Lecture 16

on

Numerical Methods

& Computer Programming

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Curve Fitting Least Squares

Fitting a straight line such as

$Y = a_o + a_1 X$

$$m a_0 + a_1 \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i$$

$$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} x_i y_i$$

Curve Fitting Least Squares

Fitting a nonlinear curve such as

$y = a_o + a_1 x + a_2 x^2$

$$m a_0 + a_1 \sum_{i=1}^{m} x_i + a_2 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} y_i$$

$$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 + a_2 \sum_{i=1}^{m} x_i^3 = \sum_{i=1}^{m} x_i y_i$$

$$a_0 \sum_{i=1}^{m} x_i^2 + a_1 \sum_{i=1}^{m} x_i^3 + a_2 \sum_{i=1}^{m} x_i^4 = \sum_{i=1}^{m} x_i^2 y_i$$

Curve Fitting Least Squares

Example 4.4 Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data

x	y
2	4.077
4	11.084
6	30.128
8	81.897
10	222.62

The given relation is $y = ae^{bx}$. Taking logarithms of both sides, we obtain

$$\ln y = \ln a + bx.$$

Setting $\ln y = Y, x = X, \ln a = a_0$ and $b = a_1$, the above relation takes the form $y = a_0 + a_1 x$, which is a straight line.

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$$ma_0 + a_1 \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i \qquad a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} x_i y_i$$

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$$ma_0 + a_1 \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i \qquad a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} x_i y_i$$

$X = x$	$Y = \ln y$	X^2	XY
2	1.405	4	2.810
4	2.405	16	9.620
6	3.405	36	20.430
8	4.405	64	35.240
10	5.405	100	54.050
30	17.025	220	122.150

Formulae (4.6) give

$$5a_0 + 30a_1 = 17.025, \qquad 30a_0 + 220a_1 = 122.150,$$

which yield the solution:

$$a_0 = 0.405 \quad \text{and} \quad a_1 = 0.5.$$

Hence

$$a = e^{a0} = e^{0.405} = 1.499 \quad \text{and} \quad b = a_1 = 0.5.$$

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Thank You

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