Lecture 15

on

Numerical Methods

& Computer Programming

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Curve Fitting Least Squares

Fitting a straight line such as

$Y = a_o + a_1 X$

$$ma_0 + a_1 \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i$$

$$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} x_i y_i$$

Curve Fitting Least Squares

Polynomial of the nth degree Let the polynomial of the nth degree,

$$Y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

be fitted to the data points (x_i, y_i), $i = 1, 2, \ldots, m$. We then have

$$\left.\begin{aligned} ma_0 + a_1 \sum_{i=1}^{m} x_i + a_2 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^n &= \sum_{i=1}^{m} y_i \\ a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^{n+1} &= \sum_{i=1}^{m} x_i y_i \\ \vdots \\ a_0 \sum_{i=1}^{m} x_i^n + a_1 \sum_{i=1}^{m} x_i^{n+1} + \cdots + a_n \sum_{i=1}^{m} x_i^{2n} &= \sum_{i=1}^{m} x_i^n y_i. \end{aligned}\right\}$$

Curve Fitting Least Squares

Fitting a nonlinear curve such as

$y = a_o + a_1 x + a_2 x^2$

$$ma_0 + a_1 \sum_{i=1}^{m} x_i + a_2 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} y_i$$

$$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 + a_2 \sum_{i=1}^{m} x_i^3 = \sum_{i=1}^{m} x_i y_i$$

$$a_0 \sum_{i=1}^{m} x_i^2 + a_1 \sum_{i=1}^{m} x_i^3 + a_2 \sum_{i=1}^{m} x_i^4 = \sum_{i=1}^{m} x_i^2 y_i$$

Curve Fitting Least Squares

Fitting a curve such as $y = a_o + a_1x + a_2x^2$

$$ma_0 + a_1\sum_{i=1}^{m} x_i + a_2\sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} y_i$$

$$a_0\sum_{i=1}^{m} x_i + a_1\sum_{i=1}^{m} x_i^2 + a_2\sum_{i=1}^{m} x_i^3 = \sum_{i=1}^{m} x_i y_i$$

$$a_0\sum_{i=1}^{m} x_i^2 + a_1\sum_{i=1}^{m} x_i^3 + a_2\sum_{i=1}^{m} x_i^4 = \sum_{i=1}^{m} x_i^2 y_i$$

Fit a polynomial of second degree to the data points given the Table below:

x	y
0.0	1.0
1.0	6.0
2.0	17.0

Curve Fitting Least Squares

$$ma_0 + a_1\sum_{i=1}^{m} x_i + a_2\sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} y_i \qquad a_0\sum_{i=1}^{m} x_i + a_1\sum_{i=1}^{m} x_i^2 + a_2\sum_{i=1}^{m} x_i^3 = \sum_{i=1}^{m} x_i y_i$$

$$a_0\sum_{i=1}^{m} x_i^2 + a_1\sum_{i=1}^{m} x_i^3 + a_2\sum_{i=1}^{m} x_i^4 = \sum_{i=1}^{m} x_i^2 y_i$$

In Eq. (4.10), we require the quantities $\Sigma x_i, \Sigma x_i^2, \Sigma x_i^3, \Sigma x_i^4, \Sigma y_i, \Sigma x_i y_i$ and $\Sigma x_i^2 y_i$. These are computed as in the following table:

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	6	1	1	1	6	6
2	17	4	8	16	34	68
3	24	5	9	17	40	74

Curve Fitting Least Squares

Fitting a curve such as $y = a_o + a_1x + a_2x^2$

In Eq. (4.10), we require the quantities $\Sigma x_i, \Sigma x_i^2, \Sigma x_i^3, \Sigma x_i^4, \Sigma y_i, \Sigma x_i y_i$ and $\Sigma x_i^2 y_i$. These are computed as in the following table:

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	6	1	1	1	6	6
2	17	4	8	16	34	68
3	24	5	9	17	40	74

Using Eqs. (4.10), we now obtain the equations

$$3a_0 + 3a_1 + 5a_2 = 24$$
$$3a_0 + 5a_1 + 9a_2 = 40$$
$$5a_0 + 9a_1 + 17a_2 = 74.$$

the solution to which is $a_0 = 1$, $a_1 = 2$ and $a_2 = 3$.

Curve Fitting Least Squares (Exercise)

Fitting a curve such as $y = a_o + a_1x + a_2x^2$

$$ma_0 + a_1\sum_{i=1}^{m} x_i + a_2\sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} y_i$$

$$a_0\sum_{i=1}^{m} x_i + a_1\sum_{i=1}^{m} x_i^2 + a_2\sum_{i=1}^{m} x_i^3 = \sum_{i=1}^{m} x_i y_i$$

$$a_0\sum_{i=1}^{m} x_i^2 + a_1\sum_{i=1}^{m} x_i^3 + a_2\sum_{i=1}^{m} x_i^4 = \sum_{i=1}^{m} x_i^2 y_i$$

Fit a polynomial of second degree to the data points given the Table below:

x	y
0	6
1	17
2	33
3	58

Curve Fitting Least Squares (think it)

Example 4.4 Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data

x	y
2	4.077
4	11.084
6	30.128
8	81.897
10	222.62

Thank You

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