

Lecture 12

On

Numerical Methods & Computer Programming

Topic: Lagrange's Interpolation

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Lagrange's Interpolation

$$f(x) = f(x_0) \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} + f(x_1) \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} + \dots + f(x_n) \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

This is called Lagrange's interpolation formula and can be used both equal and unequal intervals.

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Lagrange's Interpolation

Certain corresponding values of x and $\log_{10}x$ are given in the Table.

x	300	304	305	307
$Y = \log_{10}x$	2.4771	2.4829	2.4843	2.4871

Find $\log_{10} 301$

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x	300	304	305	307
$Y = \log_{10}x$	2.4771	2.4829	2.4843	2.4871

$$\begin{aligned} \log_{10} 301 &= \frac{(-3)(-4)(-6)}{(-4)(-5)(-7)}(2.4771) + \frac{(1)(-4)(-6)}{(4)(-1)(-3)}(2.4829) \\ &+ \frac{(1)(-3)(-6)}{(5)(1)(-2)}(2.4843) + \frac{(1)(-3)(-4)}{(7)(3)(2)}(2.4871) \\ &= 1.2739 + 4.9658 - 4.4717 + 0.7106 \\ &= 2.4786. \end{aligned}$$

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Lagrange's Interpolation

The function $y = \sin x$ is tabulated below

x	$y = \sin x$
0	0
$\pi/4$	0.70711
$\pi/2$	1.0

Find $\sin \pi/6$.

Lagrange's Interpolation

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Find $\sin \pi/6$.

$$\begin{aligned} \sin \frac{\pi}{6} &\approx \frac{(\pi/6 - 0)(\pi/6 - \pi/2)}{(\pi/4 - 0)(\pi/4 - \pi/2)}(0.70711) + \frac{(\pi/6 - 0)(\pi/6 - \pi/4)}{(\pi/2 - 0)(\pi/2 - \pi/4)}(1) \\ &= \frac{8}{9}(0.70711) - \frac{1}{9} \\ &= \frac{4.65688}{9} \\ &= 0.51743. \end{aligned}$$

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Thank
you

