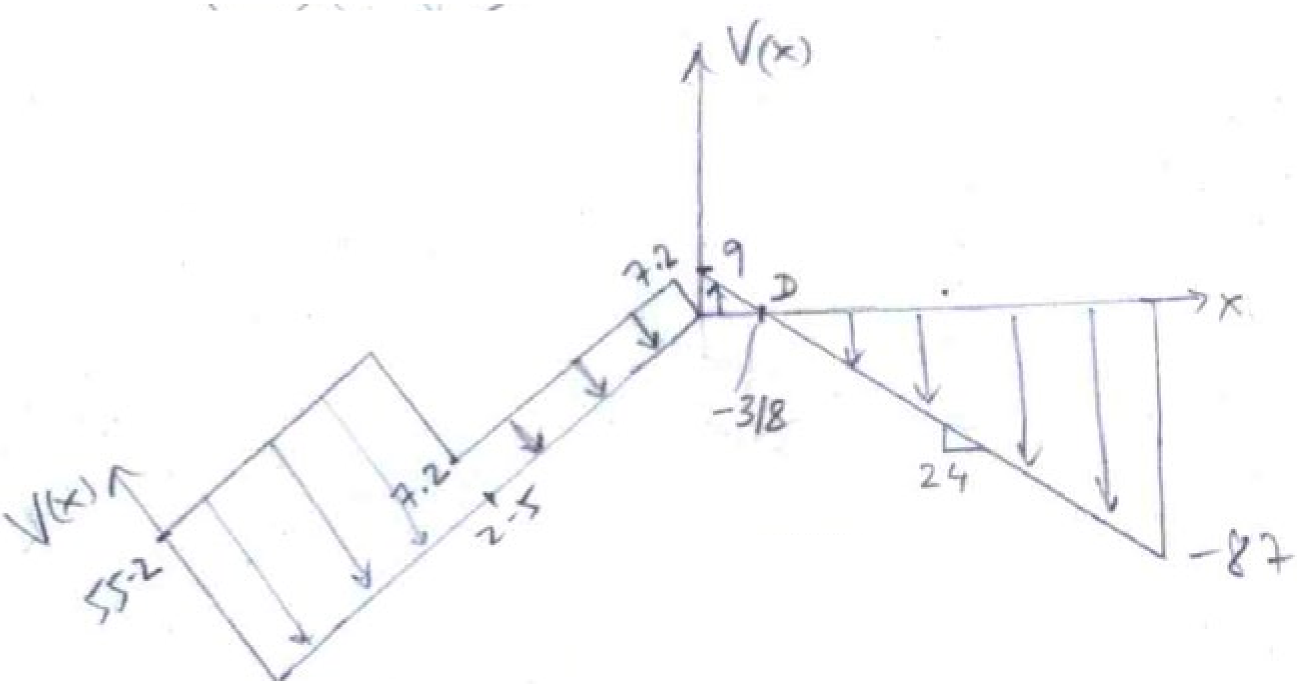
$M_B = 55.2*\ 5 - 48*2.5 = 156$ kN-m



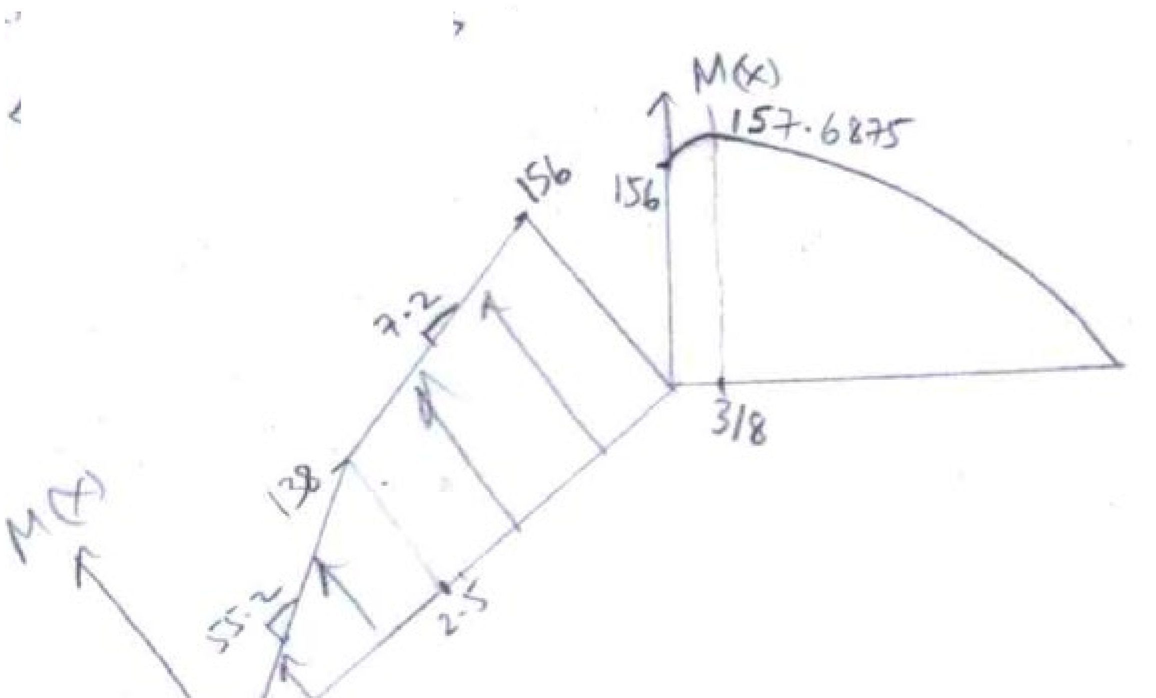
$F_B = 7.2/\cos\theta = 9$ kN $F_B = 87 - 24*4 = 9$ kN

