


Lecture 10
On
Numerical Methods & Computer Programming

Topic: Central Difference Interpolation

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CENTRAL DIFFERENCE

3.3.3 Central Differences


The central difference operator δ is defined by the relations

$$y_1 - y_0 = \delta y_{1/2}, \quad y_2 - y_1 = \delta y_{3/2}, \dots, \quad y_n - y_{n-1} = \delta y_{n-1/2}.$$

Similarly, higher-order central differences can be defined. With the values of x and y as in the preceding two tables, a central difference table can be formed:

Table 3.3 Central Difference Table

x	y	δ	δ^2	δ^3	δ^4	δ^5	δ^6
x_0	y_0						
		$\delta y_{1/2}$					
x_1	y_1		$\delta^2 y_1$				
		$\delta y_{3/2}$		$\delta^3 y_{3/2}$			
x_2	y_2		$\delta^2 y_2$		$\delta^4 y_2$		
		$\delta y_{5/2}$		$\delta^3 y_{5/2}$		$\delta^5 y_{5/2}$	
x_3	y_3		$\delta^2 y_3$		$\delta^4 y_3$		$\delta^6 y_3$
		$\delta y_{7/2}$		$\delta^3 y_{7/2}$		$\delta^5 y_{7/2}$	
x_4	y_4		$\delta^2 y_4$		$\delta^4 y_4$		
		$\delta y_{9/2}$		$\delta^3 y_{9/2}$			
x_5	y_5		$\delta^2 y_5$				
		$\delta y_{11/2}$					
x_6	y_6						




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3.7 CENTRAL DIFFERENCE INTERPOLATION FORMULAE

In the preceding section, we derived and discussed Newton's forward and backward interpolation formulae, which are applicable for interpolation near the beginning and end respectively, of tabulated values. We shall, in the present section, discuss the central difference formulae which are most suited for interpolation near the middle of a tabulated set. The central difference operator δ was already introduced in Section 3.3.3.

The most important central difference formulae are those due to Stirling, Bessel and Everett. These will be discussed in Sections 3.7.2, 3.7.3 and 3.7.4, respectively. Gauss's formulae, introduced in Section 3.7.1 below, are of interest from a theoretical stand-point only.



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Gauss' Forward Formula

$$y_p = y_0 + G_1 \Delta y_0 + G_2 \Delta^2 y_{-1} + G_3 \Delta^3 y_{-1} + G_4 \Delta^4 y_{-2} + \dots,$$


$$G_1 = p,$$

$$G_2 = \frac{p(p-1)}{2!},$$

$$G_3 = \frac{(p+1)p(p-1)}{3!},$$

$$G_4 = \frac{(p+1)p(p-1)(p-2)}{4!}.$$

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
x_{-3}	y_{-3}						
		Δy_{-3}					
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$				
		Δy_{-2}		$\Delta^3 y_{-3}$			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$		$\Delta^6 y_{-3}$
		Δy_0		$\Delta^3 y_{-1}$		$\Delta^5 y_{-2}$	
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$		
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$				
		Δy_2					
x_3	y_3						



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Gauss' Forward Formula

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
x_{-3}	y_{-3}						
x_{-2}	y_{-2}	Δy_{-3}					
x_{-1}	y_{-1}	Δy_{-2}	$\Delta^2 y_{-3}$				
x_0	y_0	Δy_{-1}	$\Delta^2 y_{-2}$	$\Delta^3 y_{-3}$			
x_1	y_1	Δy_0	$\Delta^2 y_{-1}$	$\Delta^3 y_{-2}$	$\Delta^4 y_{-3}$		
x_2	y_2	Δy_1	$\Delta^2 y_0$	$\Delta^3 y_{-1}$	$\Delta^4 y_{-2}$	$\Delta^5 y_{-3}$	
x_3	y_3	Δy_2	$\Delta^2 y_1$	$\Delta^3 y_0$	$\Delta^4 y_{-1}$	$\Delta^5 y_{-2}$	$\Delta^6 y_{-3}$

Gauss' Forward Formula

Example 3.9 From the following table, find the value of $e^{1.17}$ using Gauss' forward formula:

x	e^x
1.00	2.7183
1.05	2.8577
1.10	3.0042
1.15	3.1582
1.20	3.3201
1.25	3.4903
1.30	3.6693

Gauss' Forward Formula

Example 3.9 From the following table, find the value of $e^{1.17}$ using Gauss' forward formula:

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1.05	2.8577
1.10	3.0042
1.15	3.1582
1.20	3.3201
1.25	3.4903
1.30	3.6693

We have

$$1.17 = 1.15 + p(0.05),$$

Gauss' Forward Formula

The difference table is given below.

x	e^x	Δ	Δ^2	Δ^3	Δ^4
1.00	2.7183				
1.05	2.8577	0.1394			
1.10	3.0042	0.1465	0.0071		
1.15	3.1582	0.1540	0.0075	0.0004	
1.20	3.3201	0.1619	0.0079	0.0004	0
1.25	3.4903	0.1702	0.0083	0.0004	0.0001
1.30	3.6693	0.1790	0.0088	0.0005	

Gauss' Forward Formula

$$y_p = y_0 + G_1 \Delta y_0 + G_2 \Delta^2 y_{-1} + G_3 \Delta^3 y_{-1} + G_4 \Delta^4 y_{-2} + \dots,$$

$$e^{1.17} = 3.1582 + \frac{2}{5}(0.1619) + \frac{(2/5)(2/5-1)}{2}(0.0079) + \frac{(2/5+1)(2/5)(2/5-1)}{6}(0.0004)$$

$$= 3.1582 + 0.0648 - 0.0009$$

$$= 3.2221.$$

$$G_1 = p,$$

$$G_2 = \frac{p(p-1)}{2!},$$

$$G_3 = \frac{(p+1)p(p-1)}{3!},$$

$$G_4 = \frac{(p+1)p(p-1)(p-2)}{4!},$$

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Gauss' Backward Formula

Gauss' backward formula can therefore be assumed to be of the form

$$y_p = y_0 + G_1' \Delta y_{-1} + G_2' \Delta^2 y_{-1} + G_3' \Delta^3 y_{-2} + G_4' \Delta^4 y_{-2} + \dots \quad (3.19)$$

where G_1', G_2', \dots have to be determined. Following the same procedure as in Gauss' forward formula, we obtain

$$G_1' = p,$$

$$G_2' = \frac{p(p+1)}{2!},$$

$$G_3' = \frac{(p+1)p(p-1)}{3!},$$

$$G_4' = \frac{(p+2)(p+1)p(p-1)}{4!},$$

$$\vdots$$

(3.20)

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
\vdots	\vdots						
x_{-1}	y_{-1}						
x_0	y_0	Δy_{-1}	$\Delta^2 y_{-1}$	$\Delta^3 y_{-2}$	$\Delta^4 y_{-2}$	$\Delta^5 y_{-3}$	$\Delta^6 y_{-3}$
x_1	y_1	Δy_0	$\Delta^2 y_{-1}$	$\Delta^3 y_{-1}$	$\Delta^4 y_{-2}$	$\Delta^5 y_{-2}$	
\vdots	\vdots						

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Stirling Formula

Taking the mean of Gauss' forward and backward formulae, we obtain

$$y_p = y_0 + p \frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{p^2}{2} \Delta^2 y_{-1} + \frac{p(p^2-1)}{3!} \frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \frac{p^2(p^2-1)}{4!} \Delta^4 y_{-2} + \dots \quad (3.21)$$

Formula (3.21) is called *Stirling's formula*.

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Thank
you



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