Lecture 11

On

Numerical Methods & Computer Programming

Topic: Stirling Formula

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CENTRAL DIFFERENCE

3.7 CENTRAL DIFFERENCE INTERPOLATION FORMULAE

In the preceding section, we derived and discussed Newton's forward and backward interpolation formulae, which are applicable for interpolation near the beginning and end respectively, of tabulated values. We shall, in the present section, discuss the central difference formulae which are most suited for interpolation near the middle of a tabulated set. The central difference operator δ was already introduced in Section 3.3.3.

The most important central difference formulae are those due to Stirling, Bessel and Everett. These will be discussed in Sections 3.7.2, 3.7.3 and 3.7.4, respectively. Gauss's formulae, introduced in Section 3.7.1 below, are of interest from a theoretical stand-point only.

Gauss' Forward Formula

$$y_p = y_0 + G_1\Delta y_0 + G_2\Delta^2 y_{-1} + G_3\Delta^3 y_{-1} + G_4\Delta^4 y_{-2} + \cdots,$$

$$\left.\begin{aligned} G_1 &= p, \\ G_2 &= \frac{p(p-1)}{2!}, \\ G_3 &= \frac{(p+1)\,p(p-1)}{3!}, \\ G_4 &= \frac{(p+1)\,p(p-1)(p-2)}{4!}. \end{aligned}\right\}$$

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
x_{-3}	y_{-3}						
		Δy_{-3}					
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$				
		Δy_{-2}		$\Delta^3 y_{-3}$			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$		$\Delta^6 y_{-3}$
		Δy_0		$\Delta^3 y_{-1}$		$\Delta^5 y_{-2}$	
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$		
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$				
		Δy_2					
x_3	y_3						

Gauss' Backward Formula

Gauss' backward formula can therefore be assumed to be of the form

$$y_p = y_0 + G_1'\Delta y_{-1} + G_2'\Delta^2 y_{-1} + G_3'\Delta^3 y_{-2} + G_4'\Delta^4 y_{-2} + \cdots \quad (3.19)$$

where G_1', G_2', ... have to be determined. Following the same procedure as in Gauss' forward formula, we obtain

$$\left.\begin{aligned} G_1' &= p, \\ G_2' &= \frac{p(p+1)}{2!}, \\ G_3' &= \frac{(p+1)\,p(p-1)}{3!}, \\ G_4' &= \frac{(p+2)(p+1)\,p(p-1)}{4!} \\ &\vdots \end{aligned}\right\} \quad (3.20)$$

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
⋮	⋮						
x_{-1}	y_{-1}						
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$	
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$		$\Delta^6 y_{-3}$
		Δy_0		$\Delta^3 y_{-1}$		$\Delta^5 y_{-2}$	
x_1	y_1						
⋮	⋮						

Stirling Formula

Taking the mean of Gauss' forward and backward formulae, we obtain

$$y_p = y_0 + p\frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{p^2}{2}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}$$

$$+ \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2} + \cdots \qquad (3.21)$$

Formula (3.21) is called *Stirling's formula.*

Stirling Formula

Example 3.10 The following table gives the values of e^x for certain equidistant values of x. Find the value of e^x when $x = 0.644$.

x	$y = e^x$
0.61	1.840431
0.62	1.858928
0.63	1.877610
0.64	1.896481
0.65	1.915541
0.66	1.934792
0.67	1.954237

Stirling Formula

The table of differences is

x	$y = e^x$	Δ	Δ^2	Δ^3	Δ^4
0.61	1.840431				
		0.018497			
0.62	1.858928		0.000185		
		0.018682		0.000004	
0.63	1.877610		0.000189		−0.000004
		0.018871		0	
0.64	1.896481		0.000189		0.000002
		0.019060		0.000002	
0.65	1.915541		0.000191		0.000001
		0.019251		0.000003	
0.66	1.934792		0.000194		
		0.019445			
0.67	1.954237				

Stirling Formula

$$p = \frac{0.644 - 0.64}{0.01} = 0.4.$$

The third difference contribution to both Stirling's and Bessel's formulae is negligible, and using Stirling's formula, we obtain

$$y(0.644) = 1.896481 + 0.4\frac{0.018871 + 0.019060}{2} + \frac{0.16}{2}(0.000189)$$

$$= 1.896481 + 0.0075862 + 0.00001512$$

$$= 1.904082,$$

Stirling Formula

Calculate the deflection of the structure at 6.25 ft distance from the fixed support as shown in figure below. The distance are in ft and the deflection are in inch.