$V_x = 9 - 24*x$ $M_x = 156 + 9*x - 24*x*x/2$

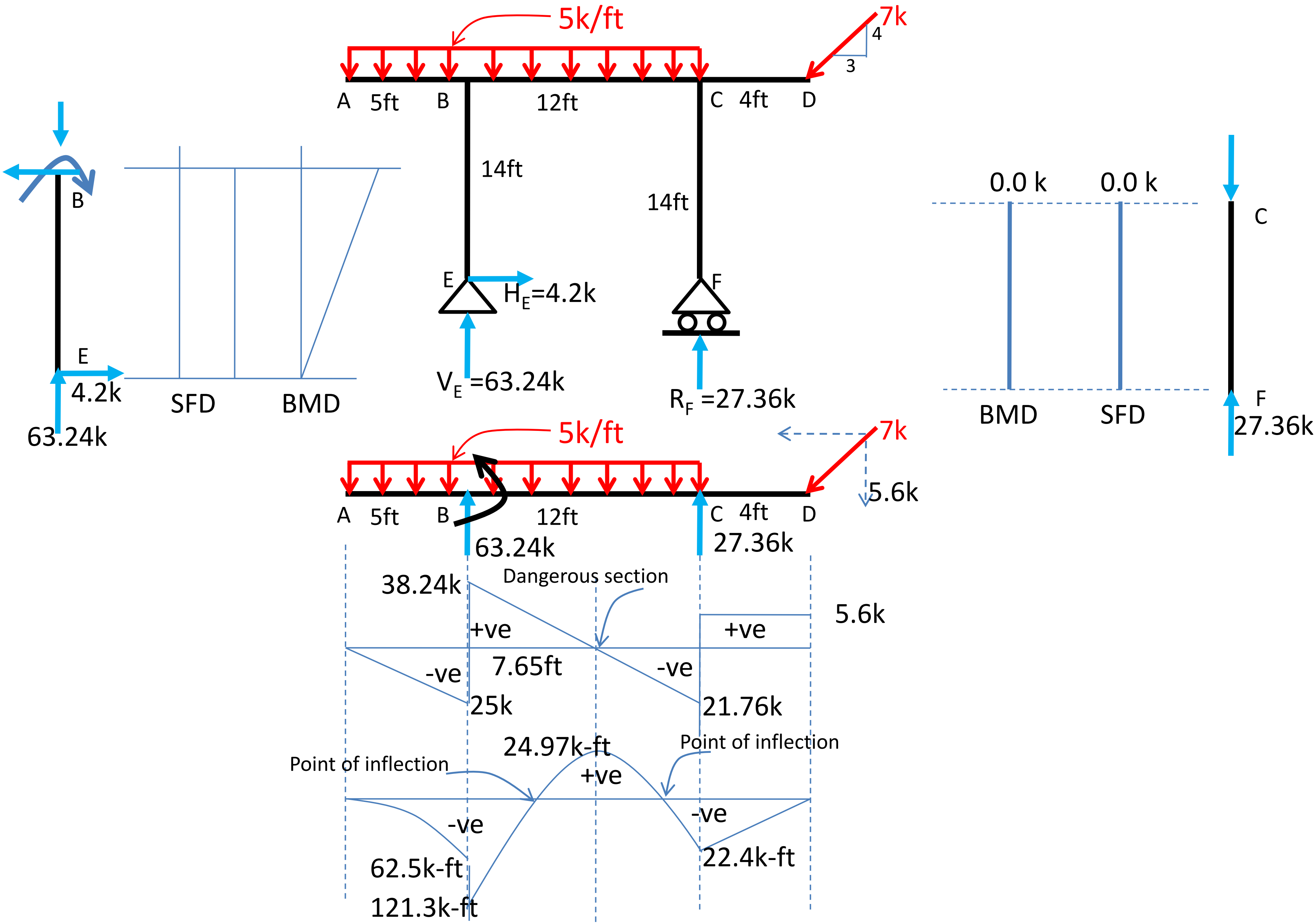

24kN/m
M
B
C
87 kN
10
-10
-30
-50
-70
-90
0
1
2
3
4
160
120
80
40
0
0
1
2
3
4


V(x)
7.2
9
D
x
-3/8
24
-87
7.2
2.5
55.2
V(x)

SFD


M(x)
157.6875
156
156
7.2
138
55.2
2.5
3/8
M(x)

BMD


5k/ft
7k
4
3
A
5ft
B
12ft
C
4ft
D
14ft
14ft
E
H_E=4.2k
F
V_E =63.24k
R_F =27.36k
B
E
4.2k
63.24k
SFD
BMD
0.0 k
0.0 k
C
F
BMD
SFD
27.36k
5k/ft
7k
5.6k
A
5ft
B
12ft
C
4ft
D
63.24k
27.36k
38.24k
Dangerous section
5.6k
+ve
+ve
7.65ft
-ve
-ve
25k
21.76k
24.97k-ft
Point of inflection
Point of inflection
+ve
-ve
-ve
62.5k-ft
22.4k-ft
121.3k-ft